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USING STATISTICS FOR PROCESS STUDY AND IMPROVEMENT

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Chapter 1

Continual Improvement and the Need for Management Education in the Study of Variation

1.1 Introduction

The objective of this manual is to provide managers and engineers of industrial organizations with a textbook for the statistical study of the systems and processes of an organization. Crucial competencies and responsibilities of managers and engineers in the future will be the ability to characterize the organization's systems with performance indicators which serve as guidance for improvement and to have the ability to direct the organization in improvement of these measures by using methodologies for assessing the sources impacting the performance measures. Knowledge of variation is critical to the effective execution of these new responsibilities. A conscious recognition of variation, an informed interpretation of the messages it contains regarding management behavior and the organizational system, and an understanding of the effects variation has on system performance will help define the different behaviors and decisions which will characterize the role of the manager engaged in systems management. It is the intent of this chapter to enlarge on this new role and to provide a motivation for an understanding of the statistical methodologies contained in this manual.

1.2 Current practices and interpretation of variation

Monitoring business unit performance by using several monthly indicators is a well known and accepted practice. An organization with multiple plant sites typically would require reports for various indicators by each plant. Depending upon the industry and tradition, numerical indicators might include monthly throughput values per direct labor hour, cost of purchased materials per unit of production, dollar amount of all goods in inventory, waste, budget variances, and number of lost time accidents. These numbers are meant to serve several purposes for corporate or division people and the site manager. Comparison against previous periods or over longer periods help reveal positive or negative trends in overall business activity, performance, and efficiency. Comparisons from one site to another are often used to gain an appreciation as to superior performance or suggestions for changes at other sites. The numbers are also used as signals as to where management should put emphasis in order to effect changes or improvements in the business. Based on what these numerical indicators seem to portend, it is a customary and often unquestioned practice to use these monthly indicators as a means of monitoring the business as well as a basis on which to act.

Any manager who has been required to report such numbers or who chooses to use such numbers to understand the current position of the business knows about the variation exhibited by these numerical measures. The manager also understands that the recorded numbers are the results of numerous activities and decisions; many of which he or she has little or no ability to influence. The manager also understands that, typically, there are numerous strategies or tactics for effecting changes in the numbers. That is, something is understood about the sources of variations in these numbers. At issue is the depth of understanding regarding the reasons why the numbers vary and the range of viable choices the manager has for affecting the outputs measured by these numbers. Since the range of potential choices for affecting these outputs will broaden as this understanding increases, knowledge of the nature of the variation in these numbers, the causes or sources of variation in the numbers, and the effect that different management behaviors would have on these numbers might be useful in realizing how improvement in the numbers might be effected.

In evaluating performance results through the use of the monthly indicators, two contexts which are too often used for making judgments are: (1) the relative size of the result when compared to some value established as a forecast, goal, standard or some other type of expectation or prediction or (2) a result of the same kind observed in a previous time period. This perspective limits the range of potential questions regarding the deviations in the numbers and therefore limits the learning and understanding that might be had from the numbers. In both contexts, the evaluation of a result does not include recognition that the particular result being evaluated is usually one in a series of similar results produced by the organizational system week to week, month to month or year to year. A purpose of the analysis must be to learn something about why it is that the numbers behave the way they do. It is in this understanding that guidance can be found for selecting and taking appropriate actions to change those things which in turn change the numbers. The appropriate actions and the selection of what to act upon depends upon management intention and is not likely to be found in the numbers themselves. By treating the current result as a member of a series of results and by making judgments in light of the variation exhibited by that series, useful information for evaluating current and past practice and for assessing degree of belief in forecasts of future results can be gained.

A powerful perspective of the variability in a series of results is expressed in terms of a model which describes common and special causes of variation. Common causes are those system sources which affect each and every organizational result and are exercised or experienced on an ongoing basis. Figure 1.1 is a plot of how measurements subject only to common cause sources of variation would behave. Although the most recent value in the series apparently shows a deterioration, that value is within the boundaries of variation seen in the historical series. It does not represent an exceptional case and is the result of the interaction of multiple causes. This insight into the behavior of the series is useful, because efforts to react to this single outcome without understanding the cause system which generated the result may not have the intended effect

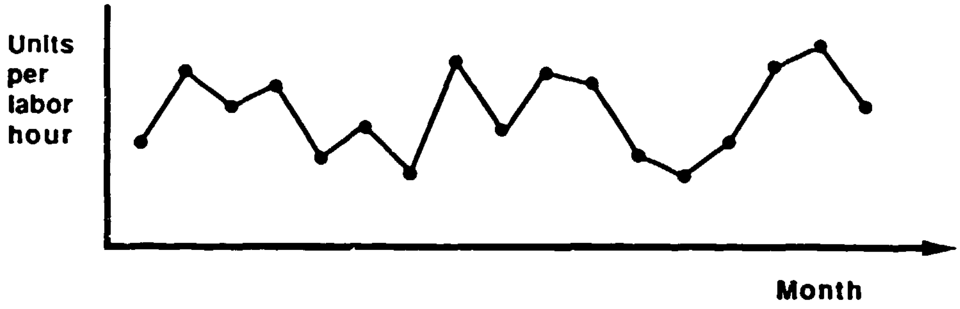


Figure 1.1

Special causes of variation are those which are thought to affect only some results. Measurements subject to special cause sources of variation in addition to common cause sources might behave as do those plotted in Figure 1.2. Given both the level and degree of variation in the series of values in this figure, the most recent value plotted in Figure 1.2 is exceptional. A special cause for the deterioration in these values can be determined to exist. Special causes occur intermittently, arising from behaviors, methods, and equipment variations that are beyond the usual experience.

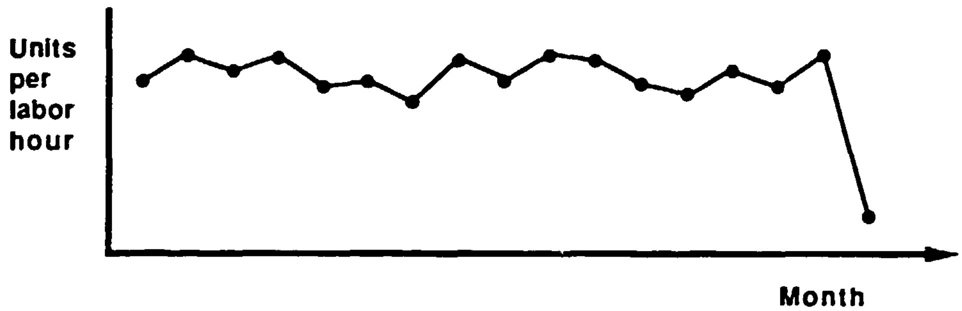


Figure 1.2

In definition, the concepts appear simple and straightforward. In practice they require study, elaboration, and insight. Understanding as to possible sources becomes essential because the model is meant to serve as to where and on what to work in order to effect changes according to managerial selected criteria. The concept and interpretation of common and special cause are meant to be used as guidelines to help identify and eliminate sources of erratic variability, as well as to provide support for focusing upon and working to improve the underlying system. There is no intention to suggest that a system subject to only common causes as the ideal or perfect system; it is the manager's job to evaluate that system relative to what is required and make judgments as to a changed system relative to defined criteria.

Typically, numerical indicators of performance are reported each month along with a standard which indicates what is required of the system. Figure 1.3 and 1.4 are plots of the same time series depicted in Figure 1.1. The line drawn on the plot indicates the standard which defines what is acceptable performance for the plant. The standard gives rise to a different view of variation than what is indicated by Figure 1.1. Without the awareness of the concept of a stable system of variation about the average, each deviation from the standard is taken to be a separate measure of the performance of the system. The concept that the system might be performing in a similar fashion over all outcomes recorded is missing.

Figure 1.3

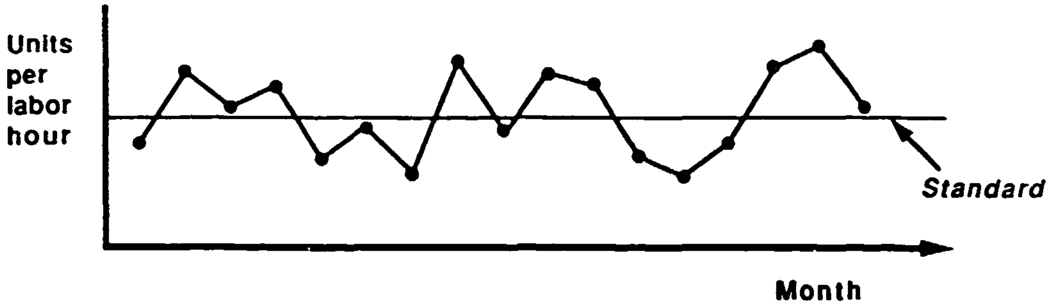


Figure 1.4

The idea of using a standard to judge current performance is prevalent in many industries. Typically, a manager required to report such performance measures will also be asked to explain deviations from the standard. Of course, explanations will be sought and found, but the usefulness of this activity should be closely questioned. Standards, of course, are set for a number of different reasons. In some instances, they are meant to describe the results of best practices. In other instances, they are meant to serve as a goal or an aspiration. In evaluating current results against standards, the purpose of the standard and its definition must be operative and be used in the evaluation. Further, the organization's capability must be considered. If the establishment of the standard has not been based on the known capabilities of the system, but possibly on the hope that setting a high standard will give the business something for which to aim, then explaining each deviation misdirects attention from examining the behavior of the system to explaining differences which may not in fact exist.

There are a number of limitations in using deviations from standard as a management signal which must be recognized and addressed. For example, if a deviation from a standard is thought of as a one time occurrence, then common causes get addressed and treated as special causes with no mention or view of special causes. Another view is recommended; this view considers the deviation as a realization from a system, a way of doing things, that may have yielded any one of several values for the deviation in that time period. This view offers the advantage of allowing history to help judge the effect of past decisions or practices and also to serve as a guide for modified or changed practices.

If each result or outcome is considered a one time event, then it becomes difficult to appreciate the concept that work is a process. Yet, work as a process, a part of a system, is a necessary point of view, since this view provides a means for interpreting the interrelationships among elements and behaviors by which the organization functions. Without a process point of view, deviations will remain one time events, carrying little insight into the behavior of the underlying processes.

A further limitation of viewing a deviation from standard as a one time event is that there is no awareness that said deviation is only one observation from an array of possible values from that system. Consequently, there is no specific awareness of the largest or smallest values which might have been observed for the deviation and therefore there is no means by which to judge the capability of the system. In addition, the manager is in no position to judge the effect as being due to special cause or as being due to common causes. Without this information, there is no strong incentive to analyze and improve the system which actually produced the deviation; indeed that possibility may not be appreciated by the manager. Without the concept of predictability and with the distribution concept missing, issues regarding the system variability and average are not considered. Without these viewpoints, the structure for management decision is weakened.

Being able to measure variation is necessary for the manager with system responsibility. By measuring variability at moments in time, the manager can know the magnitude or size of the variation and can track that measure over time, over a variety of changing conditions, and know something about the stability of the variation. Tracking system variation is not only for the purpose of controlling and moderating the system, although that may be a valuable correlate. The purpose is to learn about the causes and effects of critical system activities.

1.3. Evaluating the Effects of Variation

Variability results from the interaction of procedures, people, material, and equipment used in creating a product or service; variation of a given magnitude is a consequence of a way of doing business. In the same way that, say, symmetry is imparted to the vase by the artisan's vision, skill, experience, and intention, variability in the part or service is created by the organizational system. In general, variation in results is not deliberately created by an organization. In fact, the sources of the variation are not always known or appreciated. For example, variability in the cost or performance of a complex mechanical or electrical part may be partly due to manufacturing and assembly capability and largely due to materials selection and the actual design itself, the selection and implementation of the manufacturing process, or the specification and procurement of part and materials. Knowledge of the effects of variation, in terms of cost, capability, or performance, would be essential in deciding if improvements were necessary or potentially beneficial. Perhaps because variability is always present, the effects of variation are not always well known or understood. With this difficulty in mind several examples are presented to provide some experience in thinking about the possible effects or variation.

Fill weight for a granular product shipped in containers with a given label weight provides an example for considering the impact of variation. There is a target value for container content. The target has been set based upon a lower specification and some understanding that process variability exists. If the filling process were managed to produce stable variation in fill weights then managers could confidently determine the target, taking into account the existing, known magnitude of variation in fill weight. When this knowledge is lacking, target selection must be influenced by the organizations lack of "confidence" in its ability to perform in a predictable fashion over time. Because a minimum value must be maintained, erratic, unpredictable, variability generally means that an even larger target value is set in order to assure the lower specification is met. Advantages of predictable and decreased variation in fill weight are easy to understand and discuss in this example. Stable variation provides knowledge as to an appropriate target value and provides information for arriving at a firm estimate of the cost associated with a given level of variability. Predictable variation provides the opportunity to consider possible benefits from working to reliably reduce the variability. Decreased variability in fill weight provides management with the possibility of lowering the targeted fill weight, thus reducing costs and increasing efficiency and throughput time. This option, however, calls for additional knowledge as to what sources affect the average, by how much, and in what direction.

The advantages of predictable and decreased variation in fill weights are easy to recognize. The preceding discussion also implies certain types of actions and analysis by process managers. Process managers should be measuring and analyzing variability in actual fill content. In order to be able to affect sustained improvement, managers would have to know those operating practices, as well as those material, equipment and environmental properties that affect variation and its stability over time, and would have to be willing to act on that knowledge. They would have to accept the charge that it is their job to do these things.

For fill weight, large, though predictable, variation in net content meant a financial loss due to the practice of overfilling to maintain a lower specification, a realization that is immediately and easily understood and appreciated by the financial people. The relationship of variation to costs and benefits are not always as easily understood. Gelatin capsules, widely used in the pharmaceutical industry, have a product characteristic which offers a different perspective on what variability might mean to management. An important property for gelatin capsules is wall thickness. The current target value for wall thickness has been specified in order to provide the material in the capsule with the necessary protection from the environment and to insure compatibility of the capsule with the customers' filling equipment. There is no need to change average wall thickness; indeed there are strong reasons for not changing the average. Because average wall thickness must remain unchanged, there is no opportunity to lower the average and save material after achieving decreased variation in wall thickness. However, suppose that current process variability consistently produced capsules having wall thickness outside of specifications. This large variability has several consequences; the capsules must be sorted to remove those of an incorrect size, thus additional material is used to manufacture capsules which cannot be shipped, and machine, as well as other resources, are used to produce product which cannot be used. The costs associated with these losses are neither easily identified by current cost accounting philosophies nor searched for by current management practices. The process may, for example, be on budget because standards allow for a given amount of waste. If the costs are not seen to exist, then the benefits may also not be recognized.

There are internal benefits which can be traced to the decreased variability. After the improved variation has been documented and proven repeatable, there will be less inspection and cycle times will be increased. The financial gains may appear to be modest. However, other, and possibly more significant, benefits are to be found external to the business. With shipments of capsules having known, predictable values within a specified range, the customer can have greater assurance that a shipment of capsules will run on his or her equipment without machine stops, leading to higher equipment utilization and improved cycle times with attendant economies. This assurance in supporting high efficiencies by verified improvement in material properties can provide competitive advantage. In addition, there may be a competitive advantage in having gained the process knowledge demonstrated by the ability to reduce variation. This improved knowledge allows the manufacturer of the capsules to be better able to respond to new information on customer needs in wall thickness, as well as to become a valued supplier of the customer.

It is significant that the benefits of decreased variation in wall thickness do not result in an immediate return to the manufacturer of capsules. Again, the manager must have understood the implications of variability in this process parameter; a narrow attitude will not uncover opportunities for gaining new customers or securing old ones. In this instance, the management group needs more than process knowledge. They must know what it is that their customers might value or where diminished sacrifice in use would offer potential gains. They must be able to correlate these attributes with internal systems and they must assume the responsibility for the stabilization and improvement of those systems. Experience indicates that financial audits are usually focused upon the more narrow, internal evaluation of costs and benefits, thus directing attention away from the range of possibilities to be had from decreased variability. Reliance on existing financial models will not identify the financial gains which may be had by managing the variation in the systems which supply what is valued by the customer. Because of the complexity involved in thinking about and considering the effects of variation, it is imperative that a manager work through what excessive and erratic variation might mean in the fulfillment of system responsibilities. These system responsibilities include knowing and working to learn further about customer concerns. For a particular system, the determination of the costs or disadvantages associated with variation must be determined by the manager. By beginning to understand the effects of variation, the manager is in a position to make an objective analysis regarding the benefits of reducing variability.

1.4. Changing Practices

Understanding the sources of variation in an output result is only one part of the story. There must be a motivation for developing and using that understanding. Working to achieve stability only follows from knowing that achieved stability is a responsibility of the manager. Decreased variability results from system change and incremental improvement. These results follow only after management understands that this is their work. The work therefore cannot be taken lightly; it is for this reason that variability must be understood in the context of a particular system. It is necessary that the manager understand clearly why it is that a particular system exists. Without this knowledge the manager is in no position to make informed judgments about variability and the benefits of its reduction, movement, or elimination. Understanding the sources of variation and the effect of variation in the context of system analysis will affect the way in which a manger approaches work.

Consider a situation in which an increased throughput rate is desired. The rationale for that objective is not questioned here; rather, the needed increase will serve as a starting point for a short discussion on tactics for obtaining the increase. Increased throughput could be attained in a variety of ways. It is the selection from among the alternatives and the rationale for choosing an alternative that forms the centerpiece of this simple example. Typically, the expectation that throughput be increased is expressed forcibly at the plant level. However, the management and staff may not have realistically considered how to achieve the requested gains. Some tactics for achieving the increase may be stated very specifically, such as working to eliminate a known bottleneck or requiring that the throughput rate of each unit in the facility be increased. However, the expressed focus may not be consistent with the actual system sources of variation that deliver the current throughput levels. An overemphasis upon result, the throughput itself, may promote practices contrary to other expectations and needs required of the business. For example, under pressure to increase throughput, a department may release poor quality material. By making this choice, the throughput rate may increase while the yield stays the same. As a further consequence, the opportunity to achieve improved throughput by increasing the ability to consistently produce high quality product at each stage of operation is foregone because of the concentration on the schedule rather than on other system parameters. There are other losses. The capability of the people has not been increased because no process knowledge has been gained. The management group has not been strengthened in acquiring different behaviors by which to manage in the future. There may be no sustained experience in working to reduce defects; therefore there is no assurance that the approach will yield appreciable benefits. There is no confidence that working systematically on the input side will result in improved throughput capability. Under pressure to attain or surpass schedule, equipment may be run without adequate maintenance, having an impact on quality and future plant abilities and well being. Shifts, treated as if they are independent production units, are goaded toward quota. The effect may be to delay appropriate maintenance or set aside the opportunity to better choose when to perform maintenance. If each shift is pressured to achieve a certain production quota, the effect is often to set one shift working against the others. Production knowledge is hoarded; appropriate work is shifted to other shifts or times by a variety of means. Each shift concentrates on getting their own quota, often emptying the line of all work in progress in order to achieve the objective. Thus a larger start-up job is left for the next shift. The idea of running the operation in a smooth, consistent fashion over the different shifts is not considered, nor the benefits in improved throughput which might result.

A different practice for working to improve throughput levels would involve understanding the sources of variation contributing to the level and variability in throughput rates, evaluating those sources, and selecting where and when to make changes to improve the existing system. Implied in this statement is that there exists a multitude of sources which affect each other as well as throughput rate. The idea of searching for one, or the most prevalent, cause of deviations from a standard is to be replaced by the idea of understanding system behavior and the variations in components of the system which affect throughput levels. In examining the total system for making product, management might begin to look at specific activities in a different respect, finding opportunity in places previously unexamined. Numerous set-up changes and within run modifications to accommodate raw material variation reveals that purchasing is not attached to the production subsystem. At least two issues surface. There is the obvious impact on productivity and efficiency of frequent set-ups and modifications. There is also evidence of system breakdown between the purchasing and production subsystems. Examining the interface between purchasing and production offers large returns for management work.

Another simple example to illustrate the contrast of past practices with practices that recognize variation and its implications is tool quality. Inconsistent tool quality leads to erratic and frequent tool changes with a consequent effect on throughput in terms of quantity and quality. However, it also indicates a management which has not paid attention to developing knowledge about tool requirements, to understanding why the tools might be different from time to time, and to working with vendors to prevent those same issues resurfacing, year after year, product after product. The sources of variation evidenced by frequent tool changes may initially be characterized by the physical properties of the tools themselves. But at a more fundamental level, these sources are seen to result from management practices and behaviors which fail to address the issues of developing adequate requirements and providing a mechanism for working with vendors.

Working on sources of variation can be focused in a variety of ways within an organization. One possible focus would be at the operational or functional level. Although this focus is necessary and desirable, it leaves unattended many of the most valuable opportunities for improvement. For example, work on improving throughput in a particular function might have been driven by noting the different capabilities of several machines which are performing the same operation. Improvements in the process would be realized by identifying this source of variation and then making changes to bring all machines into similar operation. Because this operational focus is both useful and necessary, the focus for working on variation may be limited to this level of the organization. In fact, a previously espoused role of management has been to empower the personnel involved with this work to investigate and implement such changes. However, this role by itself is inadequate to address many of the practices and behaviors which might be the larger drivers of variation. Working to insure that all machines operated in a consistent manner may have provided a valuable improvement, but perhaps it is more important to address the management behaviors and actions that allowed the machines to operate inconsistently. Possibly, there existed no system to bring new machines on line in accordance with the other machines. Purchasing would need to become involved with this aspect. Or maybe there exist no present procedures is this or other operations to be watching the effects of these types of differences; management would need to address the existing process for developing standard operating procedures. It is possible that the difference could be addressed by changing maintenance practices. Again, it is management's role to understand these issues and address them. The manner in which management addresses these issues across functions is also important. Just as the role of empowering people is not sufficient to manage the variation in the business, neither is the role of serving as a facilitator of improvement work across functions. Although it would be short-sighted to conclude that this approach has not provided valuable improvements, the passive nature of the role of facilitator will not be adequate to address the issues confronting management. The content of the work of management may be enhanced by knowledge of the existence, quantification, and effects of variation. The context within which this work takes place is the systems management approach.

1.5. A System Perspective of Managing Variation

The above examples of what variation might mean to a manager are relatively obvious. More general examples, requiring a system perception, are presented in following paragraphs. These examples offer more general reasons for studying variation. These examples appear to plead for the system view, a more comprehensive and useful perspective for the manager. This is true. But, upon reflection, these examples also rest upon variability issues, knowing the magnitude and the sources and the predictability of variation, and understanding that the system must be addressed by addressing these issues.

The view that results can be usefully managed by examining deviations from standard has been seen to have a number of limitations. Accompanying this view there is often the perception that there is one cause for what it is that has been observed to deviate. Actions taken from these particular points of view, and without system knowledge, are likely to have other consequences than that intended. By identifying and acting on only one cause, underlying causes do not get addressed, either wholly or in part. This in turn may have the consequence of moving the variation from one part of the business to other parts, perhaps in a different form and possibly at a different time. That may be an appropriate strategy, but also it may have unintended and negative effects on other system parameters which in turn may impair performance in terms of cost, quality or delivery. Another result, one which generally goes unnoticed, is that no new knowledge has been accumulated to support system change or improvement or to support future decisions.

Acting on a single type of cause can have unintended effects when potential effects on other results and on the cause system that generates the collection of results are not taken into account. As an example, consider a situation in which shipping dates are not being met. Deviations from committed dates are observed, attention is drawn to these deviations, and a program to improve these results is initiated. Performance to shipping dates will improve, at least until attention turns to other concerns. What happened to achieve this improvement and the affect on other parts of the business may be any one of or a combination of things, depending upon the mixture of people and departments brought in on the issue. Shipments may leave on schedule after the program with its attendant publicity goes into action because materials of questionable quality have been shipped. Or, perhaps overtime has been incurred, or other crews have been added or more capacity is available because maintenance is delayed or canceled. All of these actions may be appropriate; they also may be inappropriate. In either category, they are also likely to be unintended. It might also be that ship dates are manipulated through an ongoing series of negotiations with customers. These dates could be met and performance marks improved, but perhaps real customer needs as to timing, quality, and quantity are not being addressed, a matter which will accrue to the disfavor and discredit of the business in the future. A system viewpoint by management would indicate that the issue must be considered more deeply than just deciding to better meet promised ship dates. The underlying system issues as to why there are difficulties in meeting shipment schedules would have to be addressed. These may be found in rather prosaic matters such as quality or assembly problems, which in turn may be related to design, equipment capabilities or maintenance, or material purchases. Inability to meet scheduled shipping dates may also be due to the process for assigning a particular order mix to the plant, or possibly to the lack of a focus as to which products to produce first for which customer. If the potential effects on other results such as labor cost, future equipment performance, quality, and future sales were not considered and attempts were made to manage, by directly affecting performance to shipping schedules, the outcome could be damaging to current and future business performance.

Another example is supplied by considering a manufacturing firm which incurs a large fraction of direct cost in the acquisition of raw materials, components, or piece parts. Since management is surely aware of these costs, attention quite naturally becomes focused on cost of incoming materials and parts. Frequently, pressure to reduce those costs results in searches for low bid price, a desirable and intended end result. However, gaining a reduction in purchase price may result in variations in other costs, many of which may not be directly linked to the purchasing function and thus may go unnoticed as a resulting effect of the search for low bid price. For example, quality of incoming material may suffer, leading to internal sorting and rework, thus resulting in an increase in production costs. In addition, inferior materials can easily decrease productivity because of difficulty in working, forming or assembling purchased parts. There are any number of other ways in which the acquisition of raw materials, components, or piece parts can contribute to system deficiencies. Costs of course must be controlled and decreased, that is a fact of managerial life and a necessity for continued success. But which costs, against what criteria, and achieved by what means are not generally addressed from the organization's perspective. If the guidelines for managing costs are entirely functional in form, then functional optimization may move the variation and attendant waste from one part of the organization to another with no overall benefit to the business.

1.6. Conclusion

As part of his or her job the manager must demonstrably know system capability for measurable parameters and to judge the suitability of that capability. The stability, average, and variability of these parameters must inform the decisions on the management of the processes or systems of the organization. Evidence of system instability indicates to the manager that either unmanaged sources of variation are impacting system performance or that management activity has an inconsistent result on performance. This in turn suggests to the knowledgeable manager that certain managerial behaviors are required to address, remove, and prevent such occurrences. Evidence of system stability sends other signals to the manager and necessitates a different management mode with a different judgmental structure. While the average strongly dominates the description of what the system can typically produce, it is the variability which confuses and misleads, perhaps generating inappropriate activity. The variability in the system results in an outcome different than that anticipated. It is necessary to understand that the magnitude of the variability and the consistency with which that magnitude is repeatable is itself a measure of system capability.

The content of this manual is aimed at equipping the manager and process engineer with the tools for process and system management. The statistical methodologies presented provide a set of analytical tools for describing process or system variability, for assessing the impact of sources of variability on process or system outcomes, and to guide the work on the management of that variability. The competence of the managers and engineers of an organization in using these tools will improve the capability of a business to deliver products and services which are valued by customers.

Chapter 2

Basic Elements of Data Collection

Whenever an evaluation is made, the result is a measurement. Evaluations are made for a variety of purposes. Products are evaluated to determine what to do with them next. Are they fit for use or not? Should they be sent on to the next stage in the normal sequence or directed elsewhere? Products, along with process data, are evaluated to judge the performance of the processes that produce them or to study the product variation with the aim of learning more about the causal factors generating the variation. Daily or weekly counts of the number of pieces of scrap or estimates of the monetary value of that scrap are made and reported. The number of flaws in a part or assembly, or the hardness of a metal component or the amount of impurity in a volume of raw material or a quantity of lubricating fluid are evaluated and recorded. Procedures are carried out to place a monetary value on the inventory on hand. Figures are developed for the performance of equipment and processing lines. Measures of degree of customer satisfaction and dissatisfaction are reported and analyzed. Prototype parts or assemblies are subjected to tests of durability and performance. Machinery is evaluated for reliability. Measures of response time for various services are recorded. Costs are calculated for a business unit. Suppliers' delivery performance is evaluated. All of these activities and a multitude of others involve some form of measurement. Any collection of activities carried out to produce qualitative or quantitative evaluations as an end result of those activities is a process of measurement. The following discussion describes the components of a measurement process and outlines some reasons for measurement and record keeping. Four typically encountered types of data and some simple methods of summarizing these data types are discussed. Then, some guidelines for construction of data collection forms are provided. The chapter concludes with a discussion of factors which affect the quality of measurements and a summary of the key points in the chapter.

2.1 Components of a measurement process and factors affecting measurement results

Whenever people operate equipment or carry out any kind of job function to act on inputs and transform them into outputs, they are involved in the operation of a process. When an evaluation of any kind is made on the operation of that process, the evaluation itself is an outcome of a measurement process. Measurement processes can be as seemingly simple as making a judgment and recording the result, or they can be extremely complex involving prescribed multi-step protocols for chemical, mechanical or electronic analysis. Regardless of the sophistication of the equipment and procedures applied to produce the products of a measurement process, all measurement processes may be characterized in terms of their common components: the **people** who carry out the measurement activity, the items measured and the physical materials used (**materials**), the **equipment** used in carrying out the measurement, and the **methods** applied in making the measurement. Given that the process of measurement also creates variation, another important potential causal factor is the **environment** within which measurement is done. Fluctuations of environmental factors such as temperature and relative humidity may be reflected in measurement fluctuation. Personnel selection and training practices, pressure to meet schedules, equipment procurement policies and practices, and a host of other managerial practices create the environment in which measurement takes place and influence the quality of a measurement process.

There are numerous factors that potentially influence the ability to produce measurements of high quality. The following discussion of some of these factors is organized by the measurement process components identified above.

Measurement equipment

Gauges and other types of measurement devices are subject to wear and deterioration due to age and use. Measurement devices should be subjected to the same kind of careful preventive maintenance applied to other critical pieces of equipment used in producing products and services. High precision measuring devices that require carefully controlled temperature, humidity, cleanliness, freedom from vibration, etc., should not be used in production environments where these conditions cannot be met. Measurement devices that require a nearly constant supply of air or power or fluid of some type may not perform as expected if those requirements for consistency are not met. Sensitivity of equipment to conditions of use should be known and taken into consideration when selecting or designing measurement equipment.

Measurement equipment should be designed so that the potential for introducing error into measurements by the difficulty of "reading" the signal produced by the equipment is low. The resolution of the measurement readout should not be greater than the measurement equipment can detect, but should be fine enough so that variation in product can be detected. For example, if product variation is on the order of thousandths of inches, then the combined resolution of the measurement equipment and its readout (dial, digital display, etc.) should be on the order of ten-thousandths of inches. If measurement equipment is used to obtain data for study of product variation, then reduced product variation will lead to a need to improve the resolution of measurement equipment so that the ability to detect variation will not be lost.

Materials

Proper procedures for measuring an item of product often include allowing the item to reach a certain state (temperature, age, etc.) before measuring it or following an established procedure for preparing the item to be measured; e.g., cleaning the item or removing lubricants from its surface or cutting it into sections, etc. Failure to follow proper procedures can introduce variation into measurement results and render them less reliable.

Materials of measurement, such as gauge masters, chemical additives, filters, and so on, require proper handling and storage to avoid changes in those materials that may in turn change measurement results. Established procedures for handling of those materials should be rigorously followed in order to prevent such changes from affecting results.

People and Methods

Methods of measurement include the way materials, equipment and the items to be measured are prepared, the way in which the equipment is used (such as physical location of the equipment relative to the item to be measured, adjustments or calibrations carried out prior to measurement, locating of probes or transducers, clamping or securing the item to be measured, applying a particular force or pressure for a specified period of time, and so on), the way the observation is made (visual judgment relative to a standard, reading a dial, etc.), and the way in which the observation is translated into a recorded result.

For each of the methods described above, there should be a clear description of the standard practice to be used. A helpful way to provide that description is a flow diagram describing each activity and decision to be carried out in the proper sequence. Inconsistency in the practice of measurement methods is very likely without careful training of the personnel who will use them. A plan for training of personnel who will make measurements should provide for ongoing maintenance training to ensure that there is not deterioration in use of the methods over time.

When plans are made to begin collecting data for process study, an important item that is often forgotten is that the introduction of new methods for data collection and analysis may amount to introducing a new technology into the process environment. There have been numerous incidences of introducing the use of control charts into production environments and expecting production people to be able to do the arithmetic of data summary and charting without any consideration of whether they possess the arithmetic and graphical skills necessary to do the work. Any plan for measurement, data summary and analysis should include assessment of the basic skill requirements of the people who will carry out the work and provision for necessary training and skill development.

Environment

Environmental factors, such as temperature, that may affect measurement results include those mentioned above. In addition, environmental effects on the ability of people to observe and evaluate may affect the quality of measurement process results. Adequacy of lighting, freedom from distractions and other factors may have important effects. Another important environmental factor may be the consistency of use of definitions of what is acceptable product and what is not. If meeting production schedules overrides all other considerations in a production facility, there may be a strong pressure to change evaluations of product day to day or even hour to hour in order to ensure that the schedule is met even though produced quality varies. Managerial policies and practices form the foundation of the social environment in which measurements are made and may have a powerful, if indirect, influence on evaluations of all kinds that take place within that environment.

The preceding description of factors that may influence the quality of measurement results is, of course, incomplete. Methods for investigation of the effects of those factors and discovery of other possible factors is presented in later chapters.

2.2 Reasons for measurement and record keeping

Evaluations of product and service outcomes may be made for various reasons at any of several points in the creation or delivery of those results. Evaluation may be carried out by conducting a census or by applying a sampling strategy. A census is accomplished by evaluating every unit in some set of product, material or service. A census often takes the form of inspection to weed out all units of product that fail to meet one or more requirements. A sampling strategy is a method for collecting a subset of units. Sampling strategies are used when evaluation destroys or alters the product or service in some critical respect. In other cases, a census is economically infeasible due to a justifiable need for high quality measurements obtainable only at considerable expense; hence, a sampling method is performed. When the objective of evaluation is to describe the statistical behavior of the variation produced by an ongoing process, there is no possibility of a census because the output of the process is never complete. Sampling strategies must be designed to characterize ongoing process behavior rather than to describe a fixed set of product, material or service.

Measurements of product or service characteristics are made for at least five reasons:

- to determine disposition of the product or service
- to develop and maintain a history of product or service results
- to assess the stability of variation in results produced by the processes creating the product or service
- to take actions to change process or input factors
- to develop an understanding of how causal factors and their interactions affect product or service results.

Irrespective of the reason for measurement, it is often difficult to carry out an evaluation so that measurements obtained over a period of time and over a variety of conditions are consistent and have the same meaning. Consistency of measurement procedures, equipment and materials is a recurring and important theme with regard to measurement processes, regardless of whether a sampling or a census strategy is used and regardless of the measurement intent. Gaining knowledge of measurement consistency is necessary; therefore, in-depth discussion of measurement consistency and further development of the concepts and methods required are deferred to Chapter 8.

One of the reasons listed above for product evaluation is **to determine disposition**; i.e., to decide what is to be done with the product: scrap it, rework it, or deliver it to the next stage. Although carrying out inspection for disposition may be dictated by economic considerations as an appropriate short term tactic, an analysis with a longer term view should, in most cases, lead to the conclusion that resources should be invested in process improvement to remove the need to inspect for disposition. However, decisions for upstream improvement need not be based solely upon benefits to be derived from removal of inspection. Internal benefits of upstream improvement that may not be easily measurable include reduction of complexity of operations, reduced record keeping requirements, smaller fluctuations in workloads for rework and repair, and many others.

Measurements of product or service characteristics may be collected **to add to a history of product or service results**. A product or service profile may be maintained for the purpose of meeting regulatory or other reporting requirements or for warranty or reliability reference. Under the assumption of measurement consistency, the history may provide information concerning the stability of variation in process results and and a basis for evaluating the ability of existing processes to meet future requirements. Although the objective of establishing and maintaining a product or service history may be entirely legitimate, some histories are established and maintained as a result of a temporary request to gather information expressed by a management group and never questioned thereafter. Numerous reports and performance records are initiated in this way. Poorly planned measurement strategies which involve large capital investment for measurement equipment.are often difficult to remove once put in place. An opportunity for removal of waste from an organization lies in questioning the intent and need for institutionalized measurement and reporting systems.

Product or service evaluation is also performed **to assess the stability of the variation produced** by the processes creating the product or service. The stability or instability of variation in the results of a given sampling and measurement strategy provides a measure of the effectiveness with which the process is engineered, managed, and operated. Effectiveness is related to the current knowledge base used for process management and the consistent use of that knowledge. The degree of ability to replicate process outcomes consistently affects the confidence that can be placed in the results of planning, purchasing, scheduling, resource allocation, and so on.

Actions taken to change process or input factors may have a temporary effect on process results. Measurements made as part of a manual or automated "control" system usually produce temporary effects on the process and its outputs. "Statistical process control" restricted to monitoring process variables or output characteristics and adjusting the process upon signal of change is a form of control system designed to maintain a steady, predictable variation in the output stream. Continuing adjustments to maintain a stable process is an integral part of such systems.

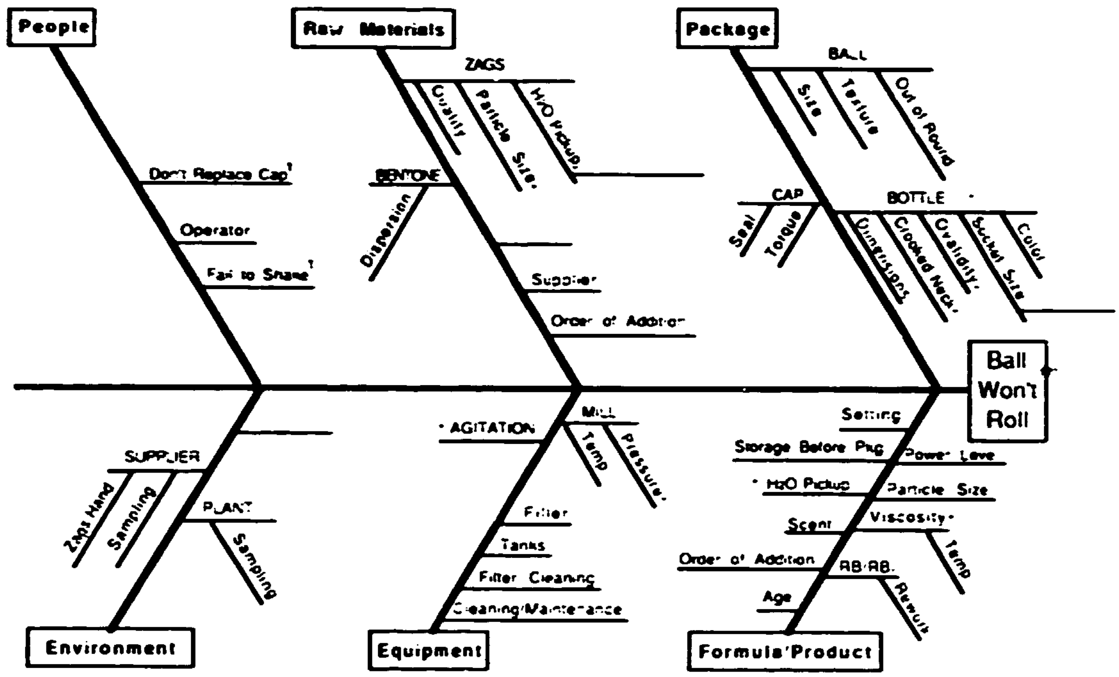
Measurement of product or service characteristics may be a part of an engineering or managerial feedback control system intended to monitor output results to gain information for input or process adjustment. In the previous sentence, the term "control" is not being used in the statistical sense, but rather in the sense of a formally designed or de facto operating procedure for process intervention. The procedure may or may not use well thought out statistical criteria to determine the need for and the magnitude of intervention and adjustment. When a process is unstable and subject to change, monitoring and intervention to maintain a steady state is an important objective. However, two critical points are often missed with regard to the design and operation of feedback systems. First, they do not usually provide for systematic learning and action to reduce variation beyond what the existing feedback mechanism can accomplish, so the best result they can produce is maintenance of the level of variation built into the process design. Secondly, many managerial feedback control systems and some engineering control systems do not appropriately recognize and interpret the "noise" present in results from any stable process or system. Therefore, there is risk that intervention and adjustment will take place for the wrong reasons and will introduce additional variation into process or system results. Introducing an understanding of stable variation into the design and operation of control systems provides potential to gain considerable improvement in results. Further, learning about causal factors that act to produce variation and subsequently acting on those factors may lead to changes in the feedback system in terms of what is measured and what intervention and adjustment strategies are used. Thus, engineering and managerial control systems are, themselves, suitable subjects of improvement activity.

Intermediate evaluations are often performed in producing physical units or in the delivery of certain services. One or more corrections or adjustments in later process steps may be needed in order to yield what is required as an end result. In this respect, upstream product and service evaluations support the production of final outcomes that better meet customer needs and requirements. In surgical procedures, a particular outcome from a physiological evaluation, performed prior to or during surgery, may require that a surgical team modify their procedures to protect the patient's welfare or to achieve the original surgical objective. Intermediate evaluations in manufacturing processes are sometimes part of a manual or automated feed forward control strategy. The feed forward strategy alters processing conditions further downstream to accommodate the condition or state of the product as it exists at the evaluation point. A feed forward control strategy may be justified, given the economics of downstream adjustment versus change in the upstream conditions and processes that generate the need for feed forward control, but a control strategy should not be adopted without consideration of the economic issues in some breadth and depth. Improvement in upstream conditions and processes may render a feed forward system unnecessary.

Finally, evaluation of product or service characteristics may be done **to develop an understanding of how causal factors and their interactions affect product or service results.** In this context, evaluation results are used for the analytical purpose of developing a deeper insight into the mechanisms that generate the product or service. The objective is to stabilize and then to improve the generating processes in order to create improved value for customers, a multi-dimensional aim that includes quality, cost, timeliness and performance in use of the product or service. When attempting to stabilize and then to improve the processes generating the product or service results, the exact causal factors influencing the process may not be known or well understood. In order to improve the results from the process, the affects of potential causal factors must be carefully analyzed. Hence, it is often necessary to hypothesize on all possible factors which may be affecting the results. Consider the manufacturing process of roll-on deodorant bottle assembly. Deodorant is discharged into the deodorant bottles and a polypropylene ball is forced into the top of the bottle. After the ball placement. samples of assemblied bottles are tested for functionality. Even though each bottle assembly is seemingly produced in the same way. inspection of the final deodorant bottles shows dispersion in the process results. A result of particular concern is that the plastic (polypropylene) ball in the top of the container does not always roll This dispersion could be caused by changes in raw materials, the equipment, work methods, or a multitude of other possible factors. Figure 2.2.1 portrays the results of brainstorming and theorizing on possible causal factors for the functional nonconformity.

Figure 2.2.1

Cause-and-Effect Diagram for Ball Not Rolling in Assemblied Deodorant Bottles



† Consumer Services Follow-Up
* Key areas to concentrate efforts

Cause-and-Effect diagrams (also referred to as fishbone diagrams) are helpful in sorting out the possible causal factors and for portraying the relationship of these potential causes with the outcome or result. To construct a cause-and-effect diagram, the characteristic or result of concern is placed in a box on the right side of the figure. This characteristic, such as type of defect, is referred to as the effect. Major factor types which may contribute to the dispersion in the results form the main branches in the diagram. In the above example, these major categories are raw materials, people, package, environment, equipment, and the formula or product. The specific factors (i.e., causes) within these major categories are then placed as stems on the corresponding branches. When using cause-and-effect diagrams in the study of factors which influence the dispersion in process results, it is important to solicit input and ideas from all persons having knowledge of the process. Another important consideration is that the cause-and-effect diagram is only a tool for sorting and describing the relationships of potential, hypothesized causes. Careful analysis of the causal relationships and the process itself should be performed prior to making any process changes. In the previous example, the possible causal factors which were decided as critical areas in which to concentrate efforts were marked by astericks. However, upon careful and further study of the process, it was determined that the major cause of the ball not rolling in the final product was the ball texture - a possible causal factor which was not hypothesized to be a critical cause of the resulting dispersion in process output.

Little has been stated in the preceding discussions concerning measurement of input characteristics, process factors or operating conditions. The previous ideas concerning the disposition of process outputs and feed forward control systems apply to the measurement of input characteristics. Since inputs are simply outputs of some prior process or intermediate results in a longer process stream, measurement of input characteristics is also done for the purpose of disposition or as a form of feed forward control. The reasons for measurement of process factors and operating conditions are the same as the reasons provided for the measurement of outputs. In some industries, the evaluation of process factors and operating conditions is a matter of meeting regulatory requirements. For example, in the pharmaceutical industry there are specifications on processing conditions, such as temperature and processing times, that must be maintained in order to satisfy regulatory requirements for acceptance of the product. These measurements of process factors and operating conditions can also serve as inputs to feedback control systems. For example, a control system for a machining process uses measurements of machine temperatures, machine forces and tool characteristics as inputs to a control algorithm. In order to develop an understanding of how causal factors and their interactions affect output characteristics, plans for measurement of inputs and process factors must address how the resulting measurements will be associated with measurements of outputs.

Even though measurement and subsequent data summary or analysis are done for multiple purposes, it is important is that those who initiate and carry out measurement activities understand why data are being collected. Data collected to maintain a historical record may not be useful for assessing process stability or for using the data in a feedback mode, because source of data and sequence of production are often lost. Thus, it is difficult to obtain information on process changes and potential causal factors. For process investigations and for the development of process knowledge, timely and specific data along with the careful documentation of process changes and potential causal factors are required. When data are collected for process investigations, the data must provide infomation on process changes and must permit investigation of hypothesized cause and effect relationships. Such information is not always seen as necessary and, therefore, may not be recorded if data collection has some purpose other than process investigation.

Measurements of process inputs, factors, conditions, and methods, taken for the purpose of process investigation, are used for more than just simple descriptions of what currently exists or of what has happened in the past. The intention of obtaining these measurements is to understand the effects of changes in material, equipment, methods or environmental conditions on process output, to judge the correctness of the current level or average performance, and to understand the potential benefits to be gained by altering the magnitude of variation. The ultimate purpose is to make changes to the process which will improve the future outcomes of that process.

2.3 Kinds of data

The results of repeated application of a measurement process, collected together for summary and analysis, form a set of data. Proper summary and analysis of the set of data require knowledge of the type of data being examined. In traditional quality control, two types of data are usually identified: attributes data and variables data. When the measurement process requires a count of items or a count of a specified quality characteristic, the resulting measurements are attributes data. Attributes data include categorical data, counts of events or items, and rank data. Variables data consist of actual measurements of a quality characteristic, such as the weight of a bar of soap. This distinction between attributes and variables data determines the method of data analysis. For example, the X-bar and R chart is often used in the analysis of variables data, whereas the p chart for fraction nonconforming is commonly used for control charting categorical data. Hence, the four major kinds of data typically encountered are:

- categorical data
- counts of events in space (area or volume), time or amount of work

• data consisting of ranks or ratings
• variables data.

Each of these kinds of data are described below. Some examples and a mention of some of the tools available for analysis that are discussed in later chapters is also provided.

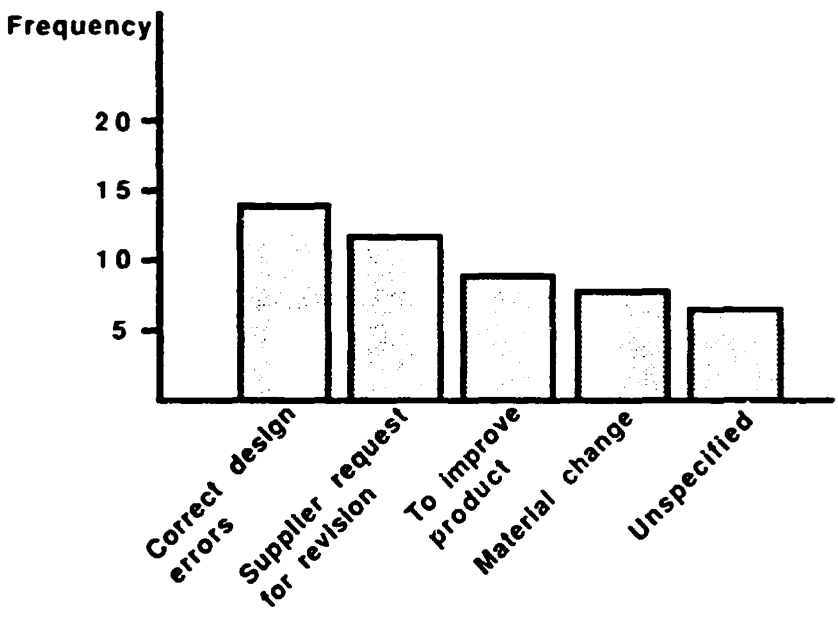
Categorical data arise by classifying each of a group of individuals or items or events into one of two or more categories. The resulting data usually consist of the number in the group that are classified into each category. Some examples of categorical data are:

- Parts in a collection are classified as scrap, repair or good. For example, the data recorded is that 3 parts are scrap, 8 parts are slated for repair and 89 are good among a total of 100 parts.
- Respondents to a survey questionnaire are classified by their age. The resulting data consist of counts of the number of respondents in each age category.
- Engineering change notices are classified by the reason for the change. The resulting data indicate that, of 50 engineering change notices, 9 changes were made as product improvement items, 12 changes were made as a result of supplier requests for revision of standards, 14 changes were made to correct design flaws or errors, 8 were made to institute material changes and the remaining 7 were made for unspecified reasons.
- An overnight package delivery service classifies a delivery as "on time" if the delivery is made before 10:30 A.M. on the specified day of delivery. A week's deliveries in a given region are classified as "on time" or "not on time" (the latter category includes lost packages). The resulting data consist of the total number of deliveries which should have been completed that week and the number of on time deliveries.
- Processing line interruptions are classified by the type of interruption. The resulting data consist of counts of the number of interruptions by type that occurred in a month. Using this type of descriptive information to guide prioritization of problem-solving work each month is fairly typical practice. Improved practice is to analyze such data as counts of events in space or time (discussed below) with the intent of distinguishing chronic kinds of problems from sporadic problems.
- Sheets of material are examined for surface defects of various kinds. The resulting data consist of counts of the number of defects of each kind found in

a given number of sheets. As in the previous example, the data consist of counts which should be analyzed with reference to an ongoing production process as counts of events in space or time.

Categorical data that consist of counts of the number of items in two or more categories are sometimes summarized using bar graphs. Figure 2.3.1 shows a bar graph (also called a Pareto diagram) of the data on engineering change notices described earlier.

Figure 2.3.1
Bar Graph of Number of Engineering Change Notices by Reason for Change



The basic statistical model used for the study of variation in data consisting of counts of the number of individuals or items in one of two categories is called the Bernoulli or binomial probability model. The generalization of this model to more than two categories produces the multinomial model. When studying variation in a sequence of counts of number of items in one of two or more categories (or the proportion of items in one category) as outcomes from an ongoing process, statistical control charts called np or p charts are usually employed.

Counts of events in space (area or volume), time or amount of work are made in a large variety of circumstances. Some of these counts, such as the number of processing line interruptions in the previous example, arise as a result of observing occurrences of some kind of event in a given period of time. Counts of the number of customer complaints per month, the number of accidents per labor hour, the number of fires per month, the number of equipment failures per thousand parts produced, the number of requests for service or number of service interruptions per unit time arise by counting the number of events in time. Counts requiring similar treatment involve the existence of flaws in two or three dimensional space or in given volumes of liquid or gaseous material. The surface defect example provided above involves counting flaws of a certain kind in a two-dimensional area. Similarly, the number of dirt particles in the paint on the hood of an automobile and the number of scratches on the journals of a crankshaft.are counts of events in a given area. An example of a volumetric count of events is the number of voids and fractures in nuclear reactor vessel welds. Levels of impurities present in food products, such as grain or prepared foods, are also evaluated. In these cases and in the case of chemical products, impurity levels are recorded for a given volume of material. Some air quality measurements are intended to reflect the concentration of various kinds of particulates per unit volume. Counts of errors in documents or financial records or assembly discrepancies in manufacturing are counts of kinds of events as well. The amount of work represented by the document , record or assembly might be an appropriate measure of "opportunity" for the errors to occur. The basic statistical model for study of variation in data consisting of counts of events in space, time or amount of work is called the Poisson probability model. When studying variation in a sequence of such counts (or functions of the counts) as outcomes from an ongoing process, statistical control charts called u charts are usually employed. The reader may refer to material on u and c charts in Chapter 4.

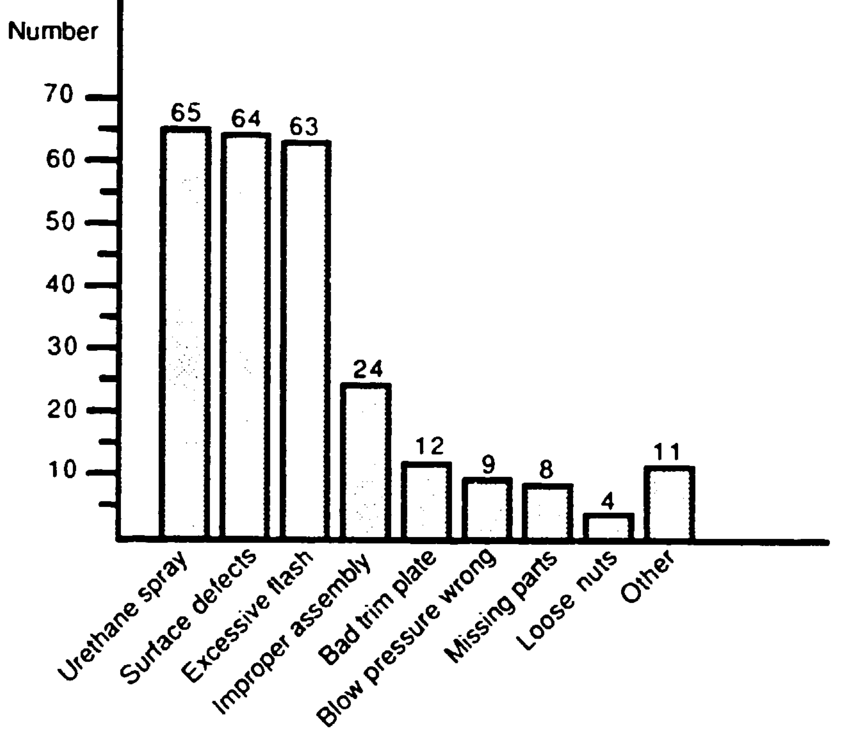
Counts of events in three or more different categories, such as defects by type, accidents by type and impurities by type, are often summarized by the use of a Pareto diagram. A Pareto diagram is a bar graph used to compare frequencies of events. A Pareto diagram is constructed by first selecting the categories to be used to summarize the counts of occurrences. Once the counts are summarized by the chosen categories, the resulting counts are ordered. If there are some infrequently occurring, unrelated categories, they may be grouped together as "other." The counts are then used to draw a bar graph with the vertical scale corresponding to frequency. Bars of heights which correspond to the category frequencies are drawn above labels on the horizontal scale that identify the categories.

The inspection of horn pad assemblies provided the data in Table 2.3.1. In addition to keeping records of number of defective items found at final inspection, records were maintained of the kinds of defects found. Summary counts of the number of defects of several different types found during inspection of 1000 horn pad assemblies were as follows (the counts have been arranged in descending order of magnitude):

Table 2.3.1
Counts of Number of Defects by Type
Obtained from Final Inspection of Ho n Pad Assemblies

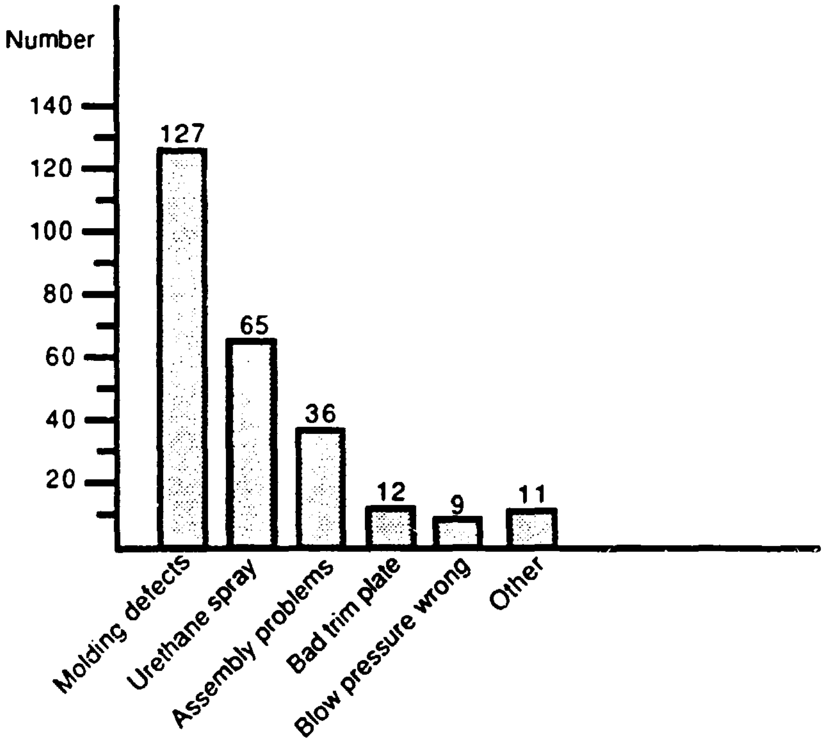
Type of Defect	Number	Percent of Total
Defective urethane spray	65	25.0
Surface defects	64	24.6
Excessive flash	63	24.2
Improper assembly	24	9.2
Bad trim plate	12	4.6
Other	11	4.2
Blow pressure wrong	9	3.5
Missing parts	8	3.1
Loose nuts	4	1.5
Total	260	

Figure 2.3.2
Pareto Diagram for Counts of
Horn Pad Assembly Defects by Type



IThe purpose for constructing the Pareto diagram for horn pad assembly defects type was to get information that would be helpful in determining what areas of the production process to work on in order to reduce the occurrence of problems in the final assembly. With this purpose in mind, examination of the data summarized in Table 2.3.1 and in Figure 2.3.2 led to a reorganization of the data. Surface defects and excessive flash were both defects produced in the pad molding process. Improper assembly, missing parts and loose nuts were connected to the assembly process. When the counts of defects were reorganized by point of origin in the production process, the Pareto diagram shown in Figure 2.3.3 resulted. Examination of that Pareto diagram leads to the obvious conclusion that the pad molding process is the point of origin of the most defects and is the most fruitful place to begin work in order to reduce the frequency of occurrence of defects.

Figure 2.3.3
Pareto Diagram for Horn Pad Assembly Defects Organized by Process Step



Pareto diagrams are used to identify the most commonly occurring or most expensive problem. The principle behind Pareto analysis is that a few kinds of problems account for most of the total occurrences or for the majority of the cost.

Prioritizing problems as a guide to improvement efforts presents some potential pitfalls that should be avoided. One pitfall is failure to recognize the existence of variation in the number of occurrences over time of problems generated by a stable process. A stable process will create varying numbers of different types of problems day to day, week to week, and so on. When there are several types of chronic problems occurring at similar rates, the ranking of types of problems or defects each time period might produce different orderings of problems, depending upon the particular time period from which the data is chosen. Data gathered over multiple time periods may be used to distinguish between problems that are chronic and those that occur sporadically. Knowledge of the average rate of occurrence over time of chronic problems and the identification of sporadically occurring problems provides a more sound basis on which to prioritize work to identify and remove causes of problems.

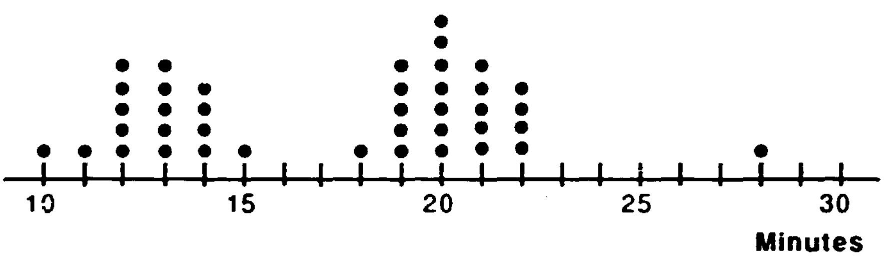
A second potential pitfall in using data on problems or defects arises in the problem solving process itself. Examining only items having a problem as a means to identify cause(s) can be misleading and can thus contribute to faulty analysis and ineffective, if not incorrect, actions on processes. If problem-free as well as problem items are produced during the same time span, some of the problem-free items should be examined to reduce the likelihood that the problem is incorrectly associated with a factor believed to be a potential cause. It is helpful to view the process as generator of both bad and good items and to develop a list of several possible factors whose influences combine to produce a large enough variation defective items. An example of the the second pitfall deals with an assembly designed to spray a liquid into a chamber. Leakage from the assembly was considered to be a critical defect. Several leaking assemblies were found by inspection of production output. Only assemblies that leaked were examined to identify the possible causes of the leakage. The leaking assemblies were found to have a particular component that was contaminated by foreign matter. Since contamination of components had been associated in the past with leakage, the product characteristic identified as the cause of leakage was taken to be component contamination. As a result, the supplier of the component was preparing to add a costly decontamination step to the component production process. Upon investigation and subsequent examination of some non-leaking assemblies produced during the same time period, it was found that non-leaking assemblies had both the same type and degree of contamination as the leaking assemblies. Since contamination existed in both leaking and non-leaking assemblies, it became necessary to determine other possible reasons for the leakage problem. Further discussion of possible causes of leakage led to investigation of variation in some dimensional characteristics of the assembly. This investigation showed that excessive variation in component dimensions had been the factor that created the leakage. Work to reduce that variation cleared up the leakage problem and removed the need for the supplier to add the decontamination step to the component production process. Of course, this example demonstrates faulty problem solving logic as well as the failure to recognize the existence of a process producing a stream of results. Both the existence of variation in the number of occurrences over time of problems generated by a stable process and the problem solving process itself should be considered when attempting to understand cause and effect relationships.

Data consisting of ranks are created by ordering a collection of individuals or items from 1 to n (n = the number of items in the group) according to some criterion (e.g., best to worst, largest to smallest, least to greatest, etc.) Such data are called **ordinal** data. Similar data arise by assigning a **rating** (e.g., excellent, good, mediocre) to an individual or item. Customer evaluations of product or service often consist of responses of this type. Although a collection of ratings such as customer evaluations can be treated as categorical data and analyzed accordingly, other special statistical methods have been devised to deal with data of this kind (Refer to Conover, W. J., 1971).[1].

Variables data arise from measurement of a quantitative characteristic of an individual, item or group. For example, measurement of a person's age, height and weight results in three measurements of the variables type. Dimensions, forces, velocities, rates of flow, temperatures, and other quantities that can possess any numerical value on a continuous scale are data of the variables type. Times, such as time required to accomplish a particular service or task are data of the variables type. Measurements of the elapsed time until the first occurrence of a failure or breakdown or the time between those occurrences are commonly used in the analysis of product reliability. In certain circumstances, count data of the two kinds mentioned earlier are most reasonably treated as variables data, but some sophistication is required to determine when this is appropriate. For example, costs are often treated as variables data.

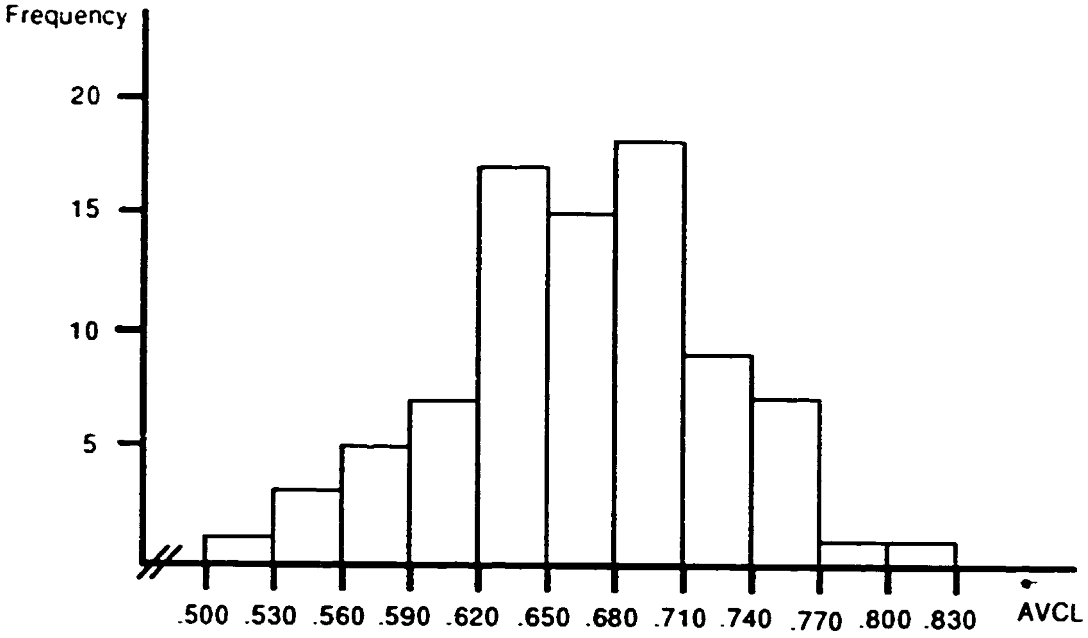
There are a variety of methods used to summarize a collection of variables data. Dot plots may be used to picture the pattern of scatter or variation of a small collection of measurements over the scale of measurement. A dot plot is constructed by drawing a horizontal scale for the range of measurement and plotting a dot for each measurement above the appropriate point on the scale. Figure 2.3.4 shows an example of a dot plot constructed from data on times required to transport shipments of parts from one plant to another in an urban area. This plot suggests that two different processes exist which result in these transport time and it shows an outlying data point.

Figure 2.3.4
Dot Plot of Transport Times for Shipments of Parts



Histograms can be used to describe the shape of distribution of a larger collection of measurements. Figure 2.3.5 is an example of a histogram constructed from 85 measurements of available chlorine (AVCL) from production samples of a consumer cleaning product.

Figure 2.3.5
Histogram Constructed from 85 Measurements of Available Chlorine in Samples of a Consumer Cleaning Product



A histogram is constructed from a collection of measurements by carrying out the following steps:

1) Find the smallest and largest measurements in the collection.
2) Divide the portion of the scale of measurement covering the smallest and largest measurements into from 5 to 20 equal, contiguous intervals that have non-overlapping end points, so that each measurement will clearly belong in only one interval. (With more data, more intervals can be used.)
3) Count the number of measurements that lie in each interval.
4) Draw a horizontal scale representing the scale of measurement and mark off the intervals on the scale.
5) Draw a vertical scale representing frequency.
6) Draw vertical bars over the intervals with heights that correspond to the number of measurements that lie in each interval.

Table 2.3.2 contains the 85 measurements whose frequency distribution is summarized in Figure 2.3.5.

Table 2.3.2
85 Measurements of Available Chlorine in Samples of a Consumer Cleaning Product

.72	.75	.67	.70	.67	.69	.69	.57
.64	.70	.68	.71	.73	.69	.75	.74
.68	.72	.66	.68	.61	.64	.53	.51
.62	.75	.77	.80	.70	.69	.69	.66
.69	.63	.59	.64	.63	.58	.53	.62
.65	.68	.71	.65	.69	.70	.66	.60
.64	.64	.61	.67	.63	.60	.63	.62
.67	.67	.64	.62	.65	.54	.58	.72
.69	.76	.75	.66	.67	.73	.69	.66
.68	.62	.58	.59	.64	.57	.62	.67
.59	.71	.72	.75	.70			

To construct the histogram shown in Figure 2.3.5, the smallest and largest measurements in the collection shown in Table 2.3.2 were found; those numbers are .51 and .80. It was decided arbitrarily that to construct about ten intervals. Dividing the range, .29, by the desired number of intervals yielded .029 as the width of each interval. Constructing equal width intervals to cover all the measurements in the set yielded the intervals: .500 - .529, .530 - .559, .560 - .589, 5.90 - .619, .620 - .649, .650 - .679, .680 - .709, .710 - .739, .740 - .769, .770 - .799 and .800 - .829. Counting the number of measurements in each of those intervals yielded the histogram shown.

An important point to keep in mind when examining histograms is that the choice of the number and width of intervals to use is arbitrary. However, the resulting picture of the shape of distribution is dependent upon that choice. Thus, interpretation of histograms should be approached with a healthy degree of caution.

Both dot plots and histograms are summaries of a collection of measurements that describe pictorially the location of the entire collection on the measurement scale and the pattern or distribution of variation in the data. If the measurements come from results or characteristics of ongoing processes, a summary of the data which ignores the time sequence in which the results were produced may obscure some of the most important information in the data - that information associated with time of occurrence. As an illustration, consider the time sequence plots shown in Figures 2.3.6 and 2.3.7. In each case, the series of measurements has been summarized by use of a histogram on the right side of the figure. (The measurement scale for the histogram is the vertical scale in each figure.)

Figure 2.3.6
Time Ordered Plot of Measurements
Taken on Successive Results of a Process

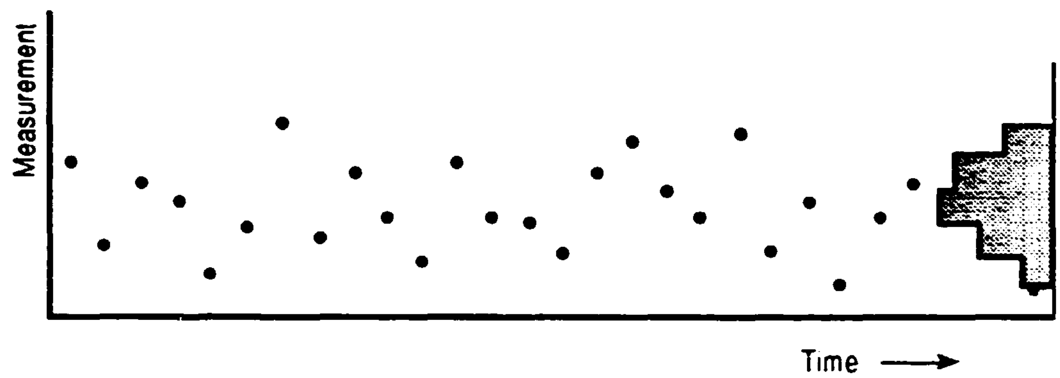


Figure 2.3.7
Time Ordered Plot of Measurements
Taken on Successive Results of a Process

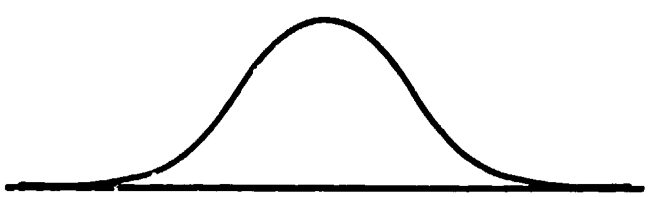
Although the summary histograms in Figures 2.3.6 and 2.3.7 are identical, the time sequence plots shown in Figures 2.3.6 and 2.3.7 show the existence of two very different processes. The series of results plotted in Figure 2.3.6 shows no particular systematic pattern. The series shown in Figure 2.3.7 exhibits a downward trend. Examination of Figure 2.3.7 leads to the conclusion that systematic change is occurring in the process producing the series of results and, consequently, that a need exists to understand the cause(s) of the change.

If only the histogram had been used to summarize the results of the process, the important information concerning the systematic change in process behavior contained in the time sequence plot would not have been discovered. Similarly, the dot plot of transport times shown in Figure 2.3.4 may obscure useful information available in the original time series.

The descriptive methods presented above are useful for summarizing a collection of existing data. However, it is important to remember that existing data are always records of the past. Historical results, by themselves, should not be used to predict future results. Knowledge of the state of statistical control of the process in question and subject matter knowledge of the causal factors which are likely to produce variation in future results are essential ingredients of good predictions.

The model most commonly used to study variation in data of the variables type is the normal probability model. The normal model has been used as a basis for numerical constants used to construct control limits for X-bar and R charts and other kinds of statistical control charts for variables data. One of the characteristics of the normal model is that the shape of distribution of measurements described by that model is symmetric with its highest point at its center. Figure 2.3.8 shows a picture of a normal curve.

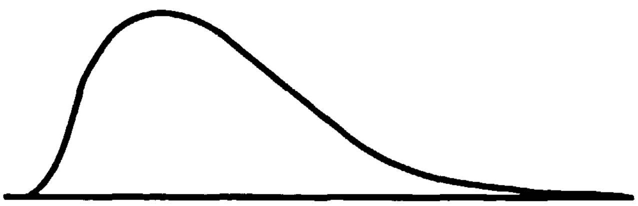
Figure 2.3.8
A Normal Curve



The picture shown in Figure 2.3.8, as well as the one shown below in Figure 2.3.9, can be interpreted as a description of a relative frequency distribution for a large supply of measurements. Such a relative frequency distribution might be thought of as a limiting relative frequency histogram for a very large number of measurements when the number of class intervals grows very large and the width of those class intervals approaches zero.

Some kinds of variables data, such as waiting times or times to failure, maxima or minima, ranges along a continuum, or chemical measures in parts per million, may tend to have distributions that are non-symmetrical. Figure 2.3.9 shows a picture of a skewed, non-symmetric distribution model.

Figure 2.3.9
A Skewed Distribution



In extreme cases of lack of symmetry, use of non-standard methods may be required. In particular, data consisting of times between occurrences of events over time may be more appropriately analyzed using methods other than those based upon the normal model.

Statistical control and a normal distribution should not be confused. Statistical control, as a concept of variation, refers to the fluctuations in results produced by a process over time, and whether or not those fluctuations are of a steady magnitude around a steady average. The accumulated results of a process producing statistically controlled variation may form a shape of distribution that bears no resemblance to a normal curve. As particular examples, values of fraction defective or counts of occurrences of events from a stable process will exhibit shapes that may be heavily skewed, depending upon the average per unit. These kinds of data arising from counts are, of course, not of the variables type. But variables data produced by a stable process do not have to form a normal distribution. Examples of kinds of non-normally distributed results were mentioned above. Some practitioners believe that a histogram constructed from an accumulation of process data over a particular time span can be used to "test" for statistical control; i.e., non-normality implies lack of statistical control or an approximately normal shape implies statistical control. Reference to Figure 2.3.7 makes it clear that when a sequence of results is summarized by a histogram, the shape produced can appear to be approximately normal, even though the sequence of results exhibits a pattern that is clearly non-random. A statistically controlled sequence should not usually exhibit systematic patterns. Although isolated extreme results that might produce an out of control signal on a control chart may appear as extremes on a histogram, the absence of such extremes on a histogram is clearly insufficient to judge the state of statistical control of process results. Control charts provide the means and operational definition to make a judgment of whether or not process results are statistically controlled.

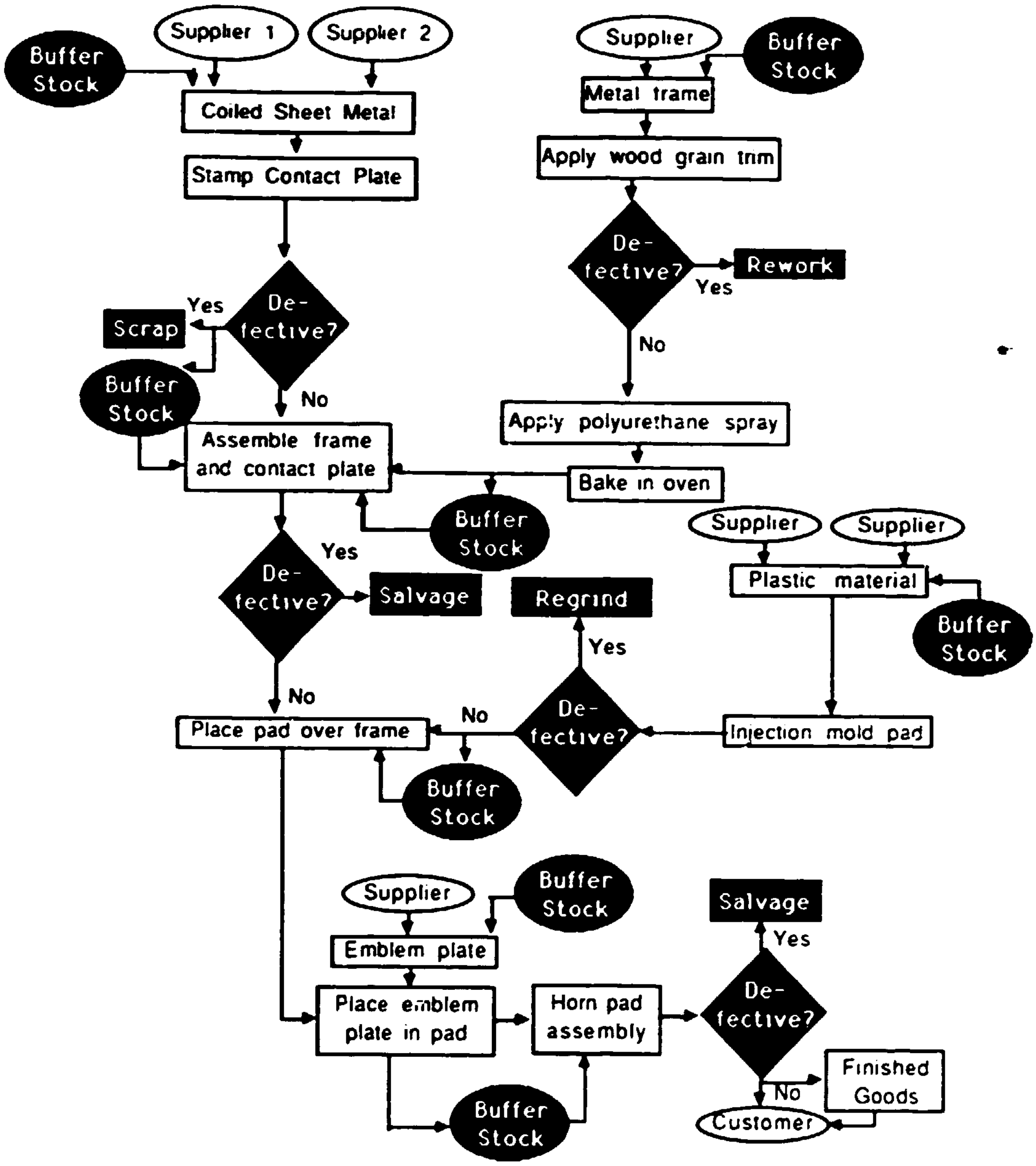
2.4 Data collection

One of the final stages in carrying out the activities of a measurement process is the recording of the measurement results. The effectiveness of the measurement results (i.e., the data collected) depends heavily on the planning of the data collection strategy. In many organizations, a large amount of historical data is collected and stored. In attempting to study processes to determine causes of dispersion in process outcomes and to make process changes which improve the process, historical data currently stored often does not contain pertinent, sufficient information. If the purpose of the measurement process is to yield data to serve as a basis for decision-making, then the reason for the data collection must dictate the collection strategy. In planning for data collection, it is necessary to understand the actual process in order to determine critical points for measurement in the process. These measurement points should correspond to process parameters which have impacts on the process outcomes. Hence, the current process should be carefully and accurately described and potential causal factors should be determined prior to data collection.

Flowcharts are useful for identifying key points in the process for study and, consequently, for determining points of measurement. Not only does the flowchart provide a schematic picture of the current operation of the process, it also serves as a useful reference for determining critical process parameters. It is often the case that redundant or unnecessary, costly process steps are identified during the flowcharting process; hence, immediate changes can be made to reduce the process complexity. Consider the flowchart in Figure 2.4.1. The shaded areas in the current process description show non-value added stages in the process as well as points of measurements .

Figure 2.4.1

Flow Diagram of Manufacturing Process of Horn Pad Assemblies



One factor that influences the effective use of measurement results is the care and attention paid to correctly recording the measurement and any important auxiliary information. Data collection forms should be designed to make correct recording of measurement results easy and efficient.and to facilitate recording error-free results, rather than to introduce the potential for error through making recording of data difficult and time consuming. A data collection form should also allow space to record the time of data collection, identification of the person who collected the data, and other auxiliary information.. When data are to be collected with the intent of developing process knowledge, the kind of auxiliary information collected depends upon current theories about causal factors that affect results. What is considered important to observe is dependent upon those theories and will change as knowledge is developed. For example, information on the source of the items measured, such as lot number for incoming supplies, machine identification for parts, time or product, et cetera, might be recorded. Observed conditions and changes that might be important to the interpretation of the data such as ambient temperatures, maintenance actions, machine adjustments, machine stops and starts may need to be recorded as well. The form should provide for flexibility in recording changing kinds of information as process knowledge is developed.

If a running record of results is to be examined for statistical control, the data collection form should also be used to maintain that record in graphical form. Figure 2.4.2 shows a data collection sheet for maintaining inspection records of counts of defects and recording their types combined with a chart format to plot a running record of those counts.

Figure 2.4.2
Data Collection Sheet
Horn Pad Assemblies

MACHINE	SHIFT	DATE 6/3	6/4	6/5	6/6	6/7	COMMENTS
1	1	A D I	A E	A A D F			
	2	B F	A D C	D E			
2	1	A E G	A I	B E			
	2	A E G	AD E	A A D E			
3	1	D D I	A B E	D			
	2	A B	E F	D D B			

MACHINE	SHIFT	DATE 6/8	6/9	6/10	6/11	6/12	COMMENTS
1	1						
	2						
2	1						
	2						
3	1						
	2						

TYPES OF DEFECTS

A defective urethane spray	D surface defects	G blow pressure wrong
B improper assembly	E excessive flash	H loose nuts
C missing parts	F bad trim plate	I other

2.5 Operational Definitions

Measurements are the information link between process results and subsequent action, whether the action be on the results themselves or the process that produced them. Communications between individuals, groups, functions, customers and suppliers include summaries and analyses of measured results. The quality and consistency of those communications and the waste incurred as a result of miscommunication depends heavily on a common understanding of what is actually measured, how it is measured and how those measurements are to be used in making decisions and taking actions.

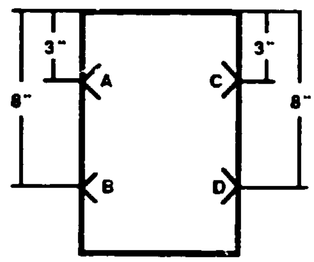
When a product is transferred from a supplier to a customer, internal or external to the enterprise, both parties involved in the transaction have a need to know something about the properties of the items being delivered for purposes of determining payment and disposition or for purposes of determining what adjustments will need to be made in order to use the material received. The information required in the transaction usually refers to characteristics of the product that must meet certain specifications in order to be usable. Specifications have little meaning and are a source of confusion and conflict if there are not operational definitions for the specified characteristics. To have an operational definition for a given characteristic, there should be: (1) a clear statement of what is considered acceptable that is communicable; i.e., can be understood the same way by all parties, (2) an agreed method of evaluating or measuring the characteristic, including equipment to be used and method of selecting the material to be measured, and (3) a criterion for deciding whether requirements are met that will result in a clear and repeatable yes or no decision. Time spent between producer and user constructing operational definitions for important characteristics will usually be repaid many times over by preventing disagreement and uncertainty in subsequent deliveries of product.

Figure 2.5.1 contains an example of a definition constructed for determining whether an 8.5" by 11" sheet of paper meets a specification on width. It should be noted that this definition is neither right nor wrong, it simply represents an agreement between the producer and the user.

Figure 2.5.1
A Definition for a Specified Characteristic

Criterion for determining whether a sheet of paper meets a specification on width of 8.5 ± .001 inches

Choose one long edge of the sheet. Orient it to the left.
Measure down the left edge from the upper left corner with a (type specified) measuring device a distance of 3 inches. Mark the point A.
Measure down the left edge a distance of 8 inches. Mark the point B.
Repeat the marking process for the right edge to obtain points C (3 inches down) and D (8 inches down).

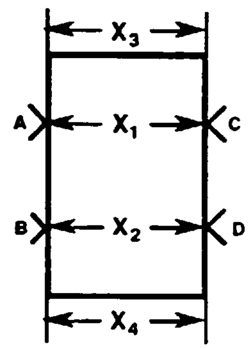


Measure the distance from A to C. Call the result X_1

Measure the distance from B to D. Call the result X_2

Measure the length of the top edge of the sheet. Call the result X_3

Measure the length of the bottom edge of the sheet. Call the result X_4



Criterion:
If all of the measurements (X_1, X_2, X_3, X_4) are numbers between 8.499 and 8.501 inclusive, then the sheet meets the width specification.
If any of the measurements is less than 8.499 or greater than 8.501, then the sheet does not meet the width specification.

2.6 Summary

The discussion of this chapter has focused on introducing some of the basic ideas associated with the subject of measurement. First, the act of making a measurement or performing an evaluation is a process that produces a product - the resulting quantitative or qualitative evaluation. The products of a measurement process are subject to the same considerations of variation and predictability as the products of any other process. Measurements are made for a variety of reasons, including determining disposition of manufactured product, developing and maintaining a history of product or service results, assessing the stability of variation in results of manufacturing or service processes, taking actions to change process or input factors and developing an understanding of the effects of various factors on product or service results. In all of those cases, the results of carrying out a measurement activity constitute data that may be characterized by kind as categorical, counts of events, ranks or ratings, or variables data. The kind of data obtained determines the kinds of methods that may be appropriately used to summarize and analyze the data. Detailed discussions of appropriate methods are provided in following chapters. Some simple descriptive tools for summarizing sets of data were introduced in this chapter, including Pareto diagrams, dot plots, and histograms. There are a variety of other such tools discussed in other statistical texts. One important point to remember when using tools for organizing and summarizing sets of data is that important information associated with the time of production or of measurement may be lost in the process of summarizing. Time ordered sequences of measurements should always be examined for the existence of systematic patterns. Another important point is that data summaries are histories of past results and do not, by themselves, provide sufficient basis for prediction of future results. There are numerous factors associated with the components of a measurement process that may influence the quality of the results and their use. One of the most important of those factors is the existence of a common understanding of how and why measurements are made and a common understanding of the actions and decisions that ought to result from making measurements and analyzing them. In the case of characteristics that are important for use of the product at the next stage, an important factor in developing that common understanding is the creation and use of operational definitions for those important characteristics.

Measurement provides a window through which the dynamics of processes and systems may be observed and studied. Clouding or distorting the information that might be conveyed through measurement is clearly undesirable and unacceptable. To reduce the degree of loss of information, knowledge of measurement process performance over time and knowledge of factors that influence measurement results in terms of their quality and predictability must be developed so that the measurement process can be improved. Methods for developing that knowledge are discussed in later chapters.

Chapter 3

Control Charts for Attributes Data
p and np Charts

As with other charting techniques which support improvement work, the effective use of p or np charts for understanding and improving the processes and systems of an organization requires competency in the application of the statistical methods used in generating and interpreting the charts. However, knowledge of these methods is not sufficient for realizing the type of improvements discussed in chapter 1. If charting techniques are to be effective, a number of other issues and concerns must be addressed prior to and concurrently with the use of these charts. Some of these issues include the intent which exists for developing the control charts, the manner in which prior process knowledge is recorded and made known to the personnel working on the process, the use which should be made of this prior knowledge in developing sampling and subgrouping plans for data collection, and the management practices which are in place to make use of the knowledge gained from the statistical study of the process.

This chapter will not begin by attempting to discuss all of these issues concurrently, but instead will start with the more prosaic aspects of constructing and interpreting p and np charts. With this background in place, a discussion of the issues which should be considered for the effective use of p and np charts will follow. Finally, several case studies which model appropriate practice in the use of these statistical techniques for continual improvement wil! be presented.

3.1 Description of attributes data requiring the use of p or np charts

The information gathered to study the behavior of a process is often obtained by counting the number of items which possess a certain characteristic or attribute. This type of attribute data, as presented in chapter [], is called categorical data. Each item examined is thought of as falling in one of two categories, those that possess the specified characteristic and those that do not. In quality improvement work, these data generally arise by counting the number of items in a collection or subgroup of items which are judged to be nonconforming according to a set of criteria describing the requirements for use. Examples of situations in which attributes data have been generated are:

1. Counts of the number of damaged cans in lots of 1000 cans. Each can was judged as being "damaged" or not.

2. Counts of the number of shafts in samples of 200 whose diameters fail to meet the stated specifications. Each shaft either met or failed to meet the specifications.

3. Daily counts of the number of assembled motors from each day's production which failed to pass a certain voltage test. Each motor was judged as passing or failing the voltage test.

4. Weekly counts of the number of invoices which contain at least one error. Each invoice contained either one or more errors or was correct.

Three possible evaluation objectives may have led to the collection of a set of attributes data. The first objective might be to collect such data simply for information purposes. As an example, a merchandiser may have recorded the number of dented cans in a lot of canned goods purchased from a producer of such food. This information was needed to arrive at the number of dented cans for which the producer will reimburse the merchandiser. The interest in obtaining this information was only to describe the quality of the lot of cans inspected.

A second reason for collecting data on nonconforming items is to build a product or service profile which may be used to describe the product or service characteristics over time. The data in Table 3.1 were collected for this purpose. The information in this table summarizes the performance of automobile engines subjected to extensive testing. At the particular plant where these data were gathered, a set of ten engines are selected from each shift's production and subjected to extensive testing; it is the results of this test which are summarized in Table 3.1. In the test, 47 different characteristics are checked on each engine. If an engine fails on any one of the 47 characteristics, it is recorded as a reject. The numbers in Table 3.1 are the counts of the number of engines rejected in a week (5 days) of production for 21 successive weeks. When attributes data are collected over time, as in the present case, the stability of the variation in the results being summarized by the data can be evaluated. If the variation appears to be stable over time, then there exists a basis for predicting the future performance of the output with regard to this characteristic.

Finally, data on nonconforming items may be generated as an aid to ongoing process or system improvement work. Attributes data are then used to gain insight into the mechanisms of the process or the dynamics of the system which generated the product being studied. In this instance, current knowledge of the process and systems is used to guide why and how data will be collected. Analysis of the attributes data is then used to update the current process/system knowledge as well as to indicate process/system improvements.

3.2 Construction of p charts

When attributes data of the type discussed in section 3.1 have been collected on a process and the information is to be used to gain process performance knowledge, an evaluation of the behavior of the variation in the data over time is necessary. P and np charts are used for making such an evaluation. Chapter [] discussed the usefulness and the importance of evaluating the stability of a process by using control charts. These concepts will be discussed further after the techniques for constructing and evaluating p charts and np charts for categorical data have been explained.

The data in Table 3.1 will be used to illustrate the construction of a p chart. The numbers recorded in this table are counts of the number of engines rejected in a week (5 days) of production for 20 successive weeks. For this set of data there are 20 subgroups, representing the 20 weeks over which the data were collected. Each of these subgroups was formed by counting the number of nonconforming engines in a subgroup containing one hundred engines.

Notation

k will denote the number of subgroups

n will denote the number of items in a subgroup

3.2.1 Plotting the points on a p chart

The construction of a p chart begins by plotting the fraction nonconforming, p, for each subgroup. The fraction nonconforming is calculated for each subgroup by dividing the number of nonconforming items in the subgroup by the number of items contained in the subgroup. An examination of Table 3.1 shows that for the 100 engines tested during the first week, there were six found to be nonconforming. So the fraction nonconforming for this first subgroup is .06. The p values for all 20 subgroups are then plotted on a chart which has the subgroup number along the horizontal axis and the range of fraction nonconforming along the vertical axis. Figure 3.1 was constructed in just this fashion. It is important to note that the data were recorded in the order in time in which they were collected and that this time ordering was maintained when the data were plotted. The value of plotting the data in this fashion will become clear in the discussion of the analysis of p charts.

3.2.2 Calculating the center line for a p chart

The center line on a control chart represents an average value of the data used to construct the chart. In order to determine the center line for a p chart, denoted by $\bar{p}$, the total number of nonconforming items in all subgroups is divided by the total number of items examined in all subgroups being used to establish the center line.

$$\bar{p} = \frac{\text{total number of nonconforming items}}{\text{total number of items inspected}}$$

When the number of items inspected in each subgroup is the same we have:

Total number of items inspected = k x n.

For the data of Table 3.1, there was a total of 131 rejected engines among the 2,100 engines inspected, so that:

$$\bar{p} = \frac{131}{2100} = .0624$$

This value of $\bar{p}$ has been used to position the center line drawn on the p chart in Figure 3.2.

3.2.3 Calculating the control limits for a p chart

The upper and lower control limits define the amount of variation which might be expected to occur in p values collected from a stable process. (The insights provided by this definition will be discussed after the calculation of these limits is illustrated.) The formulas used to calculate the upper and lower control limits are, respectively:

$$UCL_p = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$LCL_p = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

where $\bar{p}$ and n are the average fraction and the subgroup size, respectively. For the data on the engine hot tests the upper control limit is found to be:

$$UCL_p = .0624 + 3\sqrt{\frac{.0624(1-.0624)}{100}} = .1350$$

The calculations for the lower control limit:

$$LCL_p = .0624 - 3\sqrt{\frac{.0624(1-.0624)}{100}}$$

result in a negative number, -.0102. Since a negative fraction of nonconforming items is impossible, the convention of saying that there is no lower control limit will be used in this text. Thus, for the engine testing data:

$$LCL_p - \text{none}$$

Figure 3.3 displays the completed control chart.

3.2.4 Summary

Construction of p charts

k denotes the number of subgroups

n refers to the number of items in a subgroup

1. Each item in a subgroup of n items is classified as either conforming or nonconforming. The number of nonconforming items is recorded for each subgroup.

2. Calculate p (fraction nonconforming) for each of the k subgroups.

$$p = \frac{\text{number of nonconforming items in the subgroup}}{n}$$

3. Calculate the center line on the chart as:

$$\bar{p} = \frac{\text{total number of nonconforming items}}{\text{total number of items inspected}}$$

4. Calculate the control limits as:

$$UCL_p = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$LCL_p = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

3.3 Statistical Control

The p chart constructed from the data on engine tests provides information about the process producing these engines. The first thing to be noticed about this chart is the variation in the values of p. The smallest p plotted is 0.03 and the largest is 0.12. What can be learned about the process from this observed variation? For example, the largest value of p, 0.12, occurred for engines made during the tenth week. Would it be useful to try to identify what was different about the tenth week of operation? The lowest value of p, 0.03, occurred during the fourth week and again during the sixteenth week. Does this value indicate that better work was done during these weeks? Answers to these questions require further concepts. It is necessary to consider what is meant by a stable process, one which is in a state of statistical control, how much variation might be expected in the outcomes of a stable process, and what these ideas mean for interpreting data collected from a process.

A stable process may be thought of as one which is consistently producing the same level of variation over time. The inescapable fact is that the number of nonconforming items produced hour to hour, day to day, and week to week will vary. However, for a given data collection strategy, the amount of variation observed in the fraction of nonconforming items produced by a stable process will be of a predictable size while that from an unstable process will not be. W. E. Deming refers to the causes of variation in a stable process as "common causes;" i.e., causes which are common to all outcomes. An unstable, out-of-control process has special causes of variation acting as well as common causes. Special causes act on some outcomes to produce additional variation in the outcomes. These concepts can be applied to process data in order to provide a basis for understanding causes of variation.

These two types of variation can be illustrated by using the engine test data. Common causes of variation in the values of the fraction of rejected engines would be those causes of a nonconforming engine which are present throughout the time in which the data was gathered. These causes may have as their sources the capability of the equipment used for engine manufacture, the maintenance practices for this equipment, the original design of the engine and alterations to this design, or the methods used throughout the production of the engine. For example, over time the metal from which the engine block is cast will be of varying degrees of hardness from block to block. This variation in the hardness will mean that machining operations will require varying amounts of time to complete. At some times throughout production, adequate machining may not be done because of the difficulty in accommodating the machining equipment to the varying amounts of hardness. Consequently, some engines will at times be improperly machined, possibly resulting in oil leakage or inadequate combustion characteristics. This type of problem might occur at any time throughout the manufacture of the engine and would thus be common to all outcomes.

Another source of variation in the fraction of nonconforming engines being produced might be due to the type of cutting tools or grinding wheels being used in the machining operation. Suppose these tools were being purchased from several suppliers; the type which is used one week may not be the same as the type used in the previous week. Differences in results caused by inadequate tools purchased from one supplier would then not affect all outcomes of the process. At the point in time when an inadequate tool is used, a marked increase in nonconforming engines occurs. This additional variation due to increased levels of nonconforming engines in the weeks that inadequate tools were used would be identified as due to a special cause.

Earlier in the discussion the question was raised as to whether the p value of 0.12 which occurred in week ten was large enough to indicate that a difference existed between the way the engines were being produced at this time as compared to the other points in time at which data were collected. This question could now be rephrased as: "Does the value of p=0.12 differ so much from the other p values recorded that there is an indication that a special cause was affecting the outcomes of the process at this time?"

Studying a process to determine whether or not it is stable is necessary in order to be able to predict the outcomes of that process. If the outcomes of a process have been studied and it has been determined that the process has been behaving in a stable fashion over time, then there exists a basis for predicting that the process may behave in a similar fashion in the future. Conversely, if a process is subject to special as well as common causes of variation, and the special causes are not identified, then prediction of the outcomes of this process is not possible. The effects of these special causes may occur at any time, hampering the ability to predict future outcomes of the process.

The ability to predict the outcomes of a process is an important tool in planning. This statement is made clear by considering the engine example. By being able to predict the proportion of conforming engines produced in a week, the cost of that production can be estimated, the schedule for engine production can be managed in an efficient manner, and the ability to estimate the number of engines which can be supplied to the division of the business assembling the automobiles is known. Certainly, cost estimates and production schedules can and are built without the knowledge of stability of process results. At issue, then, is the improved ability to estimate and schedule with this knowledge. The benefit of this improvement is appreciated by a brief consideration of the effects of, say, poor scheduling. Schedules must necessarily be built with or without the ability to run the process in a stable manner. If there exists a large discrepancy in the schedule and the ability to meet the schedule, ways will be introduced to handle these discrepancies. These may take the form of overtime to complete the schedule, hold-ups at the next stage of production, and negotiated allotments with customers. If these ways of handling poor scheduling ability become too painful, the business may choose to increase the estimated scheduling time of a job. In other words, the lack of confidence in being able to run to schedule, may result in large estimates of schedule time as a way of avoiding the consequences of unmet schedules.

As well as being an important tool for business planning, studying the stability of a process is an important step in a quality improvement program, since work on process improvement requires this kind of process knowledge. The ability to work to improve a process requires not only the engineering knowledge of the process mechanisms, but also the knowledge of the outcomes which result from these process mechanisms. These two aspects of process knowledge, understanding the process mechanisms and understanding their affects on process results, are most usefully used to support each other in process improvement. In the current example of the data on engine testing, little knowledge of the mechanisms which produce engines was used in guiding the acquisition of the data. These data were simply acquired at the end of the production line which produced these engines. Consequently, the ability to use this kind of information to understand the underlying process will be limited. However, the direction in which the analysis of these data would focus future work is an important concept, which can be usefully described with this example.

Suppose, for example, that the information collected from a process indicated that the process was unstable. The first step in working to gain process knowledge would be to identify the special cause(s) acting to create the instability. If a special cause results in increased levels of nonconforming items, what should be done to remove this cause? If the special cause is acting to reduce the level of nonconforming items produced, what can be done to insure that the special cause becomes part of the standard operating procedure for the process? In the context of the engine test example, if it was determined that a value of p=0.12 differed so much from the other p values that a special cause appeared to be operating, work on the process would entail first identifying what that special cause was and then working to eliminate the cause. In addition, suppose it was determined that a value of p=0.03 was so much smaller than the other p values recorded that a special cause must be acting to reduce the level of nonconforming engines. Work on the process would then proceed by attempting to make the special cause a part of the ongoing operation of the process.

Alternatively, a conclusion that a process is stable and subject to only common causes leads to the recognition that only process changes which act on one or more common causes will result in improvement. In order to improve a stable process, it will be necessary to collect information on the process in a manner which will allow the impact of common causes to be studied. It is at this point that it becomes evident why the data collected on engines would most likely be insufficient to support process improvement. The data as collected provide little insight into the underlying causes of variation in the production process. Knowledge of the process which would make that process accessible to study would need to have been developed in order to try to understand and impact the common causes of variation.

Distinguishing between common and special causes is important in order to know what kinds of actions would be appropriate to take to improve the process; in addition, it is also an important distinction to make in order to avoid actions which would be inappropriate. Without knowing whether or not the p value of 0.12 indicated a special cause was operating, one might proceed as if there were an indication of a special cause. If the process were actually in control, it is doubtful that any actions taken could be effective in improving the process. At worst, such actions could, by overadjustment, produce more variation in the outcomes of the process.

The completed p chart in Figure 3.3 shows that none of the points plotted fall outside of the control limits. Since the control limits on the p chart indicate the amount of allowable variation in the plotted points if the process is stable, the conclusion reached from examining this chart is that there is no evidence of instability in the process, in other words, that the process appears to be in statistical control.

3.4 Collection of Attribute Data to Support Process Study

In a plant which produces single serving, frozen meat pies, called pot pies, past evaluation of the process producing the pie crusts showed that a large number of these crusts were being scrapped after they had been put into metal trays, but before the filling had been added. The data used to evaluate the performance of the line producing bottom crusts had been obtained from the inspection station at the end of the line which produces the bottom crusts. Current practice was to inspect 100% of the bottom crusts produced in order to remove from the production line any crusts which had obvious breaks in the dough or which did not completely cover the metal tray which contains the crust. The line producing bottom crusts was run for two shifts each day. Analysis of the data gathered from these two shifts indicated that the process was currently in control and producing about 18% nonconforming crusts.

The management group which met to begin considering ways to improve the pot pie production process did not agree that the large number of nonconforming crusts being produced constituted a "quality" problem. Some members of the group believed that there did not exist a quality problem, since the crusts are 100% inspected and any nonconforming crusts found are removed from the production line and the dough from these crusts is reused in the process. However, other members of the group held that the plants inability to produce good crusts without rework resulted in additional expense, since machine and operator time was used to produce unusable product. Further, they pointed out that other characteristics, such as taste and texture of the crust, were deteriorated by the process of reworking the dough and so rework should not be considered an appropriate solution.

To support the work on the pie crust line, the management decided to gather together the current process knowledge. Figure 3.4 is the flow chart of the pot pie production and Figure 3.5 provides a more detailed description of the portion of the production process where the bottom crusts are formed. In addition, two members of the management group met with several of the operators of the pot pie line to create the cause and effect diagram which is displayed in Figure 3.6. This cause and effect diagram led to a lengthy discussion about which causes listed were actually active in producing nonconforming pie crusts. It was decided at this juncture that some of the differing opinions about the reasons for nonconforming crusts was that the 18% figure on nonconforming crusts provided incomplete information about the current capability of the pot pie line. In particular, there was little knowledge about whether there were more nonconforming crusts on some days rather than others. If this were true, than the group would investigate more closely those causes which were present on some days but not others. Further, the group had no knowledge about whether the number of nonconforming crusts was larger at some times of the day rather than at other times. Again, knowledge of the behavior of the process during a day would provide direction as to what might and might not be a factor in causing nonconforming crusts. The point was also made during these discussions that a more thorough knowledge of how the process was currently operating would also provide a baseline by which to judge future improvements in the process.

In order to better understand the process, the management group decided to collect information on the number of crusts discarded throughout a days operation. The flow chart in Figure 3.5 describes the current operation of the process and illustrates where in the process data collection was performed. The process of producing the pie crusts begins with the dough being extruded from the vat in which it is mixed into a thick sheet of dough. From this point the dough is pulled into the forming machine, which rolls the dough into a thin sheet. The dough is cut into wide strips by the forming machine; the strip then covers 24 metal trays. The dough is pressed into the pie plates and excess dough is trimmed. The forming of a group of 24 crusts is referred to as a cycle, and the operator who runs the forming machine controls the time at which the cycles occur. This same operator examines the pie crusts after the forming operation to see that there are no visible breaks in the dough and that all metal trays are completely covered by dough. It is the large number of crusts which are discarded at this point of the operation which has caused concern. Therefore, it was decided to collect information on nonconforming crusts at this point of the process.

3.5. Construction of np charts

In the study of the current behavior of the pot pie line, the management group decided to collect information throughout the day and over several days time on the number of nonconforming crusts being produced. To this end it was decided to have the operator who was inspecting the crusts record the number of nonconforming crusts produced in four consecutive cycles of the machine forming the crusts. The number of nonconforming crusts produced in these four cycles was to be recorded. The operation of this line only occurs on one shift, (the day shift) each day. The operator of this line was instructed to count and record the number of nonconforming crusts produced by the four consecutive cycles of the machine once each hour. This data collection was carried out for one week (five working days.) These data have been recorded in Table 3.2.

A p chart could be used to evaluate the information contained in these data. However, in this case the subgroup size was 96. (It should be recalled that each subgroup was formed by counting the number of nonconforming crusts in four cycles of 24 crusts.) The management group decided it would be simpler to plot the number of nonconforming crusts, rather than the fraction nonconforming. When the number of nonconforming items is to be plotted on a chart, then an np chart is the appropriate technique for analyzing the information on the plot. It is important to notice that the information contained on the np chart is just the same as if the choice had been to construct a p chart with the data. The decision as to whether to use a p chart or an np chart when, as in the present case, each subgroup contains the same number of items examined, is merely a matter of convenience.

3.5.1 Plotting the points on an np chart

Table 3.2 contains information on the time of inspection, the number in a subgroup, and the number of nonconforming crusts in a subgroup. On an np chart, it is the number of nonconforming items which are plotted. These values have been plotted in time order on the chart in Figure 3.7. Again, the subgroup numbers appear along the horizontal axis and now the range of the number of nonconforming appears along the vertical axis. The analysis of the information contained in this plot about the pot pie line proceeds in the same fashion as for a p chart. A center line is calculated and then upper and lower control limits are calculated in order to describe the amount of variation which should occur in the plotted points if the data were obtained from a stable process.

3.5.2 Calculating the center line and control limits for an np chart

On an np chart, the value used to the draw the center line is the average number of nonconforming items in a subgroup. This value, denoted by $n\bar{p}$, is the total number of nonconforming items in all subgroups divided by the number of subgroups.

$$n\bar{p} = \frac{\text{total number of nonconforming items}}{\text{number of subgroups}}$$

For the data of Table 3.2, there was a total of 579 nonconforming crusts in the 35 subgroups, so that:

$$n\bar{p} = \frac{579}{35} = 16.5429$$

This value of $n\bar{p}$ has been used to position the center line in Figure 3.8.

The calculation of the upper and lower control limits for an np chart require that the value of $\bar{p}$ be calculated. This value is, of course, calculated just as described in Section 3.2.2. Alternatively, $\bar{p}$ could be calculated by dividing $n\bar{p}$ by n, the subgroup size.

$$\bar{p} = \frac{16.5429}{96} = .1723$$

The upper (UCL) and lower (LCL) control limit are then calculated as follows:

$$UCL_{np} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$LCL_{np} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$$

For the data in Table 3.2, the upper and lower control limits are:

$$UCL_{np} = 16.5429 + 3\sqrt{16.5429(1-.1723)} = 27.6439$$

$$LCL_{np} = 16.5429 - 3\sqrt{16.5429(1-.1723)} = 5.4419$$

These values have been used to draw the upper and lower control limits in Figure 3.8. The completed chart shows no indication of a special cause operating. From the available data, it appears that hour-to-hour throughout the week studied, that the proportion of nonconforming pie crusts appears to be stable with about 17% nonconforming crusts being produced.

The summary of the results of the np chart at first appears to provide the same information as that contained in the initial statement about the pot pie line, that it was presently producing about 18% nonconforming. But there is additional guidance to be gained from the np chart. In order to begin improving on the current capabilities of the pot pie line, a direction for study of the line needs to be set. The np chart can be used to indicate which direction might be most fruitful. For example, the points on the chart were collected hour-to-hour over a week's period; and the indication from the chart is that the process is stable when judged in this fashion. Consequently, those sources identified on the cause and effect diagram which would act to create differences between days or between hours might not provide the more useful areas for study at this juncture. Since the chart reports that hour-to-hour and day-to-day the process is behaving consistently, there is no evidence that a special cause is acting at some hours and not at others. This knowledge provides the basis for the judgment that working to detect why some hours or days differ from others would not be useful. Alternatively, those causes on the chart which might be thought to affect nonconforming crusts throughout an hour need to be further explored. The cause and effect diagram might be used to aid in identifying which causes might be acting consistently within an hour to produce nonconforming crusts.

3.6 Selecting the Subgroup

In the study of the pie crusts, each subgroup on the np charts consisted of four groups of 24 crusts. These four groups came from four consecutive cycles of the machine which formed 24 pie crusts at each operation. The 35 subgroups on the chart in Figure 3.8 were collected at various points in time throughout one day. By selecting the subgroups in this fashion, any changes in the process which were detected on the control chart as a special cause would be changes that occurred between subgroups. On the other hand, any causes which were acting during the time of collection of a single subgroup would not be indicated as a special cause on the control chart. Thus, the manner in which the subgroups are selected determines those causes of change in number of nonconforming items which can be detected as special causes on a control chart. What is learned about the operation of the process from a set of data collected from the process depends on how the data were gathered. The next stage in the study of the pie crust process illustrates this idea.

One of the possible causes identified on the cause and effect diagram for nonconforming pie crusts was that tins occupying an end position when the dough was placed over the tins were more likely to be defective for the following reasons: the dough was too thin at this location, or the dough was not properly rolled out and so didn't cover the pie tins completely, or the dough at the ends dried more quickly than the dough which covered the middle positions. If rejected pie tins are associated with position, then it would be expected that different proportions of tins are rejected at the two positions. Two questions then arise:

1. Are the number of nonconforming crusts formed at the two locations consistent (or stable) over time, and

2. How do the number of rejected crusts at the two locations compare?

The previous subgrouping strategy was not designed to provide answers to these questions as each subgroup contained both middle and end crusts. In order to determine if the end positions did, indeed, produce a higher proportion of defective crusts, a different subgrouping strategy was devised. The subgroups were selected so that some subgroups contained only "end" crusts and some contained just "middle" crusts.

Collecting data using this method required a much larger investment of time, since each crust had to be identified by its location in addition to noting whether it was nonconforming. Since the line was not going to be stopped while this data collection activity was proceeding, the data were not entered on a chart as it was gathered but rather recorded on the type of form shown in

Figure 3.9. The circles on the form correspond to the 24 locations of the pie tins when they are being covered with dough. The person collecting the data would then make an "X" on the form to indicate which location contained a nonconforming pie crust. As before, this information was recorded on four consecutive runs every half hour. The first set of completed forms are shown in Figure 3.10. However, unlike the first set of data collected on the line the data were now summarized by position. The places which have been labeled 1 through 4 and 21 through 24 were considered to be the end positions. The crusts in these eight positions in the first four consecutive runs formed the first subgroup. The number of nonconforming crusts in this first subgroup is given in Table 3.3. The sample size for this subgroup is n = 4 x 8 = 32. The second subgroup consists of the remaining 4 x 16 = 64 "middle" crusts from the same four consecutive operations from which the first subgroup was collected. The remaining data in Table 3.3 were recorded in a like fashion. The odd numbered subgroups were formed from end crusts and have subgroup sizes of 32. The even numbered subgroups were formed from the "middle" crusts and have subgroup sizes of 64. The 40 subgroups were collected over one shift of operation. Because the subgroups in these data do not all have a common value for n, the subgroup size, a control chart for varying n will be constructed for the data.

3.7 Construction of p charts when n varies

If a set of data on nonconforming items has varying subgroup sizes then the appropriate chart for analyzing these data is a p chart. Plotting the points and drawing the center line on the chart is done just as it was for a p chart when all subgroup sizes were the same. For the data in Table 3.3, the values for fraction nonconforming, p, have been plotted by subgroup on the chart in Figure 3.11. The center line has also been drawn on the chart. Just as described in section 3.2.2, the center line, , was calculated by dividing the total number of nonconforming items in all subgroups by the total number of items examined. In the twenty subgroups with 32 crusts there were 208 nonconforming crusts and in the twenty subgroups with 64 crusts there were 114 nonconforming crusts, thus:

$$\bar{p} = \frac{208+114}{(32x20+64x20)} = .1677$$

The formulae for upper and lower control limits for p charts which appeared in section 3.2.3 were a function of n, the subgroup size. A way of interpreting this information would be to say that the amount of expected variation in fraction nonconforming measured on a stable process depends on the number of items in a subgroup. Thus, when the subgroup sizes in a collection of data are different, such as in the data in Table 3.3, it is not appropriate to use the same control limits to assess the variation in the fraction nonconforming. Instead, the control limits used for each p plotted will depend on the number of items which were used to calculate that value of p. Thus for those points which were based on 32 crusts, the upper and lower control limits will be:

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$UCL = .1677 + 3\sqrt{\frac{.1677(1-.1677)}{32}}$$

$$= .1981$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$LCL = .1677 - 3\sqrt{\frac{.1677(1-.1677)}{32}}$$

LCL - none

For those points which were calculated as the fraction nonconforming in 64 crusts, the upper and lower control limits will be:

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$UCL = .1677 + 3\sqrt{\frac{.1677(1-.1677)}{64}}$$

$$= .3081$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$LCL = .1677 - 3\sqrt{\frac{.1677(1-.1677)}{64}}$$

$$= .0276$$

These limits have been used to construct the p chart in Figure 3.12. The odd-numbered subgroups had subgroup sizes of 32, thus the upper control limit for these points is .3658 with no lower control limit. The remaining points had a subgroup size of 64. The control limits for these points are .3081 and .0276.

The chart in Figure 3.12 shows several points above the upper control limit. These correspond to subgroups 19, 23, 27, 31, 35, 37, and 39. In addition, the p value for subgroup 20 is below the lower control limit. Since all of the points above the upper control limit are for subgroups of "end" crusts, it appears that "end" crusts are inconsistent when evaluated against "middle" crusts, or, in other words that the "end" crusts tend to have larger nonconforming rates then "middle" crusts. The strategy used for detecting this difference is worth reviewing, as it provides a good introduction to using control charts for process study. A possible source of variation in nonconforming crusts was identified as due to tin position. In order to investigate this suspicion a subgrouping strategy was used which consisted of "end" crusts in some subgroups and "middle" crusts in others. When the points associated with the "end" crusts appear to be out of control then there is evidence to believe that the special cause acting is in some fashion connected with "end" crusts.

3.8 Detecting Non-random Patterns on Control Charts

In the previous section a control chart which contained subgroups of both "end" and "middle" crusts was constructed. Since the chart indicated a higher nonconforming rate among "end" crusts, it is necessary to construct separate control charts for the two types of crusts. Clearly, future work to improve the crust formation will involve understanding why more nonconforming crusts are produced on the ends rather than at the middle. Yet how this work proceeds should depend on whether the number of nonconforming crusts when considered separately by type are stable over time or not. If, for example, the number of nonconforming "end" crusts are out-of-control, then work would proceed by attempting to identify those sources which may be present at some times and not others and thus result in inconsistent levels of nonconforming crusts. On the other hand, if the process producing "end" crusts appears stable, work would proceed in a different fashion. Then it would be necessary to consider how to discover and study the effects of those common cause sources which are present throughout the production of "end" crusts.

Figure 3.13 and 3.14 contain p charts of the fraction nonconforming "end" crusts and "middle" crusts, respectively. It is left to the reader to verify that the center line and control limits on these charts are correct. The behavior of the chart in Figure 3.13 should be noted. It appears as though the number of nonconforming "end" crusts is increasing over time since there are many points below the center line on the left of the chart and more above the line on the right side which corresponds to later time periods. The runs test referred to in Chapter [] should be used at this stage to make a decision about whether this suspicious behavior is evidence of nonrandom behavior. Since there is a run of nine points (points 1 through nine, in fact) below the center line, there appears to be a tendency for the number of nonconforming "end" crusts to increase over time. Efforts should now be made to attempt to identify the cause or causes which are acting to produce the effect of an increasing trend in the number of nonconforming pie crusts. Of course, nothing on the chart indicates why the perceived systematic pattern occurred. The ability, or "luck," in identifying such causes will in part depend on the observation powers of manager, technical people, and operators and their ability to correlate these observations with how the process work is performed.

3.9 Studying Cause and Effect Relationships

The ability to detect special causes is enhanced by having more than just operational level people performing data collection as well as by collecting additional information from the process. Noting on the chart such things as changes in material, changes in operators, line shut downs, etc. may provide some guidance in searching for special causes. Of course such possible causes will only be noted if thought is given before the data is collected to those process factors which might have some effect on the output. Although development of a complete list of possible causes for variation or unacceptable results prior to data collection may be unlikely, the exercise of doing preliminary cause and effect analysis will markedly increase the chance of successfully identifying causes. The collective knowledge of all people who work in some way with the process is invaluable in the attempt to identify causes.

Previous work on the pie crust process had suggested some possible causes for the gradual increase in the number of nonconforming "end" crusts throughout the day on which the above data were collected. Two of these suggested causes were:

1. Changing environmental conditions. At the start of the day, the moisture level in the dough is set by adjusting the rates at which dry and liquid ingredients flow into the mixing vat. The moisture level is thought to be critical as dough that is too dry will tear easily. However, as the temperature and humidity in the plant change throughout the day, the set-up rates may not maintain the moisture level appropriately. Other data sets, collected daily, might provide information on this speculation.

2. Recycling of excess and scrap dough. The operation of cutting the dough to fit the tins in which they're molded generates a certain amount of scrap dough. Current practice has this dough being returned to the mixture from which other pie crusts are then rolled. Additionally, dough from nonconforming pie crusts is also returned to the mixture. There was some concern that this practice might cause deterioration of the dough throughout the day, resulting in increased numbers of nonconforming "end" crusts.

Although these two causes were identified as possible reasons for the trend in nonconforming pie crusts, it was not clear whether either one of them was actually acting to produce the trend noted. Since the next step in the work to understand the process would be to study the effect of these causes, additional time could usefully be spent trying to identify other possible reasons for the observed increase in nonconforming crusts. Armed with the knowledge of the existence of a trend, other possible causes may be suggested. In fact, identifying additional possible causes may help in the study of ones already identified. For example, it might be decided to study the effect of recycling of dough by discontinuing this practice and watching to see whether the trend persists. It might then be the case that no trend is observed and this change is attributed to the discontinuing of the recycling. However, if on the days when the study was conducted, the ambient temperature and humidity stayed fairly constant, then the conclusion that the change in the process was due to discontinuing the recycling is now brought into question. Studying the cessation of recycling under a variety of temperature and humidity conditions is now indicated. Thus, identifying other possible causes for the trend would be useful for identifying those sets of conditions over which the process should be observed in order to better understand the process.

The approach which the team studying the pie crust line took to study the trend in nonconforming "end" crusts was to gather several days of data from the line before the recycling was stopped. Although the environmental conditions were not controlled, the changes in the temperature and humidity were noted throughout the day. These same conditions were also noted on several successive days on which the recycling was discontinued. The data on "end" crusts in Table 3.4 were gathered on one of the days on which the recycling was discontinued. (The data came from the same pie crust line and was collected in the same fashion as the data in Table 3.3.) Although the information is not reproduced here, the temperature and humidity were judged to change in a fashion similar to the earlier days when a trend in the process was again noted. Figure 3.15 contains an np chart of the data from Table 3.4. It is left for the reader to check that the center line and control limits given in this chart are correct. In addition, the reader will want to verify the fact that an application of the runs tests indicates that there is no evidence of a trend in these data.

The fact that no trend was noted on the chart in Figure 3.15 supports the assumption that the recycling of the dough causes an increase in the number of nonconforming "end" crusts. This result raises the question about what can and should be done with this information. Two issues should be raised at this point:

Issue 1: What kind of confirmation work needs to be done on this recycling study?

The issue of performing confirmation studies is important not only in the present context, but is a recurring issue in much of the work directed at continual improvement of a process. Confirmation studies are studies which are performed repeatedly under a variety of conditions to determine if the improvements noted in earlier studies are maintained under a variety of conditions. Too often an effect which is noted at one point in time, such as the reduction in nonconforming crusts when recycling is halted, may not reoccur in later studies. It sometimes happens that the increased focus on a particular area results in improvements in that area, improvements which may not be sustained after attention is directed somewhere else. These improvements may be incorrectly attributed to some change made to the process rather than to the increased attention. This type of incorrect attribution may be avoided by performing confirmation studies. In addition, it was earlier mentioned that other causes may be acting on the pie crust line to cause a trend in the number of nonconforming "end" crusts. If all such causes are not understood, and surely they won't be, then the improvement noted may have been due to one of these less understood causes. Just as confirmation studies will help detect improvements that come simply from increased attention to a process, they will also be useful in determining if an improvement occurred for some reason other than the one originally proposed. If confirmation studies conducted under a variety of conditions don't show that the same level of improvement can be maintained by the proposed change, then additional work to identify other causes which may be acting would be indicated. In the present example on the pie crust line, confirmation studies might consist of continuing to collect data over a larger span of time which should include a greater variety of operating conditions as well as performing similar studies on other shifts and other lines to determine if the recycling is actually responsible for an increasing level of nonconforming pie crusts.

Issue 2: What kind of process improvements are indicated in view of the fact that stopping the recycling meant that the number of nonconforming crusts did not increase throughout the shift observed?

The answer to the above question will depend on the further study of the process which is now indicated. Some of the points to be considered would be:

What is there about the returned dough that contributes to the problem?

How much dough is currently being recycled and at what rate is it recycled?

Why was recycling performed? Was it a cost saving measure or a way of improving the scheduling of the operation?

Can the recycled dough be reconstituted?

What are the effects of recycling on other dough characteristics such as taste and texture of the dough?

Would the same type of improvement in the process occur if the amount of recycling were reduced but not stopped?

The above questions ask about technical process knowledge, the current operating procedures of the process, and the basis for the decisions made about the management of the process. The wide range of technical expertise and management responsibility necessary to direct and evaluate the work on these questions demands that management lead these efforts.

3.10 Supporting ideas for the effective use of p and np charts

The ideas discussed in this session are intended to provide the reader with some thoughts on how the use of charts, and p and np charts in particular, can be most effectively integrated into a program for continually improving the activities of a business system. These ideas should be read as suggestions for serious consideration, rather than as rules not to be broken. Clearly there will be situations where the suggestions can not or should not be applied. However, it is vital that the practioner go through the exercise of understanding as much as possible about a process before collecting data and in making appropriate decisions about how information should be collected on the process. Thus, the practioner will have a clear idea of the value and the limitations of the information gained from the use of charts for process study.

Idea 1: A process flow diagram should be in place.

Before a set of data is collected for process study, the process needs to be understood in terms of how it affects and is affected by the larger processes and systems of which the process under study is a part. This activity is particularly important for those at management levels who set the priority for the process study. Different alternatives for process improvement may result from the investigation and the process flow diagram will aid in the determination of whether proposed improvements are consistent with other priorities. Furthermore, potential process improvements will have to be evaluated in terms of their impact on other aspects of the process, as it is necessary not to introduce larger problems in other aspects of the process as a result of any currently proposed improvement.

In view of the above comments about the role of the process flow diagram, it should be clear that a diagram which simply lists the order in which operations are intended to occur in the process is insufficient. The flow chart should describe how the process currently operates. Even if a flow chart exists when attention is being focused on the process, the chart will most likely need to be revised and updated.

Idea 2: An operational definition of the specifications, characteristics, or other attributes under study should be in place.

A number of benefits accrue if managers, at appropriate levels, are involved in selecting and defining those characteristics to be measured. One such benefit is that the value of studying these characteristics exists because management has set the priority for studying them based on known information about attributes necessary to deliver customer value. The team working on the process is also strengthened when members understand why the operational definition and its consistent application is important. In addition, the validity of the measurements and the reliability of outcome descriptions, conclusions, and recommendations are all strengthened if the management group has thought far enough ahead to assure themselves that training in these matters has been addressed.

Procedures should be in place to update or revise the definitions and specification as product is changed to meet different customer requirements. As customer response, in terms of changing expectations or better information, is evaluated, the need to revise definitions and standards should be considered and steps taken to assure that operational definitions are revised to reflect these changes. Operational definitions will need to be checked and tested on a regular basis and verification provided that inspection is being properly and consistently performed by the different shifts and departments involved.

Proper attention to operational definitions is vital. If inspection procedures are inconsistently applied or if records are inexact or incomplete, then nothing useful can come from the application of a p chart.

Idea 3: Plans for using the various types of information gained from a p chart should be in place.

The management and supervisory direction necessary to work on identifying, understanding, and preventing special causes must be in place. Without this direction, it will almost surely be the case that fundamental, or root, causes do not get addressed. "Band-aid" problem fixing, based upon restricted resources and, perhaps limited understanding of the technological aspects of the special causes are then the likely result. Management will need to put in place plans for systematic study of the process. For example, a suspect list of "special causes" could be built and effort directed towards understanding these. Individuals to be involved in the work should also be identified by management. In selecting these individuals management will need to consider what kinds of authority, responsibility, skills, and knowledge will be needed. In addition, the types of problems which may be identified by this work and the selection and implementation of solutions will require that departmental and functional representation be considered in selecting the individuals to be involved in the work.

The importance of the above advice regarding management's involvement becomes crucial when one considers the fact that what may be identified as a "special" cause at a micro level in the process is actually a common cause in a larger, more complicated management of engineering system. For example, a special cause may be identified as a "maintenance" problem when, in fact, there is a system issue which limits correct and consistent maintenance practices in various ways. As another example, an untrained operator or supervisor indicates a larger training and educational issue.

Other elements of a plan for process study should consider the cause-and-effect relationships of the characteristic under study. Since the purpose of the work is to improve the process, bringing a process into stable operation is only the first step. Subgroup formation and sample frequency need to be considered in light of the intent to understand cause-and-effect relationships. This approach will require knowledge as to how a p-chart is used to delineate the variation within and between subgroups. This knowledge will need to be correlated with insight into how a process operates and with what sources of variation the work team wants or needs to address.

Idea 4: P-charts will be most effective for process study if fraction nonconforming for only one specification, standard, or nonconformity is included on a chart.

A fundamental mistake is often made in resource allocation and expenditure when managers or work teams proceed by reporting on numerous specifications or characteristics on one chart. In terms of time, effort and other resources, data collection and basic "charting" is by far the least expensive. Problem identification, determination of causes, and testing and implementing solutions is the more greedy absorber of resources. In an attempt to make life easier, managers will often direct the construction of a few charts with multiple characteristics on a single chart. However, when many types of nonconforming attributes are included on a single chart, the usefulness of the chart for gaining process knowledge is severely limited. Increasing of decreasing trends in the fraction nonconforming for one characteristic may be disguised or offset by contrary movements in other other characteristics. Furthermore, values for fraction nonconforming indicating the presence of special causes are not easily understood or interpreted in terms of the process because the value itself does not provide a reason for the rejection in that subgroup or subgroups. Numerous common cause systems and various types of special causes may be active but the chart will provide little information about any one of these. With multiple characteristics on one chart, the time spent on inspection and data collection efforts will most likely be wasted since the form of data presentation does not aid in problem identification, hides problems, and disguises opportunities to learn about underlying common and special cause systems.

There are surely exceptions to the above policy. For example, before one can begin to select opportunities for improving process or product, one must have an idea of what typical problems might be occurring and with what relative frequency they occur. A p-chart which includes all types of nonconforming attributes may be constructed to report on the current state of a process. A Pareto chart may be used to support this initial investigation in order to communicate some of the larger problems. Of course judgment will have to be used in the priorities which may be set from this work. The severity of respective problems will have to be questioned, their impact considered from the customer's viewpoint, and their possible long run nature evaluated.

Idea 5: Where technically and economically feasible, inspection for the purposes of process and product improvement should be converted to a variables format.

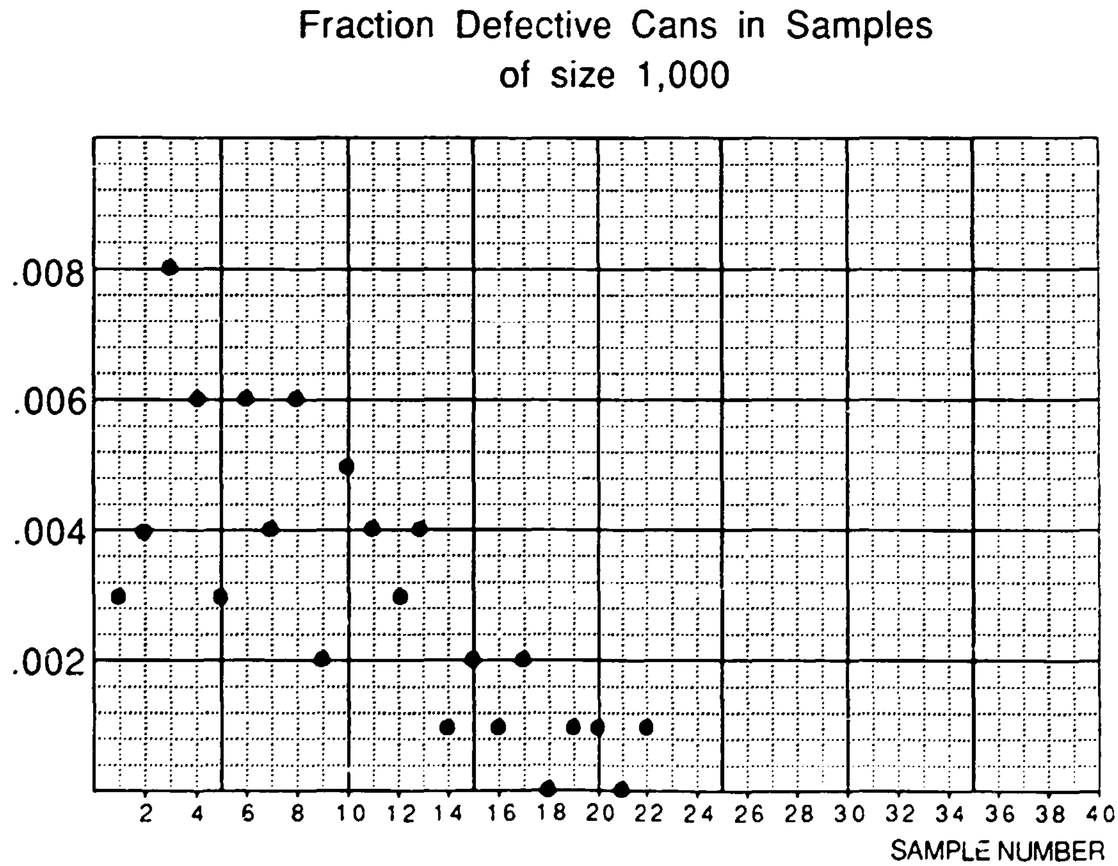
Sometimes work on quality and productivity depends on making a measurement, comparing it to specifications, and then recording "conforming" or "nonconforming" as the response. In such instances valuable information is discarded. Although the p chart does provide information about an inability to meet specifications, the information is too vague for work on process improvement; there is no clue as to whether the problem is in too much variation, an incorrect average, or in both characteristics. Evaluating process performance will require working on the sources of variation in the measurements, information which is not available when only conformance to specifications is recorded.

1. At a food packaging plant, a recent increase in the number of customer claims concerning defective cans has prompted your investigation into the situation. An initial concern was whether or not the inspection process is stable and predictable. You have been informed that operational definitions have been developed and an inspection training program has been implemented in the recent past. Assuming, therefore, that the inspection process is consistent, your attention turned to the packaging process. Knowing that the sealed metal containers pass through a final inspection, where they are checked for proper can height, label application, vacuum, and other surface characteristics, you have requested data on the proportion of defective cans found in final inspection. You are provided with the following:

FRACTIONS OF DEFECTS IN 22 SAMPLES OF 1000 CANS EACH
(Total # of Defective Units = 67)

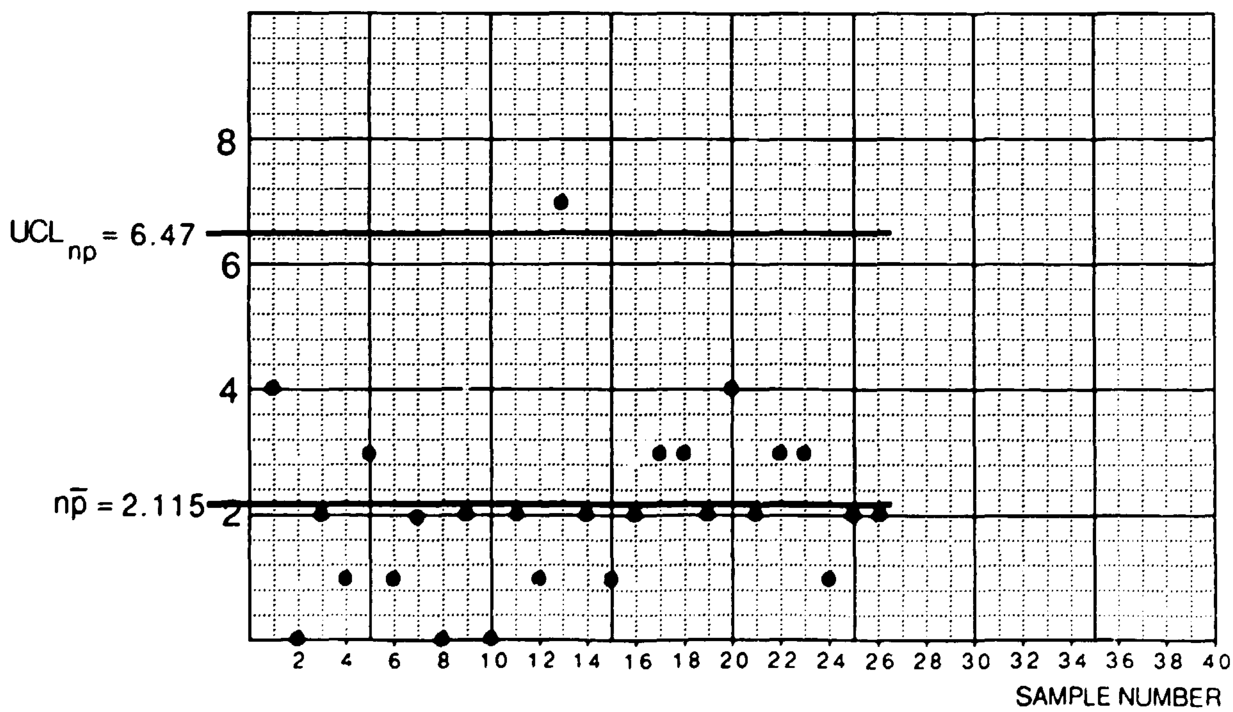
.003	.004	.008	.006	.003	.006	.004	.006	.002
.005	.004	.003	.004	.001	.002	.001	.002	.000
.001	.001	.000	.001					

a. Construct an appropriate control chart for the data.
b. Comment on the manner in which these data were obtained.
c. In order to determine the current status of the process and to analyze the possible causes for the increase in defective cans, what will be your next course of action?



2. The Shift 2 supervisor has expressed his belief that the number of defective cans are higher in Shift 1 than in his own shift. In order to check this claim, you request that, on each of 26 successive shifts, 700 cans are to be inspected, and the number of defective cans in each sample recorded. You are later presented with the following data and control chart:

Sample	Shift	# Defective	Sample	Shift	# Defective
1	1	4	14	2	2
2	2	0	15	1	1
3	1	2	16	2	2
4	2	1	17	1	3
5	1	3	18	2	3
6	2	1	19	1	2
7	1	2	20	2	4
8	2	0	21	1	2
9	1	2	22	2	3
10	2	0	23	1	3
11	1	2	24	2	1
12	2	1	25	1	2
13	1	7	26	2	2



2 con't.

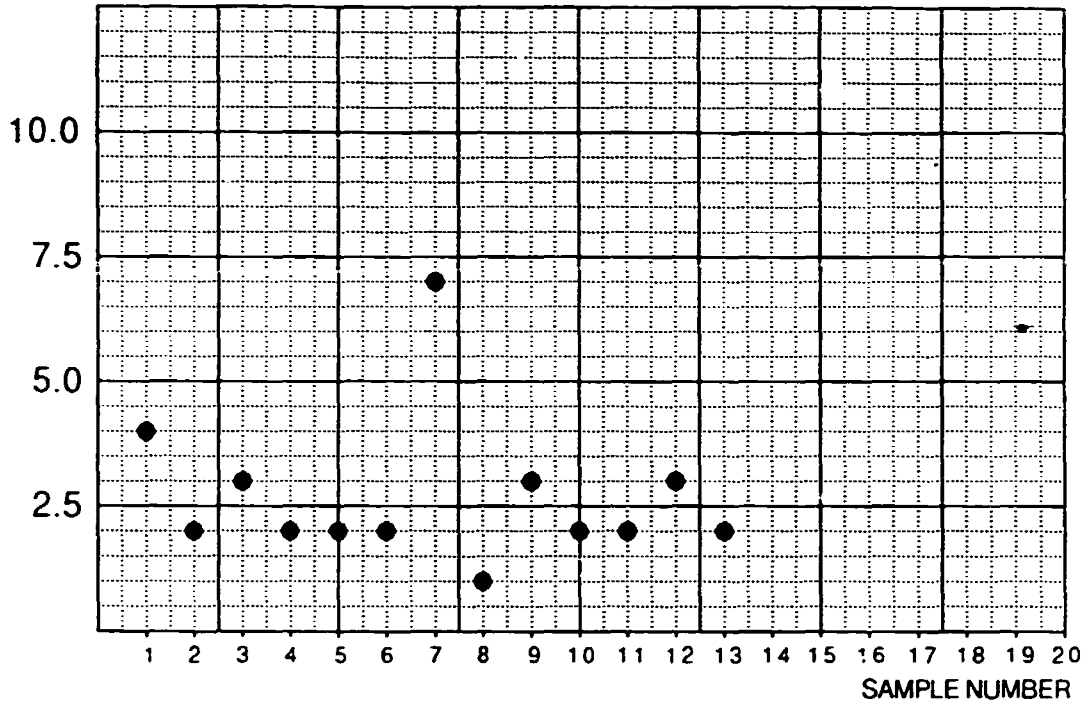
a. Based on your analysis of this chart, what would your conclusions be concerning the stability of the process?

b. Construct separate sets of control charts for the 2 different shifts.

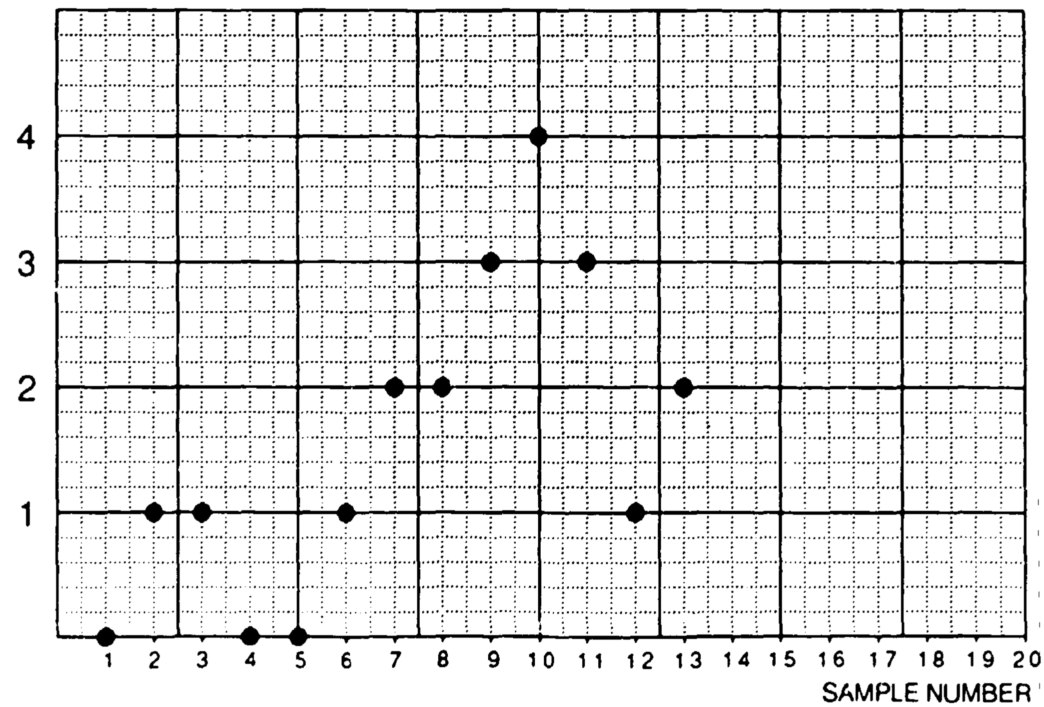
c. As a manager, how would you respond to the Shift 2 supervisor's claim?

d. Are there any reasons that make you skeptical about the data and, consequently, about the information provided in these control charts?

Number of Defective Cans - Shift 1
(Sample Size = 700)



Number of Defective Cans - Shift 2
(Sample Size = 700)



3. In an assessment services company, customized assessment instruments and plans are developed. Of great concern to the project leaders is the large proportion of rework required due to customer complaints. 16 projects are examined each week for 25 weeks. The number of projects in each week which required rework are:

Week	Number of Projects Requiring Rework	Week	Number of Projects Requiring Rework
1	1	14	2
2	3	15	3
3	5	16	5
4	0	17	0
5	6	18	3
6	2	19	4
7	4	20	5
8	1	21	6
9	0	22	2
10	3	23	3
11	5	24	1
12	4	25	2
13	4	Total	74

a. If a p-chart were constructed from these data, what would be the centerline and control limits for the chart?

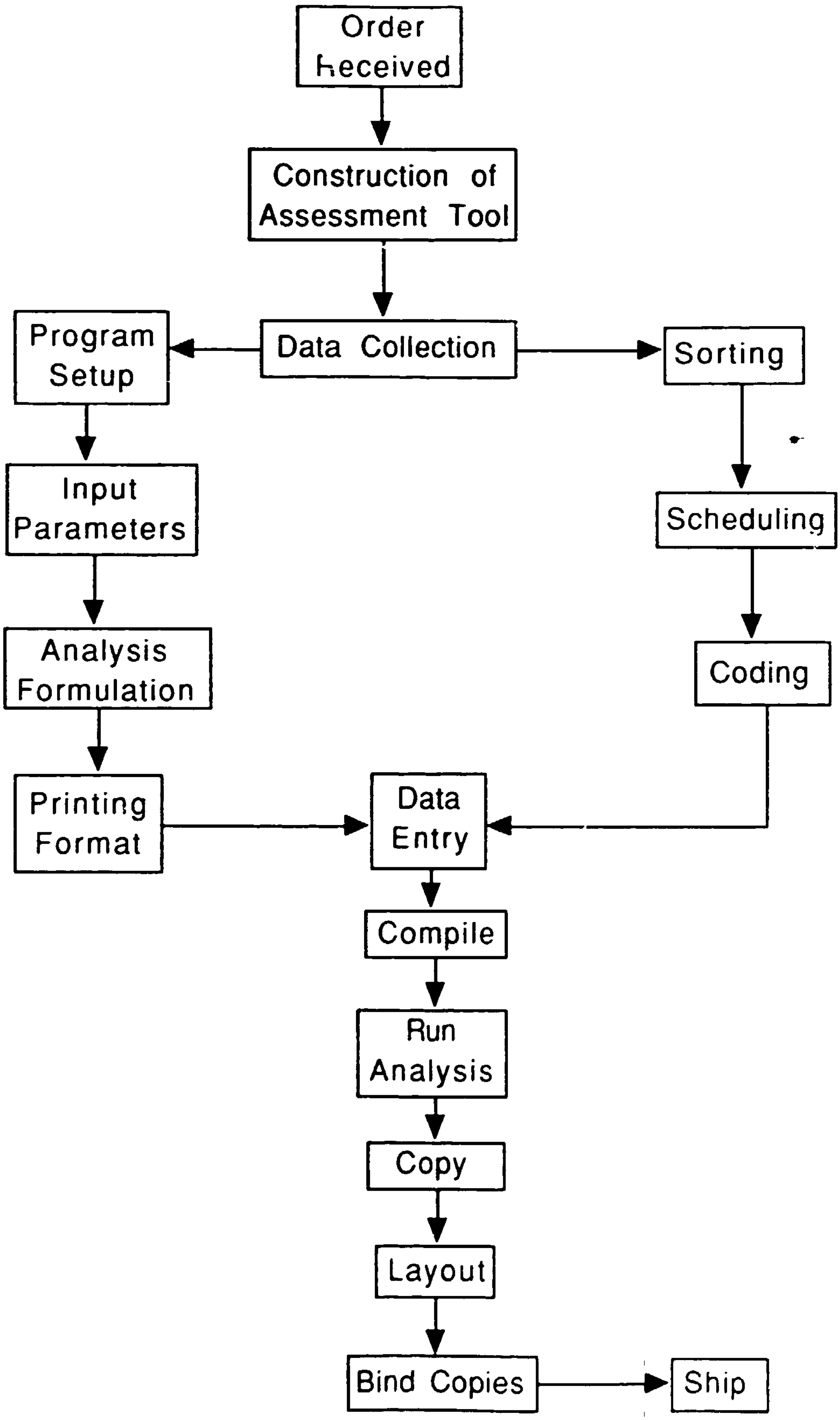
b. Deciding to focus on the types of errors that cause the rework requirements, the rejected projects were broken down by the causes of rework. Perform a Pareto Analysis to help identify the major causes.

Reason	Number
Data Entry	13
Coding	8
Print	7
Scheduling	9
Copy	21
Compiling	9
Other	7

c. If the control chart had revealed that the process is out of control, what concerns might you have about the use of a Pareto Analysis to focus your improvement effort?

d. A flow diagram of the process is provided for you on the following page. Make suggestions with regard to methods of sampling that would allow a team to focus their analysis on the main causes of rework.

Flow Chart for Problem 3



4. In your company, individual work center performance is measured on the basis of whether or not the processes are kept in control. After your area manufactures crankshafts, they are sent to the line where they are used in engine assemblies. Due to several complaints you have received from the line concerning defective crankshafts, you decide to investigate your process for control through use of a control chart. The data are obtained by 100% visual inspection of 24 lots of varying sizes. Out of a total of 5544 crankshafts inspected, 367 of the units are found to be nonconforming. Data for two particular points on the chart are given below

Lot	Lot Size	Number Nonconforming
22	200	23
23	275	5

a. Calculate the values which would be plotted on the control chart and the control limits for these two lots.
b. Discuss the impacts that methods of performance appraisal can have on the use of data to improve systems.

5. A manufacturer is making an effort to reduce the fraction defective of his products. Machine 75 appears to be operating in control at an average fraction defective of 0.10. Just before sample number 42 was taken, a part on a sensitive location of the machine was replaced. The following sample results had been obtained before and after the part replacement:

Sample Number	Number Inspected	Number Rejected
40	49	10
41	225	25
42	100	15
43	100	20
44	225	45
45	225	29

Did the replacement of the part have any real effect on the working of the machine, and if so, what?

TABLE 3.2
Number of nonconforming crusts in four consecutive cycles of 24 crusts

Date	Time	Number in subgroup	Number Nonconforming
11-08	8:00 a	96	15
	9:00 a	96	16
	10:00 a	96	17
	11:00 a	96	20
	12:00 a	96	12
	1:00 p	96	19
	2:00 p	96	13
11-09	8:00 a	96	18
	9:00 a	96	27
	10:00 a	96	16
	11:00 a	96	21
	12:00 a	96	22
	1:00 p	96	20
	2:00 p	96	19
11-10	8:00 a	96	9
	9:00 a	96	14
	10:00 a	96	18
	11:00 a	96	23
	12:00 a	96	19
	1:00 p	96	12
	2:00 p	96	13
11-11	8:00 a	96	15
	9:00 a	96	16
	10:00 a	96	18
	11:00 a	96	18
	12:00 a	96	16
	1:00 p	96	12
	2:00 p	96	16
11-12	8:00 a	96	11
	9:00 a	96	17
	10:00 a	96	11
	11:00 a	96	18
	12:00 a	96	15
	1:00 p	96	14
	2:00 p	96	19
			579

Table 3.3

Number of nonconforming end crusts and middle crusts in four consecutive cyles

Subgroup	n	np	p	Subgroup	n	np	p
1	32	9	.281	2	64	2	.031
3	32	10	.313	4	64	9	.141
5	32	8	.250	6	64	5	.078
7	32	9	.281	8	64	4	.063
9	32	8	.250	10	64	9	.141
11	32	6	.188	12	64	9	.141
13	32	7	.219	14	64	6	.094
15	32	9	.281	16	64	6	.094
17	32	10	.313	18	64	6	.094
19	32	12	.375	20	64	1	.016
21	32	11	.344	22	64	7	.109
23	32	12	.375	24	64	5	.078
25	32	10	.313	26	64	7	.109
27	32	14	.438	28	64	3	.047
29	32	10	.313	30	64	7	.109
31	32	13	.406	32	64	2	.031
33	32	10	.313	34	64	8	.125
35	32	13	..406	36	64	6	.094
37	32	14	.438	38	64	5	.078
39	32	13	.406	40	64	7	.109
		208				114	

FIGURE 3.1

PLOT OF PROPORTION OF REJECTED ENGINES

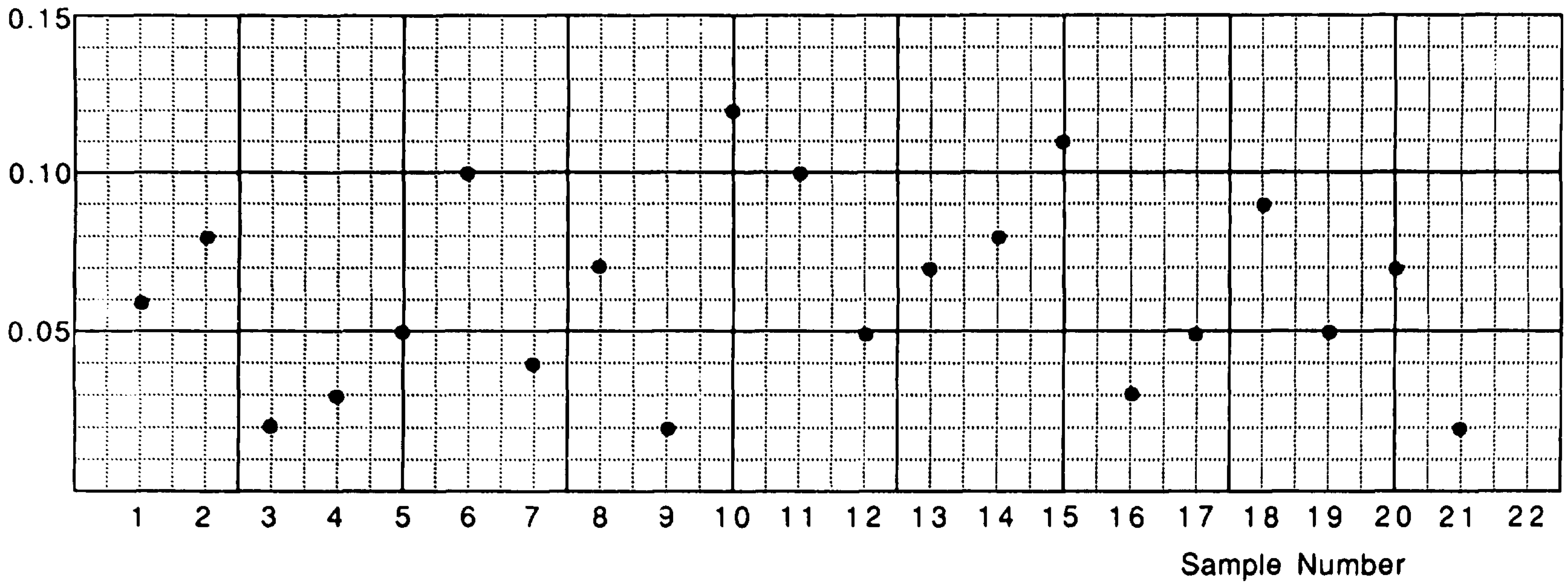


FIGURE 3.2

PLOT OF PROPORTION OF REJECTED ENGINES

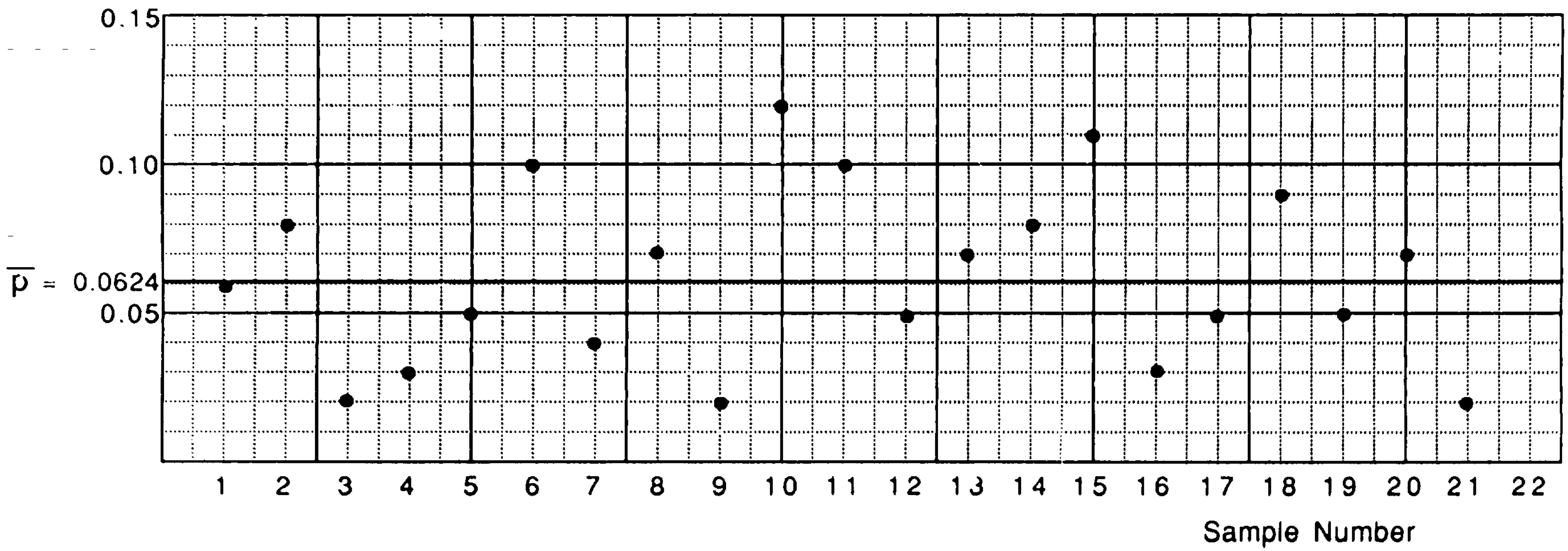


FIGURE 3.3

PLOT OF PROPORTION OF REJECTED ENGINES

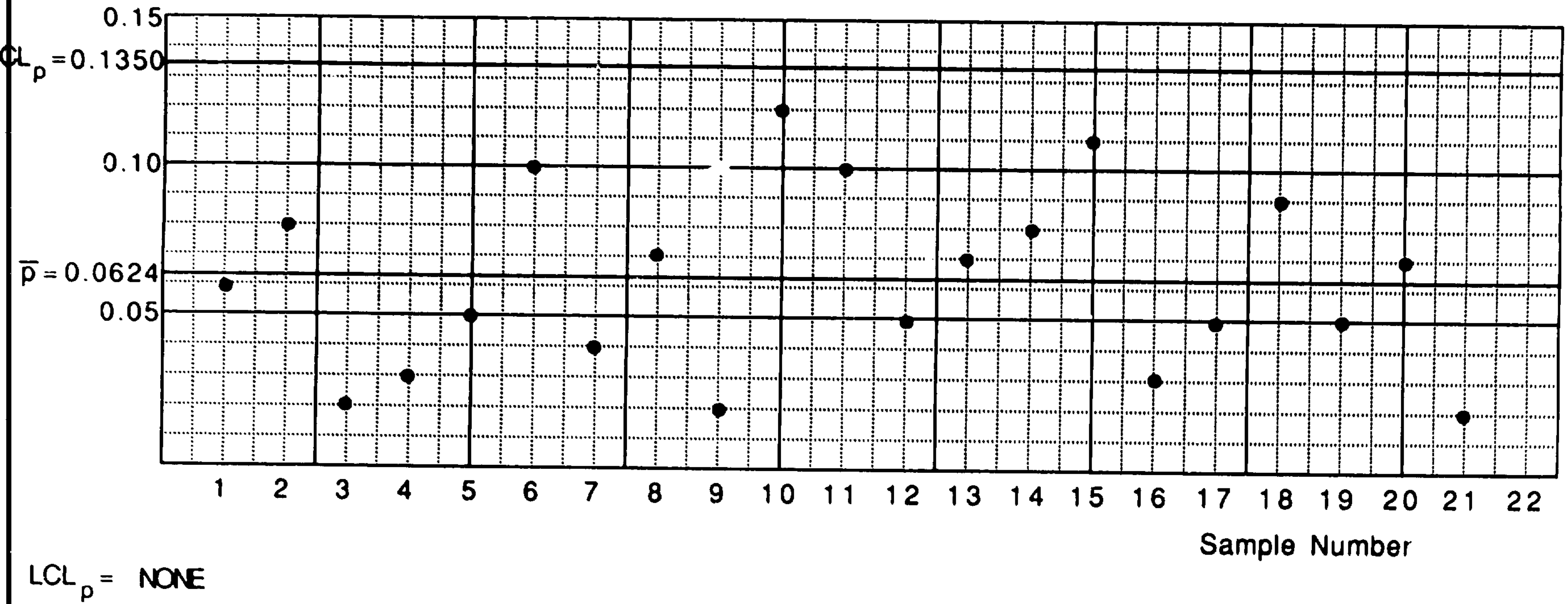


FIGURE 3.4

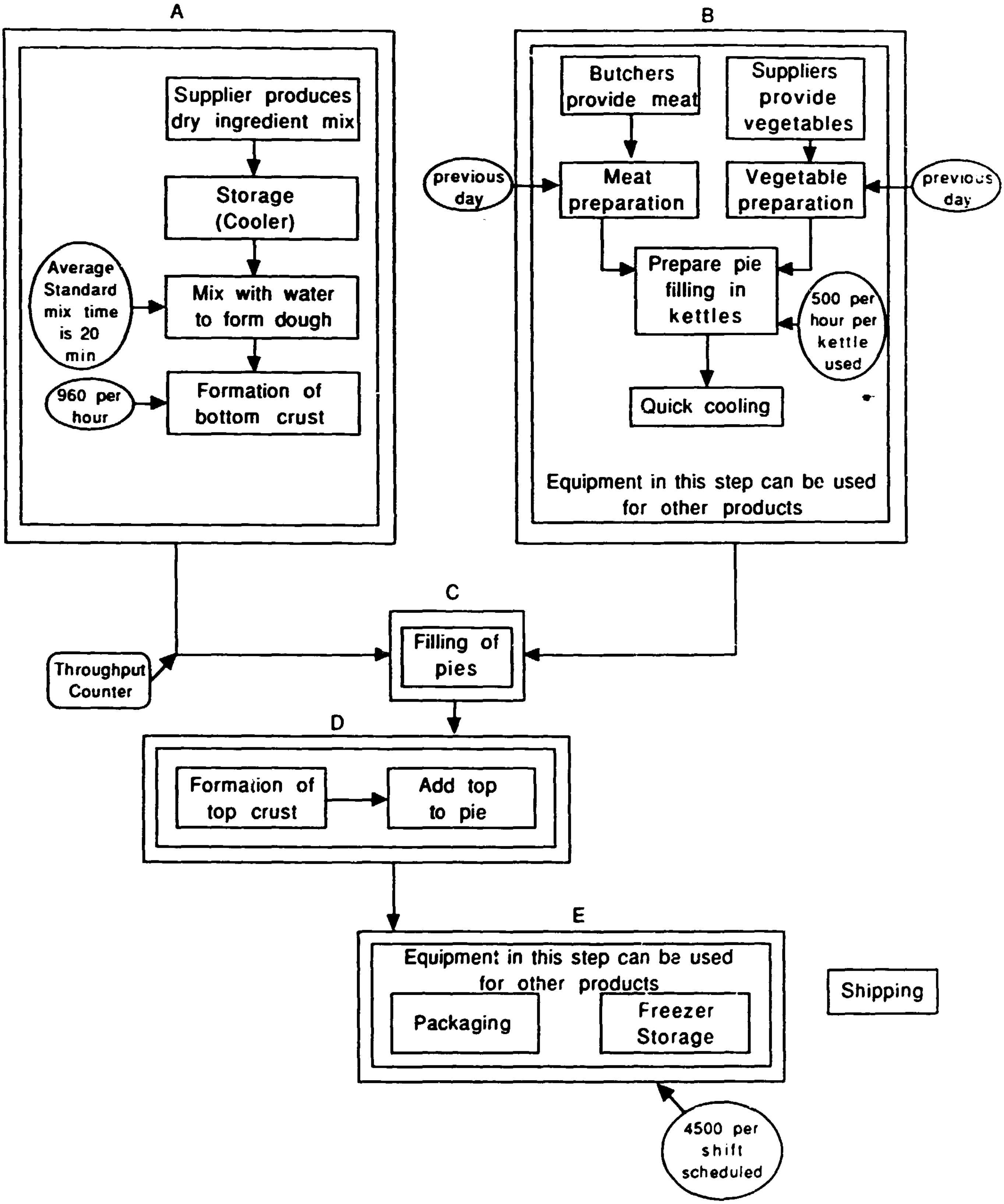

A
Supplier produces dry ingredient mix
Storage (Cooler)
Average Standard mix time is 20 min
Mix with water to form dough
960 per hour
Formation of bottom crust
B
Butchers provide meat
Suppliers provide vegetables
previous day
Meat preparation
Vegetable preparation
previous day
Prepare pie filling in kettles
500 per hour per kettle used
Quick cooling
Equipment in this step can be used for other products
C
Throughput Counter
Filling of pies
D
Formation of top crust
Add top to pie
E
Equipment in this step can be used for other products
Packaging
Freezer Storage
Shipping
4500 per shift scheduled

FIGURE 3.5

A

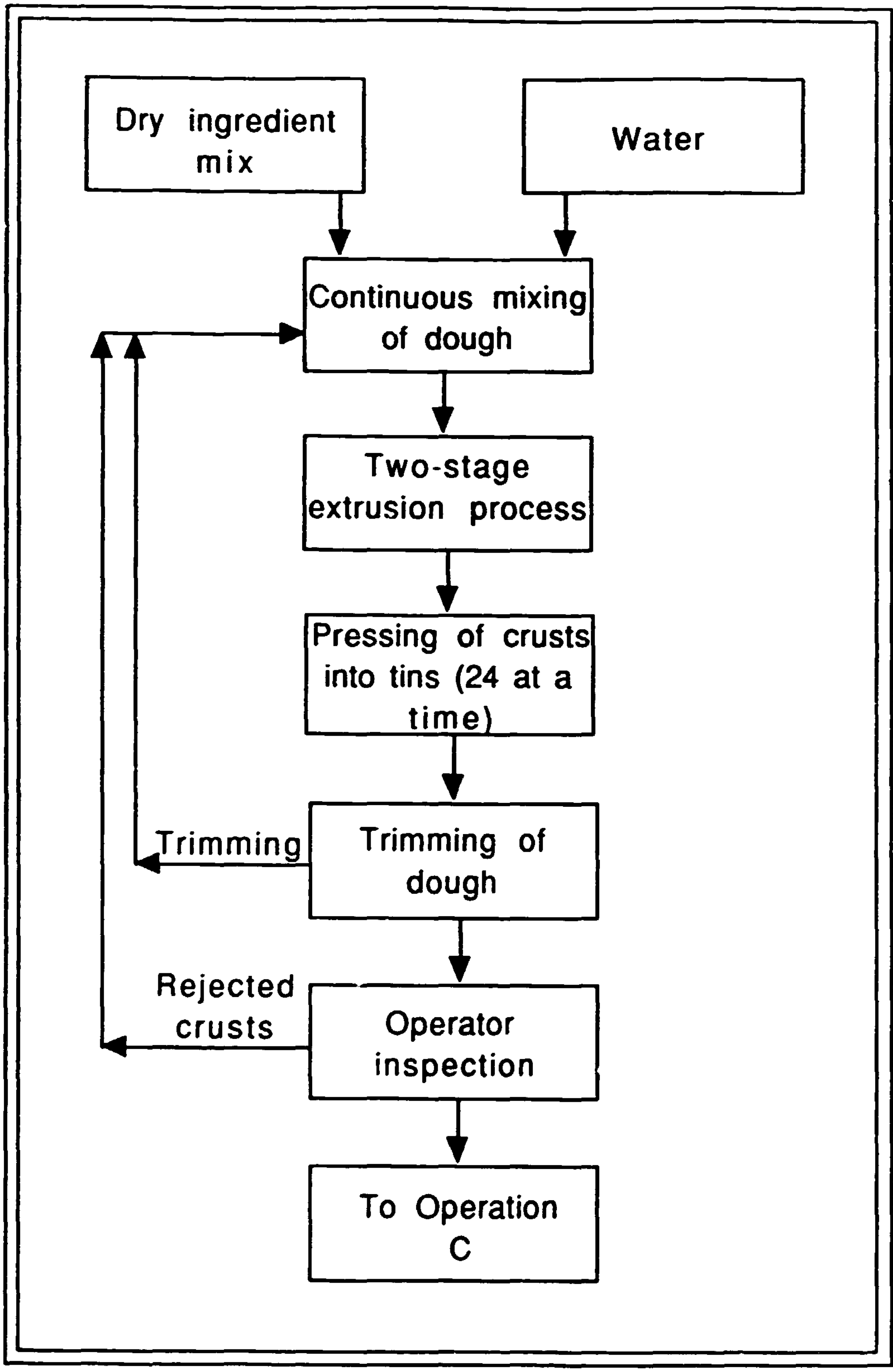
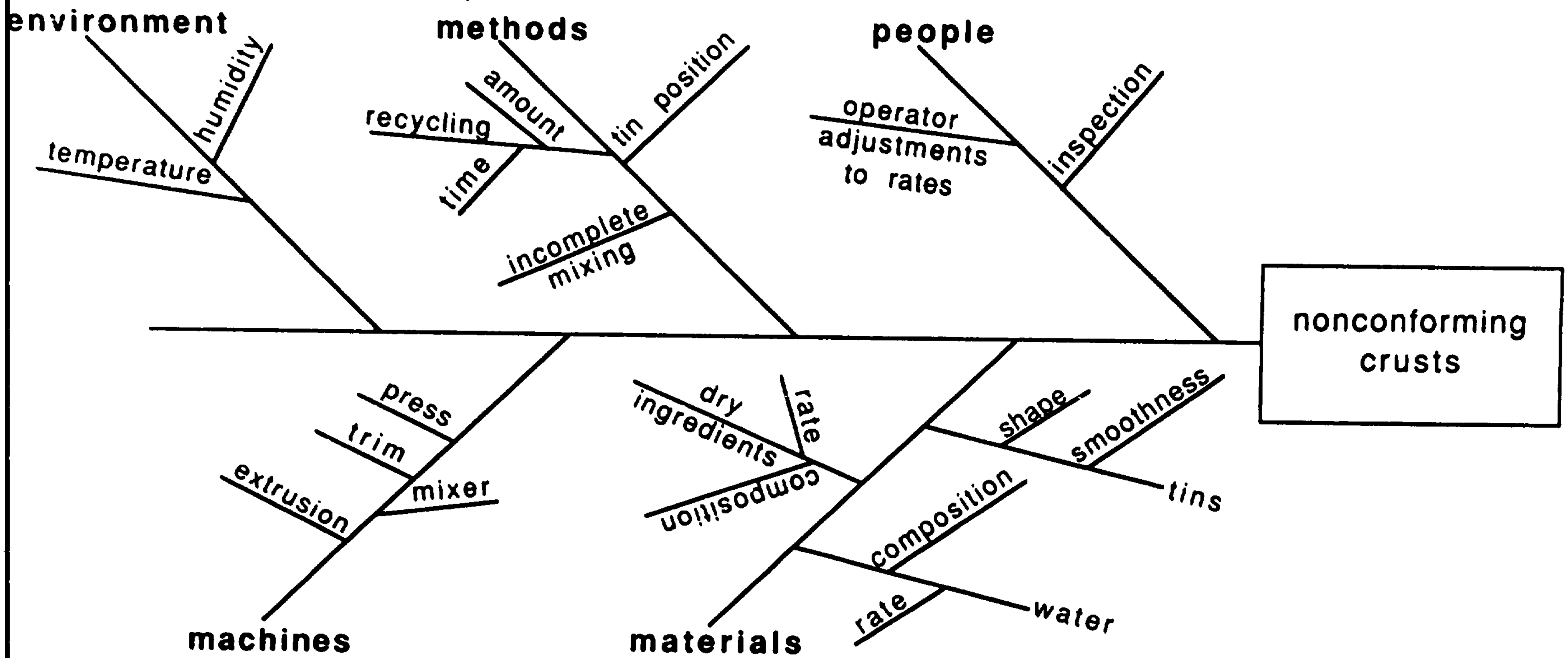

Dry ingredient mix
Water
Continuous mixing of dough
Two-stage extrusion process
Pressing of crusts into tins (24 at a time)
Trimming
Trimming of dough
Rejected crusts
Operator inspection
To Operation C

FIGURE 3.6



environment
humidity
temperature
methods
amount
recycling
time
tin position
incomplete mixing
people
operator adjustments to rates
inspection
nonconforming crusts
machines
press
trim
extrusion
mixer
materials
dry ingredients
rate
composition
shape
smoothness
tins
composition
rate
water

FIGURE 3.7

PLOT OF NUMBER NONCONFORMING

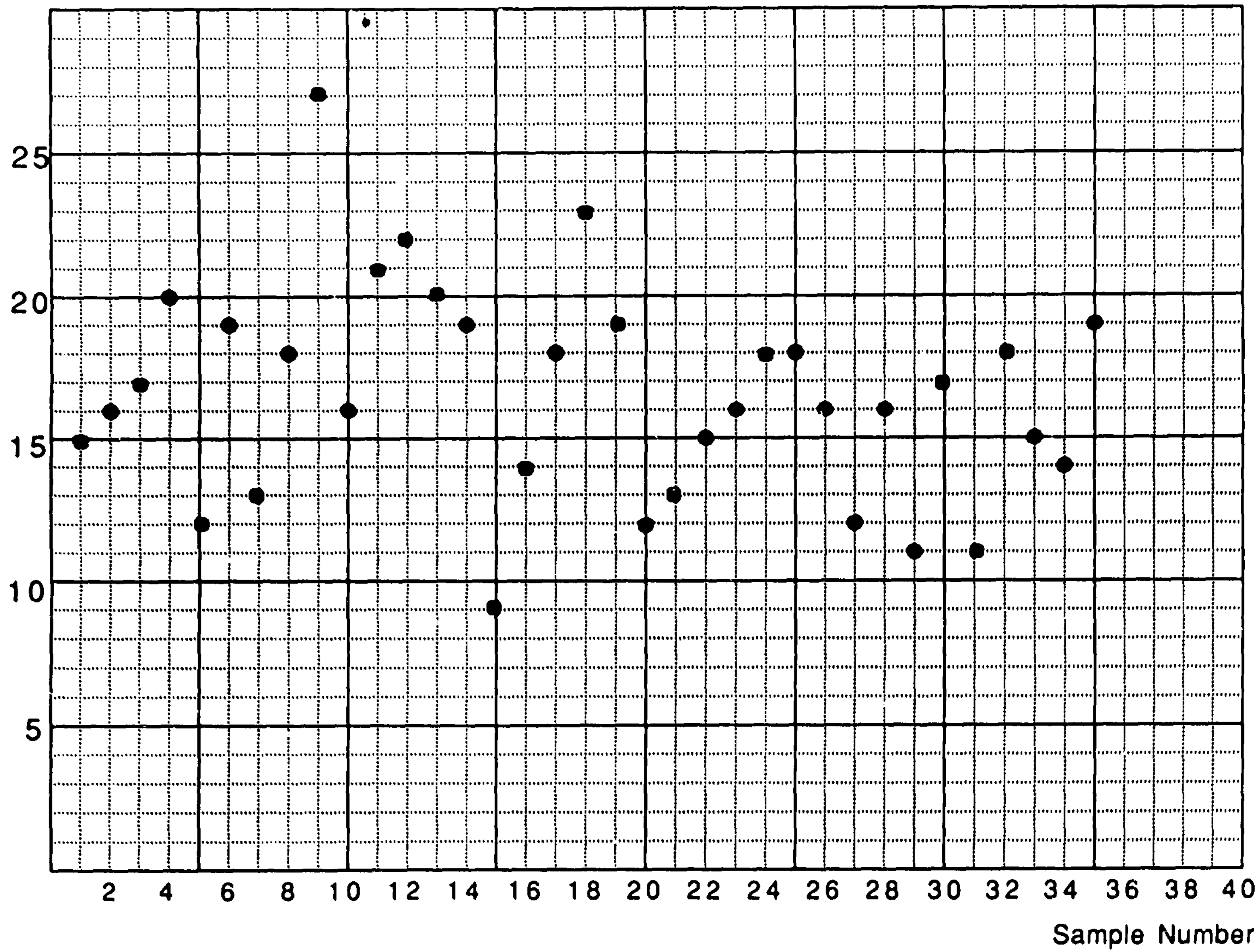
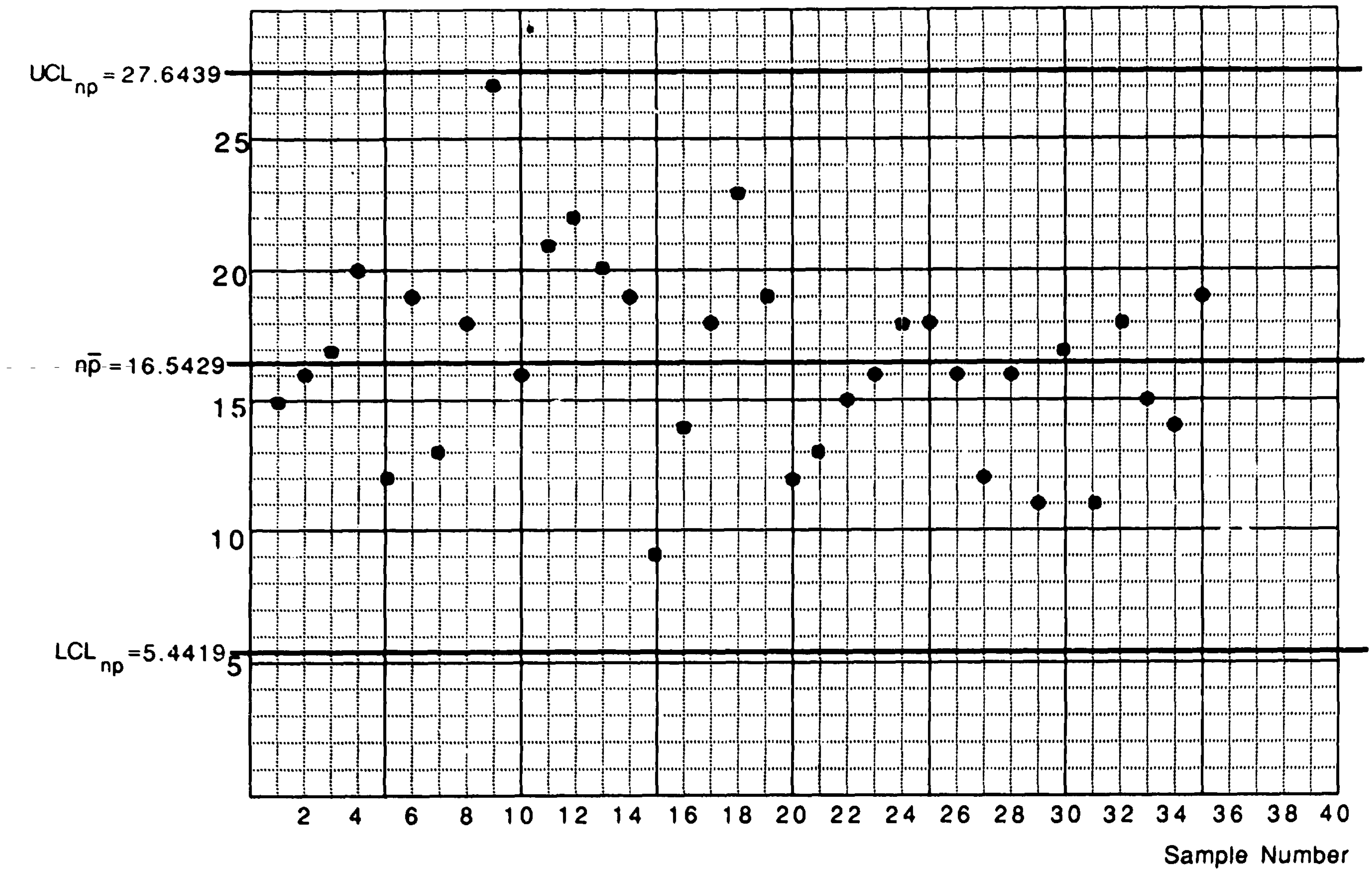


FIGURE 3.8
COMPLETED NP CHART



INSPECTION TIME 1 3 5 7 9 11 13 15 17 19 21 23

____

INSPECTOR

______________ 2 4 6 8 10 12 14 16 18 20 22 24

INSPECTION TIME 1 3 5 7 9 11 13 15 17 19 21 23

____

INSPECTOR

______________ 2 4 6 8 10 12 14 16 18 20 22 24

INSPECTION TIME 1 3 5 7 9 11 13 15 17 19 21 23

____

INSPECTOR

______________ 2 4 6 8 10 12 14 16 18 20 22 24

INSPECTION TIME 1 3 5 7 9 11 13 15 17 19 21 23

____

INSPECTOR

______________ 2 4 6 8 10 12 14 16 18 20 22 24

INSPECTION TIME ______________

1 3 5 7 9 11 13 15 17 19 21 23

INSPECTOR ______________

2 4 6 8 10 12 14 16 18 20 22 24

INSPECTION TIME ______________

1 3 5 7 9 11 13 15 17 19 21 23

INSPECTOR ______________

2 4 6 8 10 12 14 16 18 20 22 24

INSPECTION TIME ______________

1 3 5 7 9 11 13 15 17 19 21 23

INSPECTOR ______________

2 4 6 8 10 12 14 16 18 20 22 24

INSPECTION TIME ______________

1 3 5 7 9 11 13 15 17 19 21 23

INSPECTOR ______________

2 4 6 8 10 12 14 16 18 20 22 24

FIGURE 3.11
PLOT OF DATA FROM TABLE 3.3

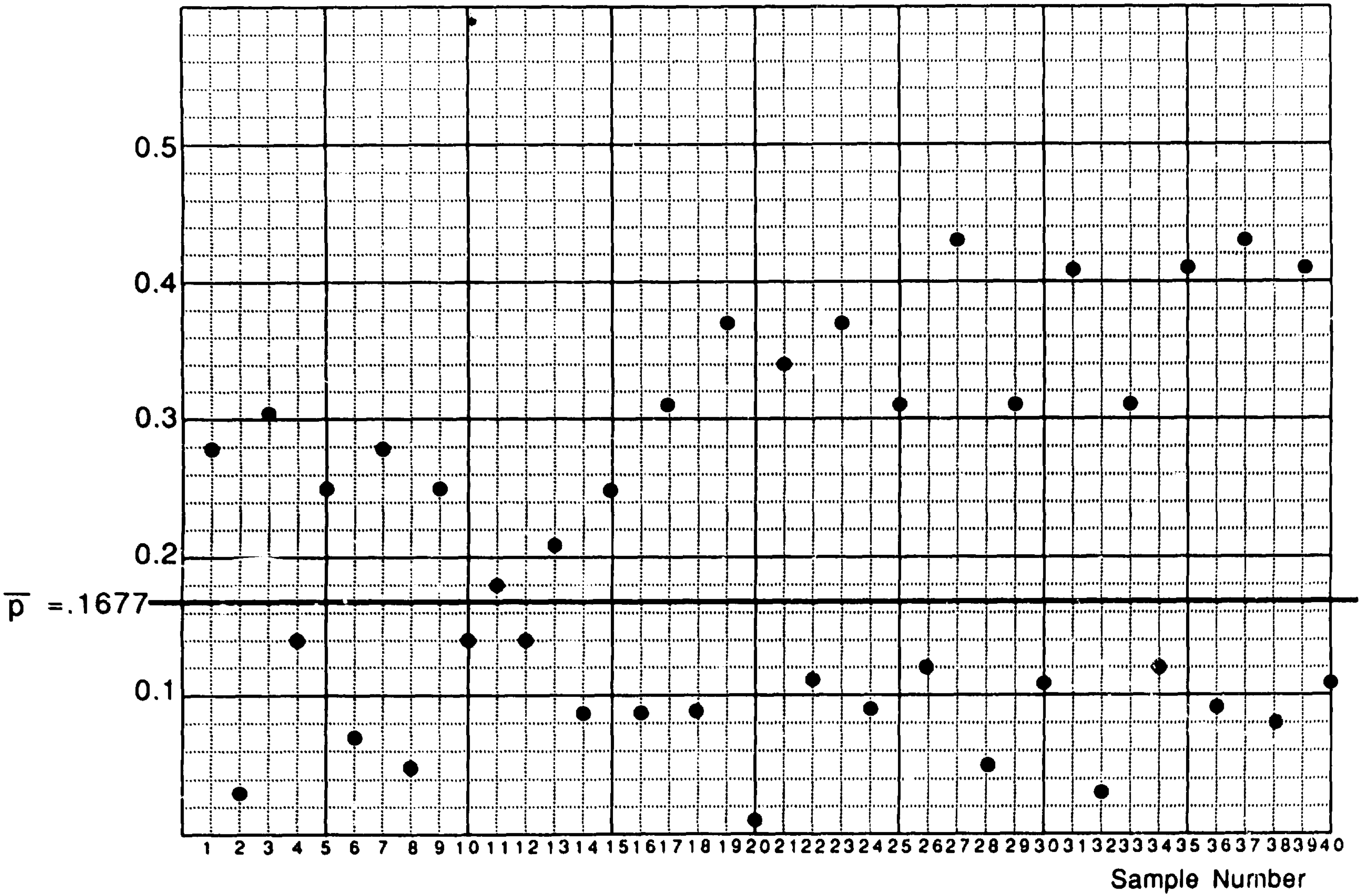


FIGURE 3.12

COMPLETED P CHART

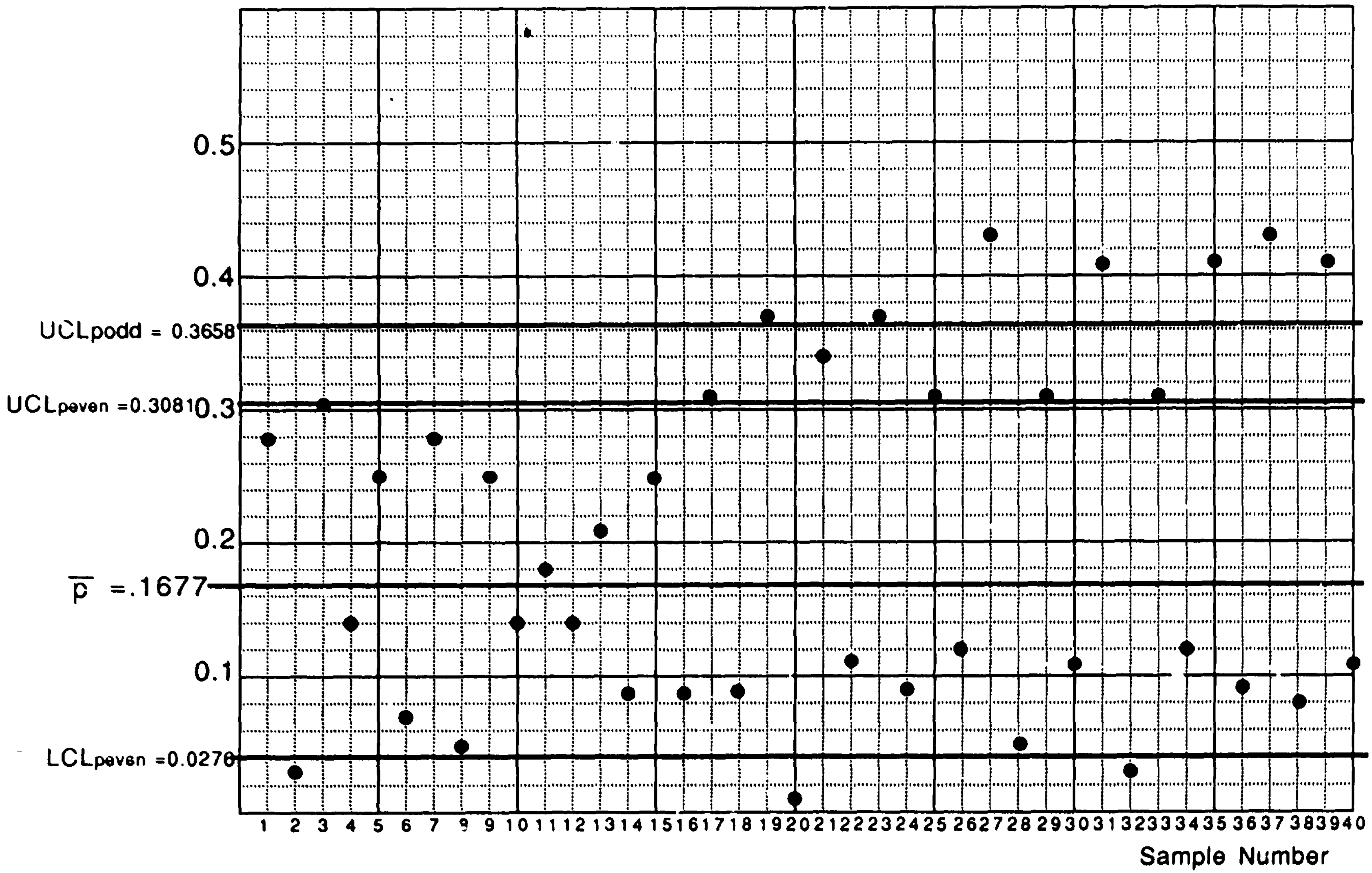


FIGURE 3.13
P CHART OF END CRUSTS ONLY

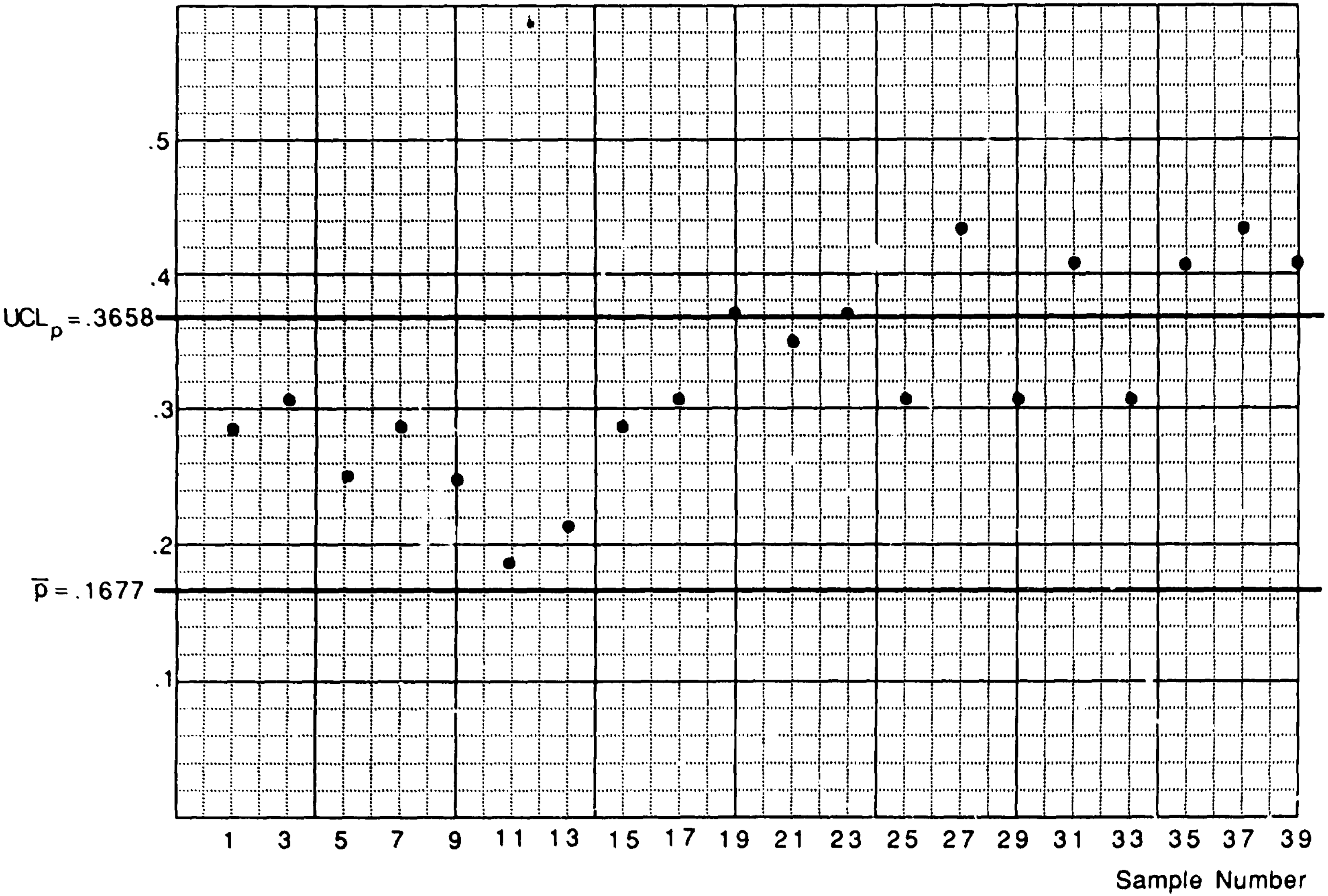
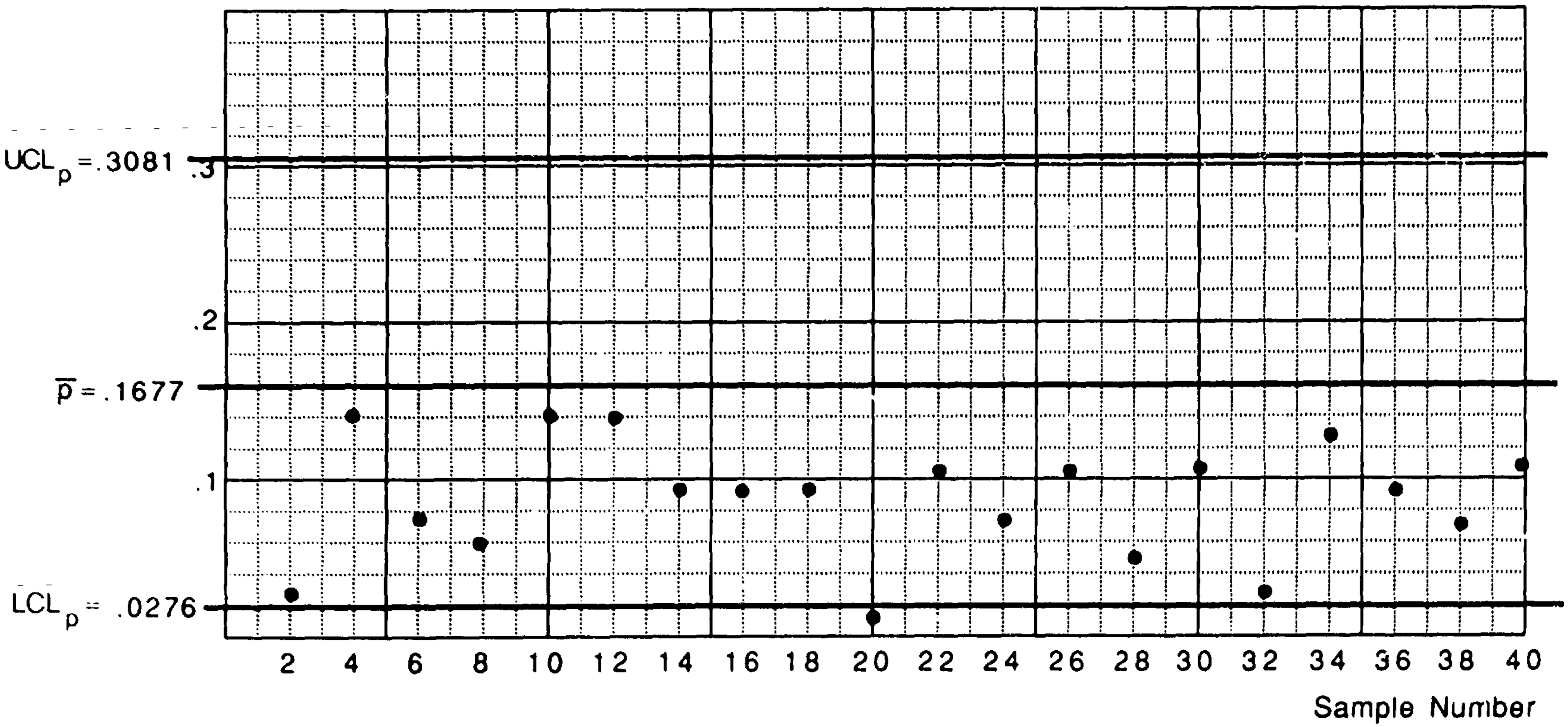


FIGURE 3.14

P CHART OF MIDDLE CRUSTS ONLY



Chapter 4

Control Charts for Attributes Data
c and u Charts

There are situations in which ongoing process data consist of counts other than the type discussed in the previous chapter. Data are collected and recorded on the number of individual flaws or defects found in a single item or collection of items of product, such as the number of surface flaws on a sheet or several sheets of material, or the number of impurities found in a volume of continuously manufactured or batch-produced product. These counts are different from those analyzed by a p or np chart because they arise by counting occurrences of some type rather than by classifying each item of product into one of two categories, acceptable or unacceptable. Counts of occurrences of events in time are also collected in business environments. For example, counts of the number of service or transmission interruptions, the number of production line stops, the number of accidents, the number of calls for service, the number of customer complaints are duly recorded and counted. All of these kinds of counts exhibit variation over time and conditions which can be studied to provide information about process and system behavior over time and conditions and to provide guidance on possible actions and the effect of these actions taken to improve process or system performance.

For many processes on which such count data are collected, stable behavior of the process over time and conditions would indicate that the data should be in control when judged by a c or u chart. And, of course, an out of control condition on one of the charts indicates some type of instability. This chapter will first describe those situations where the use of the c chart or the u chart is appropriate. Subsequently, examples illustrating the construction of these charts will be given. The later part of the chapter will be devoted to illustrating the use of c charts and u charts for process study and improvement.

4.1 Description of attributes data requiring the use of c or u charts

Counts of occurrences of events in time or of nonconformities on a unit of material often can be analyzed by the use of c or u charts. Whether such an analysis is appropriate depends on the manner in which such occurrences might happen if only random or common cause sources of variation were present. The four criteria listed below summarize what must be true about the nature of the occurrences of events or nonconformities if analysis by a c or u chart is to be used. The examples which follow illustrate further the use of these criteria.

The type of situation for which random behavior is characterized by the upper and lower control limits on a c or u chart is when counts of occurrences meet the following criteria:

Criteria 1 : The counts are independent of each other.

Criteria 2: The number of possible occurrences is large.

Criteria 3: The chance of an occurrence at any one time or place is small.

Criteria 4: The expected number of occurrences is proportional to the amount of time or material which is included in an inspection unit.

The study of counts of flaws on sheets of manufactured materials provides an illustration of the application of the above four criteria. For example, in the manufacture of aluminum cans it is critical that there not be any holes in the aluminum prior to manufacture. At one plant where aluminum cans are manufactured, past experience has shown that large holes do not appear in the aluminum sheets received from the supplier of aluminum, but that pin holes occasionally occur. The aluminum is thus 100% inspected prior to use for can manufacture and the number of pin holes found in a sheet of aluminum is counted. The use of a c or u chart to understand the behavior of the process which makes aluminum involves considering the anticipated behavior of the occurrence of pin-holes if only common cause sources of variation were present. The presence of only common cause sources of variation in this case would mean that one would expect to see pin holes scattered in a random fashion on the sheet of aluminum. If some special cause were active, this might have the impact of isolating the pin holes to possibly one area of the sheet of aluminum. If pin holes did tend to occur in such clusters, then the detection of a pin hole at one position would make the possibility of others in the vicinity of that position quite likely. Criteria one for using a c or u chart captures the idea of this type of random behavior. If only common cause sources of variation are present, then the occurrence or non-occurrence of a pin hole at a particular location is independent of knowing whether other pin holes have occurred in the vicinity. Instead, pin holes would tend to be randomly scattered across the sheet.

If one considered a sheet of aluminum and thought about the number of possible places a pin hole could occur, it is easy to see that criteria two would be met. In fact, if a sheet of aluminum was subdivided into many pieces, each one of these pieces might possibly contain a pin hole. However, on every one of these pieces surely there would not actually be a pin hole, hence criteria 3 would be met. The important concept in these two criteria (criteria two and three) is that there exists a large opportunity for pin holes, although the anticipated number would generally be fairly small.

If the sheet of aluminum were to be divided in half, the anticipated number of occurrences of pin holes on one half would be the same as the other. Or if two sheets of aluminum were inspected, about twice as many pin holes would be anticipated than if only one sheet were inspected. This is the idea behind criteria four. The number of anticipated or expected pin holes is proportional to the amount of aluminum which is inspected. Of course, the consistency of the number of anticipated pin holes from one sheet to the next describes the behavior one would expect to see if the process were stable over time. Departures from this behavior, as in observing a very large number on one sheet, would indicate the presence of a special cause. It would then be useful to direct attention towards identifying what different conditions, materials, etc. may have occurred to cause increased numbers of pin holes.

Another situation where a c or u chart might be used is in counting the number of production line stops in, say, a shift. In the example of pin holes in aluminum, the unit of inspection was a sheet of aluminum. In the current example, the unit of inspection would be a unit of time, in particular, one shift. If the production line is shut down at planned intervals, say every four hours for a tool change, then using a c or u chart to summarize this information would not be appropriate nor useful. Instead, the c or u chart would be considered for studying such a situation when randomly occurring events are causing some difficulty which may result in stopping the production line. An example might be in the production of paper where a number of different causes, maybe acting together or separately, result in breaks in the paper. The times at which breaks might occur cannot be predicted, but rather occur at random. The four criteria listed for the use of c or u chart would then be used as an attempt to describe stable behavior for occurrence of breaks.

The four criteria for using c and u charts can again be examined as an aid to understanding how these charts can be used for process study when occurrences of events are being counted and a unit of time is the inspection unit. Criteria one stated that the counts of occurrences should be independent. In the present situation, this would imply that if one production stop occurred, it would not mean that one or more production stops are more or less likely to occur in the near term. The next production stop would occur at random sometime in the future, with the same chance of occurrence as if the previous one had not occurred. Now when considering the use of a c or u chart, a question about whether such an assumption is correct might be raised. This question might be posed as "Is it possible that production stops tend to occur in clusters when certain conditions are present?" Believing that such might be the case would not imply that a c or u chart should not be used. Rather, the c or u chart provides an objective method for studying the process to provide some guidance to the question posed. Limits on a c or u chart would indicate the number of occurrences that could be expected to occur if the process were behaving according to the given criteria. Unusual behavior on the chart, such as runs or points outside the limits, would indicate that nonrandom behavior, possibly of the kind posed by the question, was occurring.

One could consider the applicability of criteria two and three in much the same way as done for examining pin holes in aluminum. If the unit of inspection for line stops was, say, a shift, one could think of dividing the shift into many small increments of time, maybe into seconds. A stop might occur in any one of these many small time increments. Thus the number of occurrences is possibly very large. However, the chance of seeing an occurrence at a large number of these time increments is very small; most of the time increments will not contain a line stop. Criteria four states that the expected number of occurrences is proportional to the amount of time which is included in an inspection unit. It will often require careful consideration of how to collect data on a process to see that this criteria is reasonably well met. For example, in collecting data on the number of line stops, it might be the case that when the production line stops it might be down for as long as an hour or two. If such were the case, it might not be reasonable to have the unit of time over which production line stops are counted be an eight-hour shift. Instead, a more appropriate unit of time for analysis might be one that corresponded to actual running time of the equipment.

4.2 Construction of c charts

Both c and u charts are used to examine the stability of a process over time when the information collected on the process is in terms of counts of occurrences. The c chart is used to study the counts of occurrences when the amount of time or material inspected at any one time remains the same over the course of the study. If the counts of occurrence are made on varying sizes of units of inspection, then a u chart is used. The data in Table 4.1 provide an example of a situation in which the amount of material inspected stays constant over the course of the study. The data in this table arose from counting the number of pin holes in each of 20 subgroups. Each subgroup was formed by inspecting 10 rolls of aluminum from a shipment received from a supplier. Each of the rolls of aluminum had the same number of feet per roll, so for each subgroup (for each ten rolls inspected,) the amount of material inspected stayed the same. The letter, c, will be used to denote the number of nonconformities in each subgroup.

Notation

k will denote the number of subgroups

c will denote the number of nonconformities in each subgroup

4.2.1 Calculating the center line and control limits for a c chart

As in the construction of a p chart, the construction of a c chart begins with plotting the data. With a c chart, the points plotted are the c's, or in other words, the counts of nonconformities. Figure 4.1 is the plot constructed from plotting the number of pin holes in the ten rolls for each of the 20 subgroups. The horizontal axis corresponds to the subgroup number and the vertical axis to the count of pin holes. The time order represented by the horizontal axis corresponds to the listing given in Table 4.1 of the subgroups. The subgroups match with the time at which the shipments were received; subgroup 1 corresponds to the inspection of a shipment which arrived at the plant prior to the material inspected for subgroup 2, etc. It should be noted, though, that it is not known by the plant whether the timing at which the shipments were received correlates to the timing of manufacture.

The center line on a c chart represents the average number of nonconformities per inspection unit. Since the number of inspection units on a c chart is the same as the number of subgroups, the average, $\bar{c}$, is simply the total number of nonconformities found divided by the number of subgroups.

$$\bar{c} = \frac{\text{total number of nonconformities}}{\text{total number of units inspected}}$$

For the data in Table 4.1, there was a total of 460 pin holes in the 20 subgroups, so that:

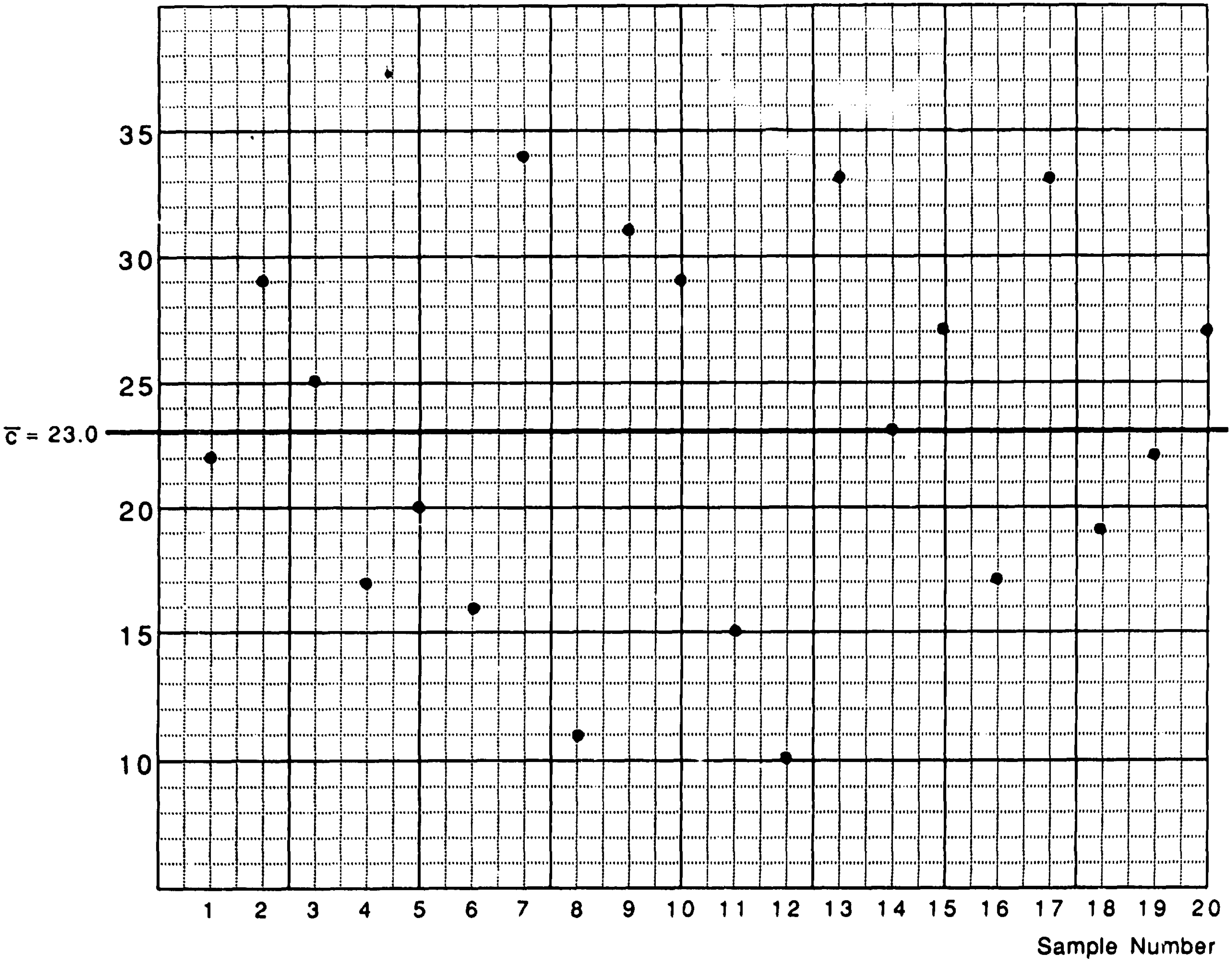
$$\bar{c} = \frac{460}{20} = 23$$

This value of $\bar{c}$ has been used to place the center line drawn on the c chart in Figure 4.1.

The upper and lower control limits for the c chart define the amount of variation that might be expected in the recorded nonconformities if only common cause sources of variation are present. Stated differently, the control limits describe the amount of variation one would expect the recorded c's to exhibit if the process were subject to the kind of random behavior captured by criteria one through four of Section 4.1. The formulas used to calculate the upper and lower control limits are, respectively:

FIGURE 4.1

PLOT OF NUMBER OF PIN HOLES IN TEN ROLLS FOR EACH OF 20 SUBGROUPS



$$UCL_c = \bar{c} + 3\sqrt{\bar{c}}$$

$$LCL_c = \bar{c} - 3\sqrt{\bar{c}}$$

where $\bar{c}$ is the average number of nonconformities. For the data on pin holes the upper control limit is found to be:

$$UCL_c = 23 + 3\sqrt{23} = 37.39$$

The calculations for the lower control limits are:

$$LCL_c = 23 - 3\sqrt{23} = 8.61$$

Figure 4.2 displays the completed control chart. Because there are no points on or outside the control limits or any trends in the plotted points, the completed chart shows no indication of a special cause operating. It would appear that the occurrence of pin holes in the aluminum appear in a random fashion over time. One subgroup of 10 rolls does not have a significantly higher or lower number of pin holes to indicate that the process may have been behaving differently when those rolls were made.

4.2.2 Summary

Construction of c charts

k denotes the number of subgroups
(k will be the same as the number of inspection units)

1. The number of nonconformities on each inspection unit (in other words, in each subgroup) is counted. The letter **c** refers to the number of nonconformities in an inspection unit. These c values are plotted on the c chart.

2. Calculate the center line on the chart as:

$$\bar{c} = \frac{\text{total number of nonconformities}}{\text{total number of units inspected}}$$

3. Calculate the control limits as:

$$UCL_c = \bar{c} + 3\sqrt{\bar{c}}$$

$$LCL_c = \bar{c} - 3\sqrt{\bar{c}}$$

4.3 Comparing Processes

The previous example about counting pin holes in rolls of aluminum was generated by a company which buys aluminum rolls in order to produce aluminum cans. The number of pin holes is of concern to the company since portions of aluminum which contain holes need to be removed before the aluminum is cut into pieces to form cans. Since the c chart showed that the process appears to be consistent, the value of $\bar{c}$, 23, provides an estimate of the average number of pin holes one would expect to see in ten rolls of aluminum. The plant which was collecting these data felt that an average of 23 per ten rolls (or 2.3 per roll) was too high a number of pin holes. A series of conversations with the supplier of aluminum was begun in order to communicate the findings of the previous study and to solicit the vendor's help in addressing the issue of reducing pin holes in aluminum.

The supplier of the equipment was surprised by the average number of pin holes being reported by the manufacturer of cans. Ongoing inspection at their plant revealed a considerably lower number of holes per roll. Inspection techniques by the two sites were different; different equipment was used by the two groups to count pin holes. In addition, the number of pin holes at the supplier's plant were counted prior to rolling the aluminum and at the can manufacturer after rolling. Thus, it was felt that there was the possibility that pin holes were being formed during the rolling or unrolling of the aluminum. In order to better understand if any discrepancies existed in the counting of pin holes and as a prelude to identifying how to rectify both the differences in counts as well as the magnitude of counts, the two groups proposed the following study. The supplier of aluminum agreed to count the number of pin holes in 10 rolls of aluminum in each shipment. These ten rolls would be tagged. As they were unwound by the can manufacturer, the number of pin holes would again be counted by the can manufacturer. Table 4.2 captures the counts of nonconformities by the aluminum supplier as well as by the manufacturer for 10 shipments of aluminum.

There are several ways that one might choose to examine the data of Table 4.2. However, since the initial question raised was whether consistent counts were being generated by the two groups counting pin holes, it would seem appropriate to first look at a plot of the data of Table 4.2 to determine if any inconsistencies in counts by the two groups are apparent. Figure 4.3 is a plot of the c's (the number of pin holes) from Table 4.2. The data have been plotted by first plotting the results from the supplier then the results found by the can manufacturer. If the center line and control limits are based on the twenty subgroups of Table 4.2, it is left to the reader to verify that the center line for the control chart will be 19.3 and the upper and lower control limits, respectively will be 32.5 and 6.1. An examination of Figure 4.3 shows that the last nine of the 10 subgroups from the supplier all fall below the center line, a condition which fails the rule of seven described in chapter 3. Thus, it would appear that the counts obtained by the supplier are lower than those obtained by the manufacturer. The reason for this discrepancy will of course require further investigation. It is possible that the equipment used by the two groups results in widely differing counts, counts different enough to generate the discrepancies seen on the c chart of Figure 4.3. However, there is also the possibility that the rolling of the aluminum by the supplier, or something in the shipping process, or the unrolling by the can manufacturer generates additional pin holes which are then counted by the can manufacturer. These questions remain to be investigated by the two groups.

As an aid to this investigation, it will be useful to examine the data generated by the supplier; in other words to plot only the 10 subgroups generated by the supplier to see whether the process appears stable. An examination of this plot will reveal whether the process supplying the aluminum to be rolled is stable. Investigation of the inspection and rolling processes subsequent to the counts made by the supplier will rely on the information discovered here. In addition, the supplier is interested in reducing the number of pin holes found by the can manufacturer, so the nature of the process supplying aluminum to be rolled will be useful in directing the work prior to the rolling operation. If the ten subgroups of counts of nonconformities made by the supplier are plotted on a chart, as in Figure 4.4, the stability of the process prior to rolling can be determined. For this c chart the reader can verify that the center line is 15.2 with an upper and lower control limit of 26.9 and 3.5, respectively. Thus the process, as currently examined, for producing aluminum appears to be operating in a stable fashion. Reductions in the number of pin holes prior to rolling will proceed by examining those sources of variation impacting the manufacture of each shipment of aluminum. At this point in the investigation, a cause and effect diagram would be useful for listing those sources which may be active to produce pin holes. Techniques for studying the effect of these sources, maybe like those used for understanding possible discrepancies in counts by supplier and manufacturer, can be employed once such a list of causes is available.

4.4 Construction of u charts

As with c charts, u charts are used to understand process behavior when the data collected on the process are counts of occurrences. The distinguishing feature of u charts is that they are used when the amount of material or the unit of time which forms a subgroup varies from one subgroup to the next. The example on occurrence of temperature excursions which follows will serve to illustrate when and why a u chart is appropriate and how to construct this type chart.

A chemical firm produces many of its products in batches. A recent focus of work at one site of the company was to begin understanding how well the temperature profile for these batch processes is managed and whether improvements in the management of these profiles promises improvement in yield and throughput of the batch processes. As attention was directed to the issue of correct processing temperatures, it was realized by the work group that there was currently little knowledge about how well the currently stated temperatures were being managed. Thus initial efforts were directed at developing operational definitions for a maintained temperature profile and at gaining understanding of how well the profiles were maintained.

The graph in Figure 4.5 illustrates what is meant by a temperature profile. The horizontal axis is the processing time in a tank for one stage of a batch process. Temperature is graphed on the vertical axis. One can see from the graph that, for this stage of the batch process, the temperature in the tank is to be raised from 75 degrees to 150 degrees in the first hour of processing, maintained at 150 degrees for 6 hours, then cooled to 100 degrees over the next 2 hours. The information captured by this line as to correct temperature for batch operation is what is referred to as a temperature profile. It should be noted that the profile drawn for the rise during the first hour and the drop during the seventh and eighth hours is more explicit than what was stated by the batch protocol. The protocol required a gradual warming and drop in temperature, but the rate of these changes was not required to be linear over time. However, in order to begin evaluating how well the protocol was being followed, specific descriptions of the manner in which these temperature changes should occur was required as well as a definition of what constituted a significant deviation from protocol. Initially, a judgment by the chemists and technical staff was made to decide how far from the profile the temperature could wander before being labeled a temperature excursion. Figure 4.6 captures the type of limits drawn around a profile in order to define how close to the profile the temperature should be held. When the temperature falls above or below these limits, this phenomenon is labeled a temperature excursion.

An initial study on temperature excursions was begun on one set of processing equipment. Over a three week period, data on the number of temperature excursions were collected. These data are summarized in Table 4.3. The second column in Table 4.3 is labeled batch type. Since the processing equipment under study is used for producing several different types of material, records of which batch was being produced have been kept. These different types of material have, of course, different temperature profiles which need to be maintained. These temperature profiles have varying levels for temperature and are of different duration. The third column in Table 4.3 contains the duration in hours of the temperature profile. The number of temperature excursions throughout the recorded hours is given in the fourth column.

These counts of number of temperature excursions provide an example of varying time units of inspection. Batch type C has 12 hours of processing time, whereas type B only has eight hours. Thus, there is a longer time period during which temperature excursions could occur with batch type C. A u chart will thus be the appropriate technique for studying the behavior of temperature excursions across the three week time period. The fifth column in Table 4.3 is labeled "u." The values in this column are the average number of excursions per hour. These were obtained by dividing the number of excursions by the number of hours of operation. The u values could be interpreted as the average number of excursions per hour. One hour is referred to as an inspection unit. It is these values, the counts of occurrences per inspection unit, which will be plotted on the u chart. Figure 4.7 is a plot of these values. The reader will note that instead of points, the letters which correspond to the type of batch have been plotted.

The technique for constructing a u chart is summarized in the following paragraph

Construction of u charts

c denotes the number of nonconformities in a subgroup

n denotes the number of inspection units in a subgroup

1. For each subgroup, calculate u, the number of nonconformities per inspection unit.

$$u = \frac{c}{n}$$

2. Plot the u's on the control chart.

3. The center line of the control chart, $\bar{u}$, is calculated by:

$$\bar{u} = \frac{\text{total number of nonconformities}}{\text{total number of units inspected}}$$

4. The upper and lower control limits for each plotted u will depend on the number of inspection units used to calculate the value of u. The formulas for the upper and lower control limits are given by:

$$\text{Upper control limit: } UCL_u = \bar{u} + 3\frac{\sqrt{\bar{u}}}{\sqrt{n}}$$

$$\text{Lower control limit: } LCL_u = \bar{u} - 3\frac{\sqrt{\bar{u}}}{\sqrt{n}}$$

The center line for the u chart on temperature excursions is given by:

$$\bar{u} = \frac{66}{196} = .3367$$

The formulas for the upper and lower control limits of a u chart are a function of n, the number of inspection units in a subgroup. Of course, this value is different for each different subgroup. In Table 4.3, the third column, labeled "hours," contains the values for n. The control limits for each of the subgroups will be calculated using the value of n, the number of inspection units, for that subgroup. For example, the first subgroup is for batch type A which took 10 hours of processing time. Thus the value for the upper and lower control limits for this subgroup are given by:

$$UCL_u = .3367 + 3\frac{\sqrt{.3367}}{\sqrt{10}} = .8872$$

$$LCL_u = .3367 - 3\frac{\sqrt{.3367}}{\sqrt{10}} - none$$

The remaining values for the upper and lower control limits for the u chart are given in Table 4.4. (The reader should check that the control limits for subgroups with 8 and 12 inspection units are correctly recorded.) Figure 4.8 contains the completed u chart. The twelfth point plotted is above the upper control limit. This fact would indicate that the ability to manage to the temperature profile is not uniform across the batches and the time studied. Of course, with the available information it is not clear whether the difference noted at the twelfth point is something different which occurred at that time point or is due to the fact that the temperature profile for batches of type A are more difficult to manage than the other batch types. Nor is it possible to resolve this issue without more specific process knowledge about the equipment which maintains the temperature profiles and how this equipment performs its role of increasing, decreasing and maintaining a set temperature. Understanding the reasons for the inconsistent behavior in the temperature profiles was a priority item with the work group at the chemical plant.

The initial intent of the study of temperature profiles was to try and connect the ability to manage the temperature profile with the yields and throughput of the batch process. Having observed that the ability to manage to the profile was inconsistent across batches, the personnel at the plant had no ability to predict conformance to temperature profile. Thus, the ability to examine the relationship between the management of the batch process and the yield rates could not be made. In fact, even if this information were available, there would not exist the ability to act on the information since the method to manage the batch process according to profile was not understood. Therefore, more information was required about the reasons for inconsistencies in the ability to manage the temperature profiles of the batch processes.

To provide additional insights into the current operation, two other analyses of the temperature excursion data were considered. The first additional analysis is captured by the u chart of Figure 4.9. In this chart the data have been arranged by batch type rather than in strict time order. The first six points on the u chart correspond to the results for batch type A, the next four are the results from batch type B, and so on. The previous control limits have been plotted on this new plot of the data. Of course, the same points are out of control on this new chart; the additional insight gained from this plot is that it would appear that batch type A is running at a higher number of average temperature excursions per hour than the other batch types. The second additional analysis provides another way of examining this suspicion.

The second analysis is captured in Figure 4.10. This u chart has the data plotted in the same manner as in Figure 4.9, by batch type. However, the centerline and limits for this chart have been calculated separately for each batch type. For example, batch type A was run for a total of 60 hours and had 35 temperature excursions in those 60 hours. Thus the center line drawn for batch type A is 0.583. The upper and lower control limits for batch type A is based on this value for u-bar. Table 4.5 has all center lines and control limits for the four batch types. The reader should check the accuracy of these numbers. The chart of figure 4.9 is actually four separate control charts which, for convenience sake, have been plotted on the same graph. At present, there isn't enough data available on any of the batches to feel confident about conclusions made from these four separate charts. Nevertheless, the plot in Figure 4.9 seems to indicate that the four batch types are consistent within themselves in terms of the number of temperature excursions experienced. Further data on each batch type would be helpful in deciding on the consistency of the excursion rates for batch types.

The conclusions reached so far about the temperature excursions are that the number of excursions are inconsistent across time and there is a suspicion that the inconsistency may be influenced by batch type. The people at the chemical plant used this information to formulate a plan for further study of the effect of temperature excursions on batch processing. The first information they felt was necessary was to determine the nature of the differences in excursion rates. Although all of the batch types had different temperature profiles, the profiles were similar in that they all called for an initial step of increasing temperature followed by maintaining the elevated temperature and then having the temperature fall. The decision was made to record temperature excursions not only be batch type but also by the stage of temperature control. In other words, for each temperature profile, the number of temperature excursions would be recordes during the heating, maintaining and cooling stages. It was thought that this type of information might provide insights into the observed inconsistency in the number of temperature excursions observed in Figure 4.9. Then, armed with a better understanding of the reasons for inconsistent behavior by batch type, the plant personnel would have the ability to reduce the number of temperature excusions.

4.5 Supporting ideas for the effective use of c and u charts

The ideas discussed in this session are intended to provide the reader with some thoughts on how c and u charts can be most effectively integrated into a program for continually improving the activities of a business system. When the aim of charting is process improvement, getting the process "in control" is at best only a first step. Process improvement will be accomplished by identifying sources of variation and then acting on those sources to reduce variation. It is with this intent in mind that the folowing ideas on using c and u charts are provided.

The use of c or u charts does not begin with the collection of data. Rather, the intent or purpose of the study needs to be well outlined. Current information about the process should be collected as a guide to what is currently known about process operation and what information is lacking. The collection of this information is aided by the construction of flow charts and cause-and-effect diagrams. The flow chart is useful in describing how the process currently operates and in providing a useful reference for thinking about where critical process parameters may be impacting process outcomes. The cause-and-effect diagram is invaluable in describing what is currently known about factors affecting process output and in listing what further information is needed about the process. Armed with information about the current process operation, data collection can be guided to provide further information on improvement opportunities.

In planning the data collection strategy, the goal of developing a subgrouping or sampling strategy should be to maximize the opportunity to learn about the sources of variation. This goal is the first consideration in determining the frequency of subgroups, the sampling locations and the number of subgroups. For example, subgroups should be collected with sufficient frequency and over a sufficient time to allow those items listed on the cause and effect diagram to change between subgroups. In addition, consideration should be given to collecting data from the different streams identified on the cause-and-effect diagram.

As plans are developed for collecting process data, which specific process nonconformities to count will be decided. People often find the practice of reporting on a number of different types of nonconformities a beguiling one since, in this manner, a number of different "problems" can be attacked at once. Consider the previous example on recording temperature excursion. It would have been possible to count other types of deviations from protocol at the same time. The number of deviations from the correct pH level, the number of deviations from the correct stirring procedure, etc., could have been included. However, this practice of including many different types of nonconformities is best avoided. In general, a chart should report only nonconformities having similar causal structures, since the gathering of all possible categories of nonconformities on one chart may hide signals that woul be visible if the categories were charted separately.

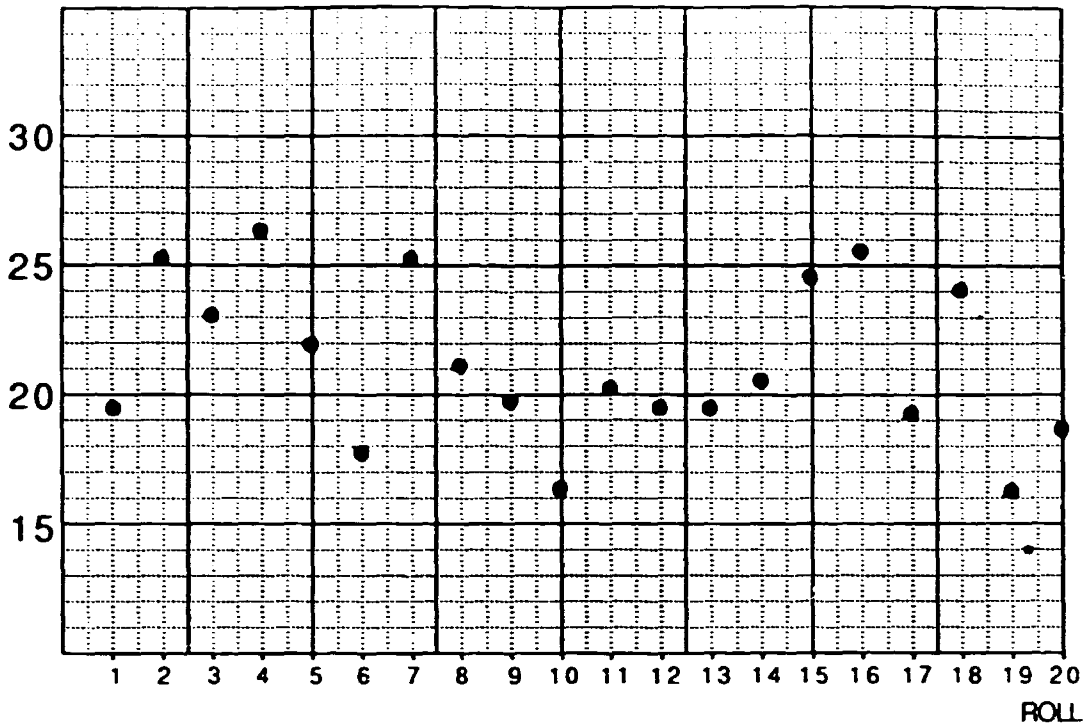
4.6 Chapter 4 Practice Problems

1. In the manufacturing of newsprint, a key issue is the uniformity of the rolls of paper. Customers expect the newsprint to be the same in terms of consistency and appearance from roll to roll. In your plant, the newsprint is manufactured by blending three pulps. The mixture is then moved to one of three paper machines where it is formed, pressed, and dried. The paper is rolled and cut into two possible lengths: 300-yard rolls and 500-yard rolls. In an attempt to determine whether or not the rolls are uniform across the two shifts, the number of nonconformities are counted on two rolls from each shift over a five-day period. An inspection unit is defined to be 100 yards.

Roll	Length	Shift	Number of Nonconformities
1	500	1	98
2	500	1	126
3	300	2	69
4	500	2	132
5	500	1	110
6	500	1	89
7	300	2	76
8	500	2	106
9	500	1	99
10	500	1	82
11	300	2	61
12	500	2	97
13	500	1	97
14	500	1	103
15	500	2	123
16	300	2	77
17	500	1	96
18	300	1	72
19	500	2	81
20	300	2	56

a. Prepare an appropriate control chart to check for evidence of lack of stability in the process.

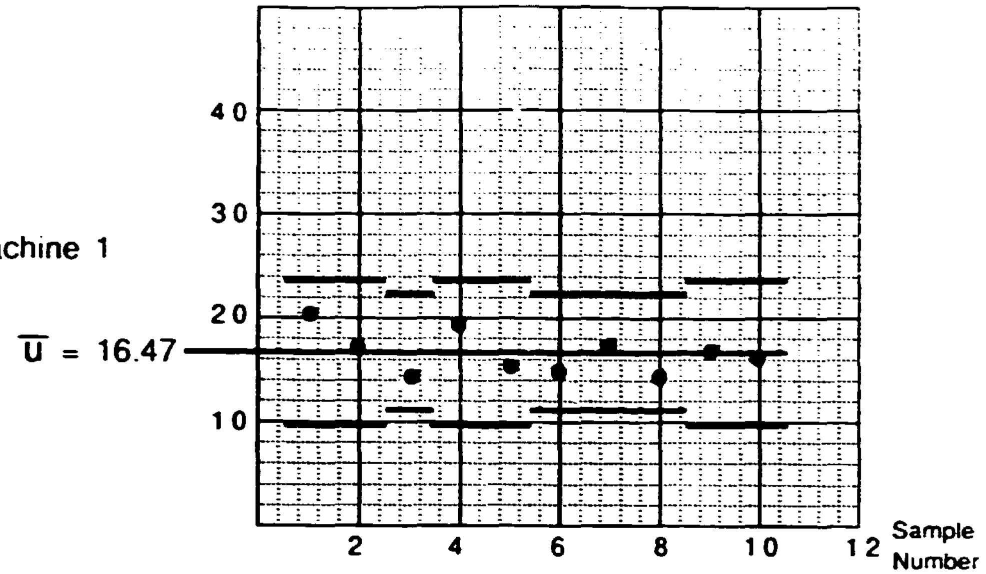
Number of Nonconformities per 100 Yards
in Rolls of Newsprint



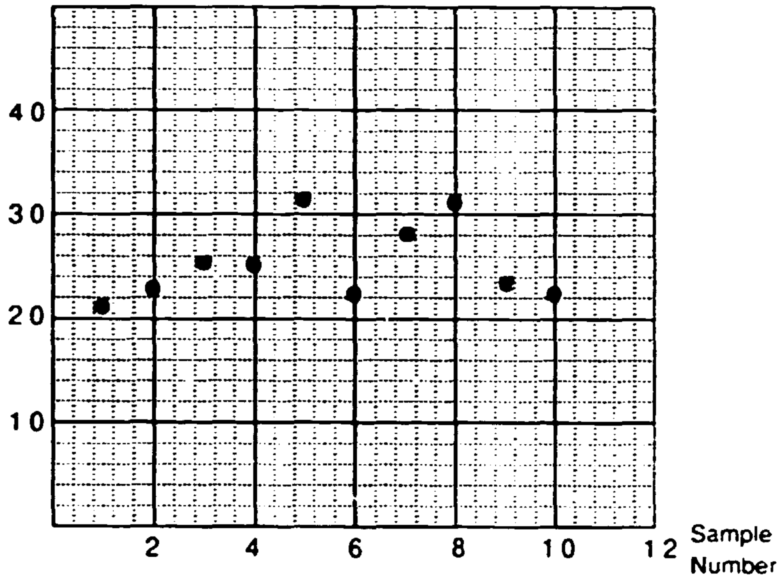
b. Based on the manner in which the samples were taken, what information about your process can your acquire from this control chart?
c. What would have been the advantages of developing a detailed flow diagram of the process and a cause-and-effect diagram prior to the collection of data?
d. In order to determine whether or not there exists a difference in the consistency and appearance of the rolls processed by the different machines, the number of nonconformities is counted on one roll per shift per machine over a period of two days. Are there any reasons why this sampling methodology might not be appropriate for obtaining the desired information? Give an example of one type of problem where this sampling procedure would not be adequate.
e. The number of nonconformities per 100-yards inspected have been plotted on separate control charts for the three different machines. Given the following data, construct the appropriate control charts for the three machines. (The control chart for machine 1 has been constructed for you.) Discuss any information provided about the process in these charts.

Roll	Length	Shift	Machine	Number of Nonconformities
1	300	1	1	61
2	500	1	2	106
3	300	1	3	70
4	300	2	1	53
5	500	2	2	115
6	300	2	3	55
7	500	1	1	72
8	500	1	2	128
9	500	1	3	90
10	300	2	1	59
11	300	2	2	76
12	300	2	3	59
13	300	1	1	46
14	300	1	2	95
15	500	1	3	94
16	500	2	1	76
17	500	2	2	113
18	300	2	3	97
19	500	1	1	88
20	300	1	2	84
21	500	1	3	96
22	500	2	1	71
23	300	2	2	94
24	500	2	3	100
25	300	1	1	51
26	300	1	2	71
27	500	1	3	94
28	300	2	1	49
29	300	2	2	67
30	500	2	3	92

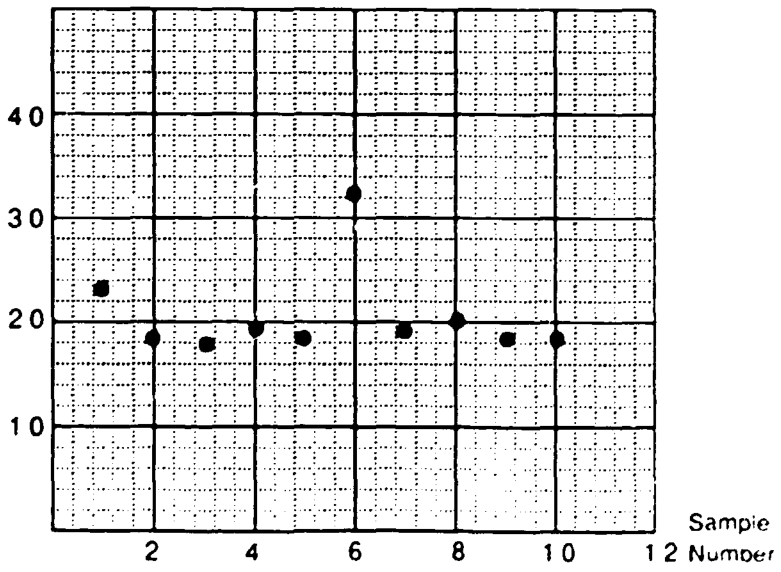
Machine 1



Machine 2



Machine 3



f. Information was also collected on possible causes of the lack of uniformity between the rolls of newsprint.

Reasons for Nonconformance	Frequency Machine 1	Machine 2	Machine 3
Holes	27	99	87
Thin Areas	128	255	293
Discolorations	206	161	122
Brightness Variation	80	84	103
Rough Areas	169	287	214
Others	16	63	28

If appropriate, perform a Pareto Analysis for each individual machine.

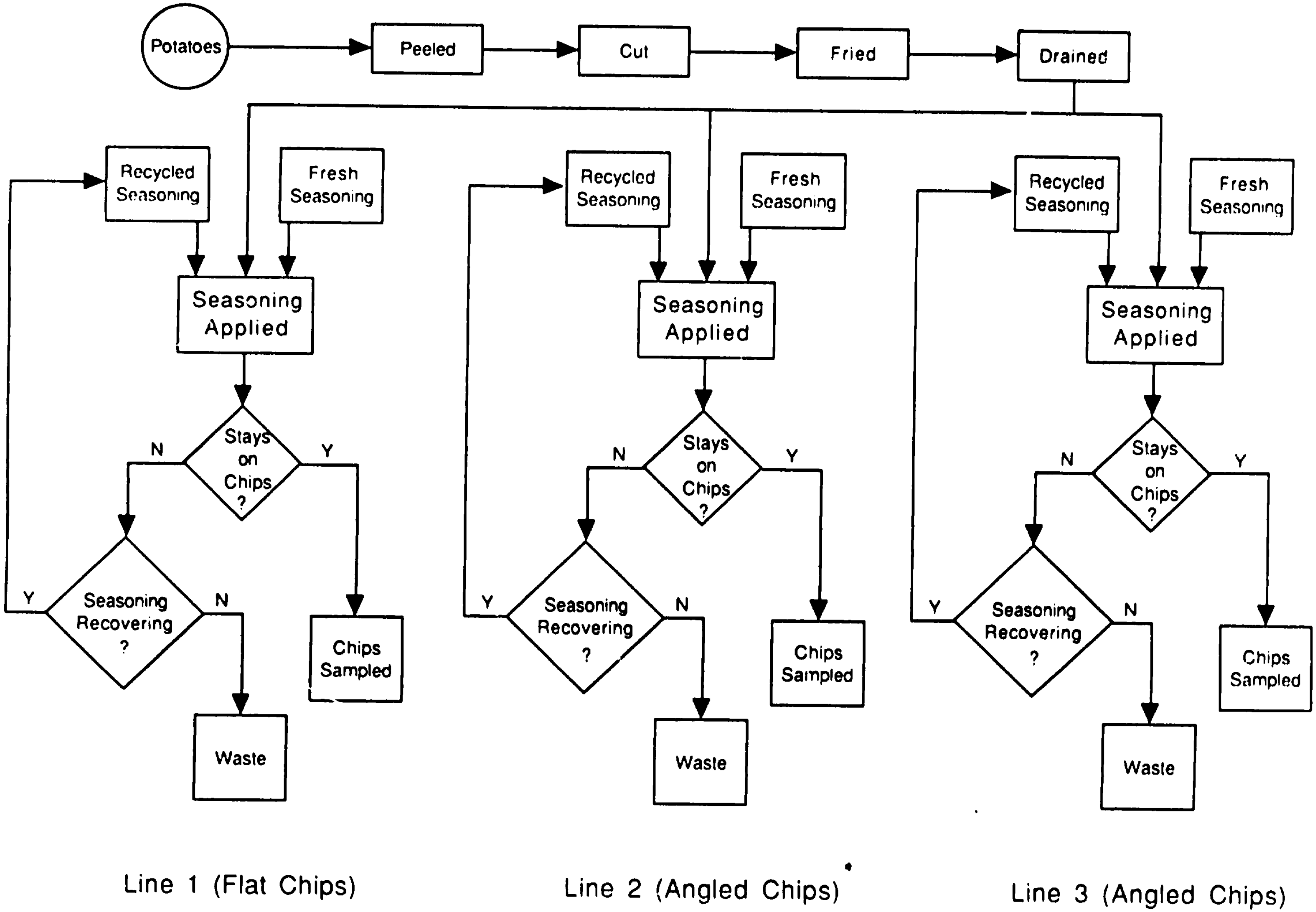
2. In an electronics manufacturing facility, radios are checked for nonconformities after the assembly process is complete. For some time, the process has been in control at an average of six nonconformities per radio.

   a. If the process were to be monitored by inspecting three radios from time to time and recording the total number of nonconformities per group, what type of control chart should be used?
   b. What should the central line and control limits be for tne chart?

3. In a potato chip plant, after the chips are drained, they enter into one of three lines where seasonings are applied. Line 1 applies seasoning to flat chips, while the other two lines season the "angled" chips. After the chips are seasoned, they are sampled by one of three inspectors on that shift. The inspectors are responsible for sampling chips out of each seasoned batch and determining whether the batches are consistent with respect to chip presentation and amount of seasoning. If the inspectors find that a series of several batches are inconsistent, they can stop the line for adjustment. The line can also stop because of machine breakdowns, change of seasonings, shortage of chips at the start of the seasoning lines, etc. You are on the work team which is investigating the stops in the production line.

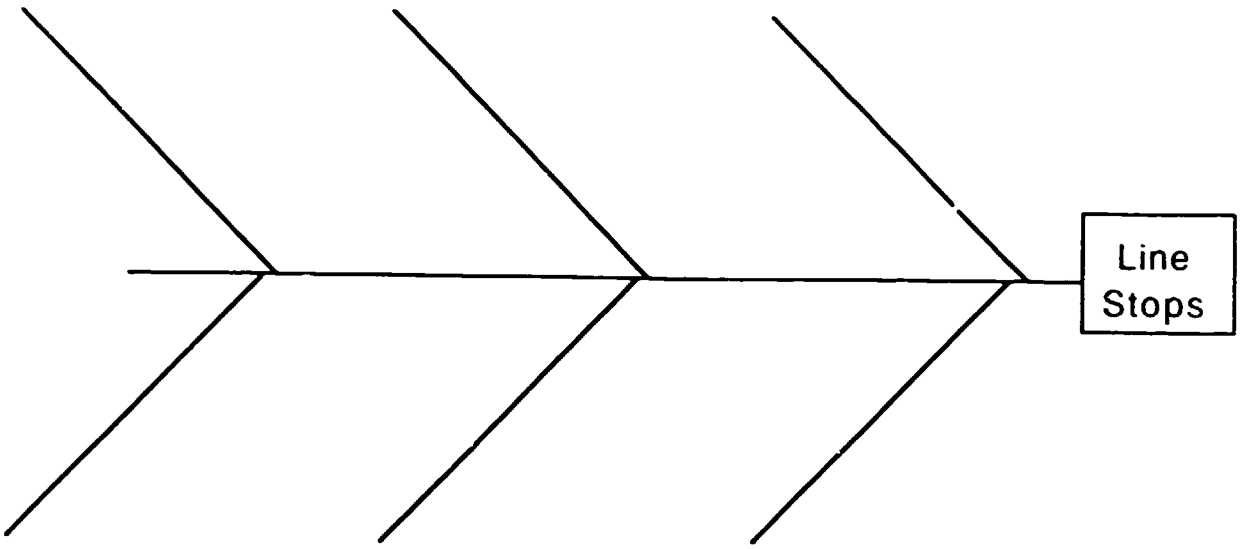
   In order to help determine where the specific causes of the line stops may be, the flow diagram of the process has been developed. In addition to the flow diagram, you have suggested that a cause-and-effect diagram would also help the team to identify possible causes of line stops. Before constructing the cause-and-effect diagram, you questioned several people on the lines. The following are comments that were made:

   > "As I see it, the line stops can't be prevented until someone changes the way the system is run. There are just too many different people who can stop the line."
   >
   > "The inspectors are the problem. There's one inspector who calls for a line stop for every little thing."
   >
   > "If we didn't always have to wait for the chips to be delivered to the line, the situation would not be so bad."
   >
   > "I can tell you where the problem is: It's with maintenance. These machines are just patched so we can keep running. If they would fix them right, it would make a big difference in the long run."



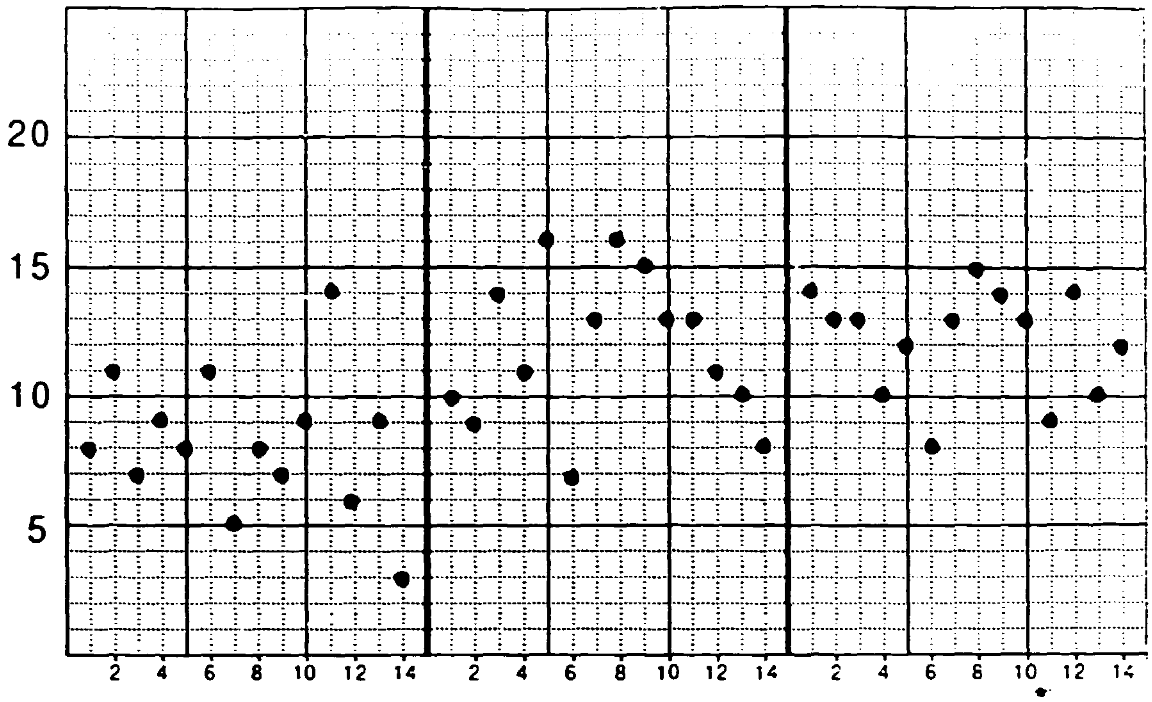
Potatoes
Peeled
Cut
Fried
Drained
Recycled Seasoning
Fresh Seasoning
Seasoning Applied
Stays on Chips ?
N
Y
Seasoning Recovering ?
Chips Sampled
Waste
Line 1 (Flat Chips)
Line 2 (Angled Chips)
Line 3 (Angled Chips)

a. Develop the following cause-and-effect diagram to help identify possible reasons for the line stops.



b. The following data report the number of line stops per day (24 hours) for each of the three lines. Construct a control chart to determine whether there exists a difference in the number of line stops between the three lines. (Put individual sets of control limits on the chart for the different lines.)

	Number of Stops		
Day	Line 1	Line 2	Line 3
1	8	10	14
2	11	9	13
3	7	14	13
4	9	11	10
5	8	16	12
6	11	7	8
7	5	13	13
8	8	16	15
9	7	15	14
10	9	13	13
11	14	13	9
12	6	11	14
13	9	10	10
14	3	8	12
	115	166	170



c. Your team has asked you to make recommendations on what the next step in the improvement effort should be. What will your recommendations be?

4. A company manufacturing oilcloth figures all cost estimates and prices on the basis of 100 square yards of oilcloth. The following data are obtained from inspection:

Lot Number	Square Yards Inspected	Total Number of Defects in Lot
1	200	5
2	250	7
3	100	3
4	90	2
5	120	4
6	80	1

Construct the appropriate preliminary control chart(s) for these data. Comment on the state of control.

Table 4.1

Number of Pin Holes in Aluminum Rolls

Subgroup	Number of Pin Holes	Subgroup	Number of Pin Holes
1	22	11	15
2	29	12	10
3	25	13	33
4	17	14	23
5	20	15	27
6	16	16	17
7	34	17	33
8	11	18	19
9	31	19	22
10	29	20	27
		Total:	460

Table 4.2

Study of Pin Hole Counts Performed by Aluminum Supplier and Can Manufacturer

Shipment	Number of Pin Holes Found by Supplier	Number of Pin Holes Found by Manufacturer
1	22	29
2	18	19
3	17	23
4	14	33
5	18	10
6	11	27
7	17	34
8	12	15
9	9	17
10	14	27
	152	234

Table 4.3

Number of Excursions from Temperature Profile

Subgroup	Batch type	Hours	Number of excursions	u
1	A	10	3	.300
2	B	8	0	0
3	C	12	4	.333
4	A	10	7	.700
5	B	8	1	.125
6	A	10	2	.200
7	C	12	7	.583
8	D	8	1	.125
9	B	8	4	.500
10	C	12	1	.083
11	D	8	1	.125
12	A	10	9	.900
13	C	12	2	.167
14	A	10	6	.600
15	B	8	2	.250
16	D	8	3	.375
17	C	12	0	0
18	D	8	2	.250
19	A	10	8	.800
20	C	12	3	.250
		196	66	

Table 4.4

Control Limits for u chart on
Number of Excursions from Temperature Profile

Batch type	Hours ...(n)...	UCL	LCL
A	10	.8872	-
B	8	.9522	-
C	12	.8392	-
D	8	.9522	-

Table 4.5
Control Limits for u chart on Number of Excursions
from Temperature Profile using Distinct Center Line Values

Batch type	Hours ...(n)...	$\bar{u}$	UCL	LCL
A	10	.583	1.307	none
B	8	.219	.715	none
C	12	.283	.744	none
D	8	.219	.715	none

FIGURE 4.2
COMPLETED CONTROL CHART

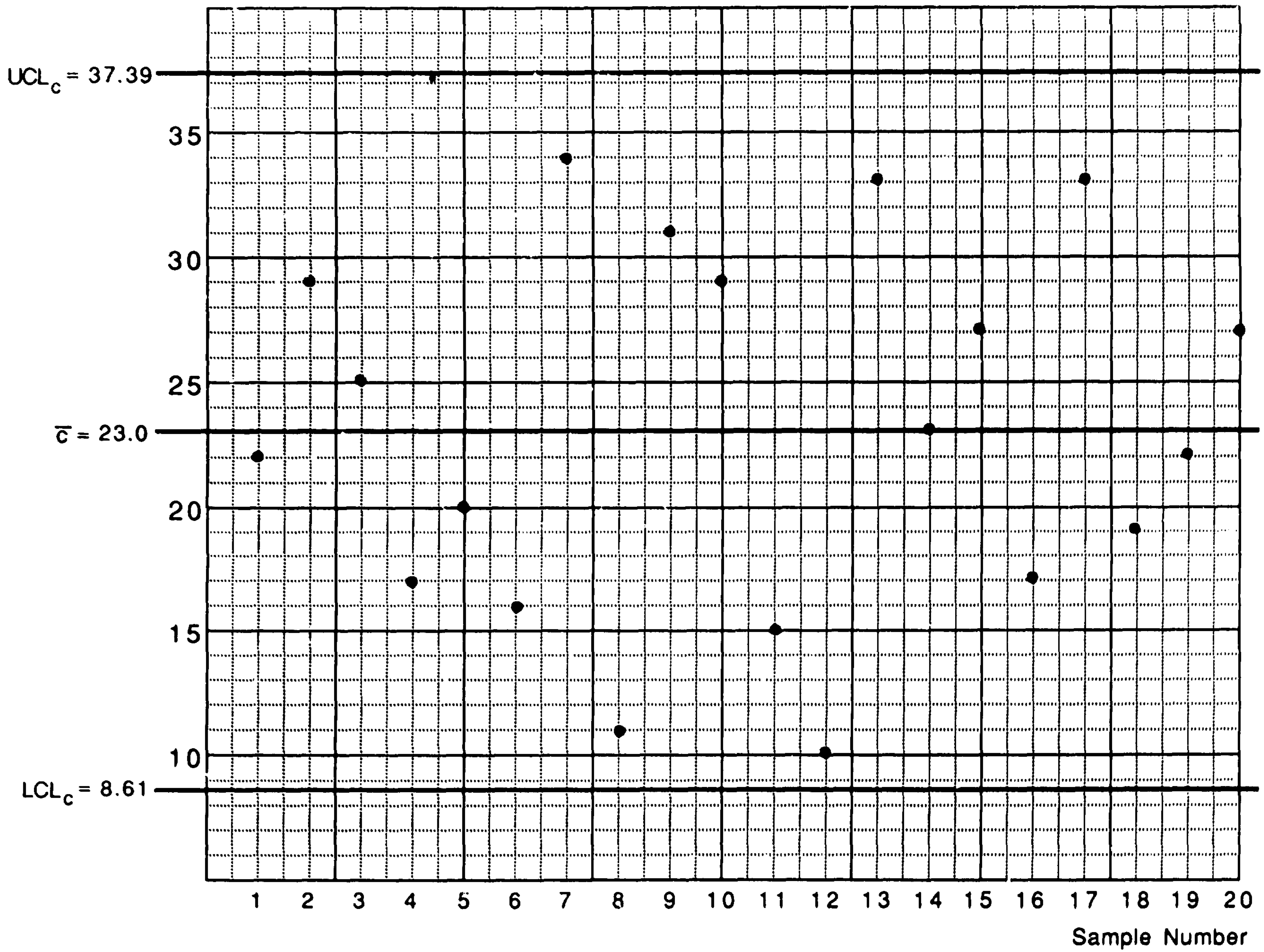


FIGURE 4.3

PLOT OF NUMBER OF PINHOLES FROM TABLE 4.2

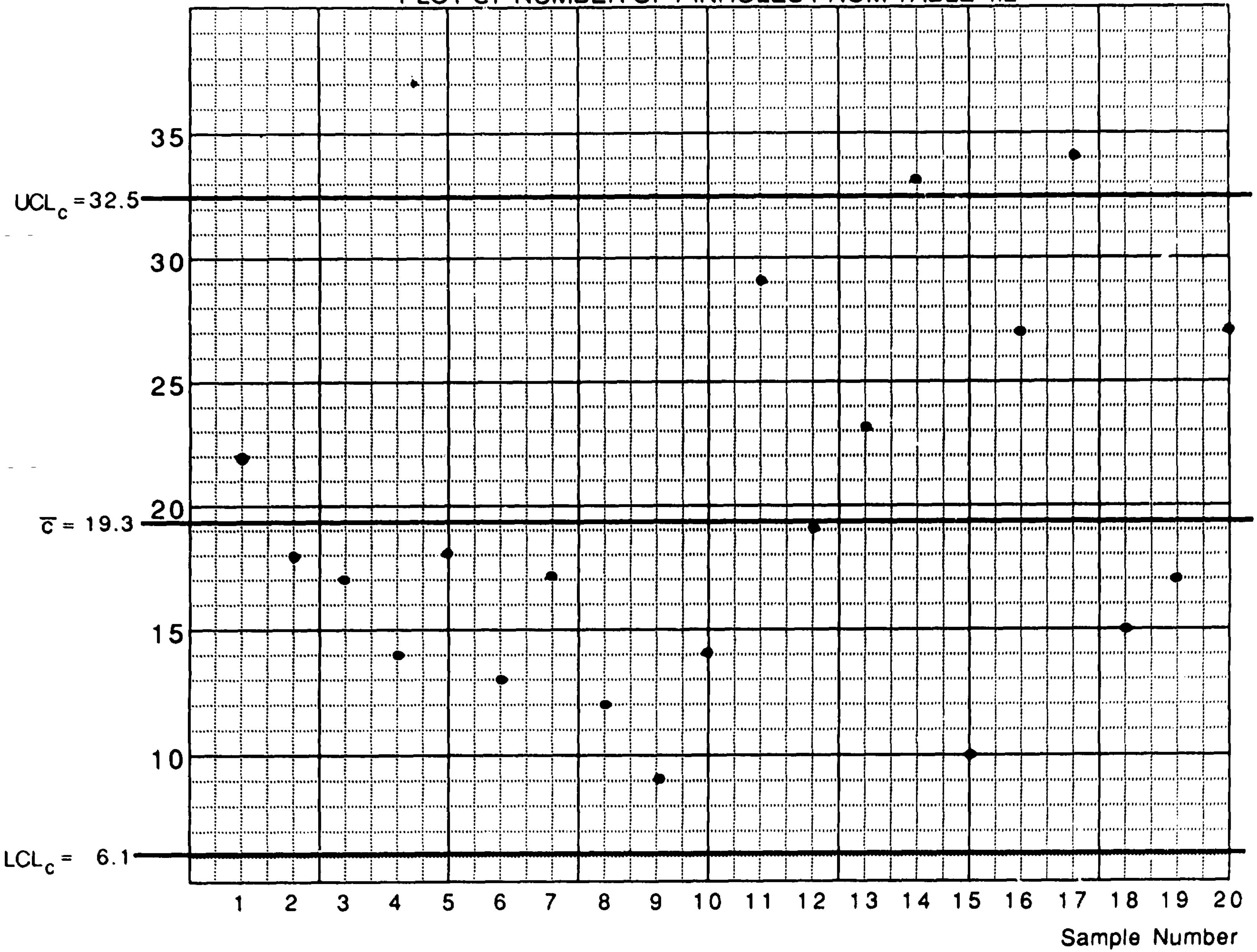


FIGURE 4.4

TEN SUBGROUPS OF COUNTS OF NONCONFORMITIES MADE BY THE SUPPLIER

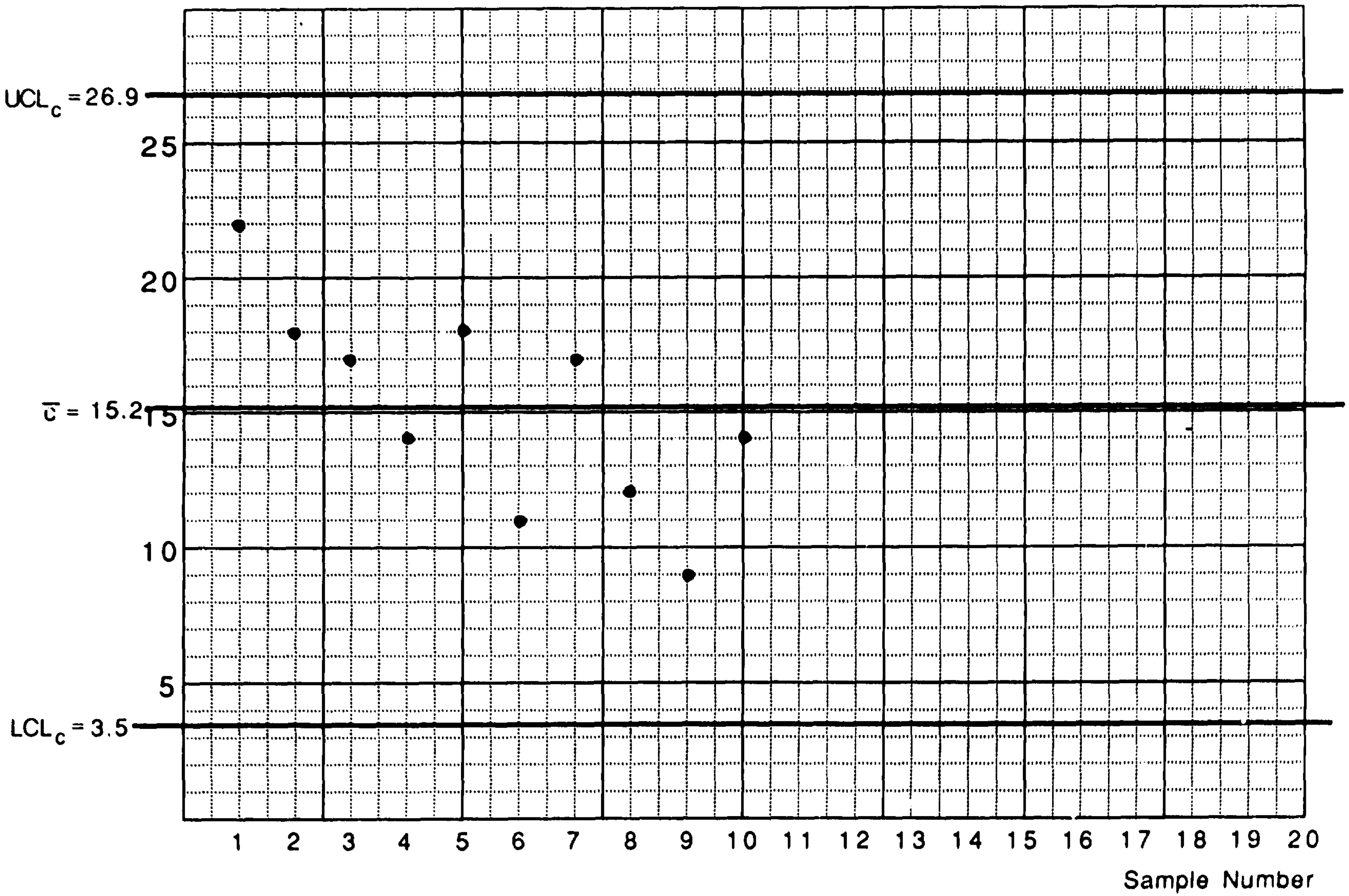


FIGURE 4.5
TEMPERATURE PROFILE

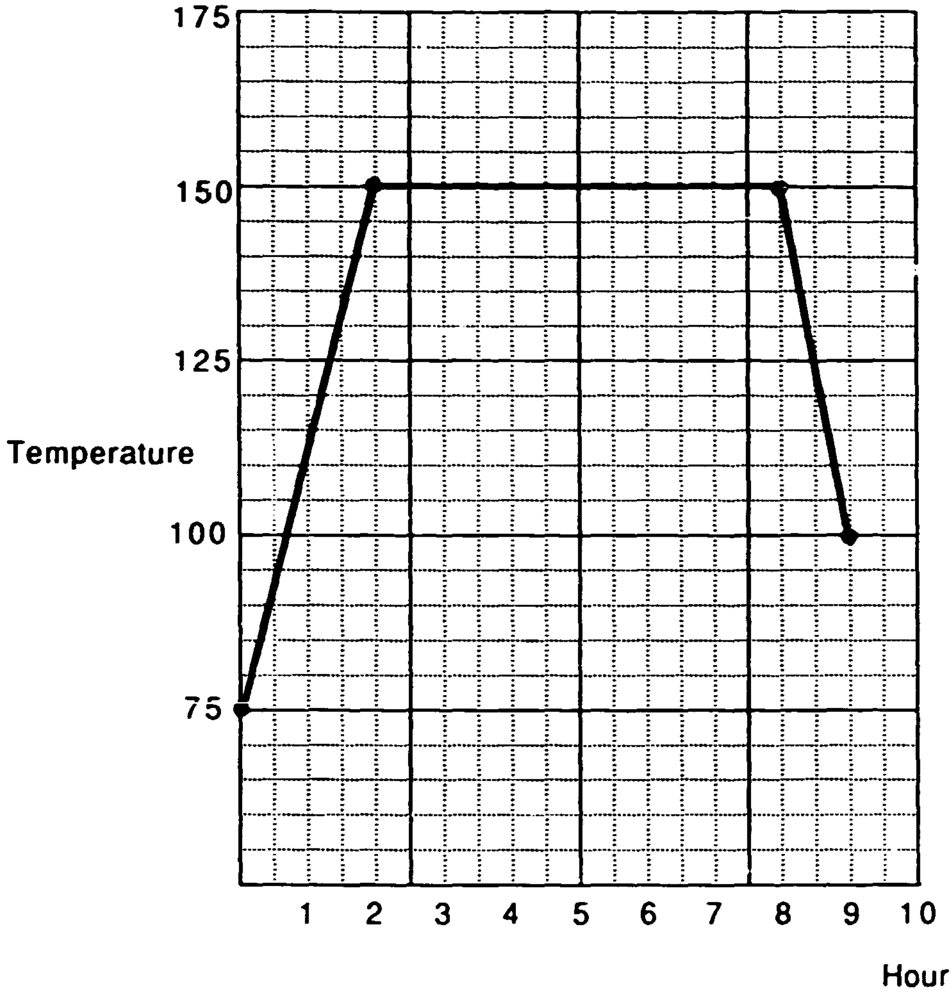


FIGURE 4.6
TEMPERATURE PROFILE WITH LIMITS

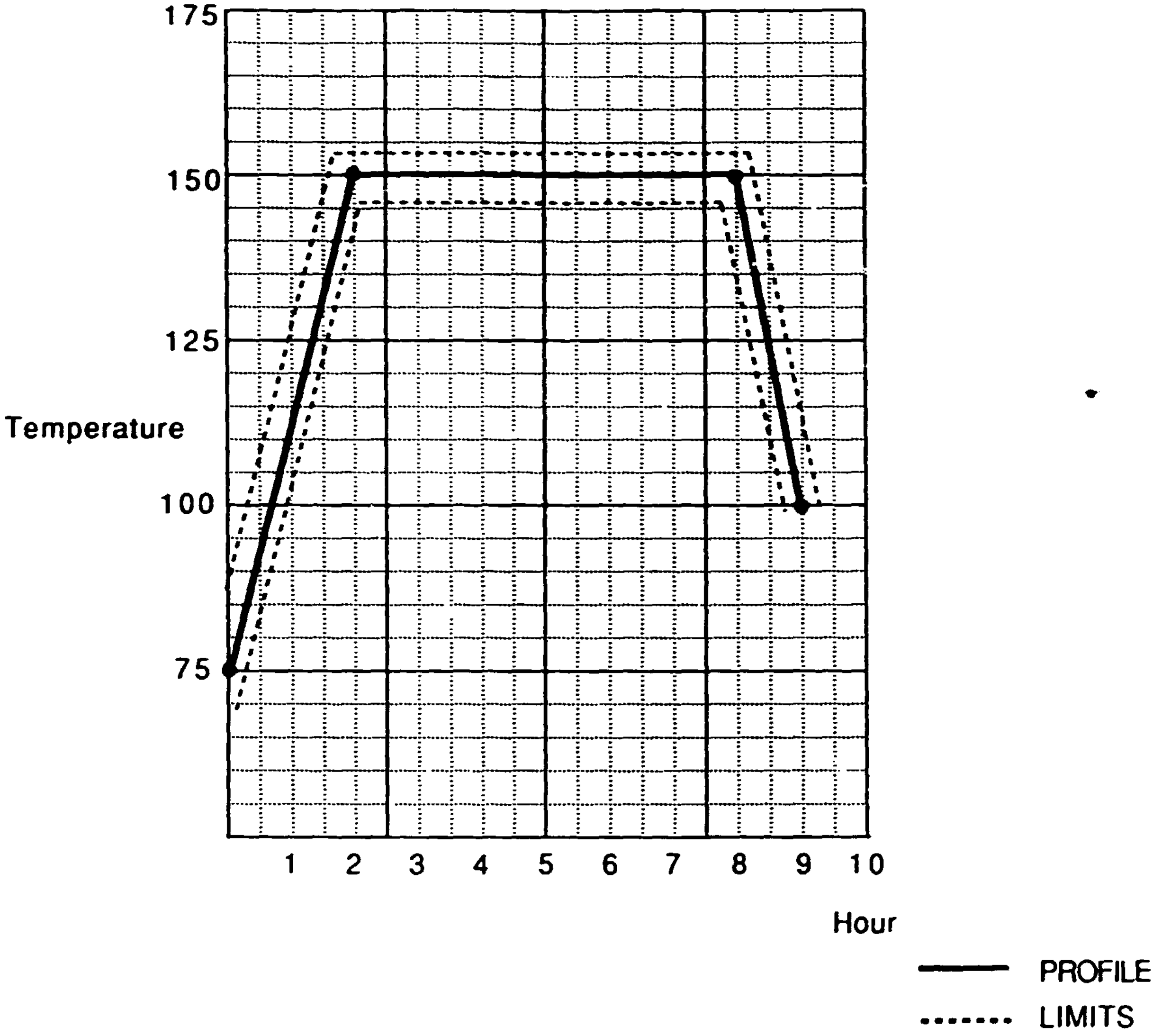


FIGURE 4.7

COUNTS OF OCCURRENCES PER INSPECTION UNIT

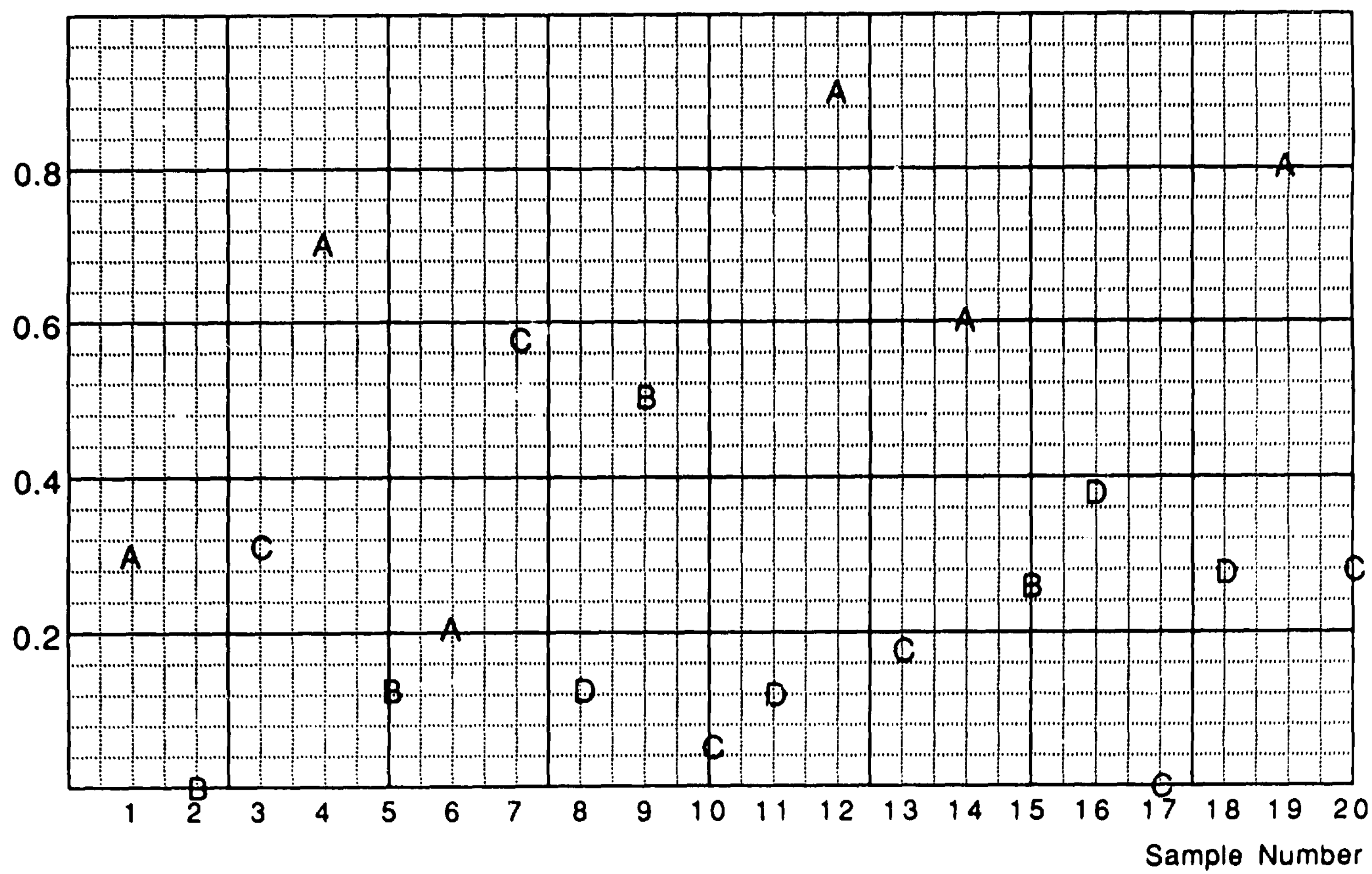


FIGURE 4.8

COMPLETED U CHART

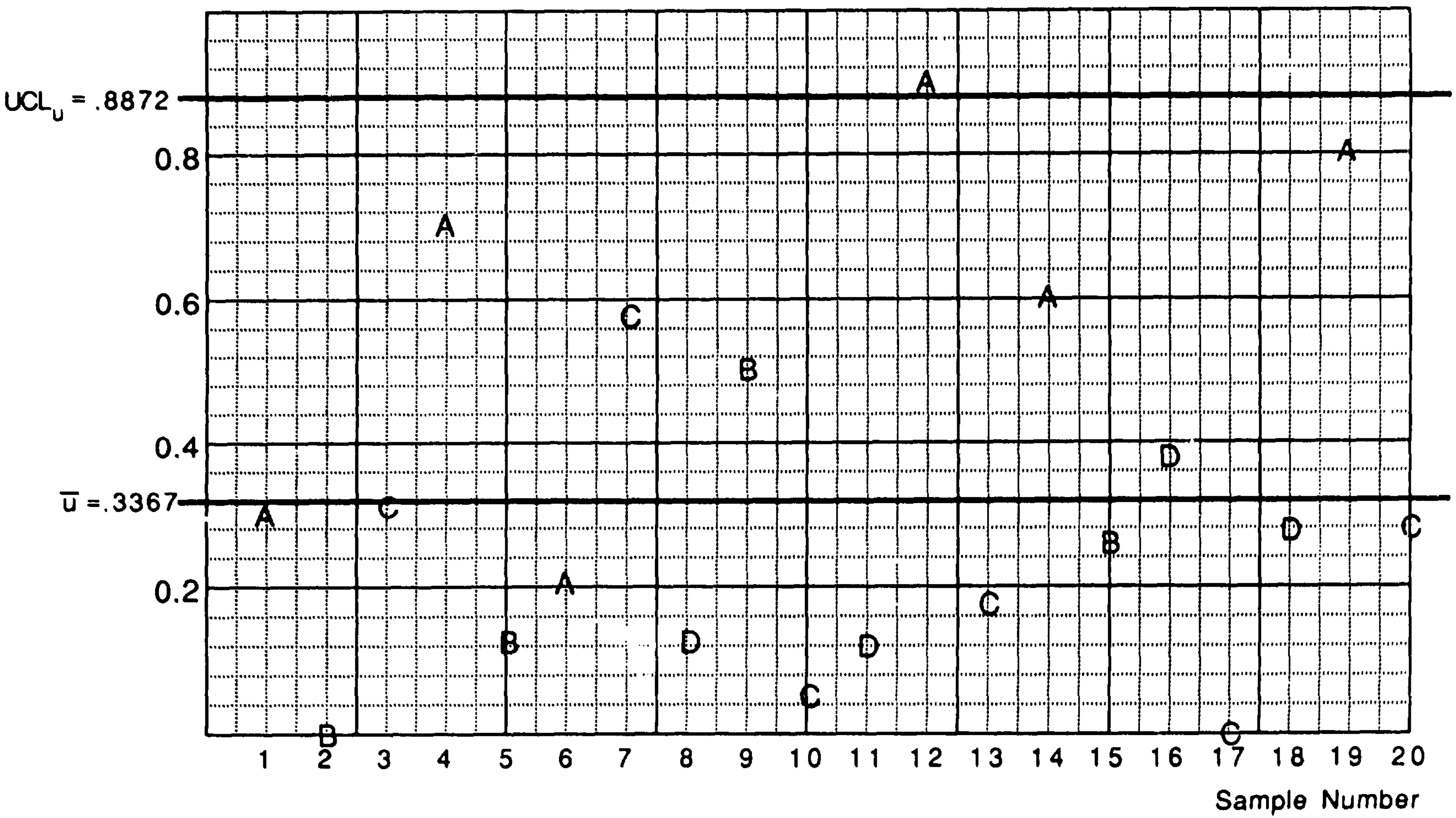


FIGURE 4.9

U CHART ARRANGED BY BATCH TYPE

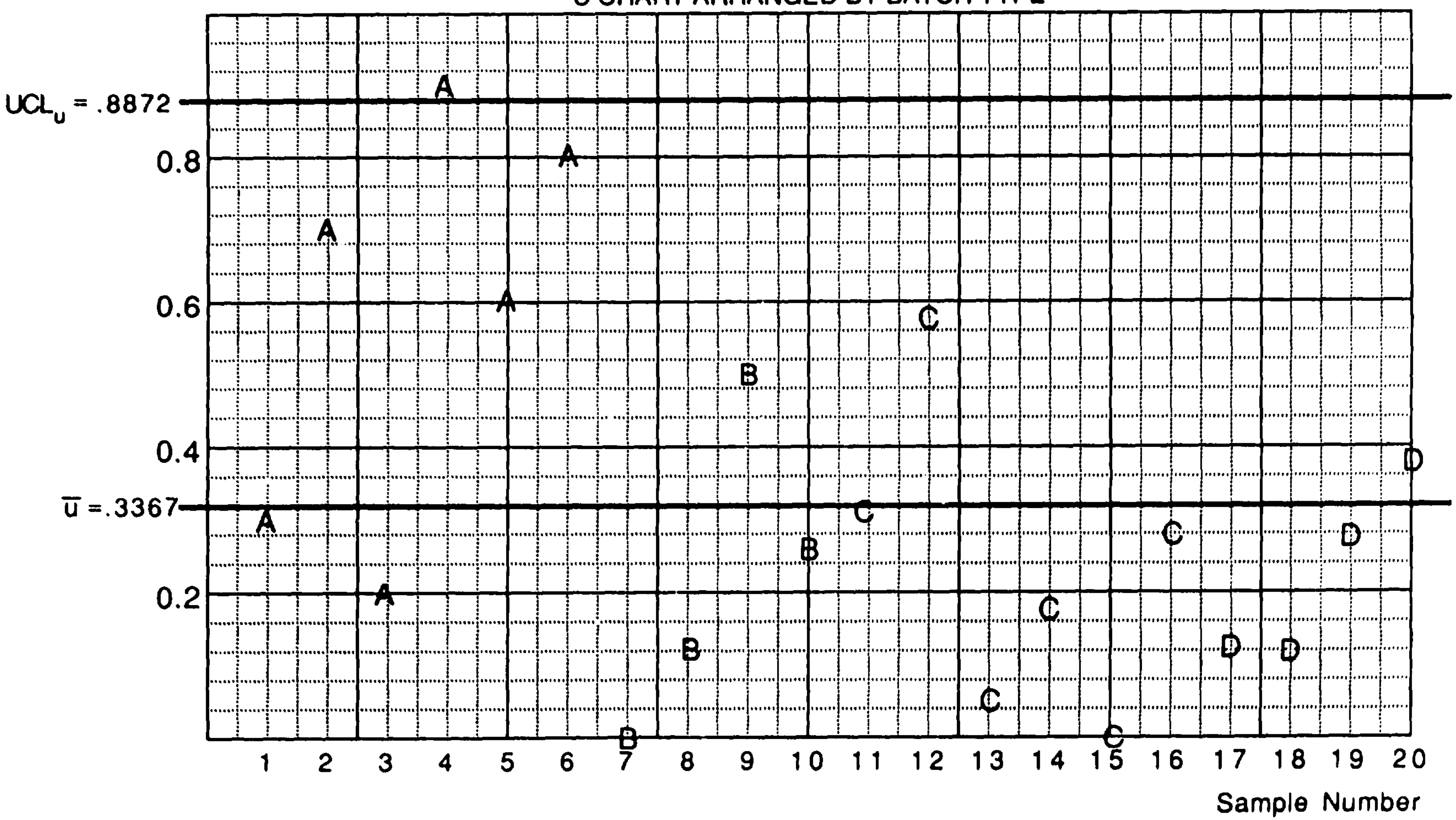
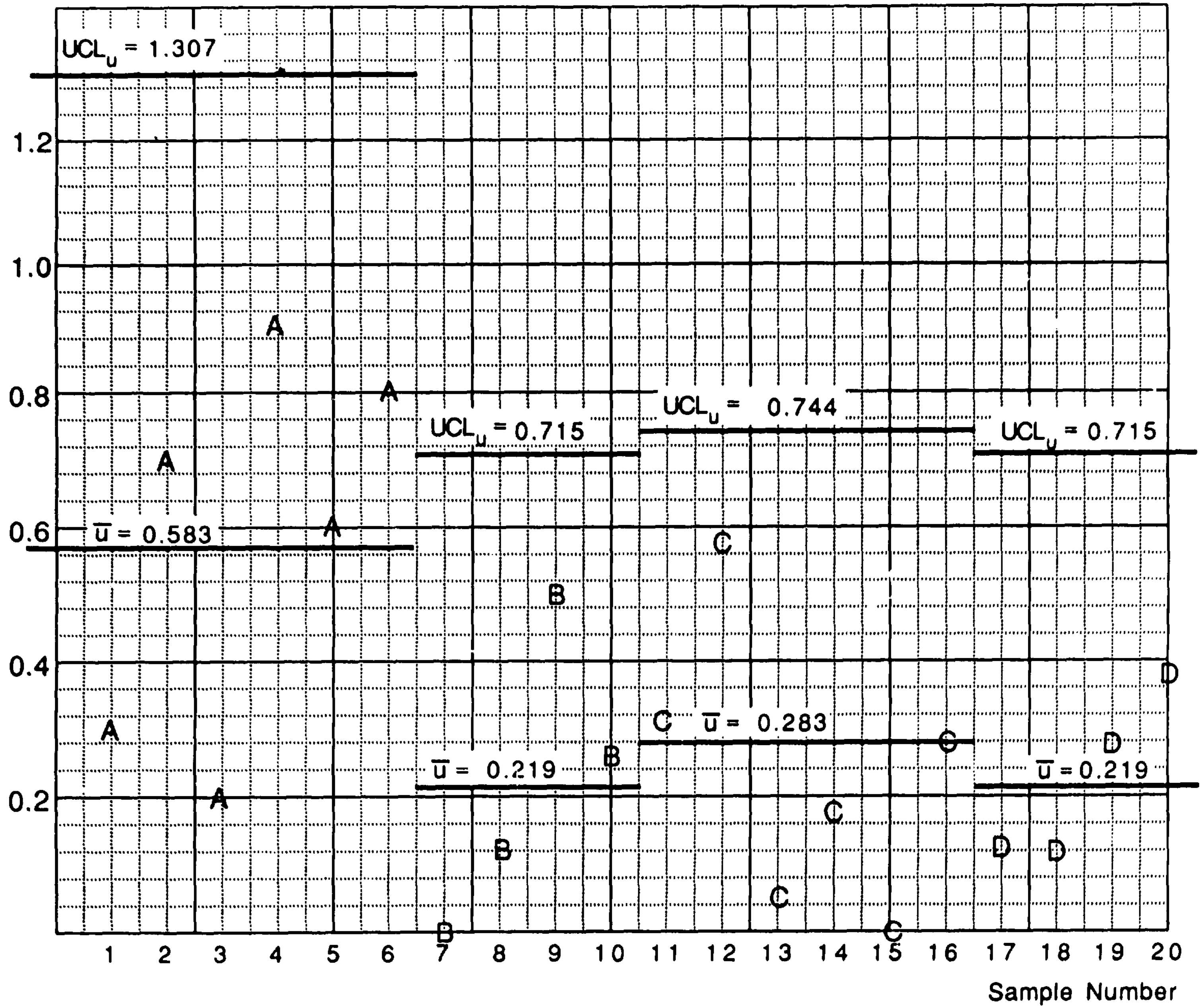


FIGURE 4.10

U CHART WITH SEPARATE CALCULATIONS BY BATCH TYPE



Chapter 5

Control Charts for Variables Data: Variability and Location

There is a wide variety of techniques for data analysis. These techniques include run charts, control charts, scatter diagrams, analysis of variance procedures, time-series techniques, and others. Selection of an appropriate technique should depend primarily upon the nature of the problem but also upon the type and quality of the numbers involved and the background of the people involved in preparing, analyzing, and acting upon the data summaries. Significantly, technique selection should depend upon the questions and issues which motivated the data collection. This chapter is directed toward an introduction to the construction and use of control charts for variables data, techniques which are particularly useful in supporting work directed toward process control and improvement.

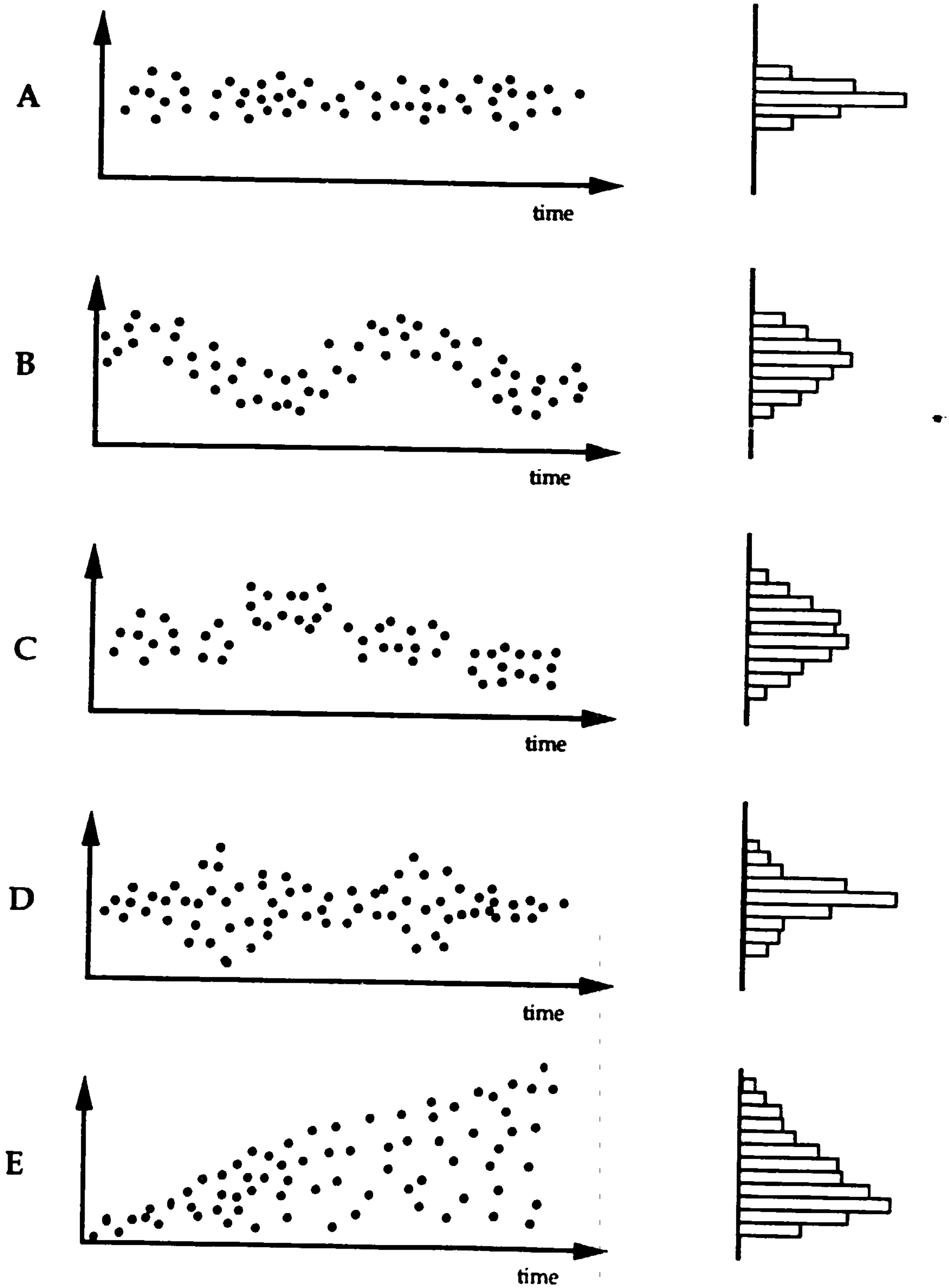
Measurement of characteristics of processes, goods and services may, as discussed previously, yield either quantitative or qualitative evaluations. Qualitative evaluations of goods and services typically result in 1) counts of the number of nonconforming or, 2) counts of the number of errors, defects, or omissions. Suggested analytical treatment of these two types of data have been discussed in preceding chapters. Quantitative evaluations or variables data, require determination of the amount or degree to which a process condition, parameter, job or service possesses a characteristic under study. For example, variables data result from measurements of density, pressure, temperature, resistance, force, hardness, or dimensions. The collection of such measurements requires that an adequate process for obtaining the measurements be in place. In this chapter, it is assumed that a measurement process has been evaluated and found to be predictable and adequate for the current purpose; the specific nature of the methodology regarding measurement process evaluation will require further discussion after some additional statistical methods have been discussed. Suffice it to say at this point that consistent measurement processes are a prerequisite for successful process analysis.

Measurement, sampling strategy, and selection of statistical techniques must support process analysis. The purpose and rationale for process investigations must be clearly understood by the manager, engineer or operator. Process investigations should be directed toward seeing and understanding changes in material, methods, and equipment and in knowing the effect that these changes have on process output. Obviously, processes undergo changes in materials, methods, or equipment from time to time; environmental changes also occur. Discovering when and under what circumstances these changes take place, and verifying the cause and effects of these changes in terms of magnitude and direction for the measurement in question is an essential responsibility in process investigation. Investigations are intended to provide specific knowledge regarding the effects of changes in various process factors upon the product characteristics being studied. The defined needs for knowledge begin to provide guidance in the way in which process data are collected and used. When organized and presented in an appropriate manner, process data provide the manager, engineer, and operator a medium through which changes taking place in the process can be seen and evaluated.

5.1 Description of Types of Variation in Process Data

The purpose of data analysis is to provide information for process control and improvement. To that end, data analysis must contribute to identifying and evaluating sources of variation in process variables and outcome results. Sources of variation reveal their effects in the magnitude of the short-term variation, in the average, and in any change in these attributes of process data. Consequently, initial data analysis should be directed toward measuring process variability and process level or location and evaluating predictability. An introductory discussion regarding these issues is to be based upon a series of short examples. Data plots from five processes, A, B, C, D, and E are shown in Figure 5.1. In each case, the vertical scale corresponds to a number line for measurements on a process input, a process parameter, or a process output. These measured values could be viscosity, length, density, flow rate, temperature, or thickness. The horizontal scale corresponds to time. Over an extended period of time, measurements on process output are made and plotted in production order. Although these examples of possible process behaviors surely are not exhaustive, they collectively display patterns and characteristics that demonstrate some of the central issues in data analysis. These issues are emphasized and discussed by comparing the time plots with respect to variability, average, and predictability.

Figure 5.1 Process Examples



First, consider the data plot representing measurements on a characteristic for process A. For convenience of discussion it is assumed that these measurements have been made on a process output. In common with all processes, process A does not provide exactly the same result each time. Process A obviously exhibits or displays variability; all the data points are not the same in value or magnitude. The magnitude and nature of this variability is of primary interest. How large is it? What causes it? Is it predictable? For process A, the range from the smallest to the largest numbers during any short time period appears to be about the same throughout the total, or "long," time period covered by these data. Because of this uniformity in variation, short-term variability is said to be predictable. In terms of a specific measurement and sampling strategy, a process is either stable and predictable or unstable and unpredictable. A conclusion regarding stability in short-term variability marks a significant finding in process investigations. Long-term variation, related to changes or shifts in process level or average, is a different issue. Consequently, another important property of process measurements is the level around which individual values tend to fluctuate. Process A, as an immediate example, appears to operate at a nearly constant average value throughout the period for which data were collected. Because of this apparent constancy in average value, the process level is said to be predictable. Because process A is judged to have almost constant short-term variability and nearly constant average; the process is said to be in statistical control.

By comparing observed properties of other processes to those of process A and by contrasting other processes, one against the other, additional insights can be had concerning important features of process data about which data analysis must provide information. Consider the nature of the short-term fluctuations for process B. For results completed within a small span of time (to be taken as a short interval on the horizontal axis) the range from the smallest to the largest measurement appears to be of about the same magnitude regardless of the selected time interval. With that being the judgment, short-term variability for process B would be described as being predictable or stable. These sentences would serve equally well as descriptors of the point to point variability displayed in the data plot for process C. For process B and C, respectively, the point to point fluctuation, described as short-term variability, is about the same size or magnitude as that displayed by process A. However, results for processes A, B, and C, obviously are not equivalent, as a quick visual comparison makes apparent. Specifically, process B does not operate at a consistent average over the long time period represented by the complete horizontal scale. The same remark applies to the measurements from process C. Process B appears to undergo smooth, incremental changes in its average; significantly, it is not possible to judge whether this apparent "cyclic" behavior replicates itself in a consistent fashion over longer periods of time. In contrast to the smoothness in changes displayed by the average for process B, process C exhibits abrupt shifts in its average. This assessment of inconsistent average for these two processes is distinctly different from that made for process A, which was judged to be operating at a consistent average. Processes B and C do not have predictable averages.

Process D also appears to be operating at a consistent average over time. However, process D differs from A, B, and C in an important way. Process D displays inconsistent short-term variability in individual measurements around its long-run average while process A apparently maintains the same magnitude for fluctuations around its average throughout this time period. Process E displays a trending average together with an increasing variation in individual measurements.

It is apparent that important characteristics in these processes are the short-term variation and the level or average around which the process tends to fluctuate. Data analysis must provide information on these characteristics of variation and its causes. Control charts are excellent for providing this information. Control charts are meant to help discover the effects of changes in processes over time. The control charts can signal when things are different. With diligent data collection and a clear understanding of measurement and sampling strategy, the charts can help provide the basis for suggesting why things are different.

Process consistency relates to fluctuations in individual values in the short term and to possible fluctuations in average over a longer period of time. Process A is said to be in control with respect to variation and average because fluctuations in individual values over a short interval seem to be of about the same magnitude throughout the time period and because the average value remains unchanged for the time period. Processes B, C, D, and E are said to be unpredictable, or "out of control". Data from B and C, respectively, reveal a consistent short-term fluctuation in individual values but an inconsistent, or changing, average. Process D maintains the same average throughout, but suffers from inconsistent fluctuations around this average. Process E is inconsistent with respect to both variability in individual values and in the average. In summary, the processes B, C, D, and E, respectively, do not display consistent behavior. Control charts can be used to provide information as to the state of control of variations in the process or its results.

It should be emphasized that changes over time in an average as well as deviations in individual values around an average contribute to total variation in process output. All values plotted for process A have been used to construct the histogram which appears to the right of the time plot for process A in Figure 5.1. Similarly, histograms for processes B, C, and D are also on the right hand side of Figure 5.1. It is easily appreciated that the values from processes B, C, and D are more variable than the values in the distribution from process A, although the fluctuations around the average during any short period of time appear to be of about the same magnitude for these four processes. All of the values represented by the plotted points for the characteristic in question may be within specifications; that is not the issue. The central issue is that the patterns for processes B, C, D, and E, with their respective changes in short-term variability and in their average, contribute to the overall variation. The thrust of the work should be to identify the reasons for this variation and then to work to reduce it.

5.2 Identifying Variation and Knowing Its Sources

Understanding variation in process output and knowing its causes is furthered in those situations where the variation exhibited by process output can be associated with potential sources of variation. Processes A through E, described in the previous section, exhibited different types of variation over time. Subsequent understanding of these processes would entail beginning to characterize the sources of variation which are creating the assignable causes of variation and those which are contributing to common cause sources of variation. The data plots aid in this analysis since they describe the manner in which the process output changes as the variable time changes. However, time itself is an ill-defined proxy variable. Time corresponds to numerous changes in processing circumstances, with any of these changes having the potential to affect the short-term variation or average of the process output. For example, component parts or raw material surely change over time and their characteristics may change in one or more respects. These changes, in turn, may generate increased fluctuations around an average or they may cause the average to shift up or down. Another example of changes which may occur over time is that management, operating, and engineering personnel change as shifts, weeks and months pass and as they change, so may the methods or techniques for operating machines, assembling components, or mixing materials. These changes may affect the variation, the average, or both. Environmental variables, such as temperature and humidity, can also change in a dramatic fashion over time. These changes may cause other changes which then affect output characteristics. Operating efficiency for process and laboratory equipment will change with the passage of time and so the state of maintenance becomes a critical issue. If process output is supposed to be the same all the time, then processes should not be affected by typical changes in important or influential variables. It becomes management's job to assure that either process output is robust to changes in process variables or, failing that, assure that process factors or variables are managed at specific averages within given specifications.

5.3 Range and X-bar Charts and Process Analysis

As process analysis begins, information similar to that displayed in the five time plots of Figure 5.1 is not typically available. The manager, engineer, or operator who begins to study the process may be said to be searching for a description of process behavior. In fact, the analysts are looking for the signs of inconsistency noted in process B, C, D or E. A process is judged to be consistent if there is no evidence in data collected over time that points to non-random behavior, which is interpreted as a signal of inconsistency. If a process is found to be inconsistent, that is, out of control, then the first responsibility is to get the process into control. Once the process is in control, the ongoing responsibility is to maintain control. Once a process is in control and the ability to maintain this control is demonstrated, then the variation of the characteristic in question is predictable. Those responsible for the process can begin to compare the current output against what is required. After control is achieved and maintained, responsibility moves toward working on the process to obtain improvement. Process improvement may be realized in many ways. In this immediate discussion, process improvement is defined as reduced variation and, if necessary, a different, more favorable average. If the process is found to be in control, then the cycle begins with maintaining control coordinated with systematic changes toward improvement. It should be emphasized that process evaluation and work is an iterative activity; it does not end. It is apparent that throughout process work, a premium is placed upon knowledge regarding possible sources of variation.

Processes tend toward instability and chaos. Processes do not tend to remain in control. Maintenance of stability becomes an important responsibility for all managers and operators.

5.4 Arithmetic for the Construction of Range and Average Charts

The data set printed in Table 5.1 is presented only for the purposes of illustrating and practicing the arithmetic necessary to complete some basic statistical techniques for analyzing variables data in process analysis. Twenty subgroups, each of size four, are shown. An example of each arithmetic step is given. It is recommended that the reader practice the calculations. Some explanatory remarks are provided at specific points in the presentation. Symbols and definitions are introduced as needed.

The numbers in Table 5.1 are meant to represent measurements of a single characteristic on a collection of parts. The data are organized in subgroups of four readings each. This implies that the four values were collected under the same conditions, or at the same point in time, or from the same source. There should be a rational basis for subgrouping the data.

There are several kinds of information wanted frcm this data set. Certain kinds of information will be required of each subgroup and other kinds of information will be needed for all subgroups. For each subgroup, the subgroup range will be computed as a measure of how much the four parts differ among themselves. The average value is also required for each subgroup. All of the subgroup ranges will be used to provide information about the stability of short-term variation and its magnitude. The subgroup averages will offer information about process centering and its predictability. For a controlled, predictable, process, the individual observations can be formed into a histogram which can be used to provide a "picture" of process output.

The symbol "n" is used to represent the number of measurements in a subgroup; in this example, n=4 because there are four measurements in a subgroup.

The symbol "k" represents the number of subgroups; in this example there are twenty distinct subgroups and so k=20.

Table 5.1
A PRACTICE DATA SET

Observations

Subgroup	X_1	X_2	X_3	X_4	R	$\bar{X}$
1	36.13	32.85	34.05	38.04	5.19	35.2675
2	38.68	34.95	32.36	33.68	6.32	34.9175
3	34.34	35.69	35.06	29.72	5.97	33.7025
4	33.37	31.73	33.45	35.58	3.85	33.5325
5	32.42	35.58	34.35	35.79	3.37	34.5350
6	30.62	34.10	34.75	36.91	6.29	34.0950
7	31.76	33.29	37.41	31.50	5.91	33.4900
8	34.94	33.79	33.68	36.90	3.22	34.8275
9	34.66	32.02	36.34	33.50	4.32	34.1300
10	39.37	34.82	33.47	31.30	8.07	34.7400
11	33.06	38.97	35.88	36.07	5.91	35.9950
12	37.42	36.39	34.68	33.52	3.90	35.5025
13	37.18	34.43	36.34	33.88	3.30	35.4575
14	32.19	34.90	36.34	33.41	4.15	34.2100
15	33.36	34.36	33.38	33.68	1.00	33.6950
16	33.22	31.18	32.95	32.51	2.04	32.4650
17	34.22	33.01	36.63	35.10	3.62	34.7400
18	32.68	33.03	38.15	35.47	5.47	34.8325
19	31.49	35.84	31.00	35.47	4.84	33.4500
20	34.08	28.97	34.76	35.53	6.56	33.3350
SUM					93.30	686.5900

5.4.1. Measure of location: the Subgroup Arithmetic Average

A subgroup value is represented by the symbol, X_i, where the i indicates the position of each number in a sequence of observations. The symbol for the average is $\overline{X}$, called X-bar. The formula for the arithmetic mean or average is:

$$\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$

where the symbol $\sum$, pronounced sigma, means "add" the values represented by the symbol X from i= 1 to i=n, with n being 4 in this example.

$$\overline{X} = \frac{36.13 + 32.85 + 34.05 + 38.04}{4} = 35.2675 \cong 35.27$$

Averages for all subgroups, or samples, are shown with the original data.

The arithmetic mean or average is the most frequently used measure of location. The arithmetic average is the balance point, or center of mass, for a collection of measurements. The average value need not be a number that occurs in the subgroup; in data set 1, for example, no observation has the value of 35.27, the subgroup average. The average value need not have an equal number of observations above and below it. Data set 3, as an example, has three values larger than the average and one smaller. In some data sets having extreme measurements, this last property can result in an average value that does not do a good job of representing the other values in the data set.

5.4.2. Measure of Subgroup Variation: the Range

Numerous methods have been developed to measure the variation in a set of numbers. For the purposes addressed in this text only one of those measures, the subgroup range will be used to characterize the variation within a subgroup. The subgroup range, indicated by the symbol "R" is defined to be the difference of the largest value and the smallest value. The range for subgroup 1 is:

$$R = 38.04 - 32.85 = 5.19$$

The range is recognized as the simplest and most direct method for measuring variation in a subgroup of size n. The value of the range depends only on the two extreme observations. The range for the remaining subgroups is reported in the original listing of the observations.

5.4.3. Construction of Range and Average Charts

The range chart is constructed and evaluated first and, if appropriate, limits are then placed on the average chart.
Values of the subgroup range in Table 5.1 are plotted in Figure 5.2. The average value of these ranges is first determined. The average range, $\bar{R}$, read as R-bar, is found from the ranges for all subgroups and, in this case, is calculated to be:

$$\bar{R} = \frac{\sum_{j=1}^{k} R_j}{k} = \frac{93.30}{20} = 4.665 \cong 4.67.$$

Lower and upper control limits for the range chart are found by calculating, respectively,

$$LCL_R = D_3 \bar{R} \text{ and } LCL_R = D_3 \bar{R}$$

where D_3 and D_4 are control chart constants that have values indexed according to the number of observations within a subgroup. Values of D_3 and D_4 are shown in Table 5.2. This table is also found in appendix A. In this example, with n=4, D_4 = 2.282 and the upper control limit for the range chart is found to be:

$$UCL_R = 2.282(4.665) = 10.646, \text{ or } 10.65.$$

For subgroups of size six or less, there will be no lower limit for ranges because the value D_3 is not defined. In this application there is no lower control limit for the range chart because n = 4.

Control charts are meant to provide information on changes in the process. Process changes are recognized by the behavior of the data points on the charts in one of two ways:

1) By the magnitude of the variation in the points. If all points are within the control limits, the process is judged not to have changed.

2) By any non-random pattern in the points. A non-random pattern indicates the presence of special or unusual events. In this text, a non-random pattern is defined to be

a) a run of 7 or more consecutive points on the same side of the average line for the chart, or

b) a trend, up or down, of 7 consecutive increases or decreases.

Other patterns, usually associated with subgrouping strategy, are discussed in Chapter 6.

There are no indications of the presence of special causes in the R chart in Figure 5.2. All the values of R are within the upper control limit; there are no values of R which equal or exceed the value for the upper control limit. There is no pattern in the data points which would suggest the presence of systematic influences. The conclusion is that the variation in the values of R is produced by common causes.

The presence of special causes would be indicated if one or more of the values of R equaled or exceeded the upper control limit or if there was an indication of a systematic pattern in the data. If the range chart contains signals of instability in within subgroup variation, then the first order of business is to do the work necessary to stabilize the variability. If there had been evidence or signals of the presence of special causes, then it would not be theoretically appropriate to place control limits on the X-bar chart. Although the X-bar values should be plotted to look for gross patterns, control limits should be placed on the X-bar chart only after the range has been stabilized. Once that is done, then analysis as to the state of the process average can be conducted.

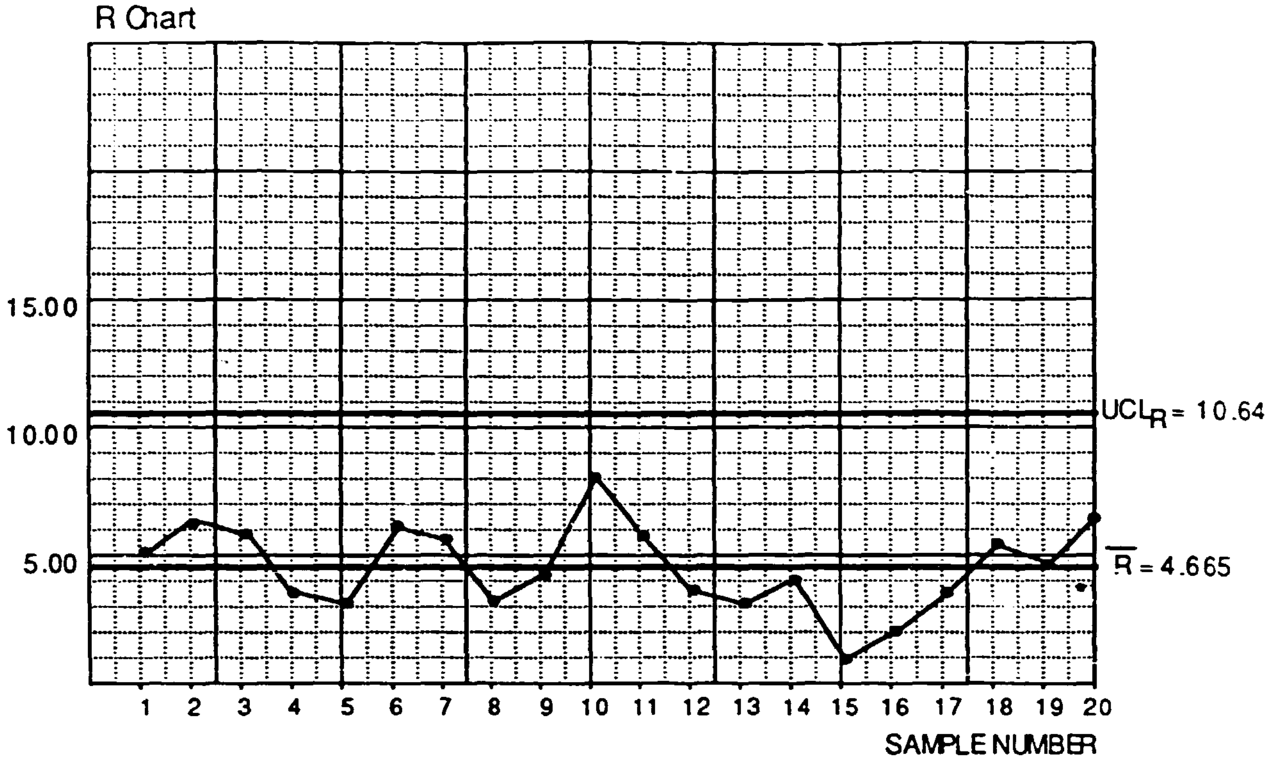
Because the range chart indicates that the process was stable with respect to short-term variation, and therefore, predictable, it is possible to put limits upon the magnitude of variation which the subgroup average values should display. As with other control charts, the X-bar chart requires a center line and control limits. The center line is taken to be the average of the subgroup averages. Formulas for the average of the averages and for the upper and lower control limits for the average chart are given below, along with the numerical results for this example problem. The average for the subgroup averages is represented by the symbol, $\overline{\overline{X}}$ (pronounced as X double bar), and is often referred to as the center line for the X-bar chart:

$$\overline{\overline{X}} = \frac{\sum_{i=1}^{k} \overline{X}_i}{k} = \frac{686.92}{20} = 34.3460$$

$$LCL_{\overline{X}} = \overline{\overline{X}} - A_2\overline{R} = 34.3460 - .729(4.665) = 30.945$$

$$UCL_{\overline{X}} = \overline{\overline{X}} + A_2\overline{R} = 34.3460 + .729(4.665) = 37.747$$

The same rules are used to evaluate the X-bar chart as were used for judging the range chart. All of the subgroup averages are within these control limits and do not display any evidence of special causes in terms of unusually large or small values. The pattern of the data points must also be examined. There is no pattern in the values for average suggesting the presence of special causes at work in the process.


R Chart
15.00
10.00
5.00
UCLR = 10.64
R = 4.665
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
SAMPLE NUMBER


X Chart
40.00
35.00
30.00
UCLX = 37.73
X = 34.33
LCLX = 30.93
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
SAMPLE NUMBER

Figure 5.2

It is useful to examine the formulas for upper and lower control limits for the X-bar chart from a process perspective. The limits on X-bar charts depend upon the average value of the range. The numerical value of the average range reveals the effect of common cause variations acting within subgroups. The subgroups are formed according to a particular sampling strategy and the numbers result from applying a given measurement process. If either the measurement process or the sampling strategy were to change, it is likely that numerical results would change. The process average is judged to be in or not in control according to the effects of common cause variations captured within subgroups.

As judged by these charts, the process is stable and predictable in terms of variability and average. It is to be immediately noted that stability, or "control" is an operational definition and says nothing about the utility of what the process is providing. A controlled process announces that the effect of material, equipment, or method changes is consistent. A controlled process is not necessarily a satisfactory process.

5.5 Summaries of Process Behavior

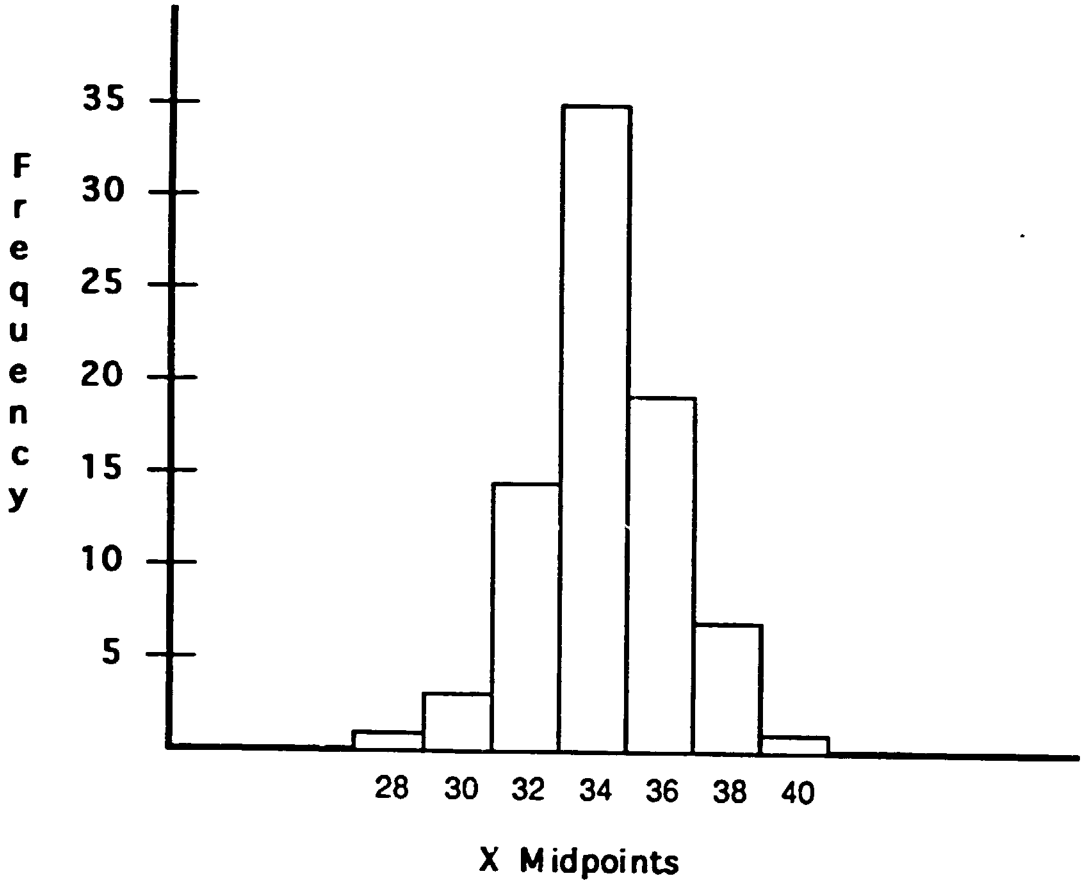
Having judged that the process is stable with respect to both variability and average, estimates of those process properties can be reported. In addition, a histogram can be constructed from the individual measurements to provide additional insight into process behavior. These three descriptors, the process average, the process standard deviation, and the process distribution not only provide useful ways of characterizing process operation but also provide descriptors by which the current outcomes delivered by a process can be compared to what is required. The information provided by these three descriptors and the comparison of these properties to current specifications on a process will be illustrated by a further examination of the previous data set.

5.5.1. Interpretation of histograms

The 80 individual measurements in the practice data set from Table 5.1 have been used to construct the histogram in Figure 5.3.

Figure 5.3

Histogram of Measurements from Table 5.1



An examination of this histogram illustrates the characteristics of a process which need to be summarized. The first of these characteristics is the shape of the distribution of measurements. Words used to describe the shape of the histogram in Figure 5.3 might be "mound-shaped" or "bell-shaped." These words capture the idea that most of the observations appear to cluster around a center value and that the observations appear to fall in a symmetric manner about that center. This behavior is fairly close to a mathematical model used by statisticians which models mound-shaped behavior in a distribution of measurements. This mathematical model is called a normal distribution. Thus, another descriptor which would typically be used for the data set in Table 5.1 is that it appears to be "normally distributed." The utility of describing the shape of a distribution of measurements in process work is linked to the shape a histogram of process measurements would be expected to have and how that might agree or not with the actual constructed histogram.

The four histograms in Figure 5.4 illustrate some different shapes which might occur in practice. Histogram A appears to have a large number of measurements either right above the lower specification limit or right below the upper specification limit. One might rather expect the measurements to taper off on either side of the histogram in a more gradual fashion. A possible reason for the observed behavior might be that material which was near the upper or lower specification limit was reworked to insure it fell within specifications. Another possible reason for the observed behavior would be that when measuring the material, values that were close to the upper or lower specification limits were remeasured until a reading was obtained which fell within specifications.

Histogram B appears to be chopped off at the upper and lower specification limits. As with histogram A, the explanation for this behavior will need to be sought by a closer examination of the process generating the results. One explanation which suggests itself is that the material being measured has been sorted prior to the place at which the measurements were collected. The material which fell above or below the specifications was then removed. If this supposed behavior were correct it would indicate that the process has more variation than can be tolerated and the solution adopted for this problem is to sort the nonconforming material. Histogram C shows a similar pattern to B, but with a more dramatic drop at the lower specification limit. Again, the suspicion might be that there is more variation in the process than can be tolerated. Furthermore, one might imagine that the material measured prior to the sort would have a normal distribution with the measured values clustering around a central value. Histogram C would then suggest that this center does not fall in the middle of the specification limits thus explaining the larger amount of material removed below the lower specification limit.

Histogram D shows two distinct peaks. A number or reasons for such behavior are possible and would need investigation. A speculation which might help such an investigation is that there are two separate processes generating these data. For example, if two machines were creating the output measured, then a possibility to be investigated is that they are each creating material centered at different values.

One of the uses of the histogram is to provide insights into process operation; another is the use of the histogram as a description of the measurements which will be produced by the process. This second use is dependent on being able to use the histogram as a predictor of process behavior, which in turn is dependent on knowing that the process is operating in a consistent manner. The histograms of Figure 5.1 provide an illustration of this last statement. The histograms for process B and for process C look similar. Their shape might be described as mound-shaped. However, because both histograms represent processes which are inconsistent over time, neither of the two histograms can be relied on to describe process outcomes. The histograms only capture the shape of process outcomes over the time frame investigated. The inconsistencies in the process mean that for a different time period, the histograms will likely show a different description of process outcomes.

5.5.2. Estimating process variation

Another look at the histogram of Figure 5.3 indicates that the variation in the measurements is another important property of the process which needs to be characterized. Examination of the histogram shows that the 80 measurements spread over a range from about 28 up to 40. The process standard deviation is a numerical measure which captures this information on process variability. Since the process which generated the data in Table 5.1 was judged to be stable, calculating a standard deviation from these numbers would provide an idea of what the standard deviation for other process values would be. The calculation of this standard deviation is given below:

$$s = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \overline{X})^2}{n-1}}$$

where here, $\overline{X}$ would be the average of the 80 observations and n would be equal to 80. The reader should check that s is found to be 2.0993.

An examination of the formula used to calculate the standard deviation helps in understanding how it is capturing information about process variability. The number calculated is based on the squared deviations about the average. The farther away a data value is from the average, the larger will be the contribution to s, the standard deviation.

Of course, the number, s, calculated above, is only based on 80 measurements. One now needs to imagine that a very large number of measurements on the process (maybe a billion!) were available and a standard deviation was calculated from these numbers. (This task is clearly a conceptual one and is discussed here to provide an understanding of what is meant by a process standard deviation.) The number which would result from this calculation will be referred to by the Greek letter, σ. This number, σ, would capture a description of the variability of the process, and will be referred to as the process standard deviation. The calculation of s from the 80 measurements would be one way to estimate σ. However, an estimate of the process standard deviation can also be found from the average range, which was calculated as $\overline{R} = 4.665$. The formula for estimating σ and the resulting value are reported as:

$$\hat{\sigma} = \frac{\overline{R}}{d_2} = \frac{4.665}{2.059} = 2.27.$$

The value for d_2 is found in Table 5.3. The symbol σ indicates the standard deviation while the symbol " ^ " indicates that the reported numerical value is an estimate of the process standard deviation. Because the range chart did not contain any signals as to abnormal behavior and because the $\overline{X}$ chart indicated a stable process average, $\hat{\sigma}$ is thought to be a reliable estimate of the process standard deviation.

The standard deviation of a process is a useful descriptor of process variation. It provides a measure by which the variation delivered by a process can be compared to what is required. Such a comparison is often made by using the standard deviation to estimate the range of measurements which will occur in the process. For processes which are normally distributed, almost all measurements will fall within a range of six standard deviations. For the data in Table 5.1, the standard deviation of the process was estimated to be 2.27. Thus, an estimate of the range over which measurements can be expected to be observed is 6x2.27=13.62. The histogram in Figure 5.3 shows that the measurements fall within a range from 28 to 40. This spread of 40-28=12 is comparable to the value estimated by six standard deviations.

5.5.3. Estimating the process average

The histograms of Figures 5.3 and 5.4 were described as having distributions of measurements which clustered around a central value. The process average is a way of capturing information about this central value. When signals as to abnormal behavior are absent from both the range and X-bar charts, the average of subgroup averages, which is the value for the center line for the X-bar chart, can be taken as an estimate of the average value for the process. For the data of Table 5.1, the process average would be reported as being 34.35, rounded from 34.346.

5.5.4 Process Capability

In its most general form, process capability refers to the capability of a process to deliver what is required. However, in response to the need to quantify "how capable" a process is, process capability often refers to whether the measured outcomes of a process exhibit small enough variation to fall within some set specification limits. If the specifications stated that the measured dimension of the data from Table 5.1 should fall between 30 and 40, then the engineering tolerance (ET) for the measurement would be:

$$ET = 40 - 30 = 10$$

The natural tolerance (NT) for the process refers to the range of measurements over which the process will produce material. Typically, the natural tolerance for the process is estimated by:

$$NT = 6\hat{\sigma}$$

As was discussed in section 5.6.2, the natural tolerance for the process described by the data in Table 5.1 would be 6x2.27=13.62. A comparison of the NT with the ET allows one to decide whether the process spread is small enough that the specifications could be met if the process were properly centered. In the present example the NT is larger than the ET so the process would be said to be "not capable."

Various capability indices have been defined as a method for reporting on the ability of a process to meet specifications. Two of the more common indices are C_p and C_{pk}. C_p is defined by the ratio:

$$C_p = \frac{ET}{NT}.$$

The C_p index is an attempt to quantify "how capable" a process is. If this number has a value of 1 or greater, the process is said to be capable. Many organizations state that a preferred value for this number is 1.33 or more.

A drawback to using C_p to report on process capability is that the index does not capture information on the process average. A process could have a large C_p ratio and yet be producing a lot of product outside of specifications if the process is not targeted. The C_{pk} index is an attempt to report on not only the affect of process variation but also process average on the ability of the process to produce what is required. C_{pk} is defined to be the smaller of the numbers C_{pU} and C_{pL}, which are calculated from the formulas:

$$C_{pU} = \frac{USL - \overline{\overline{X}}}{3\sigma} \qquad C_{pL} = \frac{\overline{\overline{X}} - LSL}{3\sigma}$$

where USL refers to the Upper Specification Limit and LSL the Lower Specification Limit. From the data in Table 5.1, the following estimates of process properties have been obtained:

$$\overline{\overline{X}} = 34.35$$
$$\hat{\sigma} = 2.27$$
$$NT = 13.62$$

The upper and lower specification limits for the process were:

$$USL = 40$$
$$LSL = 30$$

From the above information, C_p would be:

$$C_p = \frac{10}{13.62} = .73$$

Of course, since C_p is less than one, the process is not capable. The number .73x100%=73% could be interpreted as "73% of the NT is used by the ET."

In order to calculate C_{pk}, C_{pU} and C_{pL} are first calculated to be:

$$C_{pU} = \frac{40 - 34.35}{3(2.27)} = 0.83 \qquad C_{pL} = \frac{34.35 - 30}{3(2.27)} = 0.64$$

Thus, C_{pk} would be .64, the smaller of these two numbers. The smaller value of C_{pk} as compared to C_p can be explained by comparing the process average to the center of the specification limits, or the nominal value. The nominal value in this instance would be 35; the process is not centered on this nominal value, but is targeted somewhat below 35 at a process average of 34.33.

5.5.5. Assumptions and limitations behind the use of capability indices

The previous section illustrated the calculation of two capability indices assuming that the process under study was operating in statistical control. One should be very skeptical about reported values of C_p and C_{pk}, as these indices may be reported when there does not exist knowledge about the stability of the process. Too often, C_p or C_{pk} are determined by a one-time application of a control chart or, even worse, by collecting, say, 30 consecutive readings from a process. If this were the case, then the reported process information, $\overline{\overline{X}}$ and $\hat{\sigma}$, cannot be relied on to summarize process behavior. These numbers are only useful if it is known that the process average and the process variation are staying stable over time. In the case where these numbers are estimated from a one-time application of control charts, then it is doubtful whether the estimated standard deviation will capture the variation in process outcomes which might occur from one run to the next, or which might occur as a result of incoming materials to the process changing, or which might result from changing crews, etc. In these cases, reliance on C_p and C_{pk} to describe process capability is a doubtful, if not worthless, practice.

The calculation of either of the two capability indices also assumes that the distribution of measurements closely resembles a normal distribution. This information on the process is only available by actually constructing a histogram of individual results to understand the shape of the distribution of measurements. Further, one would need to closely question an organizational practice which required a value of C_p or C_{pk} to be reported on all measured outcomes from a process. Many measurements on a process could not reasonably be expected to behave according to a normal distribution. The amount of impurities or time to breakage are two examples where process measurements typically have a skewed distribution.

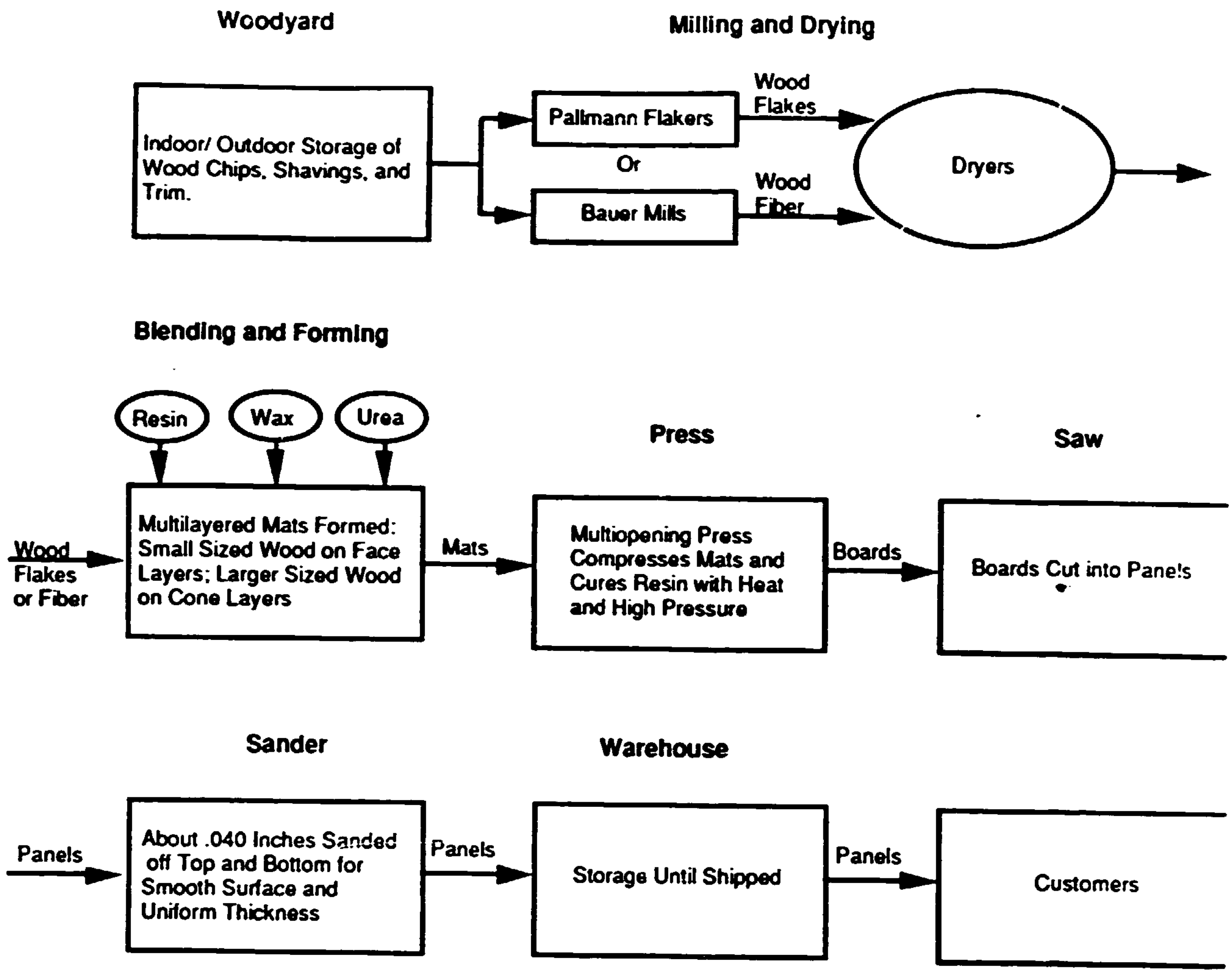
Capability indices provide summary measures of how current process performance compares to some stated specifications. However, their use as an aid to directing or prioritizing improvement efforts is limited. For example, when C_p or C_{pk} are reported as summary results of a process, the focus of study is often on the results of the process, rather than on the characteristics which need to be studied to improve the results. The sources of variation contributing to process results needs to be worked and studied. Yet, implicit in the use of capability indices is that the correct characteristics of a process to be measured are known. This knowledge will only come from considerable process understanding. Even if there exists sound reasons for investigating the characteristic under study, the behavior of this characteristic, in terms of the ability to target the characteristic at the correct average and maintain the characteristics at small levels of variability, are not well described by either of the indices.

It is, unfortunately, common practice in many organizations to state goals for C_p or C_{pk} for many, if not all, processes. In light of the above limitations and assumptions about the use of the indices, such practices do not provide the kind of management necessary to improve process behavior. Such goals for C_p or C_{pk} (like goals about 6-sigma) are just that, goals. They provide no direction on how or where to work to improve the value provided by a process.

5.6 An Example of the Process Use of Range and X-bar Charts

Particle board is an industrial product which is produced and sold by manufacturing plants or divisions to other industrial companies. These companies then process these boards and create products which go to other industrial users or to final consumers. In common with other industrial products, particle boards have numerous important features, characteristics, or attributes. These range from physical properties, such as internal bond, to required characteristics of surface conditions, such as flatness, smoothness, and moisture retention.

The manufacturing process takes wood chips and other materials and converts these into a finished product. A block flow diagram of a manufacturing process for particle board is shown on an accompanying page. The process begins with the specification and purchase of wood on the front end. Raw materials are moved through milling and drying, through blending, where resin, wax, and urea are added, and through the forming machine where multi-layered mats are put down and taken to the press where the mats are pressed into a particle board. Boards are then allowed to cool, after which they are sawed and sanded and packaged and then moved into storage.



Woodyard
Milling and Drying
Indoor/ Outdoor Storage of Wood Chips, Shavings, and Trim.
Pallmann Flakers
Or
Bauer Mills
Wood Flakes
Wood Fiber
Dryers
Blending and Forming
Resin
Wax
Urea
Wood Flakes or Fiber
Multilayered Mats Formed: Small Sized Wood on Face Layers; Larger Sized Wood on Cone Layers
Mats
Press
Multiopening Press Compresses Mats and Cures Resin with Heat and High Pressure
Boards
Saw
Boards Cut into Panels
Sander
Panels
About .040 Inches Sanded off Top and Bottom for Smooth Surface and Uniform Thickness
Panels
Warehouse
Storage Until Shipped
Panels
Customers

The flow diagram depicts the physical process flow; however, important aspects of the process are not represented or displayed on that diagram. How the work is to be performed is not represented; raw material requirements and specifications, and equipment conditions are not described or stated. Flow rates and volume parameters are not defined and neither are the required in-process characteristics as the material moves from one department to another. Operational definitions have to be provided for process parameters, work methods, machine settings, and material characteristics. These comments are only to reinforce the statement that flow diagrams, in and of themselves, are incomplete. Additional description is needed. It is among these elements that explanations for special or assignable cause are often found. Oftentimes, process information of this type can not immediately be had or displayed upon demand. It frequently is compiled only after managers, engineers, and operators begin to do the work and discover that information is lacking in one or more respects.

In this brief application, the measurement is on mat weight after the completed mat emerges from the former (a machine which "lays" down the chips to create a mat to be pressed). For purposes of this discussion, the mats may be pictured as being several inches think, several feet wide, and twenty feet long. The ability of the former to lay down the correct amount of material in a correct and uniform pattern is important for creating consistent final board properties after a correct press operation. Process operations upstream from the former affect the ability of the former to correctly and consistently produce conforming mats. Consistency of material properties, moisture and density, are important in achieving target mat weight, which changes from product to product. There will in fact be variations within mats and between mats. Variations within mats are created by certain process factors; variations between mats are created by these and other process factors. The general point being made is that managers, engineers, and operators must immediately begin thinking about their process in terms of variations, within part, part to part, product source to product source, time to time, and in other useful ways. The concept of variations and process reasons for the variations are essential to process analysis. Knowledge of process reasons for variations is almost always incomplete; process improvement begins with this understanding and this incompleteness.

Measurements on mat weight for process evaluation are obtained by automatic measurement methods which read and store mat weight in pounds per cubic foot. For this example problem, one reading per mat is used to represent the mat weight. The sampling plan used creates one subgroup of five mat weights (n=5) each hour by taking one mat every twelve minutes. Weights for individual mats are recorded and the range and average of the subgroup of five each hour are computed and reported. Here, as in any and all uses of charts for process analysis, a distinction must be made as to those factors that affect the within subgroup variation and those factors that tend to affect variations between subgroups. In this application, within subgroup variations are created by the way in which process factors fluctuate or act within an hour's time. Between subgroup variations are created by normal process variations and by those process factors which tend to become active in unusual ways from hour to hour.

In overly simple terms, points out of control on the range are driven by unusual variations or inconsistencies in those causes which tend to operate within subgroups. The average value, R-bar, of the effect of these sources, is used to judge the variations in the X-bar chart. The formulas for the control limits on the X-bar chart make specific use of the average range. Out of control conditions on the X-bar chart are recognized by variations on that chart that exceed the average variation experienced within subgroups; again, the use of the average range in the control limits for X-bar should be noted. Inconsistencies or abnormalities for causes or factors that tend to become active between subgroups result in out of control conditions on the X-bar chart. Sources or factors of this general type or nature are described as driving long-term variations. A clear implication of these ideas is that the responsible manager understand the sampling and measurement strategy relative to the factors or causes shown on cause-effect diagrams. The effectiveness of chart applications is diminished by a lack of understanding regarding where the variations in particular causes are likely to be revealed. Subgrouping strategies and further discussion regarding subgrouping and its importance will be discussed in a later chapter. However, it is important to begin to develop the ideas as to within subgroup and between subgroup variations and their causal structure early in learning about correct and productive use of charting techniques. The reason for knowing these causal structures must be clear to the process manager. Knowing the causal structure and identifying factors as tending to act within and between subgroups aids in the development and practice of identifying and removing assignable causes, but it also constructively builds knowledge regarding variation sources and supports informed decision making for achieving effective process improvement.

As previously discussed, subgroup range values reflect certain sources of variation which must be understood according to the sampling strategy and according to the manner in which process variations behave. For this example where the sampling scheme selects five mats per hour, the within subgroup variations are created by those process sources producing within mat variations and mat to mat variations over the course of one hour. Hour after hour, the deviations in the five mats yield range values for each subgroup. These values fluctuate according to causes operating consistently within an hour. An assignable cause may result in the deviations among five mats being unusually large. Unusually large deviations of course are recognized by points on or above the control limit on the range chart. Within mat variations are perhaps most influenced by process settings, maintenance status, and operational condition. Mat to mat variations occurring within one hour are affected most significantly on a "common" cause basis by "short-term" changes in material characteristics, in particular density and moisture changes. Change in material characteristics are created, in turn, by properties of incoming wood materials, screening and milling equipment and practices, and the addition of resins and waxes. The magnitude of the average range value, computed from hourly range values taken over an extended period of time, reflect the effect of these process conditions. The stability, or lack of same, indicates the consistency with which the process creates material of the same bulk density, and of consistent moisture, and the consistency of the mix and application of resin and wax. Inconsistency, or lack of control, in the range chart indicates that process factors which act on the process within an hour's span of time are not consistently practiced or achieved. Again, reasons for this inconsistency would be found in methods, equipment, and material variations and the inconsistent practice and operation of these factors. Common cause variations, measured by the average range for a controlled process, are those things which produce variations on the range chart hour after hour. Obviously, the purpose of these somewhat detailed points is not to teach the reader how to make particle board; the purpose is to demonstrate ways of thought which are necessary for process control and improvement.

The X-bar chart, which contains values for hourly subgroup averages, measures those process effects which play out in the short and long term In this application, fluctuations of the averages for five mat weights are affected by what happens within any one hour and by those process inputs that tend to reveal their effects in periods of time that exceed one hour.

Successful practice in applying these charts to process control, study, and improvement requires that the manager, engineer, and operator begin to know and understand those process reasons, shown on the cause-effect diagram, that can and do reveal themselves in common and special causes in the within subgroup sampling scheme. The variations from hour to hour are represented by movements in the points on the X-bar chart. These variations are judged to be stable (predictable) when they do not exceed the variation captured by the within subgroup variation. The average value for the range represents the magnitude of the within subgroup variation.

Data for demonstration and discussion of the above points are plotted on the accompanying page. Results are plotted in two formats. On the left side of the page, the sample data are plotted in production sequence, hour by hour. On the right side of the page the data are plotted by crew identification with correct production sequence maintained within data plot for a crew.

The range chart is plotted first. That chart, on the left side of the accompanying page, indicates that the variations from mat to mat within an hour are consistent. That judgment is made because of the absence of signals as to the presence of assignable causes;

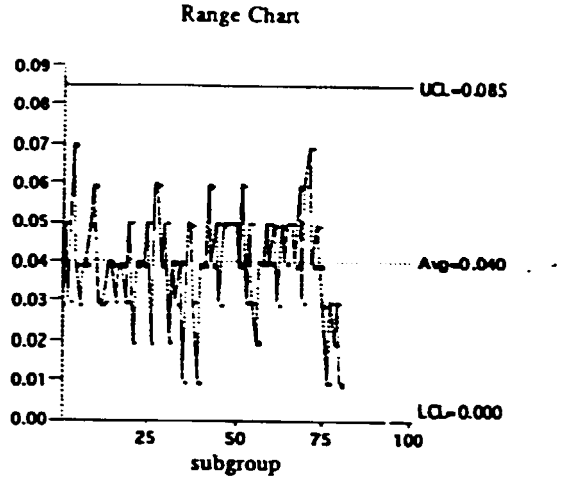
there are no points on or outside the limits and there are no specific patterns in the plotted points. The average range is 0.04. Consistency does not say that the average range is small or large; consistency does not imply that customer requirements are or are not being met. The judgment is simply a statistical one that reports on the consistency of the effects of the observed variations typically occurring within one hour for mats separated by twelve minutes. The range data indicate that the same within hour deviations in mat weight are achieved and maintained regardless as to which crew is operating the process.

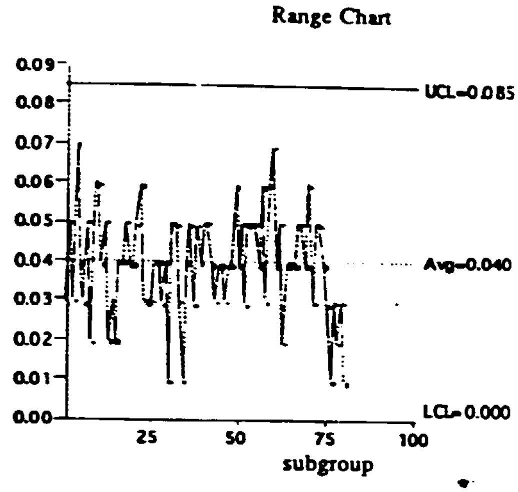
The X-bar chart contains signals as to the presence of assignable causes. There are numerous points outside the control limits for this chart. Variations in X-bar are driven by those kinds of things which produce within subgroup variations and, potentially, by those things which occur between subgroups that produce variations larger than experienced within subgroups. In general, the presence of assignable causes affecting subgroup averages is recognized by observed variations exceeding that predicted or captured by the average range. The plotted points themselves contain no clue as to why large variations exist. Reasoned experience, careful logs and notes, other data, or sample data organized according to different criteria than that originally used will be necessary to begin to suggest reasons for the larger variations. Process knowledge and understanding may be gained by then verifying the suggested reasons for unstable variation through further study and data collection.

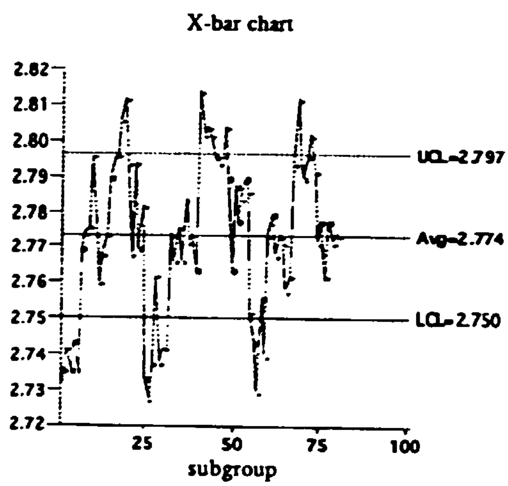
In this instance, the original data on mat weights are retained on an hourly basis, but are plotted by crew. The revised data are again plotted by range and by average. The pattern in the range chart does not reveal any differences for within hour deviations for the different crew operations. However, the X-bar chart clearly reveals that crews operate to different average values. Use of the common value for average range to put limits on the respective X-bar charts provides other information. Within themselves the crews operate in a stable fashion, although the crews, as stated, run or operate at different average values for mat weight.

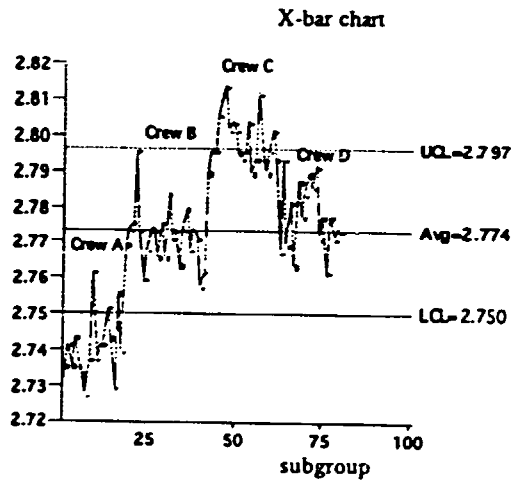
Up to this point, the following information has been gained about the mat weights. First, within hour deviations in mat weight are consistent, regardless of operating condition or crew. Secondly, the plant as a production source, produces mats that are not in control with respect to the average mat weight; the presence of an assignable cause has been identified. Third, crews produce mats to different average weights. Fourth, each crew, with respect to itself, produces consistent mat weights. It would be too easy to claim that the reason for the assignable cause can be associated with crew practices. While that may be the case, the reasons as to why crews run in a consistent fashion to different averages remains to be discovered. It may be a lack of uniform operating procedures; it may be due to differences in raw material which can be handled in other ways, it may be due to differences in equipment conditions above and beyond what individual crew management can affect. Process investigation, conjecture, experimentation, and verification remain to be completed.

Natural Order **Ordered by Crew**









5.7 Effective use of R and X-bar charts

In considering applications of R and X-bar charts, the manager should have in mind the purpose or purposes which the chart is to serve. These and other types of SPC charts can be employed in various ways and for various means. These applications, as discussed elsewhere, derive from several intentions. These purposes may be briefly described as:

1) The charts serve as a signaling system which indicates the presence of assignable causes.

2) The charts are used to help judge the effect of deliberate process changes. The plotted information provides data for comparing process outcomes before and after a process change. The charts provide evidence as to the magnitude, direction, and stability of the effects of the process change.

3) After a process change is in place and verified as to its effect, the charts provide an ongoing confirmation that the change is maintained. In this respect, the charts support holding or maintaining a gain achieved by a previous process change. Over a period of time, of course, this purpose merges into that of maintaining control.

4) The charts provide a data representation by which operators, engineers, and managers can begin to discover, evaluate, and know the effect of various sources of variation. The purpose of knowing is the intention to apply the knowledge to improve the process. The intention is to understand the sources of variation and their impact upon the quality variable, and to gain information as to which sources should be attacked in order to reduce or eliminate their effect upon variation, either short or long term.

The purposes listed above are not naturally mutually exclusive. For the knowledgeable manager, engineer, or operator, the charts can serve several purposes simultaneously. In that spirit, these various purposes deserve further discussion, individually and collectively.

Obviously, one purpose is to assist an operator, engineer, or manager in maintaining good control over a process. Control is here defined as the identification and removal of assignable causes, assignable causes which are defined by a particular sampling or subgrouping strategy. In these applications, baseline data are collected, out of control conditions are identified, and work is begun with the purpose of preventing these assignable causes from occurring in the future. In practice, there is often confusion regarding this purpose; engineers or operators may see a process go out of control, make an offsetting change in one or more process parameters and mistakenly consider that "control" has been gained. This is in fact a tactical and logical error in both process control and improvement work. Practices of this type build a culture that works against process control and improvement. Obviously, it may be necessary to make such offsetting changes, but it is absolutely necessary that the accountable management, engineering and operator group look for, verify, and eliminate the root causes of those out of control conditions. Management has the responsibility to assure that this practice in correct process management be in place.

Correct practice in organizing and doing work for obtaining and managing process control will involve an ongoing study of the process. Managers, with the assistance of engineers and operators, will have constructed verified process flow diagrams indicating how, when, and where work is done. Cause and effect diagrams are constructed. These diagrams contain verified information regarding relationships of critical input variables and the outcome of interest. Items listed on the cause-effect diagram call for specific work; this work is to describe the nature, type, and degree of the relationship between each item and the required effect. In particular, work is indicated and required where information regarding relationships or effects is missing or where suspicions about the actual existence of these relationships and effects are unconfirmed. Priority can be assigned to these investigations on the basis of managerial criteria. Charting work should have confirmed operating variations and levels for key input variables. Work methods are examined, tested, verified, standardized, and practiced in a consistent manner. Maintenance issues will have been addressed, corrected, and improved.

It will be necessary to plot and study process outcome results. However, it would be insufficient, and possibly misleading, to only plot and study data taken from inputs and practices which are known to be important. There are several reasons for this. First, all critical variables, key methods, and protocols may not be known at any particular stage of process analysis. In the absence of knowledge or experienced suspicion, nothing remains but to observe and control "everything," an impossible situation. Second, it may be that all input variables and parameters are maintained at recommended levels and that known important methods are practiced according to recognized standards, but this information does not by itself provide data by which outcomes or results may be observed and judged. It will not be possible to know the effects of process parameters without studying the outcome variables. Although it will not be possible, in general, to verify a causal relationship by observational practices alone, observation, conjecture, and correlation are essential for building process knowledge. Suggestions, experiments, and verification follow. Third, it will not be possible to identify variations in materials, machines, or practices which contribute to the presence of out of control conditions without observing outcome variables.

As the type of work described in the previous paragraph proceeds, the process will be made predictable; that is, it will be brought into a state of control. Values for the quality, or output, variable are gradually described by the range and average chart in terms of achieved variability and average. As the process becomes stable or predictable in terms of these values, information regarding standard deviation, average, and histogram shape begin to be credible. Information obtained in these endeavors will be used in establishing control and rescue protocol and will provide knowledge to support process control. Knowledge gained in this work will afford some information on which decisions for process improvement work can be based.

Predictability, or control, is an ongoing concern and must be constantly addressed. Processes of course do not naturally remain in control. Data collected and plotted in an ongoing manner support good control: the plotted points enable one to see where the process is or where it might be headed. The plotted points reveal the presence of assignable causes, according to a defined sampling strategy. The removal of these assignable causes is necessary in order to effectively gain control over the process. The simplicity and directness of the preceding statements give no hint of the discipline and amount of work required. Successful applied practice of these ideas requires that several considerations be in place. Managerial intention to run these processes in control must be communicated and appropriate behavior practiced. Line supervisors, operators, and process engineers must know how to measure, sample, evaluate, and act and must know that these duties are expected of them. Verified process knowledge must be in place, deployed, and consistently practiced; in this interest, extensive process knowledge must be built. For example, knowledge must be in place regarding past and current behavior of key input variables and work practices. The effects of variations in these variables or methods will have to be known in order to know which ones are critical for close tolerances and standard practices. Practical operating knowledge must be in place, or must be developed, regarding potential or actual out of control conditions and correct changes or adjustments in appropriate inputs or practices be tested and verified. Successful process management over a period of time begins to assure that these practices and knowledge be in place.

Charts on outcome variables are often assumed to be capable of serving as part of an engineering "feedback" mechanism. This is sometimes practical and useful, but it is very much process dependent. It is often thought that the purpose of "charting" is to represent the current process condition; the process is out of control or is tending toward that condition. Used in this way, the charts serve as part of a "feedback" mechanism for a manager, engineer, or operator as to the control status of the ongoing process. But, results in this mode are often disappointing. The charts are thought to provide clues as to when to leave the process alone and when to make an adjustment in those situations where the charted information can be realistically used as a feedback mechanism. In general, the sampling measurement methodology which provides data for these charts and the statistical limits provided are not useful as part of an engineering feedback control system. The assumptions underlying such suggested uses are vast, unstated, and unconfirmed. The limits, for example, say nothing about which process variables should be manipulated in order to maintain a certain level. Other knowledge and association may be missing; the statistical limits previously defined are not related to the time lag or other process dynamics, all of which are necessary in order to construct and operate a successful engineering feedback mechanism. Even rewarding practice of the above may be self-defeating in an improvement sense. People become confused as to the rationale for using the charts. Control as defined in this monograph does not mean "keep it between the limits." Control, again, means identifying assignable causes, often by changing sampling and measurement strategy, and learning the root causes of reasons for those assignable events, and then removing or permanently nullifying the effects of those causes.

A mature and experienced process view appreciates that charts on process output represent checking with the results. The chart on the outcome variable is a check on process management through the quality of the outcome. The process to be controlled is revealed by those items listed on the cause-effect diagram. Output or outcome data enable one to "see" if appropriate conditions are maintained on critical cause variables. Process control, however, is maintained by managing the crucial cause variables at appropriate values and conditions. These critical variables should be plotted to reveal their behavior and to provide information by which to check the effect of their behavior on the outcome variable. Out of control or potential out of control conditions are corrected or circumvented by correction of these primary cause variables. Successful control often requires that specifications on material, methods, and machines be evaluated, verified, and then consistently followed or practiced.

For purposes of control, the dominant issue for management, engineering, and operations is "Are the essential cause factors known and are they managed correctly and consistently?" which is where appropriate energy and direction should be given. Understanding clearly the management of the common cause system provides the base of support from which process improvement flows. It is in these terms that the charts are best used for "control." Process management using only results data without knowledge of the common cause system provides for only reactive behavior in a control sense and suggests that knowledge and impetus for improvement will be lacking.

As indicated in the purposes described previously, SPC charts can serve as powerful barometers of the effectiveness of process change. However, process change must be supported by successful process control, since the ability to judgment of the sustained effects of a process change will be uncertain in the presence of an unpredictable, or out of control, process. Effective charting discipline for process evaluation requires that process information be plotted over time periods during which both common causes and special causes have the opportunity to make themselves known. Verification of consistently practiced process knowledge is provided by charts which remain in good control over these changing conditions. Baseline data may indicate that assignable causes must be removed from the process before the effects of a process change can be studied.

Once the process is in good control according to a defined sampling strategy, process changes may be recommended to affect sources of common cause variations; the intent of these changes may be to reduce short-term variation, to move the average to a more favorable value, or to remove long-term shifts in run averages. The specific nature of the intended effect should be defined. Process changes, of course, are rooted in machine changes or revised machine parameter tolerances, changes in material, material characteristics or specifications, or in revised work methods or protocols.

Established charts on range and average will characterize or describe the stable process. The previously established baseline data are then used for judging the effect of process changes. The range and average charts contain data that represent the process as it is prior to a specific process change. After the process change is made, data are collected and plotted with the measurement and sampling strategy remaining unchanged. The "new" data are plotted directly onto the established range and average charts. The effect of changes can be evaluated by comparing the new data against the established process data. The data plots following the change reveal the effect of the process change. Several outcomes are possible; some of these are described in the following discussion. The charts may reveal that the process change had:

a. No effect on the process. Evidence for this conclusion is had by data plots on either or both of the previously established range or average charts that are very similar to data plotted prior to the process change. The new data will be consistent with the old and the charts will not reveal signs of sustained process change.

b. The anticipated effect on either the short-term variation or on the average.

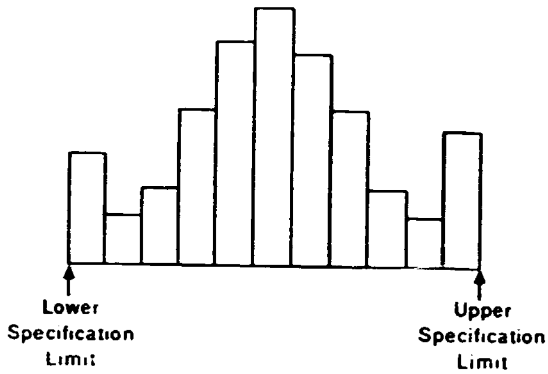
A decrease in short-term (within subgroup) variation is revealed by data on subgroup ranges that plot at a lower average value than the average range on the previously plotted R chart. The new data would suggest that the average value for the range had been decreased.

A shift in the average or mean value to the new, required level is revealed by plotting the new data on the X-bar chart. A process moved to a new level will yield data that plot in ways that reveal "out of control" signals on the established average chart; complete transfer success of the process change requires that the revised process yield data that plot in control but centered about the new average value.

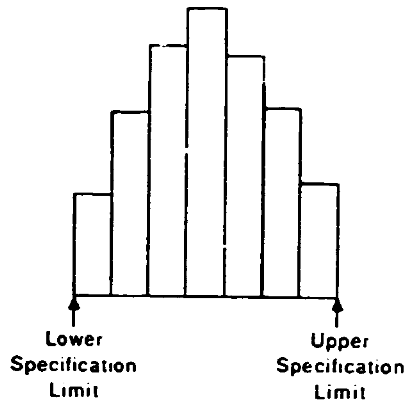
c. A deteriorating effect on the process. It may be that the process change has thrown the previous process "out of control," but has not resulted in a process that is stable or predictable about any value. In this case, it may be that an improved process would result if the new process could be successfully practiced, that is, brought into control. Again, work would be required to address and remove the "new" assignable causes that have appeared in connection with the changed process. The change may have created process deterioration in other respects; perhaps the short-term variation has become larger or the average has shifted but to a value not anticipated or wanted.

Figure 5.4

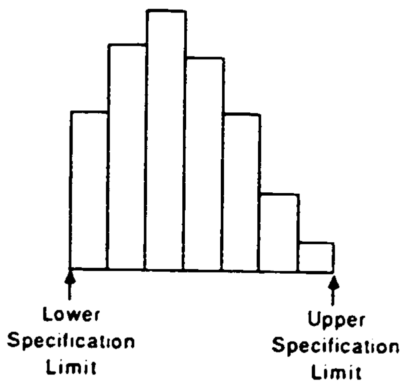
Histogram Examples



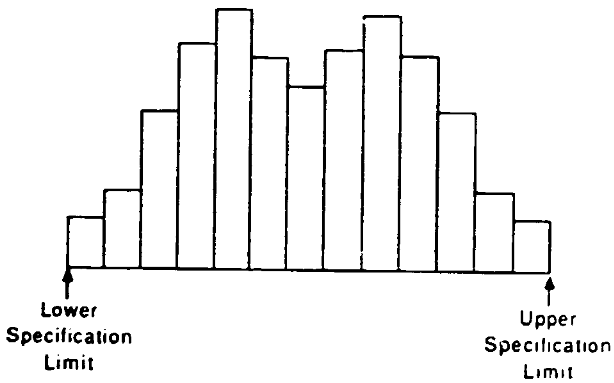
Histogram A



Histogram B



Histogram C

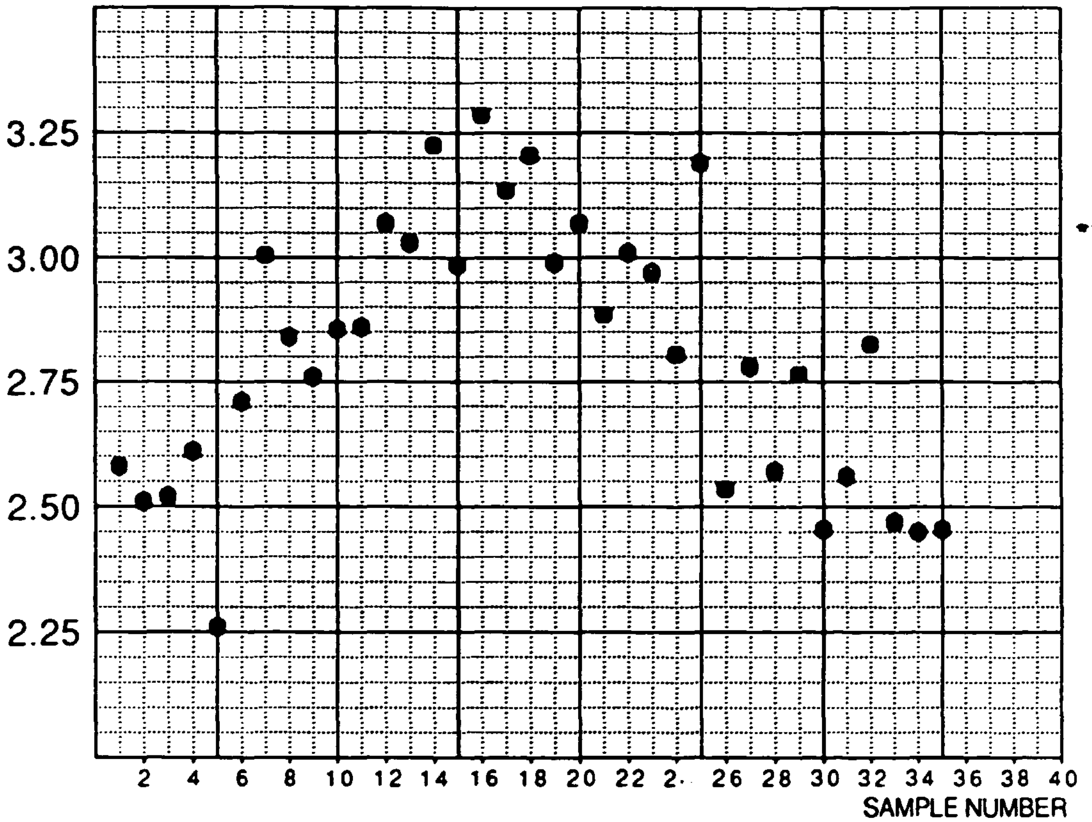


Histogram D

5.8 Chapter 5 Practice Problems

1. In the bearing manufacturing division of your company, one of the grinders, Machine NO. 2325, is believed to be unstable with respect to the bearing sizes produced. In order to learn about the variability and the average of the process, 35 measurements (five per day) of the bearing diameters are taken over a week. Based on the calculated average and standard deviation of the 35 measurements, the average diameter of the bearings is estimated to be 2.81 inches, and the standard deviation of the process is estimated to be .266 inches.

   A plot of the individual measurements taken over time is shown below.



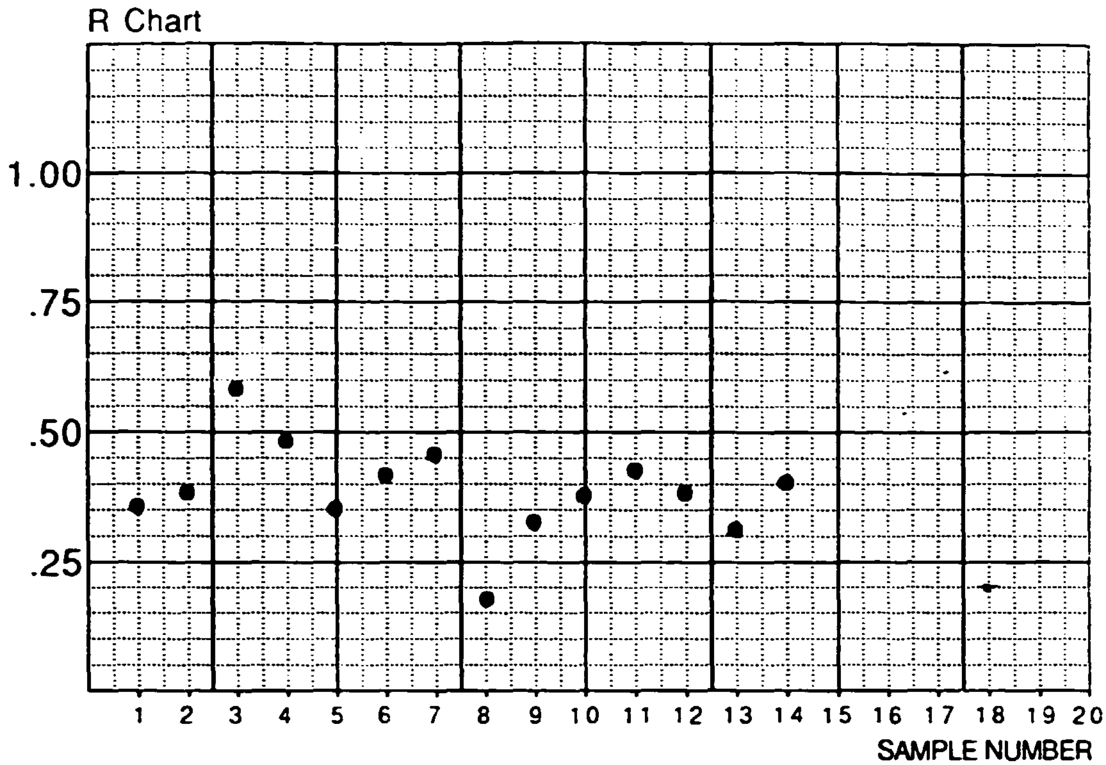
   a. What information about the process, that is revealed in the plot, is lost when the data are combined into one large data set?
   b. Is the calcuiated standard deviation of .266 inches an appropriate estimate of the process standard deviation? Explain.

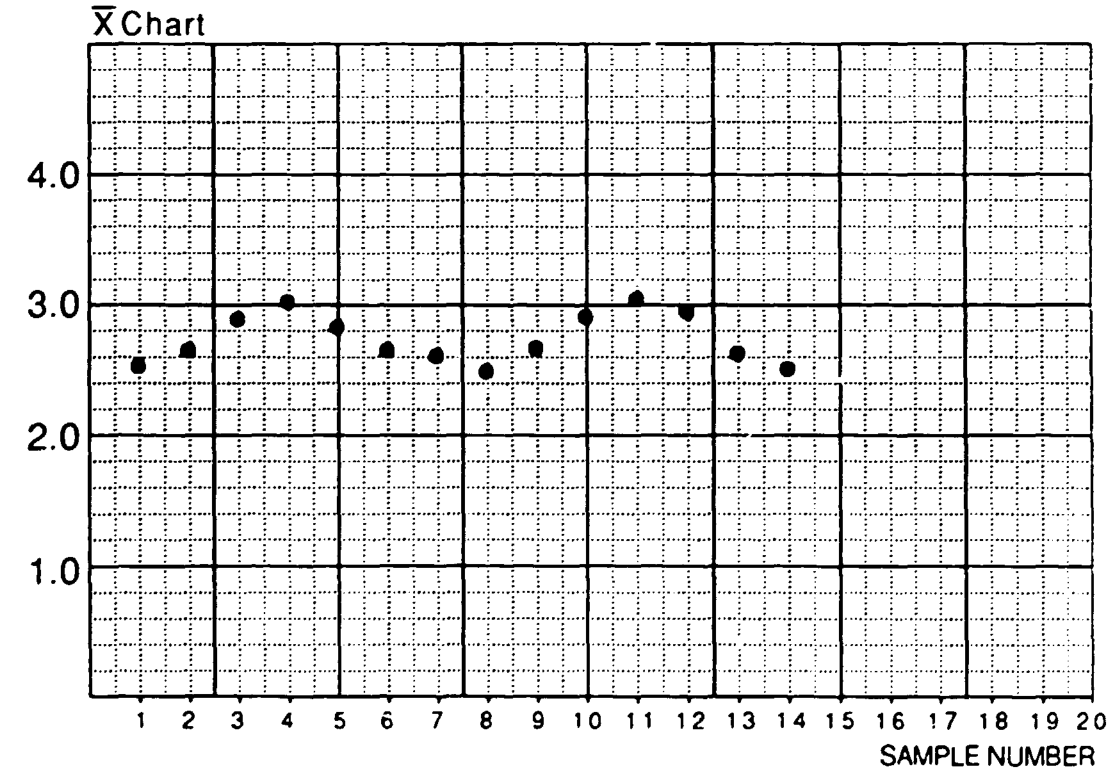
c. In order to learn more about the variability and the average of the process, five successive measurements of Machine 2325 bearing diameters are taken daily for two weeks:

Measurements

Day	1	2	3	4	5	$\bar{X}$	Range
1	2.763	2.583	2.401	2.614	2.469	2.566	.362
2	2.532	2.837	2.878	2.497	2.659	2.681	.381
3	2.538	3.114	3.122	2.893	2.836	2.901	.584
4	3.033	3.226	2.886	2.846	3.344	3.067	.498
5	2.909	2.819	2.808	2.637	2.989	2.832	.352
6	2.714	2.654	2.512	2.557	2.936	2.675	.424
7	2.829	2.697	2.363	2.754	2.437	2.616	.466
8	2.583	2.503	2.518	2.609	2.431	2.529	.178
9	2.582	2.918	2.809	2.734	2.798	2.768	.336
10	2.861	3.014	3.089	2.718	2.922	2.921	.371
11	3.293	3.147	2.859	2.962	3.177	3.088	.434
12	2.892	3.006	2.973	2.811	3.197	2.976	.386
13	2.542	2.780	2.573	2.767	2.455	2.623	.325
14	2.558	2.836	2.427	2.455	2.462	2.548	.409
						2.7707	.3933

Construct appropriate control chart(s) to look for changing variation and a changing process average. Blank control charts are provided for you.

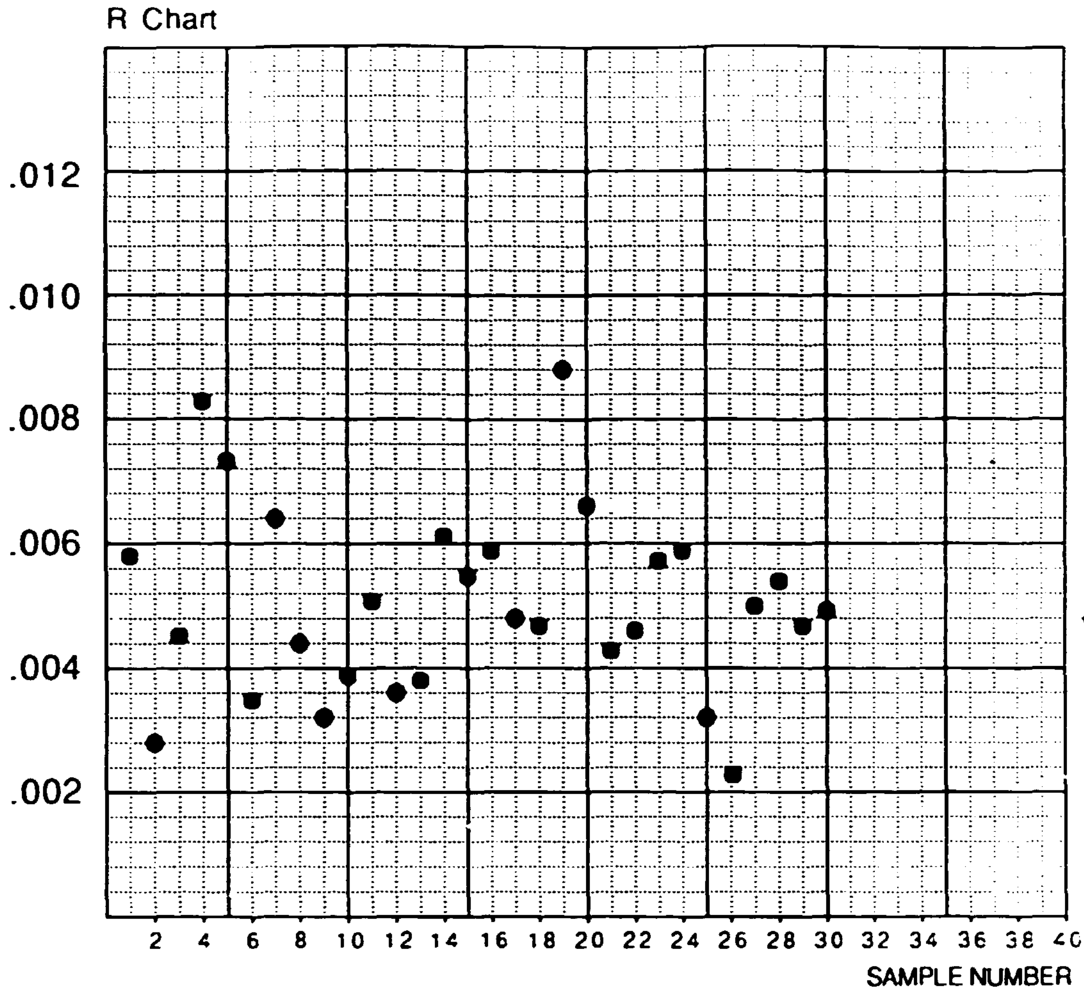


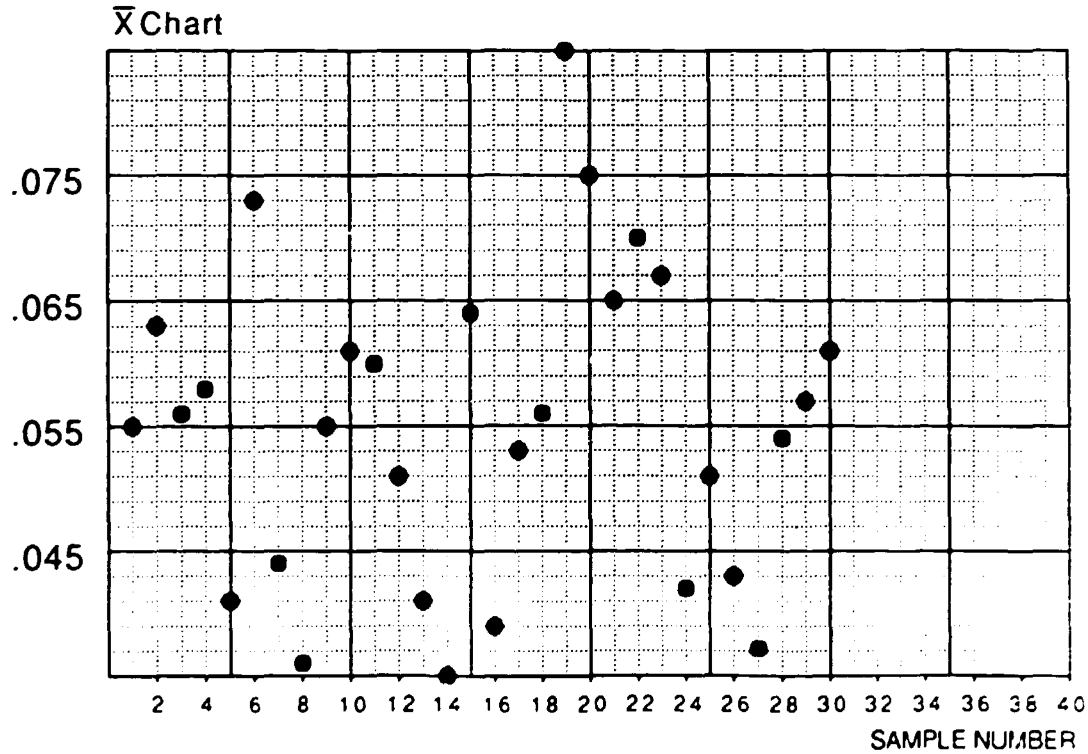


d. Based on the way in which the data were subgrouped, what information is provided in the control chart(s) about the process?

2. A chemical product is produced in large batches. An important characteristic of the product is the level of active chlorine in the material. It is suspected that there exist two major sources of variation in the process which affect the level of active chlorine in the product. One possible source of variation is that the mix of ingredients is not the same from batch to batch. Another potential source is that the batches may not be thoroughly mixed. In order to study the active chlorine levels, three samples from each of 30 successive batches are taken. The levels of active chlorine in the 3 samples are measured. The range and the average of the 3 samples are calculated and plotted on control charts for ranges and means. The sum of the sample means is 1.648 and the sum of the sample ranges is .151.

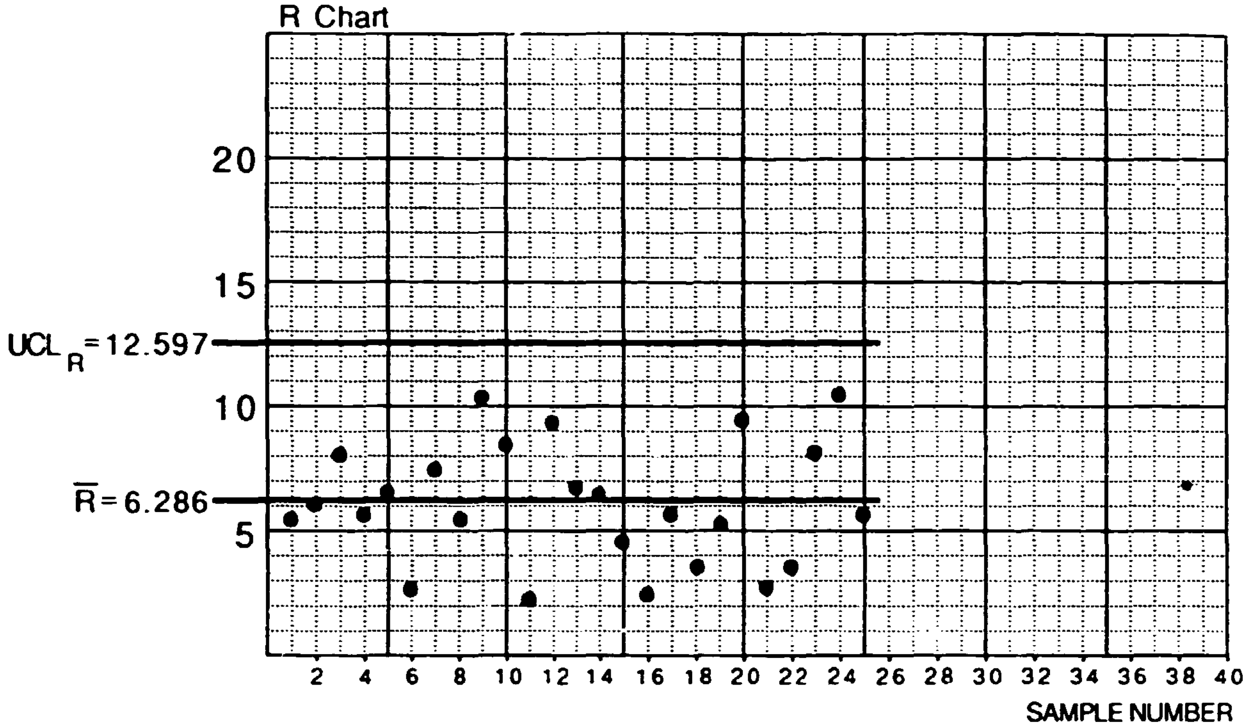
a. Compute the control limits for the range chart and, if appropriate, the X-bar chart. Put the control limits on the provided charts of the data.
b. Which of the two major sources of variation is reflected in the Range values?
c. Which source of variation would tend to cause the range of variation in the subgroup averages to be larger than that predicted by the control limits on the X-bar chart?
d. From visual inspection of the charts, which of the two sources of variation appears to be causing the most variation in the product?

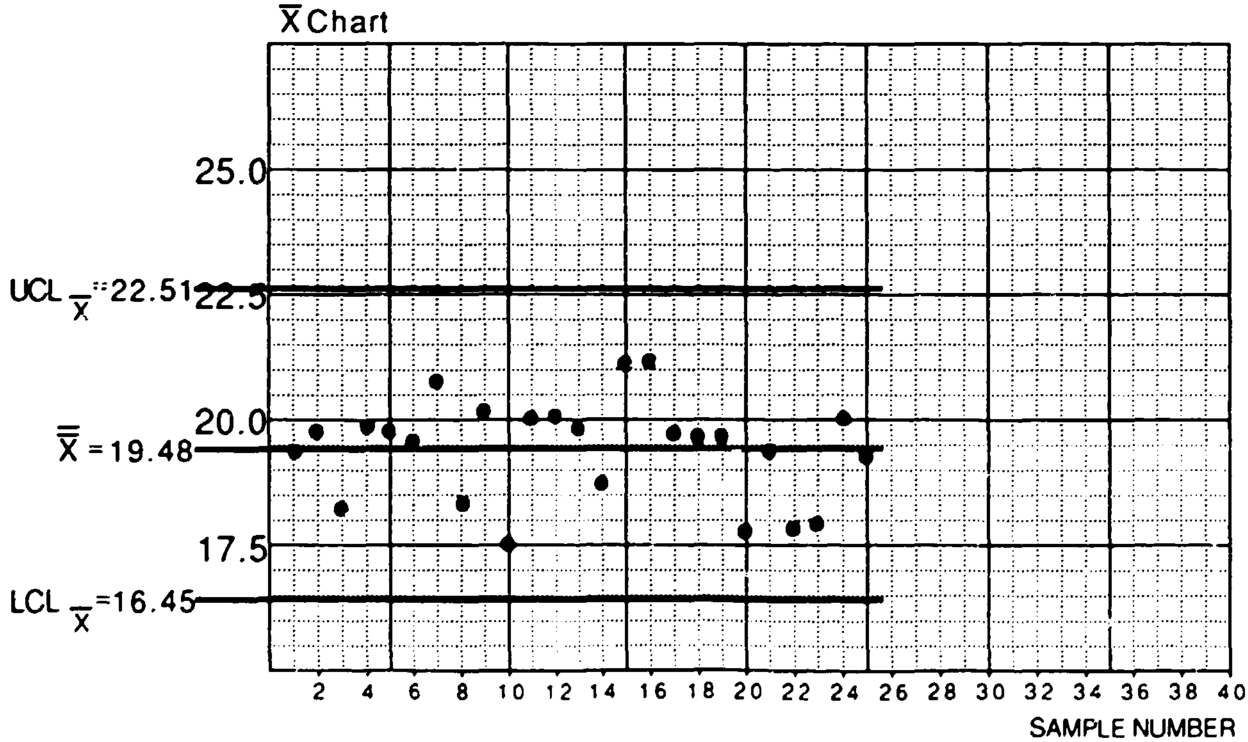

R Chart
.012
.010
.008
.006
.004
.002
2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40
SAMPLE NUMBER


$\overline{X}$ Chart
.075
.065
.055
.045
2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40
SAMPLE NUMBER

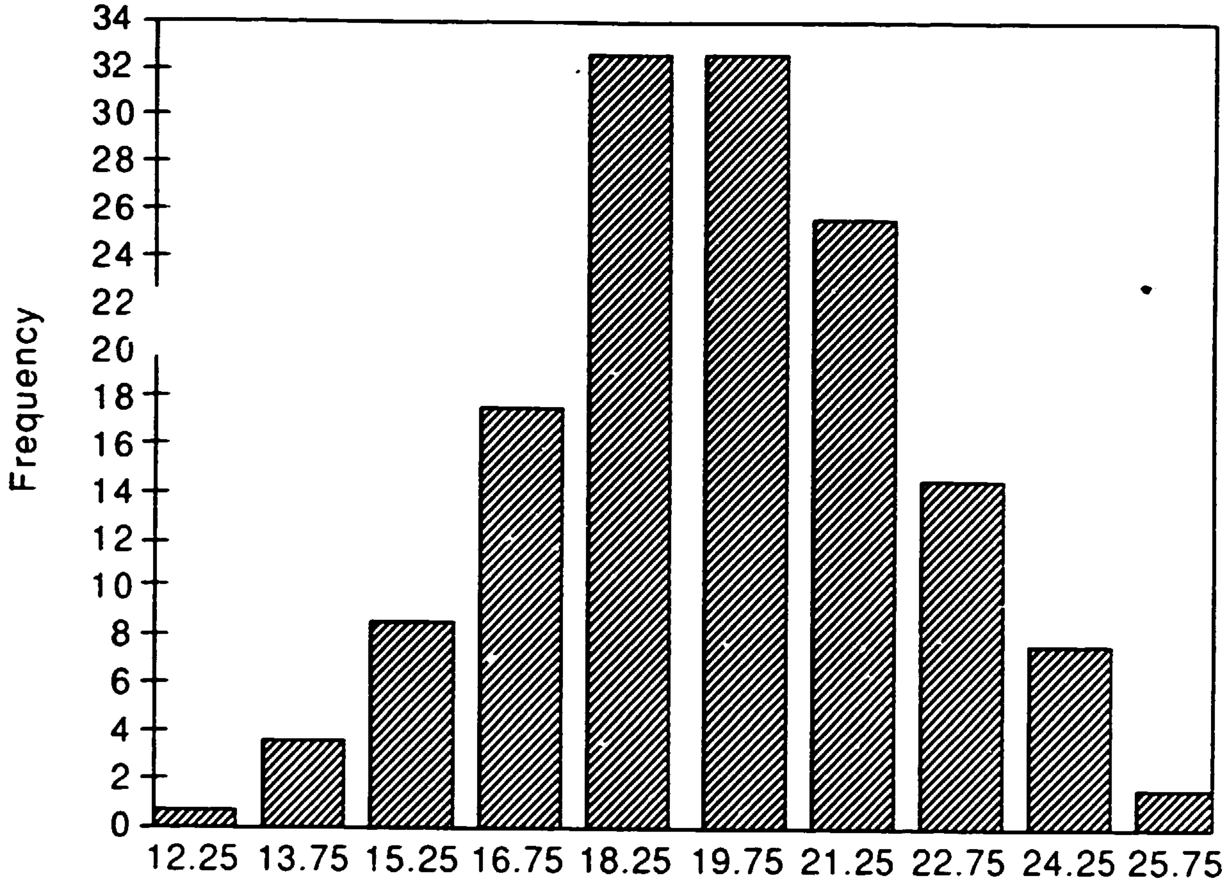
3. On a liquid filling line, empty bottles are removed from a case, filled, and then capped. At present, the capping system is not meeting the required cap torque specifications of 18.0 ± 3.0. The torque is measured after the cap has been on the bottle for at least one hour, in order to simulate the force a consumer will have to apply to remove the bottle cap. The bottles that have cap torques greater than 22 or less than 14 cannot be shipped and must be sent back through the capping system. 25 samples of 6 consecutive bottles are taken from the filling line, and the cap torque is measured on each bottle. The control charts constructed from these data and a histogram of the measurements are provided below.

   a. If appropriate, estimate the process standard deviation.
   b. Is this process capable of meeting specifications?


R Chart
20
15
$UCL_R = 12.597$
10
$\overline{R} = 6.286$
5
2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40
SAMPLE NUMBER


$\overline{X}$ Chart
25.0
$UCL_{\overline{X}} = 22.51$
22.5
20.0
$\overline{\overline{X}} = 19.48$
17.5
$LCL_{\overline{X}} = 16.45$
2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40
SAMPLE NUMBER

Histogram
of Cap Torque Measurements



4. In the production of steel bars, square billet stock is heated and rolled. After the bars are cut to length and straightened, they are shipped to one of two converters where they are sized. Your plant requires the following size specifications to be met by the two converters: 8.75 ± .025. After the steel bars are sized at the converters, they are returned to your plant, where they are stored in lots. The lots generally consist of a mixture of the steel bars sized at the two converters. Prior to shipping to your customers, a final inspection is performed where a sample of bars is inspected for defects and size data is collected on the sample. Because of recent claims by customers that the steel bars are not meeting the size specifications, 10 bars are randomly selected from each of 20 lots and measured.

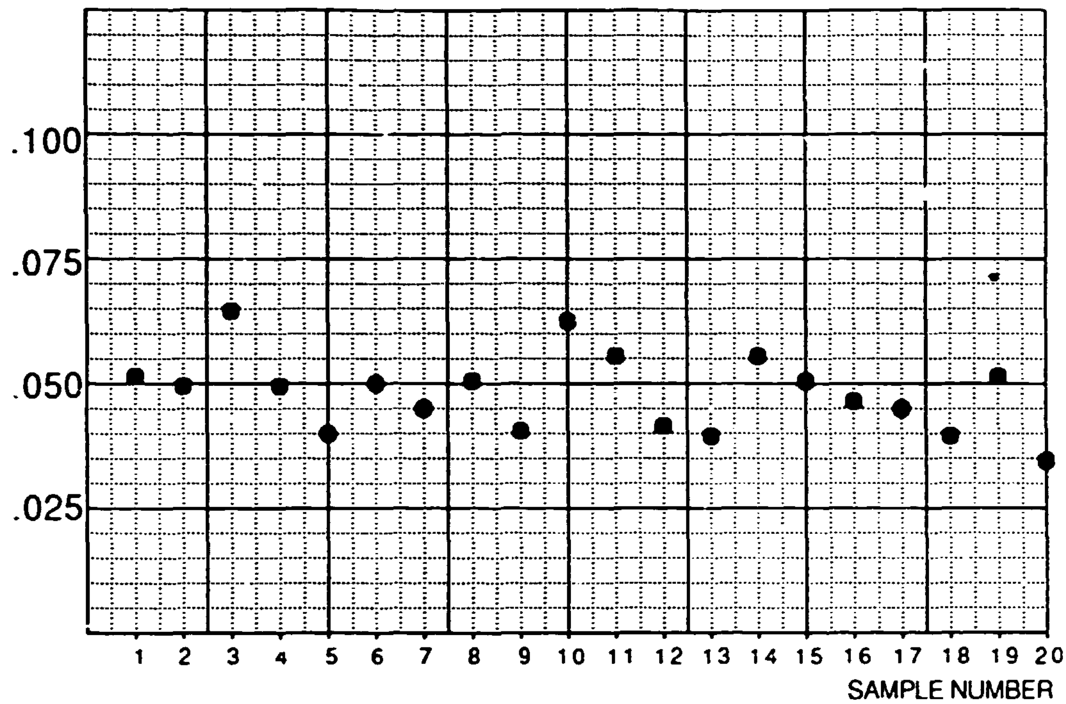
Measurement

1	2	3	4	5	6	7	8	9	10	X-bar	Range
8.760	8.749	8.743	8.793	8.772	8.779	8.743	8.741	8.779	8.775	8.7634	0.052
8.780	8.781	8.743	8.757	8.760	8.778	8.760	8.732	8.767	8.742	8.7600	0.049
8.757	8.779	8.753	8.768	8.722	8.786	8.770	8.775	8.779	8.774	8.7663	0.064
8.779	8.769	8.730	8.771	8.774	8.774	8.758	8.752	8.746	8.779	8.7632	0.049
8.772	8.749	8.753	8.755	8.784	8.775	8.789	8.778	8.784	8.751	8.7690	0.040
8.764	8.772	8.755	8.760	8.787	8.766	8.785	8.783	8.752	8.737	8.7661	0.050
8.738	8.774	8.783	8.781	8.762	8.746	8.777	8.780	8.771	8.776	8.7688	0.045
8.785	8.736	8.782	8.787	8.760	8.756	8.737	8.762	8.745	8.754	8.7604	0.051
8.782	8.785	8.775	8.761	8.781	8.777	8.782	8.751	8.750	8.744	8.7688	0.041
8.725	8.788	8.776	8.778	8.779	8.740	8.760	8.749	8.765	8.776	8.7636	0.063
8.784	8.779	8.782	8.728	8.758	8.741	8.750	8.756	8.757	8.780	8.7615	0.056
8.778	8.779	8.783	8.775	8.747	8.746	8.788	8.782	8.779	8.776	8.7733	0.042
8.781	8.745	8.748	8.748	8.742	8.761	8.779	8.765	8.780	8.776	8.7625	0.039
8.794	8.743	8.768	8.757	8.747	8.789	8.772	8.774	8.799	8.775	8.7718	0.056
8.759	8.745	8.739	8.755	8.747	8.790	8.774	8.788	8.766	8.785	8.7648	0.051
8.759	8.758	8.773	8.750	8.740	8.783	8.787	8.768	8.773	8.783	8.7674	0.047
8.774	8.751	8.737	8.732	8.749	8.740	8.754	8.773	8.777	8.775	8.7562	0.045
8.770	8.770	8.755	8.763	8.773	8.767	8.754	8.780	8.778	8.741	8.7651	0.039
8.750	8.759	8.739	8.752	8.759	8.791	8.783	8.773	8.755	8.739	8.7600	0.052
8.752	8.757	8.784	8.781	8.772	8.785	8.768	8.751	8.755	8.758	8.7663	0.034
										175.2985	0.965

a. Construct the appropriate control charts to analyze the stability of the process. Is the variability of the sizing process in control? Is the process average in control?

b. If appropriate, compute an estimate of the process standard deviation. Determine if the process is capable of meeting the specifications.

R Chart



$\overline{X}$ Chart

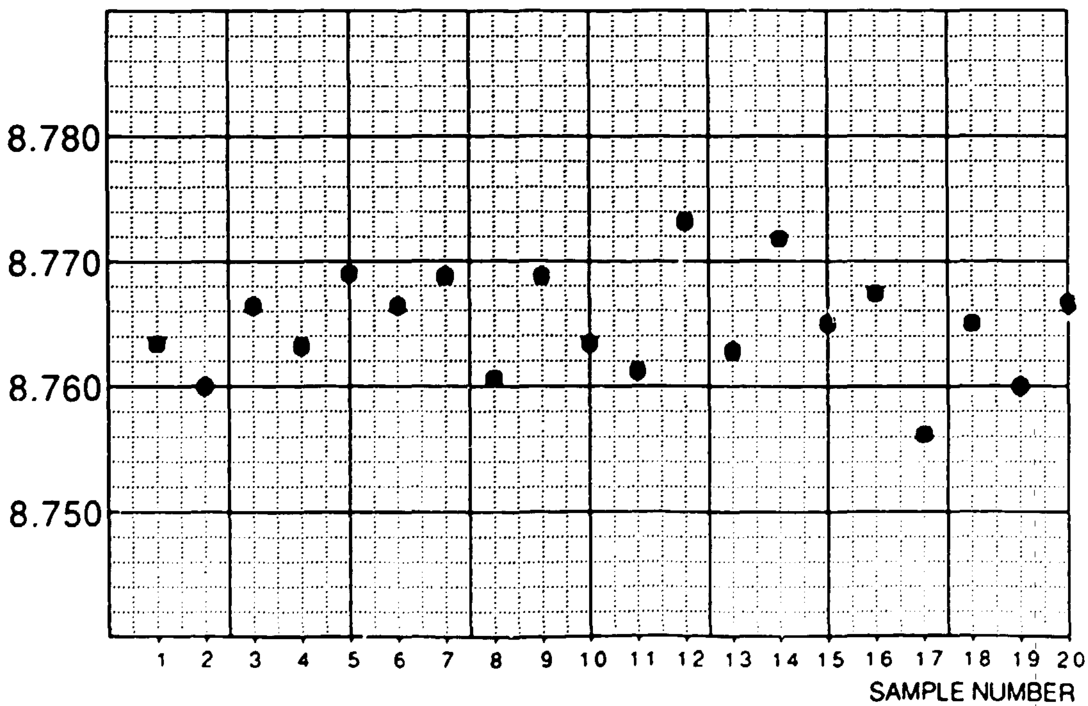


Table 5.2

Factors For Use With $\overline{X}$ and Range Charts

Number of Observations in Subgroup n	Factor for $\overline{X}$ Chart A_2	Factors for Range Charts LCL D_3	Factors for Range Charts UCL D_4	Factor for Estimating σ d_2
2	1.880		3.267	1.128
3	1.023		2.574	1.693
4	0.729		2.282	2.059
5	0.577		2.114	2.326
6	0.483		2.004	2.534
7	0.419	0.076	1.924	2.704
8	0.373	0.136	1.864	2.847
9	0.337	0.184	1.816	2.970
10	0.308	0.223	1.777	3.078
11	0.285	0.256	1.744	3.173
12	0.266	0.284	1.716	3.258
13	0.249	0.308	1.692	3.336
14	0.235	0.329	1.671	3.407
15	0.223	0.348	1.652	3.472
16	0.212	0.364	1.636	3.532
17	0.203	0.379	1.621	3.588
18	0.194	0.392	1.608	3.640
19	0.187	0.404	1.596	3.689
20	0.180	0.414	1.586	3.735
21	0.173	0.425	1.575	3.778
22	0.167	0.434	1.566	3.819
23	0.162	0.443	1.557	3.858
24	0.157	0.452	1.548	3.895
25	0.153	0.459	1.541	3.931

Chapter 6

Variables Data, Continued
The Moving Range and Individual's Chart

Before beginning a discussion of control charts for individual measurements, it will be useful to review some of the basic tenants underlying the use of control charts for process improvement. Statistical control charts for ranges and averages of subgroups of size 'n' are based upon a sampling strategy that recognizes a common cause system of variation generating or creating the variation seen in each group of 'n' measurements, or, in other words, a common set of causes operating within the respective subgroups. This common cause system of variation is thought of as being causes which produce variation subgroup after subgroup and which have approximately the same effect upon individual parts or material whose measurements make up the subgroup.

The creation of the range chart allows the manager, engineer, or operator to examine the predictability and magnitude of the common cause system supposedly operating within the subgroups. As evidenced by the fact that the average subgroup range is used to construct limits for the average chart, the variation between subgroups, seen in the deviations from one subgroup average to another, is evaluated against the sources, or causes, creating within subgroup variation. Process abnormalities which are active between subgroups are likely to be detected by having one or more values of the subgroup averages outside of the control limits. These points are taken to be influenced by causes not captured or represented by the variation within subgroups and provide evidence that assignable causes are, or were, present in the process.

It is the removal of these assignable causes that permits the process to be brought "into statistical control." In general, however, this necessary activity of finding and removing from recurrence the assignable causes does not result in process improvement. Process improvement is usually gained by first reducing long-term consistent deviations in averages and then by reducing the magnitude of within subgroup variations.

6.1 Analysis of process data with subgroups of size 1

There are numerous processes where it is not feasible to create rational subgroups of two or more measurements on a process. Examples are found in processes where measurements are expensive or require long periods of time to obtain. Other situations are found in processes where a measurement is available only hourly, daily, weekly, or less frequently. In these situations, it may not be possible to form homogeneous groups of measurements for subgrouping purposes. These situations may be found in manufacturing, service, or administrative processes. In a manufacturing setting, two examples in which the formation of homogeneous subgroups may not be possible are, one, when large, homogeneous batches of material are produced and two when records on process parameters such as temperature variability and level, pressure values, and amperage are to be studied . Administrative examples would include the study of average overtime hours per full-time employee, daily utilization rates of equipment or other resources, accounting data on shipments and orders, monthly number of items produced per direct labor hour, and monthly deviations of actual sales from forecast. In each of these examples, manufacturing and administrative, only one observation is available to represent a given set of circumstances.

In those situations which have only one measurement available at any one condition, there is no way to compute a subgroup range. In the attempt to use short-term variations as baseline variability against which to judge long-term process movements, a compromise is made. This short-term variation is calculated from the average of the absolute values of the deviations between two consecutive values. In turn, this average deviation is used to calculate limits for the chart of individual values. Calculations are demonstrated by using data in the following example.

6.2 Construction of moving range and individuals charts

In common with other types of charts for process investigation, initial data must first be collected to help baseline a process. This example will deal with that initial data set. The data are taken from a batch process producing a sterilized, concentrated, baby formula. While there are several important product characteristics, only one, the Brookfield viscosity in centipoise is reported in this example. Specifications on this property are 900 ± 100. Data from 20 consecutive batches are reported in Table 6.1.

The sampling scheme for the data of Table 6.1 required that one specimen (sample) be taken from a randomly selected position in the completed batch. Thus, each reported number represents a finished batch. These individual numbers, one per batch, constitute the numbers that will be plotted onto the "individuals" chart. That chart requires an average or center line and control limits. The average value for the individual values is computed like any average. The control limits for the chart for individual values are computed from information on the chart for moving ranges; this computation is discussed in the following paragraph.

As indicated earlier, the moving range is the absolute difference between consecutive values. The first moving range recorded in Table 6.1 is the difference between the first two values. So this first moving range, or MR, is:

$$MR = 976 - 805 = 171$$

Once the moving ranges have been calculated for the baseline data set, these values are plotted onto the moving range chart. It should be noted that there will be one fewer moving ranges than there are individual values in the data set. The average value for the moving range can be computed as with any other average; sum the moving ranges and divide by the number of moving ranges. This average moving range, or $\overline{MR}$,will be:

$$\overline{MR} = \frac{1477}{19} = 77.7368.$$

The upper control limit for the chart for moving ranges is computed as if subgroups of size two were used, yielding an upper control of

$$UCL_{MR} = D_4\overline{MR} = 3.267(77.7368) = 253.97$$

A completed moving range chart is given in Figure 7.1. All values on the moving range chart are within the control limits, which provides evidence that the short-term variation is stable across the time of the study. Since the average moving range captures the magnitude of the short-term variation, this average will be used to provide a basis for judging the stability of the individual readings of the batch process. The quantity:

$$\frac{\overline{MR}}{d_2}$$

describes the standard deviation expected in the individual readings if the short-term variation captures all the variations affecting the individual viscosity readings. The limits on the individuals chart reflect this short-term component of variation. If additional sources of variation occur over the long-term (long-term necessarily defined by the time span covered by the baseline data collected) this additional variation should show up as an out-of-control signal on the chart of individual values. The control limits for the individuals chart are computed from the following formulas:

$$UCL_x = \overline{X} + 3\,(\overline{MR}\ /\ d_2)$$
$$LCL_x = \overline{X} - 3\,(\overline{MR}\ /\ d_2)$$

The control chart constant, d_2 is found in the table of control chart constants from Appendix A. The value of d_2 =1.128 is for subgroups of size two and is consistent with the use of D_4 for subgroups of size two. The center line and upper and lower control limits for the individual readings on viscosity are given by:

$$\text{Center Line} = \overline{X} = \frac{17879}{20} = 893.95$$

$$UCL_X = 893.95 + 3\left(\frac{77.7368}{1.128}\right) = 1100.70$$
$$LCL_X = 893.95 - 3\left(\frac{77.7368}{1.128}\right) = 687.20$$

The completed chart of individual viscosity readings also appears in Figure 7.1. All of the points lie within the control limits and an applicatiions of the runs test does not indicate any nonrandom behavior in the points.

6.3 Effective use of moving range and individual charts

As with the successful use of other statistical charts for process management and improvement, the manager must understand the sampling strategy relative to the process causes that create the outcome values. The sampling strategy for this process was to select one sample from each completed batch; so one value was recorded per batch. The individual observation then represents the batch. The moving range captures the effects of several sources of variation: measurement, within batch, and batch to batch. Information on the magnitude and specific sources of measurement variation can be had from studies of the measurement process used to provide viscosity readings. (The data collection and methods for studying the variation due to the measurement process are discussed in Chapter 8.) The sources contributing to within batch variation and batch to batch are process based and process created; it is likely that these variations are created by two general sets of variations, those kinds of things that tend to make one batch differ from another and these kinds of things that tend to result in non-homogeneous batches. An example of the first could be found in the amounts and densities of raw material used to manufacture a batch; these might vary from batch to batch in unappreciated and unmeasured ways and they will tend to result in batches being different. There are other process practices and methods which are equipment and personnel related that could be added to the list of sources of variation which make one batch different from another. Once a particular batch is created, there are also process sources that result in batches that are not homogeneous and this within batch variability will itself have a certain magnitude and possibly certain fluctuations. The random sample selected from the batch and the measurement made on that specimen assures that variations in the individual values are due to all of these causes. The moving range reflects these variations.

The average value of the moving range indicates the average deviation from one batch to another for this process. The upper control limit for the moving range chart provides information on the magnitude by which consecutive batches could differ from each. Points above the upper control limit for the moving range chart reveal abnormally large shifts or changes from one batch to the immediately following batch. It is recommended that the usual runs or patterns tests not be applied to the moving range chart in checking for stability of short-term variations. Moving ranges are calculated from consecutive observations and so share an observation in common with another moving range; this characteristic prevents these values from being independent, a mathematical prerequisite for appropriate use of the usual runs tests or rules. The lack of independence between values of the moving range does not significantly affect estimates of process variability based upon the average value for the moving range.

Control limits on the individuals chart provide the manager with the information to judge whether the process is subject to variations exceeding those acting in the short-term, which are represented by the average moving range.

Table 6.1

Viscosity Readings

Batch	Viscosity	Moving Range
1	805	-
2	976	171
3	901	75
4	929	28
5	927	2
6	942	15
7	904	38
8	804	100
9	874	70
10	944	70
11	850	94
12	941	91
13	992	51
14	795	197
15	952	157
16	832	120
17	809	23
18	878	69
19	936	58
20	888	48
Totals	17879	1477

Chapter 7

Subgrouping and Components of Variance

In the study and improvement of systems and processes, the value of statistical analysis depends on the knowledge of those who use the methods, the rationale and purpose of the analysis, the alignment of its use with the objectives of managers and nonmanagers regarding systems which support quality, delivery, and cost, and how timely and relevant the data collected for process study are. These characterizations of the value provided by a statistical analysis, in turn, depend on the sampling or subgrouping strategy by which data on the process are collected. Data collection and its statistical analysis will only aid process improvement if they provide information for directing work efforts towards those causes which result in unsatisfactory process performance. A statistical analysis can only provide this direction if the intent to provide this direction was understood prior to collecting process data and if this intent guided the manner in which data on the process was obtained. The sampling or subgrouping strategy is thus the major determinant of the quality of information provided by a statistical analysis.

A poor sampling strategy would be one for which there is little understanding of how the data were actually collected, or for which the output from several sources of product are mixed in manners which may not even be well understood, or when the data were gathered at only one point in time and thus causes factors present only at some times can't be studied, or when the data are collected without due consideration for the activity and effect of the sources listed on a cause-and-effect diagram. Typically, data routinely collected at inspection stations will suffer from these characteristics of a poor sampling strategy. Since this kind of inspection data is usually collected for judging process output and without regard for the sources creating good or poor quality output, it will not generally be useful for providing information on the causes affecting process outcomes.

The deliberate study of processes with the intent of working on improving them is usually a two stage effort. These stages might be called the control stage and the improvement stage. In process study, the first responsibility is to study the process to know how to operate the process in a consistent fashion in the presence of the cause factors which may be affecting the process. This statement implies that the cause factors and the nature of their effects are understood. After a demonstrated ability to run the process in a consistent fashion, it is the responsibility of management to address methods for improving the process. This improvement may take the form of reducing product variation or of improving the ability to target the process. The subject of the present chapter is to discuss subgrouping and analysis techniques which will aid in identifying and understanding the contribution of sources of variation to product variation.

7.1 Understanding the components contributing to total variation

Work to address the reduction of process variation depends on the ability to understand the factors contributing to the total process variation. This understanding requires the ability to identify the nature and magnitude of the sources, or components, acting to contribute to total variation. For example, in the production of a porous membrane, a characteristic of critical interest is the variation in the pore size of the membrane. The material used to create the membrane may be responsible for much of this variation, but the nature of that contribution to variation would need to be understood in order to direct work on reducing variation. In this instance, the material used to produce membranes is received in large containers. Is the variation in pore size largely affected by the differences from one container to another, or is it something about the material within each and every container which results in variability in the pore size? To answer this question information on the relative contribution of within container and between container sources to variation would need to be understood.

As another example, consider the total variation which is observed in the fill weights of containers coming off of a filling operation. The filling operation has three filling machines and each of the machines has four heads. In assessing the magnitude of the variation in the fill weights, it is discovered that the variation is way too large. An understanding of how to act to decrease this variation will require knowledge of the relative contributions of the differences in filling heads and differences in filling machines to the total observed variation.

A third example where understanding the relative contribution to total variation is important is in the production of material in a batch process. Of critical importance in this batch process is the variation in particle size of the material produced. To know how to act to reduce that variation an understanding of the sources for the variation is required. Is the large variation in particle size observed within each batch? Or is it that within a batch particle size is fairly uniform, but each batch differs so much from the last that the resulting variation across batches is large? Collecting data to answer these questions will require a sampling strategy which identifies what the relative contributions of the within batch and between batch sources of variation are to total variation.

The following section discusses a technique for collecting and analyzing data to understand the contributions components of variation make to total variation. Although the example used to illustrate the study of components of variation comes from a batch process, the value of the technique is not confined to batch processes. The examples above of producing a porous membrane and of evaluating a filling operation provide two other situations where components of variance studies might be used.

7.2 Study of a Batch Process

A work team has been organized to study the variation of a given batch process. They have constructed a flow process chart and identified the quality characteristics of interest for the product. One of the characteristics thought to affect final product quality was viscosity of the batch at an intermediate stage of processing. The specified range for the viscosity measurement was from 83 to 85. At the time the work on the process began there was little knowledge about how well the viscosity was being maintained, so the team decided to determine the current process status by taking one sample from each batch in order to evaluate the current level of variation and the current average of the process. Initial data for the 20 batches studied are given in Table 7.1. Figure 7.1 contains a moving range and individuals chart constructed from these data. Since both charts are in control, the viscosity appears to be consistent. . .n estimate of the standard deviation of viscosity reveal an unacceptably high level of variation.

$$\hat{\sigma}_t = \frac{\overline{MR}}{d_2} = \frac{0.605}{1.128} = 0.536$$

The natural tolerance, NT, of the process is found to be:

$$NT = 6(0.536) = 3.216$$

Since the engineering tolerance, ET, is only 2, the variation in the process is much wider than the stated requirements for viscosity. Because of this large variation, the work team decided to try and identify the sources contributing to the large variation in viscosity readings.

7.2.1 Sources affecting total variation in a batch process

In order to direct improvement efforts aimed at reducing the variation in viscosity readings, the work team needed a better understanding of the sources acting to affect total variation. Figure 7.2 is a graphical illustration of two possible scenarios which may be occurring in the batch process. In the scenario on the left side of the page, there is little difference in the viscosity measurements for batches one, two, and three; each of the three batches has about the same range and centers at about the same average. Thus, the explanation for the total variation in the combined output from the three batches is because of the large variation within each of the three batches. In the scenario depicted on the right side of the page, there is a smaller level of within batch variation for each of the three batches depicted. However, in this scenario there is a larger difference between the three batches. The variation in the combined output in scenario 2 is similar to that for scenario 1. But in scenario 2 the explanation for the large total variation in the combined output is because of the large difference <u>between</u> batches.

Understanding how the sources or variation, within batch and between batch, are contributing to the total variation is important in directing where work efforts should be concentrated in order to reduce total variation; different causal structures will be responsible for the within variation as compared to the between variation. For example, mixing practices of the batch process might be responsible for variations observed within a batch, but would have little effect on differences observed on the level of viscosity from one batch to the next. And large variations in the amount of a viscous ingredient added in the initial stage of production of each of the batches might result in large differences in viscosity from one batch to the next without impacting the within batch variation. Therefore, if scenario 1 in Figure 7.2 summarizes the impact of within and between batch variation on total variation, attention would be directed towards those sources contributing to the large within batch variability. A similar conclusion would be reached if scenario 2 described the relative contribution of within and between batch sources to total variation.

The task of understanding the effect of within and between batch sources of variation will require the development of a data collection, or a subgrouping strategy, to capture the effects of these two sources of variation. So in order to understand within batch variation, multiple readings of viscosity from each of several successive batches will be required. And to assess differences from one batch to another, readings from many batches will be needed.

7.2.2 Assessing the stability and magnitude of within batch sources of variation

The work team decided to collect five samples from randomly selected locations from each of twenty successive batches produced. From these data, they intended to, first, describe the stability and magnitude of the within batch component of variation. The averages and ranges of the five measurements from each batch are recorded in Table 7.2. Before considering an analysis of these data, it will be instructive to consider the sources of variation which are captured by the ranges. Since each of the five measurements in a subgroup came from the same batch of material, the magnitudes of the ranges reflect those sources of variation active <u>within</u> a batch of material. (Large levels of measurement variation would also act to increase the magnitudes of the ranges. Discussion of the effects of measurement variation, however, are delayed until chapter 8.) Of course, the amount of variation within a batch may not be consistent from one batch to the next. Thus a range chart will be required to decide whether this component of variation, the within batch component, is stable across the 20 batches included in the study. This range chart appears in Figure 7.3.

Since the range chart of Figure 7.3 is in control, there is no evidence that the within batch variation is inconsistent. The magnitude of the within batch variation can be estimated by calculating an estimate of the standard deviation for within batch variation. This component is found by:

$$\hat{\sigma}_{within} = \hat{\sigma}_w = \frac{\overline{R}}{d_2} = \frac{0.775}{2.326} = 0.333$$

Care should be exercised in interpreting this standard deviation; it does not represent the total variation in the batch process Clearly, this component, the within batch standard deviation, does not capture all of the variation in the process, since the between batch component of variation also impacts total variation observed in the combined output from the batch process.

7.2.3. Assessing the stability and magnitude of between batch sources of variation

The data collection strategy used to generate the data in Table 7.2 can also provide information on the between batch sources of variation. This information is available by analyzing the behavior of the batch averages. Since each average is an estimate of the average level of a batch, the differences between the batch levels will be reflected in the variation observed in the averages. Thus, an analysis of the batch averages will provide information on the between batch component of variation. The appropriate method for performing this analysis needs further discussion.

Typically, when evaluating the stability of a process level using an X-bar chart, the control limits for this chart are calculated using the average range from the R chart. However, in the present instance, this average range only captures within batch sources of variation. Yet, the intent in analyzing the batch averages is to evaluate the stability of the between batch source of variation. Thus, the limits on the X-bar chart should reflect common cause sources of variation affecting the between batch component of variation. In other words, what is required is a technique for deciding whether the between batch component of variation is stable over time or whether at some time or times a large difference in one or more batches can be concluded to be the result of a special cause acting on the batch levels.

In order to arrive at a meaningful way of evaluating the stability of the between batch component of variation, moving ranges of the batch averages have been calculated. Table 7.3 contains these moving ranges. The data in Table 7.3 includes the batch averages which already appeared in Table 7.2. Thus, the moving ranges calculated from these batch averages capture the short-term variation which is observed between batches. Of course, short-term batch-to-batch variation is not the only source of variation captured by the moving ranges. The batch averages are also be subject to variation from the sources of variation driving the within batch component of variation. Therefore, a description of the sources of variation affecting the moving ranges would be that the magnitude of the moving ranges reflects both the within batch and the short-term between batch sources of variation.

Figure 7.4 contains the completed moving range chart. The calculations necessary for completing this chart are provided in the box below.

Calculations for the moving range chart
of Figure 7.4

Since two averages are used for calculating each moving range, constants for samples of size n=2 are used.

$\overline{MR} = 0.606$

$UCL_{MR} = D_4\overline{MR} = 3.267(0.606 = 1.98)$

LCL_{MR} – none

Since the moving range chart is in control, it can be concluded that short-term between batch variation appears to be stable. Using the average moving range, the control limits for the chart of batch averages can now be calculated. The calculations necessary to complete this chart appear in the box below.

Calculations for the X-bar chart
of Figure 7.4

The standard deviation for batch averages is found from:

$$\frac{\overline{MR}}{d_2} = \frac{0.606}{1.128} = 0.537$$

The center line for the X-bar chart is the average of the X-bars.

$$\overline{\overline{X}} = 83.95$$

The control limits for the X-bar chart are found by adding and subtracting from $\overline{\overline{X}}$ three standard deviations of the batch averages.

3(0.537)=1.611

$$UCL_{\overline{X}} = 83.95 + 1.61 = 85.56$$

$$LCL_{\overline{X}} = 83.95 - 1.61 = 82.34$$

Since the completed X-bar chart is in control, it can be concluded that the between batch component of variation is stable over time. Stated another way, neither the short-term nor the long-term batch-to-batch component of variation appears to be subject to special cause sources of variation.

Now, just as with the within batch component of variation, it will be useful to have an estimate of the between batch component of variation. This estimate, $\hat{\sigma}_b$, will be based on the calculated standard deviation for the batch averages. The formula for calculating the standard deviation of the between batch component of variation is given below:

$$\hat{\sigma}_b = \sqrt{\left(\frac{\overline{MR}}{d_2}\right)^2 - \frac{\hat{\sigma}_w^2}{n}}$$

where the value used for n here is the number of observations used to calculate each batch average; thus, n is equal to five. The within batch standard deviation, $\hat{\sigma}_w$, was previously estimated to be 0.333. So, the between batch standard deviation is found to be:

$$\hat{\sigma}_b = \sqrt{(.537)^2 - \frac{(0.333)^2}{5}} = .516$$

7.2.4. Describing the contribution of within and between batch sources of variation to total variation

Using the estimates of within and between batch variation, an estimate of the total variation of the batch process can be made. This estimate uses the fact that the total variance of the batch process is found by summing the variances of the within and between batch components. Thus, the standard deviation for the batch process is found by:

$$\hat{\sigma}_t = \sqrt{\hat{\sigma}_b^2 + \hat{\sigma}_w^2} = \sqrt{(.516)^2 + (.333)^2} = .614$$

This standard deviation, $\hat{\sigma}_t$, is an estimate of the same quantity as the process standard deviation estimated from the moving range chart of Figure 7.1. The process standard deviation estimated from the moving range chart in Figure 7.1 was based on taking one sample from each of 20 successive batches. The process variation calculated above was found by first estimating the two components contributing to the total process variation and then using these two components to arrive at an estimate of total process variation.

Another way of capturing the relationship between total batch process variation and the within and between batch components of variation would be to write the relationship in terms of the total, the between, and the within variances. In other words, the relationship could be described by:

$$\hat{\sigma}_t^2 = \hat{\sigma}_b^2 + \hat{\sigma}_w^2$$

The values of the above variances are found to be:

$$\hat{\sigma}_t^2 = .377145$$
$$\hat{\sigma}_b^2 = .266256$$
$$\hat{\sigma}_w^2 = .110889$$

The description of total, between, and within variation in terms of the variances is useful as it provides a way of thinking of the relative contribution of the components to the total variation. In this example, the within contribution accounts for about 30% of the total variation. This percentage is found by:

$$\frac{\hat{\sigma}_w^2}{\hat{\sigma}_t^2} \times 100\% = \frac{.110889}{.377145} \times 100\% = 29.4\%$$

Since 30% of the total variation can be attributed to within batch variation, 70% of the total variation can be accounted for by the differences between batches. This statement impl.es that reducing the variation in viscosity readings between batches promises a greater reduction in the variation of the batch process than working on within batch sources of variation. The work team addressing the variation in viscosity readings, would now be directed to examine the causal structure affecting batch-to-batch variation.

7.3. Summary of the analysis of components of variance

The analysis of the components of variance for the viscosity readings from a batch process required several stages of plots and calculations. To aid the reader in recalling the flow of the work in analyzing the components of variance, a flow chart of the steps of the analysis are captured in Figure 7.5. As a means for summarizing the work done on viscosity readings and for providing a description of the work required for studying components of variance in other situation, a step-by-step discussion of this flow chart is provided.

Step 1 of the flow chart contains a description of the type of data required for analyzing the between and within components of variation contributing to total variation. In studying a batch process, the data necessary are repeated readings of 2 or more from each batch. The letter $\underline{n}$ in the flow chart refers to the number of readings collected from each batch. The $\underline{n}$ readings from each batch form the subgroups; k subgroups are formed. In other words, repeated readings are taken from each of k batches. How the k batches studied are selected is an important decision, since this decision will affect the kind of information that is obtained about between batch variation. At least 20 batches should be selected. However, more important than the consideration of the total number to be studied, say 20, is how the 20 are selected. The k subgroups should be chosen so that the sources of variation thought to affect batch variation have a chance to be active. For example, it may be supposed that a change in raw material source will affect between batch variation. But if the k subgroups do not include batches from different raw material sources, then the data will not capture the effect that raw material variation has on between batch variation.

Although the first as well as succeeding steps are written as a description of studying batch processes, the same data collection strategy can be used for studying within and between components for other types of processes as well. For example, in section 7.1 a process for making porous membranes was discussed. In this process it was of interest to understand the variation in pre size which can be attributed to container-to-container variation and that which can be attributed to within container variations. Data collected to understand these two sources would consist of n repeated membranes made from the same container being studied for pore size; and these repeated readings on pore size would need to made for k different containers.

Once the data are available for analysis, the next step, step 2 on the flow chart, is to construct a range chart from the ranges of each of the k subgroups. The constant used to find the correct value of D_4 is n, the number of readings in each subgroup. The range chart is used to evaluate the stability of the within batch, or within subgroup, component of variation. If this range chart is out-of-control, then work needs to be directed at discovering why the within subgroup variation is inconsistent. A cause-and-effect diagram which captures those sources of variation which affect within subgroup variation will be helpful. If the range chart is out-of-control, the cause-and-effect diagram can be explored to provide some direction for investigating those sources which may be acting intermittently to create unstable variation within the batches.

If the range chart is out-of-control, it is not possible to continue the analysis to investigate the between batch component of variation. Stated in another way, it is inappropriate to construct the moving range chart of the batch averages if the range chart for within subgroup variation is out-of-control. There are two ways that the reasons for not doing the moving range chart can be though about. One is that statistical theory does not provide a way for evaluating the moving range chart of batch averages when the within subgroup variation is unstable. The second way of considering the issue is from a more pragmatic view point. Since the range chart was out-of-control, the personnel working on the process already have provided a direction for future work; the current responsibility is to discover and correct the reasons for the unstable within subgroup variation.

Step 3 on the flow chart is the estimation of the within subgroup or within batch standard deviation. (Of course, it would not be appropriate to estimate this component of variation if the range chart had been out-of-control.) The value of d2 used in the formula of step 3 corresponds to n, the number of readings in each of the subgroups.

Step 4 of the flow chart is to construct a moving range chart from the subgroup or batch averages. It should be noted that the value of D_4 used to calculate the upper control limit corresponds to the one for samples of size 2. This value is used since there are two numbers (two averages) used to calculate each moving range. This moving range chart is done in order to evaluate the stability of the short-term batch-to-batch variation. If the flow chart in Figure 7.5 were being used to evaluate the variation in the pore size of the porous membrane, then the moving range chart would be a method for evaluating the stability of the short-term between container variation. In both cases, if the moving range chart is out-of-control, then work should be directed towards identifying the reasons for the observed instability. For example, if the moving ranges of the average viscosity readings had shown evidence of inconsistency, attention would be directed towards identifying why at some times one or more batches is very different from the others. And if the moving range chart is out-of-control, then it would be inappropriate to construct the X-bar chart.

Step 5 in the flow diagram is to construct the X-bar chart. This chart gives information on the stability of long-term between subgroup, or between batch, variation. For example, if there were trends or cycles in the batch averages, this X-bar chart should show evidence of this special cause.

If the X-bar chart is in control, steps 6 and 7 of the flow chart can be completed. These steps provide the formulas for calculating the between and total components of variation. The final step, step 8, provides the formula for calculating the percentage of total process variation due to within subgroup sources and between subgroup sources of variation. These percentages are helpful in prioritizing work for improving the process. If within sources of variation are the major contributor to total variation, then attention needs to be directed at those causes affecting within variation. For example, in the viscosity readings on a batch process, investigating within sources of variation might involve an investigation of the mixing procedures of the batches since poor mixing might be one of the reasons for large variations in viscosity within a batch.

7.4 Chapter 7 Practice Problems

1. Bottles of shampoo are filled on an automatic line using a five-head filler. You have been assigned to the team responsible for analyzing the process, determining process capabilities, and recommending ways in which to improve the process. In order to begin a study of the fill weights, you have decided to check the calibration of the scale and to determine whether or not the measurement process is stable. Three members of your team have suggested the following different subgrouping methods for this purpose.

    1. A subgroup is to consist of the weights of five consecutive bottles from one head.

    2. One bottle should be selected from each head. The subgroup consists of four weights of the same bottle from one head.

    3. A subgroup should consist of one bottle selected from each of the five heads, and each bottle should be weighed once.

        a. What are the sources of within subgroup variation in the subgrouping method (1)?

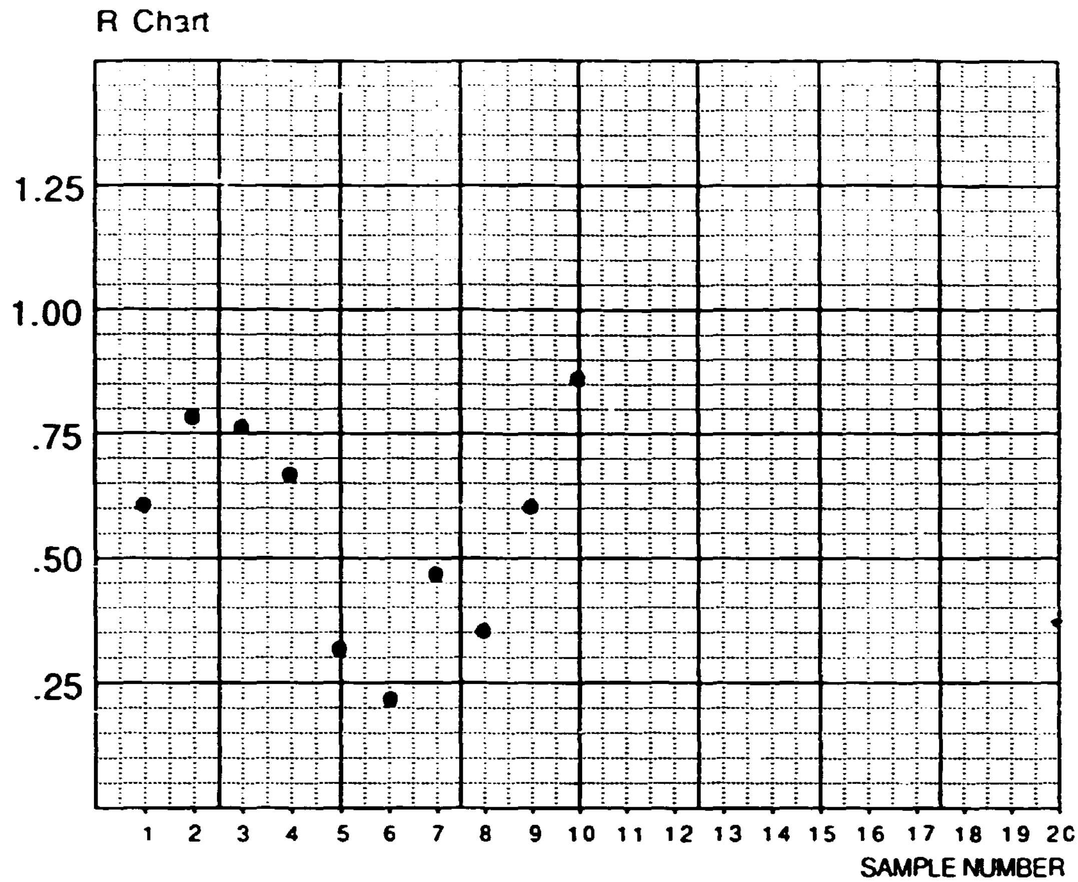
        Sources of within variation in (2)?

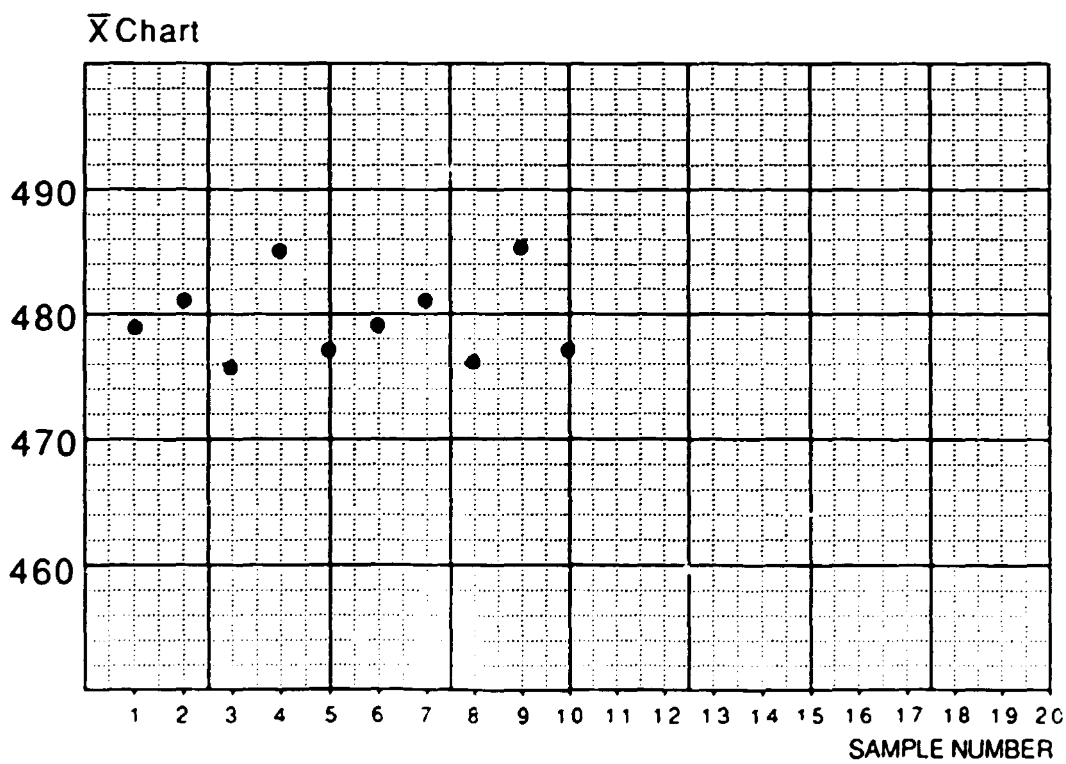
        In (3)?

The team has decided to use the second subgrouping method, whereby a bottle is chosen from one head, and four repeat measurements of the weight are made.

Sample Number	Head	Measurement 1	2	3	4	$\overline{X}$	R
1	1	479.17	478.93	479.54	479.12	479.190	0.61
2	2	481.13	481.75	480.97	481.21	481.265	0.78
3	3	475.71	476.24	475.95	475.48	475.845	0.76
4	4	485.37	485.17	484.70	484.89	485.033	0.67
5	5	476.80	477.12	477.07	477.10	477.023	0.32
6	1	479.01	479.23	479.12	479.10	479.115*	0.22
7	2	481.16	480.83	481.10	481.30	481.098	0.47
8	3	475.82	476.14	476.17	475.93	476.015	0.35
9	4	485.08	485.29	485.52	484.92	485.203	0.60
10	5	477.11	476.68	477.32	477.54	477.163	0.86
					Average	479.695	0.564

b. Construct an R chart from these data. (The subgroup ranges have been plotted for you.)
c. What sources of variation are captured in the R chart?
d. If appropriate, estimate the within subgroup variation?
e. Construct the X-bar chart based on the R-bar from the above R chart. Discuss the issue related to using the value of R-bar to put the limits on the X-bar chart for this subgrouping strategy.
f. Estimate the percentage of the total variation that is due to the within subgroup variation.
g. What does the above information tell you about the variability of your measurement process?

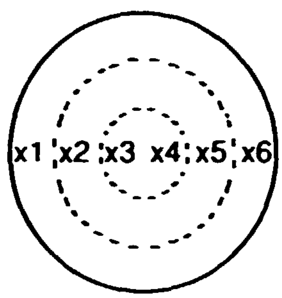

R Chart
1.25
1.00
.75
.50
.25
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
SAMPLE NUMBER


X̄ Chart
490
480
470
460
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
SAMPLE NUMBER

2. The variability of the glue delivered to paper manufacturers for use in the paper machines affects the manufacturer's ability to deliver quality paper to their customers. After meeting with representatives from various departments, you believe that a possible cause of the variation in the glue is an inconsistent glue make-up. Because of the short shelf life of glue, a new batch is not mixed until over 50% of the previous batch has been used. The time between the mixing of two batches is approximately one week. Since the ph of the glue is largely affected by the glue make-up, it is decided to measure the ph in samples taken from different batches. The specifications on the ph of the glue are 6.7 ± .5.

After the mixing of the glue is completed, a sample is taken from each of six locations across the mix tank. Thus, each subgroup consists of 6 ph measurements, taken at different locations, from one batch. Twelve different batches are sampled over a twelve week period.

Top View of Mix Tank

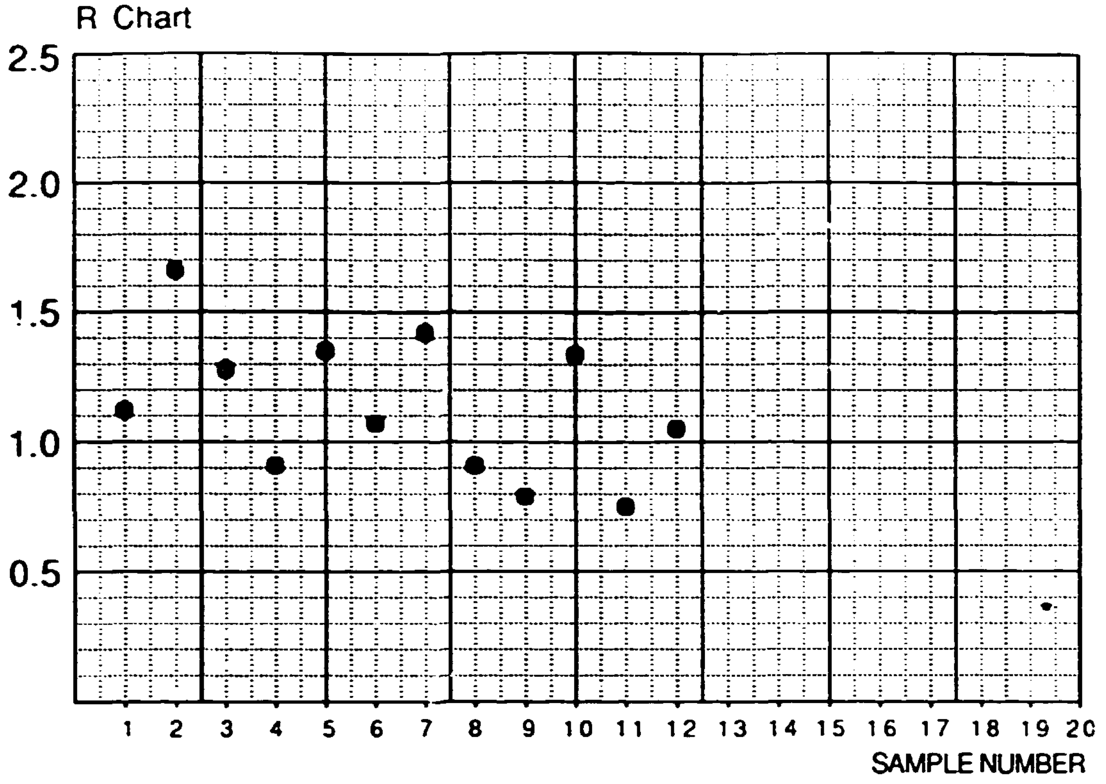


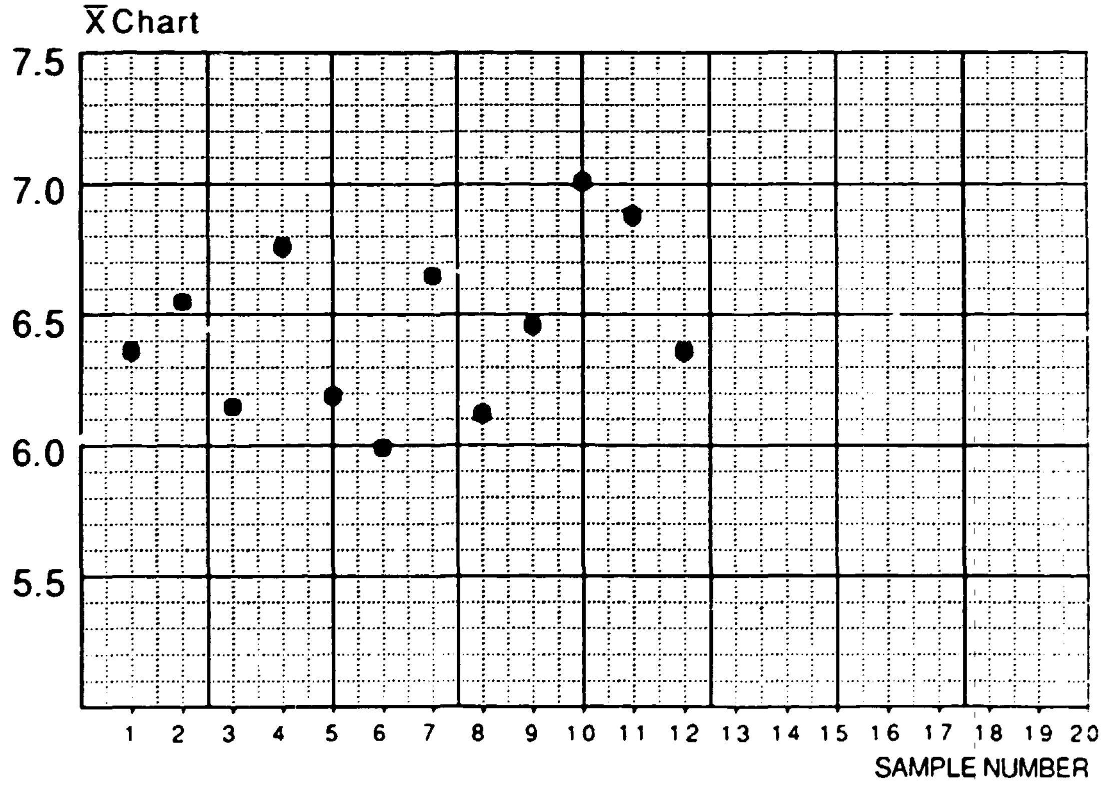
a. What are the sources of variation within each subgroup?
   What are the sources of variation between the subgroups?
b. Construct an R chart from the measurements. What can you conclude about your process from this chart?
c. What is the percent of total variation due to the within batch variation? Based on this percentage, recommend where the next steps in your improvement effort should be focused.

PH Measurements

Batch	X1	X2	X3	X4	X5	X6	Mean	Range
1	5.78	6.37	6.62	6.91	6.53	6.03	6.373	1.13
2	5.65	6.58	7.05	7.31	6.67	6.14	6.567	1.66
3	5.48	5.83	6.63	6.76	6.53	5.64	6.145	1.28
4	6.31	6.64	7.13	7.24	6.82	6.44	6.763	0.93
5	5.62	5.86	6.65	6.97	6.36	5.71	6.195	1.35
6	5.35	5.98	6.24	6.42	6.23	5.49	5.952	1.07
7	5.77	6.71	7.01	7.19	6.99	6.24	6.652	1.42
8	5.60	6.22	6.36	6.52	6.36	5.76	6.137	0.92
9	6.02	6.40	6.76	6.81	6.59	6.21	6.465	0.79
10	6.34	7.12	7.23	7.68	6.94	6.73	7.007	1.34
11	6.58	6.88	6.91	7.33	6.95	6.67	6.887	0.75
12	5.87	6.33	6.59	6.92	6.37	6.17	6.375	1.05
						Average	6.460	1.141

PH Measurements





Histogram of PH Measurements

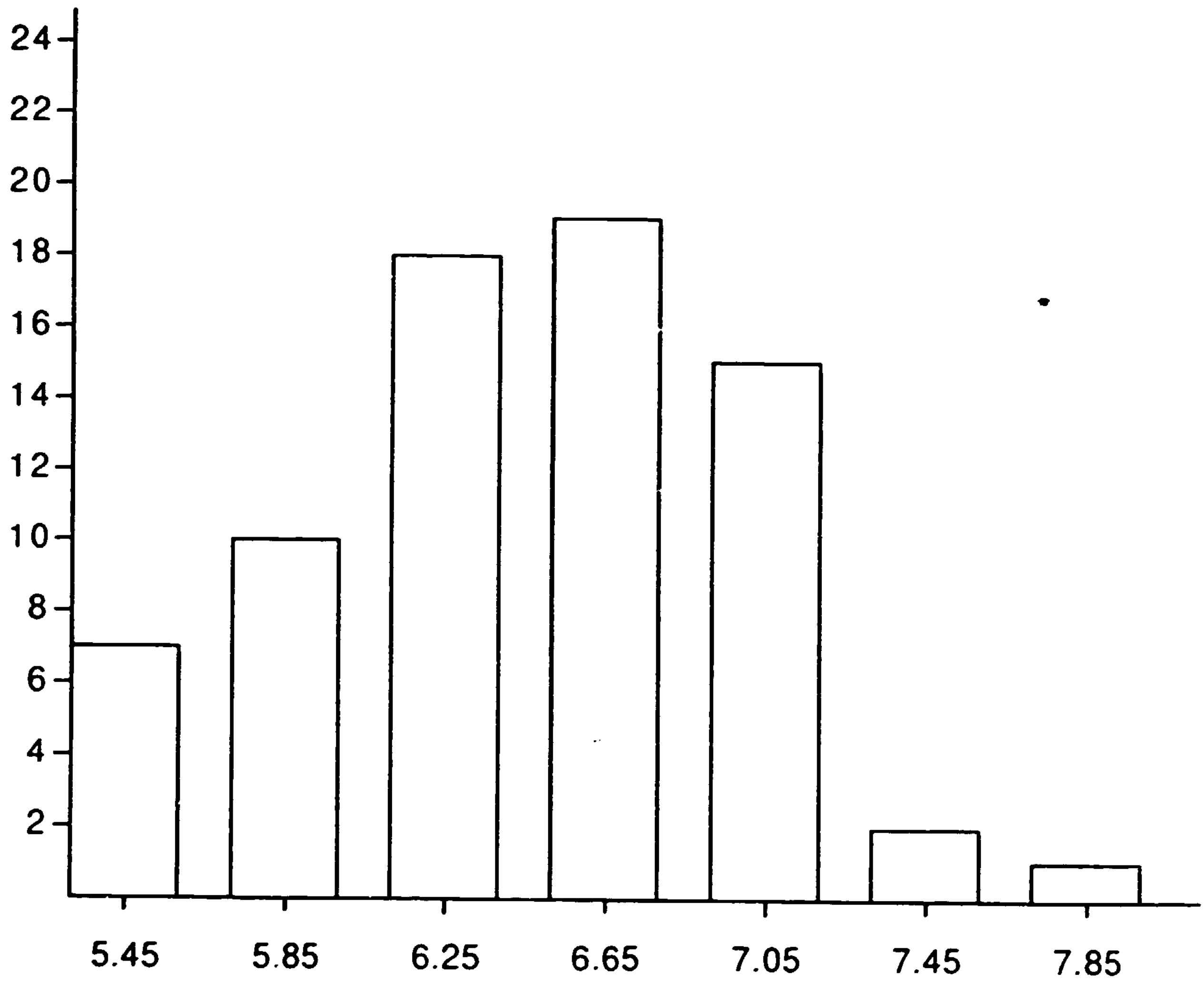


Table 7.1

Viscosity Measurements for a Batch Process

Batch	X	MR
1	84.6	-
2	84.6	0
3	84.1	0.5
4	83.7	0.4
5	84.0	0.3
6	84.1	0.1
7	84.1	0
8	83.1	1.0
9	83.0	0.1
10	84.3	1.3
11	83.9	0.4
12	83.7	0.2
13	83.9	0.2
14	84.4	0.5
15	82.6	1.8
16	84.2	1.6
17	83.7	0.5
18	84.8	1.1
19	83.8	1.0
20	84.3	0.5

Table 7.2

Averages and Ranges of 5 Viscosity Readings
Selected from Each of 20 Batches

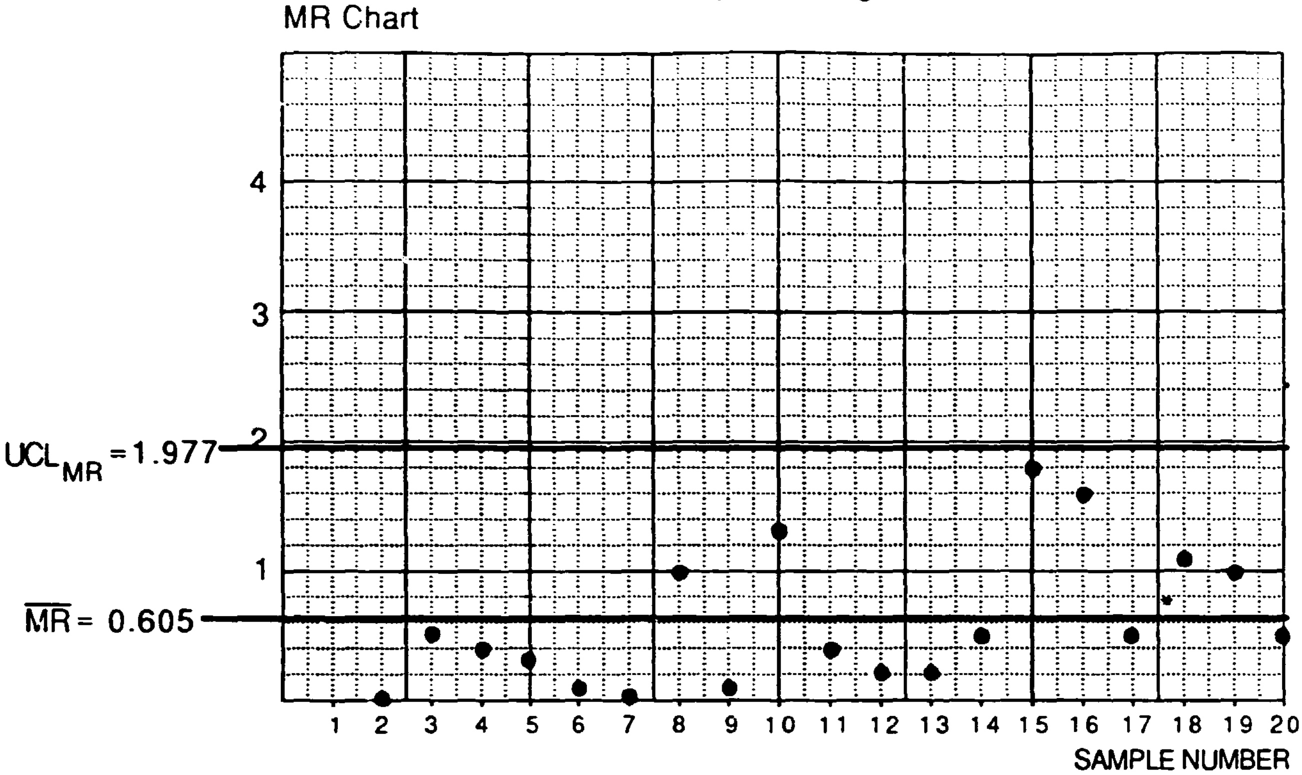
Batch	Average	Range
1	84.04	0.4
2	83.96	0.4
3	83.52	0.7
4	84.70	0.8
5	83.40	0.8
6	84.22	1.2
7	84.36	0.7
8	83.58	1.3
9	84.00	0.4
10	84.58	0.9
11	84.30	0.8
12	83.18	0.5
13	83.94	0.8
14	84.82	1.1
15	83.82	0.8
16	84.14	0.5
17	83.64	0.9
18	83.68	1 0
19	83.90	0.5
20	83.24	1.0

Table 7.3

Moving Ranges for Averages of Viscosity Readings

Batch	Average	Moving Range
1	84.04	-
2	83.96	0.08
3	83.52	0.44
4	84.70	1.18
5	83.40	1.30
6	84.22	0.82
7	84.36	0.14
8	83.58	0.78
9	84.00	0.42
10	84.58	0.58
11	84.30	0.28
12	83.18	1.12
13	83.94	0.76
14	84.82	0.88
15	83.82	1.00
16	84.14	0.32
17	83.64	0.50
18	83.68	0.04
19	83.90	0.22
20	83.24	0.66

Figure 7.1
Viscosity Readings



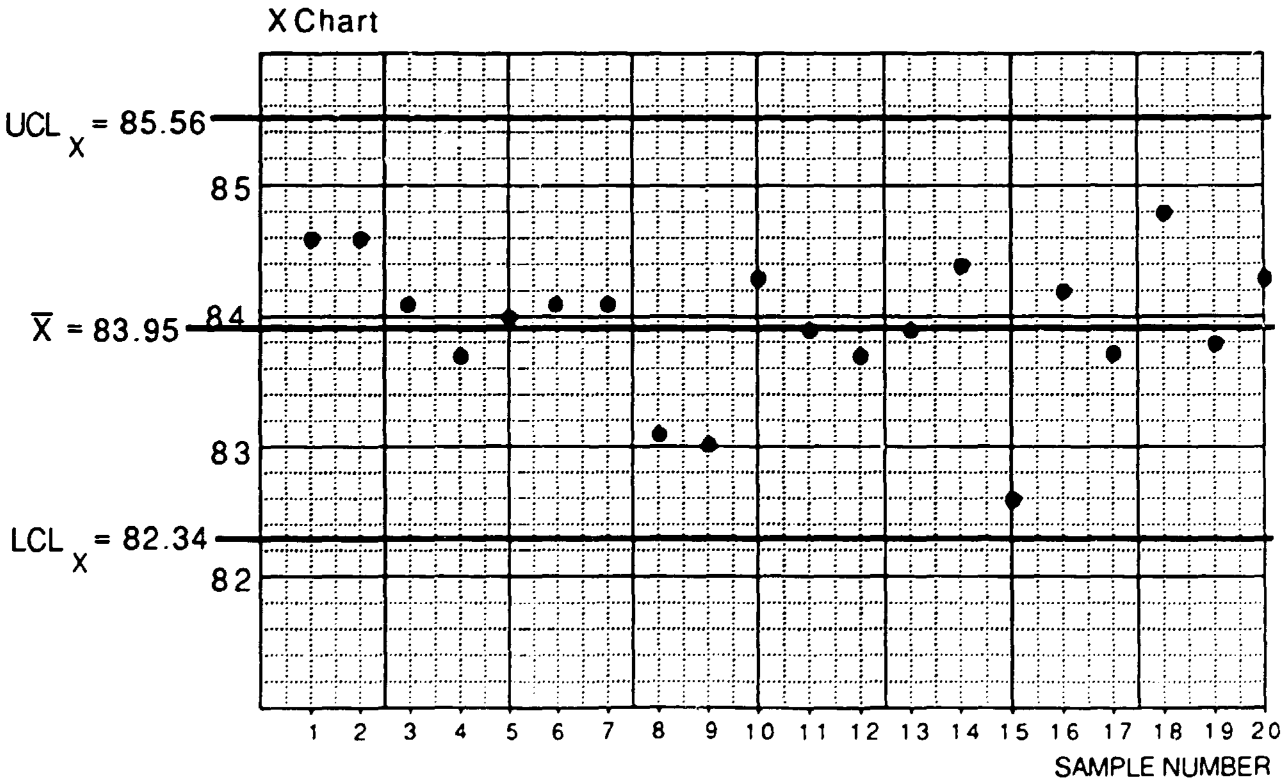


Figure 7.2
Two Explanations for the Creation of Total Variation in Viscosity Measurements

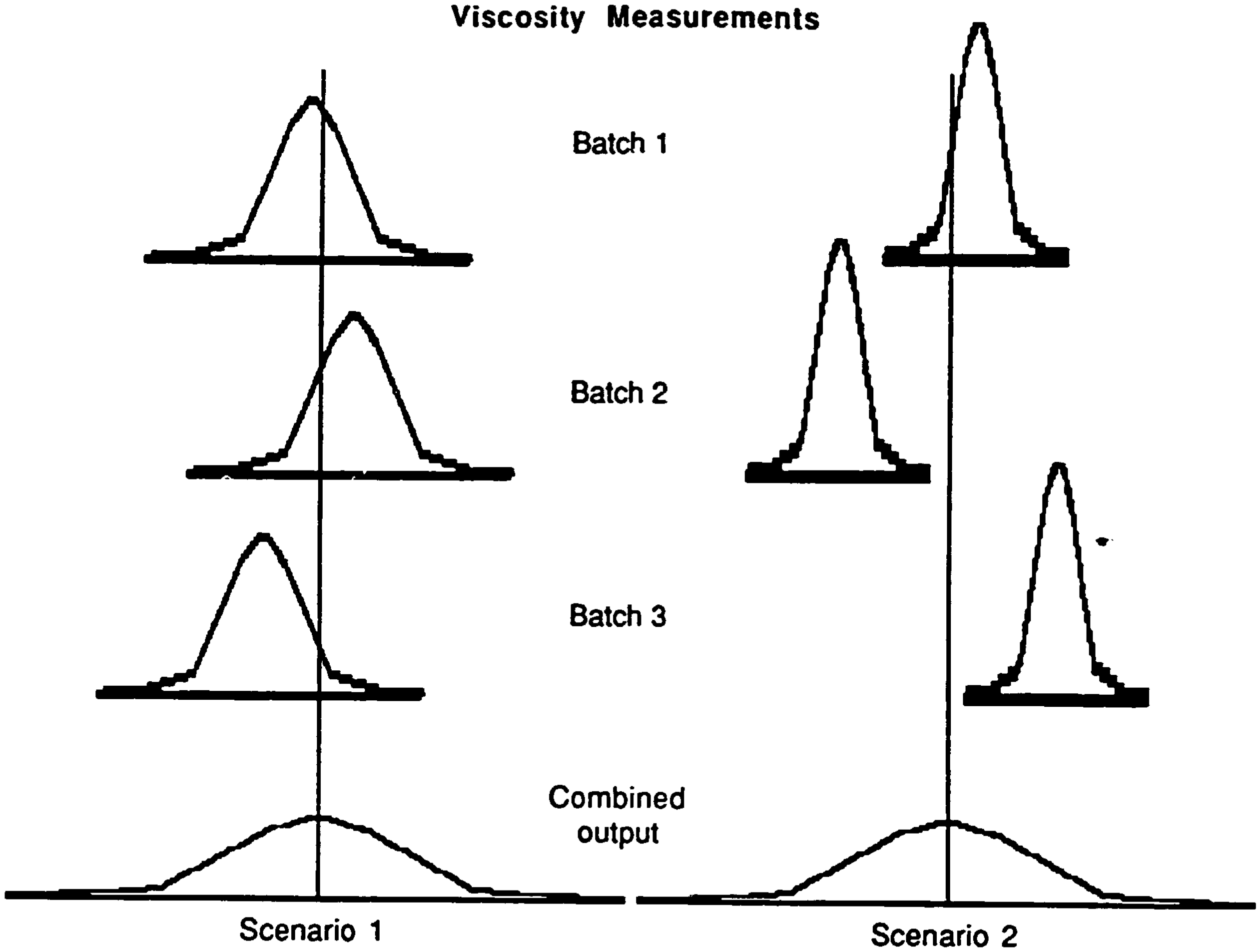


Figure 7.3
Viscosity

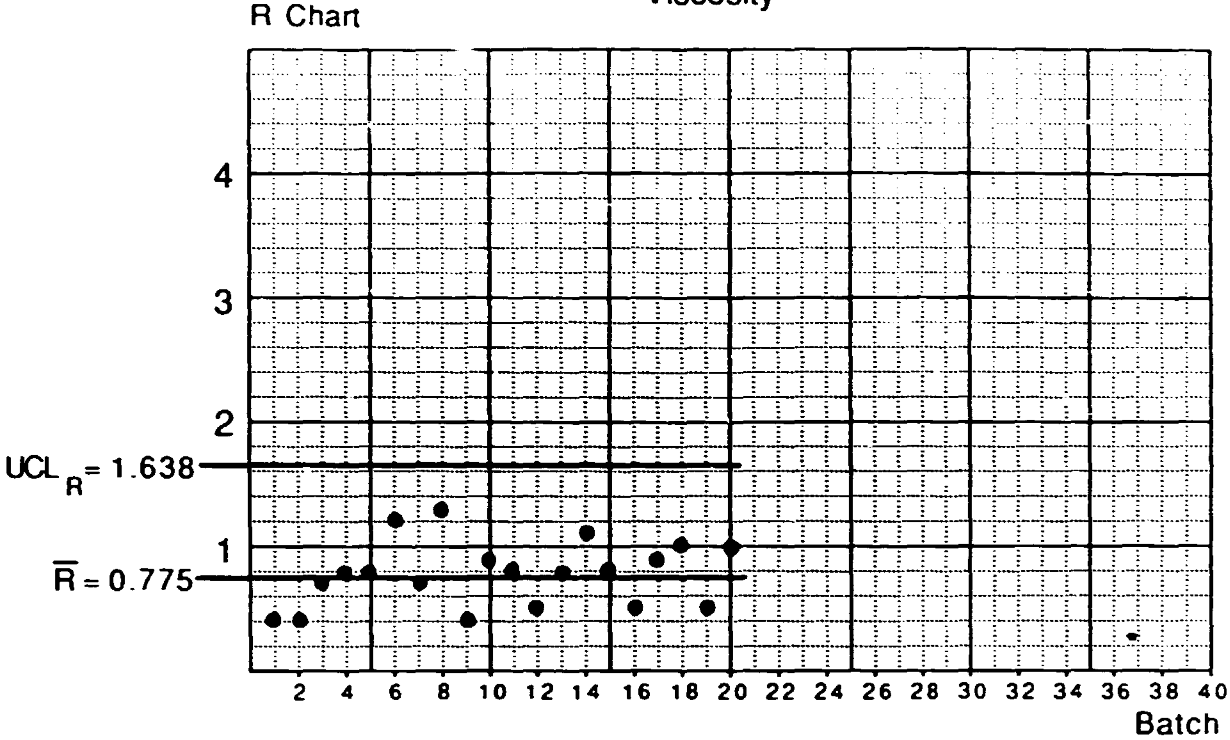


Figure 7.4
Moving Range Chart
(range based on batch averages)

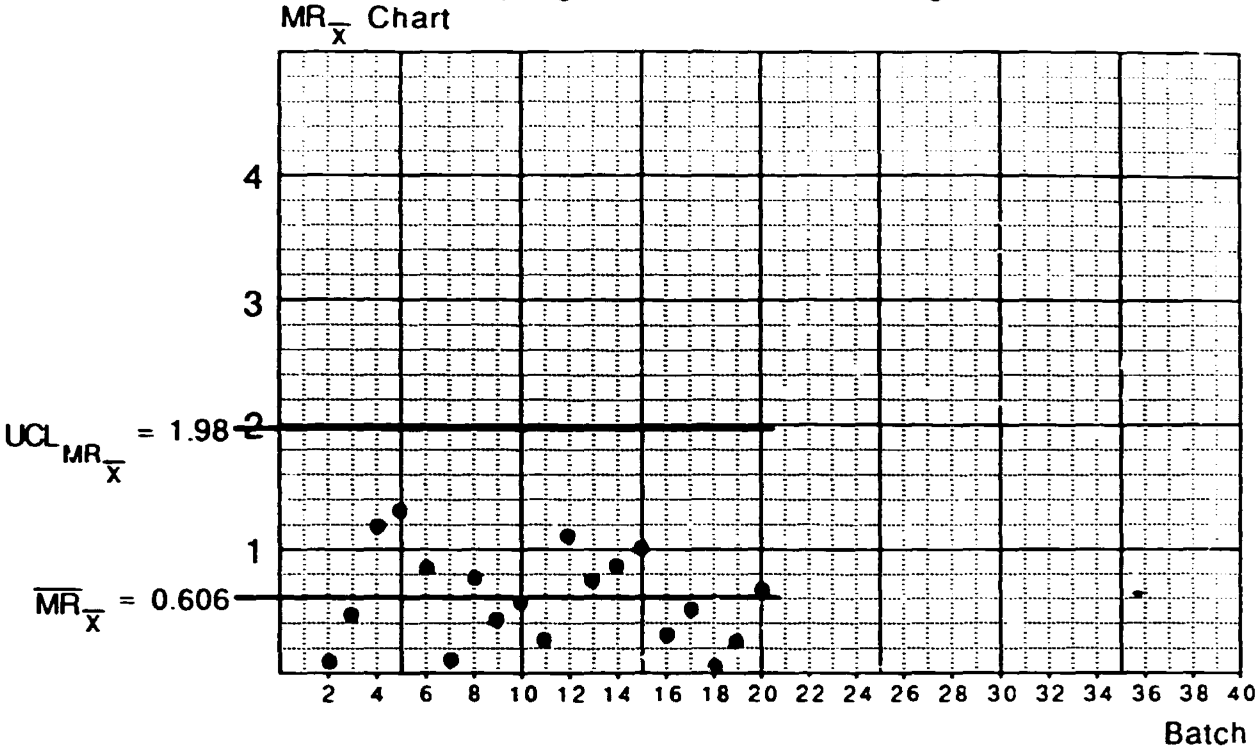


Chart for Batch Averages
(limits based on average range between batches)

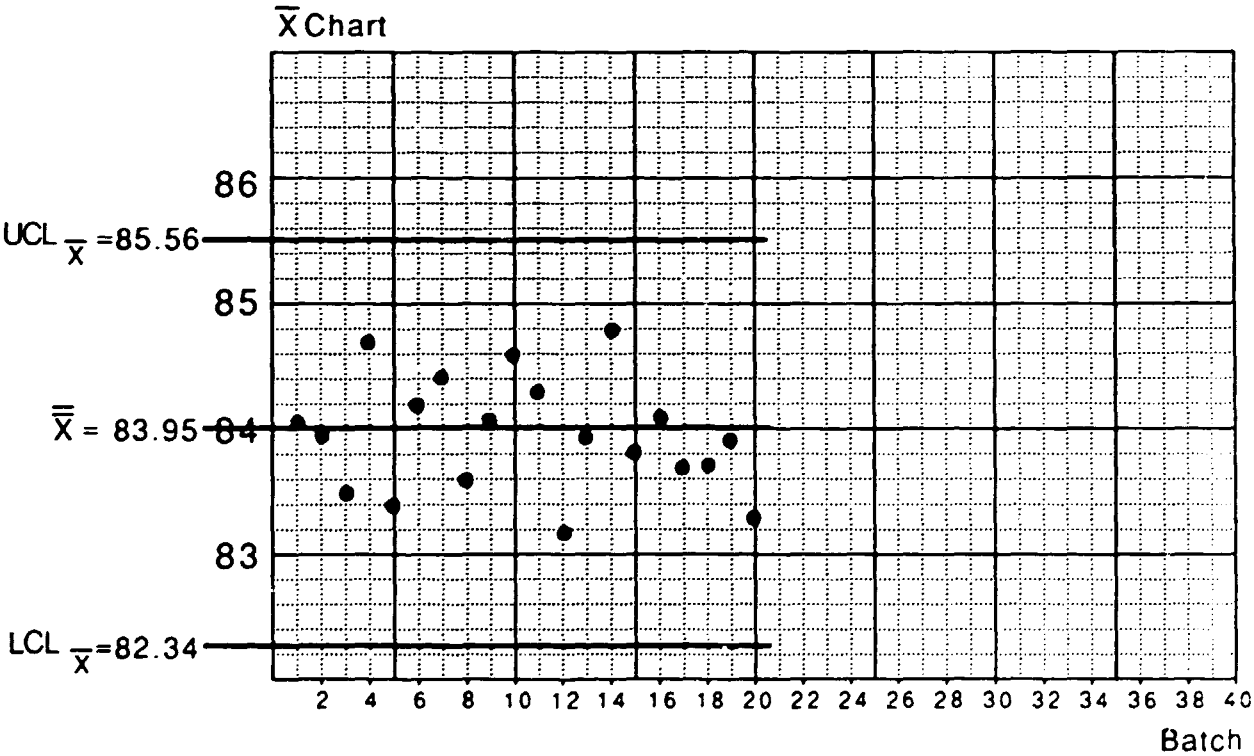
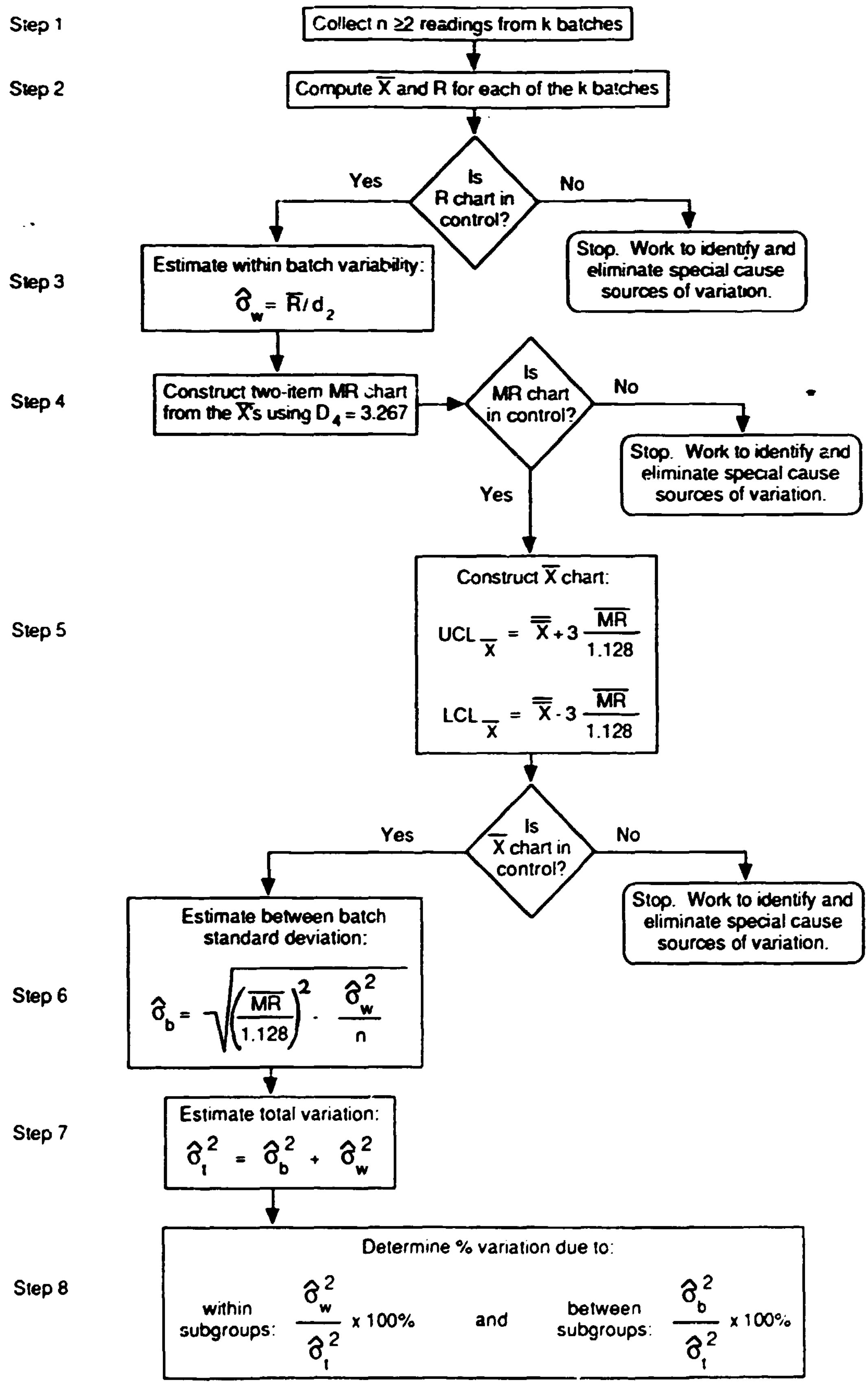


Figure 7.5
Flow Chart for
Analyzing Components of Variance
in a Batch Process



Chapter 8

Measurement Processes

Any time measurements are made on a production process, there is a measurement process being relied on to deliver acceptable quality measurements. Like a production process, the act of making measurements should also be considered as a process and examined over time and conditions to evaluate how well it delivers its intended output. In the case of a measurement process, the intended output is, of course, a measured value. The output of a measurement process will undoubtedly exhibit variation. As with a production process, it is critical to understand the nature of this variation. It will be necessary to understand the magnitude and stability over time and conditions of the measurement variation. It will be necessary to know whether the measurement process consistently measures at the same average value and to provide a judgment about the adequacy of the average value delivered by a measurement process.

The variation and average delivered by a measurement process have profound effects on the information that measured values from a production process will provide about production. If a measurement process exhibits unstable variation, then the interpretation of measurements from a production process will be hampered. If the instability in the variation of the measurement process goes undetected, the unstable variation may be attributed to an instability in the production process being measured. If a measurement process has stable, yet large variation, then the ability to understand production process variation will be reduced. And if this large measurement variation is not known, the large variation may be attributed to the production process. If a measurement process does not deliver measurements at a consistent level, then there will not exist the ability to evaluate the average of a production process. For these reasons, knowing the characteristics of the process used to generate measurements is an unavoidable first step in collecting measurements to describe a production process. The example provided in the next section illustrates the use of previously developed techniques to understand and describe a measurement process.

8.1. Evaluating a Measurement Process

A chemical manufacturing firm relies on measurements from their analytical laboratory as a check on the purity levels of a herbicide, called here product M. As part of ongoing practices to insure the quality of the measurement process, the laboratory routinely calculates the purity level of a sample (called a standard) which contains a known amount (96 units) of product M. Each time the laboratory runs an assay to determine the amount of M in one or more production samples, a determination of the amount of M in the standard is also made. Since product M is a stable compound, the actual amount of M measured in the standard sample should not change. Table 8.1 contains the measured amount of M in the standard for the 30 most recent assays. The variation in these numbers is not, of course, due to changes in the purity of material. Instead, the observed variation reports on the behavior of the measurement process.

Before examining control charts of the data in Table 8.1, it will be useful to consider how sources of variation affecting the measurement process will be captured by the above data. Some of the sources of variation affecting the measurement process might be:

The analytical laboratory contains four different chromatographs, any one of which might be used in the assay for product M.

Even were the same device used for each assay, the device might read differently from one assay to the next.

Any one of 8 different laboratory technicians may perform the assay.

The assay requires a fairly involved sample preparation. Even if only one technician performed the assay, there would be at least slight differences in the sample preparation which would produce variation in results.

The reagents used to do the assay may change over time.

At infrequent intervals the equipment itself changes, since the chromatographs have columns which need to be repacked.

Table 8.1
Amount of "Product M" in Standard Sample

Date	Assay value	Moving Range
07/26	95.9	----
07/27	95.7	0.2
07/29	96.7	1.0
08/04	95.8	0.9
08/05	96.9	1.1
08/07	95.5	1.4
08/10	96.8	1.3
08/10	96.0	0.8
08/11	95.6	0.4
08/12	96.4	0.8
08/14	96.0	0.4
08/14	95.2	0.8
08/16	96.2	1.0
08/16	96.0	0.2
08/16	95.2	0.8
08/18	95.5	0.3
08/19	96.2	0.7
08/20	96.1	0.1
08/20	96.7	0.6
08/21	96.2	0.5
08/22	96.2	0.0
08/22	96.5	0.3
08/24	95.3	1.2
08/26	96.0	0.7
08/27	95.5	0.5
08/28	95.5	0.0
08/29	96.0	0.5
09/02	97.2	1.2
09/04	95.3	1.9
09/05	96.4	1.1

Moving ranges for the measured values for the standard sample have also been recorded in Table 8.1. Since an assay may have been performed by any of several technicians on any of several devices, differences in devices or technicians would contribute to the magnitudes of the ranges. Furthermore, changes due to different sample preparation, or set-ups, would also contribute to the magnitude of the moving ranges.

The moving ranges have been plotted on a control chart in Figure 8.1. (It is left to the reader to verify that the center line and upper control limit have been correctly determined.) Since the moving range chart is in control, there is no evidence that assay-to-assay variation is inconsistent across the range of the data examined. In view of the identified sources of variation contributing to the ranges, it could be concluded that set-up variations, operator variations and device-to-device variations are fairly consistent over the time which the data was collected.

Since the moving range chart was in control, limits can be constructed for the individuals or X chart. However, before examining the completed X chart, it will again be useful to consider what sources of variation would drive the X values. If the reagents used to conduct the assay deteriorated over time, this might result in a trend in the values for the standard. Or, if the column were repacked during the course of the study, it might be that the assay read the standard to a different average value. The completed X chart in Figure 8.1 does not indicate any such trends or shifts in the average reading over time. The measurement process appears to be reading to a stable average of 95.8, a number very close to 96, the actual amount contained in the standard.

Since both the moving range chart and the X chart are in control, it is possible to estimate a standard deviation from the average of the moving ranges. This estimate would be the value of $\overline{MR} / d_2 = .62$ used to calculate the control limits for the X chart. This standard deviation describes something about the variation in the measurement process; it is instructive to consider the sources of variation in this measurement process captured by this value. Since the samples are run by an analyst on any machine and the values are separated in time by at least a day, the standard deviation could be thought of as describing the measurement variation of the laboratory. It would reflect the amount of variation contributed to measured values on product that is due to the measurement variation of the laboratory. Clearly, a choice was made by the manager of the analytical laboratory to summarize measurement variation in this fashion. An alternative procedure would have been to have the standard sample always measured on the same device. If this had been the case, then the magnitude of the standard deviation estimated might reasonably be expected to be smaller. But if product is routinely measured on any device, such an estimate would not accurately reflect the amount of variation contributed by laboratory measurements.

Another practice in use for estimating measurement variation is to run, say, 30 or more standard samples during the same assay. In other words, the sample preparation for the thirty samples would be done at the same time and determinations of the amount of product M in the samples would be reported. There would be little value in placing these numbers on control charts, as they are not measured over time; however, s, the sample standard deviation could be calculated from these numbers. This standard deviation would provide an estimate of within assay variation, but only, of course, for the one assay involved. Nothing would be known about the stability of the within assay variation over time, nor would the effect of different operators or devices be captured by the sample standard deviation. This practice of estimating within-assay variation might be useful for understanding sources of variation in the measurement process, but it clearly can not be used as a description of measurement variation.

The thirty measurements in Table 8.1 provide an understanding of the behavior of the measurement process over a little less than two months time. Good laboratory practice would consist of continuing to analyze the standard sample frequently, if not with every assay. The variation in the measurement process may not continue to stay stable. New operators or changes in equipment, reagents or procedures could cause changes in this variation. The practice of continuing to monitor the measurement variation is necessary if the measurements made on production material are to be relied on.

The above analysis has only described the current operation of the measurement process. Whether the variation in this process is small enough, will depend on what variation is required both for process improvement work as well as for judging the fitness of production material. The next section discusses methods for characterizing the current operation of the process. The following section provides techniques for deciding on the adequacy of a measurement process.

8.2 Characterizing a measurement process

In beginning to characterize a measurement process, a good place to start, as with any process, is a consideration of the sources of variation contributing to measurement variation. In arriving at a list of possible sources of variation, it is necessary to consider the conditions under which measurements will actually be made. In the example in the previous section, measurements in the laboratory were made by several operators, using several devices. Reagents which were used in the assay would change over time; in addition, the device itself was subject to changes over time. A characterization of that measurement process then requires that these sources be accounted for. In particular, summaries of the behavior of the measurement process must first address the stability of these sources of variation.

As another example, consider the measurement of the weight of a bar of hand soap. As part of the ongoing check of the scale used to perform this measurement, a standard weight known to weigh 5 ounces is purchased. Twice each shift, the standard is weighed on the scale used for weighing the bars of soap. The recorded measurements from this study for the past 40 shifts have been captured in Table 8.2. These data have been used to construct the X-bar and R charts in Figure 8.2. The fact that the R chart is in control, indicates that the variation in the measured weights within a shift remains consistent across the duration of the study. Since the X-bar chart is also in control, the average weight read by the scale appears to be consistent across the time of the study. The changes in operators, environment, or methods across time does not appear to result in an unstable measurement process.

Table 8.2
Measured weights of 5 ounce standard

Subgroup	X1	X2	R	$\bar{X}$	Subgroup	X1	X2	R	$\bar{X}$
1	4.948	4.943	.005	4.9455	21	4.985	5.039	.054	5.0120
2	4.971	5.046	.075	5.0085	22	5.029	5.050	.021	5.0395
3	5.019	5.001	.018	5.0100	23	5.017	5.012	.005	5.0145
4	5.018	5.018	0	5.0180	24	5.044	5.020	.024	5.0320
5	4.942	5.016	.074	4.9790	25	5.059	4.975	.084	5.0170
6	5.029	5.025	.004	5.0270	26	5.014	5.024	.010	5.0190
7	5.025	5.002	.023	5.0135	27	5.028	5.000	.028	5.0140
8	4.995	4.942	.053	4.9685	28	5.028	4.990	.038	5.0090
9	5.041	4.985	.056	5.0130	29	5.026	4.967	.059	4.9965
10	5.023	5.026	.003	5.0245	30	5.021	5.001	.020	5.0110
11	5.016	4.970	.046	4.9930	31	5.066	4.987	.079	5.0265
12	4.999	5.024	.025	5.0115	32	5.009	4.927	.082	4.9680
13	4.994	5.055	.061	5.0245	33	5.009	5.001	.008	5.0050
14	4.974	4.937	.037	4.9555	34	5.028	4.966	.062	4.9970
15	5.022	5.031	.009	5.0265	35	4.959	4.990	.031	4.9745
16	5.051	4.959	.092	5.0050	36	5.019	5.013	.006	5.0160
17	5.014	4.946	.068	4.9800	37	5.048	4.959	.089	5.0035
18	5.019	4.987	.032	5.0030	38	5.061	4.951	.110	5.0060
19	4.975	5.022	.047	4.9985	39	5.021	4.982	.039	5.0015
20	5.021	4.993	.028	5.0070	40	4.951	4.984	.033	4.9675

Since the use of the scale for weighing the standard weight appears to result in a stable measurement process, a histogram constructed from the 80 measurements of Table 8.2 will be helpful in understanding descriptions of a measurement process. This histogram appears in Figure 8.3. The histogram provides a visual representation of the variation that would result when the same piece of material (in this case a 5 ounce standard) is weighed repeatedly. Of particular interest is a characterization of the center or average of this distribution and the spread or range of the distribution. In characterizing measurement processes, the concept of accuracy refers to how close on average the measurement process delivers the value of the known standard. The concept of precision captures the amount of variation delivered by a measurement process.

8.2.1 Precision of a measurement process

Figure 8.4 contains histograms from four different measurement processes. For each of these processes, a five ounce standard was weighed repeatedly over a variety of time and conditions and the measurement process was found to deliver stable levels of variation and average. Histograms A and B show that the measurement processes which generated these measurements have less variability than those which generated C and D. Thus, measurement processes A and B are said to be more <u>precise</u> than processes C and D. The precision of a measurement process refers to the level of variation which would be delivered by repeated measurements of the same unit of product or material.

In constructing the histograms of Figure 8.4, only a single standard weighing five ounces was used to characterize the measurement process. Thus, the statements about the precision of the measurement processes should be qualified. The precision of the measurement process has only been determined when the weight of the object measured is five ounces. It is entirely possible that, say, measurement process A results in significantly larger variation when the object measured weighs seven ounces as opposed to five ounces. The information about the precision of the measurement process has only been determined for weights of five ounces. Thus, if the weights of soap from production varies over a wide range, it should be understood that no knowledge has been provided about the precision of the measurement process over this range. And this knowledge can only, in the final analysis, be had by actually evaluating the precision at various levels throughout the range of weights delivered by the soap making process. In addition, because the measurement precision may be change with changing level of product values, the choice of the weight(s) used for a standard(s) should be selected with care. A reasonable choice would be to use a standard which measures close to the target value of the production process.

The data of Table 9.2 resulted in an in control range chart with a value of $\overline{R}$ equal to .04095. Thus, the precision of the process can be made by estimated from the estimated measurement process standard deviation. For this example, the precision would be estimated by:

$$\hat{\sigma}_{\epsilon} = \frac{\overline{R}}{d_2} = \frac{.04095}{1.128} = .03630$$

8.2.2. Accuracy of a measurement process

The four histograms of Figure 8.4 also illustrate the concept of accuracy. Histograms A and C both are centered over 5 ounces, the actual weight of the standard measured. Since the average delivered by these two measurement processes is almost the same as the quantity measured, measurement processes A and C are said to be accurate. The accuracy of a measurement process refers to the ability of the process to deliver, on average, the recognized value of a standard. The measurement processes which generated histograms B and D are not accurate, and, in fact, are said to be biased. The fact that B and D are biased, however, does not indicate that these measurement processes can not be used for evaluating product characteristics. If the level of bias is understood, the measured values can be adjusted by the amount of the bias.

Just as in the case of measurement process precision, care should be exercised in extrapolating the accuracy of a measurement process when measuring one value (here that value is 5 ounces) to the accuracy which may or may not be observed when other weights are measured. Again, unfortunately, the accuracy of a measurement process over the range of possible production values can only be determined by actually investigating the measurement process over the indicated range of values.

The centerline from the X-bar chart for the data of Table 9.2 provides an estimate of the average value that the measurement process delivers when measuring a 5 ounce standard. Since this value is 5.00355, the measurement process is seen to be accurate.

8.3 The effect of measurement variation on process study

If the amount of measurement variation is large, then understanding, and subsequently improving, process behavior is difficult. An examination of the formula which explains the relationship between measurement variation and the variation measured on a process illustrates the difficulty encountered. This formula is given by:

$$\sigma_m^2 = \sigma_p^2 + \sigma_e^2$$

where

σ_m^2 is the variance of values measured on a production process
σ_p^2 is the variance of the output of the production process

and

σ_e^2 is the variance of the measurement process.

In words, the above equation states that the variance of values measured on output from a production process is not just equal to the actual variance of the output, but is also increased by the variance of the measurement process.

Figure 8.5 provides a graphical illustration of how large measurement variation can hinder process study. Plot I of Figure 8.5 is a time plot of the density of a plastic part produced at a plant. In this first plot, the measurement variation, σ_e^2, is known from measurement studies to be small. Two sources of raw material are used for this plastic part. The letter A is used to plot the density of a part produced from raw material A and the letter B is used for a part from raw material B. The different behavior of the raw materials is readily apparent from plot I. Although both processes, that resulting from material A and that from B appear to be stable, the average densities from the two materials are clearly different. Stated another way, the differences in the two raw materials are a source of variation contributing to the variability in density of parts produced at the plant. By identifying the differences in raw material as an important source of variation, action is then possible for reducing the variation in the density of the plastic part. This action might be to use only one of the two types of raw material in the production of this part.

Plot II of Figure 8.5 is a similar plot of the measured density of plastic parts, but the measurement process used to measure the densities was known to have large variation. From plot II, it can be seen that the differences in the average densities from parts produced from the two different raw materials is masked by the measurement variation; the difference in average density that was apparent in plot I, is not as readily detected in plot II. The ability to understand the sources of variation affecting product output is hindered by large measurement variation.

Since the magnitude of measurement variation impacts the ability to obtain useful information about a process, it is necessary to have some criteria for judging the adequacy of a measurement process relative to its variation. Two methods are suggested in this manual. The first is to calculate the percent of variation in measured product output which is due to measurement variation. The second involves the use of control charts to describe the ability of a measurement process to discriminate between different product output values.

8.3.1 Measurement variation as a percent of measured output variation

In section 8.1, an ongoing system for studying the measurement process for the determination of the purity of product M was described. An examination of part of that data led to the conclusion that the measurement process was stable over the time; the measurement standard deviation was estimated to be 0.62. The same measurement process, of course, is supplying data to production on the purity levels of every batch produced. These data are routinely plotted and an analysis of the moving range and X-bar charts constructed from this data shows that the process is in statistical control with an estimated process average of 94.4 and as estimated process standard deviation of 1.6. This standard deviation of 1.6 captures both the variation in purity levels as well as measurement variation. Using the notation developed earlier, the above information on process and measurement variation could be summarized by:

$$\hat{\sigma}_\epsilon = 0.62 \qquad \hat{\sigma}_\epsilon^2 = 0.3844$$

$$\hat{\sigma}_m = 1.60 \qquad \hat{\sigma}_m^2 = 2.56$$

The variance of the measured purity, $\hat{\sigma}^2_m$, is the sum of the variance of the actual product plus the measurement variance, $\hat{\sigma}^2_\epsilon$. Or, another way to think of this relationship, is that

$$\frac{\hat{\sigma}^2_\epsilon}{\hat{\sigma}^2_m} \text{x}100\% = \frac{.3833}{2.56} \text{x}100\% = 15.0\%$$

of the variation observed in measured purity levels can be attributed to measurement variation. Is 15% too large a percentage? Although there is no one, correct answer to this question, a value of 10% or smaller for the percentage of variation due to measurement is generally accepted as indicating that the measurement process will be adequate for studying process variation. Since for the process under study, measurement variation accounts for 15% of the total variation observed, the adequacy of the measurement process should be questioned. Further study of the sources of variation which affect the measurement process could be undertaken. Since the measurement variation includes instrument to instrument variation, it might be useful to set aside one instrument for always performing this assay and work to further reduce the variation contributed by that instrument. However, it might be that work to reduce measurement variation is unsuccessful. In this case, the laboratory personnel could reconfigure the process for making measurements by always performing multiple (2 or more) assays on both the standards used to evaluate measurement variation as well as on every product sample. The reported levels of purity would then be the average of the multiple readings. If n multiple readings are obtained for each sample, then the measurement variance should be reduced by a factor of $\sqrt{n}$. It will not, of course, be sufficient to assume that the measurement variance is decreased by this amount. A re-evaluation of the measurement process which includes collecting multiple readings on every specimen of material needs to be performed.

8.3.2 Using control charts to evaluate the discrimination of measurement processes

Large measurement variation reduces the ability to evaluate the characteristics of a product and, thus, the process producing the product. One method for evaluating the effects of measurement variation on measured product variation is to evaluate the percentage of observed product variation due to measurement variation. Another method is to use control charts to capture the ability of the measurement process to discriminate between product outcomes. This method was used by a customer and vendor who were disagreeing on the measurement of an important quality characteristic. The vendor always reported an average and standard deviation of an outgoing lot of material. When the lot arrived at the customer's plant site, some parts were selected from the incoming material and an evaluation of the average and standard deviation of the lot was performed. Part of the reasons for the disagreement was that the difference in the results reported by the vendor and those obtained by the customer were at times dramatic. Although the source of the reasons for the disagreement were not clear, the possibility that the source might in part be due to sampling differences or measurement methods was considered.

As a way to begin to narrow down the reasons for the differences in the characteristics reported on the lots, managers from the two plant sites agreed to conduct a measurement study of the characteristic in question. The study was performed on twenty parts selected at random from a manufactured lot. The twenty parts were measured at one time at the vendors facility. They were then measured a second time on the following day. After the two measurements were made on the twenty parts, the parts were then transported to the customer's facility and passed through the inspection procedure on each of two successive days. (The characteristic of interest is not changed by test, transport, or time.) The results of this measurement study are reported in Table 8.3. This table also summarizes the range of the measurements of the same part for both the vendor's and customer's measurements as well as the average measurement obtained by both the vendor and customer. Before looking at a statistical analysis of the data, it will be instructive to consider the sources of variation captured by the ranges and averages.

Table 8.3

Comparison of Two Measurement Processes

	Vendor's process				Customer's process			
Part	Measurements		Range	Average	Measurements		Range	Average
1	28.5	32.9	4.4	30.70	34.7	36.1	1.4	35.40
2	32.5	28.1	4.4	30.30	32.3	30.7	1.6	31.50
3	28.0	22.8	5.2	25.40	29.9	30.8	0.9	30.35
4	41.4	50.5	9.1	45.95	43.9	42.7	1.2	43.30
5	54.6	54.7	0.1	54.65	47.5	50.5	3.0	49.00
6	37.5	46.8	9.3	42.15	45.7	43.5	2.2	44.60
7	36.8	38.3	1.5	37.55	44.8	44.1	0.7	44.45
8	32.4	40.3	7.9	36.35	36.7	36.5	0.2	36.60
9	41.0	40.4	0.6	40.70	41.1	41.3	0.2	41.20
10	37.0	40.4	3.4	38.70	33.9	36.3	2.4	35.10
11	53.5	45.4	8.1	49.45	40.7	42.0	1.3	41.35
12	35.3	34.5	0.8	34.90	36.3	36.3	0.0	36.30
13	34.2	38.7	4.5	36.45	39.7	38.7	1.0	39.20
14	49.9	46.2	3.7	48.05	45.0	46.5	1.5	45.75
15	46.4	39.7	6.7	43.05	48.9	45.9	3.0	47.40
16	37.0	37.3	0.3	37.15	36.3	37.1	0.8	36.70
17	46.9	45.9	1.0	46.40	47.0	46.5	0.5	46.75
18	39.8	25.9	13.9	32.85	31.3	30.0	1.3	30.65
19	46.3	42.3	4.0	44.30	46.4	43.4	3.0	44.90
20	35.5	41.3	5.8	38.40	41.2	40.0	1.2	40.60

The ranges reported on the measurements obtained by the vendor capture information about the precision of the vendor's measurement process. Each of these ranges describes the difference observed from one day to the next when the same part is measured. Thus, by constructing a range chart from these ranges, the stability of measurement variation across parts can be evaluated. If different size parts resulted in different measurement variation, then this should be picked up as a special cause on the range chart. In other words, the range chart allows for a check on the stability of the measurement process across parts, but not across time. Care will need to be exercised in applying these results to the vendor's measurement process, since there is no basis on which to say that the measurement variation observed is stable across time and conditions. The recommendation would be that the vendor, if he is not already doing so, should be monitoring his measurement process across time and conditions. Of course, the same information on the customer's measurement variation is supplied in Table 8.3. Figure 8.6 contains the two range charts constructed from these data.

Both of the range charts in Figure 8.6 are in control. In other words, both the vendor's and customer's measurement process appear to deliver consistent measurement variation across the twenty parts used in the study. However, even though both processes are consistent, differences in the measurement processes are immediately apparent by examining the two charts. The average range for the vendor's measurement process is considerably larger than that of the customer. An examination of the respective average charts for the vendor and customer provide a way of evaluating the effect which the additional measurement variation from the vendor's measurement process has on the ability to evaluate product characteristics.

Before looking at the two X-bar charts, it is again instructive to consider the sources of variation affecting the X-bars. Consider the averages calculated by the vendor. Each of the averages results from two measurements made on one of the twenty parts. Although some of the differences in these twenty averages would be due to the variation from the measurement process, it would be expected that the averages would exhibit variation because of the variation in the twenty parts. In fact, if the measurement process used to measure the parts had negligible variation, then all the variability in the averages would be due to part-to-part differences.

In constructing the X-bar charts of Figure 8.7, the average range from the respective range charts has been used to calculate the upper and lower control limits. In other words, the only source of variation included in the determination of the control limits for the X-bar charts is measurement variation. This would imply that any additional variation beyond measurement variation affecting by the X-bars should result in an out-of-control signal on the X-bar chart. Since the X-bars are subject to part-to-part variation, as well as measurement variation, it would be expected that the X-bar charts would be out-of control.

An examination of the two X-bar charts shows that both of the charts are out-of-control. The conclusion to be reached from these charts is that both the vendor's and customer's measurement processes are capable of discriminating between parts. Since only measurement variation was used to construct the control limits on the X-bar charts, the differences between parts was observed on the X-bar chart; in fact the part-to-part differences were picked up as a special cause on the X-bar charts. Again, it is useful to note the differences seen in the two X-bar charts. The X-bar chart from the vendor's study does not have as many points outside the control limits as that of the customer; nor are the points as far outside the control limits on the vendor's chart. This qualitative difference can be summarized by concluding that, because of the smaller variation experienced by the customer's measurement process, this process has a greater ability to discriminate among parts than the vendor's measurement process.

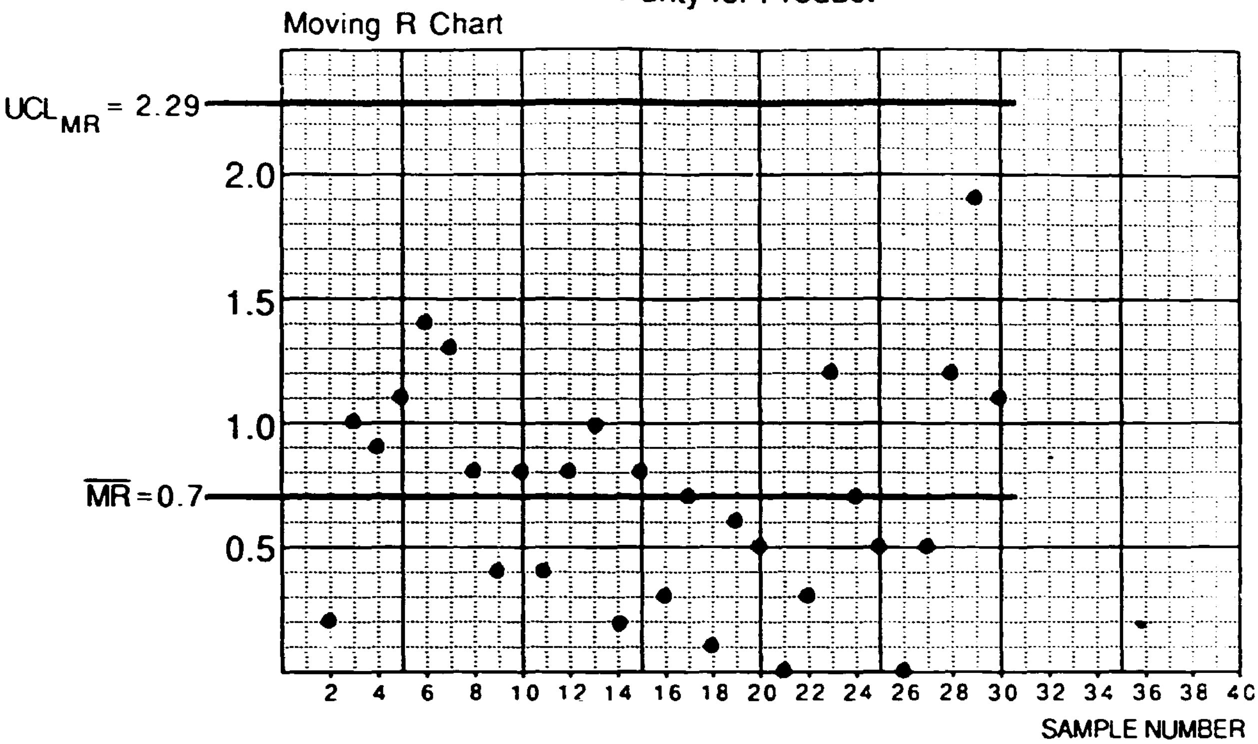
8.4 Issues in the study of measurement processes

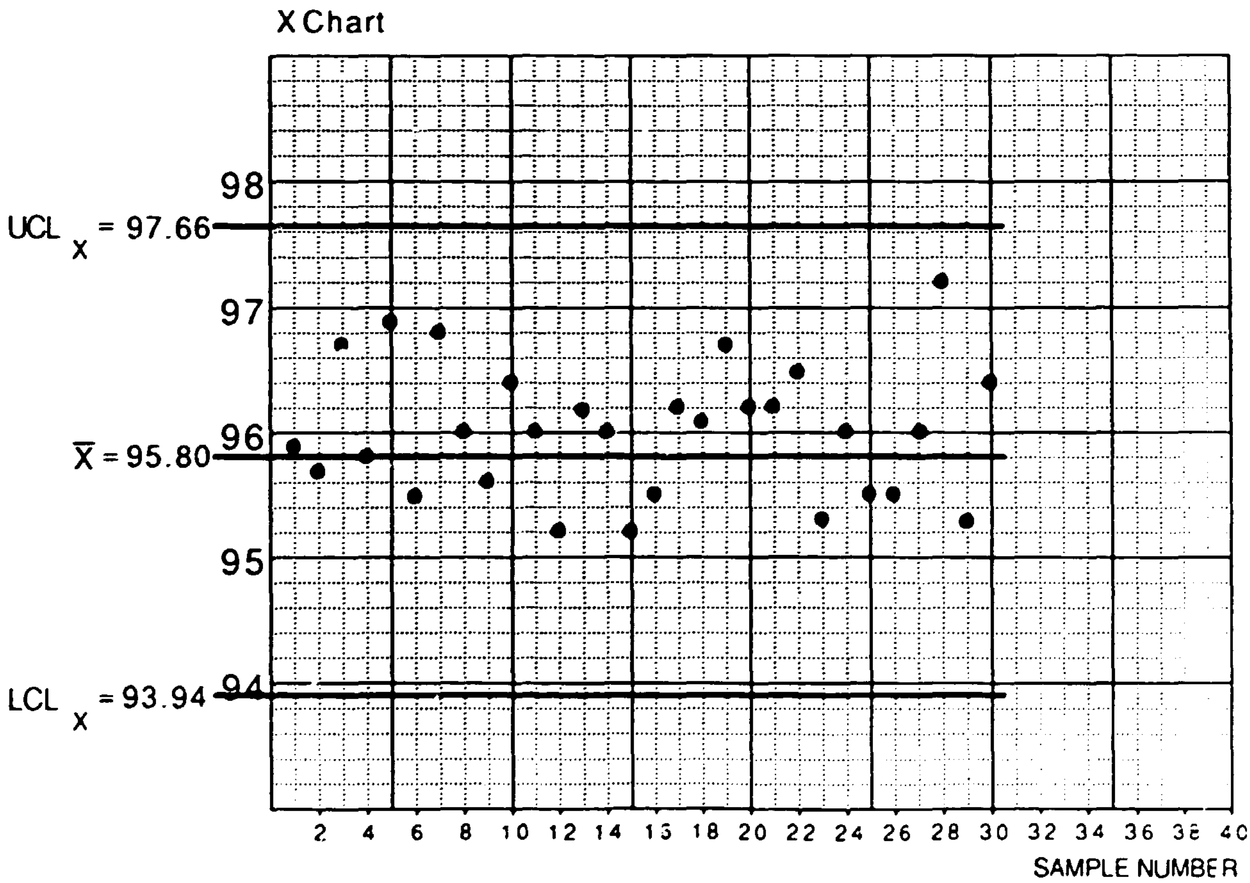
Whenever data are collected on a process, a measurement process is relied on to provide that data. The characteristics of a measurement process are thus critical to the successful use of data to study the processes and systems of a business organization. Just as with any other process, the quality of the measurement process needs to be understood. Possibly this understanding will lead to the awareness that the current measurement process is inadequate and needs to be improved.

In describing a strategy for studying a measurement process, the investigation of the process should begin with a description of the manner in which the process operates and a description of the sources of variation impacting the measurement process. As with other process study, process flow diagrams and cause-and-effect diagrams may be useful ways of capturing this information. Once the information about the sources of variation affecting a measurement process are in place, a data collection plan can be devised to understand the operation of the measurement process. The data collection and analysis should try to capture the effect that the identified sources of variation have on the measurement process. For example, in the study of the measurement of purity levels in section 8.1, the data collected about the measurement process attempted to capture the effect that different operators, different measuring devices, different reagents , and equipment changes had on the measurement of purity in a herbicide. The data collection should support the ability to evaluate the stability of the variation in measurements over time and if the measurement process is determined to be stable, to evaluate the precision, accuracy, and discrimination of the measurement process.

The studies of measurement processes in this chapter have focused on the designing of measurement studies. There is a risk, then, that the reader of this manual will conclude that such studies need only be performed once to understand the quality of the measurement process. Instead, it is important that in designing measurement studies, an ongoing evaluation of a measurement process be put in place. As materials, personnel, techniques, etc., change over time, a measurement process will, of course, have a tendency to change as well. An ongoing practice of monitoring a measurement process needs to be put in place if the measurements are to be relied on to report on the behavior of product and process outcomes.

Figure 8.1
Purity for Product





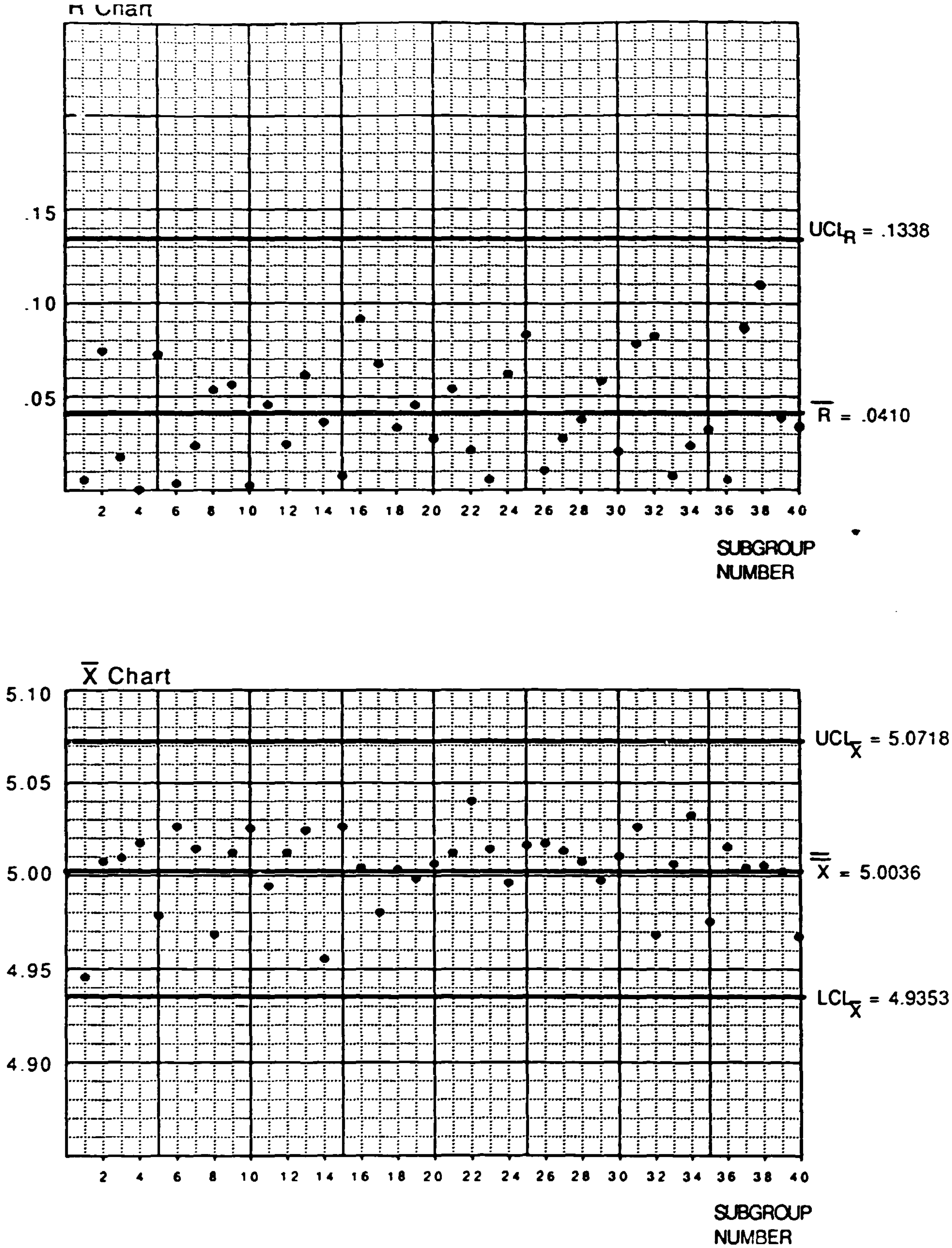


Figure 8.2

Control Charts for Weights of Standard

Figure 8.3
Measurements of Standard Weights

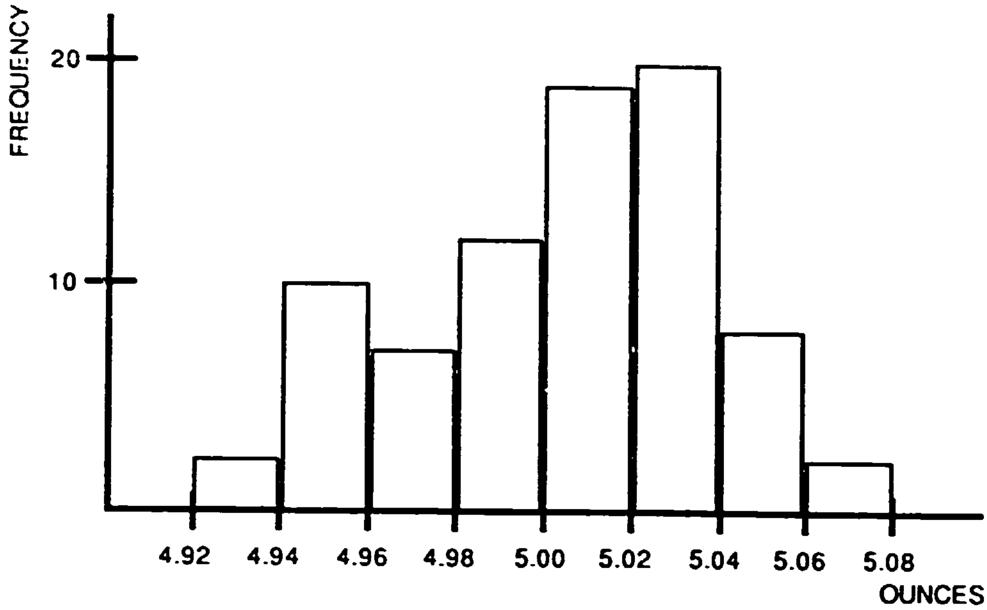
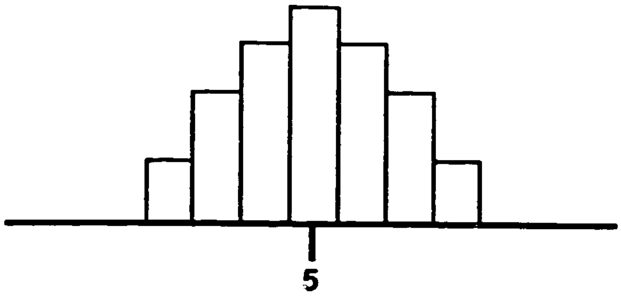
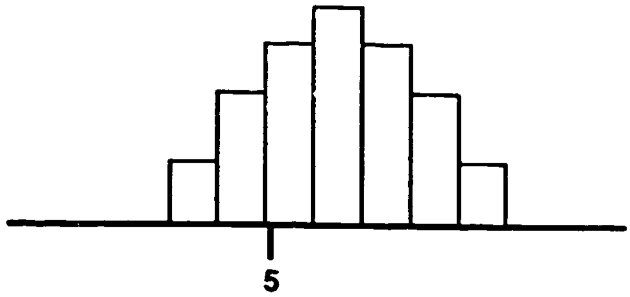


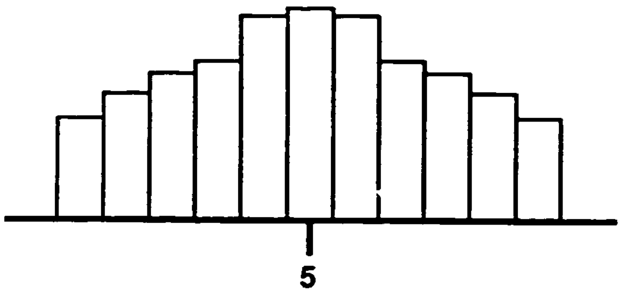
Figure 8.4



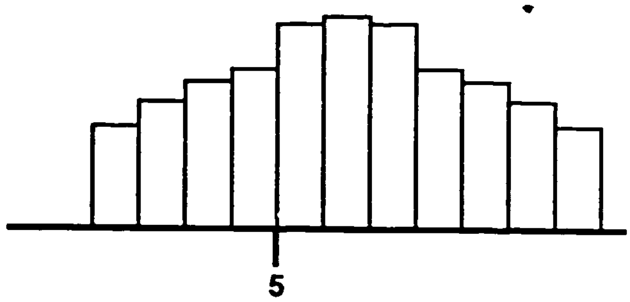
Histogram A



Histogram B



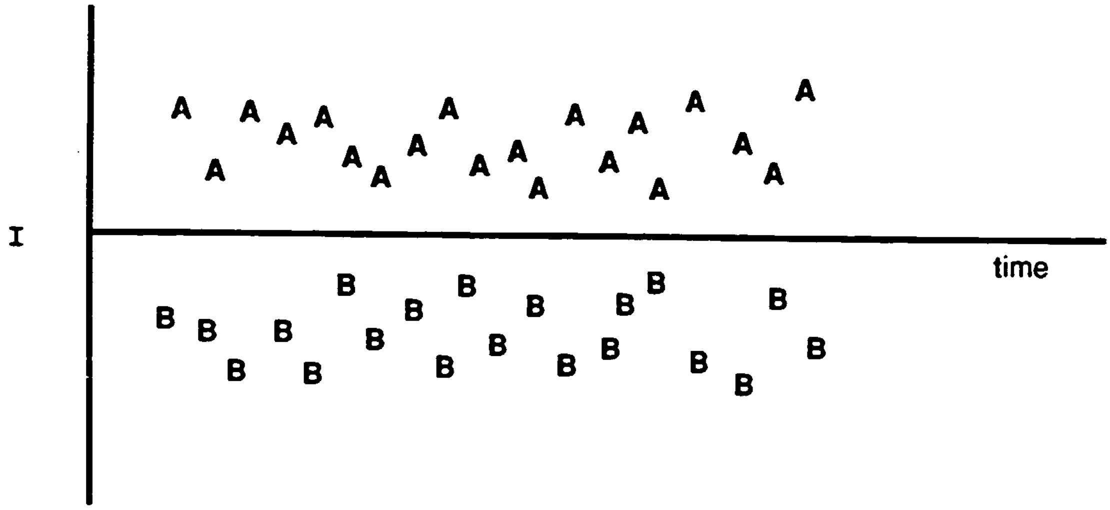
Histogram C



Histogram D

Figure 8.5

Time plot of product density with small measurement variation



Time plot of product density with large measurement variation

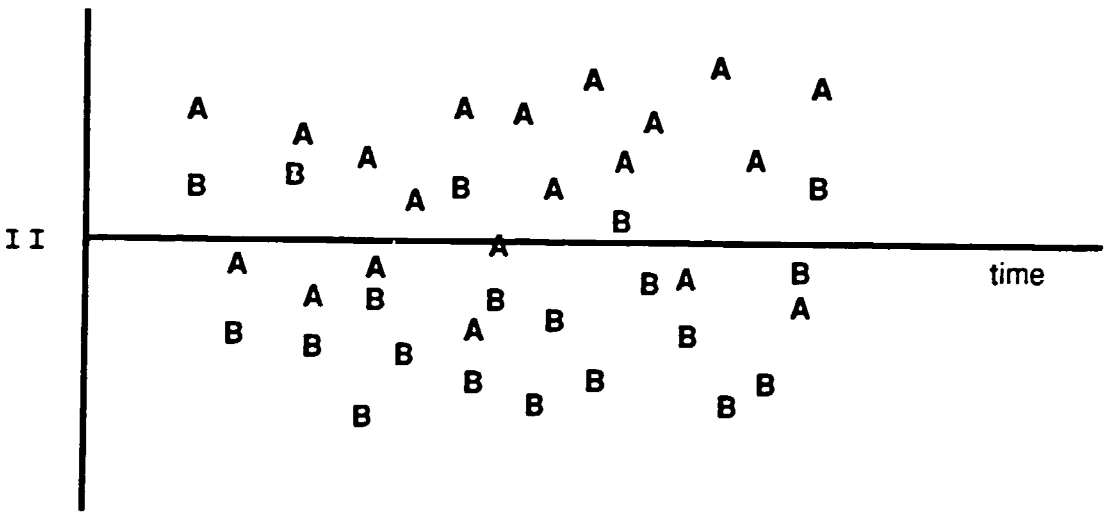


Figure 8.6
Range Charts for Two Measurement Processes

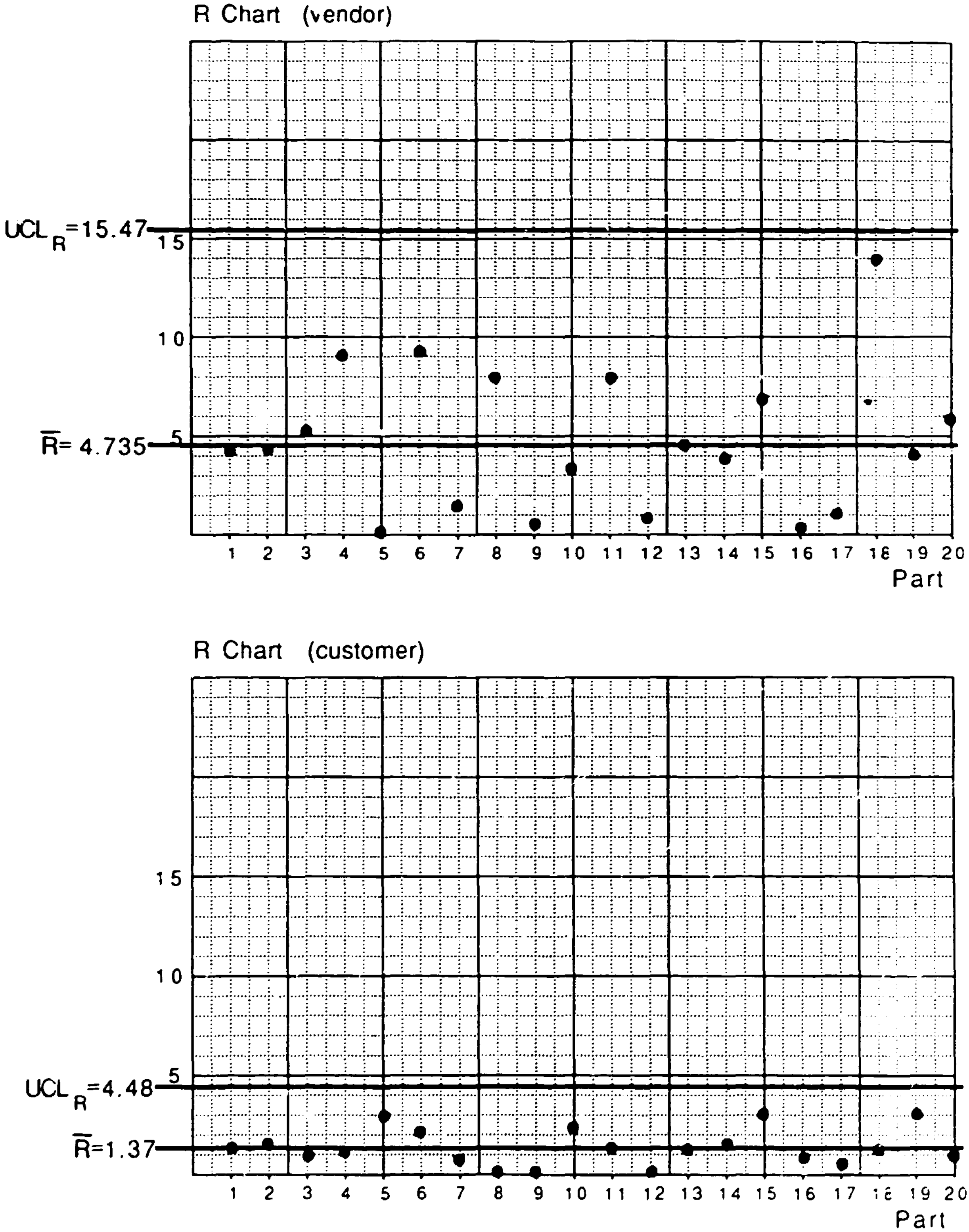
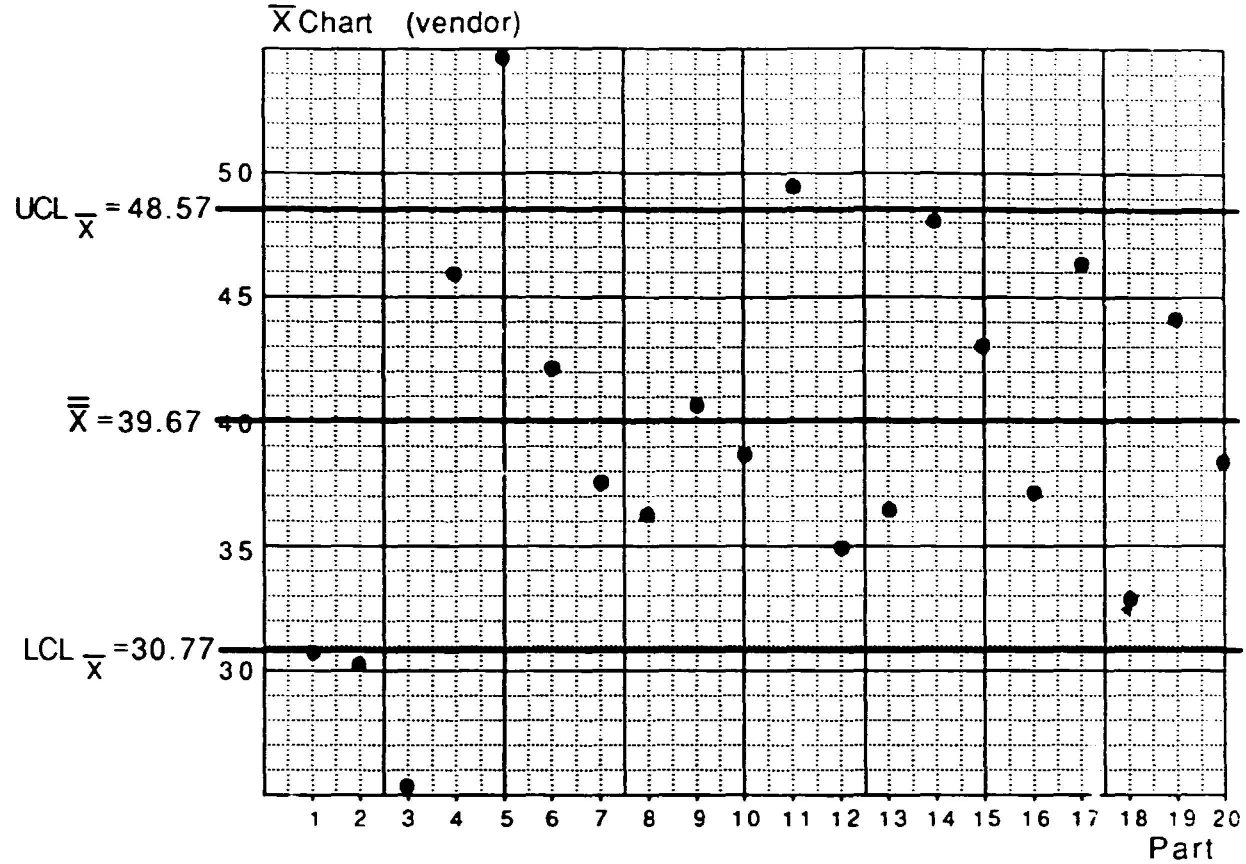
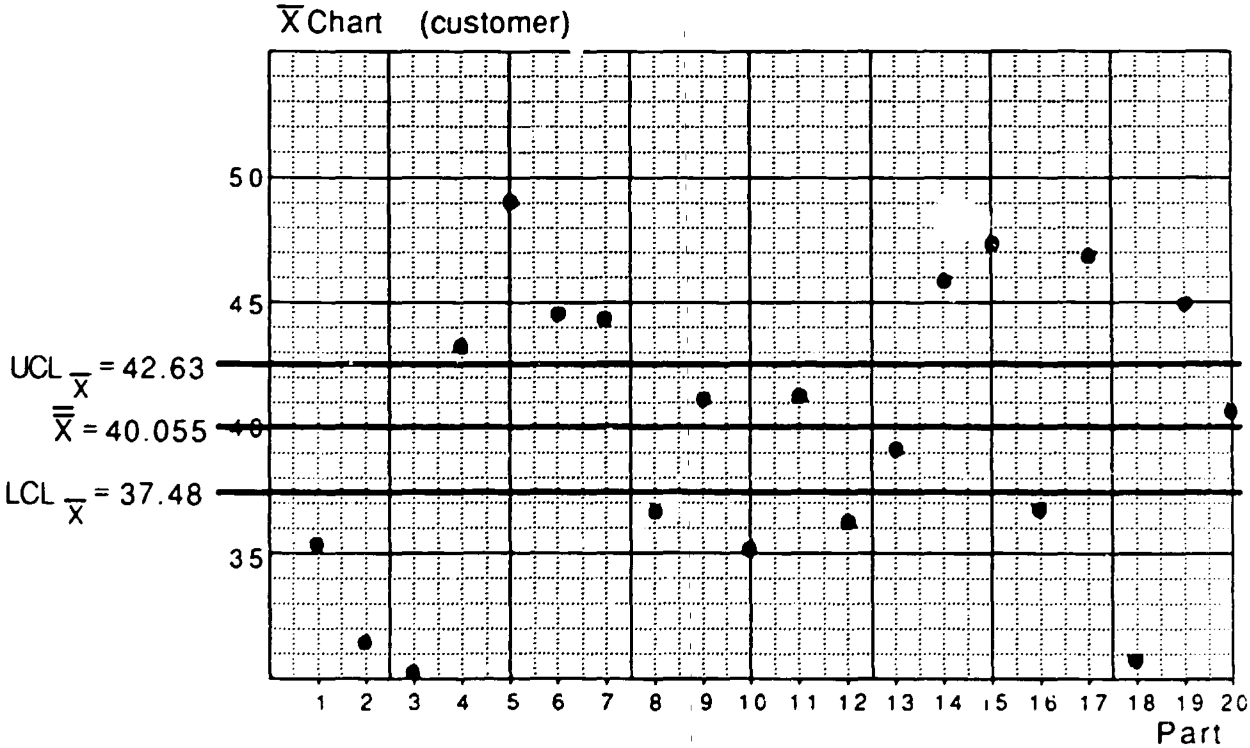


Figure 8.7
Charts for Part Averages





Factors For Use With $\overline{X}$ and Range Charts

Number of Observations in Subgroup n	Factor for $\overline{X}$ Chart A_2	Factors for Range Charts LCL D_3	Factors for Range Charts UCL D_4	Factor for Estimating σ d_2
2	1.880		3.267	1.128
3	1.023		2.574	1.693
4	0.729		2.282	2.059
5	0.577		2.114	2.326
6	0.483		2.004	2.534
7	0.419	0.076	1.924	2.704
8	0.373	0.136	1.864	2.847
9	0.337	0.184	1.816	2.970
10	0.308	0.223	1.777	3.078
11	0.285	0.256	1.744	3.173
12	0.266	0.284	1.716	3.258
13	0.249	0.308	1.692	3.336
14	0.235	0.329	1.671	3.407
15	0.223	0.348	1.652	3.472
16	0.212	0.364	1.636	3.532
17	0.203	0.379	1.621	3.588
18	0.194	0.392	1.608	3.640
19	0.187	0.404	1.596	3.689
20	0.180	0.414	1.586	3.735
21	0.173	0.425	1.575	3.778
22	0.167	0.434	1.566	3.819
23	0.162	0.443	1.557	3.858
24	0.157	0.452	1.548	3.895
25	0.153	0.459	1.541	3.931

Chapter 3

1. a $\overline{p} = \frac{67}{22000} = .003045$

$$UCL_p = .003045 + 3\sqrt{\frac{.003045\ (1-.003045)}{1000}}$$

$$= .0030345 + .00523$$

$$= .008275$$

$$LCL_p = .003045 - .00523$$

-NONE

All points inside control limits.

b. Comments about the data:

- No information on how the data were collected.
- Are the data time ordered? Without this knowledge, the runs tests are inappropriate.
- No information on the way the data were subgrouped.
- The most easily obtained data do not always provide sufficient information about the process.

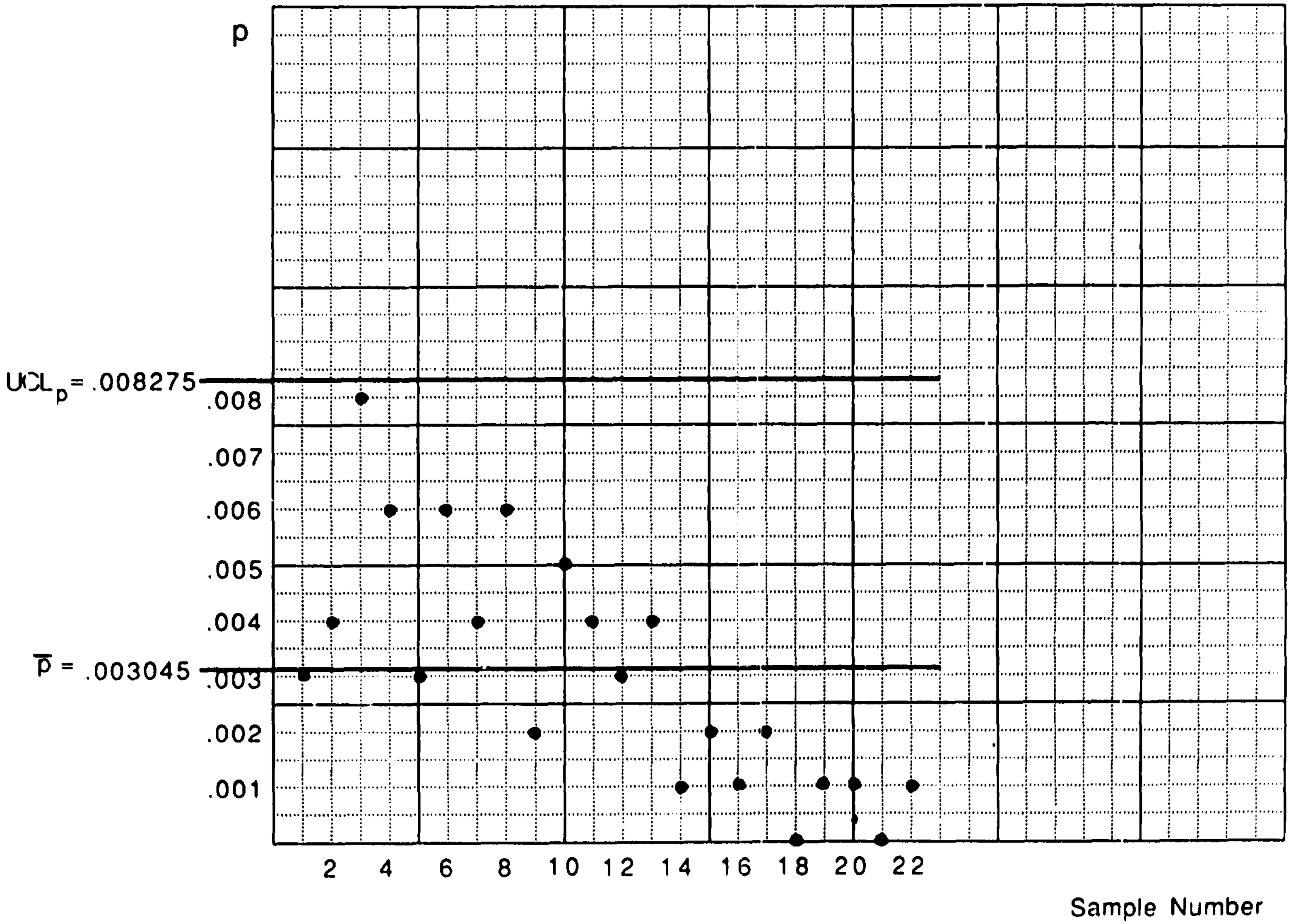
c. Next course of action:

- PLAN -- Draw process flow chart in order to define the process.

  Analyze the potential causes of defective cans by a cause-and-effect diagram.

  Develop a data collection strategy to provide desired information.

1. (cont.)


p
UCL_p = .008275
$\overline{p}$ = .003045
.008
.007
.006
.005
.004
.003
.002
.001
2 4 6 8 10 12 14 16 18 20 22
Sample Number

2. a Process is not stable.
Point outside the control limits.

b. n = number of measurements in the sample
= 700

$$\bar{p} = \frac{\text{Number of Defective Cans}}{\text{Total Number of Cans Inspected}}$$

$$UCL_{np} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$LCL_{np} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$$

SHIFT 1:

$$\bar{p} = \frac{35}{13(700)} = \frac{35}{9100} = .003846$$

$$n\bar{p} = 700\ (.003846) = 2.6923$$

$$UCL_{np} = 2.6923 + 3\sqrt{2.6923(1-.003846)}$$

$$= 2.6923 + 3(1.6377) = 2.6923 + 4.913$$

$$= 7.6053$$

$$LCL_{np} = 2.6923 - 3\sqrt{2.6923(1-.003846)}$$

$$= 2.6923 - 4.913$$

- NONE

SHIFT 2:

$$\bar{p} = \frac{20}{9100} = .002198$$

$$n\bar{p} = 700\ (.002198) = 1.53846$$

$$UCL_{np} = 1.53846 + 3\sqrt{1.53846(1-.002198)}$$

$$= 1.53846 + 3(1.23898) = 1.53846 + 3.71695$$

$$= 5.2554$$

$$LCL_{np} = 1.53846 - 3\sqrt{1.53846(1-.002198)}$$

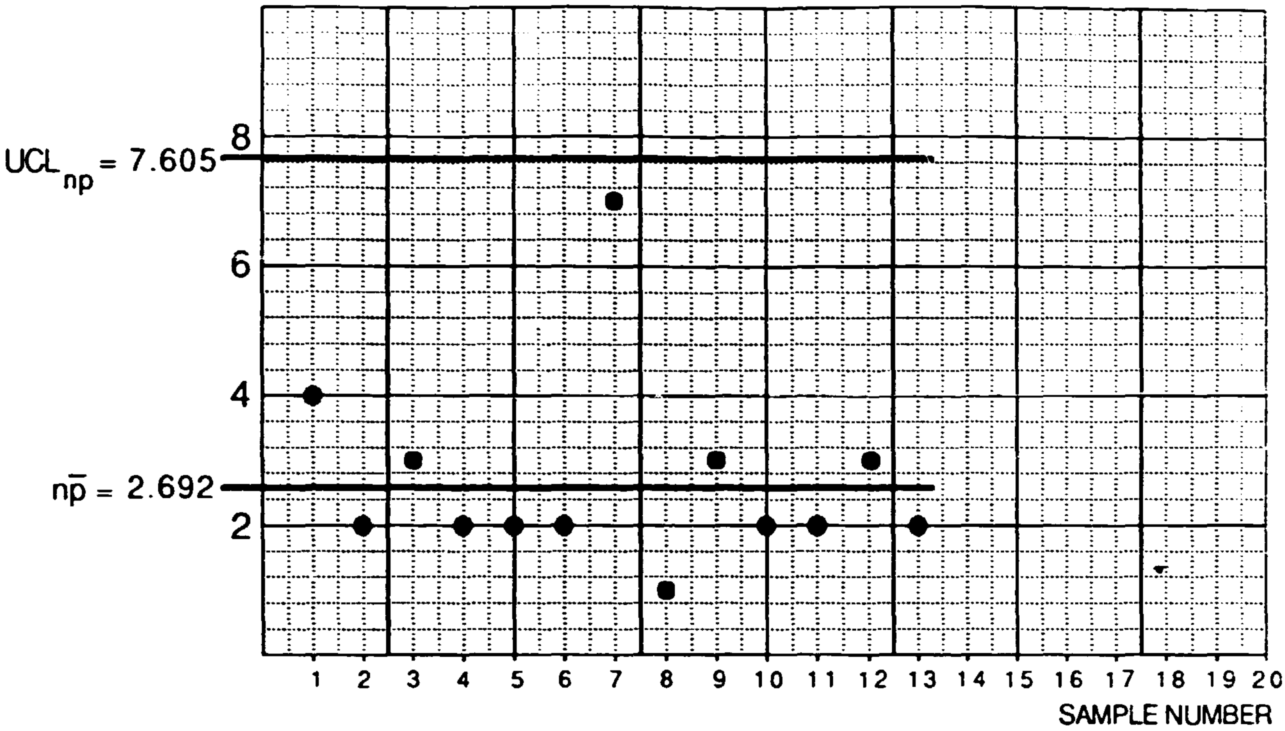
$$= 1.53846 - 3.71695$$

- NONE

2 (cont.)

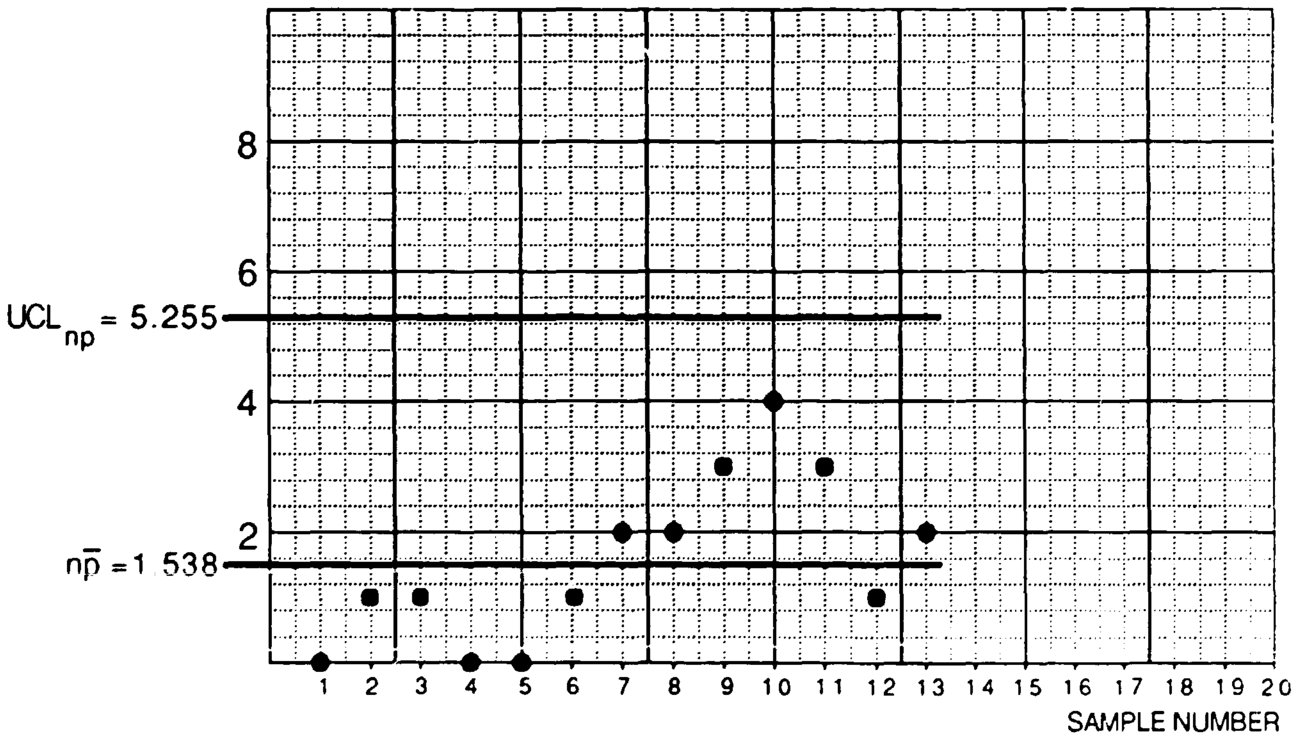
Number of Defective Cans - Shift 1
(Sample Size = 700)

np Chart



Number of Defective Cans - Shift 2
(Sample Size = 700)

np Chart



2. c. Shift 1 process is stable and predictable.
Shift 2 process is stable and predictable.
Therefore, the average proportions of defective cans between the two shifts can be compared. It appears that Shift 1 has a lower fraction of defective cans produced.

d. Questions that you might want to ask concerning the data:

- Who took the measurements?

- Is the inspection process consistent between the two shifts?

- Are there operational definitions?

3. a 25 samples
n = sample size = 16
p chart:

Central line:

$$\overline{p} = \frac{\text{Number of projects requiring rework}}{\text{Number of projects inspected}}$$

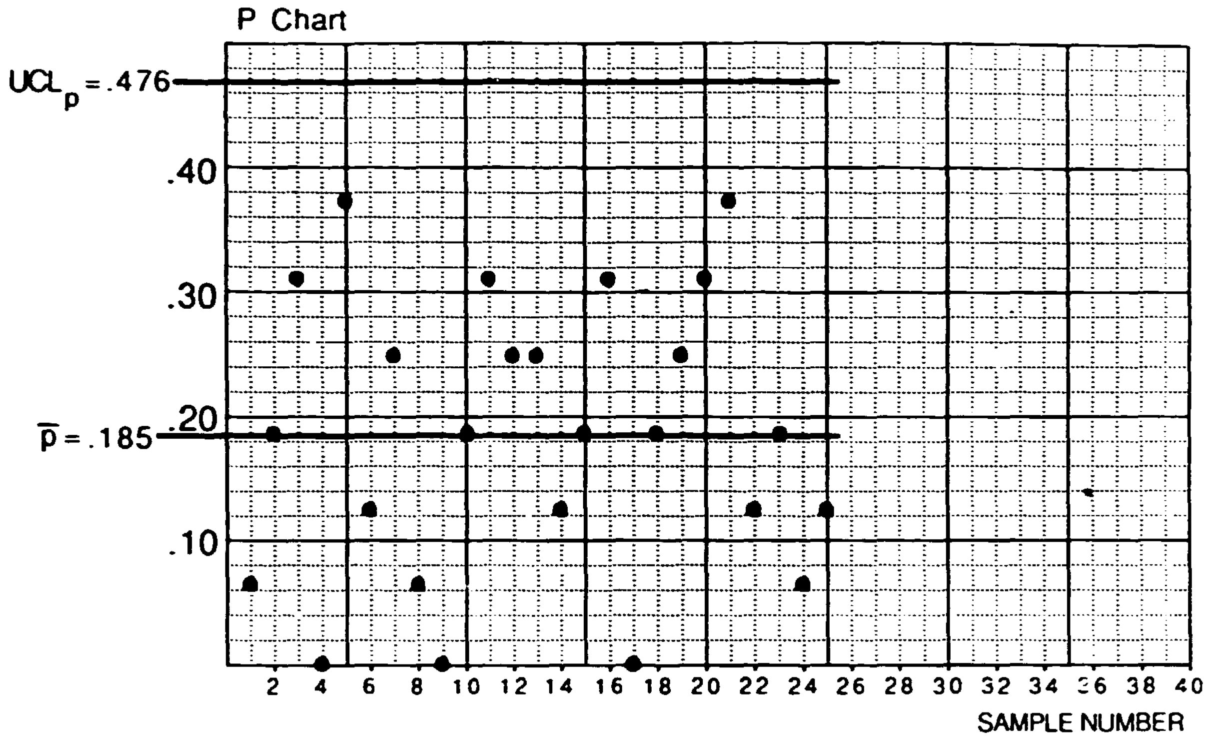
$$= \frac{74}{25(16)} = \frac{74}{400} = .1850$$

Control Limits: $\overline{p} \pm 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}}$

$$UCL_p = .1850 + 3\sqrt{\frac{(.1850)(1-.1850)}{16}}$$

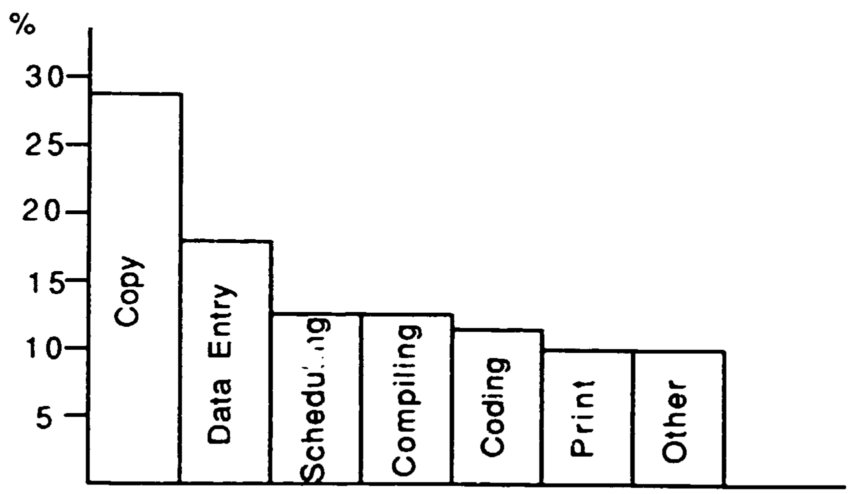
$= .1850 + 3(.0971)$
$= .1850 + .2912$
$= .4762$

$LCL_p = .1850 - .2912$
$=$ NONE


P Chart
UCL p = .476
.40
.30
.20
p = .185
.10
2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40
SAMPLE NUMBER

3.b. Pareto Analysis

Reason	Number	Percent
Copy	21	28.38 %
Data Entry	13	17.57
Scheduling	9	12.16
Compiling	9	12.16
Coding	8	10.81
Print	7	9.46
Other	7	9.46
	74	



4.a.

$$\bar{p} = \frac{367}{5544} = .0662$$

Lot 22

$n = 200$

$p = 23/200 = .1150$

Control Limits:

$$.0662 + 3\sqrt{\frac{.0662(1-.0662)}{200}}$$

$UCL_p = .0662 + .0527$
$= .1189$

$LCL_p = .0135$

All Points Inside Control Limits

Lot 23

$n = 275$

$p = 5/275 = .0182$

Control Limits:

$$.0662 + 3\sqrt{\frac{.0662(1-.0662)}{275}}$$

$UCL_p = .0662 + .0450$
$= .1112$

$LCL_p = .0212$

Points Outside Control Limits

b. Impacts methods of performance have on use of data to improve system:

- Possible misrepresentation of data.

- Use of data to support individual performance instead of process performance.

5. Using $\overline{p}$ = .10, control limits should be:

$$.10 \pm \frac{3\sqrt{.10(.90)}}{\sqrt{n}} \qquad 3\sqrt{10(.90)} = .90$$

Sample Number	n	p	$\frac{.90}{\sqrt{n}}$	UCL_p	LCL_p
40	49	.2041	.1286	.2286	——
41	225	.1111	.0600	.1600	.0400
42	100	.1500	.0900	.1900	.0100
43	100	.2000	0900	.1900	.0100 *
44	225	.2000	.0600	.1600	.0400 *
45	225	.1289	.0600	.1600	.0400 *

* indicates a point outside the control limits.

1. a u chart -- variable subgroup size

Inspection unit -- 100 yards

$$\text{Center Line: } \bar{u} = \frac{\text{Total number of nonconformities}}{\text{Total number of inspection units}}$$

$$= \frac{1850}{88} = 21.0227$$

$$\text{Control limits: } \bar{u} \pm 3\sqrt{\frac{\bar{u}}{n}}$$

For 300 yard roll: $n = 3$

$$UCL_u = 21.0227 + 3\sqrt{\frac{21.0227}{3}}$$

$$= 21.0227 + 7.9415$$
$$= 28.9642$$

$$LCL_u = 21.0227 - 7.9415$$
$$= 13.0812$$

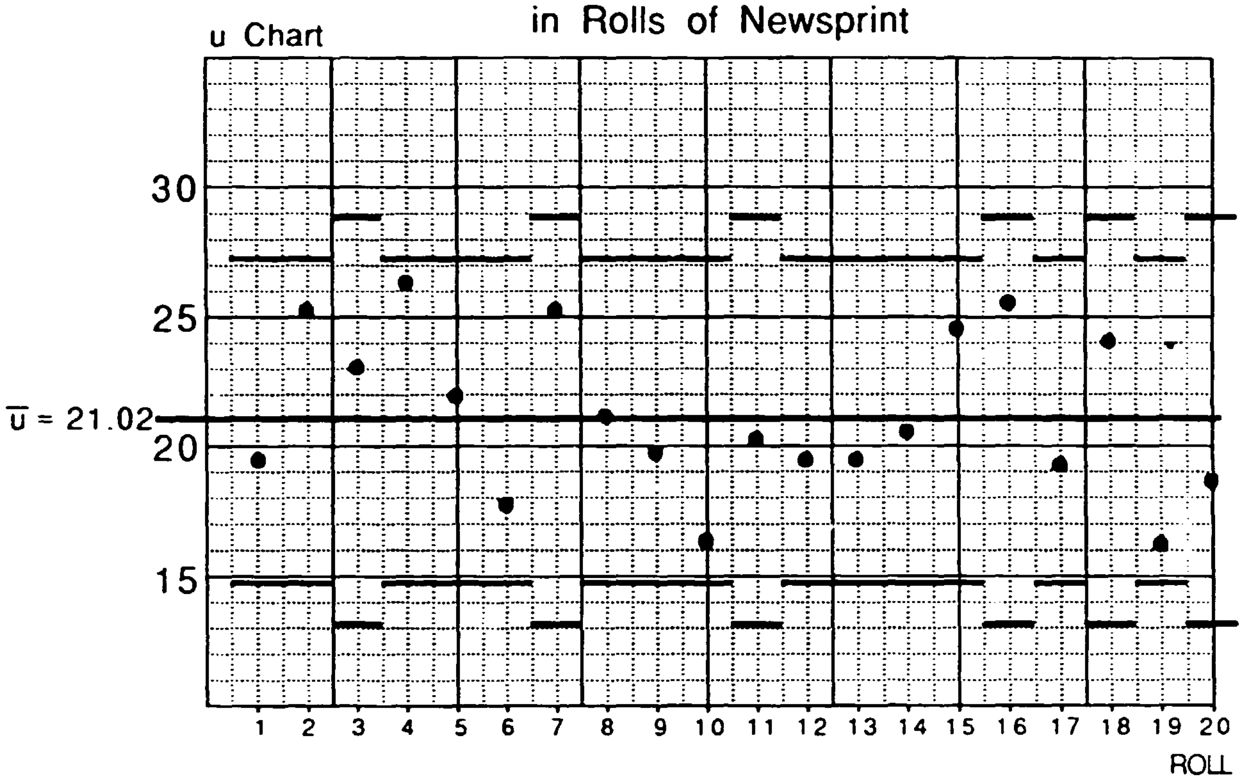
For 500 yard roll: $n = 5$

$$UCL_u = 21.0227 + 3\sqrt{\frac{21.0227}{5}}$$

$$= 21.0227 + 6.1515$$
$$= 27.1742$$

$$LCL_u = 21.0227 - 6.1515$$
$$= 14.8712$$

Number of Nonconformities per 100 Yards
in Rolls of Newsprint



1. b. Process appears to be stable and predictable.

   No obvious difference in the average number of nonconformities per 100-yards of newsprint between the two shifts.

   c. Process flow chart helps to define the process and show the relationships between the different sections of the process.

   Cause-and-Effect diagram helps to identify possible causes of nonconformities in the newsprint.

   These two diagrams help in the planning of a data collection strategy.

   d. Objective: to determine if differences exist between the three machines.

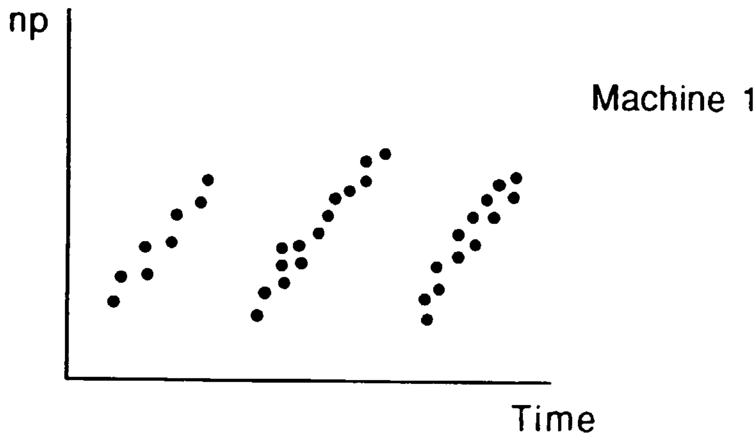
   Sampling Procedure: One roll per shift per machine over a two-day period.

   Potential Problems:
   - *Not enough data--too short of a time-period.
   - *No trends over time can be detected
   - *Shift-to-Shift differences may hide between machine differences, since there is only one count of nonconformities per machine per shift.
   - *Within shift differences cannot be detected.

1d. (cont'd)

Examples of a problem where this sampling procedure would be inadequate:

- The number of nonconformities in the paper processed by Machine 1 increases between routine maintenance.



- Raw material changes have a larger impact on the performance of one machine than on the performance of the others.

- Over-adjustments of a machine by an operator within a shift.

1.e. Machine 2

Roll	Length	# Nonconformities	n
1	500	106	5
2	500	115	5
3	500	128	5
4	300	76	3
5	300	95	3
6	500	113	5
7	300	84	3
8	300	94	3
9	300	71	3
10	300	67	3
		949	38

$$\text{Central Line} = \bar{u} = \frac{\text{Total Number of Nonconformities}}{\text{Total Number of Units Inspected}}$$

$$= \frac{949}{38}$$

$$= 24.9737$$

Control Limits for n=3:

$$UCL_u = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}} = 24.9737 + 3\sqrt{\frac{24.9737}{3}}$$

$$= 24.9737 + 8.6557$$

$$= 33.6294$$

$$LCL_u = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}} = 24.9737 - 8.6557$$

$$= 16.3180$$

For n=5:

$$UCL_u = 24.9737 + 3\sqrt{\frac{24.9737}{5}}$$

$$= 24.9737 + 6.7047 = 31.6784$$

$$LCL_u = 24.9737 - 6.7047 = 18.2690$$

1.e. (cont'd) Machine 3

Roll	Length	# Nonconformities	n
1	300	70	3
2	300	55	3
3	500	90	5
4	300	59	3
5	500	94	5
6	300	97	3
7	500	96	5
8	500	100	5
9	500	94	5
10	500	92	5
		847	42

Central Line:

$$\overline{u} = \frac{847}{42} = 20.1667$$

Controi Limits for n=3:

$$UCL_u = \overline{u} + 3\sqrt{\frac{\overline{u}}{n}} = 20.1667 + 3\sqrt{\frac{20.1667}{3}}$$

$$= 20.1667 + 7.7782$$

$$= 27.9449$$

$$LCL_u = \overline{u} - 3\sqrt{\frac{\overline{u}}{n}} = 20.1667 - 7.7782$$

$$= 12.3885$$

For n=5:

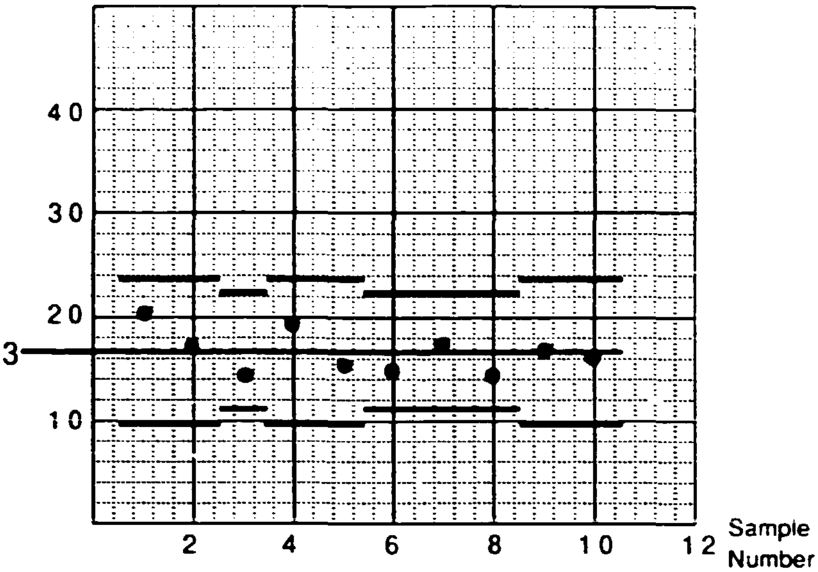
$$UCL_u = 20.1667 + 3\sqrt{\frac{20.1667}{5}}$$

$$= 20.1667 + 6.0250 = 26.1917$$

$$LCL_u = 20.1667 - 6.0250 = 14.1417$$

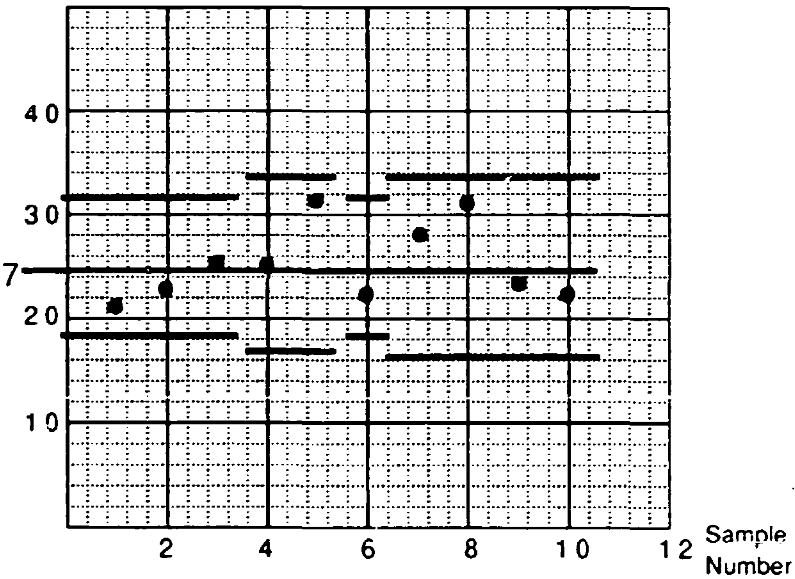
Machine 1

$\overline{U}$ = 16.7733



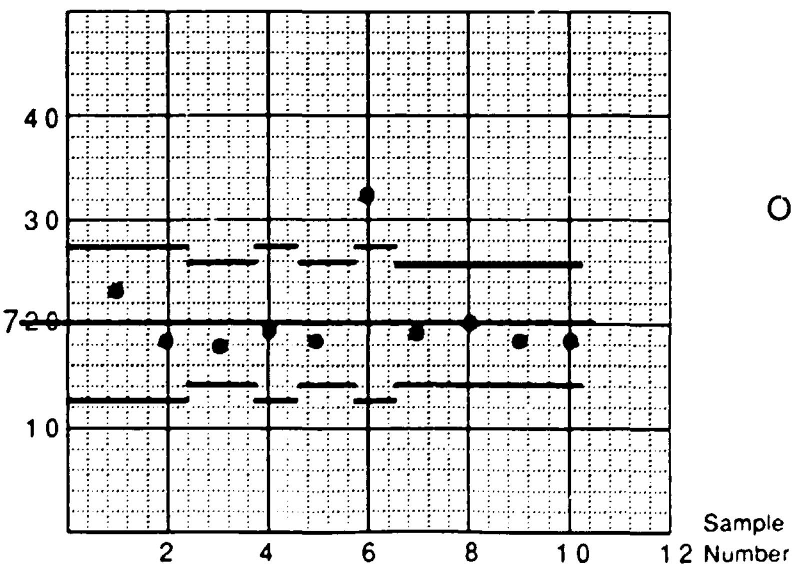
Machine 2

$\overline{U}$ = 24.9737



Machine 3

$\overline{U}$ = 20.1667



Out of Control

1.f.

Note: A Pareto Analysis is inappropriate for Machine 3, since the process is out of control.

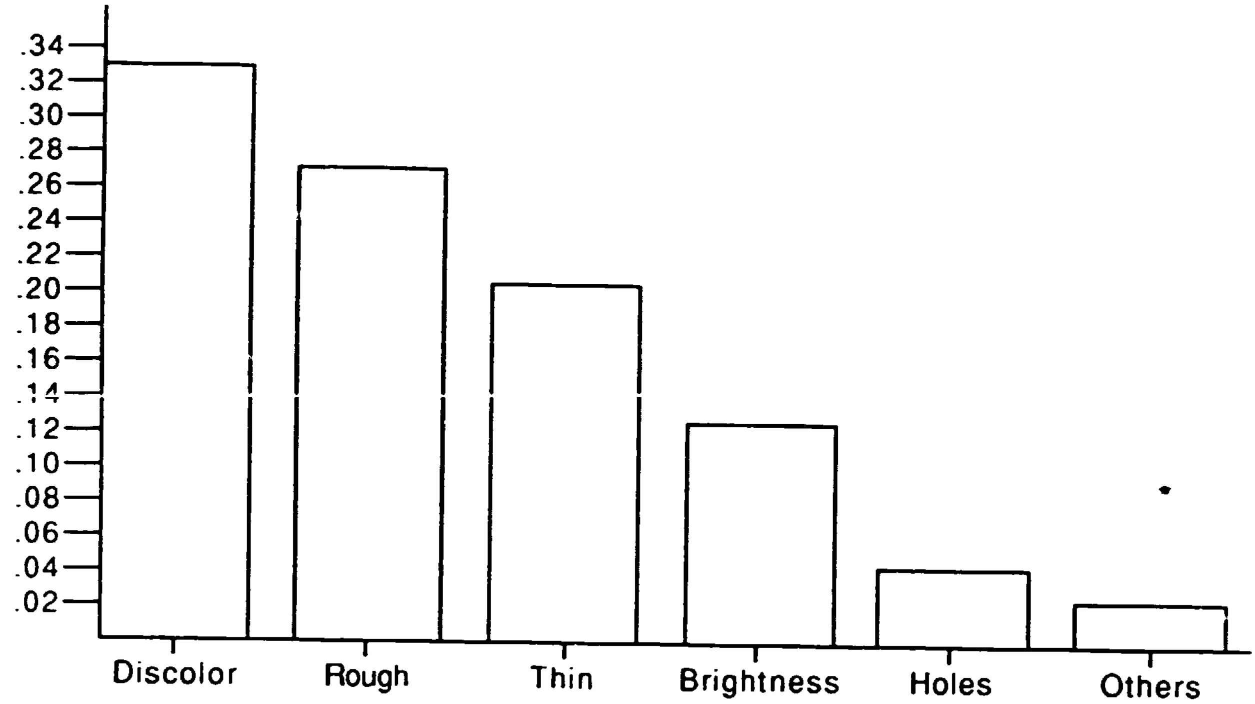
Machine 1:

Reasons for Nonconformance	Frequency	Percent
Discolorations	206	32.91
Rough Areas	169	27.00
Thin Areas	128	20.45
Brightness Variation	80	12.78
Holes	27	4.31
Others	16	2.56
	626	

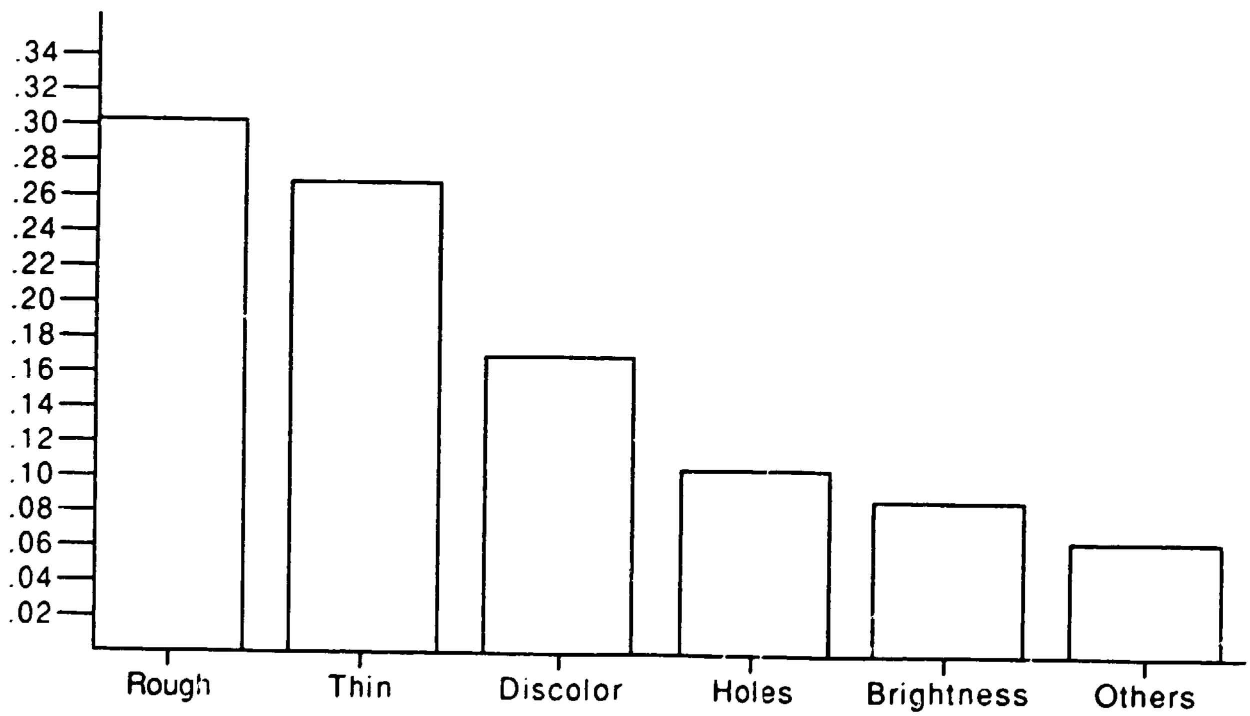
Machine 2:

Reasons for Nonconformance	Frequency	Percent
Rough Areas	287	30.24
Thin Areas	255	26.87
Discolorations	161	16.97
Holes	99	10.43
Brightness Variation	84	8.85
Others	63	6.64
	949	

Pareto Analysis
Machine 1



Pareto Analysis
Machine 2



2. average = 6 per radio

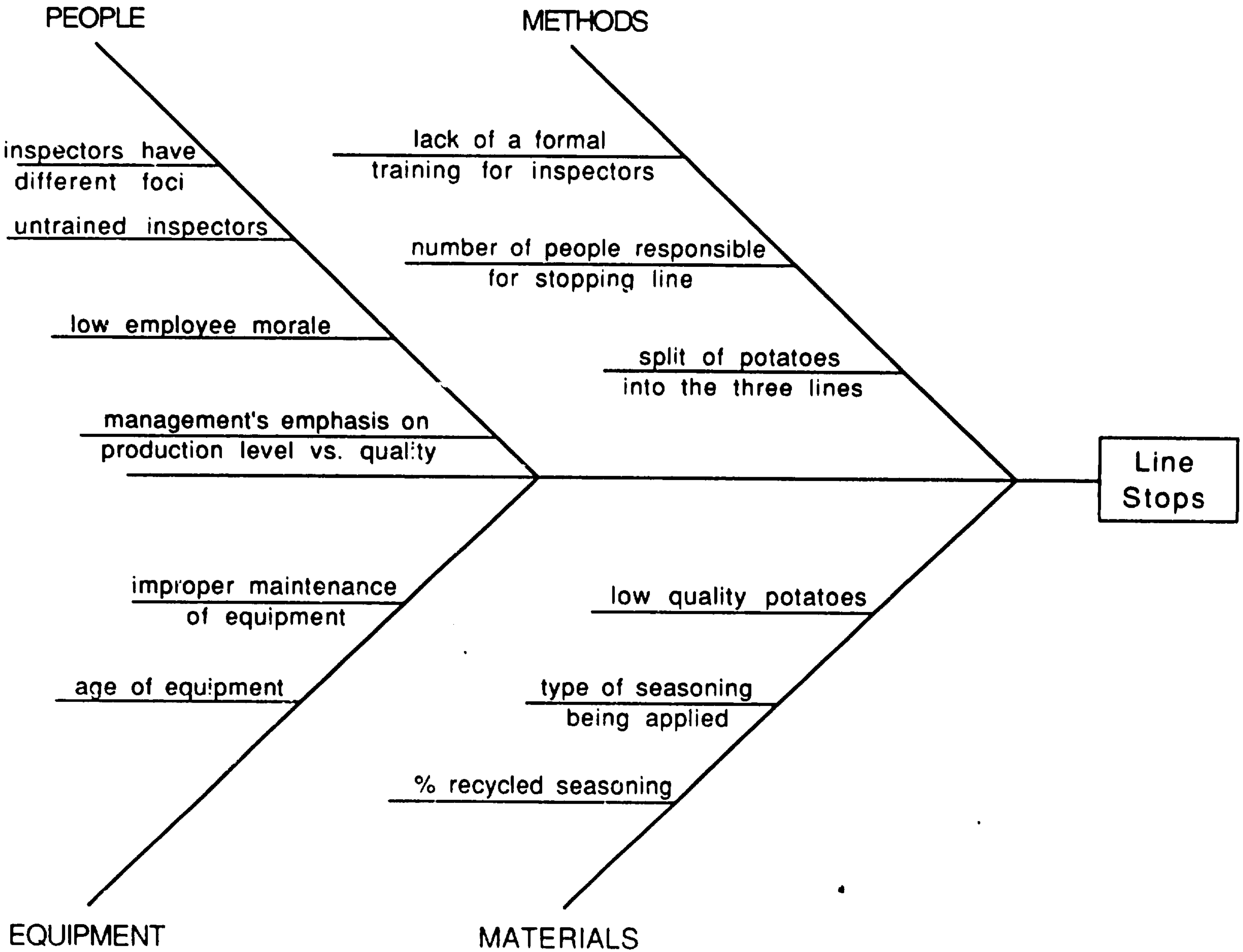
iu = 3 radios

Use c chart:

$\overline{c} = 18$

$UCL_c = 18 + 3\sqrt{18} = 30.7279$

$LCL_c = 18 - 12.7279 = 5.2721$


PEOPLE
METHODS
inspectors have different foci
untrained inspectors
low employee morale
management's emphasis on production level vs. quality
lack of a formal training for inspectors
number of people responsible for stopping line
split of potatoes into the three lines
Line Stops
improper maintenance of equipment
age of equipment
low quality potatoes
type of seasoning being applied
% recycled seasoning
EQUIPMENT
MATERIALS

3.b. c Chart

Inspection Unit = 1 day

$$\text{Central Line} = \bar{c} = \frac{\text{Number of Line Stops}}{\text{Total Number of Days}}$$

Control Limits: $\bar{c} \pm 3\sqrt{\bar{c}}$

Line 1

$$\bar{c} = \frac{115}{14} = 8.2143$$

$$UCL_c = 8.2143 + 3\sqrt{8.2143}$$

$$= 8.2143 + 8.5982$$

$$= 16.8125$$

LCL_c = NONE

Line 2

$$\bar{c} = \frac{166}{14} = 11.8571$$

$$UCL_c = 11.8571 + 3\sqrt{11.8571}$$

$$= 22.8173$$

$$LCL_c = 1.5269$$

Line 3

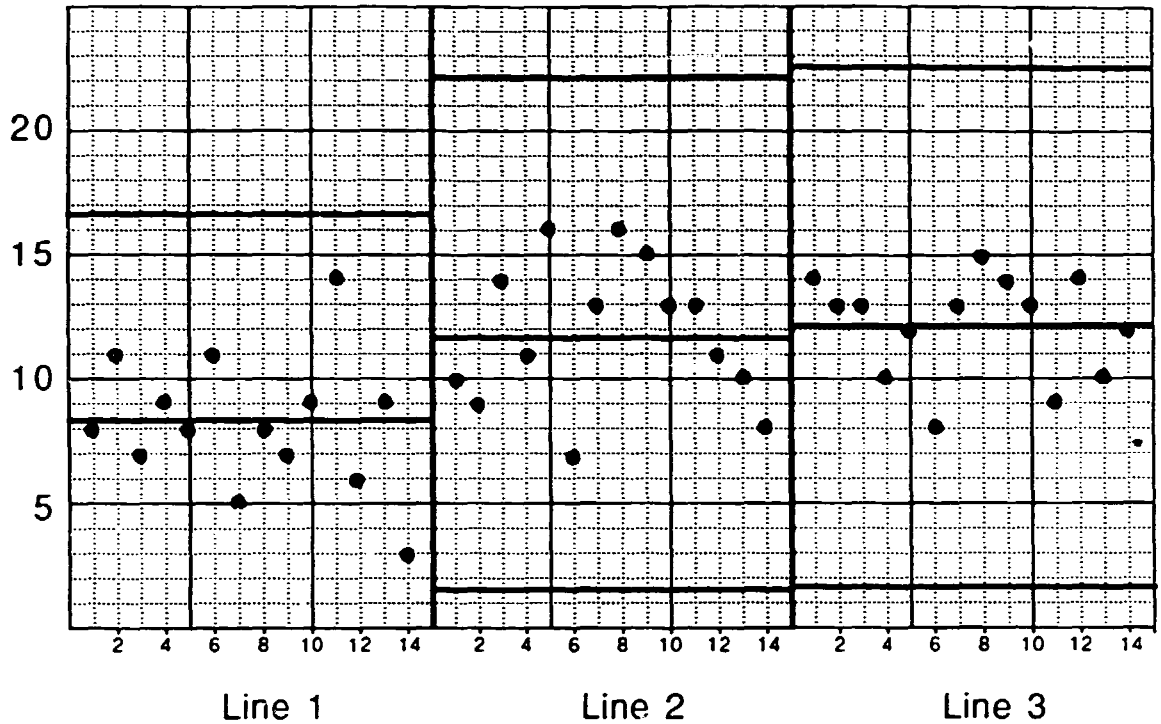
$$\bar{c} = \frac{170}{14} = 12.1429$$

$$UCL_c = 12.1429 + 3\sqrt{12.1429}$$

$$= 22.5969$$

$$LCL_c = 1.6889$$

Problem 3b.



c. Focus on reducing number line stops on lines 2 and 3.

Note: Since there are no special causes acting on the processes, the improvement effort must be focused on the system.

4. Using 100 square yards as an I. U.

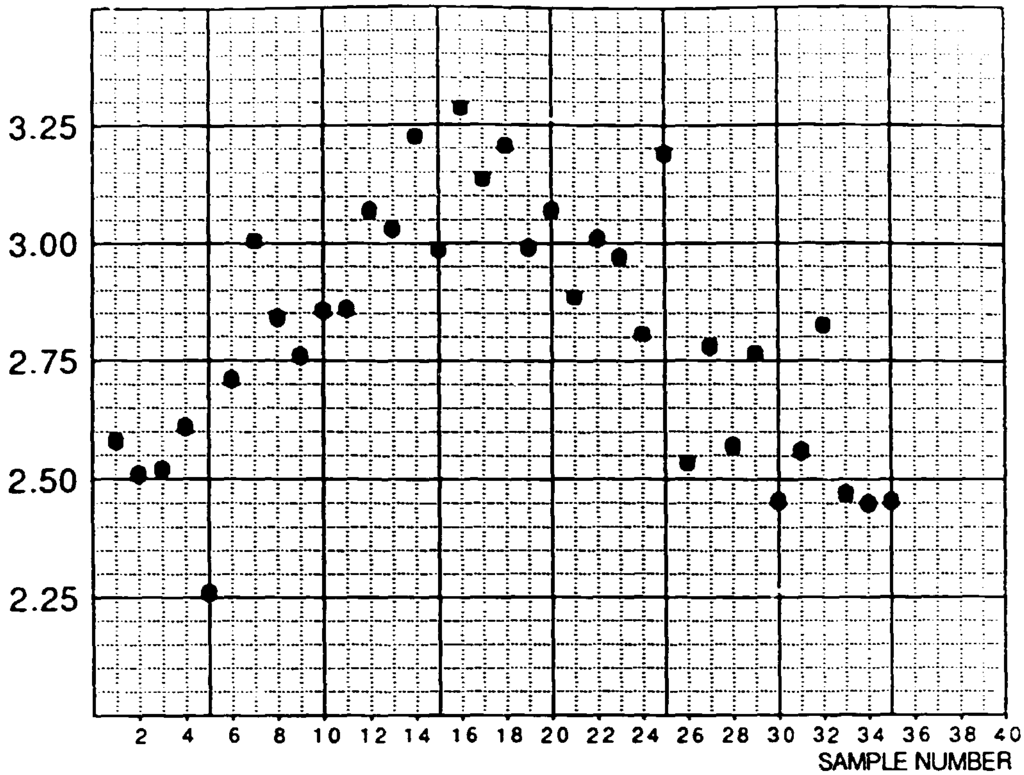
Lot No.	n	c	$u = c/n$	$\frac{4.855}{\sqrt{n}}$	UCL_u	LCL_u
1	2.00	5	2.50	3.433	6.052	–
2	2.50	7	2.80	3.071	5.690	–
3	1.00	3	3.00	4.855	7.474	–
4	0.90	2	2.22	5.118	7.737	–
5	1.20	4	3.33	4.432	7.051	–
6	0.80	1	1.25	5.428	8.047	–
	8.40	22				

$$\bar{u} = \frac{22}{8.4} = 2.619$$

Control Limits: $\bar{u} \pm \frac{3\sqrt{\bar{u}}}{\sqrt{n}}$

$$3\sqrt{\bar{u}} = 3\sqrt{2.619} = 4.8550$$

1.



How does the scale of a graph influence the conclusions drawn from that graph? Since you can alter the appearance of a graph based on the way in which it is scaled, you cannot rely solely on visual interpretation of graphs to make strong conclusions about a process.

a) When data are combined into one large data set to estimate the process average and standard deviation, the information about a trend in the process; i.e. a changing process average, is lost.

b) The standard deviation of all 35 measurements (.266) is not an appropriate estimate of the process standard deviation. The average is not in control, and one can not estimate the process standard deviation for an out of control process. The value .266 is an overestimate of the short-term standard deviation, which appears to be somewhat stable. The value .266 might very well underestimate the standard deviation of values taken from this process over a period of time longer than seven weeks. This depends on the behavior of the average and short-term variation over time.

If the process were in control, then 2.81 inches may be representative of the true process average. But, since the trending process would fail the runs test, then 2.81 inches would not be representative of the true process average.

1.c.

	Measurement						
Day	1	2	3	4	5	$\overline{X}$	Range
1	2.763	2.583	2.401	2.614	2.469	2.566	0.362
2	2.532	2.837	2.878	2.497	2.659	2.681	0.381
3	2.538	3.114	3.122	2.893	2.836	2.901	0.584
4	3.033	3.226	2.886	2.846	3.344	3.067	0.498
5	2.909	2.819	2.808	2.637	2.989	2.832	0.352
6	2.714	2.654	2.512	2.557	2.936	2.675	0.424
7	2.829	2.697	2.363	2.754	2.437	2.616	0.466
8	2.583	2.503	2.518	2.609	2.431	2.529	0.178
9	2.582	2.918	2.809	2.734	2.798	2.768	0.336
10	2.861	3.014	3.089	2.718	2.922	2.921	0.371
11	3.293	3.147	2.859	2.962	3.177	3.088	0.434
12	2.892	3.006	2.973	2.811	3.197	2.976	0.386
13	2.542	2.780	2.573	2.767	2.455	2.623	0.325
14	2.558	2.836	2.427	2.455	2.462	2.548	0.409
						38.791	5.506

R-Chart

$$\text{Center Line} = \frac{\text{Sum of Ranges for Subgroups}}{\text{Number of Subgroups}}$$

$$= \frac{5.506}{14} = .3933$$

$UCL_R = D_4\overline{R} = 2.114(.3933) = .8314$ $\qquad$ $LCL_R = \text{None}$

$\overline{X}$-Chart

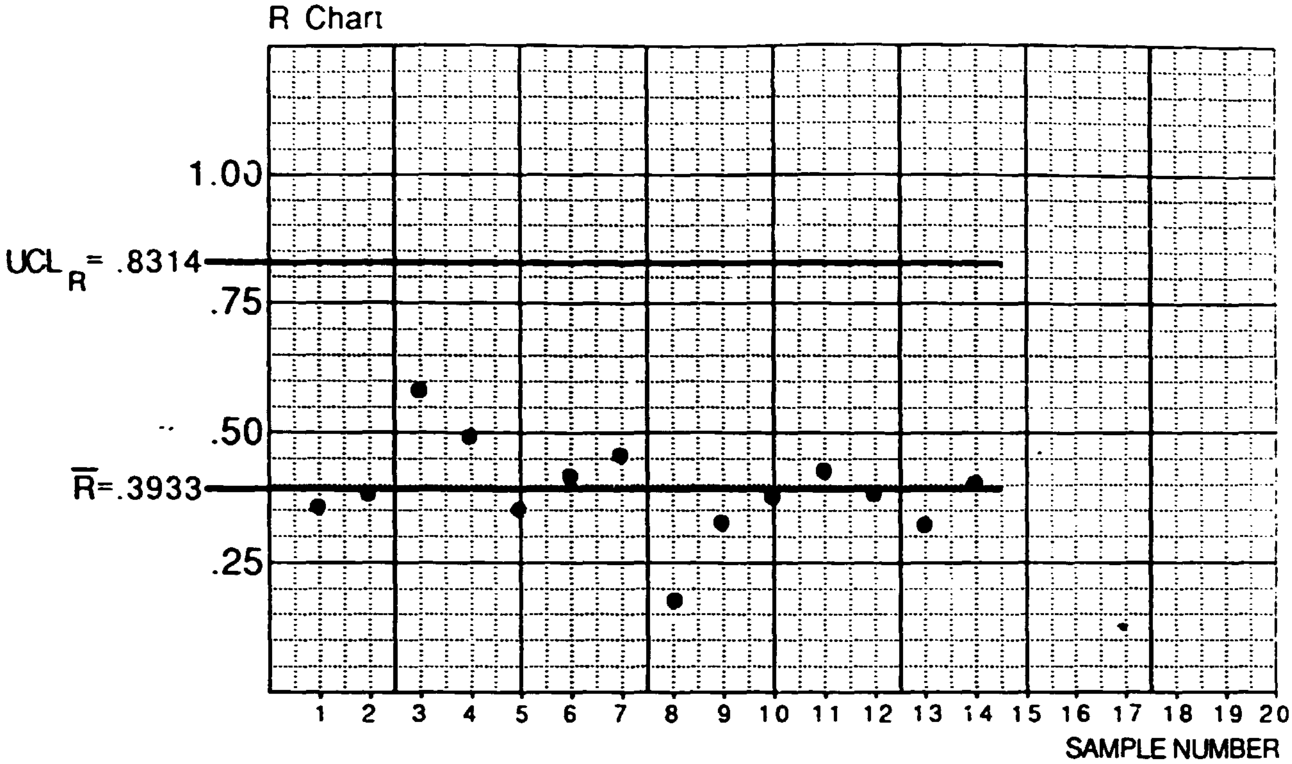
$$\text{Center Line} = \overline{\overline{X}} = \frac{\text{Sum of } \overline{X}\text{'s for Subgroups}}{\text{Number of Subgroups}} = \frac{38.791}{14} = 2.7708$$

$$UCL_{\overline{X}} = \overline{\overline{X}} + A_2\overline{R}$$
$$= 2.7708 + .577(.3933)$$
$$= 2.7708 + .2269$$
$$= 2.9977$$

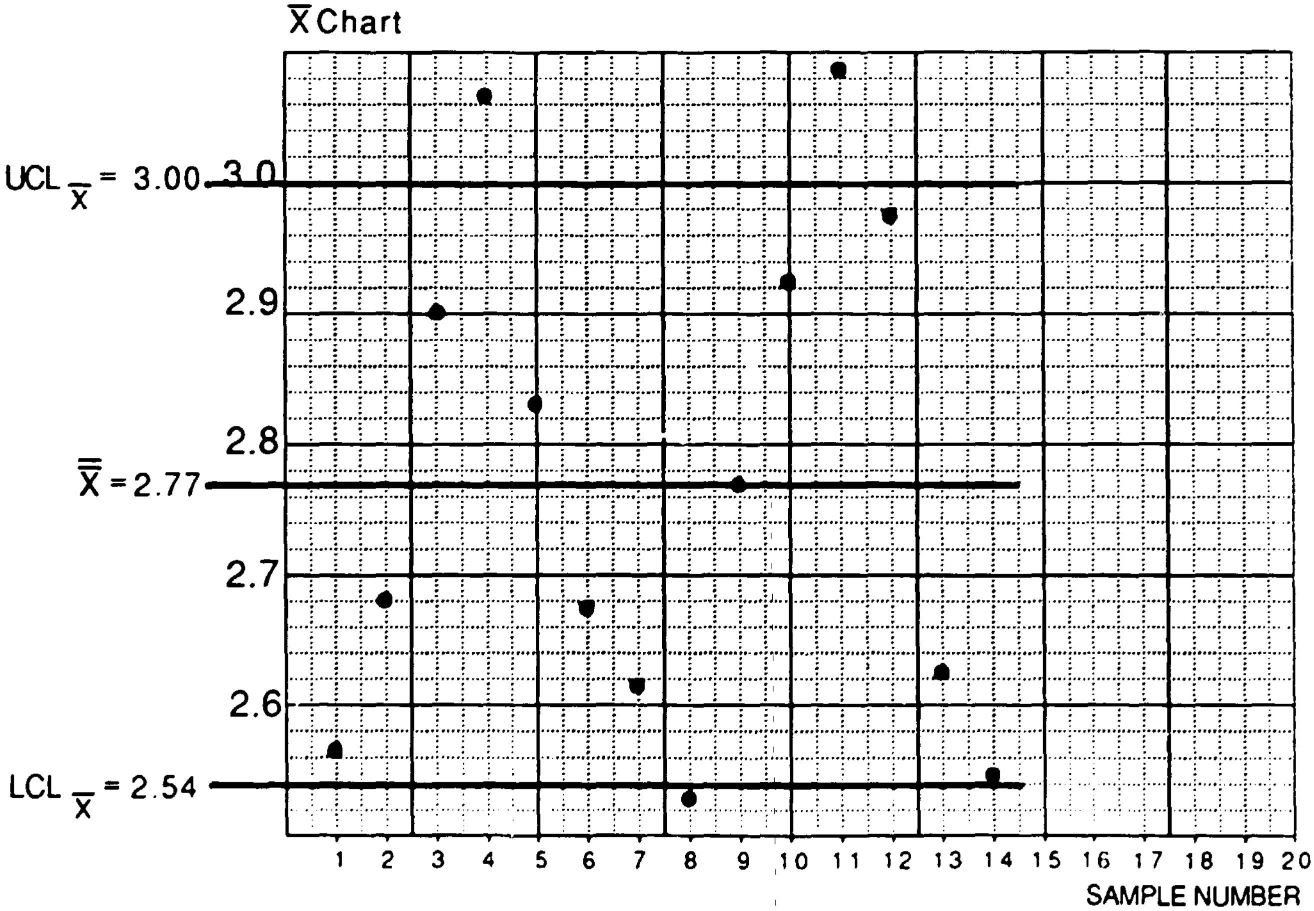
$$LCL_{\overline{X}} = \overline{\overline{X}} - A_2\overline{R}$$
$$= 2.7708 - .2269$$
$$= 2.5439$$

1.c.

4-24



R Chart
1.00
UCL R = .8314
.75
.50
R̄=.3933
.25
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
SAMPLE NUMBER



X̄ Chart
UCL X̄ = 3.00
3.0
2.9
2.8
X̿ = 2.77
2.7
2.6
LCL X̄ = 2.54
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
SAMPLE NUMBER

2.a. Range Chart

$$\text{Center Line} = \overline{R} = \frac{.151}{30} = .00503$$

$$UCL_{\overline{R}} = D_4\overline{R} = 2.574(.00503) = .0129$$

$$LCL_R = \text{None}$$

$\overline{X}$-Chart

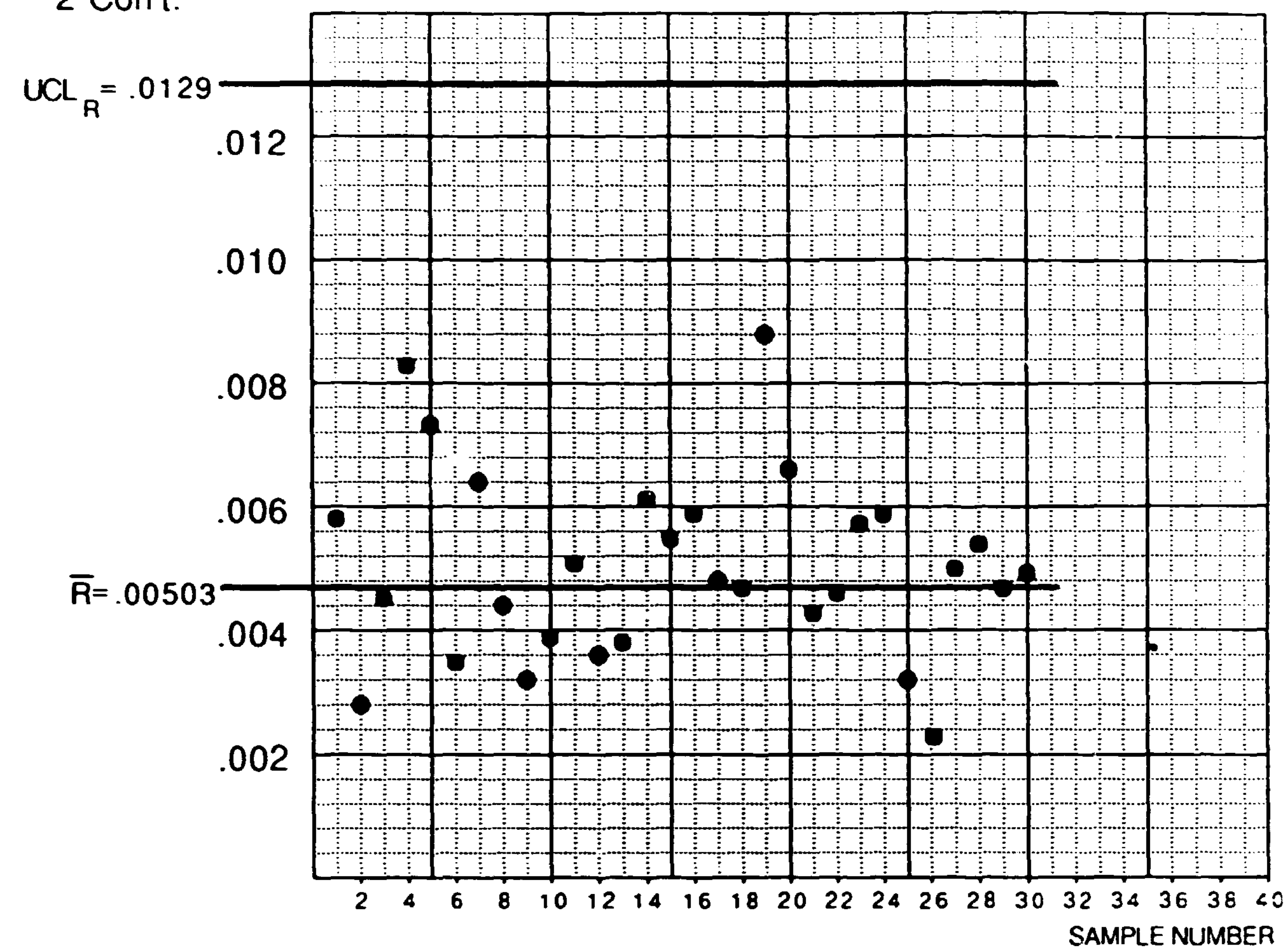
$$\text{Center Line} = \overline{\overline{X}} = \frac{1.648}{30} = .05493$$

$$\begin{aligned} UCL_{\overline{X}} &= \overline{\overline{X}} + A_2\overline{R} \\ &= .05493 + 1.023(.00503) \\ &= .05493 + .0051 \\ &= .0601 \end{aligned}$$

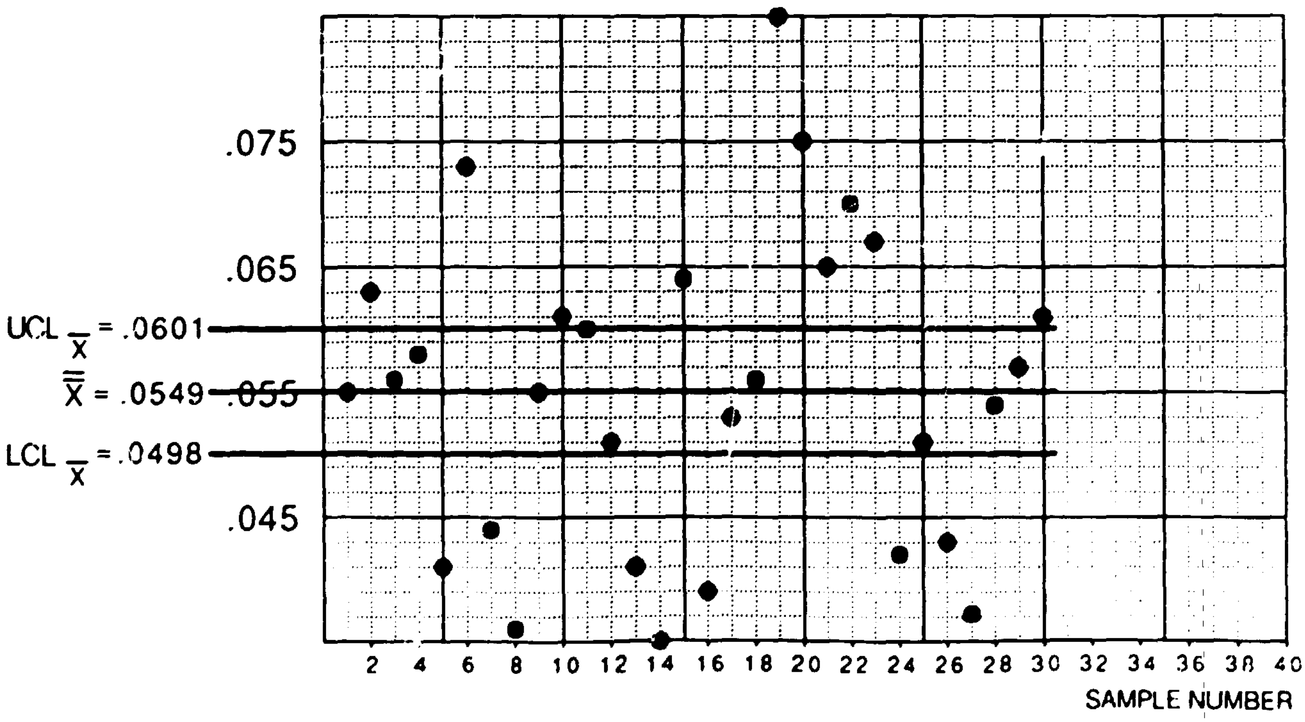
$$\begin{aligned} LCL_{\overline{X}} &= \overline{\overline{X}} - A_2\overline{R} \\ &= .05493 - .0051 \\ &= .0498 \end{aligned}$$

2 Con't.

R Chart



$\overline{X}$ Chart



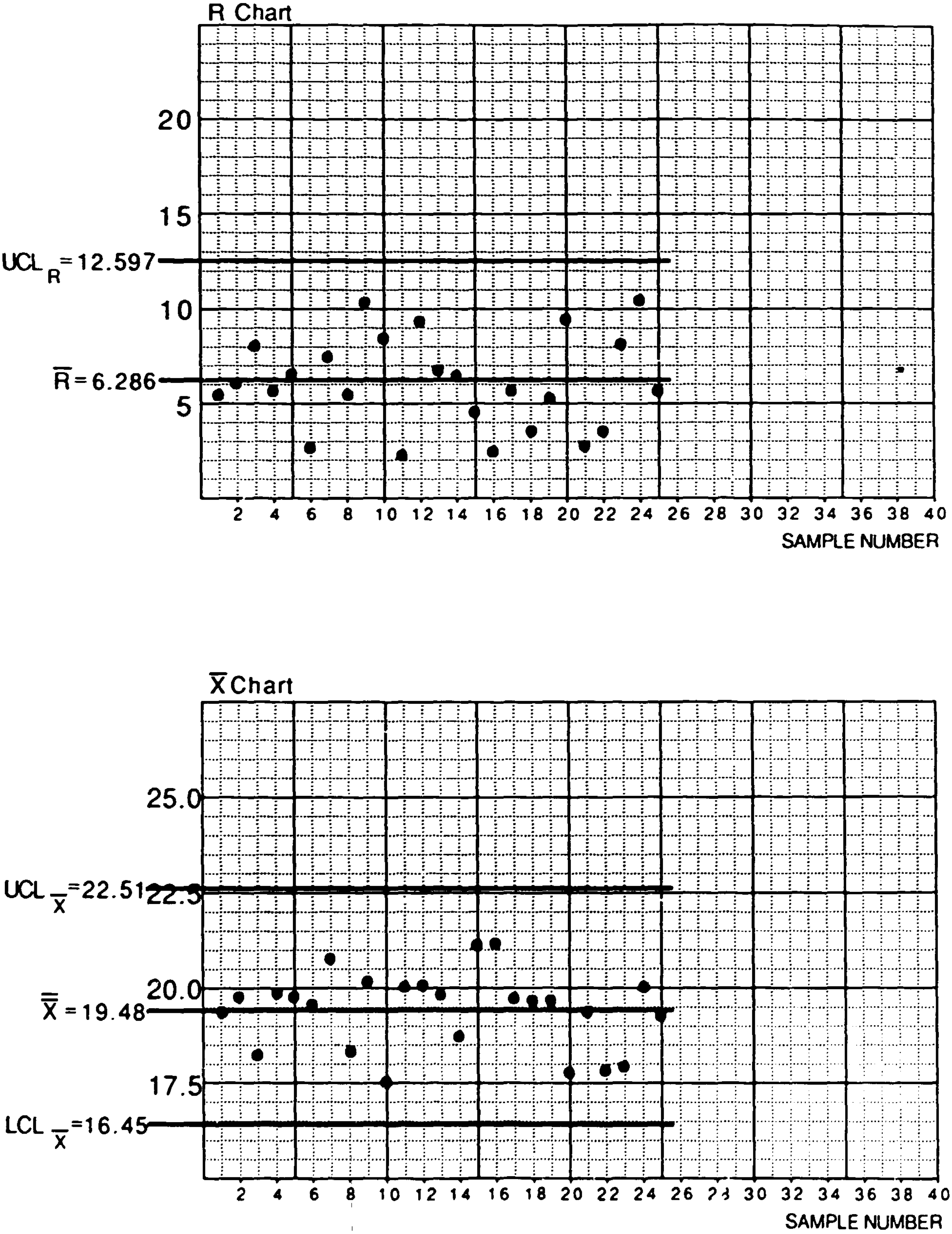
2. b. Within batch variation; i.e., nonhomogeneous batch, is reflected in the Range values.

   c. The between batch variation, or the variation due to the ingredients being different between batches, would cause the range of variation in the subgroup averages to be larger than that predicted by the control limits on the X-bar chart.

   d. The variation between the batches appears to be causing the most variation in the process.

3.

Cap Torque Measurements



3. (cont.)

a. Since the process is stable (i.e., both the R chart and the $\overline{X}$ chart are in control), it is appropriate to estimate the process standard deviation.

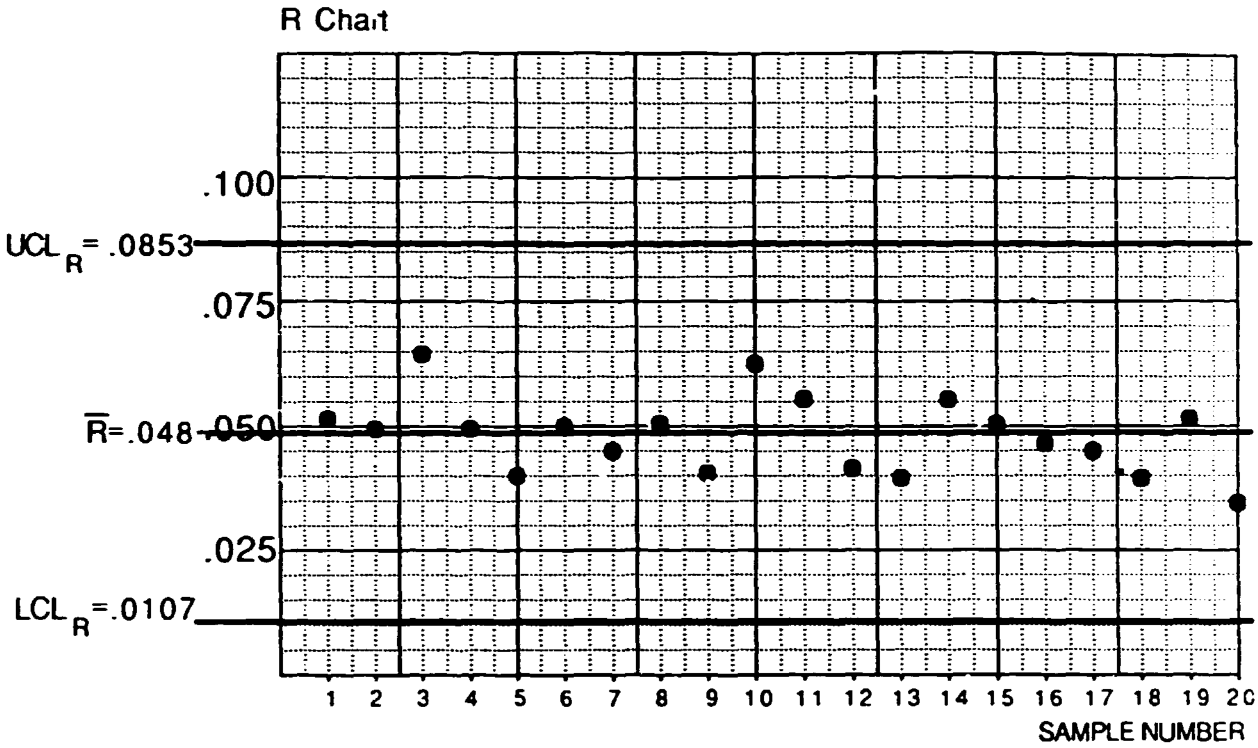
$$\hat{\sigma}_X = \overline{R}/d_2 = 6.286/2.534 = 2.481$$

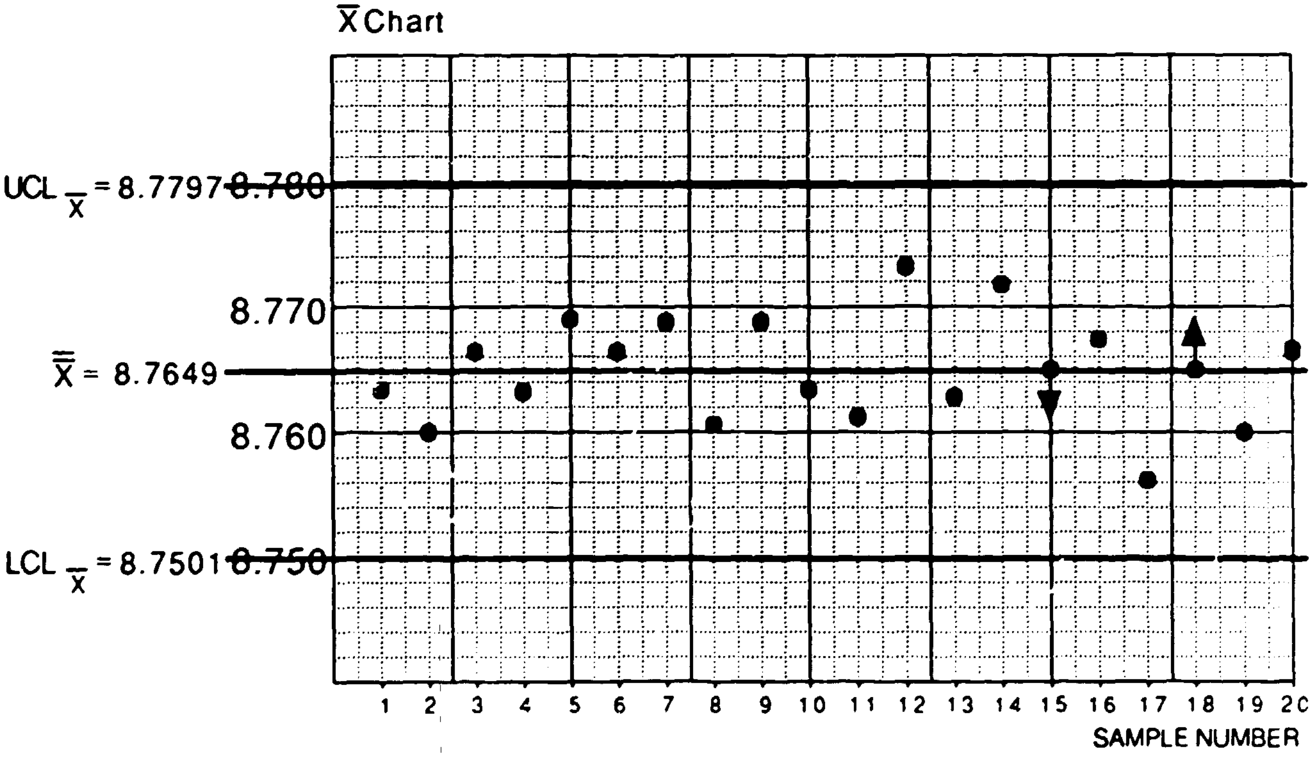
b. NT = 6(2.481) = 14.886
ET = 21 - 15 = 6

Since NT > ET, this process is not capable of meeting specifications.

4.a.

Size Data on Steel Bars





4. (cont.)

b. Estimate of Process Standard Deviation:

$$\hat{\sigma}_X = \bar{R}/d_2 = \frac{.048}{3.078} = .0156$$

NT = 6(.0156) = .0936

ET = .05

Process is not capable.

Chapter 7

1. a. Three Suggested Subgrouping Methods:

1. A subgroup is to consist of the weights of five consecutive bottles from one head.

   Sources of within subgroup variation:

   ** Within head variation-- lack of homogeneity.
   ** Bottle-to-Bottle variation within the same head.
   ** Measurement variation.

2. One bottle should be selected from each head. The subgroup consists of four weights of the same bottle from one head.

   Sources of within subgroup variation:

   ** Measurement variation.

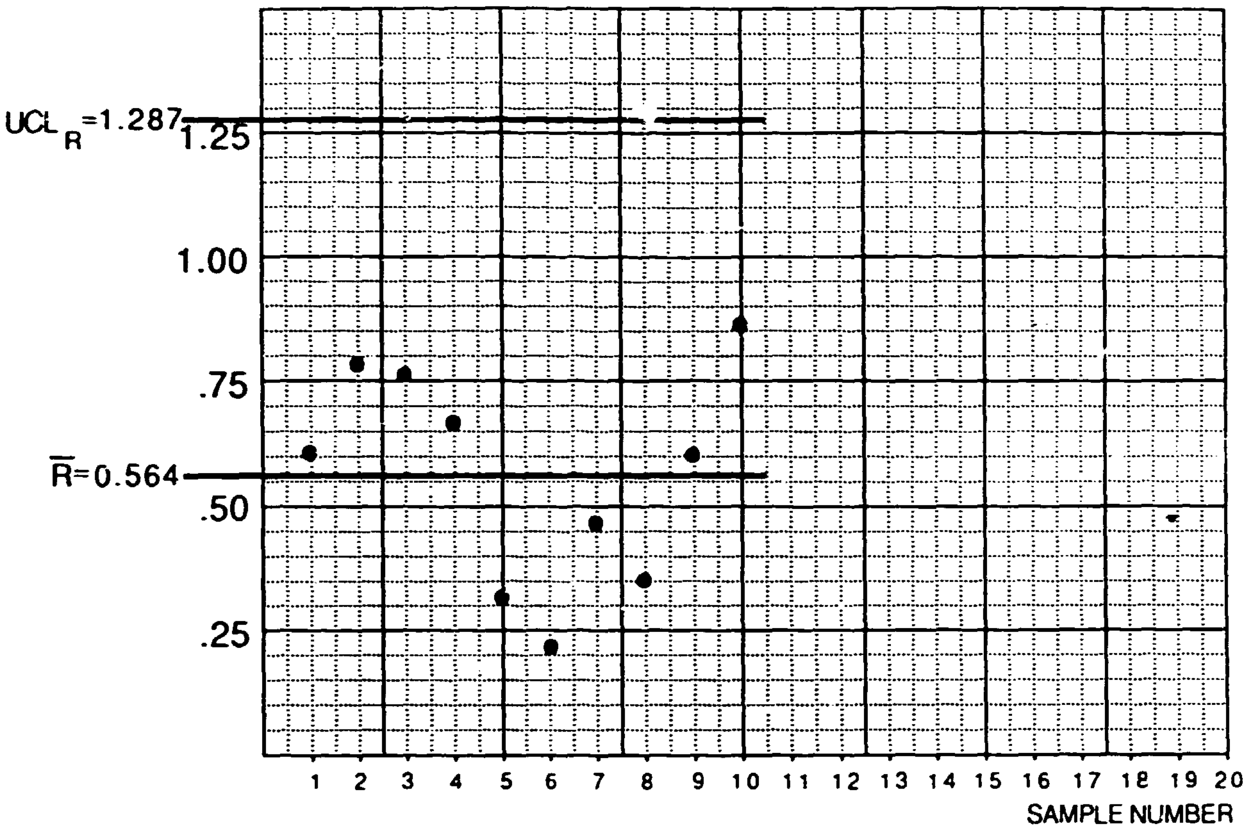
3. A subgroup should consist of one bottle selected from each of the five heads, and each bottle should be weighed once.

   Sources of within subgroup variation:

   ** Head-to-head variation.
   ** Measurement variation.
   ** Bottle-to-Bottle variation.

1. b.

R Chart



Centerline $= \overline{R} = 0.564$

$UCL_R = D_4\overline{R} = 2.282\ (0.564) = 1.287$

$LCL_R = D_3\overline{R} =$ None

c. Sources of variation captured in the R-chart:

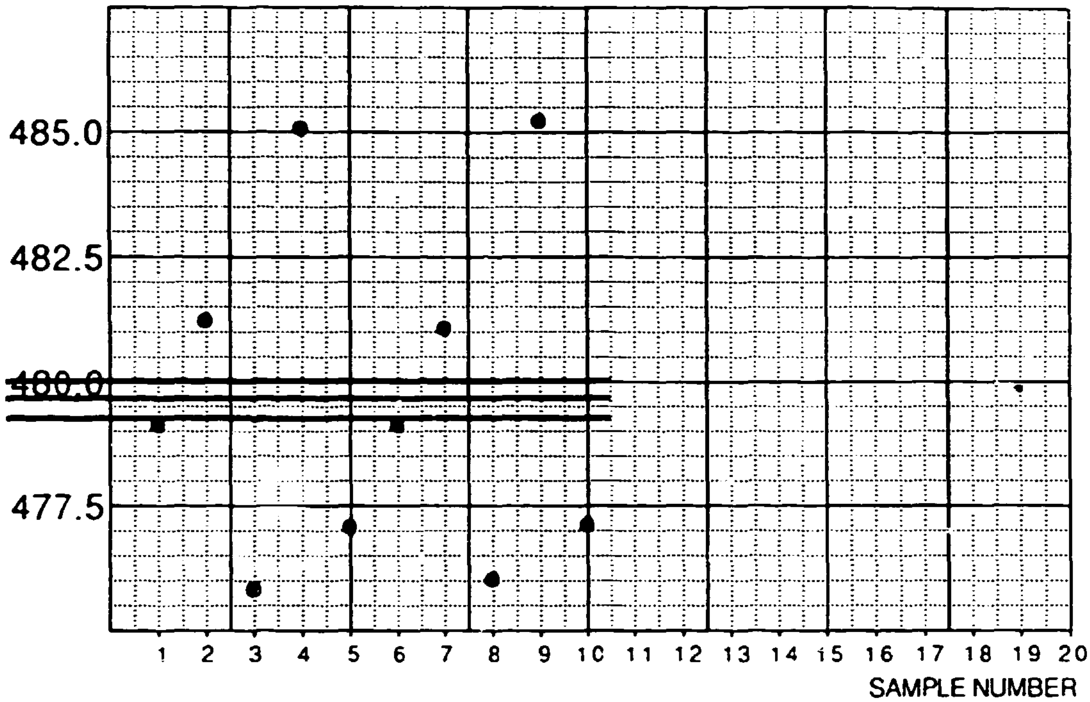
Within subgroup variation -- Measurement variation.

1d. Estimating the within subgroup variation:

Since the R-chart is in control. it is appropriate to estimate the within subgroup variation.

$$\hat{\sigma}_W = \overline{R}/d_2 = 0.564/2.059 = 0.274$$

1.e. $\overline{X}$ Chart



Central Line: $\overline{\overline{X}} = 479.695$

$$\begin{aligned} UCL_{\overline{X}} = \overline{\overline{X}} + A_2\overline{R} &= 479.695 + (0.729)(0.564) \\ &= 479.695 + .4112 \\ &= 480.1062 \end{aligned}$$

$$\begin{aligned} LCL_{\overline{X}} = \overline{\overline{X}} - A_2\overline{R} &= 479.695 - .4112 \\ &= 479.2838 \end{aligned}$$

Issue related to using the value of $\overline{R}$ to put control limits on the $\overline{X}$ chart:

Only the within subgroup variation is taken into consideration. There are other sources of variation (i.e., head-to-head, bottle-to-bottle) that should be considered a part of the system, but are not captured in the within subgroup variation.

1.f.

Sample Number	Head	1	2	3	4	$\overline{X}$	$MR_{\overline{X}}$
1	1	479.17	478.93	479.54	479.12	479.190	---
2	2	481.13	481.75	480.97	481.21	481.265	2.075
3	3	475.71	476.24	475.95	475.48	475.845	5.42
4	4	485.37	485.17	484.70	484.89	485.033	9.188
5	5	476.80	477.12	477.07	477.10	477.023	8.01
6	1	479.01	479.23	479.12	479.10	479.115	2.092
7	2	481.16	480.83	481.10	481.30	481.098	1.983
8	3	475.82	476.14	476.17	475.93	476.015	5.083
9	4	485.08	485.29	485.52	484.92	485.203	9.188
10	5	477.11	476.68	477.32	477.54	477.163	8.04
						4,796.95	51.079

Calculations for Measuring Between Subgroup Variation

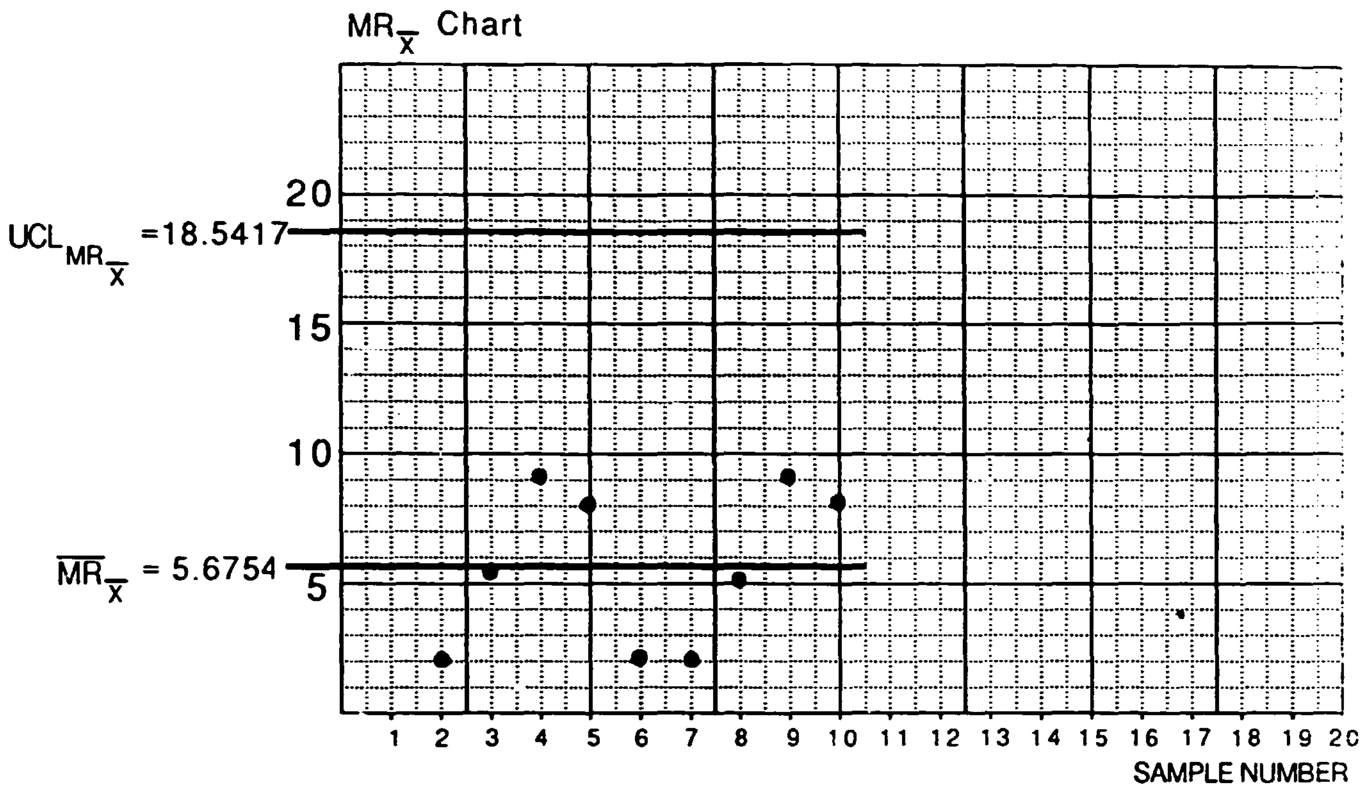
$$\overline{MR}_{\overline{X}} = \frac{\Sigma MR_{\overline{X}}}{k-1} = \frac{51.079}{9} = 5.6754$$

$$\boxed{\begin{aligned} UCL_{MR_{\overline{X}}} &= D_4 \overline{MR}_{\overline{X}} = (3.267)(5.6754) = 18.5417 \\ LCL_{MR_{\overline{X}}} &= D_3 \overline{MR}_{\overline{X}} = \text{None} \end{aligned}}$$

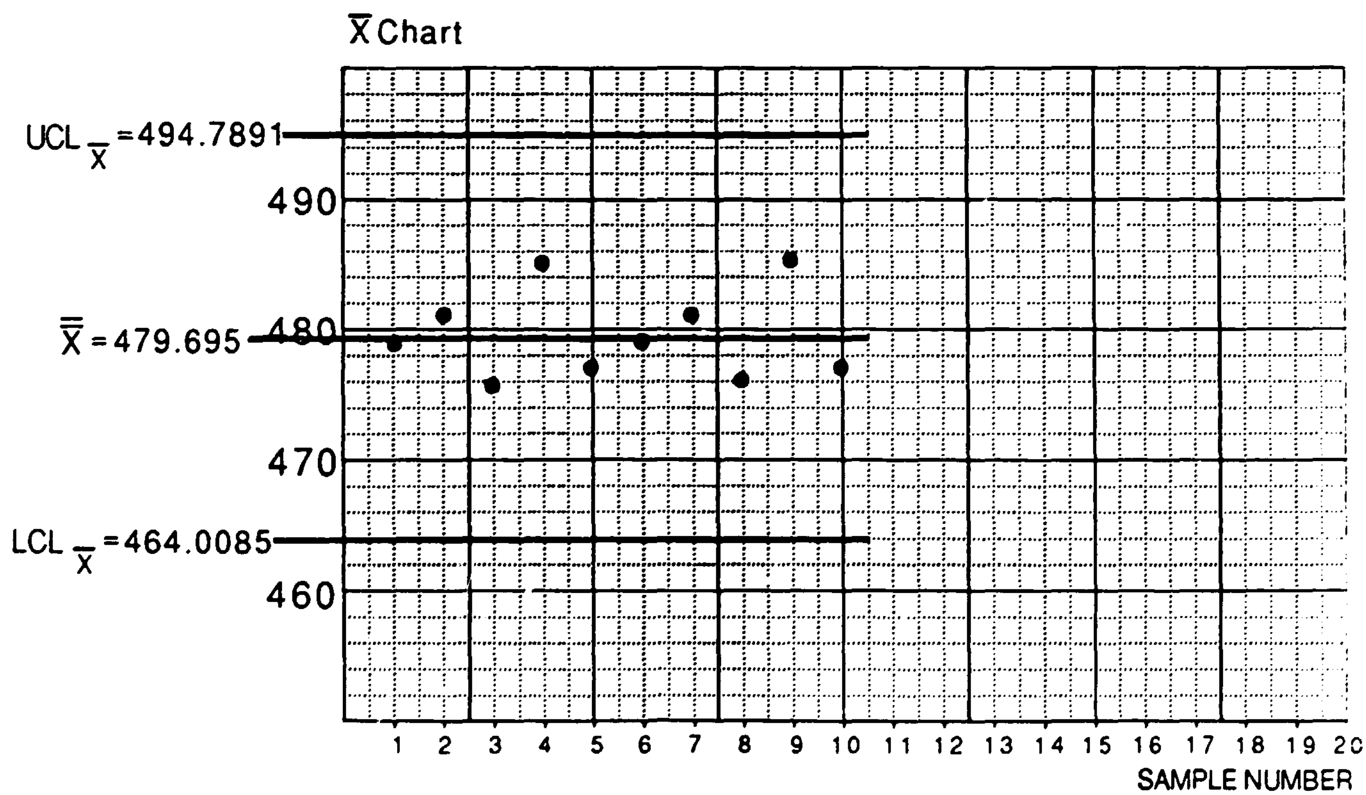
$$\overline{\overline{X}} = \frac{\Sigma \overline{X}}{k} = \frac{4796.95}{10} = 479.695$$

$$\boxed{\begin{aligned} UCL_{\overline{X}} &= \overline{\overline{X}} + 3\left(\frac{\overline{MR}_{\overline{X}}}{d_2}\right) = 479.695 + 3\left(\frac{5.6754}{1.128}\right) = 494.7891 \\ LCL_{\overline{X}} &= \overline{\overline{X}} - 3\left(\frac{\overline{MR}_{\overline{X}}}{d_2}\right) = 479.695 - 3(5.0314) = 464.0085 \end{aligned}}$$

1.f.



$MR_{\overline{X}}$ Chart
$UCL_{MR_{\overline{X}}}$ =18.5417
$\overline{MR}_{\overline{X}}$ = 5.6754
20
15
10
5
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
SAMPLE NUMBER



$\overline{X}$ Chart
$UCL_{\overline{X}}$ =494.7891
$\overline{\overline{X}}$ = 479.695
$LCL_{\overline{X}}$ =464.0085
490
480
470
460
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
SAMPLE NUMBER