



**TOGETHER**  
*for a sustainable future*

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18411

Brazil.RDS-20  
0590

Meanwhile,  
in the cross direction.....

does the profile tend to remain  
about the same over time?

*Consultant: Richard SANDERS  
Backup off. Mr. Ballance, A.D./STAT*

The subgroup range in this application reflects variation in the cross direction. It has been suggested that the value of R can be used to measure the profile across the board. Discussion should follow.

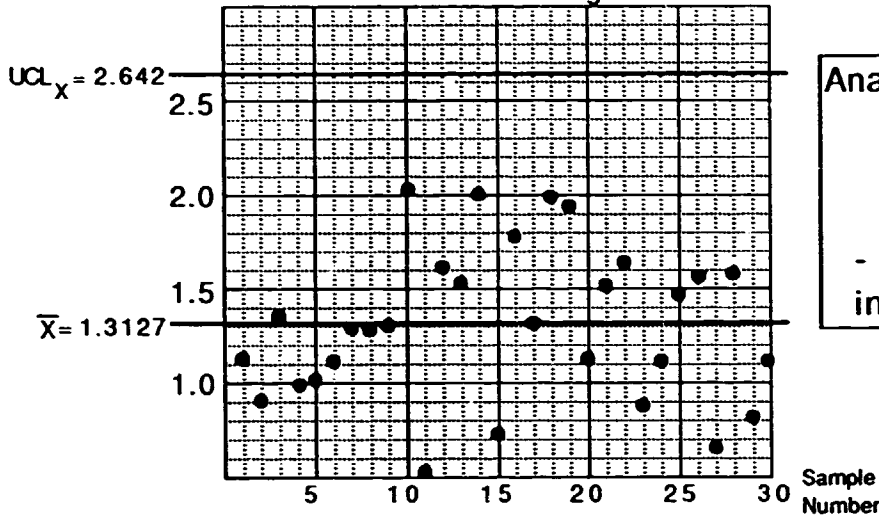
An appropriate analysis on the subgroup range values can provide some help in answering the question.

Use the 30 values of R as individual measurements on a property; at each subgroup, there is a profile and the value of R is a measure on that product/process characteristic. The plan is to use each value of R as a single point measure and apply the technique of the two-item moving range and moving average and an individuals' chart to that column of numbers.

Analysis of the Range as  
a Profile Characteristic  
-Range Used in an Individuals' Chart-

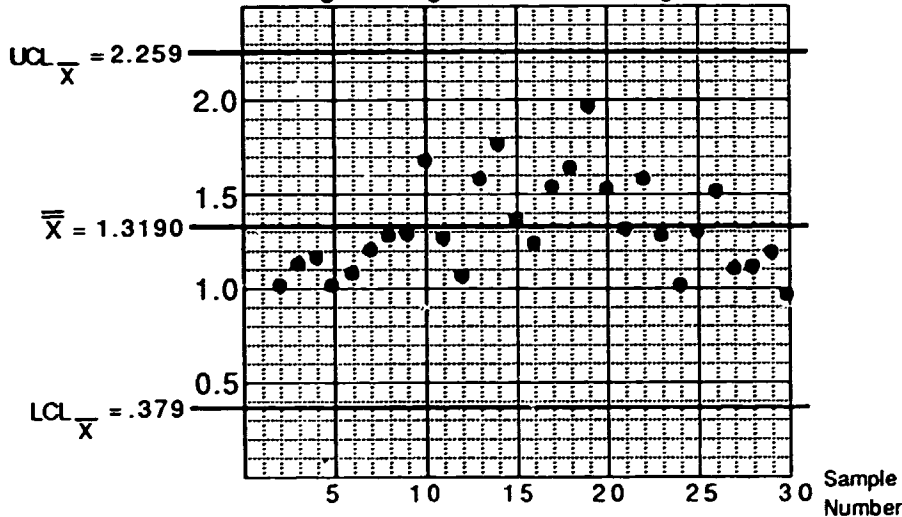
Obs	X	$X_{i-1}$	R	$\bar{X}_i$
1	1.14	.	.	.
2	0.90	1.14	0.24	1.020
3	1.36	0.90	0.46	1.130
4	0.99	1.36	0.37	1.175
5	1.03	0.99	0.04	1.010
6	1.13	1.03	0.10	1.080
7	1.29	1.13	0.16	1.210
8	1.27	1.29	0.02	1.280
9	1.30	1.27	0.03	1.285
10	2.03	1.30	0.73	1.665
11	0.52	2.03	1.51	1.275
12	1.62	0.52	1.10	1.070
13	1.54	1.62	0.08	1.580
14	2.00	1.54	0.46	1.770
15	0.73	2.00	1.27	1.365
16	1.78	0.73	1.05	1.255
17	1.32	1.78	0.46	1.550
18	1.99	1.32	0.67	1.655
19	1.95	1.99	0.04	1.970
20	1.12	1.95	0.83	1.535
21	1.51	1.12	0.39	1.315
22	1.65	1.51	0.14	1.580
23	0.88	1.65	0.77	1.265
24	1.14	0.88	0.26	1.010
25	1.46	1.14	0.32	1.300
26	1.56	1.46	0.10	1.510
27	0.66	1.56	0.90	1.110
28	1.58	0.66	0.92	1.120
29	0.81	1.58	0.77	1.195
30	1.12	0.81	0.31	0.965

Individuals' Chart for Ranges

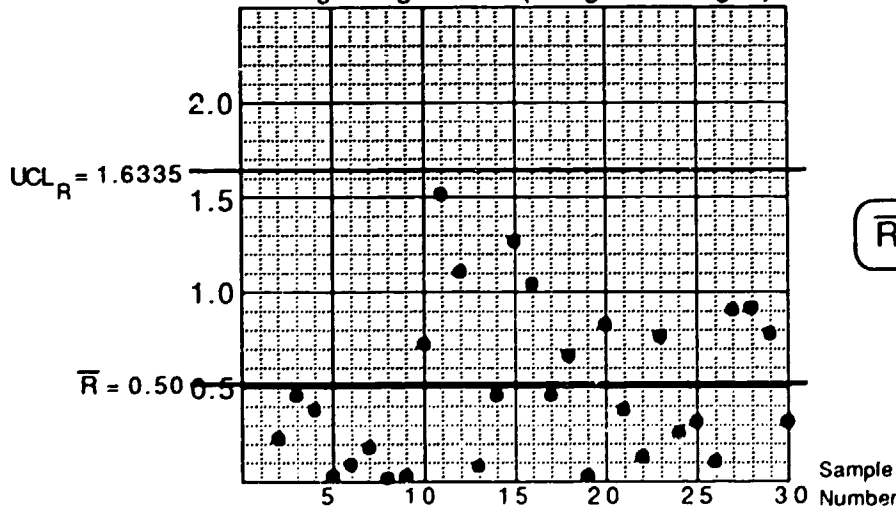


Analysis of the Range  
as a Profile  
Characteristic  
- Range used in an  
individuals' chart-

Moving Average Chart for Ranges



Moving Range Chart (Range of Ranges)

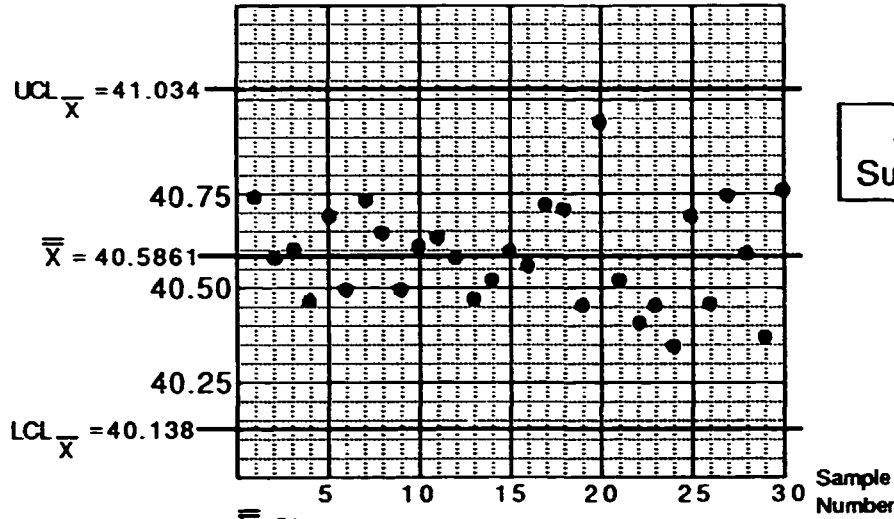


$\bar{R}/d_2 = .4432$

## Analysis of the Subgroup Averages

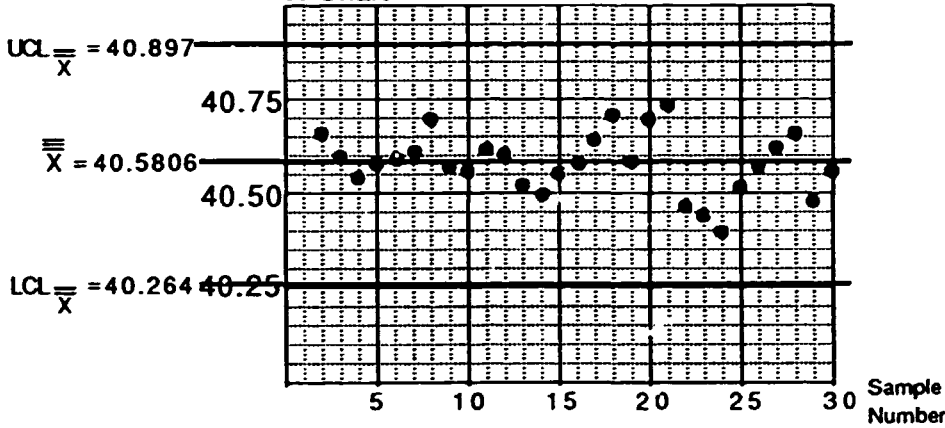
Obs	$\bar{X}$	$\bar{X}_{i-1}$	$R_{\bar{X}}$	$\bar{\bar{X}}_i$
1	40.7400	.	.	.
2	40.5875	40.7400	0.1525	40.6638
3	40.6025	40.5875	0.0150	40.5950
4	40.4700	40.6025	0.1325	40.5362
5	40.6825	40.4700	0.2125	40.5763
6	40.4950	40.6825	0.1875	40.5887
7	40.7325	40.4950	0.2375	40.6137
8	40.6475	40.7325	0.0850	40.6900
9	40.4975	40.6475	0.1500	40.5725
10	40.6075	40.4975	0.1100	40.5525
11	40.6350	40.6075	0.0275	40.6213
12	40.5775	40.6350	0.0575	40.6063
13	40.4725	40.5775	0.1050	40.5250
14	40.5225	40.4725	0.0500	40.4975
15	40.5950	40.5225	0.0725	40.5588
16	40.5600	40.5950	0.0350	40.5775
17	40.7250	40.5600	0.1650	40.6425
18	40.7075	40.7250	0.0175	40.7163
19	40.4525	40.7075	0.2550	40.5800
20	40.9450	40.4525	0.4925	40.6988
21	40.5225	40.9450	0.4225	40.7338
22	40.4050	40.5225	0.1175	40.4638
23	40.4550	40.4050	0.0500	40.4300
24	40.3425	40.4550	0.1125	40.3987
25	40.6900	40.3425	0.3475	40.5162
26	40.4575	40.6900	0.2325	40.5738
27	40.7475	40.4575	0.2900	40.6025
28	40.5825	40.7475	0.1650	40.6650
29	40.3725	40.5825	0.2100	40.4775
30	40.7525	40.3725	0.3800	40.5625

$\bar{X}$  Chart

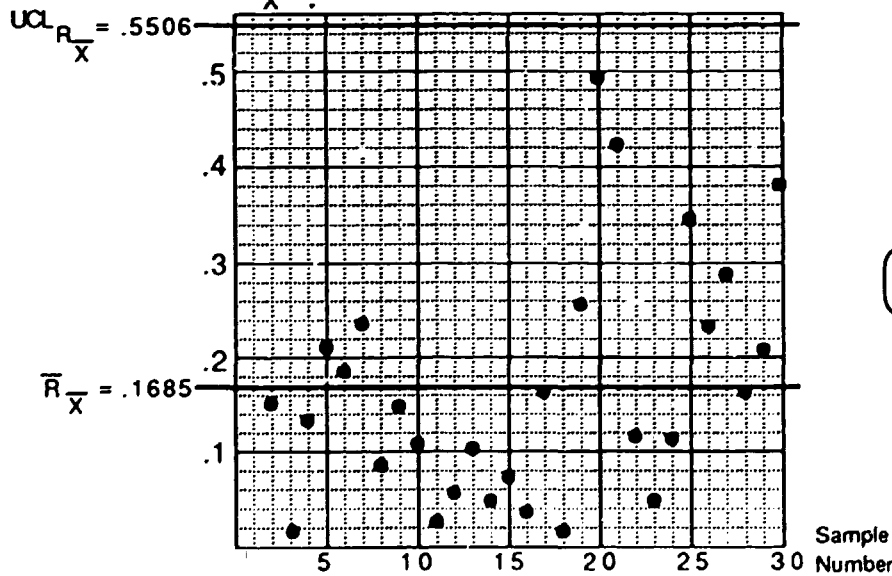


Analysis of the  
Subgroup Averages

$\bar{X}$  Chart



$R_{\bar{X}}$  Chart



$\bar{R}_{\bar{X}}/d_2 = .1494$

## Subgrouping Issues: Coil, Ribbon, or Sheet

In the paper, aluminum, steel, glass, fiberboard industries and others, measurements on caliper, density, or other product characteristics are frequently made across the coil, sheet, or ribbon and in the machine direction. These numbers require analysis as to variation, average, stability, and capability.

The discussion and examples that follow are meant to address some of the issues which are important in evaluating these and similar processes.

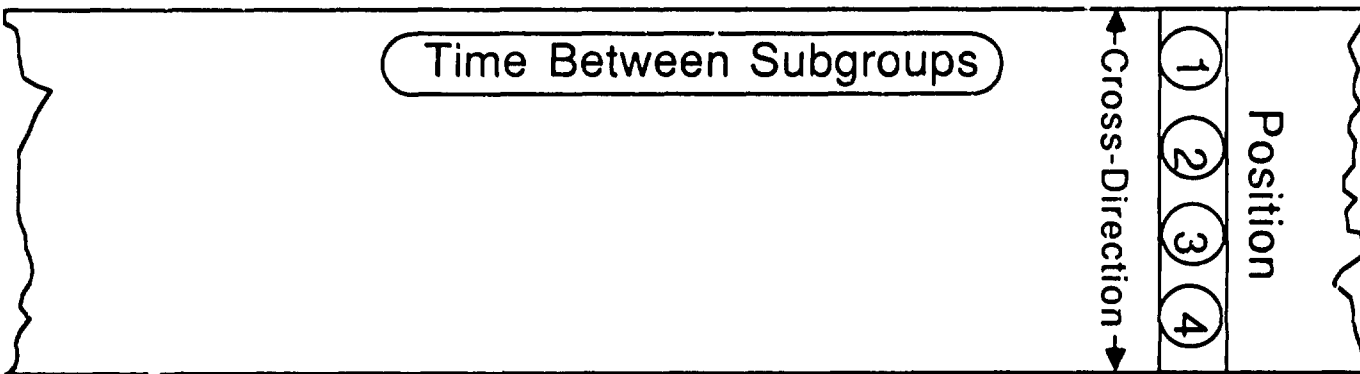




materials, equipment, shifts



Sampling Interval



The data set shown on a following page contains information on the following:

Positions, numbered 1, 2, 3, and 4, in the cross direction.

Variation in these numbers reflect things "happening" at the same time.

Subgroups or groups, 30 in this example, in the machine direction.

Variation in these numbers, for any one position, represent the effect of things "happening" at different times:

material changes,  
equipment changes or differences

Measurements are made in the cross direction in order to evaluate any positional differences that might exist, either in the variability, average, or both. In some products, metal rolling for example, specific positional differences are engineered into the process.

The amount of time or product flow between subgroups in the machine direction for purpose of product analysis may need to be explored. The subgroups should not be so close together that the numbers at any one position differ only by measurement variability; neither should they be so far apart that it becomes impossible to identify the reasons as to why the numbers might vary or be out of control. Spacing should be selected to obtain an estimate of process variation or to estimate cyclic patterns that may exist.

The data set shown on the next page contains measurements on caliper on particle board that have been collected by a work group as part of ongoing process studies. Some of the objectives of this work has to do with evaluations as to the variability and average of caliper. The boards are supposed to be 'flat,' that is, caliper should have the same value in both the machine direction and in the cross direction.

There are concerns regarding an appropriate analysis of this data. A strong-minded and dominant individual has proposed that the four measurements on caliper made in the cross direction at the same time be used as the subgroup for SPC analysis. Using that proposal, report on

- 1) the range and average for the first several subgroups, and
- 2) the control charts for range and average, and
- 3) how this method of subgrouping relates to items on the cause/effect diagram.

Finally, do you consider the above analysis to be appropriate? What questions or concerns affected your judgement and decision?

## Caliper Four Measurements in the Cross Direction

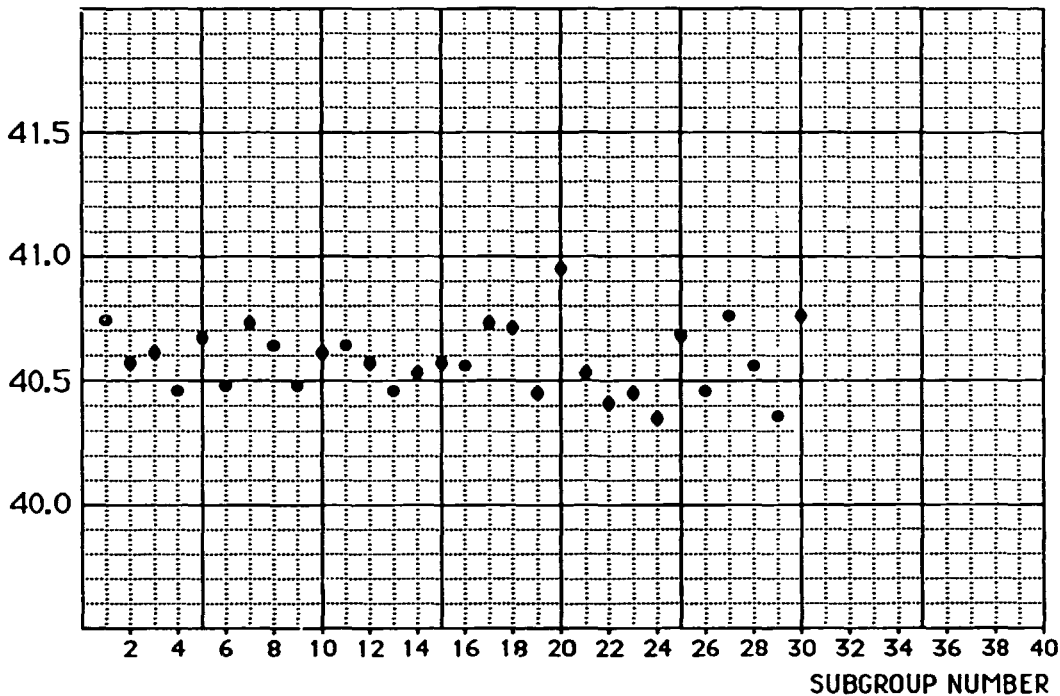
Subgroup	$X_1$	$X_2$	$X_3$	$X_4$	R	$\bar{X}$
1	41.18	40.54	40.05	41.19		
2	40.68	40.55	40.01	40.91		
3	41.11	39.82	40.30	41.18		
4	40.93	40.27	39.94	40.74		
5	40.87	40.21	40.41	41.24	1.03	40.6825
6	40.59	40.16	40.05	41.18	1.13	40.4950
7	41.04	40.46	40.07	41.36	1.29	40.7325
8	41.08	39.97	40.30	41.24	1.27	40.6475
9	41.04	40.15	39.75	41.05	1.30	40.4975
10	40.80	40.42	39.59	41.62	2.03	40.6075
11	40.67	40.53	40.41	40.93	0.52	40.6350
12	41.14	39.52	40.80	40.85	1.62	40.5775
13	41.15	40.26	39.61	40.87	1.54	40.4725
14	41.48	39.48	40.14	40.99	2.00	40.5225
15	40.82	40.25	40.33	40.98	0.73	40.5950
16	41.04	39.62	40.18	41.40	1.78	40.5600
17	40.82	40.56	40.10	41.42	1.32	40.7250
18	41.42	39.43	40.73	41.25	1.99	40.7075
19	41.17	39.23	40.23	41.18	1.95	40.4525
20	41.23	40.35	40.73	41.47	1.12	40.9450
21	41.35	39.84	39.98	40.92	1.51	40.5225
22	40.93	39.28	40.62	40.79	1.65	40.4050
23	40.61	40.05	40.23	40.93	0.88	40.4550
24	40.76	39.93	39.77	40.91	1.14	40.3425
25	41.14	40.20	39.98	41.44	1.46	40.6900
26	41.04	39.72	39.79	41.28	1.56	40.4575
27	40.86	40.32	40.98	40.83	0.66	40.7475
28	41.47	39.89	39.99	40.98	1.58	40.5825
29	40.49	40.15	40.02	40.83	0.81	40.3725
30	41.20	40.08	40.79	40.94	1.12	40.7525
					$\Sigma R = 39.38$	$\Sigma \bar{X} = 1217.58$

## Caliper Four Measurements in the Cross Direction

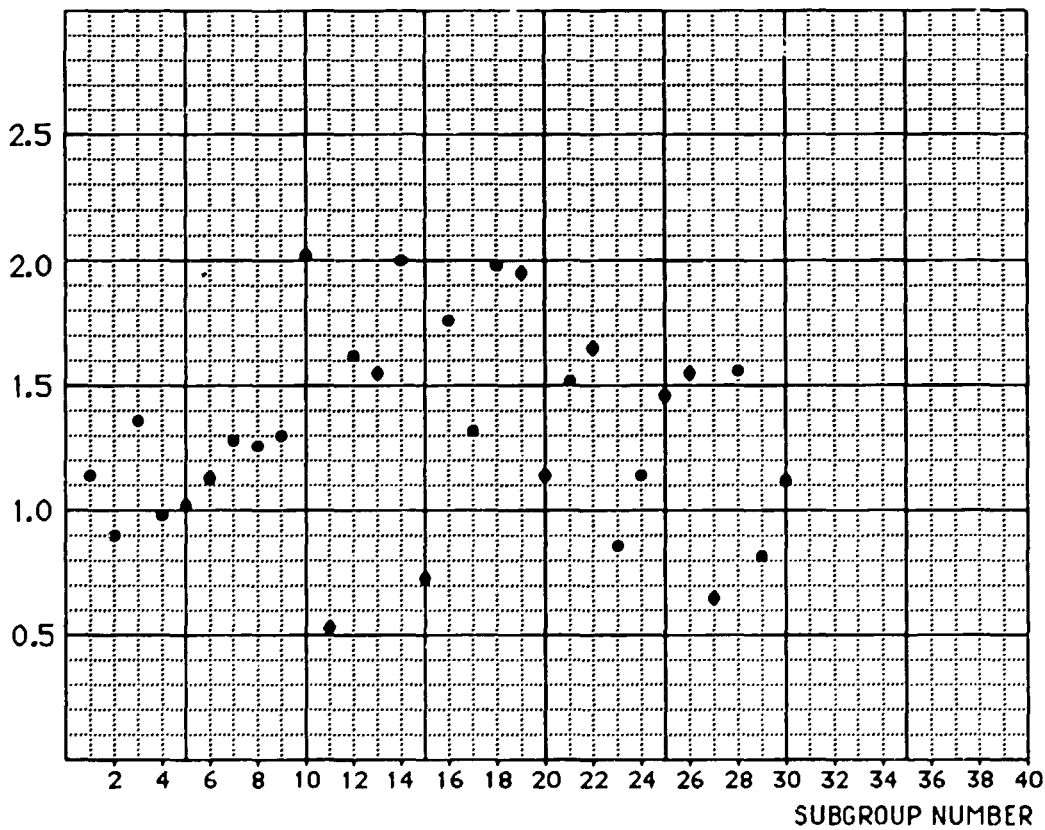
Subgroup	$X_1$	$X_2$	$X_3$	$X_4$	R	$\bar{X}$
1	41.18	40.54	40.05	41.19	1.14	40.7400
2	40.88	40.55	40.01	40.91	0.90	40.5875
3	41.11	39.82	40.30	41.18	1.36	40.6025
4	40.93	40.27	39.94	40.74	0.99	40.4700
5	40.87	40.21	40.41	41.24	1.03	40.6825
6	40.59	40.16	40.05	41.18	1.13	40.4950
7	41.04	40.46	40.07	41.36	1.29	40.7325
8	41.08	39.97	40.30	41.24	1.27	40.6475
9	41.04	40.15	39.75	41.05	1.30	40.4975
10	40.80	40.42	39.59	41.62	2.03	40.6075
11	40.67	40.53	40.41	40.93	0.52	40.6350
12	41.14	39.52	40.80	40.85	1.62	40.5775
13	41.15	40.26	39.61	40.87	1.54	40.4725
14	41.48	39.48	40.14	40.99	2.00	40.5225
15	40.82	40.25	40.33	40.98	0.73	40.5950
16	41.04	39.62	40.18	41.40	1.78	40.5600
17	40.82	40.56	40.10	41.42	1.32	40.7250
18	41.42	39.43	40.73	41.25	1.99	40.7075
19	41.17	39.23	40.23	41.18	1.95	40.4525
20	41.23	40.35	40.73	41.47	1.12	40.9450
21	41.35	39.84	39.98	40.92	1.51	40.5225
22	40.93	39.28	40.62	40.79	1.65	40.4050
23	40.61	40.05	40.23	40.93	0.88	40.4550
24	40.76	39.93	39.77	40.91	1.14	40.3425
25	41.14	40.20	39.98	41.44	1.46	40.6900
26	41.04	39.72	39.79	41.28	1.56	40.4575
27	40.86	40.32	40.98	40.83	0.66	40.7475
28	41.47	39.89	39.99	40.98	1.58	40.5825
29	40.49	40.15	40.02	40.83	0.81	40.3725
30	41.20	40.08	40.79	40.94	1.12	40.7525
					$\Sigma R = 39.38$	$\Sigma \bar{X} = 1217.58$

Caliper  
Four Measurements in the  
Cross Direction

$\bar{X}$  Chart

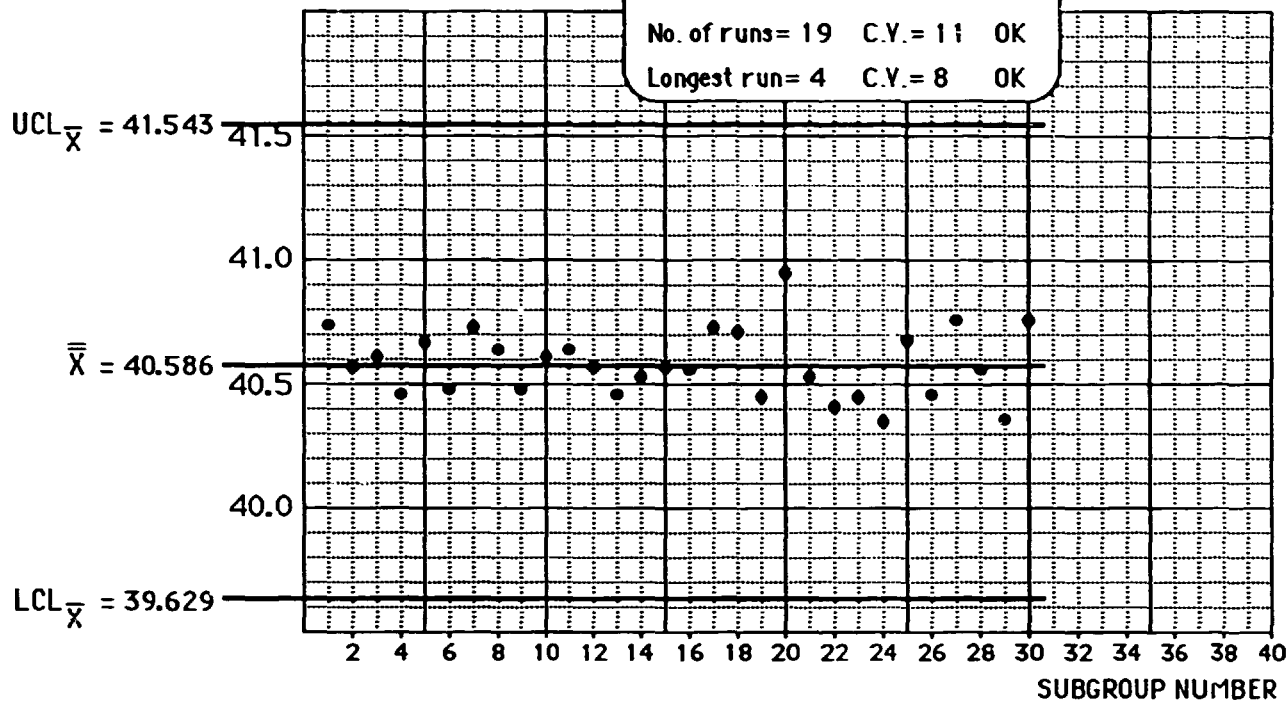


R Chart



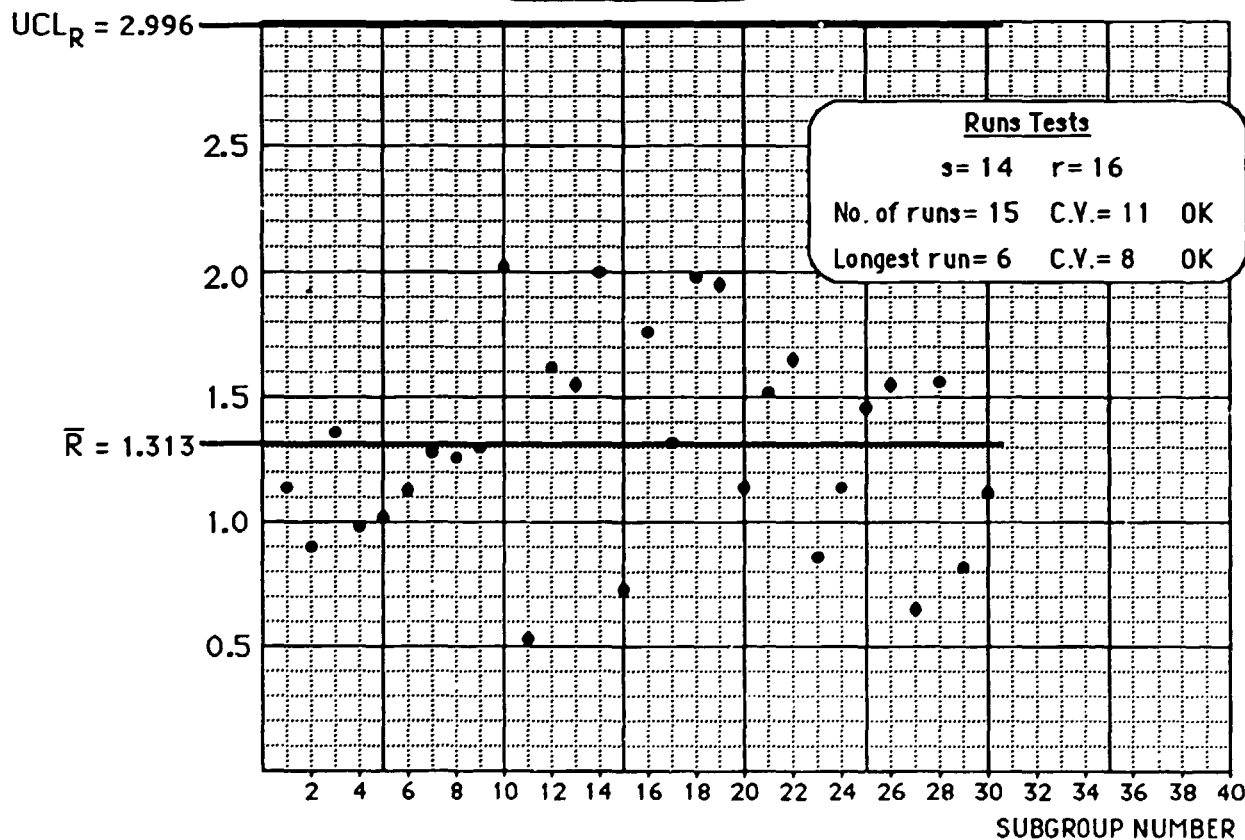
$\bar{X}$  Chart

**Runs Tests**  
 $s = 15$   $r = 15$   
 No. of runs = 19 C.V. = 11 OK  
 Longest run = 4 C.V. = 8 OK



R Chart

$\bar{R}/d_2 = .6375$



**Runs Tests**  
 $s = 14$   $r = 16$   
 No. of runs = 15 C.V. = 11 OK  
 Longest run = 6 C.V. = 8 OK

For the 30 averages,

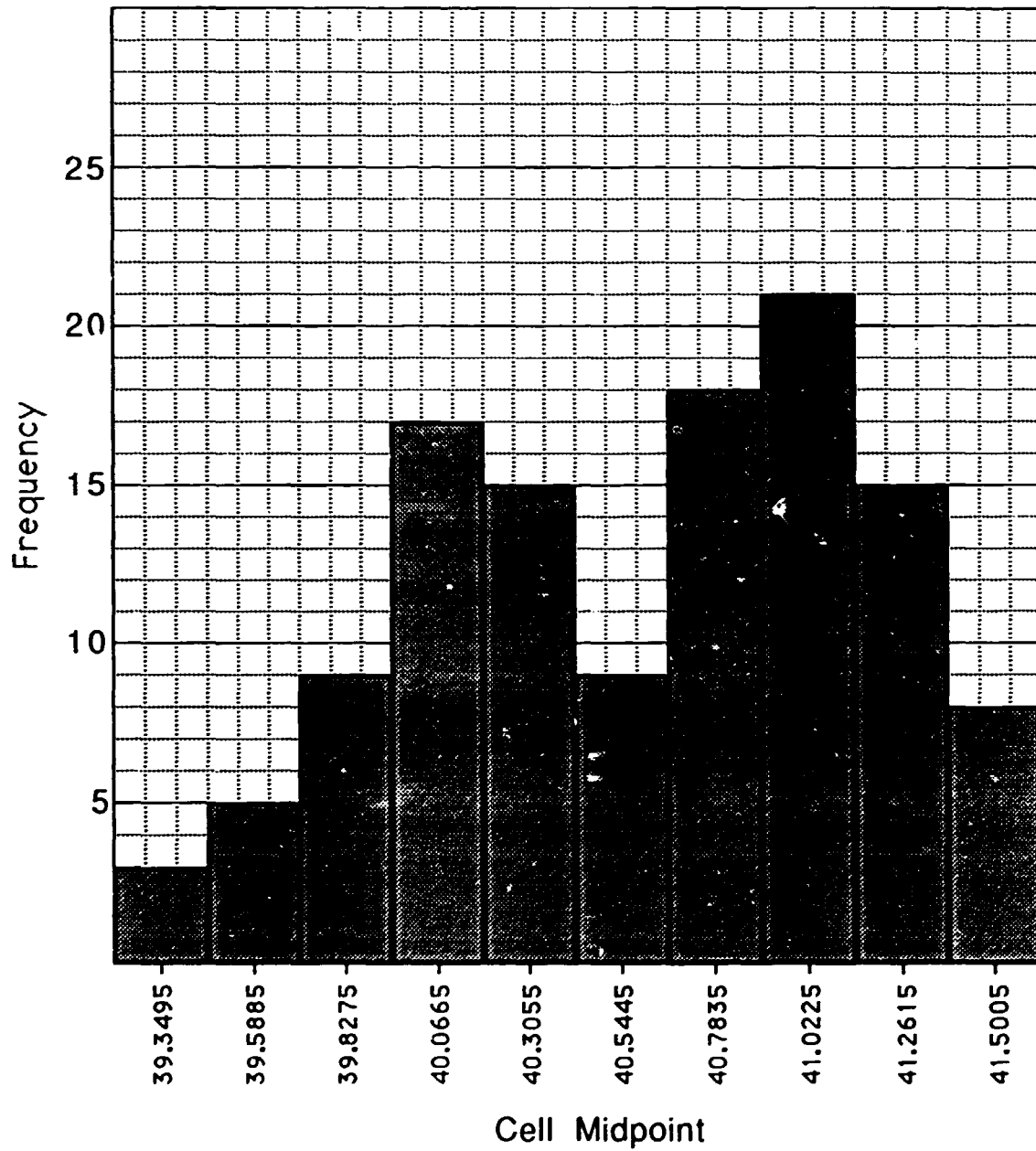
$$\bar{\bar{X}} = 40.5861 \quad s_{\bar{X}} = .135122$$

For all 120 observations,

$$\bar{X} = 40.5861 \quad s = .567855$$



### Histogram for Caliper





It has been determined that in general the previous method of subgrouping and analysis is inappropriate.

These same measurements, however, can be used in a different arrangement to provide some useful information about the output from this process.

The analysis is broken down into several parts or stages:

1. Each position is analyzed individually.

**The 30 observations at each position are to be evaluated by using a two-item moving range and average chart and an individuals' or X chart. You will be asked to help in that analysis.**

2. It is of interest to decide if the individual positions have the same variation in the machine direction.

**This analysis requires that each position be in control with respect to the moving range chart reported on in the first step.**

3. The positions may also be compared as to their respective averages.

**Stability as to individual variations is required. Having control on variation, the position averages can be examined.**

4. It can very frequently be useful to be able to determine if the profile in the cross direction remains stable over time.

**Each subgroup of 4 in the cross direction has a value for range. The range can be taken as a measurement of the profile. A rough indicator as to the stability of the profile can be had by treating each value of R as an individual measurement or observation, that is, a value of X, and performing the usual analysis.**

5. Lastly, it may be useful at times to evaluate the subgroup averages, where the subgroups contain observations in the cross direction.

5. Lastly, it may be useful at times to evaluate the subgroup averages, where the subgroups contain observations in the cross direction.

**Each value of the subgroup average is taken to be a single value; analysis is, again, to be by an individuals' chart.**

The values for range have been computed and reported. Those values were in control for the respective positions.

To compare the range values for the different positions, the average range is reported and upper control limit for subgroup size 2 is computed.

## Position 1

Obs	X	$X_{i-1}$	R	$\bar{X}_i$
1	41.18	.		
2	40.88	41.18		
3	41.11	40.88		
4	40.93	41.11		
5	40.87	40.93	0.06	40.900
6	40.59	40.87	0.28	40.730
7	41.04	40.59	0.45	40.815
8	41.08	41.04	0.04	41.060
9	41.04	41.08	0.04	41.060
10	40.80	41.04	0.24	40.920
11	40.67	40.80	0.13	40.735
12	41.14	40.67	0.47	40.905
13	41.15	41.14	0.01	41.145
14	41.48	41.15	0.33	41.315
15	40.82	41.48	0.66	41.150
16	41.04	40.82	0.22	40.930
17	40.82	41.04	0.22	40.930
18	41.42	40.82	0.60	41.120
19	41.17	41.42	0.25	41.295
20	41.23	41.17	0.06	41.200
21	41.35	41.23	0.12	41.290
22	40.93	41.35	0.42	41.140
23	40.61	40.93	0.32	40.770
24	40.76	40.61	0.15	40.685
25	41.14	40.76	0.38	40.950
26	41.04	41.14	0.10	41.090
27	40.86	41.04	0.18	40.950
28	41.47	40.86	0.61	41.165
29	40.49	41.47	0.98	40.980
30	<u>41.20</u>	40.49	<u>0.71</u>	<u>40.845</u>
	$\Sigma X = 1230.31$		$\Sigma R = 8.74$	$\Sigma \bar{X} = 1189.12$

### Position 1

Obs	X	$X_{i-1}$	R	$\bar{X}_i$
1	41.18	.	.	.
2	40.88	41.18	0.30	41.030
3	41.11	40.88	0.23	40.995
4	40.93	41.11	0.18	41.020
5	40.87	40.93	0.06	40.900
6	40.59	40.87	0.28	40.730
7	41.04	40.59	0.45	40.815
8	41.08	41.04	0.04	41.060
9	41.04	41.08	0.04	41.060
10	40.80	41.04	0.24	40.920
11	40.67	40.80	0.13	40.735
12	41.14	40.67	0.47	40.905
13	41.15	41.14	0.01	41.145
14	41.48	41.15	0.33	41.315
15	40.82	41.48	0.66	41.150
16	41.04	40.82	0.22	40.930
17	40.82	41.04	0.22	40.930
18	41.42	40.82	0.60	41.120
19	41.17	41.42	0.25	41.295
20	41.23	41.17	0.06	41.200
21	41.35	41.23	0.12	41.290
22	40.93	41.35	0.42	41.140
23	40.61	40.93	0.32	40.770
24	40.76	40.61	0.15	40.685
25	41.14	40.76	0.38	40.950
26	41.04	41.14	0.10	41.090
27	40.86	41.04	0.18	40.950
28	41.47	40.86	0.61	41.165
29	40.49	41.47	0.98	40.980
30	<u>41.20</u>	40.49	<u>0.71</u>	<u>40.845</u>
	$\Sigma X = 1230.31$		$\Sigma R = 8.74$	$\Sigma \bar{X} = 1189.12$

$$\bar{R} = .3014$$

$$UCL_R = .9846$$

$$\bar{R}/d_2 = .2672$$

$$\bar{X} = 41.0103$$

$$UCL_X = 41.812$$

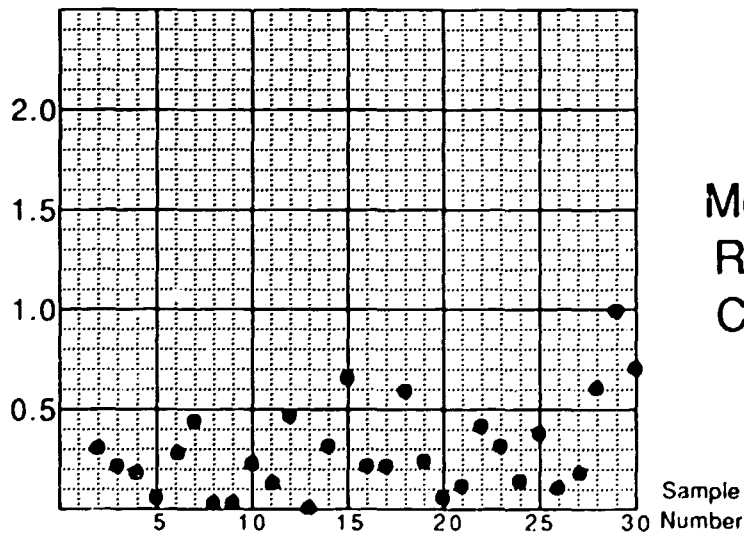
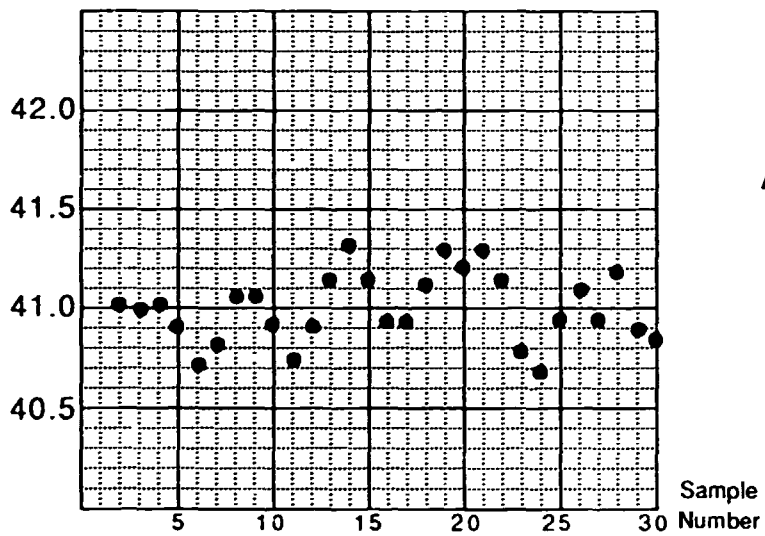
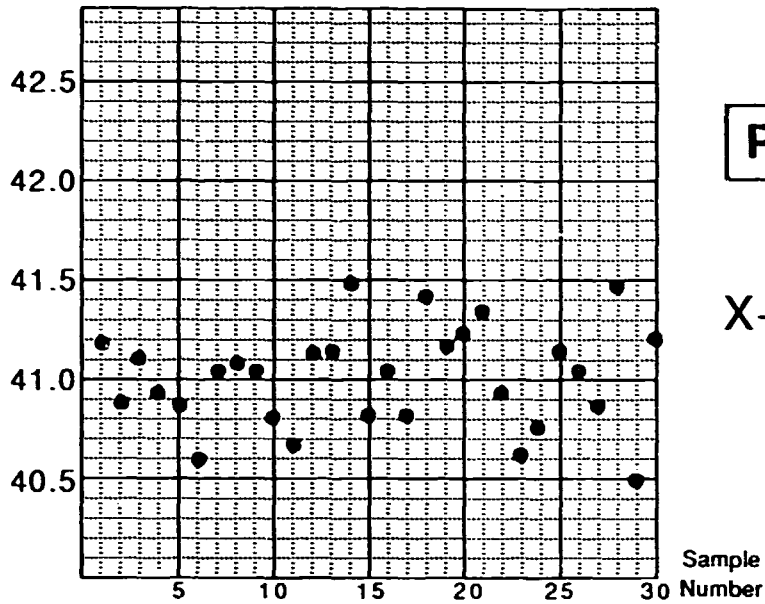
$$LCL_X = 40.209$$

$$\bar{\bar{X}} = 41.0041$$

$$UCL_{\bar{X}} = 41.571$$

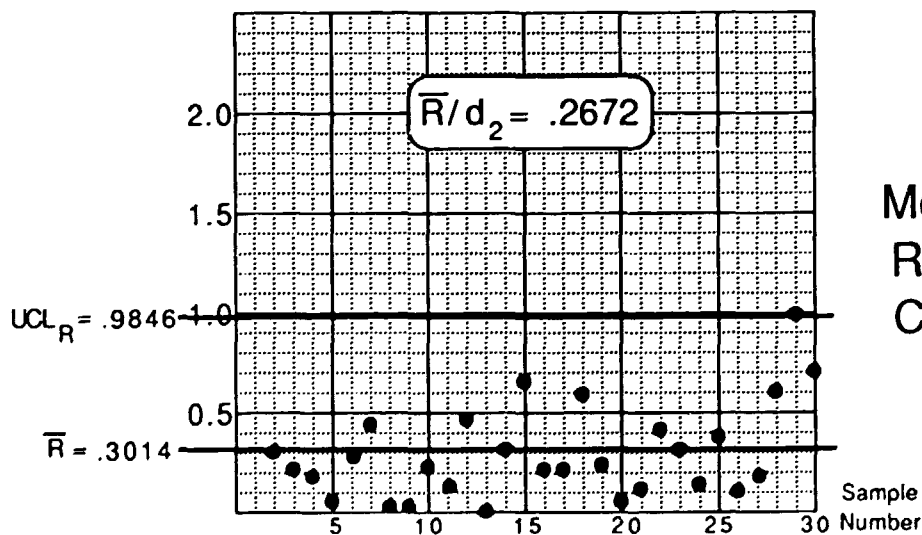
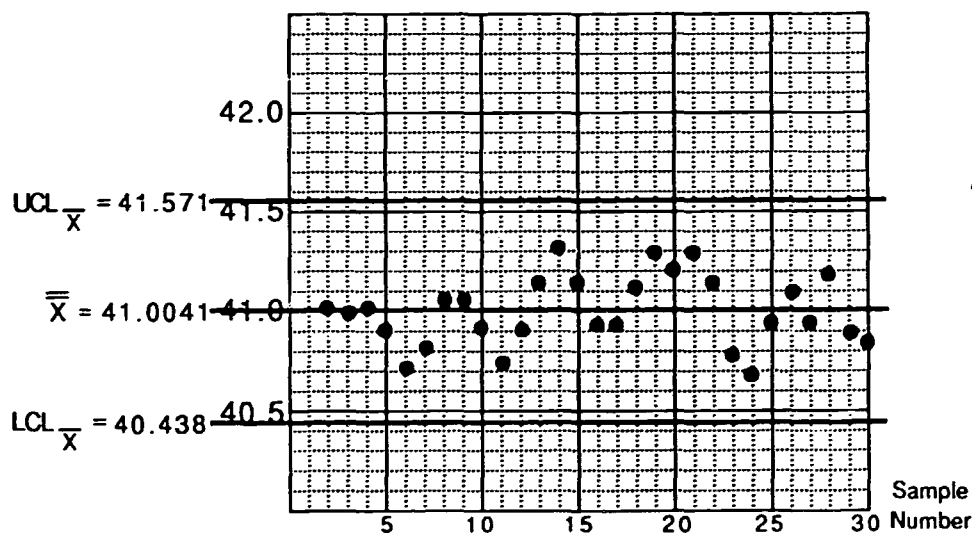
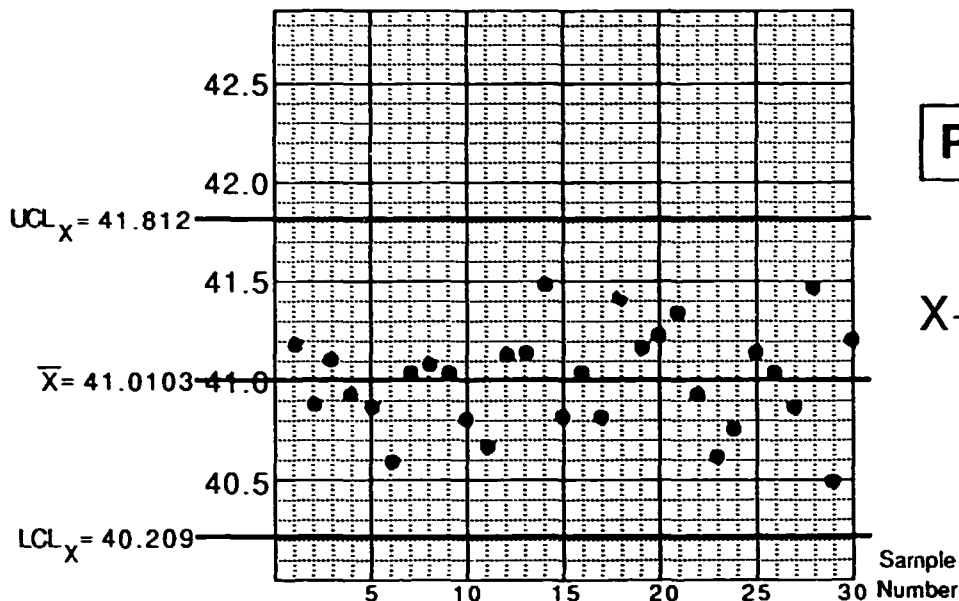
$$LCL_{\bar{X}} = 40.438$$

**Position 1**





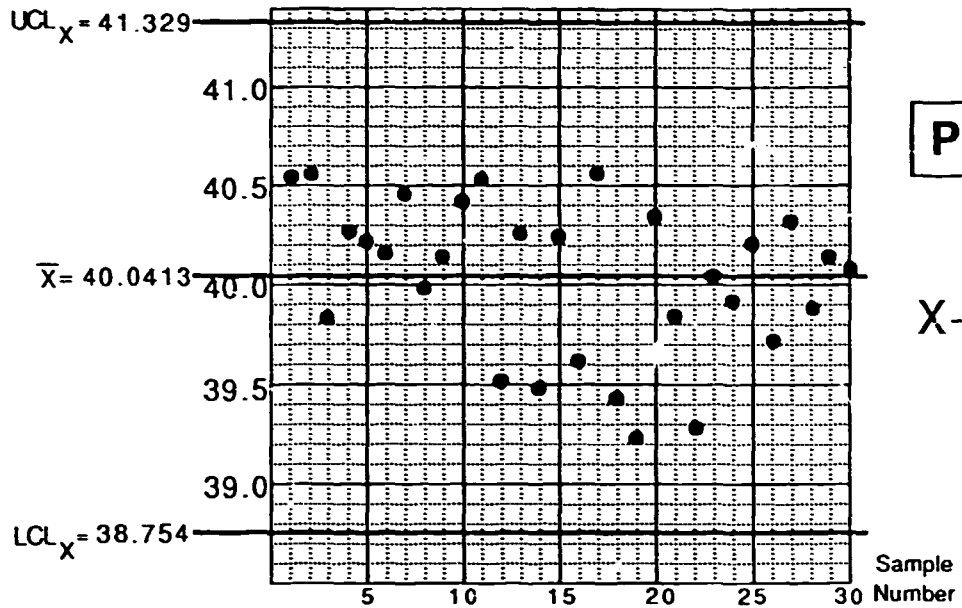
**Position 1**



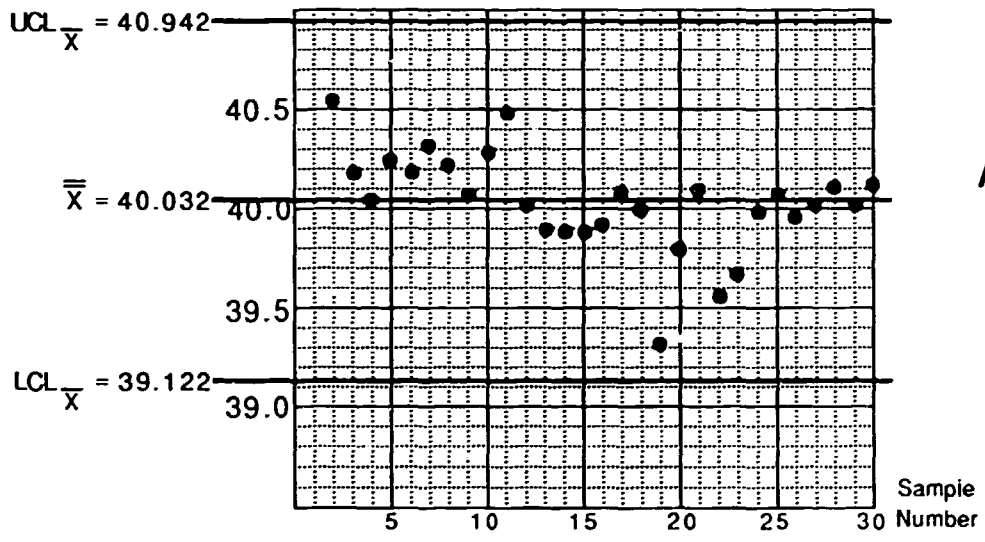
## Position 2

Obs	X	$X_{i-1}$	R	$\bar{X}_i$
1	40.54	.	.	.
2	40.55	40.54	0.01	40.545
3	39.82	40.55	0.73	40.185
4	40.27	39.82	0.45	40.045
5	40.21	40.27	0.06	40.240
6	40.16	40.21	0.05	40.185
7	40.46	40.16	0.30	40.310
8	39.97	40.46	0.49	40.215
9	40.15	39.97	0.18	40.060
10	40.42	40.15	0.27	40.285
11	40.53	40.42	0.11	40.475
12	39.52	40.53	1.01	40.025
13	40.26	39.52	0.74	39.890
14	39.48	40.26	0.78	39.870
15	40.25	39.48	0.77	39.865
16	39.62	40.25	0.63	39.935
17	40.56	39.62	0.94	40.090
18	39.43	40.56	1.13	39.995
19	39.23	39.43	0.20	39.330
20	40.35	39.23	1.12	39.790
21	39.84	40.35	0.51	40.095
22	39.28	39.84	0.56	39.560
23	40.05	39.28	0.77	39.665
24	39.93	40.05	0.12	39.990
25	40.20	39.93	0.27	40.065
26	39.72	40.20	0.48	39.960
27	40.32	39.72	0.60	40.020
28	39.89	40.32	0.43	40.105
29	40.15	39.89	0.26	40.020
30	40.08	40.15	0.07	40.115

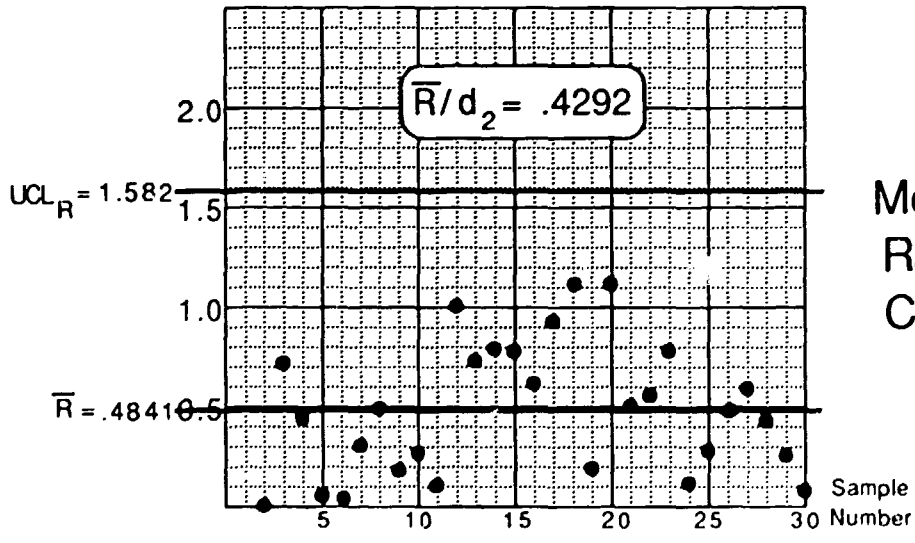
**Position 2**



X-Chart



Moving  
Average  
Chart

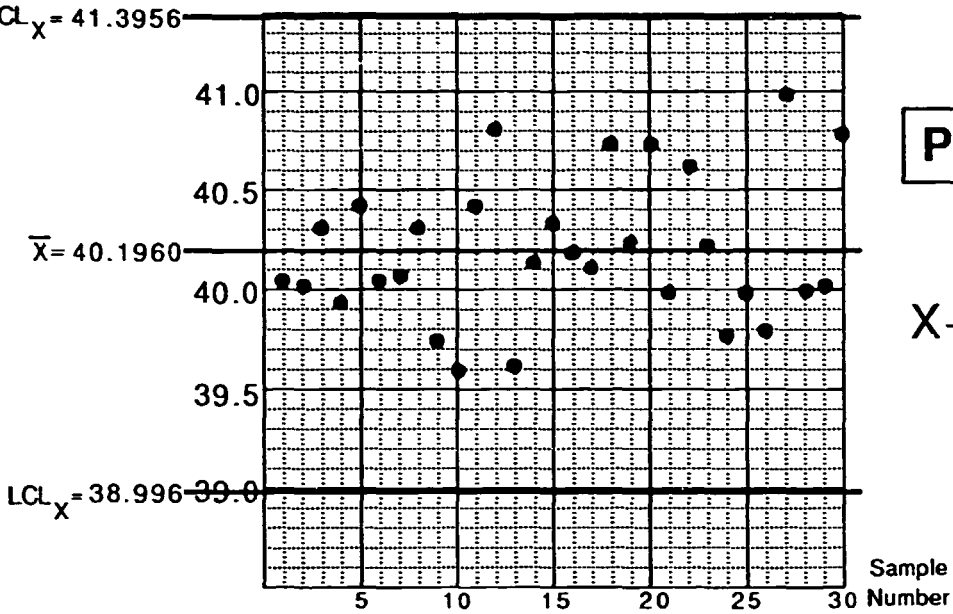


Moving  
Range  
Chart

## Position 3

Obs	X	$X_{i-1}$	R	$\bar{X}_i$
1	40.05	.	.	.
2	40.01	40.05	0.04	40.030
3	40.30	40.01	0.29	40.155
4	39.94	40.30	0.36	40.120
5	40.41	39.94	0.47	40.175
6	40.05	40.41	0.36	40.230
7	40.07	40.05	0.02	40.060
8	40.30	40.07	0.23	40.185
9	39.75	40.30	0.55	40.025
10	39.59	39.75	0.16	39.670
11	40.41	39.59	0.82	40.000
12	40.80	40.41	0.39	40.605
13	39.61	40.80	1.19	40.205
14	40.14	39.61	0.53	39.875
15	40.33	40.14	0.19	40.235
16	40.18	40.33	0.15	40.255
17	40.10	40.18	0.08	40.140
18	40.73	40.10	0.63	40.415
19	40.23	40.73	0.50	40.480
20	40.73	40.23	0.50	40.480
21	39.98	40.73	0.75	40.355
22	40.62	39.98	0.64	40.300
23	40.23	40.62	0.39	40.425
24	39.77	40.23	0.46	40.000
25	39.98	39.77	0.21	39.875
26	39.79	39.98	0.19	39.885
27	40.98	39.79	1.19	40.385
28	39.99	40.98	0.99	40.485
29	40.02	39.99	0.03	40.005
30	40.79	40.02	0.77	40.405

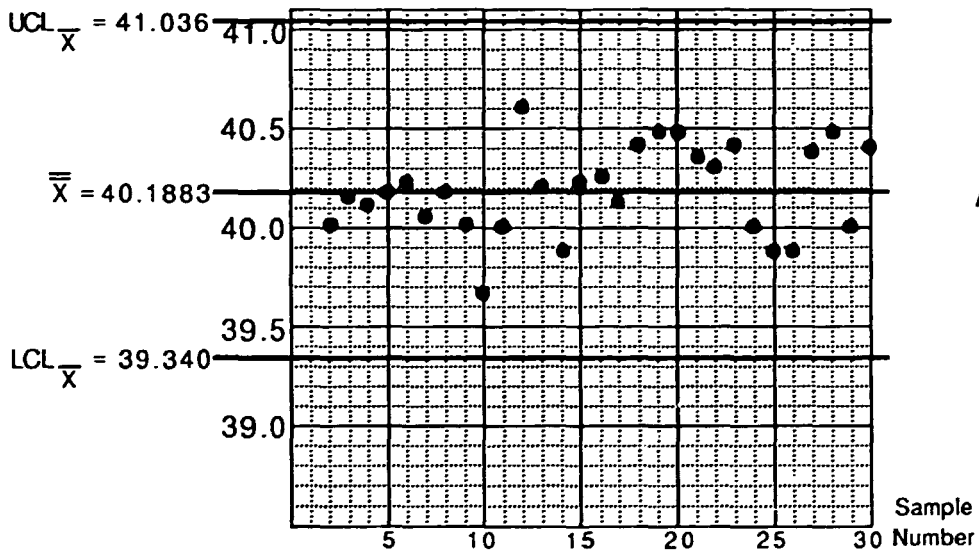
$UCL_{\bar{X}} = 41.3956$



**Position 3**

X-Chart

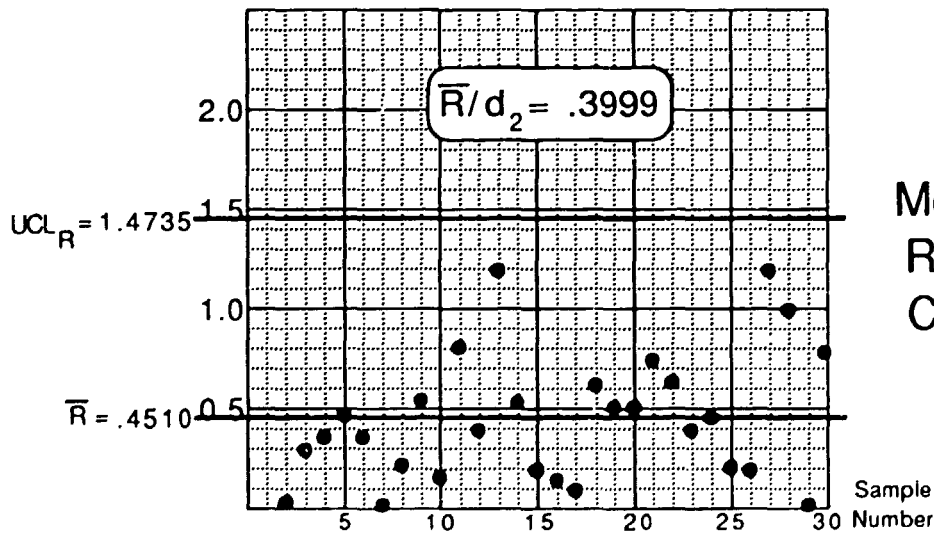
$UCL_{\bar{X}} = 41.036$



Moving Average Chart

$LCL_{\bar{X}} = 39.340$

$\bar{R}/d_2 = .3999$

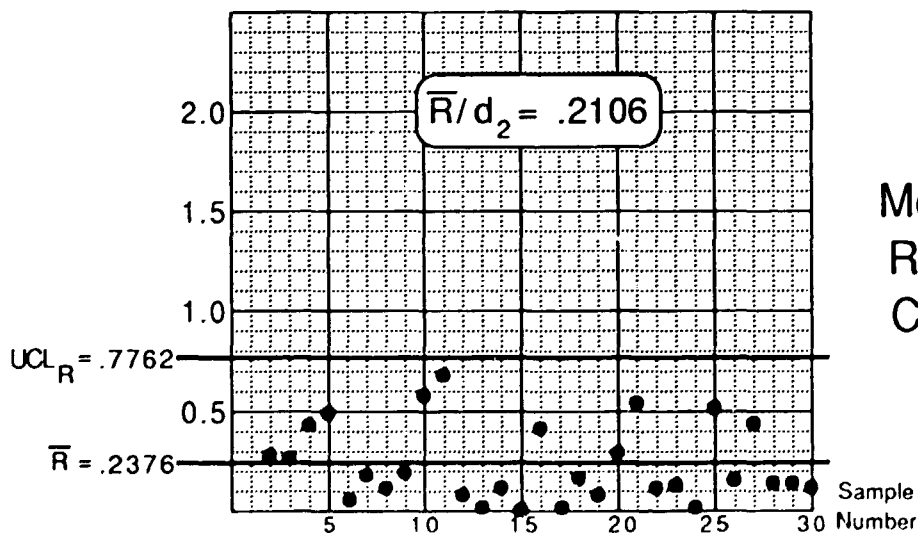
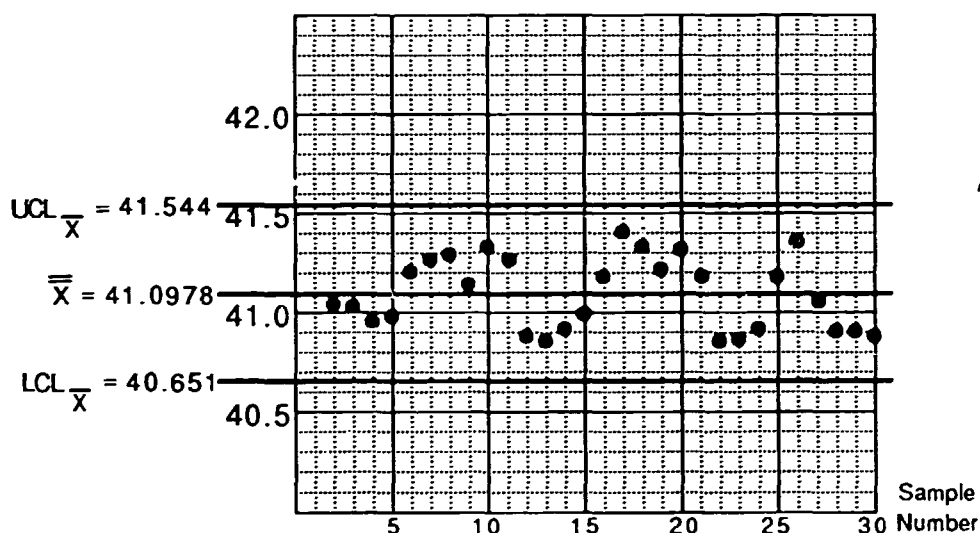
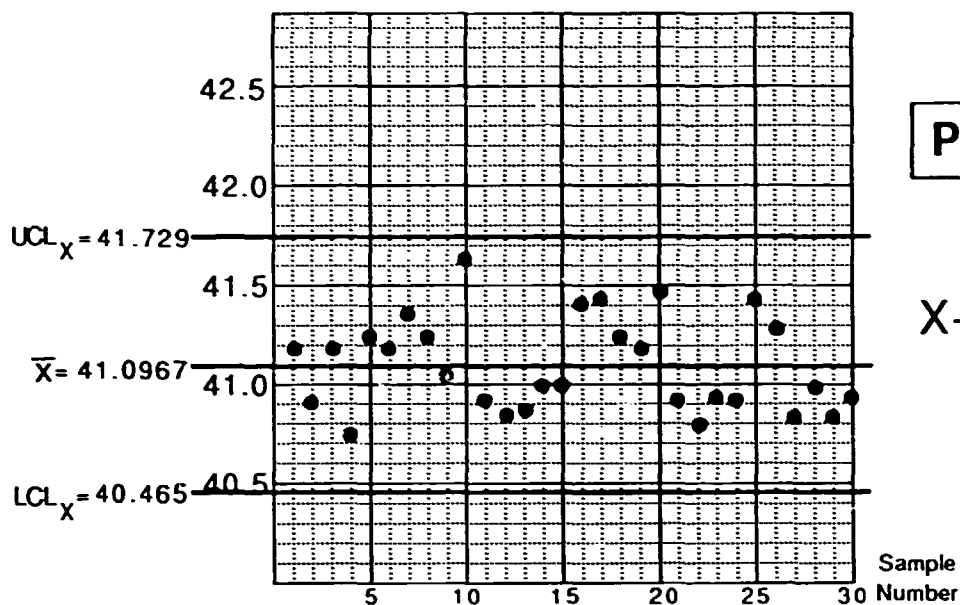


Moving Range Chart

## Position 4

Obs	X	$X_{i-1}$	R	$\bar{X}_i$
1	41.19	.	.	.
2	40.91	41.19	0.28	41.050
3	41.18	40.91	0.27	41.045
4	40.74	41.18	0.44	40.960
5	41.24	40.74	0.50	40.990
6	41.18	41.24	0.06	41.210
7	41.36	41.18	0.18	41.270
8	41.24	41.36	0.12	41.300
9	41.05	41.24	0.19	41.145
10	41.62	41.05	0.57	41.335
11	40.93	41.62	0.69	41.275
12	40.85	40.93	0.08	40.890
13	40.87	40.85	0.02	40.860
14	40.99	40.87	0.12	40.930
15	40.98	40.99	0.01	40.985
16	41.40	40.98	0.42	41.190
17	41.42	41.40	0.02	41.410
18	41.25	41.42	0.17	41.335
19	41.18	41.25	0.07	41.215
20	41.47	41.18	0.29	41.325
21	40.92	41.47	0.55	41.195
22	40.79	40.92	0.13	40.855
23	40.93	40.79	0.14	40.860
24	40.91	40.93	0.02	40.920
25	41.44	40.91	0.53	41.175
26	41.28	41.44	0.16	41.360
27	40.83	41.28	0.45	41.055
28	40.98	40.83	0.15	40.905
29	40.83	40.98	0.15	40.905
30	40.94	40.83	0.11	40.885

**Position 4**



Do all positions have the same variation in the machine direction?

Do the positions seem to run to the same average?



Do all positions have the same variation in the machine direction?

Do the positions seem to run to the same average?

**The two-item moving range for each position has been used to evaluate the stability and magnitude of the within position variability.**

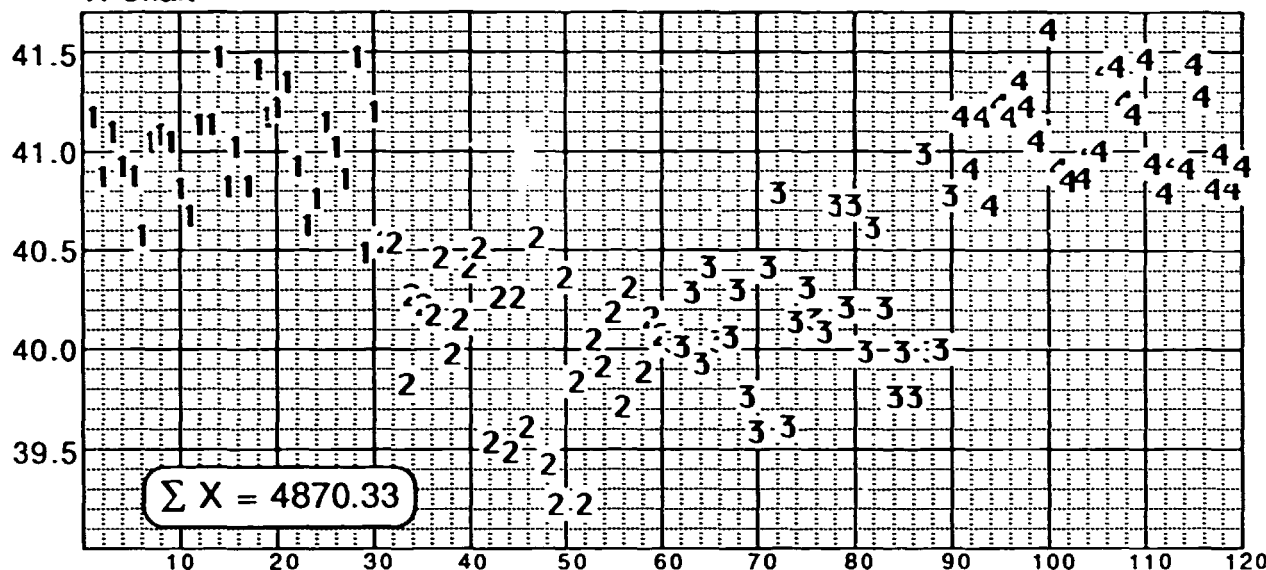
**Those values of range can be used to compare within position variability for all positions. Values of the two-item moving range have been plotted on the following page where the plotting symbol shows the magnitude of the range and identifies the position.**

**Values for the two-item moving averages for the respective positions and the individual measurements on caliper have also been plotted.**

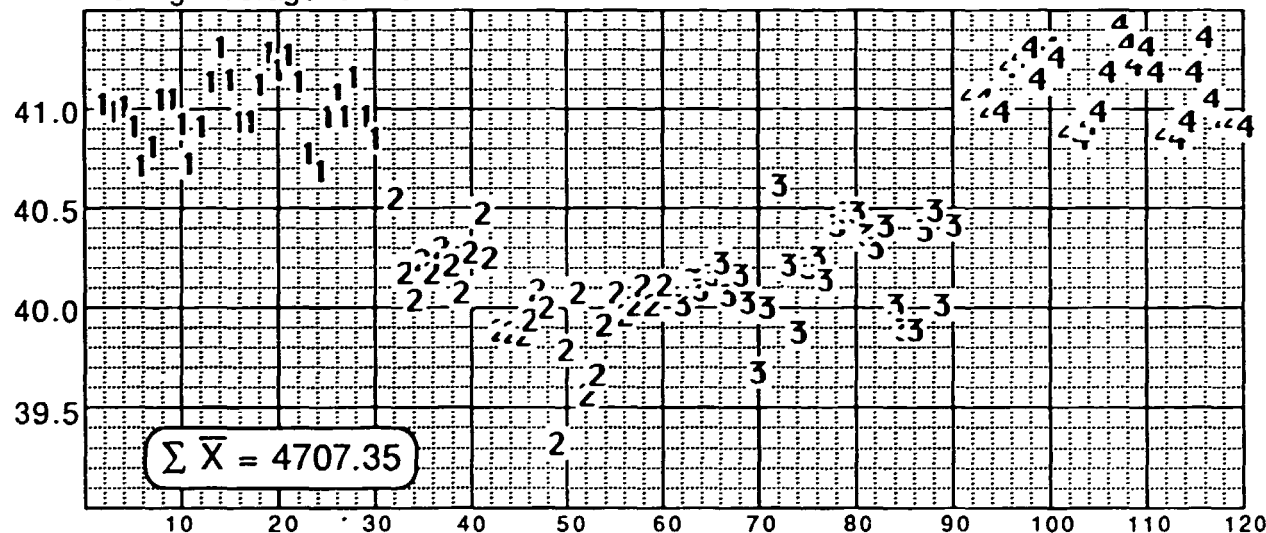
**There are 116 values of the range and average and 120 individual observations.**

# Data for Caliper, Plotted by Position

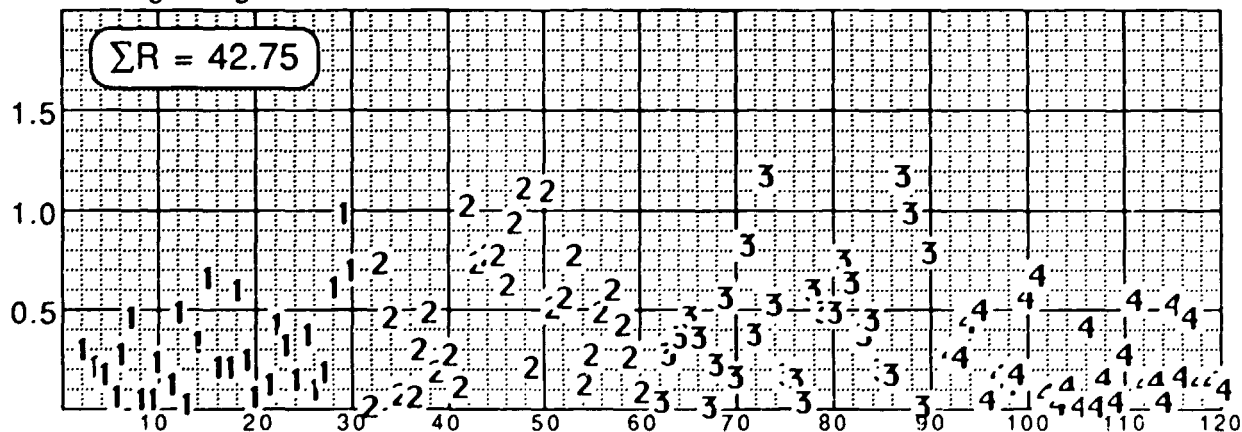
X Chart



Moving Average Chart

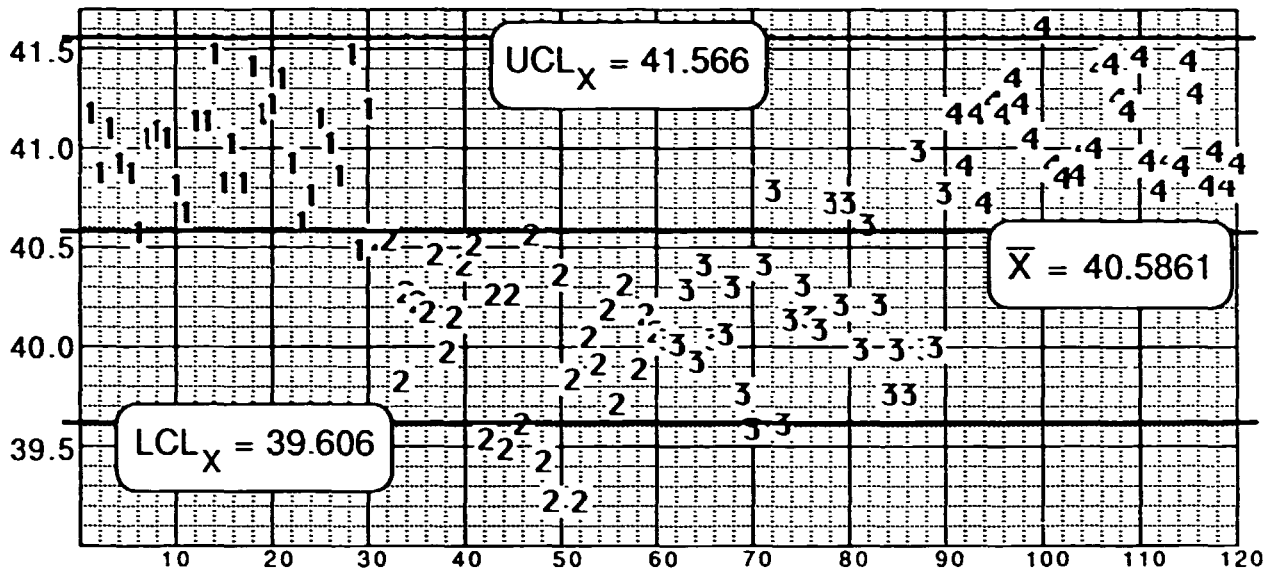


Moving Range Chart

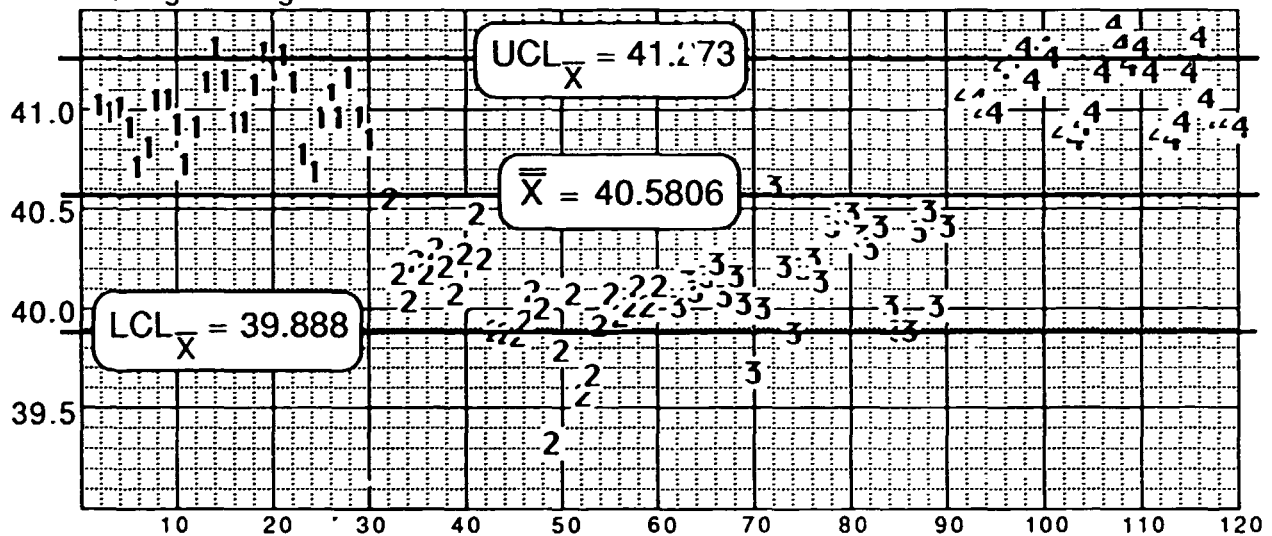


# Data for Caliper, Plotted by Position

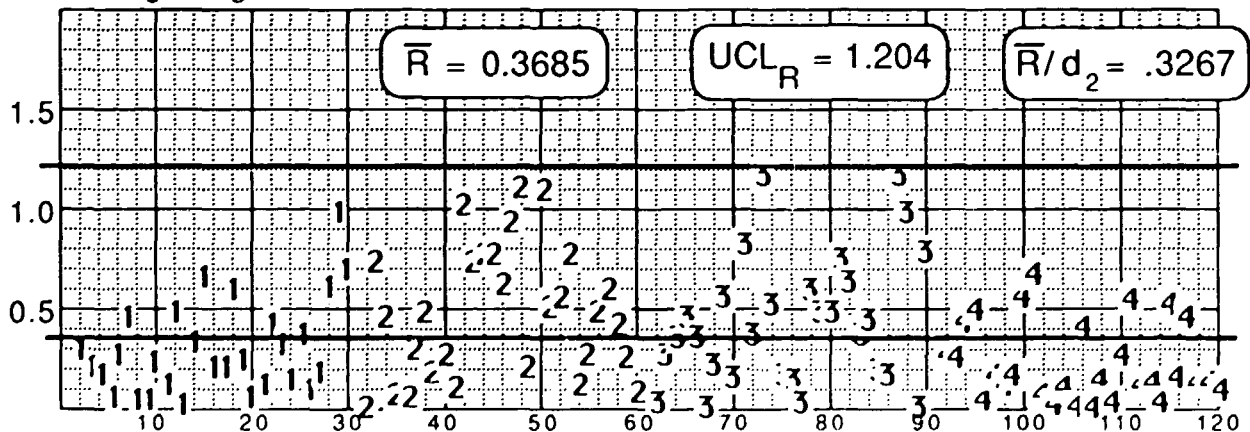
X Chart



Moving Average Chart



Moving Range Chart



Meanwhile,  
in the cross direction.....

does the profile tend to remain  
about the same over time?

The subgroup range in this application reflects variation in the cross direction. It has been suggested that the value of R can be used to measure the profile across the board. Discussion should follow.

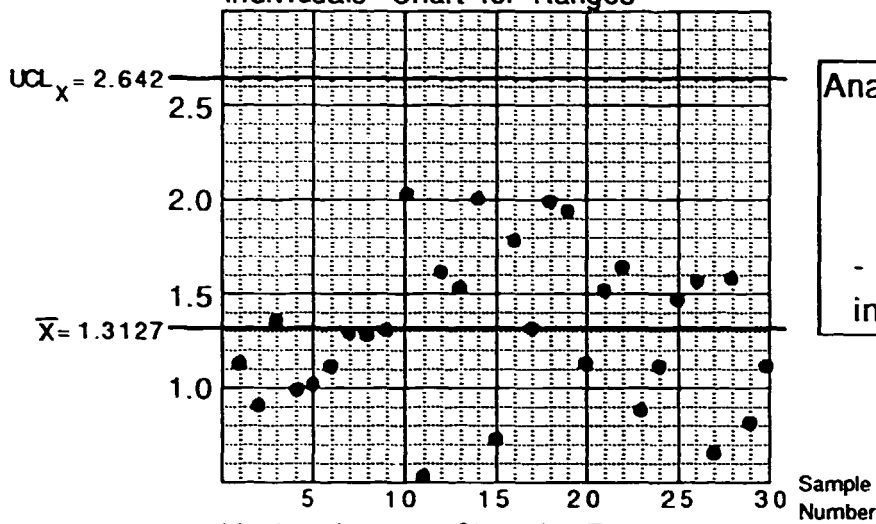
An appropriate analysis on the subgroup range values can provide some help in answering the question.

Use the 30 values of R as individual measurements on a property; at each subgroup, there is a profile and the value of R is a measure on that product/process characteristic. The plan is to use each value of R as a single point measure and apply the technique of the two-item moving range and moving average and an individuals' chart to that column of numbers.

Analysis of the Range as  
a Profile Characteristic  
-Range Used in an Individuals' Chart-

Obs	X	$X_{i-1}$	R	$\bar{X}_i$
1	1.14	.	.	.
2	0.90	1.14	0.24	1.020
3	1.36	0.90	0.46	1.130
4	0.99	1.36	0.37	1.175
5	1.03	0.99	0.04	1.010
6	1.13	1.03	0.10	1.080
7	1.29	1.13	0.16	1.210
8	1.27	1.29	0.02	1.280
9	1.30	1.27	0.03	1.285
10	2.03	1.30	0.73	1.665
11	0.52	2.03	1.51	1.275
12	1.62	0.52	1.10	1.070
13	1.54	1.62	0.08	1.580
14	2.00	1.54	0.46	1.770
15	0.73	2.00	1.27	1.365
16	1.78	0.73	1.05	1.255
17	1.32	1.78	0.46	1.550
18	1.99	1.32	0.67	1.655
19	1.95	1.99	0.04	1.970
20	1.12	1.95	0.83	1.535
21	1.51	1.12	0.39	1.315
22	1.65	1.51	0.14	1.580
23	0.88	1.65	0.77	1.265
24	1.14	0.88	0.26	1.010
25	1.46	1.14	0.32	1.300
26	1.56	1.46	0.10	1.510
27	0.66	1.56	0.90	1.110
28	1.58	0.66	0.92	1.120
29	0.81	1.58	0.77	1.195
30	1.12	0.81	0.31	0.965

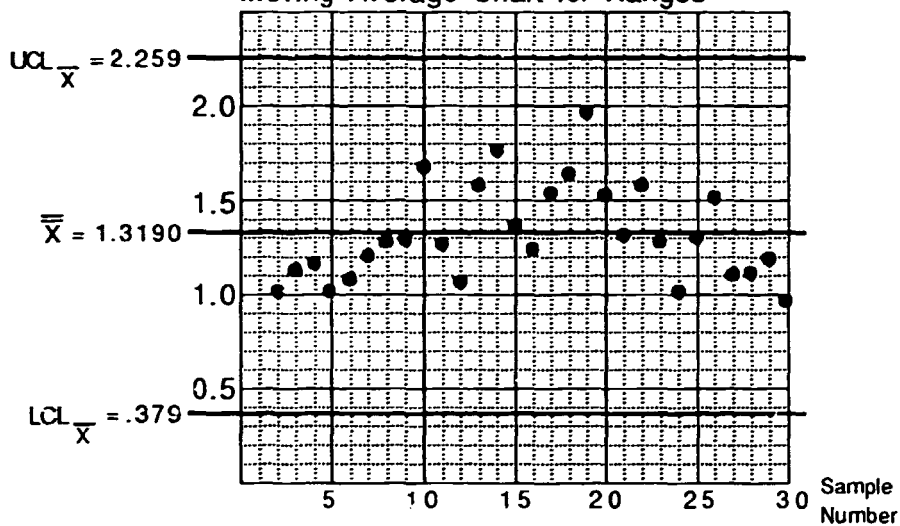
Individuals' Chart for Ranges



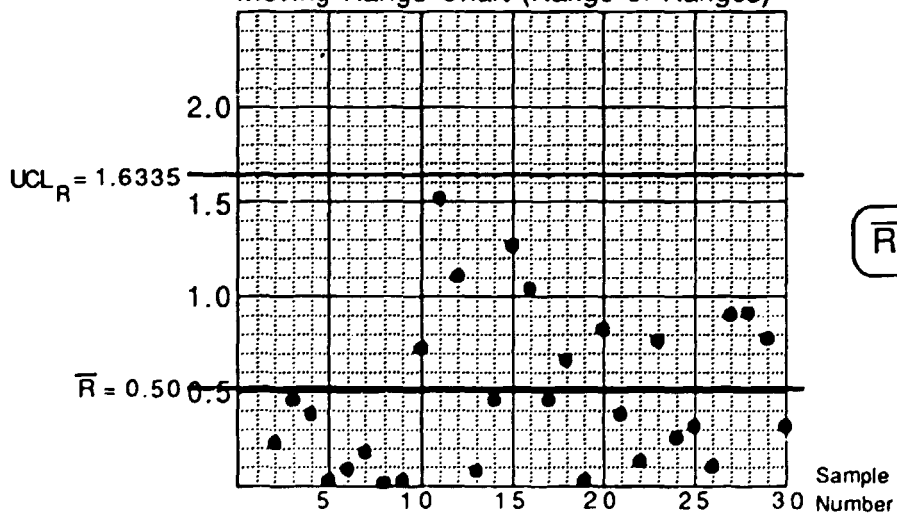
Analysis of the Range  
as a Profile  
Characteristic

- Range used in an  
individuals' chart-

Moving Average Chart for Ranges



Moving Range Chart (Range of Ranges)



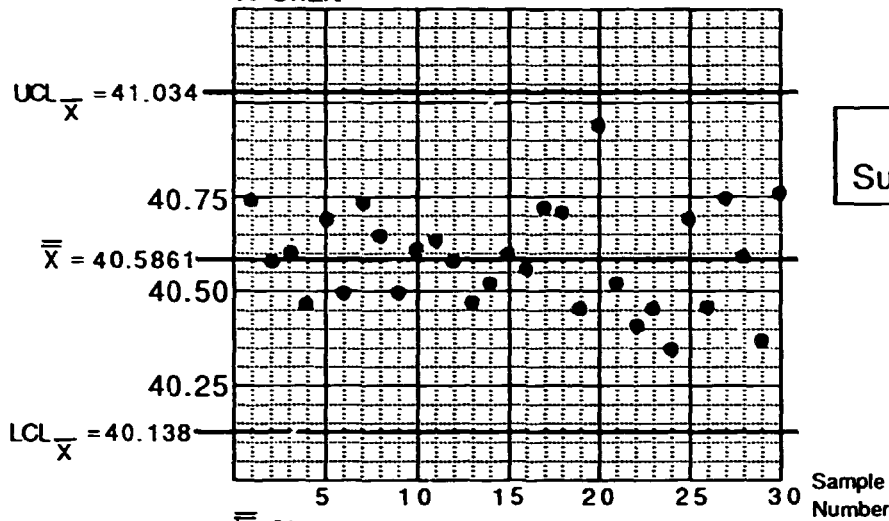
$\bar{R}/d_2 = .4432$

## Analysis of the Subgroup Averages

Obs	$\bar{X}$	$\bar{X}_{i-1}$	$R_{\bar{X}}$	$\bar{X}_i$
1	40.7400	.	.	.
2	40.5875	40.7400	0.1525	40.6638
3	40.6025	40.5875	0.0150	40.5950
4	40.4700	40.6025	0.1325	40.5362
5	40.6825	40.4700	0.2125	40.5763
6	40.4950	40.6825	0.1875	40.5887
7	40.7325	40.4950	0.2375	40.6137
8	40.6475	40.7325	0.0850	40.6900
9	40.4975	40.6475	0.1500	40.5725
10	40.6075	40.4975	0.1100	40.5525
11	40.6350	40.6075	0.0275	40.6213
12	40.5775	40.6350	0.0575	40.6063
13	40.4725	40.5775	0.1050	40.5250
14	40.5225	40.4725	0.0500	40.4975
15	40.5950	40.5225	0.0725	40.5588
16	40.5600	40.5950	0.0350	40.5775
17	40.7250	40.5600	0.1650	40.6425
18	40.7075	40.7250	0.0175	40.7163
19	40.4525	40.7075	0.2550	40.5800
20	40.9450	40.4525	0.4925	40.6988
21	40.5225	40.9450	0.4225	40.7338
22	40.4050	40.5225	0.1175	40.4638
23	40.4550	40.4050	0.0500	40.4300
24	40.3425	40.4550	0.1125	40.3987
25	40.6900	40.3425	0.3475	40.5162
26	40.4575	40.6900	0.2325	40.5738
27	40.7475	40.4575	0.2900	40.6025
28	40.5825	40.7475	0.1650	40.6650
29	40.3725	40.5825	0.2100	40.4775
30	40.7525	40.3725	0.3800	40.5625

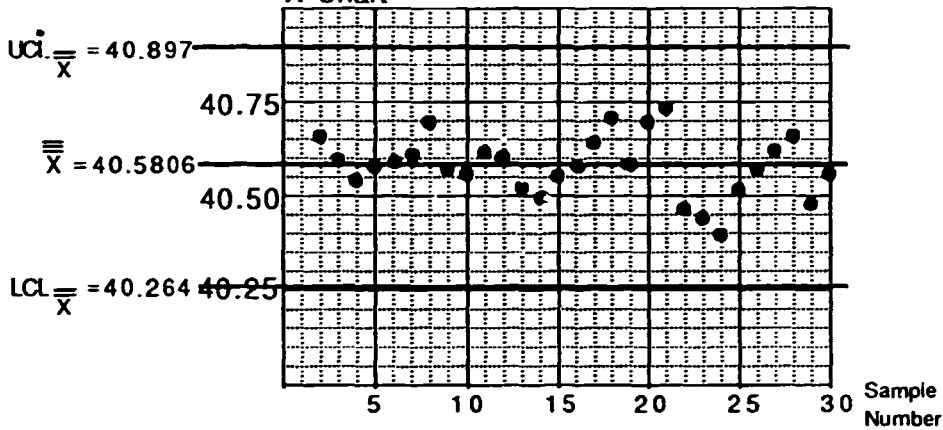


$\bar{X}$  Chart

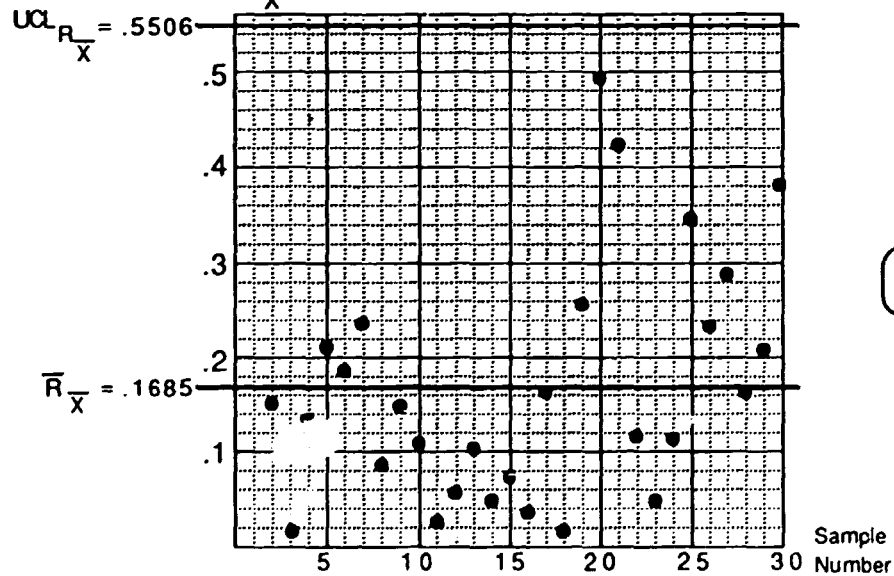


Analysis of the  
Subgroup Averages

$\bar{X}$  Chart



$R_{\bar{X}}$  Chart



$\bar{R}_{\bar{X}}/d_2 = .1494$