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17909

COMPUTER AIDED DESIGN AND COMPUTER AIDED MANUFACTURING CENTRE

DP/SRL/86/014

SRI LANKA

Technical report: Mathematical Modelling \*

Prepared for the Government of Sri Lanka  
by the United Nations Industrial Development Organization  
acting as executing agency for the United Nations Development Programme

Based on the work of Maciej Bojsak  
Expert in CAD/CAM-MME

Backstopping officer: P. Prijpratama, Engineering Industries Branch

United Nations Industrial Development Organization  
Vienna

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\* Mention of company names and commercial products does not imply endorsement of UNIDO. This document has not been edited.

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**ABSTRACT**

The mission has been undertaken under the project "The Establishment of a Computer Aided Design and Computer Aided Manufacturing Centre", DP/SRL/86/014.

The immediate objective of the mission was to assist the University of Moratuwa, Department of Mechanical Engineering in the development of training capabilities in the field of Mathematical Modelling and Design Analysis.

The mission lasted one month, from 29th August to 28th September 1989.

**ACKNOWLEDGEMENT**

The author would like to express his thanks and appreciation to the Vice Chancellor of Moratuwa University Professor Patuwathavithane, the Dean of the Faculty of Engineering Professor C.L.K. Tennekoon, The Head of Department of Mechanical Engineering Professor P.A. de Silva and to the staff members for their interest, involvement and kind assistance to fulfilling the mission.

Special thanks are addressed to Professor P.A. de Silva and his staff for their efforts to overcome uneasy circumstances and for making possible the intensive work.

The author would also like to express his warm thanks to the Senior Industrial Development Field Advisor Mr. J.P.Gorski.

## FINDINGS AND RECOMMENDATIONS

### A. Findings

1. After discussions with the staff members of the Department of Mechanical Engineering, and the industrial engineers from both public and private sectors, involved in the design, production and maintenance of various types of machinery and equipment it is evident that exists real demand for design analyses in several essential fields.

Some of the fields that were identified are:

Agricultural machinery and equipment,  
Machinery for plantation industry,  
Equipment for food and beverages industry,  
Equipment for electricity distribution,  
Equipment for water supply and drainage,  
Moulds for plastic industry,  
Moulds for rubber industry,

All sectors where spare parts  
are manufactured (transport  
services, paper industry, oil industry etc).

2. Based on recent discussions and meetings the following institutions are eager to use facilities and services that would be provided by the CAD/CAM Centre as well as to develop their own resources.

Brown and Co. Ltd.,  
Central Engineering Consultancy Bureau,  
Ceylon Government Railway,  
Ceylon Institute of Scientific and  
Industrial Research,  
Ceylon Petroleum Corporation,  
Ceylon State Hardware Corporation,  
Ceylon Steel Corporation,  
Colombo Commercial Co. Ltd.,  
Colombo Dockyard Ltd.,  
Engineering Consultants Ltd.,  
Hemas Marketing Ltd.,  
Jinasena and Co. Ltd.,  
National Engineering Research and Development Centre,  
Samuel and Sons Ltd.,  
Sri Lanka Central Transport Board,  
Sri Lanka Ports Authority,  
Sri Lanka Tyre Corporation,  
St. Anthony's Industries,  
State Engineering Corporation of Sri Lanka,  
Walker Sons and Co. Ltd.

3. During the lectures and trainings the attendents (both from the University and Industry) show that their mathematical background is on a high level. Most of them are currently using one or more high level languages such as Fortran, Basic, Pascal or C and are also applying some software packages. Therefore, they wouldn't have dificro'ties in using application software for mathematical modelling and design analysis.
4. A number of books on mathematical modelling and design analysis is currently availble at the library. However, more books, recently published are needed and should be provided as soon possible (see Annex 1 point A).
5. Unfortunately, there is a serious lack of periodicals on mathematical modelling and design analysis. Some essential titles should be subscribed as soon as possible (see Annex 1 point B).
6. For the last few months a lot of new computer equipment has been installed at the Computer Centre of the University. Some of it like 32 bit minicomputer micro VAX3600 and workstation VAX 2000 could be used by CAD/CAM Centre.
7. Despite these deliveries, there is a serious lack of tool - software as well as application software. This can be a serious obstacle in full utilisation of existing computer hardware capabilities.

#### B. Recommendations.

1. It is advisable to utilize new hardware and software capabilities existing at the University Computer Centre through establishing a link to CAD/CAM Centre. For this purpose it is necessary to purchase some additional equipment (see Annex 2).
2. During this year it is necessary to enhance current hardware capabilities of the CAD/CAM Centre by purchasing new models of micro computers of the IBM-PC line and plotter. (see Annex 3).
3. It is necessary to purchase as soon as possible the application software for mathematical modelling and design analysis. (see Annex 3).
4. Some industry problems concerning design anaylsis (of tools, dies, moulds, etc.) should be selected for solving in the assistance of the Design Analysis Expert, who according to the programme should arrive in January 1990.

## I. OBJECTIVE OF THE ACTIVITIES AND DUTIES.

### A. Objective

The objective to the mission was to assist the University of Moratuwa, Department of Mechanical Engineering in the development of training capabilities in Computer Aided Design (Mathematical Modelling and Design Analysis).

### B. Duties

1. To provide lectures on computer assisted mathematical modelling methods and train local staff in application of mathematical modelling to design analysis.
2. To elaborate a proper training programme for local staff and prepare, in cooperation with the counterpart, a detailed training programme for fellowship holders.
3. To assist in elaboration of training methodologies, syllabi and lecture notes for courses for students and industrial engineers.
4. To prepare a report, setting out the findings and recommendations for follow-up actions which should be taken.

## II. DESCRIPTION OF ACTIVITIES

### A. Lectures

The author has delivered a course with exercises on mathematical modelling and its applications to design analysis. The topics covered in the lectures are given in Annex 4.

The lectures were attended by about twenty mechanical and electronic engineers recruiting from both University and Industry.

The following institutions were represented:

Department of Mechanical Engineering,  
 Department of Electrical Engineering,  
 Department of Computer Science & Engineering,  
 Department of Chemical Engineering,  
 Open University of Sri Lanka,  
 Ceylon Steel Corporation,  
 Hervey Fab Ltd.,  
 Nixdorf Computers.



The participants who have direct access to microcomputer in the work place or privately can use the course material to solve problems that they may come across in their day-to-day activities.

To ensure more successful results the Department of Mechanical Engineering has already made arrangements to make available a printed set of lecture notes for the attendants who wish to have them.

#### **B. Training**

To get practise in the topics that were of immediate interest in the lectures the participants were engaged in solving their own problems, during the afternoon sessions.

To this effect they used software delivered by the author.

The software consists of two parts :

1. Numerical Methods package developed by Professor A. Constantinides from the State University of New Jersey, USA. The programs written in BASIC language are interactive and user-friendly (self explained). The list of programs is given in Annex 5.
2. Finite Element programme FIRFEP (First Finite Element Program) developed by the author. This is a simple educational program written in FORTRAN language which can be utilized for solving plane stress problems using linear triangular element. The listing of the program with instructions on how to use it and test example are given in Annex 6.

Above mentioned software can be used in future for demonstration and education, as well as for solving some engineering problems.

#### **C. Programmes**

The author, assisted by the staff of the Department of Mechanical Engineering, University of Moratuwa, has elaborated syllabi and extensive lecture notes for courses on Mathematical Modelling. The lecture notes will be published by CAD/CAM Centre as educational aids for students and field engineers.

**D. Workshop**

A workshop on Mathematical Modelling in Design Analysis was prepared for industry engineers and invited participants (see Annex 7). Unfortunately, due to unforeseen circumstances the workshop had to be postponed.

## RECOMMENDED LITERATURE.

## A. BOOKS

1. WELLSTEAD P.E.  
"INTRODUCTION TO PHYSICAL SYSTEM MODELLING"  
ACADEMIC PRESS, 1979
2. BLUNDELL A.  
"BOND GRAPHS FOR MODELLING ENGINEERING SYSTEMS"  
ELLIS HORWOOD LTD. ,1982
3. REDDY  
"APPLIED FUNCTIONAL ANALYSIS AND VARIATIONAL METHODS  
IN ENGINEERING"  
McGRAW HILL
4. PRZEMIENIECKI  
"THEORY OF MATRIX STURCTURAL ANALYSIS"  
McGRAW HILL
5. RICE J.R.  
"NUMERICAL METHODS, SOFTWARE AND ANALYSIS"  
McGRAW HILL, 1983
6. CONSTANTINIDES A.  
"APPLIED NUMERICAL METHODS WITH PERSONAL COMPUTERS"  
McGRAW HILL,1988
7. HAPRA S.C., CANALE R.P.  
"NUMERICAL METHODS FOR ENGINEERS WITH PERSONAL COMPUTER  
APPLICATIONS"  
McGRAW HILL, 1985
8. ROBINSON J.  
"UNDERSTANDING THE FINITE ELEMENTS"  
ROBINSON AND ASSOCIATES
9. REDDY  
"AN INTRODUCTION TO THE FINITE ELEMENT METHOD"  
McGRAW HILL
10. BATHE K.J.  
"FINITE ELEMENT PROCEDURES IN ENGINEERING ANALYSIS"  
PRENTICE HALL, 1982

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11. ZIENKIEWICZ O.C.  
"THE FINITE ELEMENT METHOD" (4TH EDITION)  
MCGRAW HILL, 1989
12. KARDESTUNCER  
"FINITE ELEMENT HANDBOOK"  
MCGRAW HILL, 1987
13. SMITH I.M., GRIFFITHS D.V.,  
"PROGRAMMING THE FINITE ELEMENT METHOD"  
J.WILEY & SONS
14. RAO S.S  
"THE FINITE ELEMENT METHOD IN ENGINEERING"  
PERGAMON PRESS, 1989
15. "GUIDELINES TO FINITE ELEMENT PRACTICE"  
NAFEMS-NEL, GLASGOW
16. MEYER CH. (EDITOR)  
"FINITE ELEMENT IDEALIZATION"  
ASCE, NEW YORK, 1987
17. COOK W.A.  
"CONCEPTS AND APPLICATIONS OF FINITE ELEMENT ANALYSIS"  
J. WILEY & SONS, 1981
18. NIKRAVESH P.E.  
"COMPUTER AIDED ANALYSIS OF MECHANICAL SYSTEMS"  
PRENTICE HALL, 1988
19. DIMAROGONAS A.  
"COMPUTER AIDED MACHINE DESIGN"  
PRENTICE HALL, 1988
20. WHITE R.E.  
"AN INTRODUCTION TO THE FINITE ELEMENT METHOD  
WITH APPLICATIONS TO THE NONLINEAR PROBLEMS"  
J. WILEY & SONS, 1985
21. BREBBIA C.A.  
"THE BOUNDARY ELEMENT METHOD FOR ENGINEERS"  
PENTECH PRESS, 1978
22. BANERJEE, BUTTERFIELD.  
"BOUNDARY ELEMENT METHODS IN ENGINEERING SCIENCE"  
MCGRAW HILL

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**B. PERIODICALS**

1. "COMPUTER METHODS IN APPLIED MECHANICS AND ENGINEERING"  
NORTH-HOLLAND, PO BOX 1991, 1000 BZ AMSTERDAM, THE NETHERLANDS.
2. "NUMERICAL METHODS IN ENGINEERING"  
J. WILEY, BAFFINS LANE, CHICHESTER, SUSSEX, ENGLAND.
3. "COMPUTERS AND STRUCTURES"  
PERGAMON PRESS, FAIRVIEW PARK, ELMSFORD, NY 10523, USA.
4. "FINITE ELEMENTS IN ANALYSIS AND DESIGN"  
NORTH-HOLLAND, PO BOX 1991, 1000 BZ, AMSTERDAM, THE NETHERLANDS.

A2 - 1

ANNEX 2

**SPECIFICATION OF EQUIPMENT NEEDED FOR LINKING UNIVERSITY  
COMPUTER CENTRE AND CAD/CAM CENTRE.  
(EXTENSION OF EXISTING DEC-ETHERNET NETWORK)**

1. STANDARD ETHERNET CABLE BNE 2
2. TRANSCEIVER- H 4005 (3 UNITS)
3. LOCAL REPEATER- DEREPAE (1 UNIT)
4. COMMUNICATION SERVER- DELNI-AE (1 UNIT)
5. TRANSCEIVER CABLE BNE 3H
6. ETHERNET CONTROLLER FOR PC XT/AT- DEPCA-AA

A3 - 1

ANNEX 3

## SPECIFICATION OF EQUIPMENT FOR CAD/CAM CENTRE (1989)

## A. HARDWARE

## 1. MICROCOMPUTER (2 UNITS)

TYPE: PC-386  
RAM: MIN 2 MB  
HARDDISK: MIN 40 MB  
MATH COPROCESSOR:  
(THESE CAN BE PURCHASED LOCALLY FROM IBM OR NIXDORF)

## 2. PLOTTER A1

MODEL HP 7580 B FROM HEWLETT PACKARD  
OR GRAPHTEC MP 9101-01 FROM GRAPHTEC

## B. SOFTWARE

## 1. DRAFTING SYSTEM

AUTOCAD VERSION 10.0 (THERE ARE MANY DEALERS ALL OVER THE  
WORLD)

## 2. SCIENTIFIC SUBROUTINE LIBRARY

ADVANCED MATH. APPLICATIONS PACK - MATHCAD  
DEALER: MATHSOFT, INC., ONE KENDALL 5, CAMBRIDGE,  
MA 02139, USA.

## 3. FINITE ELEMENT SYSTEM WITH PRE AND POST PROCESSORS

1. SAP 86 - LEVEL 1 (EDUCATIONAL)
2. SAP 86 - LEVEL 4 (PRO 386-2)
3. MTAB\*PRE- PREPROCESSOR
4. MTAB\*POST-POSTPROCESSOR

## DEALER:

STRUCTURAL ANALYSIS INC.,  
1701 DIRECTORS BLVD.,  
SUITE 360, AUSTIN, TEXAS 78744, USA.

CAD/CAM PROJECT

## WORK SCHEDULE FOR MATHEMATICAL MODELLING EXPERT

|                |       | Morning  | Afternoon               |
|----------------|-------|--|-------------------------|
| SEPTEMBER 1989 |       | 9.00 AM - 12.00 noon   | 1.30 pm - 3.30 pm       |
| Friday         | 01 st | Discussion with UNDP/UNIDO officials, preparation of the lecture programme                     |                         |
| Saturday       | 02 nd | WEEKEND  |                         |
| Sunday         | 03 rd | Preparation of lecture notes   |                         |
| Monday         | 04 th | Arrival at the department<br>Discussion with the University staff members                      |                         |
| Tuesday        | 05 th | Lecture 1  | Discussion/Consultation |
| Wednesday      | 06 th | Lecture 2  | Discussion/Consultation |
| Thursday       | 07 th | Lecture 3  | Discussion/Consultation |
| Friday         | 08 th | Lecture 4  | Discussion/Consultation |
| Saturday       | 09 th | WEEKEND  |                         |
| Sunday         | 10 th | Preparation of course syllabi and lecture notes for students and Industrial Engineers          |                         |
| Monday         | 11 th | Lecture 5  | Training                |
| Tuesday        | 12 th | Lecture 6  | Training                |
| Wednesday      | 13 th | Lecture 7  | Training                |
| Thursday       | 14 th | Lecture 8  | Training                |
| Friday         | 15 th | Lecture 9  | Training                |
| Saturday       | 16 th | WEEKEND  |                         |
| Sunday         | 17 th | Preparation of training programme for local staff, Industrial Engineers and Fellowship Holders |                         |
| Monday         | 18 th | Lecture 10   | Discussion/Consultation |
| Tuesday        | 19 th | Preparation for the workshop   |                         |
| Wednesday      | 20 th | Workshop for Industrialists, Day 1   |                         |
| Thursday       | 21 st | Workshop for Industrialists, Day 2   |                         |
| Friday         | 22 nd | Lecture 11   | Discussion/Consultation |
| Saturday       | 23 rd | WEEKEND  |                         |
| Sunday         | 24 th | Preparation of report to UNDP  |                         |
| Monday         | 25 th | Finalisation of report to UNDP and Departure   |                         |



**PROGRAMME OF LECTURES ON MATHEMATICAL MODELLING****LECTURER: PROFESSOR. M. BOSSAK (UNIDO EXPERT)****VENUE : SEMINAR ROOM, DEPARTMENT OF MECHANICAL ENGINEERING**

| <b>LECTURE NO.</b> | <b>Topics</b>  |
|--------------------|--|
| 1.                 | The role of Mathematical Modelling in the modern Computer Aided Design.  |
| 2.                 | Phases in Mathematical Modelling. Approximation methods. Weighted residuals method. Variational formulations.  |
| 3.                 | Finite Element Method (FEM). The idea of the method Definition of a finite element-shapes, nodes, nodal values, approximation functions. Examples of different finite elements.  |
| 4.                 | Coordinate systems - global, local boundary, material. Transformations of values from one to another coordinate system.  |
| 5.                 | Linear static analysis. Relations between known and unknown nodal values for the element (stiffness matrix). Relation (in global coordinates) between known and unknown nodal values for the whole system of elements (global stiffness matrix). |
| 6.                 | Boundary conditions. System of equations and its properties. Solution of the system of equations. Calculation of other desired values. Principle of virtual work. General formula for matrices describing element properties.                    |
| 7.                 | Linear dynamic analysis. Frames of reference. Loads, mass forces, elastic forces, damping forces. Equation of motion.  |
| 8.                 | Eigenvalue problems. Initial value problems - direct integration - mode superposition method. Finite element systems. Pre and postprocessors.  |
| 9.                 | Basic information on nonlinear analysis. Classification of nonlinear problems. Equation of equilibrium. Solution of nonlinear equations.   |
| 10.                | Basic information on Boundary Element Method (BEM).  |
| 11.                | Basic information on Bond Graph Method (BGM).  |



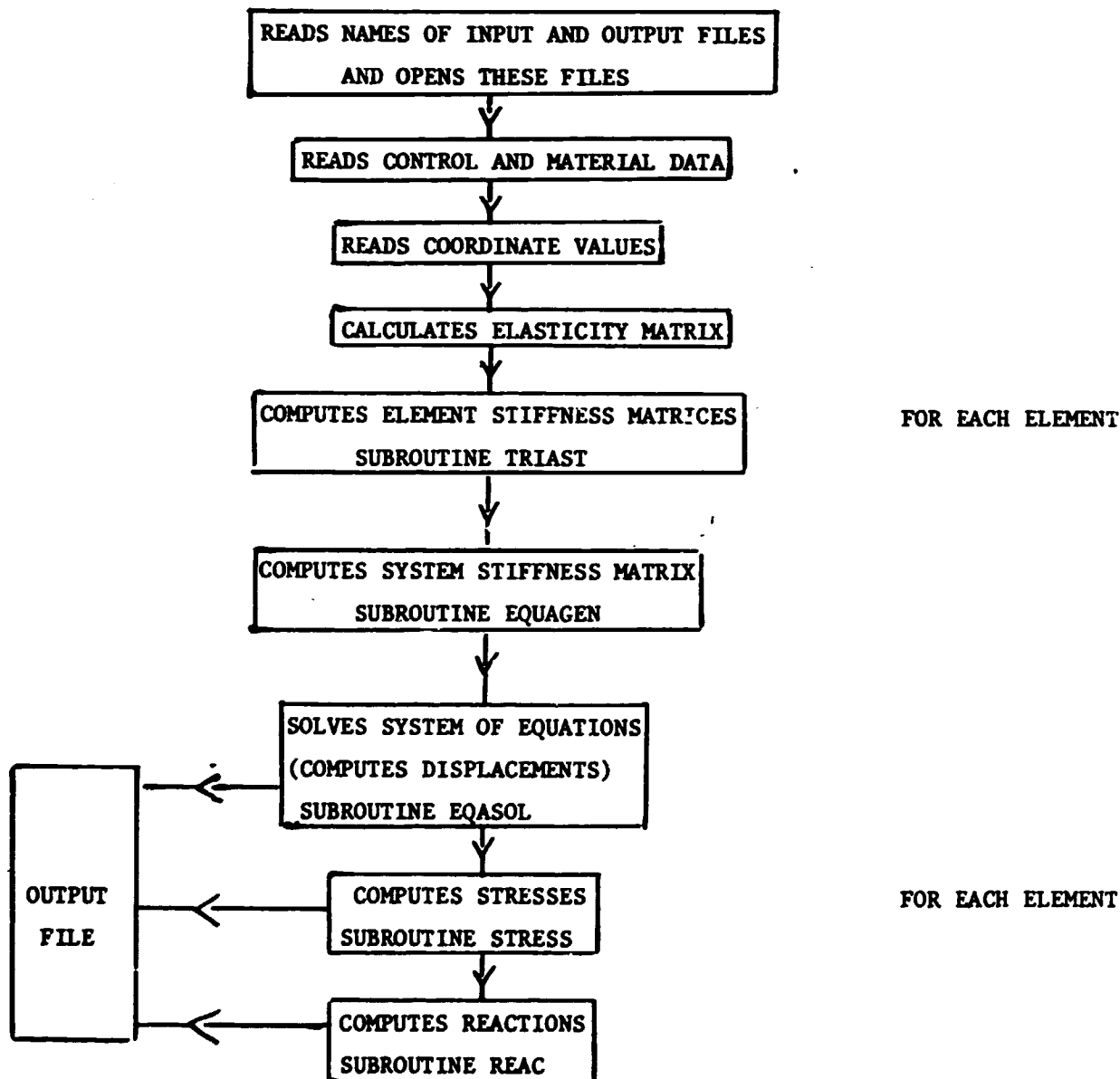
A5 - 2

10. EIGEN.BAS                    CALCULATIONS OF EIGEN VALUES  
AND EIGEN VECTORS USING THE  
PADDEEV-LEVERRIER/NEWTON-  
RAPHSON/GAUSS METHODS.
11. QR.BAS                      CALCULATION OF EIGEN VALUES  
AND EIGEN VECTORS USING  
ELEMENTARY SIMILARITY TRANS-  
FORMATIONS TO CONVERT THE  
MATRIX TO HESSENBERG FORM,  
AND THE QR ALGORITHM WITH  
PLANE ROTATIONS.
12. INTEGR.BAS                 INTEGRATION FORMULAS:  
TRAPEZOIDAL, SIMPSON'S 1/3 AND  
SIMPSON'S 3/8 RULES.
13. ODE.BAS                    FOURTH ORDER RUNGE-KUTTA AND  
EULER PREDICTOR-CORRECTOR METHODS  
FOR INTEGRATING SIMULTANEOUS  
ORDINARY DIFFERENTIAL EQUATIONS.
14. BOUNDARY.BAS              BOUNDARY VALUE PROBLEMS: THE NEWTON  
METHOD.
15. ELLIPTIC.BAS              ELLIPTIC PARTIAL DIFFERENTIAL  
EQUATIONS.
16. PARABOL.BAS               PARABOLIC PARTIAL DIFFERENTIAL  
EQUATIONS.

PROGRAM FIRFEP (FIRst Finite Element Program)

FITFEP is simple (educational) finite element program for solving plane stress problems. It uses three nodes triangular element (linear displacements, constant stresses).

STRUCTURE OF THE PROGRAM.



THE FOLLOWING DATA ARE REQUIRED:

1. CONTROL AND MATERIAL DATA

ONE CARD FORMAT(6I5,E10.2,2F5.2)

NEN,NDF,NON,NOE,NBC,NLN,E,PR,TH

NEN - NUMBER OF ELEMENT NODES = 3

NDF - NUMBER OF DEGREES OF FREEDOM AT EACH NODE = 2

NON - TOTAL NUMBER OF NODES

NOE - TOTAL NUMBER OF ELEMENTS

NBC - TOTAL NUMBER OF NODES WITH BOUNDARY CONDITIONS

NLN - TOTAL NUMBER OF LOADED NODES

E - YOUNG'S MODULUS

PR - POISSON'S RATIO

TH - ELEMENT THICKNESS

2. NODAL COORDINATES

NON CARDS FORMAT(I5,2G10.2)

(N(I),(CORD(I,J),J=1,2),I=1,NON)

N(I) - NODE NUMBER

CORD(I,J) - X,Y NODE COORDINATES

3. LOADS

NLN CARDS FORMAT(I5,2G10.2)

(NF(I),(RHS(I,J),J=1,2),I=1,NLN)

NF(I) - NODE NUMBER WHERE LOAD IS ACTING

RHS(I,J) - X,Y COMPONENTS OF THE LOAD

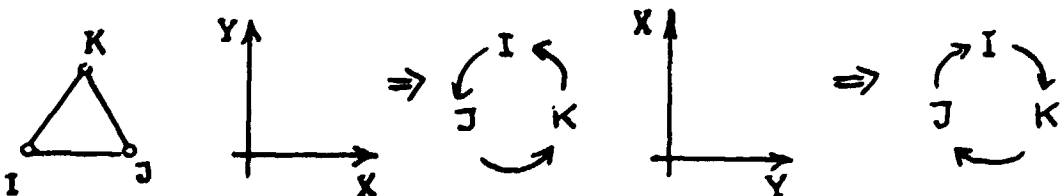
4. TOPOLOGY

NOE CARDS FORMAT(3I5)

I,J,K

I,J,K - ELEMENT NODE NUMBERS

REMARK: ORDER OF THE NUMBERS MUST BE ADEQUATE TO THE SYSTEM  
OF COORDINATES



A6 - 3

## 5. TOPOLOGY

NBC CARDS FORMAT(3I5)

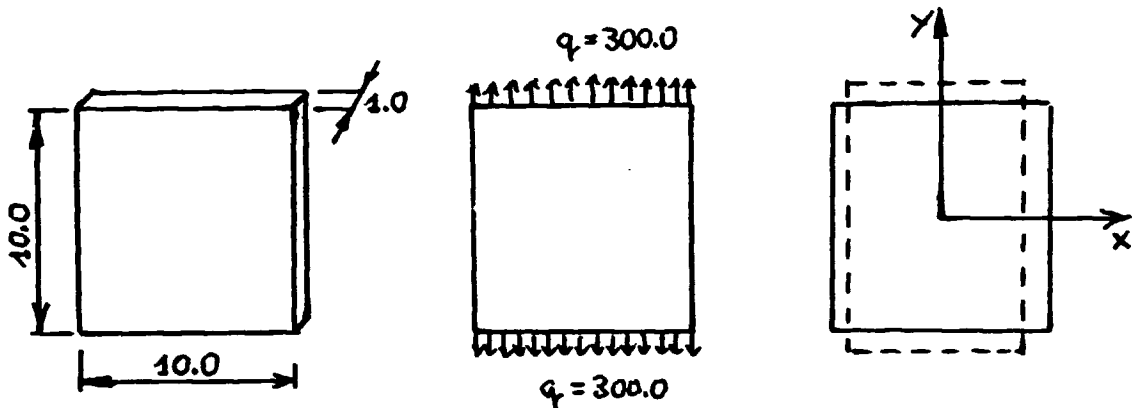
NN,NX,NY

NN - NUMBER OF THE NODE WITH BOUNDARY CONDITIONS

NX - X- DIRECTION (0=FREE, 1=FIXED)

NY - Y- DIRECTION (0=FREE, 1=FIXED)

## TEST EXAMPLE



$$E = 2.0 \times 10^6, \nu = 0.3$$

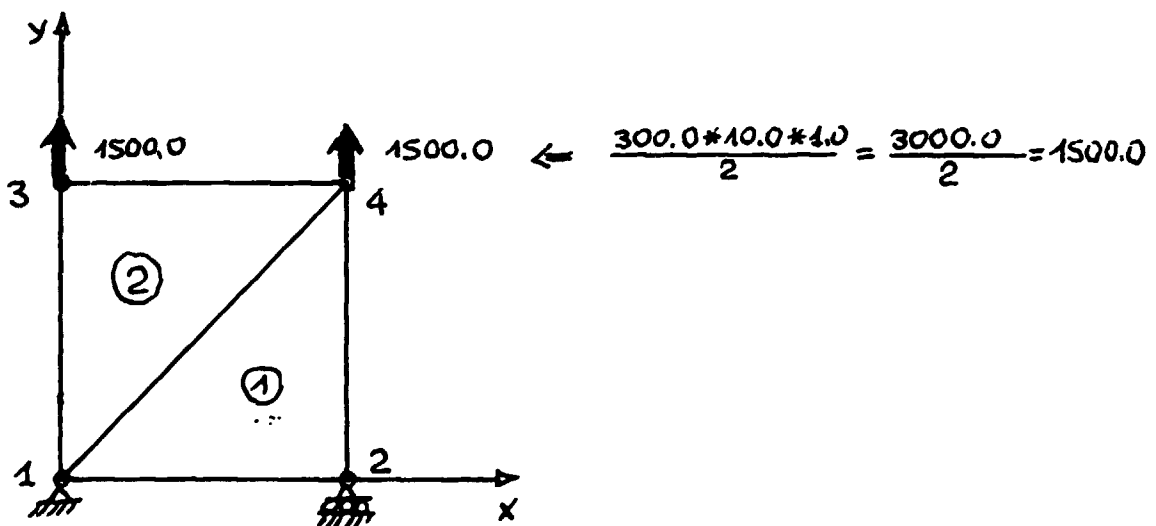
$$\sigma_y = \frac{300.0 \times 10.0 \times 1.0}{10.0 \times 4.0} = 300.0 \Rightarrow \epsilon_y = \frac{\sigma_y}{E} = \frac{300.0}{2.0 \times 10^6} = 1.5 \times 10^{-4}$$

$$\epsilon_x = -\nu \epsilon_y = -0.3 \times 1.5 \times 10^{-4} = 4.5 \times 10^{-5}$$

$$\Delta y = \epsilon_y \times L = 1.5 \times 10^{-4} \times 10.0 = 1.5 \times 10^{-3}$$

$$\Delta x = \epsilon_x \times L = 4.5 \times 10^{-5} \times 10.0 = 4.5 \times 10^{-4}$$

## FINITE ELEMENT MODEL



## BOUNDARY CONDITIONS:

$$\text{NODE 1, } U_1=0, \quad V_1=0$$

$$\text{NODE 2, } \quad V_2=0$$

## DATA FOR TEST

## NAMES OF INPUT AND OUTPUT FILES

INPUT FILE - FIRE1.DAT

OUTPUT FILE - FIRE1.RES

## 1. CONTROL AND MATERIAL DATA

3,2,4,2,2,2,2.0E6,0.3,1.0

## 2. NODAL COORDINATES

1, 0.,0.

2,10.,0.

3, 0.,10.

4,10.,i0.

## 3. LOADS

3,0.,1500.

4,0.,1500.

## 4. TOPOLOGY

1,2,4

1,4,3

## 5. BOUNDARY CONDITIONS

1,1,1

2,0,1

RESULTS OF CALCULATIONS WILL BE WRITTEN IN OUTPUT FILE FIRTE1.RES



```

C
C   program firfef
C
C   first finite element program
C
C   nen - number of element nodes
C   ndf - number of degrees of freedom at the node
C   noe - total number of elements
C   non - total number of nodes
C   nbc - number of nodes with boundary conditions
C   nin - number of loaded elements
C   E   - Young's modulus
C   pr  - Poisson's ratio
C   th  - element thickness
C   cord- nodal coordinates
C   d   - elasticity matrix
C
C
C
C
C   IMPLICIT REAL*8 (A-H,O-Z)
C   COMMON NEN, NDF, NOE, NON, NBC, TH
C   COMMON/CORD/ CORD(60,2)
C   COMMON/D/ D(3,3)
C   COMMON/LOAD/ NLN, NF(15), RHS(15,2)
C   COMMON/NOEL/ NOEL
C   DIMENSION N(60)
C   CHARACTER*30 NAME
C   CHARACTER*80 NAZWA
C
C
C   reading names of input and output files and opening this files
C
C
C   WRITE(0,1)
C 1 FORMAT(/' input file name : '\)
C   READ(0,2) NAME
C 2 FORMAT(A30)
C   OPEN(UNIT=5,FILE=NAME,STATUS='OLD',MODE='READ')
C   WRITE(0,3)
C 3 FORMAT(/' output file name : '\)
C   READ(0,2) NAME
C   OPEN(UNIT=6,FILE=NAME,STATUS='NEW',MODE='WRITE')
C
C
C
C   opening temporary files
C
C
C   OPEN(UNIT=1,FORM='UNFORMATTED',STATUS='SCRATCH')
C   OPEN(UNIT=2,FORM='UNFORMATTED',STATUS='SCRATCH')
C   OPEN(UNIT=3,FORM='UNFORMATTED',STATUS='SCRATCH')
C
C
C   problem's name
C
C   READ(5,100)NAZWA
C   WRITE(6,101)NAZWA
C
C
C   control and material data
C
C   READ(5,102)NEN,NDF,NON,NOE,NBC,NLN,E,PR,TH

```

```
WRITE(6,103)NEN,NDF,NOE,NOE,NEC,NLN,E,PR,TH
```

```
C
C
C
```

```
coordinates
```

```
READ(5,105)(N(I),(CORD(I,J),J=1,2),I=1,NOE)
WRITE(6,104)
WRITE(6,106)(N(I),(CORD(I,J),J=1,2),I=1,NOE)
```

```
C
C
C
```

```
loads
```

```
READ(5,105)(NF(I),(RHS(I,J),J=1,2),I=1,NLN)
WRITE(6,107)
WRITE(6,106)(NF(I),(RHS(I,J),J=1,2),I=1,NLN)
```

```
C
C
C
```

```
elasticity matrix calculation
```

```
GASH=E/(1.-PR*PR)
D(3,3)=0.5*GASH*(1.-PR)
D(1,1)=GASH
D(2,2)=D(1,1)
D(1,2)=PR*GASH
D(2,1)=D(1,2)
D(3,1)=0.
D(1,3)=0.
D(2,3)=0.
D(3,2)=0.
```

```
C
C
C
```

```
REWIND 1
REWIND 2
REWIND 3
```

```
C
C
C
```

```
computing element's stiffness matrix
```

```
WRITE(6,108)
```

```
C
```

```
DO 10 I=1,NOE
  NOEL=I
  CALL TRIAST
```

```
10 CONTINUE
```

```
C
```

```
REWIND 1
REWIND 2
```

```
C
C
C
```

```
computing system's stiffness matrix
```

```
CALL EQAGEN
```

```
C
```

```
solving system of equations
```

```
C
```

```
CALL EQASOL
```

```
C
```

```
computing stresses
```

```
C
```

```
WRITE(6,109)
DO 20 K=1,NOE
  NOEL=K
  CALL STRESS
```

```
20 CONTINUE
```

```
C
```

```
REWIND 3
```

```
C
```

```
computing reactions
```

```
C
```

```
CALL REAC
```

```
C
```

```
C
```

```

100 FORMAT(A80)
101 FORMAT(1H1///5X,'firfef problem : ',A80)
102 FORMAT(6I5,E10.2,2F5.2)
103 FORMAT(///15X,'control data  '///
      * 20X,'number of element nodes           =' ,I3/
      * 20X,'number of degrees of freedom at the node =' ,I3/
      1 20X,'number of nodes           =' ,I3/
      2 20X,'number of elements =' ,I3/
      3 20X,'number of nodes with boundary conditions =' ,I3/
      4 20X,'number of loaded nodes           =' ,I3///
      5 15X,'material data  '///
      6 20X,'Young's modulus           =' ,E10.3/
      7 20X,'Poisson ratio           =' ,F6.3/
      8 20X,'element thickness           =' ,F6.3)
104 FORMAT(///15X,'coordinates of nodes'//
      1 15X,'node           x - direction           y - direction'//)
105 FORMAT(15,2G10.2)
106 FORMAT(17X,I2,6X,F10.4,6X,F10.4)
107 FORMAT(///15X,'loads'//
      1 15X,'node           x - direction           y - direction'//)
108 FORMAT(///15X,'topology '///15X,'element node's number '//)
109 FORMAT(///15X,'stresses'//
      1 20X ELEMENT,2X,'sigma x',6X,'sigma y',6X,'sigma xy',5X,
      2 20X 'sigma max',4X,'sigma min',4X,'angle'//)

C
      CLOSE(UNIT=1)
      CLOSE(UNIT=2)
      CLOSE(UNIT=3)
      CLOSE(UNIT=5)
      STOP ' --- ok ---'
      END

C
C
C
C
      SUBROUTINE TRIAST

C
C
C
C
      generates stiffness matrix for triangle element

C
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON NEN,NDF,NOE,NON,NBC,TH
      COMMON/CORD/ CORD(60,2)
      COMMON/D/ D(3,3)
      COMMON/NOEL/ NOEL
      DIMENSION ELST(6,6),LDES(6),BETA(3),GAMA(3),B(3,6),DL(3,3),DF

C
      topology

C
      READ(5,110) I,J,K
      WRITE(6,111) NOEL,I,J,K

C
      calculation of beta and gama coefficients

C
      BETA(1)=CORD(J,2)-CORD(K,2)
      GAMA(1)=CORD(K,1)-CORD(J,1)
      BETA(2)=CORD(K,2)-CORD(I,2)
      GAMA(2)=CORD(I,1)-CORD(K,1)
      BETA(3)=CORD(I,2)-CORD(J,2)
      GAMA(3)=CORD(J,1)-CORD(I,1)
      AREA=0.5*DABS(GAMA(3)*BETA(2)-GAMA(2)*BETA(3))

C
      generation of the strain-displacement matrix B

```

```

DO 100 L=1,3          25
M=2*L-1
N=M+1
B(1,M)=BETA(L)
B(1,N)=0.
B(2,M)=0.
B(2,N)=GAMA(L)
B(3,M)=GAMA(L)
B(3,N)=BETA(L)
100 CONTINUE
C
C   generation of the matrix DL
C
WS=0.25*TH/AREA
DO 200 L=1,3
DO 200 M=1,3
DL(L,M)=WS*D(L,M)
200 CONTINUE
C
C   generation of the matrix product D*B
C
DO 300 L=1,3
DO 300 M=1,6
DB(L,M)=0.
DO 300 N=1,3
DB(L,M)=DB(L,M)+DL(L,N)*B(N,M)
300 CONTINUE
C
C   generation of the stiffness matrix K=BT*DB
C
DO 400 L=1,6
DO 400 M=1,6
ELST(L,M)=0.
DO 400 N=1,3
ELST(L,M)=ELST(L,M)+B(N,L)*DB(N,M)
400 CONTINUE
C
LDES(1)=2*I-1
LDES(2)=2*I
LDES(3)=2*J-1
LDES(4)=2*J
LDES(5)=2*K-1
LDES(6)=2*K
C
C   writing to the storage
C
WRITE(1)(LDES(I),I=1,6),((ELST(I,J),J=1,6),I=1,6)
WRITE(2)(LDES(I),I=1,6),((DB(I,J),J=1,6),I=1,3)
C
110 FORMAT(3I5)
111 FORMAT(18X,12,5X,3I3)
RETURN
END
C
C
C
C
C   SUBROUTINE EQAGEN
C
C
C
C
C   generates the system of equations

```

```

C
  IMPLICIT REAL*8 (A-H,O-Z)
  COMMON NEN,NDF,NOE,NOX,NBC,TH
  COMMON/LOAD/NLN,NF(15),RHS(15,2)
  COMMON/UR/ GRAN(120,120),GRANR(120),NBAND,NSSW,NBCC(10)
  DIMENSION LDES(6),ELST(6,6)

C
  DO 100 I=1,120
  DO 200 J=1,120
  GRAN(I,J)=0.
200 CONTINUE
  GRANR(I)=0.0
100 CONTINUE
  NSSE=NEN*NDF
  NBAND=0
  DO 300 NE=1,NOE

C
  reading from the storage
C
  READ(1)(LDES(I),I=1,6),((ELST(I,J),J=1,6),I=1,6)

C
  generation of the equations coefficients
C
  DO 400 I=1,NSSE
  JD=LDES(I)
  IF(NBAND.LT.JD) NBAND=JD
  DO 500 J=1,NSSE
  KD=LDES(J)
  GRAN(JD,KD)=GRAN(JD,KD)+ELST(I,J)
500 CONTINUE
400 CONTINUE
300 CONTINUE

C
  writing to the storage
C
  WRITE(3) ((GRAN(I,J),J=1,120),I=1,120)

C
  DO 550 I=1,NLN
  J=NF(I)
  JJ=2*J
  JJ1=JJ-1
  GRANR(JJ1)=RHS(I,1)
  GRANR(JJ)=RHS(I,2)
550 CONTINUE

C
  taking into account boundary conditions
C
  WRITE(6,120)
  I=1
  DO 600 J=1,NBC
  READ(5,121) NN,NX,NY
  WRITE(6,122) NN,NX,NY
  NBCC(I)=2*NN*NX-1
  IF(NX.EQ.1) I=I+1
  NBCC(I)=2*NN*NY
  IF(NY.EQ.1) I=I+1
600 CONTINUE
  NSSW=I-1
  DO 700 J=1,NSSW
  K=NBCC(J)+1-J
  NBAND=NBAND-1
  NBAND1=NBAND+1
  DO 800 L=K,NBAND
  LI=L+1
  GRANR(L)=GRANR(L1)
  DO 800 M=1,NBAND1

```



```

      DO 900 I=1,NBAND
      J=2*I-1
      K=2*I
      WRITE(6,131) I,DISP(J),DISP(K)
900 CONTINUE
C
130 FORMAT(///,15X,'displacement of nodes ',//,6X,'node
      1x - direction      y - direction')
131 FORMAT(8X,I2,14X,G12.5,5X,G12.5)
      RETURN
      END
C
C
C
C
      SUBROUTINE STRESS
C
C
C
C
      calculates stresses
C
C
C
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON NEN,NDF,NGE,NGN,NBC,TH
      COMMON/NOEL/ NOEL
      COMMON/SOLV/ DISP(120)
      DIMENSION G(6),SIG(3),LDES(6),DR(3,6)
C
      reading from the storage
C
      READ(2) (LDES(I),I=1,6),((DR(I,J),J=1,6),I=1,3)
      DO 100 I=1,6
      K=LDES(I)
100  G(I)=DISP(K)
      DO 200 I=1,3
      DO 200 J=1,6
200  DR(I,J)=DR(I,J)*2./TH
      DO 300 I=1,3
      SIG(I)=0.
      DO 300 J=1,6
300  SIG(I)=SIG(I)+DR(I,J)*G(J)
C
      computing of main stresses
C
      C=(SIG(1)+SIG(2))/2.
      BB=(SIG(1)-SIG(2))/2.
      A=DSORT(BB*BB+SIG(3)*SIG(3))
      SMAX=C+A
      SMIN=C-A
      IF(SIG(2).EQ.SMIN) GO TO 700
      ANG=57.29578*ATAN(SIG(3)/(SIG(2)-SMIN))
      GO TO 210
700  ANG=90.
210  CONTINUE
C
      printing stresses
C
      WRITE(6,140) NOEL,(SIG(I),I=1,3),SMAX,SMIN,ANG
C
140  FORMAT(2X,I3,2X,5G13.5,F8.3)
      RETURN
      END

```

C

C

C

C

C

C

SUBROUTINE REAC

C

C

C

C

C

C

computing reactions

IMPLICIT REAL\*8 (A-H,O-Z)

COMMON NEN,NDF,NOE,NON,NBC,TH

COMMON/SOLV/ DISP(120)

COMMON/UR/ GRAN(120,120),GRANR(120),NBAND,NSSW,NBCC(10)

C

C

C

reading from the storage

READ(3) ((GRAN(I,J),J=1,120),I=1,120)

NP=NDF\*NON

DO 100 I=1,120

GRANR(I)=0.

DO 100 J=1,120

100 GRANR(I)=GRANR(I)+GRAN(I,J)\*DISP(J)

NP1=NP/2

DO 200 I=1,NP1

J=2\*I-1

K=2\*I

200 CONTINUE

IW = 0

WRITE(6,150)

DO 300 I = 1, NSSW

II = I + IW

L = NBCC(II)

IF(L/2.EQ.L/2.) GOTO 10

RX = GRANR(L)

L1=NBCC(II+1)

NR = L1/2

IF (L1.NE.L+1) GOTO 20

RY = GRANR(L1)

WRITE(6,12) NR, RX, RY

GOTO 40

10 RY=GRANR(L)

NR=L/2

WRITE(6,11) NR, RY

GOTO 50

20 NR=(L+1)/2

WRITE(6,13) NR, RX

GOTO 50

40 IW=IW+1

50 IF(I+IW.EQ.NSSW) GO TO 60

300 CONTINUE

60 CONTINUE

WRITE(6,14)

C

150 FORMAT(///,15X,'support reactions ',//,10X,' node',10X,  
# 'x - direction',3X,'y - direction',/)

11 FORM=T(10X,13,11X,' no support',10X,F11.4)

12 FORMAT(10X,13,11X,F11.4,10X,F11.4)

13 FORMAT(10X,13,11X,F11.4,10X,' no support')

14 FORMAT(1X,//)

RETURN

END