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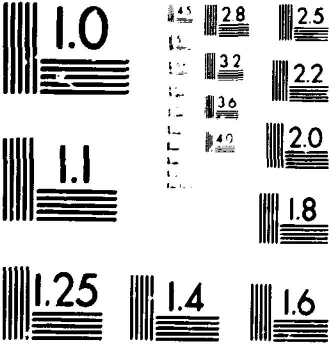

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PROCEEDINGS
OF THE SEVENTH INTERNATIONAL CONFERENCE
ON INPUT-OUTPUT TECHNIQUES

UNITED NATIONS INDUSTRIAL DEVELOPMENT ORGANIZATION
Vienna

PROCEEDINGS OF THE SEVENTH INTERNATIONAL CONFERENCE ON INPUT-OUTPUT TECHNIQUES

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Introduction

The Seventh International Input-Output Conference was held from 9 to 13 April 1979 in the Kongresshaus, Innsbruck, Austria, and was jointly sponsored by Austria and the United Nations Industrial Development Organization (UNIDO). It was attended by about 400 economists, econometricians and statisticians from throughout the world. Over 100 papers (a complete list is given later) were included under the following headings:

Global and Multi-National Models
Patterns of Economic Growth in Developing Countries
Income Distribution and Social Welfare
Europe in 1985 and After
Input-Output in Large Econometric Models
Planning and Optimization
Estimation, Adjustment and Comparison

Input-Output (I/O) analysis is now a mature tool of applied economic research and also an important element of national accounts statistics (in particular since the adoption of the revised System of National Accounts by the United Nations in 1968). The papers presented demonstrated the flexibility of I/O analysis, and that the basic framework can be adjusted to problems in various fields of economic research and can be incorporated into large econometric models.

The theme of the Conference was "Changes in the Structure of the World Economy". Reflecting this, a trend of internationalization of I/O analysis was observed at the Conference. The ILO method was originally designed for the analysis of the interdependence of activities within a single national economy. It continues to be valuable in this area, but is now increasingly employed to examine economies in relation to one another, either from the point of view of their structures, or of the relationships between changes in activity at national and international levels. Accordingly, I/O-based regional and global models have been developed, and several papers dealing with them are included here.

A further aspect of the internationalization of I/O analysis is the subject of standardization of I/O data, in order to permit comparative analysis of the structures of different economies. The problems of such standardization are being given increasing attention. To date, standardization has been principally carried out for the tables of the developed countries, but it is to be expected that future work will concentrate more on the developing countries, for which basic

I/O data is increasingly available. Such comparative studies have raised interesting methodological problems (such as the need to create a common value basis for the market and centrally planned economies) and have also demonstrated the impact of differences both in methods of compilation and of definition on the international comparability of I/O data.

One effect of the Conference was to bring out very clearly the importance of accumulated I/O data. Long time series of comparable I/O tables are an excellent basis for the investigation of various aspects of structural changes related to economic growth at various levels of development. Such data also allow a synthesis of international comparisons and analysis of structural change. Most importantly, it becomes possible to improve understanding of the stability and variability of I/O coefficients among countries and over time, and thus to improve projection of coefficients.

The importance given to those two trends, however, does not mean that other more traditional applications of I/O analysis were not treated at the Conference. The list of topics and of papers gives a good indication of the flexibility of the method.

UNIDO, as one of the sponsors of the Conference, together with the International Programme Planning Committee, has undertaken to publish a selection of papers presented in Innsbruck. However, the choice, among more than one hundred papers, was not easy. The selection of papers tends to concentrate on the main fields of interest of UNIDO, and special emphasis was placed on international analyses, structural change and national planning areas. A balanced representation of applications to different economic systems was also a criterion. Accordingly, this publication should be considered as a selection of those conference papers which deal mainly with growth, economic structure and industrialization, and which are expected to be of interest to those who are close to economic policy making.

The publication is divided into three parts. The first part includes international I/O studies, and is divided into two groups of papers. The first group deals with I/O global or intercountry models; the second group deals with international trade and co-operation. Two teams working on world models are represented. The World Bank uses a world I/O model as background for the Bank's regular World Development Reports. Studies from United Nations Headquarters in New York are long-term oriented, and focus on investigation of future development alternatives and potential growth bottle-necks. The group of papers on world models also contains an intercountry I/O model which depicts the interdependence of economic structures in south-eastern Asia, and this work has been carried out for the Economic and Social Commission for Asia and the Pacific. Another system—of interrelated dynamic I/O models sharing a similar or even identical framework, which are linked by a matrix of international trade—is being developed in close co-operation with research institutions in several countries, the original model having been developed in the University of Maryland, in the United States of America. All studies in the first part have certain common features, such as a dynamic approach to the analysis of interdependence of countries or regions, the incorporation of foreign trade matrices into the network of I/O models, and attempts to find a common value base for data which are originally priced in different national currency units.

The second group of papers includes papers which are less homogeneous. They all deal with foreign trade and co-operation among countries, but they approach this topic from different angles. One is the distribution of development aid and its impact on the international division of labour, the other the search for optimal patterns of international economic co-operation.

A more methodologically oriented paper on an econometric model of international trade deals mainly with shifts in trade shares among the developed countries. Another analysis deals with the impact of population on the pattern of regional development for the developing countries of south-eastern Asia.

The second part of the book is devoted to national applications of I/O analysis. This part is divided into three groups of papers, the first of which includes economy-wide national models; the second, studies on the interrelation between economic growth and structural change; and the third, sectoral prices and income distribution.

The group of papers on national models is a relatively representative selection of the state of the art in different parts of the world. The Cambridge IDIOM dynamic I/O model is intended to be a generalized system applicable to most national economies. The contributions from Hungary and the Union of Soviet Socialist Republics give some insight into the use of dynamic I/O models in two countries with centrally planned economies. The social accounting matrix for Egypt is an example of this extension of I/O concepts in a developing country. Another paper deals with the application of I/O analysis to a large and rapidly changing economy, China.

Economic growth is always a process of structural change, and I/O analysis, which puts such strong emphasis on interindustry flows, is a most useful tool for its investigation. The group of papers on this topic includes four papers with one common feature: they all relate certain changes in the economic structure to certain sources (or factors), for example to changes in technology, changes in demand, changes in international competitiveness etc. The analytical approaches used all show the complexity of the process of structural change. The topic of the fifth contribution, dealing with the experience of the RAS method in the updating of I/O tables is different, but the message of the paper is very important. It shows that little can be expected from mechanical projections of changes in input coefficients and that much more attention should be given to other information.

The third group of papers of this part includes papers on sectoral prices and income distribution problems. The three papers included represent, to a large extent, what could be called advanced applied I/O research. The topics of price formation, related to intersectoral terms of trade, and of income distribution, are very important, but too difficult to be investigated by simple models. All three I/O models presented here are therefore rather sophisticated. All of them are rather reports on research progress than final statements on the topics.

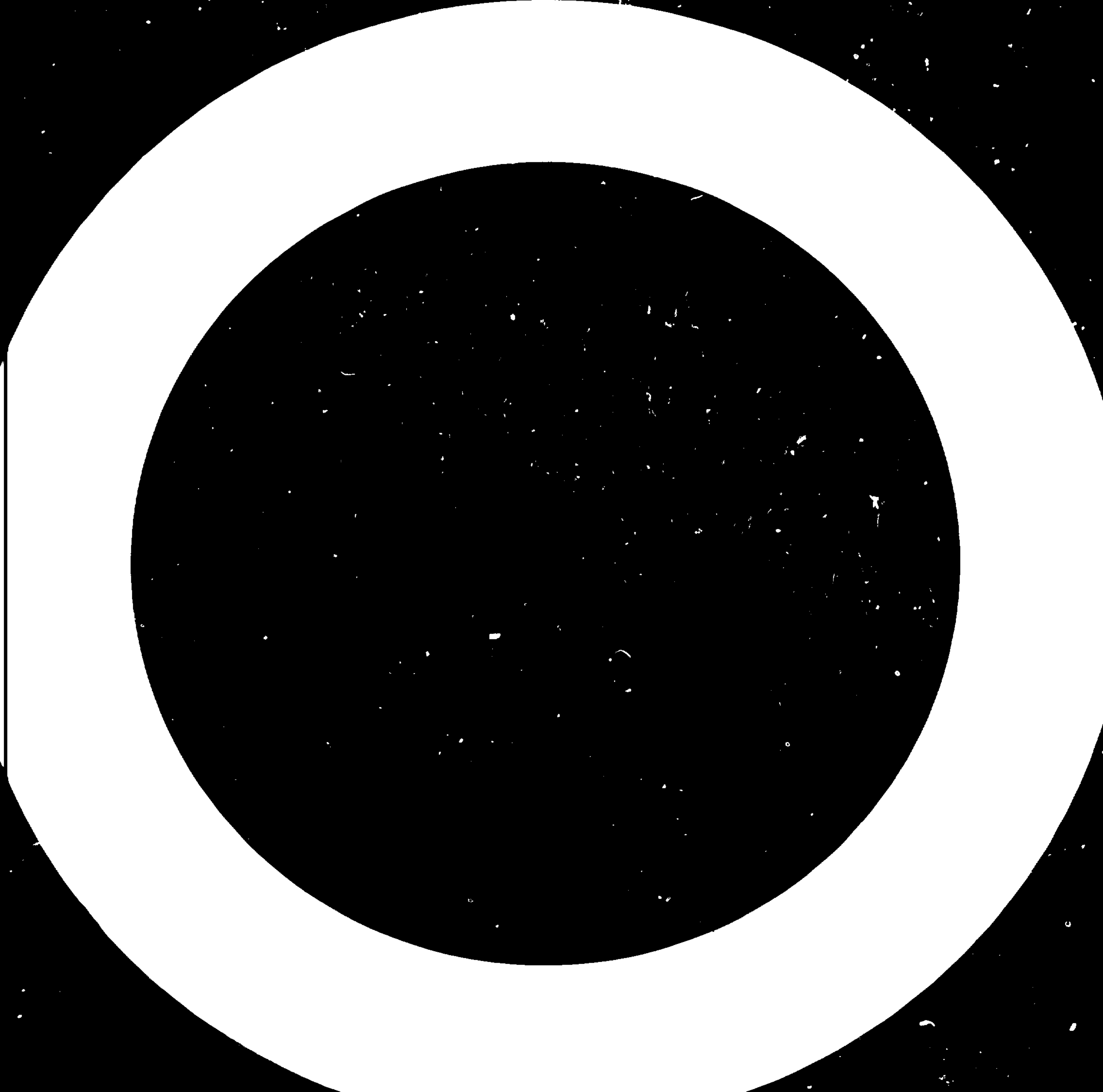
The last morning of the Conference was devoted to a round table discussion at which a paper by Richard Stone, on the present state of I/O analysis, was introduced. The paper is reproduced in the third part of the publication. Although it is the last paper in the volume, it is felt that it summarizes many of the issues, and could therefore be read as an introduction to this publication.

This volume was assembled and edited by the UNIDO secretariat. Jiri Skolka assisted in its preparation. The opinions expressed are those of the individual authors.

Part one

International input-output

World models



United Nations global modelling: experimental projections on the basis of alternative procedures

Antonio Maria Costa *

The United Nations input-output model

The I/O version of the United Nations model is an important example of non-optimizing world models. It partitions the world into 15 economic regions: (1) North America; (2) Latin America (medium income); (3) Latin America (low income); (4) Europe (high income); (5) Europe (medium income); (6) Soviet Union; (7) Eastern Europe; (8) Asia (centrally planned); (9) Asia (high income); (10) Asia (low income); (11) Middle East-Africa (oil producers); (12) Africa (arid); (13) Africa (tropical); (14) Africa (medium income); and (15) Oceania.[1] Each region is treated as a homogeneous whole, with region-specific sub-models.

Each of these sub-models consists of an I/O system with 175 linear equations and 269 variables, of which 229 are region-specific. The remaining equations cover interregional trade and payments, and enter into the world linkage system. The total world model contains 2,710 linear relationships in 4,035 variables, with the resulting degrees of freedom taken up by exogenous variables. Since it is possible—within certain limits—to shift individual variables between the endogenous and exogenous categories, the system can be flexibly used to address a broad range of policy problems.

The I/O components of the regional models are conventional. For each region there are 43 industries. Agriculture accounts for 5 of them, mineral resources account for 10, manufacturing for 22, and services for 6. The data problems encountered in the process of assembling the I/O data were enormous. The input coefficients for Europe, North America, Latin America, Japan and the Soviet Union were obtained from national sources. Construction of the input coefficients for the centrally planned region of Asia, most of which is China, and Africa was extremely difficult, because of paucity of data. Cross-country regression analyses were used to estimate input coefficients when adequate specific data were not available.

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[1]For a detailed review of the country coverage in each region see Leontief [1].

The United Nations I/O model consists of three systems of relationships:

(a) System I (regional sub-models). This system measures gross output and factor requirements corresponding to given levels of final demand. Therefore, it provides a specification of each regional economic structure;

(b) System II (dynamic methodology). This system generates a growth path of each region which is discrete and computed through a dynamically recursive procedure. Therefore, it guarantees consistency over time of each regional model's solutions;

(c) System III (world closures). This system uses trade and other payment flows to integrate the regional economies into a world system. Therefore, it guarantees consistency over space for all 15 regional solutions.

System I (regional sub-models)

Each regional sub-model consists of blocks of linear relationships which cover production, import demand, and all major national accounts variables. Among these are:

(a) The supply of commodities and services. Economic activities are disaggregated into five broad groups including processing of renewable primary products; processing of non-renewable resources; manufacturing of tradeable commodities; construction; and provision of services. Their gross supply is given by an identical I/O specification:

$$X_i = \sum_j A_{ij} X_j + \sum_s A_{is} X_s + \sum_a A_{ia} X_a + c_i C + g_i G + i_i I + (E_i - M_i) \quad (1)$$

for $i = 1 \ldots 25$ and $\sum_i c_i = \sum_i g_i = \sum_i i_i = 1$

where:

X_j = the interindustrial output in processing;
X_s = the interindustrial output in resource extracting;
X_a = the interindustrial output in pollution abatement activities;
C = private consumption;
I = investment;
E_i = export;
M = import;
G = public consumption.

(b) Supply of primary resources is given by:

$$X_s = \sum_j A_{sj} X_j + (E_s - M_s) \quad (2)$$

for $s = 1 \ldots 10$.

Extracting activities do not take into account domestic final demand usage, nor intermediate requirements in the primary industries. They only account for the flow of products into processing and manufacturing.

(c) Pollution abatement activity is given by:

$$X_a = (\sum_j A_{aj} X_j) + (\sum_j A_{as} X_s) + c_a C - Q_a^* \qquad (3)$$

for a = 1...8.

Total abatement requirement (X_a) of pollutants is measured as the difference between the level of pollution created by economic activities and the household sector, minus the emission of abatable pollutants (Q_a^*). Total regional emission Q is given by the sum of the abatable (Q_a^*) and the non-abatable (Q_n^*) pollutants which are set exogenously:

$$Q = \sum_a Q_a^* + \sum_n Q_n^* \qquad (4)$$

(d) Demand of primary factors is given by:

$$L = \sum_j l_j X_j + \sum_s l_s X_s + \sum_a l_a X_a + l_c C + l_g G + \sum_a l_a I_a \qquad (5)$$

$$K_q = \sum_j k_{qj} X_j + \sum_s k_{qs} X_j + \sum_a k_{qa} X_a + k_q C \qquad (6)$$

$$T = \sum_j t_j X_j + \sum_s t_s X_s + t_c C \qquad (7)$$

Each region's total demand for labour (L), fixed capital ($K_q, q = 1,2$ to distinguish between plants and equipment) and cultivated land (T) covers requirements by production activities and by final demand.

(e) Private and public consumption is given by:

$$C = Y^* - (I + G + \Delta S + (E - M)) \qquad (8)$$

$$G = g Y^* \qquad (9)$$

Private and public consumption are expressed as a residual and a fixed share g respectively of each gross regional product which, in the control solution, are set exogenously as Y^*. Private consumption is then disaggregated into a basket of commodities and services on the basis of an exogenously set share parameter c_i:

$$C_i = c_i C \qquad (10)$$

for i = 1...43, and $\sum_i c_i = 1$.

(f) Investment expenditure is given as:

$$I = I_e + I_p + p_e I_l + p_g I_g \qquad (11)$$

where p_e and p_g are exogenous conversion parameters to find the value level of physical quantities (acres of land).

Investment expenditure is thus given by the sum of capital formation in equipment (I_e), plants (I_p), land reclamation (I_l), and irrigation (I_g). The variables I_l and I_g, are expressed in physical terms. Each of these components is obtained as the increment of capital requirements between any two time periods, after depreciation. Because of this intertemporal feature, specification of these investment functions is included below in system II.

(g) Imports of commodities and resources are given by:

$$M = \sum_j p_j M_j + \sum_s p_s M_s \tag{12}$$

$$M_j = m_j X_j \tag{13}$$

$$M_s = X_s - \sum_j A_{sj} X_j - E_s - C_s - I_s - G_s \geqslant 0 \tag{14}$$

Purchases from abroad are measured in value terms by multiplying import volumes of processed goods (M_j) and of primary resources (M_s) by exogenously given price parameters (p_j and p_s). Import demands by industry (M_j) are derived on the basis of historical propensities m_j in gross supplies by industry X_j. Input demands for primary resources (M_s), which are constrained to be non-negative, are obtained as residuals between gross domestic supply X_s minus intermediate and final uses.

(h) Foreign capital and aid flows (KF_n) are given by the difference between investment-driven outflows (KF_e) and regional inflows (KF_m), set as a fixed share of total world capital movements:

$$KF_n = KF_e - KF_m \tag{15}$$

where $KF_e = k_e I$.

A symmetric approach is used to measure aid flows. Inflows are driven by gross regional product levels (Y^*) and supply is given as a fixed share of total world aid.

$$AF_n = AF_e - AF_m \tag{16}$$

where $AF_m = a_m Y^*$.

(i) The saving gap is given by:

$$SG = s\,Y - (I + \Delta S) + (AF_n + KF_n) \tag{17}$$

The difference between gross regional savings (given as a fixed share of regional gross national products), and regional capital formation ($I + \Delta S$), with net aid and capital flows ($AF_n + CF_n$), gives the total saving gap for each region.

(j) Income on foreign investment is given by:

$$FY = b\,SFA \tag{18}$$

Income on foreign investment is measured on the basis of a predetermined rate of return (b) applied to the total stock of foreign assets within the region (SFA).

System II (dynamic methodology)

The model is made dynamically recursive by means of a set of difference relations applied to the system's stock variables over discrete time periods (10-year intervals).

(a) Investment expenditure is given by:

$$I_{q,t} = a_{q,t}(1 - d)K_{q,t-1} \tag{19}$$

for q = 1, 2.

The demand level of each type of investment expenditure is produced by the difference in the capital requirements during production ($K_{q,t}$) between two terminal years corrected by depreciation allowances, d, and the stock-flow conversion coefficients for plants and equipment.

(b) Inventory changes are given by:

$$\Delta S_t = \sum_j \beta_j (X_{i,t} - X_{i,t-1}) + \sum_s \beta_s (X_{s,t} - X_{s,t-1}) \tag{20}$$

Similarly, for each type of economic activity inventory changes are measured as predetermined shares of incremental output between any two time periods.

(c) Stock of foreign assets is given by:

$$SFA_t = SFA_{t-1} + B_t \tag{21}$$

The old stock is corrected by the region's balance of payments in the current period.

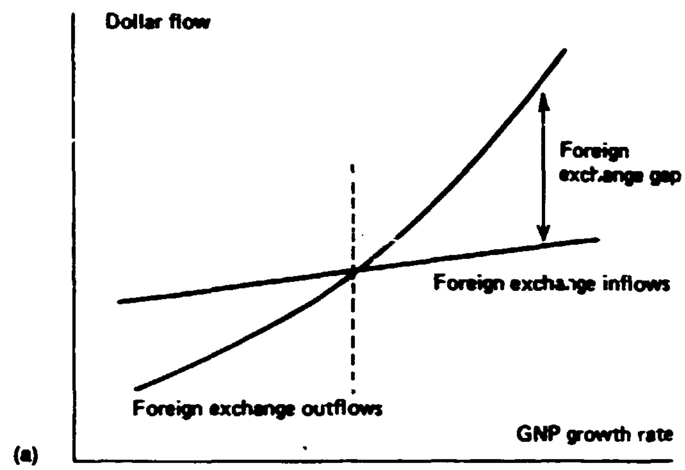
(d) Population levels are given by regional rates of growth, by region, estimated by the population division of the United Nations and introduced in the model exogenously. Their projection was done using the cohort-component method.

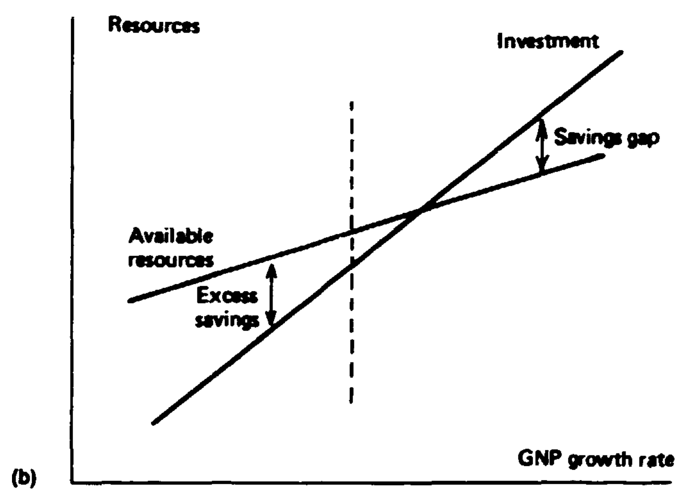
(e) In the control solution of the model as well as in a scoreof other scenarios gross regional products (Y^*) are exogenous. Their per capita level in each region has been set at different levels to test alternative world development paths. In a simulation they were set in such a way as to reduce the per capita income gap between the developing and the developed regions roughly by half.

System III (world closures)

The linkage relationships used in the United Nations I/O world model bring into line the internal and external variables through consideration of both the foreign exchange and the savings constraints of the type described in the two-gap models. The role of these constraints in income determination is illustrated in figure 1, taken from Carter and Petri [2]. Figure 1 *(a)* shows the foreign exchange

Figure 1. Disequilibrium conditions between the internal and the external gap in developing countries according to the input-output version of the world model of the United Nations





imbalances which correspond to alternative income growth paths. Figure 1 *(b)* shows excess demand and excess supply of savings at different growth rates of GNP. Since the two savings and foreign exchange equilibrium positions do not occur at the same vertical intercept on the horizontal axis, a set of world closures is used to supplement internal sources of financing by means of external flows.

The linkage relationships used to guarantee overall balancing of world flows of various types of US dollar denominated trade and payments are expressed on the basis of historically determined weights.

(a) Regional exports of resources and manufacture are not measured as import demand functions of other regions but rather as shares e_j of total world trade ($WTRD_j$) of any given commodity j or primary resource s. Therefore, regional sales abroad are:

$$E_j^r = e_j^r \, WTRD_j \tag{22}$$

where $WTRD_j = \sum_r M_j^r$ and $\sum_r e_j^r = 1$

$$E_s^r = e_s^r \, WTRD_s \tag{23}$$

where $WTRD_s = \sum_r M_s^r$ and $\sum_r e_s^r = 1$.

(b) Foreign capital inflows and foreign aid outflows are treated symmetrically on the basis of the same world pool notion:

$$KF_m^r = k_m^r \, WKF \tag{24}$$

where $WKF = \sum_r KF_e^r$ and $\sum_r k_m^r = 1$, and

$$AF_e^r = a_e^r \, WAID \tag{25}$$

where $WAID = \sum_r AF_m^r$ and $\sum_r a_e^r = 1$.

(c) Each regional balance of payments is given by:

$$B^r = \sum_j p_j(E_j^r - M_j^r) + \sum_j p_j(E_j^r - M_j^r) + CF_n^r + AF_n^r + FY_n^r \tag{26}$$

The input-output control solution and its characteristics

The several scenarios analysed in *The Future of the World Economy* [1] embody alternative growth assumptions about gross regional products. These assumptions are the main control variable of the model.

One basic scenario used growth targets set by the International Development Strategy of the United Nations. Because of divergent growth trends in population and income, the per capita income growth for both the developed and the developing regions converged to the same yearly rate of 3.5 per cent which corresponds to the long-term trends for industrial countries. This left the 1970 income gap between the two groups of countries unchanged in the projected period (1970–2000).

In another scenario the per capita income growth in the South was fixed at 4.9 per cent, while the corresponding rate for the developed countries was set below historical records, at 3.0 per cent. These rates could reduce roughly by half, by the end of the century, the income gap between the poor and the rich countries. Other scenarios simulated after publication of the original report reflect ongoing policy research within the United Nations and its agencies on development issues.

Selection of appropriate levels for the system's control variables and the testing of different scenarios has been the first stage in the use of the model to sketch alternative projections of the world economy. Research has since continued to study ways and means to utilize the unique wealth of data available for the model and its structural flexibility. A variety of considerations has been made on how to take advantage of this flexibility and, through limited respecification of the model's structure, to enhance its ability to address new concerns. A sample of the major points which emerged in the process is listed below.

(a) Primary factors. Because of the model's demand orientation, there are no explicit factor constraints to production. As primary input requirements are determined on the basis of output levels and factor intensities with supply assumed to be infinitely elastic in each region, the issue of the feasibility of alternative growth paths cannot be addressed.

(b) Balance of trade. Imports increase in proportion to outputs of industries, and exports increase according to exogenous shares in a world trade pool. Since developing regions' income growth is assumed to increase so as to reduce the existing income gap while their export shares are frozen at historical levels (the converse holds for developed countries), the results produce huge trade balance disequilibria.

(c) Adjustment mechanisms. Balances of payments do not provide constraints upon growth in the model nor do they trigger behavioural or structural changes. As a consequence some resource-endowed regions are projected to reach staggering surpluses by the end of the century. The underlying philosophy is very close to a primary resources theory of value, which jams any long-run mechanism to moderate the drastic effects of industrial terms of trade changes.

(d) Behavioural criteria. Construction of a development strategy implies the investigation of different behavioural criteria so as to provide a choice among alternatives. However, the model's I/O structure cannot make allowance for multiple technologies, consumption baskets and investment policies. As a consequence, the desirability of alternative courses of action can be assessed at best by trial and error procedures.

(e) Price and value relations. Because of its stress on physical flows (the "primal" problem), a comprehensive price mechanism (the "dual" problem) is missing in the model. However, the evaluation of factor and commodity prices is required in order to measure the relative efficiency of production within any given region, and to study the effectiveness of policies to alter existing terms of trade among regions and products.

(f) Trade mechanism. Trade is regulated by the model's world closures which drain and refill a world trade pool for each of the tradeable goods included in the model. However, in order to study inter- and intra-regional commercial strategies and the structural changes underlying them, bilateral trade flows have to be incorporated explicitly in the world model. In addition, trade volumes have to be made sensitive to regional, industrial and commodity terms of trade variations as well as to changing income conditions.

Because of these and other considerations an optimization version of the United Nations model has been under construction and testing since 1979.

The DYNAMICO programming model

Introduction and overview of the model

The general concern of providing the United Nations with effective and flexible tools for policy analysis and design has stimulated the probing of several, complementary, econometric simulation procedures. Among the techniques recently tested, the decomposed multi-level version of recursive programming was singled out for its explanatory ability and computational flexibility.

The resulting modelling framework, DYNAMICO, provides information on inputs and outputs (activity levels and factor requirements) as well as price relations (terms of trade, interest rates, and in general exchange values) at the regional as well as the global level. Because of its programming nature the model also produces an order-of-magnitude assessment of the opportunity cost of alternative resource management policies in the economies under consideration. Its final results reflect the interaction between primal and dual variables. These results shed light on the production and exchange linkages within each regional system and among systems, and provide some indication about the allocative implications of different world trade and development patterns.

Information on these socio-economic and technological interrelationships is of critical import in the study of the transmission mechanism of economic phenomena over time, space and production sectors. The model's next most striking feature is its ability to handle region-specific development priorities. Since each regional model is solved under several strategy functions and on the basis of alternative socio-economic policy assumptions, the coalescence of development patterns in the world at large into arbitrarily identical moulds is avoided. At the same time, because allowance is made for the supply of goods and services by alternative input-mixes, the model displays some analytical capability also in addressing the issue of endogenous technologies. The ultimate objective of the research is to provide an instrument able to contribute to the definition of national and international policies which could enable rich and poor nations to pursue development in line with their dynamic, long-term comparative advantage in an integrated world economic system.

DYNAMICO is based on a geographical partition of the world into 5 developed and 5 developing regions. The 10 regions are each treated as a homogeneous whole on the basis of a representative matrix of intermediate and final flows of commodities and primary resources, and of a vector of policy targets.

Developed country or region	*Numerical code*
North America	1
Western Europe	2
USSR	3
Eastern Europe	4
Japan and Oceania	5

Developing region	
Latin America	6
Oil exporting economies of North Africa and the Middle East	7
Other Africa	8
Developing countries of Asia	9
Centrally planned economies of Asia	10

The industry disaggregation is suitable for addressing some major socio-economic concerns. (A system with a different disaggregation is under study at the Academy of Sciences of the USSR in Novosibirsk [3].)

Renewable primary products, excluding grains
Grains
Non-renewable primary products, excluding fuels
Petroleum
Energy products, excluding petroleum
Intermediate goods
Consumer goods
Investment goods
Construction
Services

The model is designed to find such a set of regional development strategies which is both feasible, namely consistent with regional resource balances, and also in some sense efficient. The solution algorithm, known for its great computational flexibility, is based on a sequence of simulations which assume rational economic behaviours (Dantzig and Wolfe [4]).

At first each region solves its own resource allocation problem in a pre-linkage format. This assumes that the rest of the world is willing to purchase or to sell commodities which, at the regional level, are in excess supply or excess demand. Outward linkages, expressed in terms of each region's import and export volumes of goods, services and capital, are thus extended. In the next stage world markets for all traded resources are tested for global consistency. Whenever the aggregate world demand of a given resource diverges from its availability, policies are proposed to stimulate either demand or supply, depending on the type of bottle-neck, and to drive net resource balances to zero. These inward linkages—expressed in terms of shadow prices or other signals—are then brought to bear upon the regional systems. The interaction of the external environment with the level and the structure of domestic activity in each region shifts the international division of labour towards new patterns.

The process is repeated for several iterations until, after a finite number of experiments, zero excess-demand conditions prevail on all commodity and financial markets. The resulting regional development strategies are then feasible and, within the world system, meet the Pareto condition for optimality. Therefore, they are also, both individually and globally, the most efficient.

DYNAMICO is based on a decomposed programming algorithm, with one post-linkage core (the master or global problem) and hundreds of pre-linkage modules (the satellite or regional structures). In order to determine the most desirable time profile in the allocation of world resources with policies, factor endowments and production structures which change over time and reflect increasing cost conditions, recursive programming is used. Among the dynamic simulation techniques, the recursive approach was selected because of its ability to capture behavioural and structural shifts from one stage of the analysis to another, over a quarter of a century. The result is a policy-oriented analytical structure useful for investigating the development potential of the world economy, where the north-south and east-west frontiers inevitably recall plurality of interests and competition of goals.

Regional models

Commodity markets

In its present version the model includes 10 types of industry outputs, each produced by several, alternative—but not mutually exclusive—technologies. The generalized production function for X_{ij}, the i^{th} output by the j^{th} technology, is given by the familiar material balances relation

$$\sum_{j}^{z(k)} X_{ij} - \sum_{k} \sum_{j}^{z(k)} A_{ijk} X_{jk} - \sum_{j} c_{ij} C_j - \sum_{j} v_{ij} I_j - g_i G - s_i DS - (E_i - M_i) \geq 0 \qquad (27)$$

for $i = 1, 2 \ldots n;\ k = 1, 2 \ldots k;\ j = 1, 2 \ldots z(k)$

where $z(k)$ is the maximum number of technologies for good k.

The interindustrial coefficient a_{ijk} represents the requirement of output from industry i into production of one unit of good k, by technology j. In addition to multiple technologies, there are also different consumption baskets C_j and several types of gross fixed investment I_j.

Each c_{ij} gives the share of the i^{th} type of good in the j^{th} type of basket. Similarly, each coefficient v_{ij} provides the share of the i^{th} type of good in the j^{th} type of investment (plant, equipment, land improvement, irrigation etc.). Shares are also used to break down government consumption G and inventory accumulation, DS. Naturally:

$$1 = \sum_{i} c_{ij} = \sum_{i} v_{ij} = \sum_{i} g_i = \sum_{i} s_i \qquad (28)$$

for $i = 1, 2 \ldots n;\ \ j = 1, 2 \ldots q$.

The above production function with alternative technologies relaxes several of the restrictive assumptions of the input-output version of the United Nations model [5].

As is widely known, although the a_{ij} coefficients used in the material balances of a pure I/O formulation are unique and invariant with respect to the level of output and relative prices, the a_{ijk} used there introduce the possibility of choosing among alternative combinations of input vectors, each of which yields a given amount of output. This is especially valuable in those applications of the model where the results are likely to be affected by a changing scale of operations in each industry: a situation encountered, for example, in the study of economic growth problems and of shifts in the pattern of comparative advantage. Generalized production functions with alternative input mixes can also handle cases of decreasing returns to scale in some technologies, but constant returns in others. In some cases the production functions are constrained so as to allow non-homothetic production situations, where the ratio of any two input-output coefficients changes with the level of production, within the same technology.

Factor markets

Shortages of capital assets place a severe constraint on the expansion of output and, in general, of economic activity. The demand of each K_i type of capital assets is given by:

$$K_i \geqslant \sum_y \sum_j^{z(y)} k_{ijy} X_{jy} = \sum_j k_{ij} C_j = k_i G + k_i UP \tag{29}$$

for $i = 1, 2 \ldots f$.

There are several types of capital assets ($f = 5$), distinguished by usage or ownership. These assets include plants, equipment, arable land, inventory, and foreign-owned capital. The requirements for each of the first four types are determined by:

(a) Production according to the j^{th} type technology in each y^{th} type of output ($y = 1, 2 \ldots m$);

(b) Varieties of final demand;

(c) The need of infrastructural construction by the urban population (UP).

Their sum cannot exceed the availability, K_i. Foreign-owned capital is treated differently, as seen below.

An identity which sums up the demand for all types of accumulation (equipment, plants, inventory, irrigation and land improvement) determines each region's total investment demand I. In another identity national savings, S, is found by adding to total investment, I, net factor income from abroad, NFY, and the net trade balance, TB:

$$I = \sum_i I_i \tag{30}$$

$$S = I + NFY + TB \tag{31}$$

The net factor income from abroad, NFY, is measured on the basis of an exogenous rate of return charged on foreign-owned capital assets.

Although the model includes several types of capital assets, the labour factor is assumed to be of a single, homogeneous skill. Total labour demand is determined by:

(a) Production according to alternative j technologies in each y^{th} type of product;

(b) Various types of final demand;

(c) The need for infrastructure by the urban population.

The equation for total labour demand is:

$$L_d \geqslant \sum_k \sum_j^{z(k)} l_{jk} X_{jk} + \sum_j l_j C_j - \sum_j l_j I_j - lG - lUP \qquad (32)$$

Total labour supply, L_s, is determined on the basis of an exogenous participation rate applied to population estimates by the Population Division of the United Nations.

The stock of arable land available in the base period, H (0), is exogenously given. Through time its supply is expanded by means of investment in irrigation and land improvement, IR, which is endogenously determined. The corresponding constraint states that total land requirement cannot exceed demand:

$$H(0) + \sum_t h\,IR(t) > \sum_k \sum_j^{z(k)} r_{jk} X_{jk}(t) \qquad (33)$$

where h is a conversion factor and r_{jk} is the unit requirement (in hectares) of land by the j^{th} sector in the k^{th} technology.

Macro-economic identities

Private consumption, C, is determined by means of a consumption function which incorporates some aspects of the permanent income hypothesis:

$$C(t) \geqslant g(\sum_{i=0} q^i GNP(t-i)) \qquad (34)$$

where t represents time periods, q are geometrically declining income weights, and g is the short-term marginal propensity to consume. By means of the Koyck transformation the consumption function can be reformulated into the much simpler relation:

$$C(t) \geqslant qC(t-1) + gGNP(t) \qquad (35)$$

Government expenditure G is set on the basis of an exogenous parameter g:

$$G \geqslant gGNP \qquad (36)$$

The gross national product, GNP, is derived as an identity on the basis of its expenditure components:

$$GNP = C + G + S + (E - M) \qquad (37)$$

which includes national savings, itself an identity, as seen above.

The trade balance summarizes the commercial position of the region at any point of time, with exogenously provided c.i.f./f.o.b. charges:

$$TB = \sum_j (pe_j E_j - pm_j M_j) \qquad (38)$$

Under pre-linkage conditions export and import prices for the j^{th} tradeable commodity, pe_j and pm_j, are assumed to be equal to unity with addition, in the case of import prices, of transportation charges. In post-linkage computations, they are endogenized and are retrieved as shadow prices generated by the world model, as shown below.

The external sector

As to be expected from a model geared to the study of the interaction of economic activities which take place in several world regions, the foreign sector commands the greatest attention. Its modelling takes into account several widely shared considerations concerning trade behaviours. In the first place, imports are significantly and positively related to movements in aggregate economic activity, within each region. In the second place, in a number of regions lack of foreign exchange places yet another major constraint on imports. The price of imported goods relative to domestic ones is also an important consideration. While the literature has shown that accurate measurement of the price elasticity of imports has proved elusive, a useful proxy is the resource requirements for import substitution.

Exports are thought to be less related to other economic activities within the region, and to depend more directly on the income of trading partners. However, this tendency is mitigated in two respects. Firstly, exports (like imports) depend on relative prices so that the price of domestic goods and services produced for export relative to the price of other regions' goods is a critical variable in explaining trade behaviours. It should be noted that, as at the present time the model uses exclusively real variables measured in constant 1970 dollars, the price of a commodity or service as used here is the opportunity cost of the resources required for its supply. Secondly, the expansion of GNP and therefore imports in a given region affect the exports (and therefore, GNP) of trading partners to the extent that exports depend on the size of GNP of the impulse-region and on cross-trade elasticities. Foreign trade multipliers are thus set in motion. Growing external demand is likely to affect the impulse-region's exports, and so on in several rounds of interaction.

Under the pre-linkage format, each regional model is constrained by nothing more than a maximum level of imports for tradeable goods and services. This level is predetermined on the basis of an import propensity coefficient m and an elasticity factor e applied to the balance of payment situation of the previous period, BP:

$$M_{j,t} \leqslant m\,X_{j,t} + e\,BP_{t-1} \qquad (39)$$

Under pre-linkage conditions exports are treated as slack variables. As nothing is known about partner-regions' import demand functions, exports take place only if there is excess supply of the given good on domestic markets.

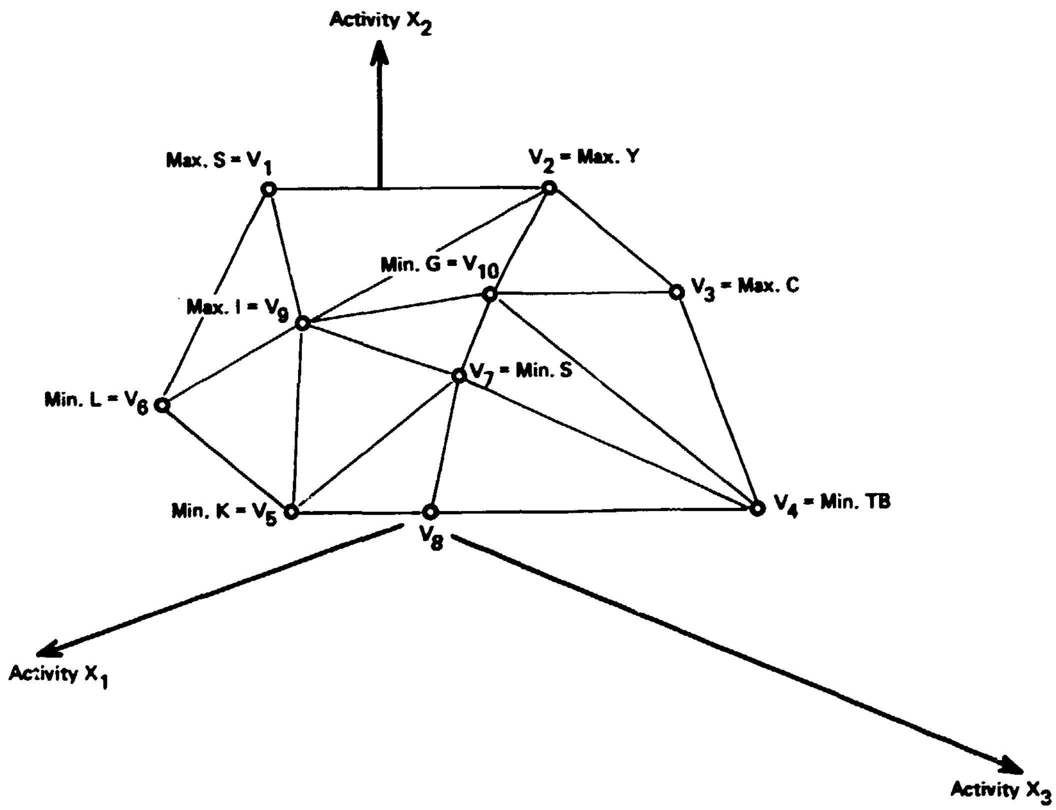
This apparent exceedingly simplified treatment of imports and, especially, of exports is greatly misleading. Extensive regional linkages are at the very core of the model.

Pre-linkage solutions

DYNAMICO is solved by means of an iterative procedure which first identifies a large number of optimal solutions for each regional model taken in isolation; these are the pre-linkage problems. Their solutions, or regional development proposals as they are often referred to, are obtained on the basis of alternative, often radically different behavioural assumptions.

The aim of this search for multiple optimal solutions is to fathom the production-possibility polyhedron which corresponds to the carrying capacity of each regional economy, and to identify as many vertices (or complexes) as possible in the set. Since each complex, as shown in figure 2, represents feasible and optimal pre-linkage solutions, any convex combination of them will also be feasible and optimal.

Figure 2. A production-possibility polyhedron, showing the carrying capacity of a region. This is a hypothetical example of complexes identified by means of multiple objective functions (pre-linkage solutions) for a 3-industry case (X_i, i = 1, 2, 3)



In the next phase a new procedure, the master or global programme, computes a series of weights to be assigned to these regional development proposals, so that world markets for all types of transactions (goods, services and financial flows) are cleared. On the basis of these weights it is then possible to compute a preliminary (first iteration) post-linkage solution of the world model.

While individually optimal, these regional solutions may not be as yet the most efficient from a global perspective, and the search for additional complexes has to continue. Before showing how the post-linkage problem is solved, it is important to provide an idea of the range of behavioural options under testing.

At the present time each regional problem, which consists typically of some 100 equations and about 120 variables, is solved under 15 objective functions. Namely a total of 150 linear programming problems are solved to scan the 10 regional production-possibility polyhedrons at any time. Over 100 additional problems are solved during the pre-link/post-link phases and added to the master problem, as iterations go by.

An example of the regional pre-linkage solutions corresponding to some of the above regional development strategies is provided in table 1.

TABLE 1. AN EXAMPLE OF PRE-LINKAGE REGIONAL SOLUTIONS
(1980 GNP in constant 1970 dollars)

Country or region	*GNP*
Developed (total)	5 230.2
North America	1 653.5
Eastern Europe	413.2
Western Europe	1 392.5
Japan and Oceania	669.9
USSR	1 101.1
Developing (total)	986.3
Oil exporting economies of North Africa and Middle East	131.6
Other Africa	96.0
Latin America	332.2
Developing countries of Asia	212.4
Centrally planned economies of Asia	214.1
GWP (gross world product)	6 216.5

The master problem

The role of the post-linkage relations, and of the master problem which consists of these relations, is to guarantee the compatibility of the external sector of all regional models. Once such compatibility is attained, one region's exports or, in general, its in-payments become a function of the imports (or out-payments) of other regions. However, since several convex combinations of regional proposals satisfy the world linkages, the master problem requires itself an optimality criterion to single out the one combination which is most desirable from a global perspective.

Among the several global development strategy functions tested so far by the world model, the following are important:

(a) Maximization of world GNP at time t;

(b) Maximization of total world employment;

(c) Minimization of the income gap between rich and poor regions.

For illustrative purposes let us select the first of these. The master problem then consists of the maximization of the weighted sum of the regional GNPs found under pre-linkage conditions by means of multiple objective functions:

$$\max. GNP^{*}_{world} = \sum_{r} \sum_{p}^{q(r)} k_r u_r^p \, GNP_r^p \tag{40}$$

for $p = 1,2 \ldots q(r)$; $r = 1,2 \ldots R$.

The u_r^p are the unknown variables of the problem. Each of them coincides with the weight to be assigned to the p^{th} development proposal for the r^{th} region. The other weights k_r are exogenously provided and used to simulate alternative policy situations in which the growth of one or another region may be stimulated (for $k_r > 0$), or impeded by the user (for $k_r < 0$). The maximand is constrained by the zero excess demand conditions on world markets for each i^{th} commodity and service:

$$\sum_{r} \sum_{p}^{q(r)} u_r^p (E_{r,i}^p - M_{r,i}^p) = 0 \tag{41}$$

for all i, $i = 1, \ldots n$

and, for the j^{th} type of payment flow:

$$\sum_{r} \sum_{p}^{q(r)} u_r^p \, KF_{r,j}^p = 0 \tag{42}$$

for all j, $j = 1, \ldots m$.

The convexity conditions require that for each region the sum of all weights u, which are constrained to be non-negative, be equal to 1:

$$\sum_{p}^{q(r)} u_r^p = 1 \tag{43}$$

for all r, $r = 1, \ldots R$.

There are as many market clearance equality constraints as there are commodity and other payment flows, and as many convexity constraints as there are regions in the world model.

The primal variables of the master problem, u_r^p, are weights required to compute, at each iteration, the linear combination of the development proposals (pre-linkage solutions) for each region. By construction the combination of the non-negativity and the convexity conditions yields:

$$0 \leqslant u_r^p \leqslant 1 \tag{44}$$

An example of such weights is found in table 2.

The dual variables of the global module are of fundamental importance. As is always the case in optimization procedures they provide the opportunity cost of primal constraints. In this context they show the gains and losses in terms of Gross World Product (GWP), the strategy function used in the master problem, resulting from the loosening up of trade and other world-wide consistency conditions. Since three types of equalities constrain the master problem's maximand, there are three types of shadow prices in this dual problem:

(a) The terms of trade (TT) of commodities and services exchanged on world markets;

(b) The implicit interest rates (IR) charged on outstanding capital balances;

(c) The resource costs (RC) required to run economic activity in each individual region at unit level.

Each of the three types of shadow prices plays a role in its own context; as they are the dual variables of equality constraints they are not constrained to be non-negative. Therefore, the sign they carry is also of fundamental importance. For example, the RC_r provide an indication of the extent to which GWP would be affected if the level of economic activity in a given region were allowed to change by a given amount. In a very loose sense, a cross-regional comparison of the RC_s provides an order-of-magnitude dimension of the contribution of each region to the world development momentum and, as may happen in some cases, the extent to which some regions benefit from it.

The terms of trade for commodities and services (TT), and the implicit interest rate for each type of outstanding capital balances (IR), provide an indication of the extent to which world GNP would be affected if trade and payment flows were rearranged so as to pursue different comparative costs and capital efficiency patterns. An example of these shadow prices is found in table 3.

Post-linkage problems

The level and the structure of economic activity in each region interact with the external sector. The explanatory power of a world model depends critically on the model's ability to address this problem. The iterative procedure which manages this interaction in DYNAMICO links serially pre-linkage and post-linkage problems by means of reaction functions and other types of feed-back mechanisms: the outward and inward linkages mentioned above. Iterations end when the GWP ceases to grow as more computations are carried out: at that point no higher world income can be hypothesized by any rearrangement of trade and payment flows among the regions covered by the model.

TABLE 2. AN EXAMPLE OF CONVEXITY WEIGHTS FOR 1980

Country or region	*Proposal number*[a]												*Total*
	2	*3*	*5*	*7*	*10*	*12*	*13*	*15*	*16*	*17*	*18*	*19*	
Developed													
North America			0.23	0.04		0.06	0.10		0.02	0.12	0.06	0.37	1.00
Eastern Europe					0.53						0.47		1.00
Western Europe		0.36		0.01				0.01	0.16			0.46	1.00
Japan and Oceania							0.08		0.72			0.20	1.00
USSR	0.10		0.36				0.54						1.00
Developing													
Oil exporting economies of North Africa and Middle East								0.90				0.10	1.00
Other Africa											1.00		1.00
Latin America					0.07				0.30		0.50	0.13	1.00
Developing countries of Asia		0.47				0.53							1.00
Centrally planned economies of Asia	0.12						0.86			0.02			1.00

[a]Missing proposals carry zero weight for all regions.

TABLE 3. AN EXAMPLE OF SHADOW PRICES FOR 1980

A. Commodities

Markets	*Terms of trade*
Agriculture	0.58 (excess demand)
Grains	0.64 (excess demand)
Raw materials	1.22 (excess demand)
Petroleum	3.03 (excess demand)
Other fuels	0.97 (excess demand)
Intermediate	0.80 (excess demand)
Consumers	0.73 (excess demand)
Investment	1.17 (excess demand)
Services	0.88 (excess demand)

B. Regional resource requirements

Region	*Requirement (billions of dollars)*
Developed	
North America	1 656.5
Eastern Europe	373.6
Western Europe	1 302.3
Japan and Oceania	589.9
USSR	981.4
Developing	
Oil exporting economies of North Africa and the Middle East	120.7
Other Africa	93.0
Latin America	292.6
Developing countries of Asia	222.3
Centrally planned economies of Asia	202.4

At each k^{th} iteration exchange and payment flows are modified through the recycling of the commodity terms of trade and the interest rate on borrowed capital, into each regional model system. Since these shadow prices represent resource requirements at the regional level for each type of tradeable commodity, the domestic level and composition of production change at each iteration, as a reflection of shifts of, and at the same time as prime mover of, comparative advantage positions through space. The shifts through time are accounted for by means of dynamic linkages, as discussed below.

At any period t the realignment of regional in- and out-payments is brought about by charging a premium on the demand of goods, services and financial assets which are in short supply on world markets, and by paying a subsidy for the supply of those items which are in excess demand. For each type of market the magnitude of the charges and the subsidies is usually identical for all regions, while substantial differences prevail among markets because of varying degrees of scarcity which characterize goods and means of payment.

With successive iterations the procedure finds new post-linkage vertices in the production-possibility polyhedron which represents the carrying capacity of each regional economy. These new solutions are based on regional development strategy functions which are a modified version of the ones used under pre-linkage conditions and, now, incorporate the charges and subsidies which prevail on world markets. For example, if the assumed behavioural criterion concerns the maximization of GNP, the maximand of the post-linkage problem for the r^{th} region at the k^{th} iteration is:

$$\text{MAX}\,\text{GNP}_r^k = \text{GNP}_r - \sum_i \text{TT}_i^k (E_{r,i} - M_{r,i}) - \sum_j \text{IR}_j^k \text{KF}_{r,j} \qquad (45)$$

for $r = 1, \ldots R$.

The last two terms on the right-hand side are the inward linkages, which include exports, E, imports, M, and capital flows, KF. This maximand is subject to constraints which concern domestic policies and resource availabilities within the region, as seen.

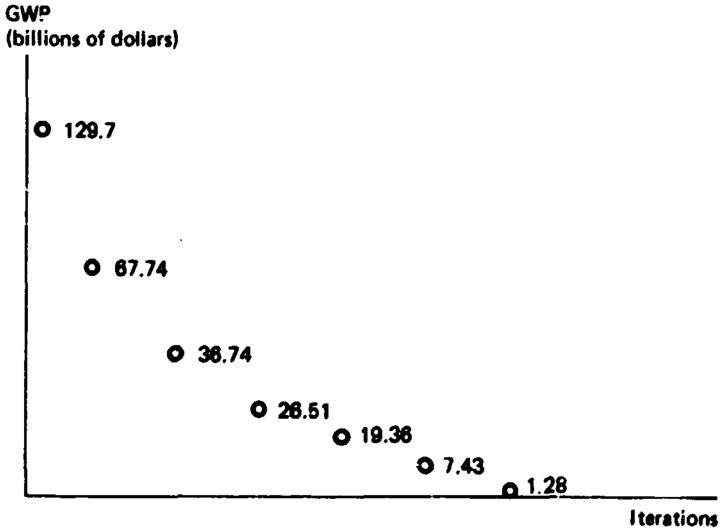
The activity levels computed by each post-linkage problem give size to new sets of outward linkages (each region's trade and payment position) which are then added to the master problem as formulated at iteration K, for recomputation of the convexity parameters u_r^p at the $k + 1^{th}$ iteration.

After solution of the new master problem, its dual variables RC_r^{k+1}, TT_r^{k+1}, and IR_j^{k+1} are recycled into the system for a new round of computations at regional level. The procedure ends when, in the given example, the increment of the world product (GWP) between two successive iterations is less than a pre-established convergence parameter δ:

$$[GWP^{k+1} - GWP^k \leq \delta]$$

As shown in figure 3 for 1980, any further rearrangement of trade and payment will not benefit any one region, without being detrimental to other regions. The

Figure 3. Decline in gains of GWP during solution. Convergence of the results for 1980



Pareto condition for optimality is thus met. Naturally nothing is said about interregional equity considerations, nor about the relative income positions of the regions covered by the system.

Dynamic linkages

In addition to linkages through space, the model includes extensive linkages through time: hence the system's name DYNAMICO. The end result is a recursive programming application of the decomposition procedure, whereby the level of the exogenous variable at any period t is retrieved from the solution values of the model at time t − 1. The control solution for 1980, the system's base year, is computed ex-post on the basis of actual information concerning these variables.

For each region the dynamic linkages concern:

Supply of factor resources
Technological change in all sectors
Extraction cost of non-renewable resources
Policy targets on final expenditure
Foreign exchange reserves
Foreign-owned production assets
Demographic behaviour

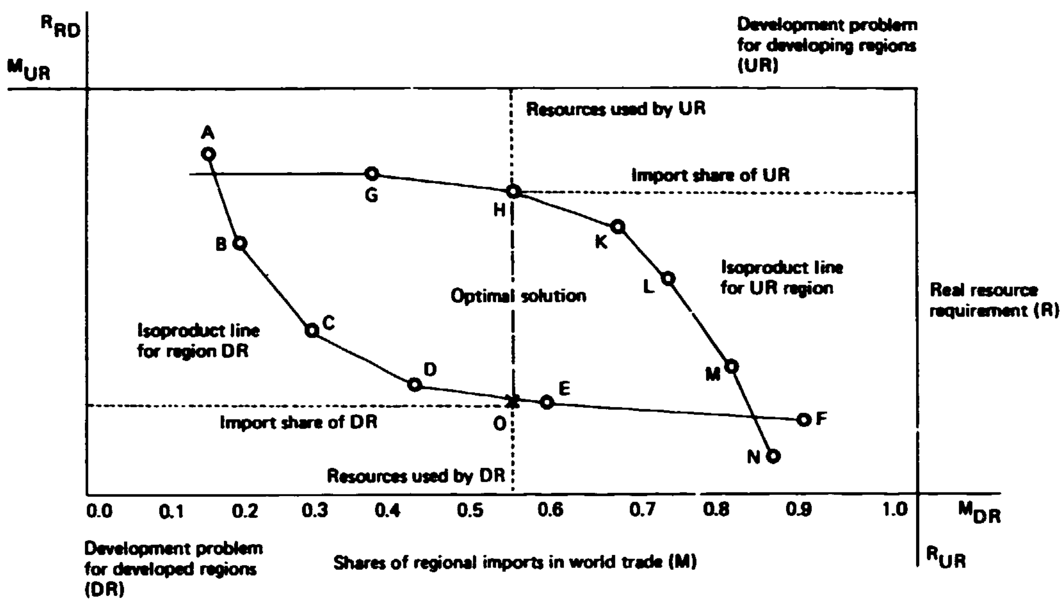
The original statistical information used in the model covers four bench-mark times, 1970, 1980, 1990 and the year 2000. For the purposes of investigating some aspects of the development trajectory of the world economy and of its regional or sectoral components, the bench-mark dates were linearly interpolated yearly, for a total of 31 observations between 1970 and the end of the century, inclusive.

In order to take into account exogenously projected rates of technological innovation and, above all, the expected increase costs in some economic activities, a series of cost step functions are used which introduce discontinuities over time in the growth of each region's production-possibility polyhedron. These cost parameters, which at the present time are computed by means of supplementary research conducted outside the main stream of the model simulation, are recycled into the system to affect the capital output/ratios in given sectors.

The geometry of the model

The decomposition procedure as applied to a single time period is simple enough to permit a graphical presentation by means of an Edgeworth box diagram, such as the one in figure 4 (Vietorisz [6]). Since by construction the diagram limits the investigation to a 2 (region) × 2 (resources) model, the five developed regions are aggregated into a single entity labelled DR, and the five developing ones into UR, to create total availability of real resources and total

Figure 4. Hypothetical Edgeworth box diagram of solutions with one master problem, two regions and two resources



world trade, which are then used to form the edges of the box. Resource volume and trade shares in each region are measured along opposite directions. Thus any point in the diagram is a simultaneous representation of four variables: real resources used by DR, and the share of imports of the regional problem for the DR (the economy of the developed regions) together with the resources and the import share of regions UR (the economy of the developing regions).

Points A, B, C, D, E and F in the diagram represent six different complexes, or basic feasible solutions that are obtained by solving the regional model for DR under several alternative strategy functions. Points G, H, K, L, M and N represent similar complexes formed from the activities of the developing region UR. Assume that all these complexes have been computed by means of the pre-linkage procedure outlined above. Could then the post-linkage problem be solved by inspection of the Edgeworth box?

Each solution represented in the box, both feasible and efficient at regional level, requires given volumes of domestic resources and yields a certain pattern of trade. In figure 4 the efficient complexes of each regional programme have been connected by a line. Points along a line connecting two complexes (such as, for example, the lines between B and C, or G and H) respresent weighted averages of the corresponding complexes. It can be verified by means of simple algebra that the resource and import requirements of any averaged complex is a weighted average of the requirements of the complexes connected by the line.

In a way the two curves of figure 4 can be regarded as generalized isoproduct functions for the two regional programmes. They describe the alternative combinations of resources and trade that would be used to meet the regional aggregate demand. Following conventional practice, the horizontal and vertical extensions of the two curves show redundant surpluses, which can be disregarded on the basis of the free disposal assumption.

In general, a world feasible solution can be obtained when one point is selected from the isoproduct line of each region, attention being paid to resource requirements. As imports (measured as a share of total world trade) are measured on the horizontal axes, when two points fall on the same vertical line total import requirements add up to the total amount of exports. When the point for regional programme DR falls to the left of the point for programme UR, excess supply for the given good on world markets is equal to the horizontal displacement between the two points. Conversely, when the point for region DR falls to the right of the point of region UR, there will be a demand bottle-neck. As both situations violate the fundamental accounting identity of world trade, an obvious strategy for selecting feasible solutions in the course of world optimization would be to choose two points that lie on the intersection of a given vertical line with each of the two regional isoproduct functions. Since real resource utilization levels are measured on the vertical axes, the vertical distance between the two points determines the resource savings which correspond to the proposed feasible solutions.

The geometry of the determination of the world optimum is now obvious. If the global strategy function consists in the minimization of resource requirements to meet given policy targets, the vertical line that maximizes the distance between the two regional isoproduct functions determines the most efficient solutions. In the present hypothetical case the optimum is determined by the line HO. Point O is a weighted average of the complexes D and E in the regional problem for DR; the weights are approximately 75 for complex E and 25 for complex D. On the other hand 100 per cent weight is assigned to proposal H for the UR region.

The geometric method of finding a solution is not applicable to larger problems containing more than two resources or two regional problems. None the less the logic of the approach does not vary: the master, or global, programme has to be formulated in terms of the strategic resources and has again to be pieced together by averaging regional complexes.

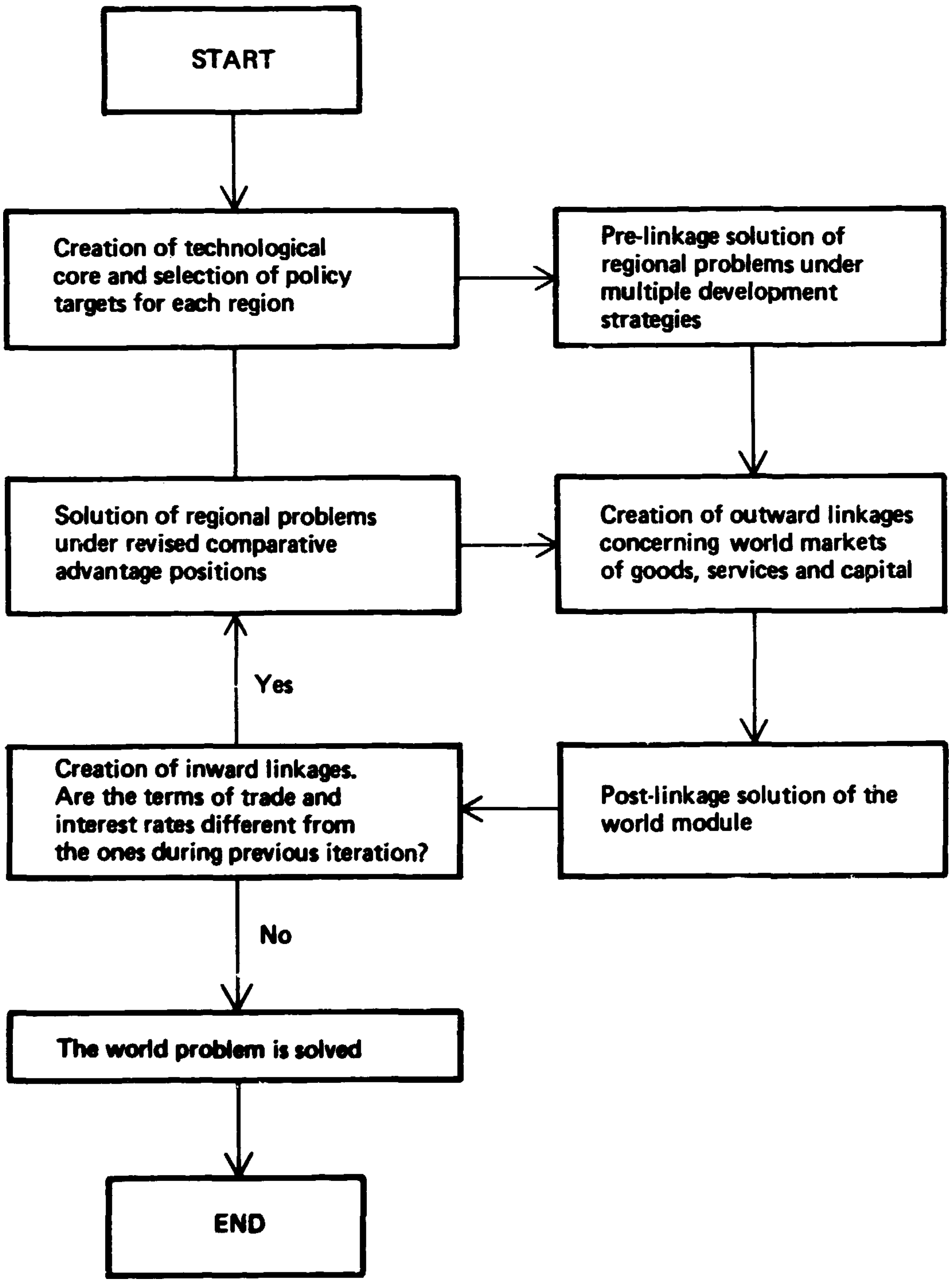
Concluding remarks

Before closing it may be helpful to assess the experience gained so far in applying the decomposition principle of linear models to the study of world development problems.

It should first be noted that the procedure discussed above is not the only one that can be utilized to co-ordinate regional development priorities within a world strategy. The economic literature has proposed several other multi-level decision-making schemes in which either the information flows follow a route different from the one shown above, or other types of signals co-ordinate the several modules of the system.

In DYNAMICO the global programme signals prices to the regional problems and the latter signal the level of resource utilization by particular development proposals back to the master programme. In other words, prices flow downwards and quantities upwards. In an alternative approach (Kornai and Liptak [7]), the master programme passes allocation of the critical resources to the individual regions. The regions, in turn, signal their own regional shadow prices for these resources to the master programme.

Figure 5. The solution procedure of DYNAMICO (the static framework)



It is essential to stress that in both instances the global problem of resource allocation is solved without interference by the centre in the technological and socio-economic choices set by each region. Namely, from a world point of view it is not necessary to standardize patterns of resource utilization and allocative priorities among the regions. Similarly, regional decision-makers do not need to be explicitly aware of the constraints which exist at the world level, although they are bound to perceive their existence eventually as manifested by the shadow prices.

From the viewpoint of the user, with the decomposition techniques it is possible to solve much bigger problems than would be otherwise possible. Problems with over 50,000 constraints are known to have been handled in this manner. Though inevitably the procedure has its own drawbacks. In actual practice it has been found that the shadow price vector tends to fluctuate a great deal, which causes somewhat erratic computational behaviour. Second, as the regional breakdown is increased, the list of development proposals tends to become very long—so long that not all can be saved in core memory of an average computer. Large-scale on-line storage is usually available but expensive, since its use tends to increase input-output more than is desirable from the viewpoint of efficient data processing. A general solution procedure for DYNAMICO is shown in figure 5.

Despite all of the above, on balance, multi-lateral planning and decomposition are possibly among the most appropriate and efficient approaches to the study of world development issues by means of global models.

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Some lines on development of the United Nations global input-output model

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Summary

The United Nations Centre for Development Planning, Projection and Policies invited the Institute of Economics and Organization of Industrial Production to participate in the study on the global I/O model so as to make use of the experience the Institute acquired in the development and the application of multi-regional models for the national economy.

This paper summarizes the first stage of work. Some feasible modifications to the global I/O model developed by a team guided by W. Leontief, which was taken as a basis for a project on the future of the world economy [1], and the findings of the test runs of a number of model versions are discussed.

The United Nations model

The global I/O model developed by a team guided by W. Leontief, which was used to forecast development of the world economy up to the year 2000, includes elements of the familiar I/O model of the national economy (static as well as dynamic), elements of the interregional I/O analysis and elements of the I/O analysis of interaction of economy and environment.

The global United Nations model is a set of regional I/O models (for groups of countries) linked together through the world matrix of trade and monetary flows.

Mathematically, the model is represented by a set of simultaneous algebraical equations having a single solution for given basic data. The existence of only one solution is due to a large number of exogenous variables such that the model has only one degree of freedom. For example, the freedom of choice among alternative world trade relations is eliminated by fixed coefficients of exports and imports. This is rather an artificial procedure since the future pattern of the international division of labour is assumed to be known before the regional output and the scale of world trade are determined.

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Long-term model runs may be carried out under different combinations of exogenous variables. The model does not contain any international mechanism for comparing alternatives of global and regional economic development from the viewpoint of their preferability. At the stages of model construction when basic data are collected and made consistent, it may be reasonable to use an I/O model. However, when the model is used, greater attention has to be paid to development alternatives and to a comparative analysis of the performance of these alternatives for particular regions and the world as a whole; thus it may prove more fruitful to use models with optimization elements.

In this study, the United Nations I/O model with its data is the basis for construction of the global optimization models (which contain conditions of equalizing regional economic levels) and of models of optimum economic interaction (with regional optimization criteria). The basic I/O model is transformed in stages. First, some artificial, rigid relationships between the variables in the basic model are removed and some fixed parameters are changed into endogenous variables so that some degrees of freedom, that is the possibility to choose among the alternatives, are gained. Secondly, conditions which restrict the sets of permissible solutions are introduced; for example, constraints relating to non-renewable resources. Thirdly, optimum regional development criteria are defined. Lastly, principles of international economic relations are formalized.

Optimization of the world economy

An optimization approach offers a number of opportunities to study the world development prospects. A greater number of economic values can be treated in such a model and shadow prices of all items of output and resources can be computed for all regions. The models of economic interaction also include prices which allow the achievement of equilibrium between supply and demand. It is possible to simulate policies of equalizing regional development levels.

However, optimization models require more data. It is necessary to introduce data on available manpower, on limits of the growth of primary sectors and on transportation costs. A number of further modelling problems arise. For example, if exports and imports are taken as unbounded variables, the range of choice of solutions is wider, but this raises a complicated problem of accounting for interregional output cross hauls. The solution of the global optimization models and those of economic interaction may not involve the cross hauls which in fact may account for a considerable portion of total flows (product-mix exchange within commodity groups). The models involving some optimization elements are also more sensitive to information on production technologies.

The world economy is a multipurpose system including countries or regions with differing concerns. Suppose that the development objectives of the s^{th} country or region could be expressed as $f_s(\bar{X}_s)$, where $\bar{X}_s$ is a development alternative of the s^{th} region. Each country or region attempts to maximize its objective function over the set of trajectories of permissible development, which is determined by its resource and technological potential, international economic relations and global resources.

The set of certain regional development paths involves vector $X = (\bar{X}_s)$ and the set of regional objective functions, $f_s(\bar{X}_s)$, gives an objective vector function of world development:

$$F(X) = [f_s(\bar{X}_s)]$$

By summing up the resource and technological potentials of all nations and of the world as a whole with regard to the global environment, one obtains a set of permissible development trajectories of the world economy, $\mathscr{X}$. The efficiency of international economic co-operation is expressed by the fact that each co-operating nation (region) attains a higher level of its particular goals than if it develops independently.

The world economy optimization model may be represented in its simplest form as an optimization problem over the set of the resource and technological potentials of the world economy development variants:

$$\max_{X \in \mathscr{X}} F(X)$$

This model neglects, for the sake of simplicity, political, social and organizational factors which constrain the mobility of the national or regional and global resources. The model shows the maximum opportunities for effective use of global resources under present and expected technologies. The degree to which these opportunities are used depends on the organization of regional economies and international economic relations.

An analysis of the model allows one to identify a set of development variants of the world economy (the Pareto set): $\mathscr{X}^* \subset \mathscr{X}$. Each element of set $\mathscr{X}^*$ is efficient in the sense that it cannot be improved in favour of any particular nation, or a group of nations, or a region, without losses for at least one nation or region. All alternatives not contained in the set $\mathscr{X}$ can be improved in favour of at least one nation or region.

The problem of selecting a world economic development strategy is thus reduced to a choice among a set of the efficient, in the sense as indicated above, alternatives. But for individual nations, the choice of the efficient alternatives is important. A deliberate choice of some world economy development strategy requires a certain compromise between the interests of individual nations or regions. In conformity with the world economy development goals set by the United Nations, any compromise of the national interests should imply an accelerated equalization of the material and cultural development levels between the developed and the developing countries.

The modifications discussed may be seen as special, but interrelated, tools for finding and comparing the world economic development paths within a set of the efficient alternatives.

A balanced state of world trade is achieved by incorporation of world balances of exports and imports into the model:

$$\sum_{S \in R} E_i^s = \sum_{S \in R} M_i^s, \ (i \in N) \qquad (1)$$

where:

E_i^s = the i[th] output item to be exported by region s;
M_i^s = the i[th] output item to be imported by region s;
N = the set of all commodities.

The shadow prices of contraints (1) may be interpreted as the exchange prices. With the help of them the trade balances may be calculated as follows:

$$\sum_{i \in N} \zeta_i (E_i^s - M_i^s) = \Delta^s \tag{2}$$

where:

Δ^s = the balance of trade for region s;
ζ_i = the exchange price.

These balances have no direct effect on the solution, but they represent important indicative indices.

The global optimization model

The "global optimization model" involves a general world economy optimization criterion that generalizes the regional criteria as well as the relationships between the regional criterion levels. Non-productive consumption values Z^s and $Z = \sum_s Z^s$ (the total sum of private consumption C and government consumption G) are taken as regional and global optimization criteria. The problem of maximizing Z is solved under additional conditions $Z^s = \lambda^s Z$, where λ^s is the share of region s in the world non-productive consumption fund ($\lambda^s \in [0, 1], \sum_s \lambda^s = 1$). This global optimization model simulates, with different values of λ^s, the opportunities and implications of alternative ways of equalizing the regional development levels (regional population's welfare). Then, beyond the framework of the given model, the questions of whether a certain alternative is realizable under existing international relations, and of how these relations should be amended in order to realize this on another alternative, have to be answered.

The model of optimum economic interaction

The "model of optimum economic interaction" chooses among a set of the efficient alternatives in rather a different way. The model is a combination of the vector optimization problem (with regional optimization criteria) with the conditions of the international economic mechanism, involving principles such as the regulation of balances of payments and price-building policies in the world market. This allows development alternatives to be found which are best for each country or region under the existing international economic mechanism.

In this model the problem for each region involves the international exchange balance of trade:

$$\sum_{i \in N} P_i (E_i^s - M_i^s) \geqslant \Delta^s \qquad (3)$$

where P_i is the exchange price for the i^{th} output item.

The optimization criterion for maximization of the fulfilment of the regional consumption programme is

$$Z^s \rightarrow \max$$

The values of the balances of trade Δ^s are fixed so that $\sum_s \Delta^s = 0$. The essence of the task of co-ordinating regional problems is to find a price system $P = (P_i)$ where the world balances of trade (1) are fulfilled.

This model is a very specific case of a wide range of models describing the economic interaction of subsystems where the economic mechanism operation principle may be not necessarily related to tasks such as finding equilibrium prices, and to balancing demand and supply.

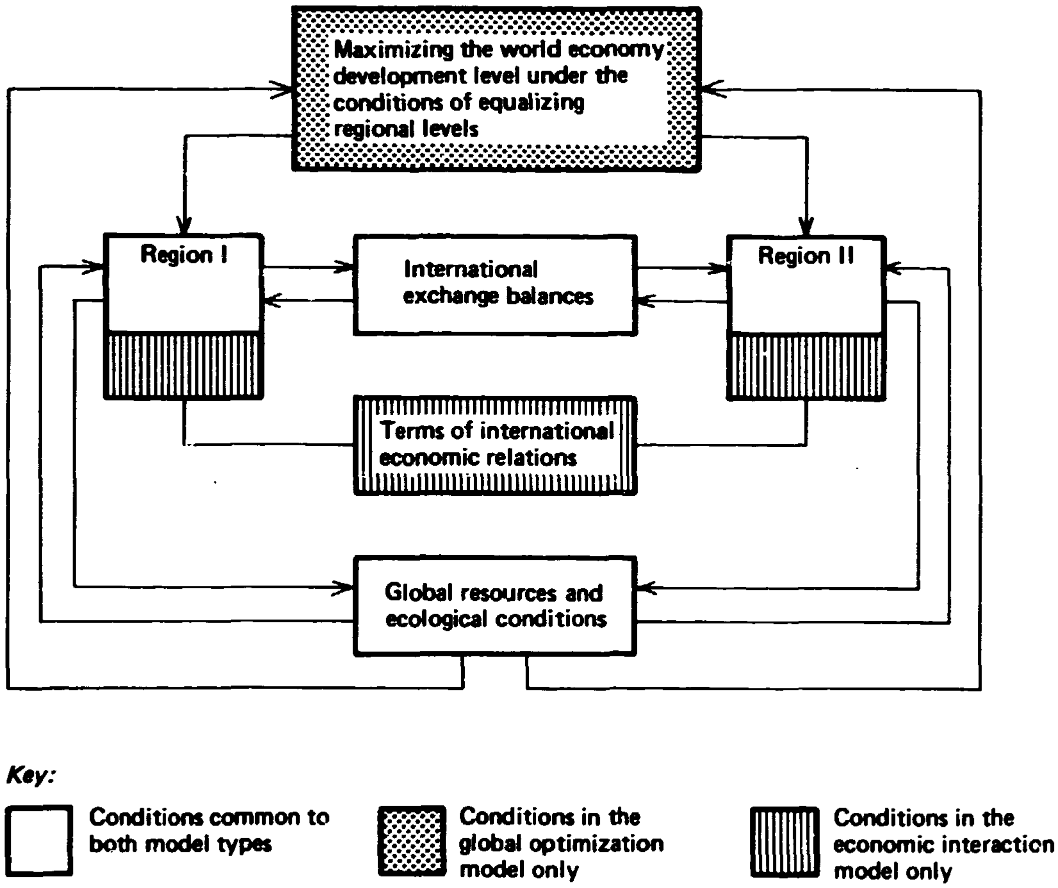
The world economy models determine the set of feasible development paths $\mathfrak{X}$ under the conditions of balances of production and distribution of transportable and non-transportable output, of balances of transportation activities, and of conditions relating to abatable and unabatable pollutants, to investment and to capital; and under constraints relating labour and output to be produced by primary industries.

The structure of both model types consists of blocks. The blocks simulating development of individual regions are bound by conditions under which the regional economic development or welfare levels are equalized, by international exchange balances, terms of international economic relations, global resource and technological constraints. Figure 1 shows the structure of the models for a case with two regions. Owing to the structural similarity, one can easily switch over from one type of the model to the other, using the same basic data.

There is an interrelation between the solutions of the models of economic interaction and global optimization. Suppose the values of constraints $Z^s = \lambda^s Z$ in the global optimization problem be positive for all s. For prices, p_i, in the economic interaction model we assume the values of constraints (1) are standardized in a certain way. On this basis the trade balances will be calculated with the help of (2). Then these prices and trade balances Δ^s will result in one of the solutions of the economic interaction model, coinciding with the solution of the global optimization problem, and vice versa. One solves the economic interaction model, with λ^s in the global optimization problem being represented by the values inversely proportional to the dual prices of constraints (3). These λ^s will be standardized so that $\sum_{S \in R} \lambda^s = 1$. Then one of the solutions of the global optimization model coincides with the solution of the economic interaction model.

Thus the two world economy model modifications complement each other. The economic interaction model runs provide indications for determining the ratios between the regional welfare programmes under the conditions of the global optimization model. And, vice versa, the dual prices and values of balances of trade resulting from the runs of the latter model may be used in building the former model.

Figure 1. Structure of the global optimization and optimum economic interaction models



In building the world economy model modifications and analysing their properties, the authors have relied upon the experience acquired in studying multi-regional interindustry models for the Union of Soviet Socialist Republics [2, 3 and 4].

Findings of the test runs of basic variants of the aggregated model

The variants of the model were run, using a model in which the world is subdivided into four economic zones: North America; other developed economies (Japan, South Africa, USSR, Europe, Oceania); Latin America; and Africa and Asia (except for economies included under other developed economies).

Five industries were identified in each economic zone. These were agriculture, extracting industries, light industry, heavy industry and services. Further, emission of two kinds of pollutants is taken into account: unabatable (pesticide) and abatable (an aggregate of seven kinds of pollutants). The conditions of monitoring the emission of unabatable pollutants are introduced into the model; the abatable pollutants are indicated by a special indicator, the sixth production sector. The main indicators are expressed in United States dollars at 1970 prices.

The test runs were carried out through several stages. First, a series of preliminary variants was computed in order to examine the consistency of different assumptions about the development of primary industries (agriculture and mining). As a result, the most optimistic assumption about the development of primary industries was selected. Then, six basic variants for the year 2000 were computed: two variants that were produced by the global optimization model and four from the economic interaction model. These variants differ in the ways in which the interregional transport costs were handled and in balances of trade. The latter conditions were for zero balance, for a balance equal to 1 per cent of the gross product of the developed regions, and for a balance equal to 2 per cent of the gross product of the developed regions.

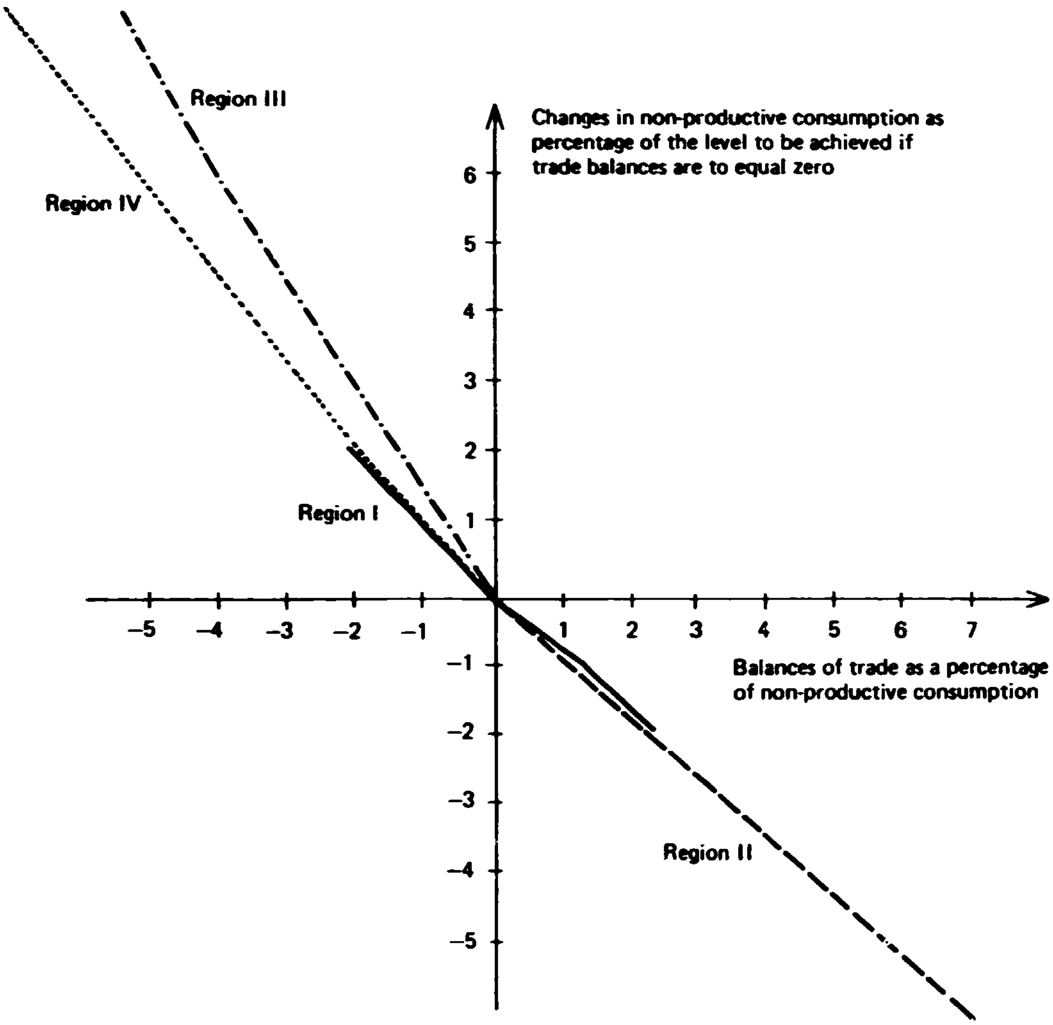
The test runs were primarily aimed at the examination of how decision behaviour is affected by the changes in various conditions and parameters of the model. The findings were compared against the background of the "central solution" in the United Nations 4 × 6 model based on the I/O model.

The computed variants produced general indicators of world economy development, such as gross product and non-productive consumption, which are higher than those of the central solution by 1.5 to 4 per cent. This minor difference from the central solution in terms of general performance indicators is due to the "narrow" range of permissible solutions, which is predetermined by the stock of basic data in use. However, with respect to the regional pattern and, in particular, to the international trade pattern, these variants differ more significantly from the central solution. This allows a number of conclusions to be drawn about how different factors affect the development trends of the world regions and especially about opportunities for equalizing the levels of their economic development.

The runs carried out show how imbalances of trade affect the volume of non-productive consumption in individual zones. Figure 2 clearly illustrates this effect. The horizontal axis shows the levels of the imbalance of trade as a percentage of non-productive consumption. The vertical axis shows the values of changes in non-productive consumption as a percentage of the level to be achieved if balances of trade are equal to zero. As can be seen from the figure, a close and proportional relation exists in all regions between the balance of trade and changes of non-productive consumption. If the change in the balance of trade is 1 per cent, then the change in consumption is nearly 0.9 per cent in regions I and II, 1.5 per cent in III and 1.2 per cent in IV. The graph was plotted through the points derived from both the solution of the global optimization problems and individual runs of some of the economic interaction model modifications.

An increase in the imbalances of trade in the developed countries by 1.0 per cent of GNP corresponds to its decrease in the developing countries by approximately 1.8 per cent of GNP. The transfer of $ 1 of GNP of the developed countries allows GNP in the developing countries to increase by nearly $ 1.4. At

Figure 2. Balances of trade versus changes in non-productive consumption



the same time, a reallocation in favour of the developing countries by means of international trade even of 2.0 per cent of GNP of the developed countries cannot solve the problem and does not essentially narrow the per capita GNP gap between the developed and the developing countries. The per caput GNP will only decrease by a multiple of 6.20 to 6.19. The non-productive consumption gap will decrease by a multiple of 6.55 to 6.17.

The findings of the world economy model runs according to the basic variants were used for a parametrical analysis of the problems for individual zones. Such an analysis is of high importance to the developing countries for which the basic variants assume very high potential growth rates of primary industries. The analysis allows the implications of the reduction in those rates to be identified and the lower limits to growth of primary industries needed to ensure satisfactory development trajectories of the economy of the developing countries to be determined. Such analysis also permits simulation of the effect of variants of the international division of labour on the economies of particular zones.

Some lines of development of the interregional world economy model

Taking into account the increase in production cost in primary industries

The analysis of the probable development of agriculture and extractive industries suggests that as accessible natural resources in most regions are depleted, unit production costs of these industries will increase with output. Therefore, it is reasonable to drop the assumption that primary industries have a proportional cost-to-output ratio.

The increase in the production costs in primary industries is dealt with by the rates of the rise in the production costs as a function of the annual average output growth. For example, for North America for the period up to 2000, with the annual output growth rate being 3 per cent so that the total growth during 30 years will be a multiple of 2.43. The production costs will increase by a multiple of 1.4 in agriculture and 3.0 in mining. These multiples for Asia and Africa are 1.3 and 1.5 respectively. Natural resources are assumed to be more abundant in the developing countries than in the developed. Due to this the relevant multiples are smaller in the developing regions than in the developed regions.

An optimistic assumption about changes in costs with regard to discoveries of new resources was examined. The rates of the rise in the costs were in this case multiplied by two thirds.

As cost increase rates are involved in the model, the linear-homogeneous cost functions for agriculture and mining are replaced by piecewise-linear functions with increasing marginal costs.

However, to maintain required output levels in primary industries it would be necessary to invest considerable physical resources, especially in the developing countries. The efficiency of the employment of material resources and of capital and labour resources would decline in such cases. The material and capital intensity of gross product increases more rapidly in the developing countries, due to their higher share of primary industries.

The rise in material inputs and capital intensity in primary industries results in an additional increase in the demand for investments in order to maintain high economic growth rates. Thus the share of investments in gross product significantly increases in all regions. The increase is from a range of 14 to 16 per cent to one of 16 to 19 per cent in the developed regions and from a range of 19 to 20 per cent in the developing regions to one of 24 to 28 per cent.

If the rise in the marginal costs in agriculture and mining is taken into account, the shadow and equilibrium prices of output of these industries rise significantly. Thus the relative shadow price of mining output rises by a multiple of 2.3 and the related equilibrium price by 2.25. The relative shadow price of mining output would increase, according to the variant with the optimistic assumption about increases in costs, by a multiple of 1.8 compared to the basic variants.

Taking into consideration the substitution of production factors and consumer goods

To analyse production processes with interchangeable resources, the production function of Cobb-Douglas is widely used. Essentially, this function may be included in the model for the light industry and for the whole assembly of employed resources and emitted pollutants. However, it is very important to take into account in the model the possibilities of substitution of two production factors—labour and fixed capital.

The production function of Cobb-Douglas that shows the output produced by the j^{th} sector as dependent upon use of labour and fixed capital has the form:

$$X_j = a_j X_l^{\alpha l} X_c^{\alpha c} \quad (4)$$

where:

X_l = labour input;
X_c = fixed capital input;
α_l and α_c = the coefficients of elasticity of output to labour and fixed capital.

The technological coefficients of the basic variants of the models strictly characterize definite relations between the input components. Based upon function (4), several production methods with varying relations between labour and capital inputs can be found for each industry. To retain the linearity of the basic resource and technological conditions, it is necessary that $\alpha_l + \alpha_c = 1$.

A similar formalization technique can be used in handling the substitution of consumer goods. To do so, it is necessary to construct a function which characterizes the total consumption depending on the consumption of particular commodities (y_i)

$$c = g \prod_{i=J} y_i^{\beta_i} \quad (5)$$

where:

y_i = the coefficient of elasticity of the consumption level to the i^{th} commodity;
J = a set of goods.[1]

If $\sum_{i \in J} \beta_i$ is taken to be equal to 1, function (5) can be replaced by consumption modes φ, describing baskets of consumer goods differing in their structure and resulting in equal consumption levels.

In order to examine the bearing of the changes in the private consumption pattern upon regional development, all industries contributing to the private consumption fund were categorized into two groups. The first group comprised

[1]Parameters g and β_i may be estimated statistically. In the computations it was assumed that $g^s = 1$ for all $s \in R$.

agriculture and light industry, satisfying basic needs. The second group comprised heavy industry and services, satisfying secondary needs. The share of mining in private consumption did not change.

Two extreme consumption programmes with varying relation between the consumed output of the two groups of industries and with the same total consumption level were designed for each region parallel to the above basis private consumption pattern. For these programmes functions $C^s = g^s y_I^{\beta_s} y_{II}^{1-\beta_s}$ were used. Ratios of the elasticity coefficients $\frac{\beta_s}{1-\beta_s}$ were given regionally as 0.8, 1.0, 1.2 or 1.5. The maximum and minimum relations of the shares of groups of industries in private consumption were given. For example, the maximum ratio of consumed output of industries 1 and 2 was taken for the region of North America as 1.0 to 3.5 and the minimum to be 1.0 to 2.5; for the region of Asia and Africa the maximum was 1.0 to 0.8 and the minimum was 1.0 to 0.2.

There are two extreme patterns of consumption, the "minimum" and "maximum". The "minimum" proved to be efficient for the developing regions, with a smaller share of these sectors in consumption for the developing regions. There seem to be two reasons for this: first, a shortage of labour in the developed countries (according to the basic data, the labour intensity of output of the second group of sectors is much higher than that of the first group; this is why the developed countries save labour by moving to the "minimum" pattern); second, the potential for agricultural growth in the developing countries limits the economic growth rates, which is aggravated by the increase in the costs in agriculture; under these conditions, it would be better for the developing regions to move to a consumption pattern with a smaller share of agricultural output.

On the whole, these model runs indicate that in the runs to be carried out, it would be reasonable to extend the range of choice of efficient non-productive consumption patterns.

Regional economic interaction models with non-equilibrium prices

The first series of the test runs were carried out using the optimum economic interaction model with equilibrium prices. It is interesting to examine situations where the prices of the economic interaction model are formed according to principles other than the principle of the endogenous determination of balancing (equilibrium) prices introduced above. World exchange prices may be deliberately regulated by intergovernmental agreements, may vary as the result of unilateral specific actions undertaken by particular regions (exports and imports), may take into account long-run implications and various situational factors, and so forth.

It should be stressed that if balances of trade Δ^s of all regions are equal to zero, fixing one of the exchange prices p_i does not modify the model solutions. This is due to the homogeneity of the model to prices.

When not all Δ^s are equal to zero, the model ceases to be homogeneous to prices. The general price level rate is in this case fixed in conformity with the levels of balances of trade Δ^s. If one fixes, without changing Δ^s, one of the prices p_i at a level other than the equilibrium one, then the solution will change. The resulting new solution will also be balanced, but it will correspond to a different way of fixing the price rate. The objective function values of the resulting new solution tend, as a rule, to increase in some regions and to decrease in other regions as compared to the basic solution.

What are some probable results of such changes? Suppose that, the values of Δ^s being constant, we raise the price of the output produced by the exporter represented by some region. Suppose also that this region will continue to be the exporter of this output in the new solution. If one price p_i increases, the total price level also tends, as a rule, to rise; this will result in a decrease in the real importance of the values Δ^s. This is profitable, i. e. giving an increase in the objective function, for the regions which had $\Delta^s > 0$ and not profitable for the regions which had $\Delta^s < 0$. If values Δ^s involved direct financial aid, then the actual weight of that aid decreases in the case under discussion.

A direct effect due to the increase in one price of exported output can be obtained only if the relative price in the new solution is increased. The opportunities for such an effect are strictly limited. The greater they are, the greater is the extent to which the exporting region monopolizes the production of the above output; and the narrower they are, the greater are the opportunities to produce this output in the importing regions or the opportunities of substitution of this output. The dependence of the magnitude of the effect upon the rise in the price is not monotonic. It may prove considerable with even a minor rise in the price and then decrease with a further increase in the price, reaching a zero, or even negative, level.

For the regions with negative Δ^s, the positive effect due to the increase in one price of the commodity p_i to be exported may be essentially upset by the negative effect of the rise in the total price level and to a decrease in the effective level of Δ^s, for example the effective value of financial aid. This was confirmed by experiment.

When $\Delta^s \neq 0$, an increase in the price of the commodity to be exported or a decrease in the price of the imported commodity has a very different impact for each region. The greater the value of Δ^s, the greater will be the difference. All the above considerations are based on the assumption that if one of the prices differs from the equilibrium level, all the other prices change so as to result in a new equilibrium state, with such a change taking place instantaneously. In reality the price changes occur with definite delays. To represent such an effect, models are needed where the price regulation process is related to a real time scale. The models discussed in this paper suggest that boosting prices in particular regions to make a gain may lead to a loss.

When two or more prices are fixed, the model properties change more radically. Equilibrium is not guaranteed and it is difficult to propose any universal approaches to solve the model. However, it is possible to examine each specific situation separately, and one is forced to surrender the attractive properties of the basic optimum economic interaction model.

In a case when all exchange prices are given exogenously, it is of interest to determine a global balanced plan such that the objective function values for individual regions deviate from the optimum levels (with given prices) by the same relative value and that that value is minimized. This value will show the extent to which local solutions are not optimal in the proposed global plan. The solution to be obtained is Pareto optimal. If the deviation value proves insignificant, the solution to be obtained may be taken as acceptable. Otherwise, an additional analysis is needed. The decisions to be made in this case may turn out to be Pareto non-optimal.

For the test findings from the economic interaction model with non-equilibrium prices, it is of interest to analyse a case where one price is fixed and regional balances of trade are given. Suppose the price of mining output is increased by 30 per cent as against the equilibrium level and the balances of trade are constant. For the model solution, changing the price determination by giving the prices of mining output does not result in significant changes in the relative prices in all industries, including mining. Thus, regions exporting mining output would not earn any immediate gain from the changes in the prices. But, the position of countries with negative trade balances becomes worse. Due to the increase in the total exchange prices by a factor of nearly 1.5, the real impact of financial aid expressed in terms of balances of trade would become less efficient.

For testing with fixed prices in all industries, the prices used in the central solution of the 4 × 6 United Nations model were fixed. Zero balances of trade were given. Thus price ratios rather than absolute prices are of real importance.

As was to be expected, the optimum solutions to the regional problems prove incompatible with given prices. Imbalances of world trade are very considerable.

To find balanced global solutions with fixed prices, the global optimization model was used. The consumption fund turned out to be only 2.35 per cent smaller than the total from the regional problems with fixed prices. The runs with fixed prices in a fixed ratio of 1.5 : 2.0 : 1.0 : 1.0 for each region gave a good balance of international trade for each region.

The consumption fund distribution changed in favour of North America by 0.3 percentage points and Latin America by 0.6 points. The share of Asia and Africa fell by 0.6 percentage points. This was due to different implications of the deviations of fixed prices from equilibrium ones from region to region. The main exporters of outputs of the agricultural and light industries, North America and Latin America, showed gains, provided fixed prices were given. Clearly, given different fixed prices, the results would have been different. Therefore, by changing prices, one can regulate the proportions in which non-productive consumption is distributed among the regions.

As already noted, the main purpose was to study the "behaviour" of the model, that is to analyse the way in which solutions are affected by changes in the model's conditions. The conclusions are of a specifically preliminary nature and will be subject to further specification as more sophisticated and detailed data are used.

Work on adjusting the main modifications of the interregional inter-industry model of the world economy based on highly aggregated data has been finished. The model is now being run with detailed information involving 15 world regions and 22 industries in each region (3 for agriculture and food, 8 resource producing, 7 manufacturing, and 4 service industries). For this model, 19 equations for macroindicators, the investment and capital items, GNP, balances of payments, governmental expenditure, per capita intake of protein and calories etc., are analysed for each region.

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Long-term projection of economic growth in ESCAP member countries

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The international community is now more and more involved in discussions related to international economic relations and development needs. The poverty-trap experienced by some Economic and Social Commission for Asia and the Pacific (ESCAP) developing countries during the 1960s and 1970s has led to increasing doubts and frustration concerning past and current development strategies.

There is now wide recognition of the need for a policy of limiting population growth rates and giving more emphasis to agriculture, particularly food supply. Rapid industrialization as well as technological transformation is important but, to be meaningful, these must be an integral part of overall development, which covers a broad spectrum of enterprises and a diversified production structure to satisfy mass demand. In general, in addition to a focus on economic growth *per se*, emphasis is being placed on specific human concerns such as elimination of mass poverty, the eradication of unemployment, and the satisfaction of basic human needs. The demand for greater equity among nations has become a demand not only for narrowing gaps in levels of living but, even more important, for a fairer sharing of power and decision making. The emphasis on mutual respect and parity in power leads to the strategy of self-reliance. Emerging diversity in developing countries, as well as the self-reliance aspect, should be fully recognized in plans of economic co-operation.

In line with international efforts to develop a new strategy, the ESCAP secretariat is presently in the process of studying possible strategies for the 1980s for ESCAP developing countries. Such strategies require a quantitative base setting out possible lower and upper limits to growth and the characteristics of, and the policies required to deal with, different scenarios, and such a base is provided by the ESCAP-FUGI model.

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ESCAP-FUGI model

The ESCAP-FUGI model has been designed to project major structural variables for the main ESCAP countries or subgroups of countries. The model was primarily developed in Japan by Professors Onishi and Kaya assisted by ESCAP staff. It is based on a former model, known as the FUGI model (the Future of Global Interdependence), which was presented at the 5th International Institute for Applied Systems Analysis (IIASA) Global Model Conference in 1977 (Kaya and others [2]). The current ESCAP-FUGI model consists of two sub-models:

The Asian macro-economic model (AMEM)
The Asian input-output model (AIOM)

Asian macro-economic model

AMEM is a dynamic macro-economic model which relates such aggregated variables as GDP, private consumption, private investment (housing and non-housing), government expenditure, bilateral trade flows, corporate profit, wages, consumer prices, wholesale prices, export and import prices, explicit deflators of government consumption, fixed equipment investment and housing investment, private foreign investment and official development assistance. Variables are presented both in constant and in current prices.

The model has a slightly different structure for different groups of countries: i. e. developing market economies (DME), advanced market economies (AME), centrally planned economies (CPE) and Asian OPEC countries (OPEC). The major difference is found in the treatment of production and expenditure. The model covers 27 regions, 15 of which are ESCAP countries and subregions (see annex I). Projections are made for each year after 1980 to 1990.

Asian input-output model

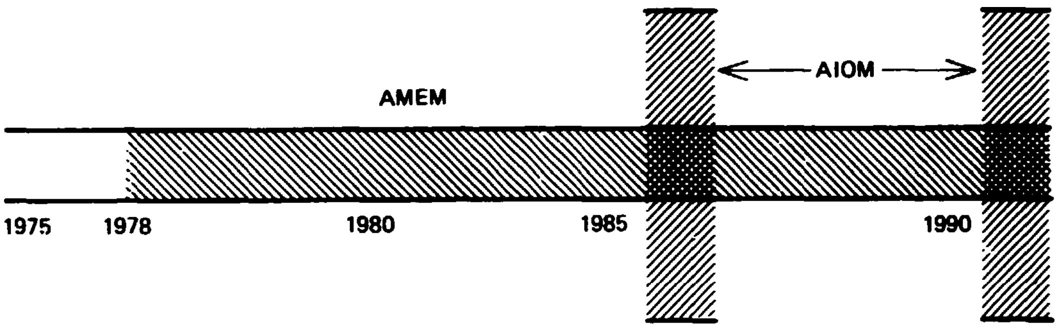
AIOM projects value-added by industry over 14 regions, 9 of which are ESCAP countries and subregions (see annex I). This model consists of 4 modules. The I/O module deals with the prediction of I/O coefficients while the final demand module deals with the prediction of the domestic final demand by industry. These predictions are made consistent with the AMEM by using AMEM predictions of GDP, per capita GDP and total final demand. The trade module gives centre estimates for exports and imports by industry with upper and lower boundaries. Again, consistency with the AMEM is achieved by controlling the total exports and imports by AMEM estimates. The linear programming module provides a method of optimizing the domestic and international allocation of resources.

On the basis of projected values such as I/O coefficients, centre trade estimates, and macro variables, the standard projection of value added over 14 regions by industry is made. Introduction of the criteria for optimization makes it possible to reallocate resources in the domain of the carefully prepared feasible boundaries.

Overall model structure

The ESCAP-FUGI model is a semi-dynamic model. The macro part is fully dynamic and the I/O part is static. The time-span between the two sub-models is shown in figure 1. The relationship is one way, that is, the results of the AMEM are reflected in the AIOM only. The converse may eventually be possible if investment data by industries become available for all the regions.

Figure 1. Time-span of sub-models AMEM and AIOM



The models AMEM and AIOM should have:

(a) Explicit consistency; thus related variables should have consistent values, for example, for gross domestic product, final demand components and exports and imports;

(b) Implicit consistency; thus consumption and production by industries resulting from AIOM should be in line with patterns of development as depicted by AMEM (this is particularly true in the estimation of I/O coefficients and in the co-ordination of macro and I/O scenarios).

Scenarios and projection results

In addition to standard trend projections it is of great value to project the future of the economies of the region by assuming certain scenarios. Later, this paper develops scenarios with respect to possible development patterns in the 1980s. These include:

(a) The effects on the development performance of ESCAP developing countries of different growth performances in the developed industrial societies, especially in so far as the latter affect the exports of the developing countries and official development assistance flows;

(b) The implications for required GDP and associated growth rates by industries if targets such as the Lima target are to be obtained;

(c) The implications for potential growth in GDP if there are constraints upon agricultural output and upon imports of agricultural projects;

(d) The possible impact of export-led growth upon GDP and employment in south Asia;

(e) The implications of growing global protectionism in the developed industrial economies.

Results of projections are presented after each scenario is specified and evaluated.

The Asian macro-economic model

The AMEM model has a different structure for different groups of countries to reflect differences in types of economies. The model is a multi-regional link model and emphasis is placed on the interactions among different regions through international trade and capital flows. The model treats 28 regions. For the estimation of parameters the method of ordinary least squares has been employed, using data from 1953 to those most recently available.

Qualitative aspects of the model

Production

The model starts with an equation for average production capacity as a distributed lag function of investment (equation 1[1]). For AME another explanatory variable, research and development expenses, is also introduced. Research and development expenses are explained by one year lagged GDP[2] (2). The production capacity, investment and research and development expenses are all expressed per employed labourer. Employment is a function of the labour force and unemployment rate. In turn, the latter is explained by wage-share in GDP, non-house investment share in GDP, and growth of the civilian labour force (4 and 7). GDP equals the average production capacity for DME, CPE and OPEC. In other words, it is assumed that the projected capacity is fully utilized in the countries belonging to these blocs. For AME, capacity utilization is determined endogenously (5) and GDP is determined by expenditure (17). The index of capacity utilization is one of the explanatory variables of non-housing investment (12).

[1]Equations 1-51 are listed later. Hereafter, numbers in parentheses refer to equations.

[2]GDP is gross domestic product if the area concerned is a country; it is gross regional product if the area concerned is a region.

Expenditure

The treatment of consumption is different for different blocs. While an explicit consumption function is introduced for AME, consumption is treated as the residual of GDP after allowing for the expenditure (9) for other blocs.

Where data are available investment is disaggregated into inventory investment, housing investment and non-housing investment. Inventory investment is explained by current GDP and its change, the interest rate, and inventories of the previous year (14). Housing investment is explained by GDP, population growth and the last year's interest rate (13). The specification of the non-housing investment equation is different for various blocs. For AME, explanatory variables are operating surplus, exports, the interest rate and the rate of capacity utilization, all of which are lagged one year. For DME and OPEC, they are GDP, exports and the availability of foreign capital, represented by the sum of various types of development assistance. For CPE it is GDP of the previous year alone.

Total exports and total imports of a region are defined as the sum of bilateral trade flows in merchandise adjusted for services and other differences (15 and 16). Bilateral trade flows, described by equation (8), are determined in a different fashion for different blocs to which importing regions belong. If an importing region belongs to AME, bilateral trade is explained by its GDP and by prices. If an importing region belongs to either DME or OPEC, the explanatory variables are its own GDP and the sum of exports and foreign capital deflated by the export price index (in other words, available foreign exchange in real terms for imports). For CPE as an importer, total exports is the only explanatory variable.

Government revenue is linearly related to GDP (11). Government consumption expenditure is related to government revenue (10).

Wages and prices

The model attempts to project unemployment rates by civilian labour force, the share of wages in GDP, and the share of non-housing investment in GDP (4). The unemployment rate gives the level of employment (7) and becomes one of the explanatory variables for wage rate determination (20). Other variables explaining the wage rate are consumer prices, labour productivity and the ratio of operating surplus to GDP. Operating surplus, in turn, influences salaries and earnings, investment, capital stock and the interest rate (19). Finally, labour productivity is defined in (21).

Prices have a direct role in some equations. For example, in exports, investment, operating surplus and wages. Another important role of prices is to transform variables expressed in constant prices to current prices. Variables in current prices have been used in the operating surplus equation; operating surplus is first explained in current prices (19) and afterwards deflated by an implicit deflator of investment (22).

Prices are represented by price indices and implicit deflators. Wholesale prices are related to such variables as import prices, real wages, labour productivity, the money supply/current income index (the ratio of money supply to GDP), and the ratio of operating surplus to current GDP (24). Consumer prices are explained by wholesale prices, wages, prices of import goods and the money supply/current income index (25). Prices of investment goods are explained by wholesale prices and the level of investment (27). The implicit deflator for housing investment is related to the deflator of private investment and wages.

Special attention is paid to a set of equations which explain export prices, import prices and exchange rates. Export prices, expressed in 1970 dollars, are presented in equation (29) as a function of several variables depending on the type of bloc. The AME region's export prices are related to the previous year's wholesale price, import prices and the world liquidity-trade index. The last variable supposedly affects prices in a manner similar to the effect of the money supply/real income index on the domestic market. For DME countries, prices of commodities are added. For CPE countries, explanatory variables are commodity prices, oil prices and import prices, whereas for OPEC countries the export price index is related to the oil price index alone.

Export prices in current dollars can be derived by using the exchange rate (30). Exchange rate movements are explained by the region's balance of payments situation (34). Import prices in current dollars are defined as the unit value index of imports (31) and import prices in national currency are calculated by using the exchange rate.

These price indices and deflators make it possible to obtain current price values of expenditure components (36 to 43). The total summation of all the components gives GDP in current prices (45) and, dividing this by GDP in constant prices (33), the GDP deflator is obtained. The definition of the trade balance is given in equation (46).

Official development assistance

The last part of the model describes the level and direction of official development assistance (ODA) and foreign investment (47 to 50). Matrices of parameters are estimated and can be adjusted for future changes in policy affecting exports and investment. ODA becomes a major policy variable in this study. Further discussion of it appears later.

The model in equational form

Production

$$\frac{(x^{**})}{l} = \bar{z}_a \left\{ \alpha + \beta \frac{\sum_{t}^{t-4} \theta_t \cdot \Delta s_p^*}{l} + \sum_{t}^{t-4} \varphi_t \cdot r_d^* \right\} + $$

$$+ (\bar{z}_b + \bar{z}_0 + \bar{z}_c) \left\{ \alpha + \beta \frac{\sum_{t}^{t-4} \theta_t \cdot \Delta s_p^*}{l} \right\} \qquad (1)$$

$$r_d^* = \alpha + \beta x_{-1}^* \qquad (2)$$

$$d_p^* \quad = \alpha + \beta s_{p-1}^* \tag{3}$$

$$u \quad = \alpha + \beta\left(\frac{w}{x_{-1}}\right) + \gamma\left(\frac{\Delta s_p^*}{x_{-1}^*}\right) + \delta\left(\frac{\bar{l}_{cs}}{\bar{l}_{cs_0}}\right) \tag{4}$$

$$x^* \quad = \psi x^{**} \tag{5}$$

$$s_p^* \quad = s_{p-1}^* + \Delta s_p^* - d_p^* \tag{6}$$

$$l \quad = (1.0 - u)\bar{l}_{cs} \tag{7}$$

Expenditure on GRP (at constant prices)

$$\begin{aligned} e_{(i,j)}^* \quad &= \bar{z}_a \mu_a \left\{\alpha + \beta x_{(j)-1}^* + \gamma\left(\frac{p_{e(i)}}{p_{m(j)}}\right)_{-1} + \delta\left(\frac{\bar{t}_m \cdot p_{m(j)}}{p_{w(j)}}\right)_{-1}\right\} + \\ &+ (\bar{z}_b \mu_b + \bar{z}_0 \mu_0)\left\{\alpha + \beta\left[e_{(j)-1}^* + \frac{1}{p_{e(j)-1}}(o_{da(j)} + \right.\right. \\ &\left.\left. + \Delta s_{op(j)} + \bar{a}_m + \bar{a}_c)_{-1}\right] + \gamma x_{(j)-1}^*\right\} + \bar{z}_c \mu_c \{\alpha + \beta e_{(j)-1}^*\} \end{aligned} \tag{8}$$

($\bar{z}$ relates to importing region (j))

$$\begin{aligned} c^* \quad &= \bar{z}\{\alpha + \beta x^* + \gamma c_{-1}^*\} + (\bar{z}_b + \bar{z}_c + \bar{z}_0)[x^* - e^* + m^* - g^* - \\ &- \Delta s_p^* - \Delta s_h^* - \Delta c_i^*] \end{aligned} \tag{9}$$

$$g^* \quad = \alpha + \beta r^* \tag{10}$$

$$r^* \quad = \alpha + \beta x_{-1}^* \tag{11}$$

$$\begin{aligned} \Delta s_p^* \quad &= \bar{z}_a \lambda_a \{\alpha + \beta y_{c-1}^* + \gamma_{-1}^* + \delta \bar{i}_{-1} + \epsilon \psi_{-1}\} + \\ &+ (\bar{z}_b \chi_b + \bar{z}_0 \chi_0)\left\{\alpha + \beta x_{-1}^* + \gamma e_{-1}^* + \delta\left[\frac{1}{p_{e(j)-1}} + \right.\right. \\ &\left.\left. + (o_{da(j)} + \Delta s_{op(j)} + \bar{a}_m + \bar{a}_c)_{-1}\right]\right\} + \\ &+ \bar{z}_c \lambda_c \{\alpha + \beta x_{-1}^*\} \end{aligned} \tag{12}$$

$$\Delta s_h^* \quad = \alpha + \beta x^* + \gamma \Delta \bar{n} + \delta \bar{i}_{-1} \tag{13}$$

$$\Delta s_i^* \quad = \alpha + \beta x^* + \gamma (x_{-1}^* - x_{-2}^*) + \delta \bar{i}_{-1} + \epsilon s_{i-1}^* \tag{14}$$

$$e^* \quad = \alpha + \beta \sum_j e_{(i,j)}^* \tag{15}$$

$$m^* \quad = \alpha + \beta \sum_i e_{(i,j)}^* \tag{16}$$

$$x^* \quad = e^* - m^* + c^* + g^* + \Delta s_p^* + \Delta s_h^* + \Delta s_i^* \text{ (for AME)} \tag{17}$$

$$s_i^* \quad = s_{i-1}^* + \Delta s_i^* \tag{18}$$

Profit-wages

$$\left(\frac{y_c}{s_p}\right) = \alpha + \beta\left(\frac{x}{s_p}\right) + \gamma\left(\frac{\omega \cdot l}{x}\right) + \delta\left(\frac{\Delta s_p}{x}\right) + \varepsilon\left(\frac{\bar{i} - l}{100}\right) \quad (19)$$

$$\omega = \alpha + \beta p_{c-1} + \gamma p_y + \delta\left(\frac{u}{u_0}\right) + \varepsilon\left(\frac{y_c}{x}\right)_{-1} \quad (20)$$

$$p_y = \left(\frac{x^*}{l}\right) / \left(\frac{x^*}{l}\right)_0 \quad (21)$$

$$y_c = y_c^* \cdot p_i \quad (22)$$

$$w = \omega \cdot l \cdot \left(\frac{w_0}{l_0}\right) \quad (23)$$

Prices

$$p_w = \alpha + \beta p_{m-1} + \gamma\left(\frac{\omega}{p_y}\right) + \delta \bar{i}_{v-1} + \varepsilon\left[\left(\frac{y_c}{x}\right)_{-1} / \left(\frac{y_c}{x}\right)_0\right] \quad (24)$$

$$p_c = \alpha + \beta p_{w-1} + \gamma \omega + \delta p_{m-1} + \varepsilon \bar{i}_{v-1} \quad (25)$$

$$p_{cg} = \alpha + \beta p_{w-1} + \gamma \omega + \delta \bar{i}_{v-1} \quad (26)$$

$$p_i = \alpha + \beta p_w + \gamma\left[\left(\frac{\Delta s_p^*}{x^*}\right) / \left(\frac{\Delta s_{p0}^*}{x_0^*}\right)\right] \quad (27)$$

$$p_h = \alpha + \beta p_i + \gamma \omega \quad (28)$$

$$\begin{aligned} p_e = {} & \bar{z}_a \{\alpha + \beta p_{v-1} + \gamma l_{qw-1} + \delta \pi_{\mu-1}\} + \\ & + \bar{z}_b \{\alpha + \beta p_{w-1} + \gamma \bar{l}_{qw-1} + \delta p_{m-1} + \varepsilon \hat{\bar{p}}_{ec}\} + \\ & + \bar{z}_c \{\alpha + \beta \bar{p}_{ec} + \gamma \bar{p}_{eE} + \delta p_{m-1}\} + \\ & + \bar{z}_0 \{\alpha + \beta \bar{p}_{ce}\} \end{aligned} \quad (29)$$

($\bar{z}$ relates to exporting region (i))

$$p_e^s = p_e / \rho_s \quad (30)$$

$$p_m^s = \sum_i [p_{e(i)}^s \cdot e_{(i,j)}^*] / [\sum_i e_{(i,j)}^*] \quad (31)$$

$$p_m = p_m^s \cdot \rho_s \quad (32)$$

$$p = \frac{x}{x^*} \quad (33)$$

$$\frac{\rho_{sdr} - \rho_{sdr,-1}}{\rho_{sdr,-1}} = \alpha\left(\frac{b}{x}\right)_{-1} \quad (34)$$

$$\rho_s = \rho_{sdr} / \rho_{sdr,USA} \quad (35)$$

Expenditure on GRP (at current prices)

$$e = p_e \cdot e^* \tag{36}$$

$$m = p_m \cdot m^* \tag{37}$$

$$c = p_c \cdot c^* \tag{38}$$

$$g = p_{cg} \cdot g^* \tag{39}$$

$$\Delta s_p = p_i \cdot \Delta s_p^* \tag{40}$$

$$\Delta s_h = p_h \cdot \Delta s_h^* \tag{41}$$

$$\Delta s_i = p_w \cdot \Delta s_i^* \tag{42}$$

$$d_p = p_i \cdot d_p^* \tag{43}$$

$$s_p = s_{p-1} + \Delta s_p - d_p \tag{44}$$

$$x = e - m + c + g + \Delta s_p + \Delta s_h + \Delta s_i \tag{45}$$

$$b = p_e \cdot e^* - p_m \cdot m^* \tag{46}$$

Official development assistance and private overseas investment

$$o_{da(i)} = \alpha_i x_i \tag{47}$$

$$\Delta s_{op(i)} = \beta_i x_i \tag{48}$$

$$o_{da(j)} = \sum_i \gamma_{i,j} o_{da(i)} \tag{49}$$

$$\Delta s_{op(j)} = \sum_i \delta_{i,j} o_{da(i)} \tag{50}$$

$$\varepsilon_{op(j)} = s_{op(j)-1} + \sum_i \Delta s_{op(j)} - \bar{r}_{d(j)} \tag{51}$$

i = AME region

j = DME region

where for each region:

- $\bar{a}_m$ = DME's official development assistance (net) received from multilateral agencies, in SDR;
- $\bar{a}_c$ = DME's official development assistance (net) received from centrally planned economy zone, in SDR;
- b = trade balance of current account;
- c, c^* = private final consumption expenditure (at current, and constant prices);
- d_p^* = depreciation of fixed capital (at constant prices);
- e, e^* = exports of goods and services (at current, and constant prices);

$e^*_{(i,j)}$ = exports of merchandise from region i to region j (at constant prices);
$\bar{f}_d$ = fade out of overseas private investment;
g, g^* = government final consumption expenditure (at current, and constant prices);
$\bar{i}$ = average interest rate on loan;
$\bar{i}_v$ = money supply/current income index;
l = employment;
$\bar{l}_{cs}$ = civilian labour force;
$\hat{l}_{qw}$ = world liquidity-trade index;
m, m^* = imports of goods and services (at current, and constant prices);
$\tilde{n}$ = population;
$o_{da(i)}$ = total official development assistance supplied by AME region i;
$o_{da(j)}$ = total official development assistance received by DME region j;
p = implicit deflator of GRP;
p_c = implicit deflator of private consumption expenditure (consumers' price index);
p_{cg} = implicit deflator of government consumption;
p_e = export price index (in 1970 \$);
$p^{\$}_e$ = export price index (in current \$);
$\bar{p}_{ec}$ = export price index of primary commodities;
$\hat{p}_{eE}$ = oil export unit index;
p_h = implicit deflator of housing investment;
p_i = implicit deflator of fixed equipment investment;
p_m = import price index (in 1970 \$);
$p^{\$}_m$ = import price index (in current \$);
p_w = implicit deflator of increase in stocks (wholesale prices index);
p_y = labour productivity index;
r^* = government current revenue (at constant prices);
r^*_d = research and development expenses (at constant prices);
s^*_i = total inventory (at constant prices);
s_p, s^*_p = fixed capital stocks (at current, and constant prices);
$\Delta s_h, \Delta s^*_h$ = housing investment (at current, and constant prices);
$\Delta s_i, \Delta s^*_i$ = increase in stocks (at current, and constant prices);
$\Delta s_{op(i)}$ = private foreign investment by AME region i;
$\Delta s_{op(j)}$ = private foreign investment in DME region j;
$\Delta s_p, \Delta s^*_p$ = non-housing investment (at current, and constant prices);
t = time;
$\bar{t}_m$ = rate of customs duty to total imports;
u = unemployment ratio;
w = compensation of employees (at current prices);
x, x^* = gross regional product (at current, and constant prices);

x^{**}	= gross regional product at average rate of capacity utilization (at constant prices);
y_c, y_c^*	= operating surplus (at current, and constant prices);
$\bar{z}$	= dummy variables: for AME: $\bar{z}_a = 1$ else $\bar{z}_a = 0$; for CPE: $\bar{z}_c = 1$ else $\bar{z}_c = 0$; for OPEC: $\bar{z}_o = 1$ else $\bar{z}_o = 0$; for DME: $\bar{z}_b = 1$ else $\bar{z}_b = 0$
ω	= index of average wage and salary per employee;
ρ_s	= foreign exchange rate index (in terms of \$);
ρ_{sdr}	= foreign exchange rate index (in terms of SDR);
ψ	= production capacity parameter which is endogenous for AME; otherwise one;
λ	= parameters of adjustment for investment (scenarios);
μ	= parameters of adjustment for trade flows (scenarios);
$\bar{}$	= exogenous variables;
*	= variables in constant prices.

Variables for population are expressed in thousands of persons; indices are based on 1970 values; other variables are expressed in millions of dollars.

Asian input-output model (AIOM)

Structure of the AIOM

Economic development implies changes in the composition of output and foreign trade. In order to depict such changes, I/O techniques initiated by Wassily Leontief provide a good foundation. The I/O model recognizes explicitly that economic activity includes far more than effort oriented towards final product alone. Total output is the sum of outputs used to satisfy the intermediate demands of the particular industries, together with those which are directed towards the fulfilment of final demand.

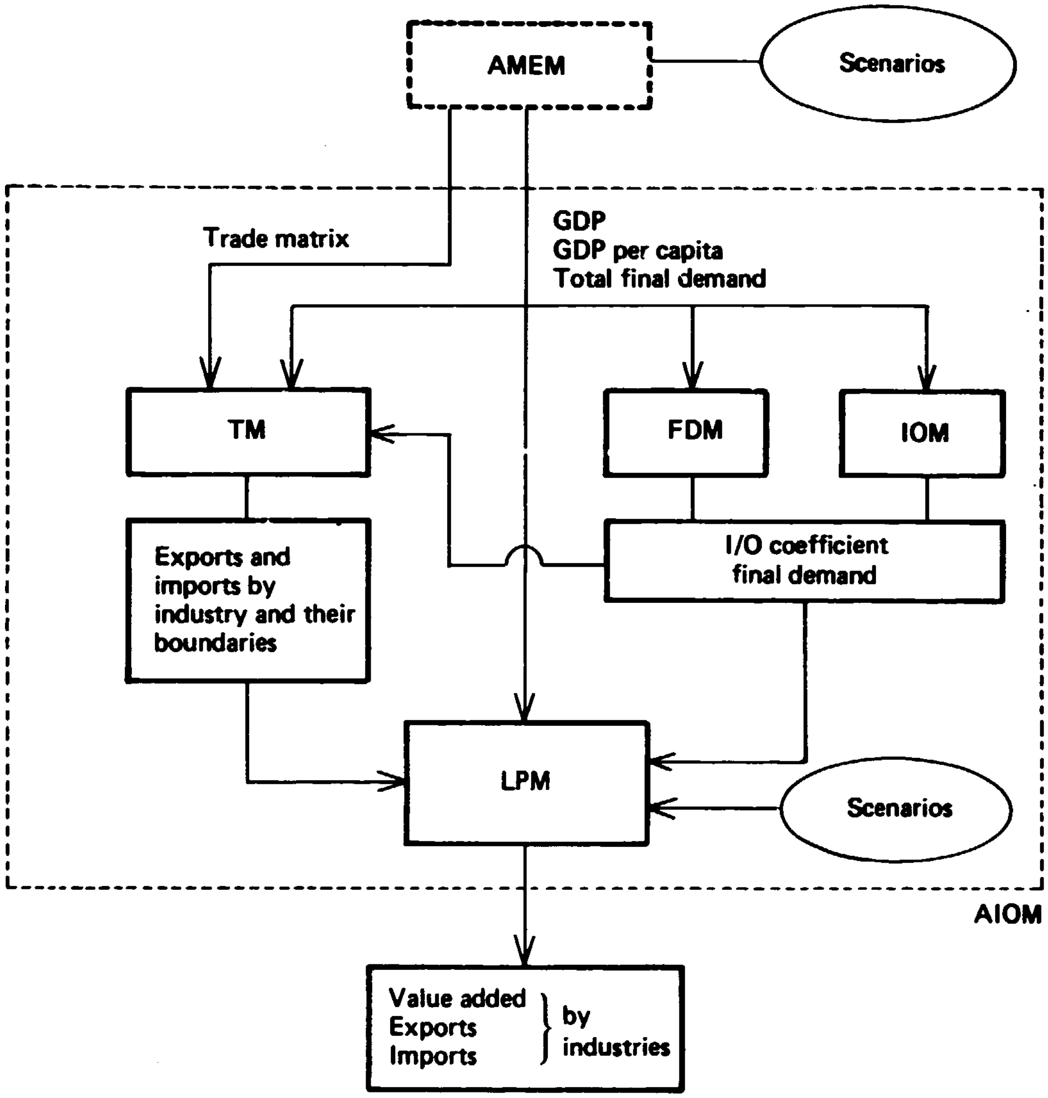
The purpose of constructing AIOM is to project the world economy in 1985 and 1990 and suggest desirable structural changes. To do so, it is necessary to have:

Projected I/O tables for 1985 and 1990
Industrial international trade estimates and their upper and lower boundaries for 1985 and 1990
An optimization scheme

The four modules of AIOM (the IOM, the FDM, the TM and the LPM) deal with the above.

The relation between each of the four modules and AMEM is illustrated in figure 2. I/O coefficients and final demands by industries are projected on the basis

Figure 2. The AIOM



of GDP, GDP per capita, and total final demand given by AMEM. Such projected I/O coefficients and final demand by industries and AMEM projections of GDP per capita and the trade matrices are fed into TM, where the regression estimates of trades by industries are modified to keep consistency with the AMEM estimates. The outputs of TM, IOM, FDM and AMEM projections of GDP per capita all enter LPM, where estimates of the optimal domestic and international allocation of resources are obtained.

The model is explained in detail in the following paragraphs: TM provides central estimates of exports and imports as well as intervals of these. When central estimates are fed into LPM, solutions can be obtained by solving I/O accounting relations. This case is described as a standard projection case later under "Standard projections for industries". When intervals of exports and imports are given by TM, LPM is used to obtain unique solutions of sectoral value added and trade figures.

Input-output module (IOM)

In analysing the structure of an economy I/O relations provide a first approximation of the real situation although I/O relations are often over simplified because of problems involving industry classifications (particularly for developing countries), the role of technology (such as fixed coefficient assumption), etc. The present model covers 14 industries for 14 regions (see annex I). It is not an easy task to aggregate I/O data for 14 industries and 14 regions for the base year 1970. It is even harder to project I/O coefficients for the target years of 1985 and 1990—yet this is the ultimate goal.

Before discussing the estimation procedure of I/O coefficients it is of interest to examine difficulties in treating country I/O tables for aggregation. Among these are:

(a) There is a lack of reliable tables for many developing countries. For some developing countries tables themselves are not available. Even if tables are available they tend to be unreliable because they often fail in achieving the consistency required for I/O tables. For example, the sum of a row does not equal the sum of the corresponding column;

(b) There are country differences in methodology of I/O tables. The most important differences are found in the treatment of imports. There are three methods of treatment: to include imports (both competing and non-competing) distributed along the rows of I/O table; again competing imports alone can be distributed; and finally domestic flows and imported goods can be separately treated;

(c) There are country differences in the specification of a table. Industry classification is different from one country to another while transactions are valued either in producer's prices or in purchaser's prices. Moreover, it is difficult to obtain tables for the same year.

These problems should be consistently treated in the aggregation procedure. Estimation or revision of I/O tables would be necessary for case *(a)*, while data modification is necessary for cases *(a)* and *(b)* to maintain consistency. Some of these problems can be overcome by the aggregation procedure adopted below.

Regional input-output table

The world is divided into 14 regions, of which India, Indonesia, Japan, the Philippines and the Republic of Korea are treated separately as single regions. For other regions which include more than two countries, a process of converting a set of country tables into a regional table is developed. The process is different from one region to another because of the regional differences in the availability of country tables: If I/O tables of all the countries in a region are available, then the regional table can be constructed as an aggregate of all the country tables by transforming them into the same specification.

For most of the regions, however, such is not the case. As a result, regional tables must be constructed with some of the standardized country tables and partial data of other countries. The following steps were taken:

(a) The first step was to construct a base table by aggregating available I/O tables of the region;

(b) The second was to find data of industrial gross outputs and value added of those countries where I/O tables are not available to modify the base table by using these partial data.

United Nations world industrial statistics provide such partial data. In equation (53) input coefficients a_{ij} and the ratio of value added v_j are modified for the region concerned as follows:

$$v_j = \frac{X_j^0 + \sum_k {}_kX_j}{Z_j^0 + \sum_k {}_kZ_j} \qquad (52)$$

$$a_{ij} = a_{ij}\frac{1 - v_j}{1 - v_j^0} \qquad (53)$$

where:

$_kX_j$ = value added of the j^{th} industry of the k^{th} country of the region;
$_kZ_j$ = gross output of the j^{th} industry of the k^{th} country of the region;
v_j = ratio of value added of the j^{th} sector of the region;
a_{ij} = input coefficients of the region;

and superscript 0 refers to the base table.

By choosing the ratio of value added and input coefficients as above the gross output and the value added for each sector of the region are consistent with the present data.

Projection of input-output tables for 1985 and 1990

There are in principle two methods of predicting I/O tables. One method is to determine parameters on the basis of information concerning initial values and the future values of the sums of rows and columns of I/O tables.

The RAS method is an example of this type.[3] This method, however, is not applicable in our model because only the macroeconomic framework, not the sums of rows and columns, is given to project future tables. The method adopted has been the application of multivariate analysis to the present data. Since the time series data of I/O tables are not available except for a few countries, the analysis is based on cross-section data.

As the economy grows the input coefficients may change for several reasons including technological change, changes in relative prices of inputs, changes in product mixes and changes in output.

[3] See, for example, R. Stone, J. Bates and M. Bacharach [1].

It may be too drastic to assume that all these effects can be approximated by the impact of economic growth on input coefficients but, remembering that our I/O tables are those of 14 industries and 14 regions, and that effects of economic growth are probably dominant, we have assumed that as the economy grows the I/O structure approximates that of countries of the same GDP level. The input coefficients are separated into two parts: one is influenced by economic growth and the other is characteristic of the region. This latter is because some parts of the input coefficients cannot be explained solely by economic growth.

In the FUGI model, which provided the foundation of the present ESCAP-FUGI model, principal component analysis (PCA) was employed in dealing with input coefficients. PCA is a convenient method when we want to describe each of many variables (in our case 14) by a linear function of a smaller number of other variables with a high degree of accuracy. It was found unsatisfactory, however, in that the fit of estimates in a few industries was poor and the correlation coefficients lower than 0.6. The estimates were often conservative, changes over time being relatively small.

To remedy these defects a new algorithm has been adopted called maximum correlation coefficient analysis (MCCA) which is an extension of canonical regression analysis. The basic concept is to find a direction in the data space along which the regressed estimate shows the best fit.

Suppose that a^o is a column vector in input coefficients of the regional table, the last component of a^o being the ratio of value added. By the nature of input coefficients

$$i'_m a^o = 1 \tag{54}$$

where i_m is a m-dimensional vector with unity as each component $(1, 1 \ldots 1)$.

a^o is disaggregated into two parts, a^c and a^r, where a^c is to be estimated with the best fit as a function of GDP per capita. a^r is determined as the difference of a^o and a^c in the base year and a^r remains constant for the projection period. a^c_i, the i^{th} component of a^c, can be expressed as a function of GDP per capita:

$$a^c_i = \psi_i(\text{GDP/P}) \tag{55}$$

We may project a^c for target years if we have GDP/P of target years. The latter being provided by AMEM, we can estimate future values of a as:

$$\hat{a} = \hat{a}^c + a^r \tag{56}$$

where a^c is given by substituting future GDP/P in equation (55). Such estimated $\hat{a}$ becomes the basis for further analysis. This procedure is carried out for each column vector of the I/O matrix.

Final demand module (FDM)

Regarding the estimation of final demand patterns, domestic final demand values for each sector are estimated for 1967 to 1973.[4] Estimations are carried out

[4]From 1967 the ISIC statistical treatment has been in its present format. In order to maintain consistency, the initial value was taken in 1967 where we employ data from *World Industrial Statistics*.

by solving the equation[5]

$$f = (I - A)V^{-1}\chi - (e - m)$$

where:

f = the column vector of final demand;
χ = the column vector of value added;
e = the column vector of exports in f.o.b.;
m = the column vector of imports in f.o.b.;
V = the diagonal matrix of ratios of value added;
A = the input coefficient matrix.

Parameters such as A and V are estimated by MCCA for each year from 1967 to 1973. Value added and trade figures are directly substituted in the above equation.[6]

Thus calculated final demand values for each industry for each region are not free from errors, which partly come from the estimation of parameters A and V and partly from statistical discrepancies of export and import data as well as value-added data.

Using such estimated final demand values for each industry and for each region we first tried regression analysis for each of these time series data as a linear function of GDP per capita of the region. The results were not satisfactory.

Much improvement has been obtained by a similar analysis where it is assumed that the coefficient of GDP per capita will be the same for each region as far as it belongs to the same bloc. AME and other blocs are explicitly differentiated. The following equation is therefore applied for AME and for other regions:

$$_k f_i / _k p = \alpha_0 + \sum_j \alpha_j \cdot D_j + \beta_k z / _k p \tag{58}$$

where:

$_k f_i$ = the domestic final demand, region k, industry i;
$_k p$ = the population, region k;
D_j = the dummy, region j;
$_k z$ = the GDP, region k.

The results are shown in tables 1 and 2 below.

In case the t-value of the coefficient of GDP per capita is less than 2.0, final demand per capita is assumed to be fixed at the 1970 level.[7] Projected final demands are adjusted proportionately to keep consistency with macro projections.

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[5]This equation is derived from the fundamental I/O accounting relations:

$$Z = AZ + f + e - m \quad \text{and} \quad \chi = VZ$$

where Z is a column vector of gross output.

[6]It should be noted, however, in the process of data transformation—mainly allocation of trade figures by industries—value added and trade figures may possess some errors.

[7]The pattern in 1970 is calculated by equation (52). The possibility of negative final demands exists, and if this occurs, such negative final demands values are changed to zero.

TABLE 1. ESTIMATION RESULTS OF FINAL DEMAND EQUATION, DEVELOPED REGIONS

Industry	*Constant term*	*Dummy variables by area or country*					*Coefficient of GDP per capita*	*t-value*	*Correlation coefficients*
		Australia	*Japan*	*United States*	*Europe*	*ECPE*			
Agriculture, forestry and fishery	0.032	−0.112	0.003	−0.058	0.015	−0.032	0.022	0.670	0.784
Mining	−0.048	0.056	0.016	−0.014	0.018	0.070	0.010	1.504	0.969
Foods, beverages and tobacco	0.140	0.044	−0.096	0.008	−0.030	0.236	0.045	2.003	0.982
Textiles, wearing apparel and leather	0.050	−0.042	0.009	0.016	−0.013	0.065	0.030	3.707	0.987
Wood products	0.049	−0.047	−0.009	0.050	−0.020	0.005	−0.007	−1.839	0.990
Chemical products	0.268	−0.235	−0.182	0.070	−0.164	−0.169	−0.011	−0.711	0.989
Metal and metal products	0.068	−0.033	−0.044	−0.117	−0.001	0.033	−0.006	−0.674	0.987
Non-electrical machinery	0.006	0.022	0.046	−0.046	−0.011	0.084	0.024	1.808	0.913
Electrical machinery	0.046	−0.047	0.022	0.038	−0.030	−0.007	0.016	2.082	0.987
Transport equipment	0.125	−0.115	−0.091	0.044	−0.082	−0.012	0.026	1.873	0.988
Construction	−0.070	0.032	0.104	−0.281	0.010	0.263	0.153	11.633	0.989
Electricity, gas and water	0.017	0.003	−0.021	0.050	−0.006	−0.032	0.008	1.612	0.992
Merchandise and services	−0.286	0.186	−0.124	−0.212	0.002	−0.668	0.622	10.149	0.997
Transportation and communication	0.006	0.025	0.023	−0.032	−0.046	−0.112	0.045	7.248	0.997

TABLE 2. ESTIMATION RESULTS OF FINAL DEMAND EQUATION, DEVELOPING REGIONS

Industry	Constant term	*Dummy variables by area or country*								Coefficient of GDP per capita	t-value	Correlation coefficients
		India	Indonesia	Malaysia, Singapore and Thailand	Philippines	Republic of Korea	Africa and Latin America	Western Asia	Other Asia			
Agriculture, forestry and fishery	0.068	−0.023	−0.037	−0.051	−0.039	0.016	0.029	−0.043	−0.032	−0.038	−0.930	0.985
Mining	0.023	−0.023	−0.022	−0.021	−0.023	−0.024	−0.002	−0.016	−0.023	−0.011	−0.385	0.900
Foods, beverages and tobacco	−0.024	0.018	0.027	0.039	0.027	0.015	0.019	0.048	0.022	0.135	2.164	0.951
Textiles, wearing apparel and leather	−0.012	0.006	0.009	0.012	0.005	0.011	−0.003	0.013	0.004	0.069	2.980	0.957
Wood products	−0.003	0.004	0.002	0.002	0.002	0.003	0.003	0.003	0.003	0.009	2.239	0.949
Chemical products	−0.047	0.038	0.036	0.030	0.038	0.037	0.002	0.011	0.036	0.114	5.098	0.933
Metal and metal products	−0.016	0.015	0.013	0.010	0.014	0.005	0.004	0.012	0.012	0.032	2.063	0.872
Non-electrical machinery	−0.013	0.012	0.011	0.011	0.012	0.012	0.003	0.007	0.009	0.042	4.353	0.950
Electrical machinery	−0.027	0.023	0.023	0.018	0.020	0.018	0.007	0.013	0.021	0.059	7.090	0.953
Transport equipment	−0.013	0.013	0.011	0.013	0.010	0.012	0.001	0.009	0.009	0.036	3.970	0.940
Construction	−0.036	0.036	0.033	0.034	0.021	0.034	0.018	0.020	0.029	0.126	3.090	0.944
Electricity, gas and water	−0.006	0.006	0.004	0.003	0.003	0.004	−0.001	0.006	0.006	0.013	1.779	0.911
Merchandise and services	−0.040	0.029	0.049	0.069	0.075	0.043	0.113	0.105	0.036	0.284	4.192	0.993
Transportation and communication	−0.031	0.030	0.028	0.028	0.022	0.028	−0.001	0.018	0.033	0.068	4.531	0.950

Sectoral trade module (TM)

The function of TM is to provide both point and interval estimates of exports and imports by industry of each region. The point estimates correspond to control values of interval estimates and must satisfy the following conditions:

(a) They should be as close as possible to the estimates obtained from the regression analysis of the past data;

(b) They should be consistent with total trade estimates given by AMEM.

To satisfy condition *(b)*:

$$\sum_{i} {}_{k}e_i = E_k \tag{59}$$

$$\sum_{i} {}_{k}m_i = M_k \tag{60}$$

where:

${}_{k}e_i$ = the estimate of the export of i^{th} industry of k^{th} region in f.o.b.;
${}_{k}m_i$ = the estimate of the import of i^{th} industry of k^{th} region in f.o.b.;
E_k, M_k = the total exports and total imports of k^{th} region estimated by AMEM.[8]

(c) They should satisfy the supply-demand balance of such goods in the global context. In equation form, this is equivalent to:

$$\sum_{k} {}_{k}e_i = \xi_k \sum_{k} {}_{k}m_i \tag{61}$$

Theoretically speaking ξ_k should be equal to one but actual ξ_k is slightly different, in most cases between 0.9 and 1.2. The average values in the last ten years are used for ξ_k in TM to keep consistency with actual data;

(d) They should not produce negative value added in any sector; so:

$$X = V(I - A)^{-1}(f + e - m) \geqslant 0 \tag{62}$$

This equation holds true for any region. For simplicity industry subscripts for regions are omitted here.

A primitive way of determining e and m to satisfy the above constraints is to solve the following optimization problem:

$$\min J = \sum_{k} \| e_k - \hat{e}_k \| + \sum_{k} \| m_k - \hat{m}_k \| \tag{63}$$

subject to constraints (59) through (62), where ˆ indicates the regression estimate and the subscript k refers to the k^{th} region.[9]

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[8]Notational changes have been made to avoid complication. E_k and M_k refer to e^* and m^* for k^{th} country of AMEM respectively.

[9]Similar approach was made by A. Nagy [2].

This is a quadratic programming problem with approximately $2 \times 14 \times 14 = 392$ variables. This is a fairly large number of variables. Instead of solving the above quadratic programming problem TM adopts the following method.

The first step is to construct the initial estimates of exports and imports of i^{th} industry for k^{th} region as:

$$ {}_k e_i^I = {}_k \hat{e}_i \frac{E_k}{\sum_i {}_k \hat{e}_i} \tag{64} $$

$$ {}_k m_i^I = {}_k \hat{m}_i \frac{M_k}{\sum_i {}_k \hat{m}_i} \tag{65} $$

These estimates obviously satisfy equations (59) and (60), where I indicates the initial estimates.

The second step is to rewrite ${}_k e_i$ and ${}_k m_i$ in terms of initial estimates as:

$$ {}_k e_i = {}_k e_i^I + {}_k e_i^I (\delta_i + f_k) \tag{66} $$

$$ {}_k m_i = {}_k m_i^I + {}_k m_i^I (\gamma_i + g_k) \tag{67} $$

where $i = 1, 2, \ldots, 14$; $k = 1, 2, \ldots, 14$.

In this step one determines residual parameters δ_i, γ_i, f_k and g_k by minimizing the following:

$$ \begin{aligned} J = {} & \sum_i (\delta_i^2 + \gamma_i^2) + \sum_k (f_k^2 + g_k^2) + \\ & + \lambda \sum_i \frac{1}{(m_i^I)^2} \left(\sum_k {}_k e_i - \xi_i \sum_k {}_k m_i\right)^2 + \\ & + \lambda \sum_i \frac{1}{(m_i^I)^2} \left\{\sum_k {}_k m_i^I (\gamma_i + g_k)\right\}^2 \end{aligned} \tag{68} $$

where $m_i^I = \sum_k {}_k m_i^I$, the world total trade of i^{th} industry products.

Minimization is carried out subject to the constraints of (59) and (60). The third and fourth terms of equation (68) are introduced so as to satisfy equation (61) without any considerable change in estimates of world trade in each sector. λ is an adjustable parameter.

The third step is to check whether the results of the above process violate the inequality constraint (62). If so, replace the calculated negative value added by a small positive number. Adjust other sectoral value added, so that the total of revised total GDP should be the same as before.

The next step is to calculate $e - m$ for each region by:

$$ e - m = (I - A) V^{-1} \tilde{x} - f \tag{69} $$

where $\tilde{x}$ is the revised x.

A final step is to modify e and m so as to satisfy (69) with as little change as possible.

This process is more economical than the quadratic programming method, as optimization in equation (68) requires the solution of a linear simultaneous equation of only $(3 \times 14) + (4 \times 14) = 98$ dimensions. The application of this process to real data has been found very successful.

It is also to be noted that both imports and exports are estimated in f.o.b. values so that trade in services and transportation may be treated explicitly in the model.

From the industry trade module (TM) point estimates of exports and imports can be obtained. Using the I/O module (IOM), on the other hand, input coefficients and final demand patterns can be obtained. In view of the basic demand-supply balance in the I/O accounting framework information is sufficient to determine value added by industries uniquely. The balance equation is:

$$x = (I - A)^{-1} V (f + e - m) \qquad (70)$$

where notations were explained in former sections and regional subscripts are omitted for simplicity sake.

Calculated value added in this way constitutes the standard industry projection. This case becomes a point of comparison. In the linear programming model intervals of exports and imports are given instead of point estimates. Therefore, it is not possible to solve for value added by industries by equation (70). To solve for value added and exports and imports by industries it is necessary to introduce an optimization procedure, which is the subject of the following paragraphs.

Linear programming module (LPM)

The LPM is used to allocate resources domestically, as manifested by value-added solutions, for industries and internationally, as manifested by exports and imports by industries in such a way that some kind of criteria is maximized under certain constraints. Linear programming is the analysis of problems in which a linear function of a number of variables is to be maximized (or minimized) when those variables are subject to a number of constraints in the form of linear inequalities.

There are two ways of evaluating LP optimization in planning.[10] The first is based on the idea that parametric programming methods provide a fairly inexpensive means of exploring the frontiers of the economy's choice set while the second view emphasizes the ability of an LP model to simulate a general equilibrium or competitive resource allocation, complete with the prices of the dual solution. Our LPM belongs to the first category.

In order to define the feasibility zone three types of constraints are introduced. The first reflects real limitation such as I/O balances and balance of payments constraints. The second is closely related with the scenarios we choose.

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[10]See, for example, chapter III by Lance Taylor, in Blitzer, Clark and Taylor [3].

For example, when one wishes to maximize manufacturing value added, one may assume that agricultural production should not decrease or that employment in manufacturing should not decrease. The third type of constraint is introduced for purely technical reasons in order to obtain realistic LP solutions. Boundaries on exports and imports and constraints on the changes in industral structure are examples of this category.

I/O balances have appeared in many places. Constraints on trade were discussed in the section on FDM while scenario constraints are to be discussed in the next section. The boundary on the changes in industrial structure is a wide one.

All these constraints make up a fairly realistic approximation of the set of feasible alternatives. The LP algorithm is an effective means to get out to the boundaries of this set. Constraints actually used, except for the second type, will be explained later.

Once one sets up the feasible zone, what is necessary is the criterion to move the economy to the boundaries. To explain it in most general terms, a total or grand objective function, J, is constructed as a weighted linear summation of some partial objectives, J_m. Such objectives as agricultural production, employment, and manufacturing value added are partial objectives. In symbol form, this is expressed as:

$$J = \sum_{m} W_m \cdot J_m \tag{71}$$

where W_m is a weight of the m^{th} objective.

The m^{th} partial objective is expressed as a linear function of sectoral value added, ${}_kx_i$, sectoral exports, ${}_ke_i$, and sectoral imports, ${}_km_i$, for all the regions.

$$J_m = \sum_{k} w_k^m \left(\sum_{i} {}_kC_i^x \cdot {}_kx_i + \sum_{i} {}_kC_i^e \cdot {}_ke_i + \sum_{i} {}_kC_i^m \cdot {}_km_i\right) \tag{72}$$

where:

w_k^m = weights assigned to the k^{th} region in the m^{th} partial objective;
${}_kC_i$ = coefficients for endogenous variables (x, e, and m) of LP.

The coefficients $\{{}_kC_i\}$ transform endogenous variables to the m^{th} objective. Suppose employment is the m^{th} objective. Then ${}_kC_i^x$ is the ratio of employment to value added of k^{th} sector of k^{th} region while ${}_kC_i^e$ and ${}_kC_i^m$ are zero. If the m^{th} objective is agricultural gross output, then ${}_kC_i^x$ is the inverse of the ratio of value added for industry 1; for other industries ${}_kC_i^x$ is zero. ${}_kC_i^e$ and ${}_kC_i^m$ are zero for all industries.

Weights (w_k^m in equation (72)) can be different from one region to another. For instance, one may assign zero weight to advanced market economies when one wishes to maximize industrial production of developing countries.

By setting up this grand objective function it is possible to optimize two or more partial objectives. However, it involves arbitrary value judgement in choosing the weights for various partial objectives. In this ESCAP-FUGI exercise we shall mainly deal with a grand objective function (J) with only one partial objective.

In sum, LPM enables us to allocate resources both domestically and internationally. The solutions of LPM depend upon:

Objective function
Constraints
Exogenous variables and parameters

All of these are affected by the scenarios. Scenarios are manifested directly in the choice and specification of objective function and constraints. The third component, part of which is the output of AMEM, indirectly depends upon scenarios because scenarios of AIOM should be consistent with those of AMEM. Scenarios are discussed below.

Scenarios and projection results

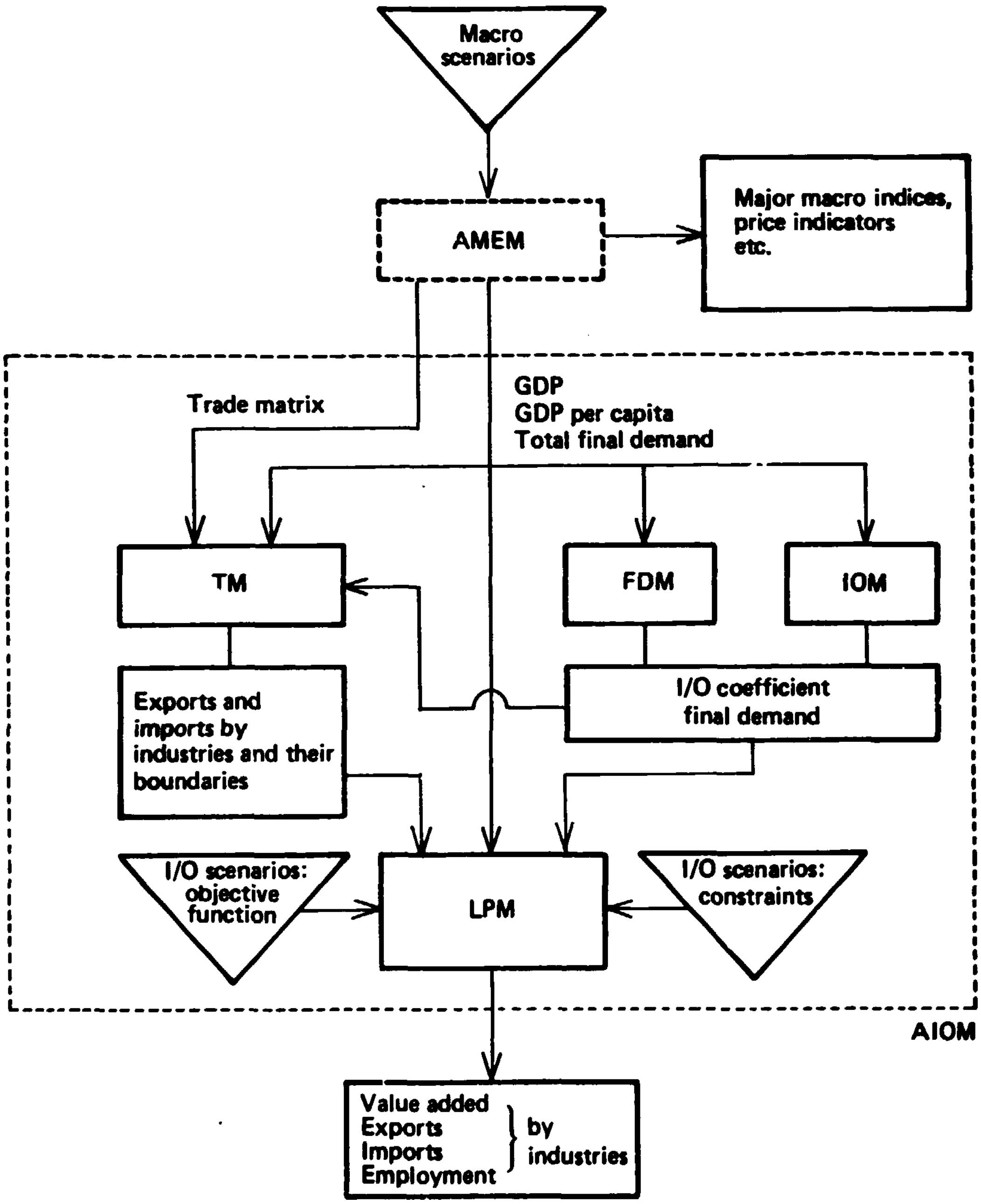
Increasing attention is being paid to the formulation of strategies for the 1980s. A conscious policy is necessary to limit population growth and give more emphasis to agricultural production. Rapid industrialization and technology transfer, though the old clichés of development policy, remain very important targets for development. However, they should be integrated in overall development policy, which should equally emphasize social, demographic, and political aspects as well as economic aspects.

In this section the possible future development of ESCAP economies in relation to the rest of the world are described by setting up scenarios which are manageable in the framework of the present model. The relationship of the scenarios generally to the fundamental structure of the model needs to be noted. As in the case of figure 2, the relation between AMEM and AIOM through common variables is presented below in figure 3 to show how scenarios come into the picture.

Figure 3 indicates that is is possible to prepare separate scenarios for AMEM and AIOM. Ideally, scenarios for two sub-models should be perfectly co-ordinated, but this is technically impossible. The best one can do is to prepare a combination of scenarios which are co-ordinated as well as possible. This point requires more explanation and will be discussed after introducing the scenarios.

The first approach to prediction is to estimate trend values for the parameters and for both exogenous and endogenous variables. This kind of prediction is different from predictions based on some scenarios such as LDC's export-led growth. In order to distinguish the latter, the present trend estimation case is called the "standard scenario". For AMEM this is merely a trend analysis. For AIOM, it is standard projection for industries. In addition to standard scenarios for AMEM and AIOM, many other scenarios are drawn up. I/O scenarios are combined with appropriate macro scenarios. The combination of AMEM standard scenario and AIOM standard scenario constitutes the basic standard scenario and one can assess the effects of other scenarios in comparison with this scenario. In this sense many of these scenarios can be seen as partial sensitivity analysis, because they try to assess the effects of changes in parts of the economic

Figure 3. Scenarios for two sub-models



structure on the rest of the economic system by analysing how much deviation occurs from the basic standard scenario.

First, macro scenarios are discussed, then I/O and combined scenarios.

Macro scenarios

Macro scenarios are formulated basically by changing parameters (or predetermined variables) and by fixing targets directly or indirectly. There are several such parameters, pre-determined variables and targets in the model. Macro scenarios are constructed by combining these components. Let us first explain these components.

Components of macro scenarios

Growth performance of developed countries

The growth performance of developed countries is a crucial element affecting the economies of the developing world. The experience of the 1970s taught us the importance of this performance. By arbitrarily assuming faster or slower economic growth of developed countries one can assess the impacts of the assumption on the developing world.

ODA

The total amount of ODA allocated to each developing country depends upon the economic growth of developed countries, their ODA-GNP ratio and their performance regarding ODA distribution. Faster economic growth will favourably affect the total amount of ODA to developing countries while the ODA-GNP ratio as a policy target is a result of the past economic performance of developed countries and the climate of world politics. Similarly, whether or not more aid should be given to the least developed countries is a matter discussed in the international political community.

Exports

The development of exports depends heavily upon the international economic and political climate as well as on export promotion efforts of individual countries. There are several issues concerning export promotion. One is export-led growth of low-income countries. As a result of their individual efforts and international co-operation to promote exports of low-income countries they may increase exports gradually. Another issue is growing protectionism. In recent years it has been clearly recognized that slower growth in developed countries adversely affects international trade. The last issue in this category is concerned with the self-reliance in international trade. When the developed countries become more protective, developing countries should protect themselves by tightening the ties among themselves. At the same time efforts to find complementarity in their trade and industrial structure have been strengthened among developing countries in recent years. One example is the strengthening ties of ASEAN. All these can be treated in our model by shifting up or down the bilateral trade equation (or by changing parameter μ in equation 8).

Doubling GDP per capita of low-income countries

The doubling of GDP per capita of low-income countries between 1980 and 2000 has been proposed in international meetings. This target is equivalent to a compound annual growth rate in GDP per capita of 3.5 per cent. The effects of this on other developing countries on the North-South gap and on the gap within developing countries are of interest. The doubling of income target is incorporated indirectly in the model by moving the non-housing investment equation by 20 per cent (see equation 12).

Prices of oil and other primary commodities

One of the proposals regarding future developments of prices of commodities including oil is to set up a system of indexation relating these prices to prices of manufactured goods. In the past, prices of manufactured goods have risen steadily while prices of the vast majority of primary commodities have experienced large fluctuations. This asymmetry has affected income and the terms of trade of primary commodity producing developing countries. In the scenarios given in table 3 oil prices are set to grow at 7 per cent per year from 1980 onwards. Similarly, commodity prices are assumed to grow at 6 per cent per year. At the later stage this assumption will be changed by *(a)* setting the growth rate of

TABLE 3. CHARACTERISTICS OF SCENARIO COMPONENTS BY GROUP

	Group		
Scenario component	*Optimistic*	*Pessimistic*	*Standard*
Growth	Developed countries have faster economic growth than trend	Slower economic growth	Trend
ODA	ODA and GDP at 0.7% target ratio	ODA/GDP ratio less than actual	Trend
	Distribution of ODA in favour of low-income countries	Unchanged distribution	Trend
Trade	Export-led growth	No export-led growth	No export-led growth
	No growing protectionism	Growing protectionism	No growing protectionism
	Self-reliance in international trade	Self-reliance	No self-reliance
Income target	Income doubling	Not doubling	Not doubling
Commodity prices	Increase in oil price 7%; other primary commodity prices 6%	Increase in oil price 7%; other primary commodity prices 6%	Increase in oil price 7%; other primary commodity prices 6%
Indexation	None	Possible	Possible[a]
Population	United Nations medium projections	United Nations medium projections	United Nations medium projections

[a]This case may be a variant of the standard case.

both oil and commodity prices at 6 per cent, and *(b)* by indexing both prices to a weighted average of export prices of advanced market economies. The indexation is expressed as:

$$p_e = \frac{p_{ec}}{\bar{p}_{e,1980}} \cdot \bar{p}_e$$

where:

$\bar{p}_e$ = the weighted average of export prices of AME countries;
p_{ec} = the commodity (or oil) price index.

For *(b)*, the weights for export prices are total exports of each advanced market economy.

Population

Population is another important element in the scenarios. Ideally this variable should be an endogenous variable but for simplicity it was decided to treat it as an exogenous variable. The medium population projections of the United Nations are used for this purpose.

The components of scenarios can be divided into three groups from the viewpoint of developing countries. These groups are the optimistic, the pessimistic and the standard (see table 3). Optimistic and pessimistic cases bound the possible growth rates of the region, and serve as reference points.

Standard macro scenario

The scenario identifies the levels of production, consumption, investment, international trade, prices etc., where trends and policies of the past continue. The results of this scenario indicate the kind and magnitude of problems which could emerge in the future if no important changes were imposed. This scenario functions as a basis for comparison from which to assess alternative policies.

Optimistic macro scenario

This scenario combines all the positive factors mentioned above. The 0.7 per cent ODA target is assumed to be fulfilled for the coming decade and more ODA is shifted in favour of low-income countries. In order to let the low-income countries take off and in order to compensate for lack of financial and other resources, this scenario envisages a development strategy giving more priority to the low-income countries. The scenario envisages a development strategy giving more ODA to the low-income countries by diverting it from higher-income developing countries. Low-income economic groupings are defined as those with per capita income less than $250 in 1976. These are Afghanistan, Bhutan and Nepal; Bangladesh and Pakistan; India; Indonesia; Sri Lanka and Maldives; Africa; and Other Asia. In order to assess this scenario we will do the following:

reduce the ODA share of non-low-income DME countries by the following scheme:

$$ODA^* = (0.95)^{t-1979}\ ODA^*_{1977} \quad (73)$$

where:

ODA* = the ratio of world ODA to the ODA given to non-low-income DME;
t = the year.

Thus, the ODA share of low-income countries will increase. This increment is proportionately distributed to each low-income country in accordance with its GDP in 1980. The weights are as follows:

Afghanistan, Bhutan and Nepal	0.0235
Bangladesh and Pakistan	0.1300
India	0.3394
Indonesia	0.0850
Sri Lanka and Maldives	0.0165
Africa	0.3736
Other Asia	0.0321

Another element constituting the optimistic scenario is concerned with trade. Export promotion is considered to be an engine for fast industrial growth. The so-called export-led growth and self-reliance can be manifested by shifting the bilateral trade flow function.[11]

The doubling of GDP per capita of low-income countries between 1980 and 2000 has been proposed in international meetings. This target is equivalent to a compound annual growth rate in GDP per capita of 3.5 per cent.

It is assumed that the prices of oil and other commodities grow at 7 and 6 per cent respectively.

Pessimistic scenario

Contrary to the optimistic scenario, the pessimistic scenario is a combination of all the negative characteristics so far mentioned.

The world economic situation at present does not look bright; high unemployment rates and low-economic growth in developed countries undoubtedly have affected exports from official development assistance flows to developing countries. To analyse the effect of different levels of economic growth in developed countries on the exports and economies in general of developing countries, when projected GDP hits the upper boundary, the upper boundary is

[11]For all DME except NICs (newly industrializing countries and areas)—Hong Kong, the Republic of Korea and Singapore—matrix elements of bilateral trade flows towards all regions are multiplied by the following scheme: 1.1 for 1980, 1.11 for 1981, ... 1.19 for 1990. In addition, to reflect the self-reliance aspect, for all DME, matrix elements of bilateral trade flows towards all DME are multiplied further by the following scheme: 1.1 for 1980, 1.11 for 1981, ... 1.19 for 1990. Hence, India's export to Malaysia, for instance, is multiplied by 1.21 for 1980, 1.23 for 1981, ... 1.32 for 1990.

reduced by 20 per cent. For example, if the projected GDP growth rate for Japan is 7.5 per cent which is the upper boundary, the upper boundary would be reduced to 6 per cent (20 per cent less than 7.5 per cent). When the projected GDP growth rate is lower than the upper boundary, the maximum projected GDP growth rate for the entire projection period would be reduced by 20 per cent and used as the new upper boundary. If such a maximum is found to be 5 per cent, then the upper boundary would be set at 4 per cent. This kind of correction is only applicable to AME.

The ODA-GDP ratio is often regarded as a policy target. When AME is in recession, it is possible that AME reduces this ratio. In the pessimistic scenario the present share is reduced by 20 per cent for all AME. For example, as Japan's share was 0.2 per cent in 1977, it is set at 0.16 per cent for the entire projection period. It should be noted here that ODA in the standard macro scenario is set at the share in 1977. There is a growing fear that the world will move back to the age of protectionism. The bilateral trade flows function is shifted down to take account of this fear. Assumption on the prices of oil and commodities are the same for the standard macro scenario.

Input-output scenarios

As in the case of AMEM, standard and other scenarios are prepared for AIOM. Point estimation of exports and imports makes it possible to solve value added for industries by the I/O accounting framework. This case becomes the standard scenario. Other I/O scenarios are depicted in the framework of the linear programming optimization problem. What to optimize (objectives) as well as under what constraints optimization should occur become very important questions. There are, however, a few common arrangements for the scenarios. Since optimization can be discriminatory to each region, one can perform optimization for both total developing countries and ESCAP developing countries in each case unless explicitly stated. Objectives can be formed in terms of value added, gross output, exports, imports, employment and labour cost.

Standard scenario

The global model has a capacity to project what the future will be under normal circumstances. By AMEM we can project the 1985 and 1990 macro figures without introducing specific assumptions on oil prices, foreign aid, and so on. Similarly in AIOM we can obtain standard estimates of outputs by industries by estimating sectoral exports and imports in a breakdown by industries. This type of estimation without optimization is the "standard scenario". In other words, the standard scenario refers to the case for finding solutions to simultaneous equations of I/O relations. Similar to the macro-standard scenario, the purpose of the I/O standard scenario is to identify projected levels of industry patterns of final demand, value added and international trade and to analyse the kind and magnitude of problems, possibly emerging in the future when the economy grows in accordance with the past trend; and to serve as a point of comparison to facilitate assessment of other scenarios such as maximization of employment in manufacturing industries.

Combination of this scenario with the macro-standard scenario gives the basis of comparison. Such a combined scenario is called the "basic-standard scenario".

Lima target

In March 1975, the Second General Conference of UNIDO adopted the Lima Declaration and Plan of Action on Industrial Development and Co-operation. The so-called Lima target was set for developing countries to obtain a share of at least 25 per cent of the total world industrial production by the year 2000, while observing that the industrial growth so obtained should be distributed among developing countries as evenly as possible. This statement is rather ambiguous. The present consensus is that "industry" excludes mining and electricity or gas or water and consists solely of manufacturing activities. The term "industrial production" is defined as net output (value added, or the manufacturing sector's contribution to GDP). The definition of "developing countries" includes only "developing market economies" listed in the *United Nations Yearbook of National Accounts Statistics.*

In AIOM the problem of the Lima target is treated indirectly. Industrial production is maximized. That is, value added by manufacturing (industries to 10), with or without constraints on agricultural production (value added) and employment. The constraint on agricultural production is such that production does not decrease from the level projected by the basic standard scenario, while that on employment is such that employment in manufacturing and service sectors should not decrease from the projected level. The last constraint is imposed because industrialization without any employment-creating effects is less desirable.

Results of LP maximization are compared with the adjusted Lima target shares of ESCAP developing countries in 1985 and 1990. Results of industrialization maximization LP gives the maximum feasible shifts among industries and international shifts in the production structure. It is of interest to see how dependent developing countries may become on the food flows from developed countries as a result of their efforts to industrialize. This scenario will be combined with the corresponding macro optimistic scenario.

Maximization of agricultural production

Although rapid industrialization still remains the king-pin of the developmental process in the poorer countries, increasing attention has been given to agriculture, particularly food supply. This scenario indicates what might happen if developing countries attain self-sufficiency in supply of food and other agricultural commodities or move towards it. One may do this with or without imposing any constraint. Because industrialization remains the key to economic development, one may keep the manufacturing production (value added) not less than the projected level in 1985 and 1990 by the combined standard scenario. The other possible constraint is that employment in manufacturing should not shift to agriculture. Alternatively, agricultural output (value added) can be maximized by squeezing the output of both manufacturing and tertiary industries.

The difficulty of treating food production is mainly one of data. In the industry classification, industry 1 includes agriculture, fishing and forestry, and industry 3 processing of food, beverages and tobacco. What one is concerned with is the food supply which is a part of industry 1. Since one cannot separate industry 1 further at the moment, one hypothesizes that food production proportionately increases where the value added of industry 1 increases. Maximization of the value added of industry 1 indirectly leads to the food-supply increment, not necessarily to the maximization of it. This scenario will be combined with macro optimistic and pessimistic scenarios.

Export maximization

For newly industrialized countries such as the Republic of Korea the export of manufactured goods is the focus of economic policy. Under this scenario we will see the effect of maximizing exports of manufactured goods of semi-developed countries as well as developing ESCAP countries. This scenario will be combined with the macro optimistic scenario.

Export minimization

Developed countries are increasingly feeling the pressure of exports from the developing countries including exports of more sophisticated manufacturing products If the developed countries move towards a hardened stand on protection the results could be a stagnation of the world economy with adverse impacts on the developing economies. In practice the countries exporting manufactures may at times be obliged to accept export restraints. This self-restraint aspect can be handled by minimizing exports of developing countries. Export of industries 3–10 are minimized for all the countries. This case is combined with the pessimistic macro scenario.

Import minimization

This may be the other side of the coin of export maximization and minimization scenarios. It is, however, interesting to observe the feasibility of trade by introducing this scenario. The import of agricultural goods for developing countries will be minimized. This will depict a situation of self-sufficiency. The corresponding macro scenario is the pessimistic scenario.

Employment

Creating employment is one of the principal goals of the development strategy. Employment in the manufacturing and tertiary industry will be maximized under this scenario. Employment in agriculture is difficult to investigate because it is difficult to distinguish unemployment from underemployment. This is combined with all the macro scenarios.

Labour cost minimization

By this scenario the ideal pattern of international division of labour by minimizing the total world cost of labour is envisaged. Shifts of production can be expected from labour-intensive industries of developed countries to those of developing countries. It should be noted that minimization is carried out under a constraint that the GDP per capita by region is fixed by AMEM. This scenario can be combined with the standard macro scenario.

Combined scenarios

Macro projections are carried out for 1985 and 1990 by setting up these scenarios. By using the macro results, AIOM tries to project structural change. I/O tables for the target years are constructed and further calculations are made based upon them. Standard projections are made by point-estimating the trade and final demand components. Other scenarios require the use of the LP optimization procedure. Given the GDP per capita of the region, LP maximization looks at first sight to be logically very strange because the results will never be reflected back into the macro projections. It is admitted that there is a jump in logic but it should be understood clearly that the full linking will be done in future work. For the macro standard scenario the corresponding I/O scenario is standard projection and this presents no problem. For the macro optimistic scenario one has a choice of two or three scenarios. The model includes the export maximization efforts. It is one of the aspects of the positive policy mix of the macro scenarios. Hence, it seems that the combination of the optimistic macro scenario and the I/O export maximization scenário depicts the very optimistic or ideal situation. In such a case there are two possible interpretations of I/O optimal solutions.

The first interpretation of the optimization results is used for scenarios other than the standard scenario; then the matching scenario for I/O gives the likely industrial structure attained during the projected period of time. For example, the macro scenario introducing export-led growth *per se* cannot tell anything about the structure and gives only aggregate results. However, it should be understood that by assuming export-led growth something should happen to change the structure to fit it. Government subsidy might be given to encourage the exports of particular commodities. Normatively, export promotion should be done from the viewpoint of industrial reallocation. In this connection LP maximization fits well. If one understands it this way, it is easy to trace the simple algorithm behind the scene.

Another way of interpreting the results of optimization is that, by changing structures, there are several ways of achieving a given per capita income. For example, for $500, several patterns are possible. The point should be clearly distinguished that for macro it is an individual country's endeavour while the I/O deals with a more global view. In most cases it is the plan of the group of developing countries. Maximization of the group's welfare does not necessarily coincide with an individual country's view.

The scenarios and their combinations are summarized below.

Scenario type	*Symbol*	*Scenario*
Macro scenarios	M (0)	Standard
	M (1)	Optimisuc
	M (2)	Pessimistic
I/O scenarios	I (0)	Standard
	I (1)	Lima target
	I (2)	Agriculture maximization
	I (3)	Export maximization
	I (4)	Export minimization
	I (5)	Import minimization
	I (6)	Employment maximization
	I (7)	Labour cost minimization

SCENARIO COMBINATIONS

Macro \ *I/O*	*I (0)*	*I (1)*	*I (2)*	*I (3)*	*I (4)*	*I (5)*	*I (6)*	*I (7)*
M (0)	×	×	×	×	×	×	×	×
M (1)	×	×	×	×			×	
M (2)	×		×		×	×	×	

Growth projections

GDP, ODA and investment

The projected rates of GDP growth are compared in table 4 with those achieved in the past. The average growth rate for all ESCAP developing countries is projected to increase but not to the extent in the 1960s. The growth of the GDP of low-income countries is projected to increase slightly, while that of the middle-income countries in expected to decrease in the standard scenario.

The projected increase in the economic growth of the low-income countries hinges on the type of scenario. ESCAP developed countries are relatively less affected than ESCAP developing countries by type of scenario. The most influential components are exports, ODA and investment. The changes in ODA and investment predicted by different scenarios are given in tables 5 and 6. As was pointed out above, ODA and investment are the key factors constituting scenarios. The extent of the changes in these will be of interest.

TABLE 4. ECONOMIES, SCENARIOS AND GROWTH RATES[a]

Economic grouping	Scenario	GNP growth rate by period (percentage) 1960–1970	1970–1975	1975–1980	1980–1985	1985–1990
ESCAP total	Standard	4.83	3.18	3.98	4.34	4.51
	Optimistic			3.98	4.49	5.06
	Pessimistic			3.98	3.95	4.05
ESCAP developed countries	Standard	4.85	3.13	3.89	4.27	4.43
	Optimistic			3.89	4.26	4.83
	Pessimistic			3.89	3.89	4.03
ESCAP developing countries	Standard	4.81	3.91	5.23	5.21	5.42
	Optimistic			5.23	7.17	7.48
	Pessimistic			5.23	4.61	4.37
ESCAP middle-income countries	Standard	7.68	7.64	8.73	7.94	7.84
	Optimistic			8.73	9.83	9.79
	Pessimistic			8.73	6.97	6.47
ESCAP low-income countries	Standard	4.04	2.57	3.65	3.66	3.75
	Optimistic			3.65	5.67	5.90
	Pessimistic			3.65	3.28	2.98
World	Standard		3.78	4.47	4.70	4.86
	Optimistic			4.47	4.97	5.48
	Pessimistic			4.47	4.29	4.27

[a]AMEM.

Table 5 shows how ODA is divided between ESCAP middle-income countries and low-income countries. Since the growth rate of ODA receipts is almost the same for the two groups and the absolute amount of ODA to the latter group is far greater than to the former group, the impact of ODA on the low-income countries is greater. The distribution of ODA among the two ESCAP

TABLE 5. RECEIPTS OF ODA
(Millions of US dollars)

Economic grouping	Scenario	1980	1985	1990
ESCAP developing countries	Standard	4372	6222 (7.3)	9204 (8.2)
	Optimistic	4372	12247 (22.9)	18001 (8.0)
	Pessimistic	4372	5119 (3.2)	7261 (7.2)
ESCAP middle-income countries	Standard	1031	1622 (9.5)	2555 (9.5)
	Optimistic	1031	2945 (23.4)	3760 (5.0)
	Pessimistic	1031	1314 (4.9)	2006 (8.8)
ESCAP low-income countries	Standard	3341	4600 (6.6)	6649 (7.7)
	Optimistic	3341	9302 (22.7)	14241 (8.9)
	Pessimistic	3341	3805 (2.6)	5255 (6.7)

Note: Figures in parentheses are compound percentage growth rates over the preceding 5-year period.

subgroups is shown in table 6. As is clear from table 6 the optimistic scenario introduces quite a contrast in the distribution of ODA.

As far as investment performance is concerned, table 7 gives a clear picture. The other side of the coin of investment performance is savings. It is vital that investment should be increased to achieve faster growth of GDP. This required investment should be covered by the injection of domestic as well as foreign savings. As shown in table 7 the optimistic case shows a very fast growth of investment especially for the low-income countries. These countries will attain a 5.8 per cent growth rate in GDP during 1980 to 1990. Considering the high population growth rate of these countries they will not achieve the target of doubling GDP.

TABLE 6. DISTRIBUTION OF ODA

(Percentage)

Economic grouping	*Scenario*	*1980*	*1985*	*1990*
ESCAP middle-income countries	Standard	23.6	26.1	27.7
	Optimistic	23.6	13.2	20.9
	Pessimistic	23.6	25.7	27.6
ESCAP low-income countries	Standard	76.4	73.9	72.3
	Optimistic	76.4	86.8	79.1
	Pessimistic	76.4	74.3	72.4

TABLE 7. INVESTMENT (1970 CONSTANT PRICE)[a]

		Compound growth rates (percentage)				
Economic grouping	*Scenario*	*1960–1970*	*1970–1975*	*1975–1980*	*1980–1985*	*1985–1990*
ESCAP total	Standard	7.9	1.1	6.2	5.3	5.0
	Optimistic				6.4	5.8
	Pessimistic				3.2	3.5
ESCAP developed countries	Standard	7.8	0.7	6.2	5.1	4.8
	Optimistic				5.2	5.0
	Pessimistic				3.1	3.4
ESCAP developing countries	Standard	9.1	7.2	6.3	6.7	6.8
	Optimistic				18.8	10.5
	Pessimistic				4.5	5.2
ESCAP middle-income countries	Standard	13.0	11.6	7.7	9.0	8.7
	Optimistic				13.5	10.4
	Pessimistic				7.1	7.3
ESCAP low-income countries	Standard	7.7	4.7	5.2	4.7	4.9
	Optimistic				22.1	10.6
	Pessimistic				2.5	3.1
World	Standard		3.6	5.5	5.6	5.4
	Optimistic				6.7	6.2
	Pessimistic				4.1	4.1

[a]AMEM.

Exports performance

The exports of developing countries are projected to grow as shown in table 8. Exports are a principal determinant of a country's foreign exchange availability, since they affect both a country's direct earnings on trade account and a country's access to the international capital markets. The growth prospects of the middle-income countries are bright in the standard and optimistic scenarios but the prospect is as bad as for other countries in the pessimistic case. The extent of the influence of the type of scenario seems very large in exports. For the ESCAP region the standard scenario projects about 6 per cent growth during the 1980s while the optimistic scenario projects about 7 per cent and the pessimistic about − 1 per cent.

Table 9 shows changing composition of exports for the world as a whole, and for developed and developing ESCAP countries respectively. It is notable that the changes over two decades for some commodity groups are significant. In both ESCAP country groups, the importance of agriculture diminishes markedly. However, whereas textile exports from developed ESCAP countries decrease proportionately to low levels, the opposite is true of the developing countries, at least up to 1985. In some of the other industries, there is comparative stability, but the importance of electrical machinery grows in both ESCAP groupings,

TABLE 8. EXPORTS[a]

		Compound growth rates (percentage)				
Economic grouping	*Scenario*	*1960–1970*	*1970–1975*	*1975–1980*	*1980–1985*	*1985–1990*
ESCAP total	Standard	7.7	6.8	5.8	5.9	6.1
	Optimistic				6.6	7.3
	Pessimistic				-0.7	-1.4
ESCAP developed countries	Standard	7.8	6.7	5.5	5.7	6.0
	Optimistic				6.3	7.1
	Pessimistic				-0.6	-1.3
ESCAP developing countries	Standard	7.1	8.6	9.0	7.8	7.5
	Optimistic				10.1	9.0
	Pessimistic				-0.8	-2.5
ESCAP middle-income countries	Standard	9.6	10.6	9.7	8.5	8.0
	Optimistic				9.9	9.1
	Pessimistic				-0.7	-2.8
ESCAP low-income countries	Standard	6.3	3.6	6.4	5.1	5.5
	Optimistic				10.9	8.7
	Pessimistic				-0.8	-1.4
World	Standard		7.2	5.6	5.9	6.1
	Optimistic				6.7	7.2
	Pessimistic				-1.7	-2.7

[a]AMEM.

TABLE 9. COMPONENTS OF EXPORTS UNDER THE BASIC STANDARD SCENARIO[a,b]

(Percentage of total exports)

Industry	*World* 1970	*World* 1985	*World* 1990	*ESCAP developing countries* 1970	*ESCAP developing countries* 1985	*ESCAP developing countries* 1990	*ESCAP developed countries* 1970	*ESCAP developed countries* 1985	*ESCAP developed countries* 1990
Agriculture, forestry and fishery	10.7	9.4	8.8	8.4	4.2	3.6	27.6	16.7	15.7
Mining	7.6	7.9	7.9	3.2	2.8	2.9	6.4	4.9	4.7
Food, beverages and tobacco	4.6	4.2	4.2	6.0	3.3	3.0	4.5	4.7	4.6
Textiles, wearing apparel and leather	7.6	8.1	8.2	10.5	4.0	3.0	23.3	31.0	31.6
Wood products	2.1	1.7	1.7	0.8	0.7	0.7	0.2	0.3	0.2
Chemical products	10.2	10.3	10.3	8.0	7.6	7.5	10.0	10.4	10.5
Metal and metal products	9.7	9.0	9.0	14.9	14.7	14.8	4.9	5.8	6.2
Non-electrical machinery	10.9	11.6	11.8	10.0	13.0	13.5	2.1	33.5	3.7
Electrical machinery	4.6	5.5	5.7	8.4	10.6	10.9	3.6	9.4	9.9
Transport equipment	8.1	9.6	9.8	12.7	19.6	20.3	1.0	1.0	1.0
Services	17.7	17.1	17.1	9.9	11.1	11.2	12.5	9.4	9.1
Transportation and communication	6.1	5.6	5.6	7.1	8.4	8.5	3.8	2.8	2.8
Total	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0

[a]1970 constant prices.
[b]AIOM.

particularly in the developing countries. Finally, a fall in the relative importance of services exports in developing countries is contrasted with a small rise in the case of the developed ESCAP countries.

Lima target

As was discussed above, the Lima target is for the developing countries (including Western Asia) to obtain a share of at least 25 per cent of the total world industrial production by the year 2000. The share in 1970 is 6.4 per cent and is projected to be 9.8 per cent in 1980. This target implies that developing countries should obtain a 12.4 per cent share in 1985 and 15.7 per cent share in 1990. This is based on the assumption of a constant growth in percentage terms. If it is interpreted in this way, the Lima target is achievable under the scenarios which combine the manufacturing maximization scenario (excluding Western Asia) or the agriculture maximization scenario with the optimistic macro scenario. These combined scenarios overshoot the targets in both 1985 and 1990. With other scenarios it is not possible to attain the Lima target as shown by figure 4.

The findings that these two scenarios make it possible to attain the Lima target indicates the importance of the close relationship between agriculture and manufacturing. In order to achieve the target for manufacturing growth it is vital to increase agricultural production—the agricultural sector is closely integrated with other sectors in developing countries in production and consumption. This is analysed further below.

However, if one assumes that the discrepancy in the share between 1980 (9.8 per cent) and 2000 (25 per cent) should be eliminated by an increase in the share with a fixed amount, then this increase should be 3.8 per cent every five years. This is hardly attainable for 1985 and far beyond our projections in the case of the most optimistic scenario for 1990 (see figure 4).

Manufacturing structure

When we examine the results in figure 5, it becomes clear that projection results are such that manufacturing ratios [12] become lower for Japan and higher for other countries and areas. The employment maximization scenario combined with the optimistic macro scenario generally shows the highest results and, strangely, manufacturing exports maximization combined with the pessimistic macro scenario sets the lowest level. The latter result implies that when one maximizes manufacturing exports of the ESCAP developing countries overall industrialization is dampened. The projection for 1990 as shown in figure 6 remains about the same as for 1985. Middle-income countries and areas such as Hong Kong, the Republic of Korea, Malaysia, Philippines, Singapore and

[12] The ratio of value added of industries 3–10 to the total value added.

Thailand will be relatively more successful in industrialization than the low-income countries such as India, Indonesia and Pakistan. It should be noted that the manufacturing ratios of the middle-income countries become much higher than that of Japan in 1985 and 1990 under any circumstances.

Agriculture

For the low-income ESCAP countries the development of their rural economies holds the key to decreasing poverty. The growth of the manufacturing industries of developing countries depends upon agriculture because a large part

Figure 4. Percentage share of developing countries in world manufacturing value added versus year for various scenarios

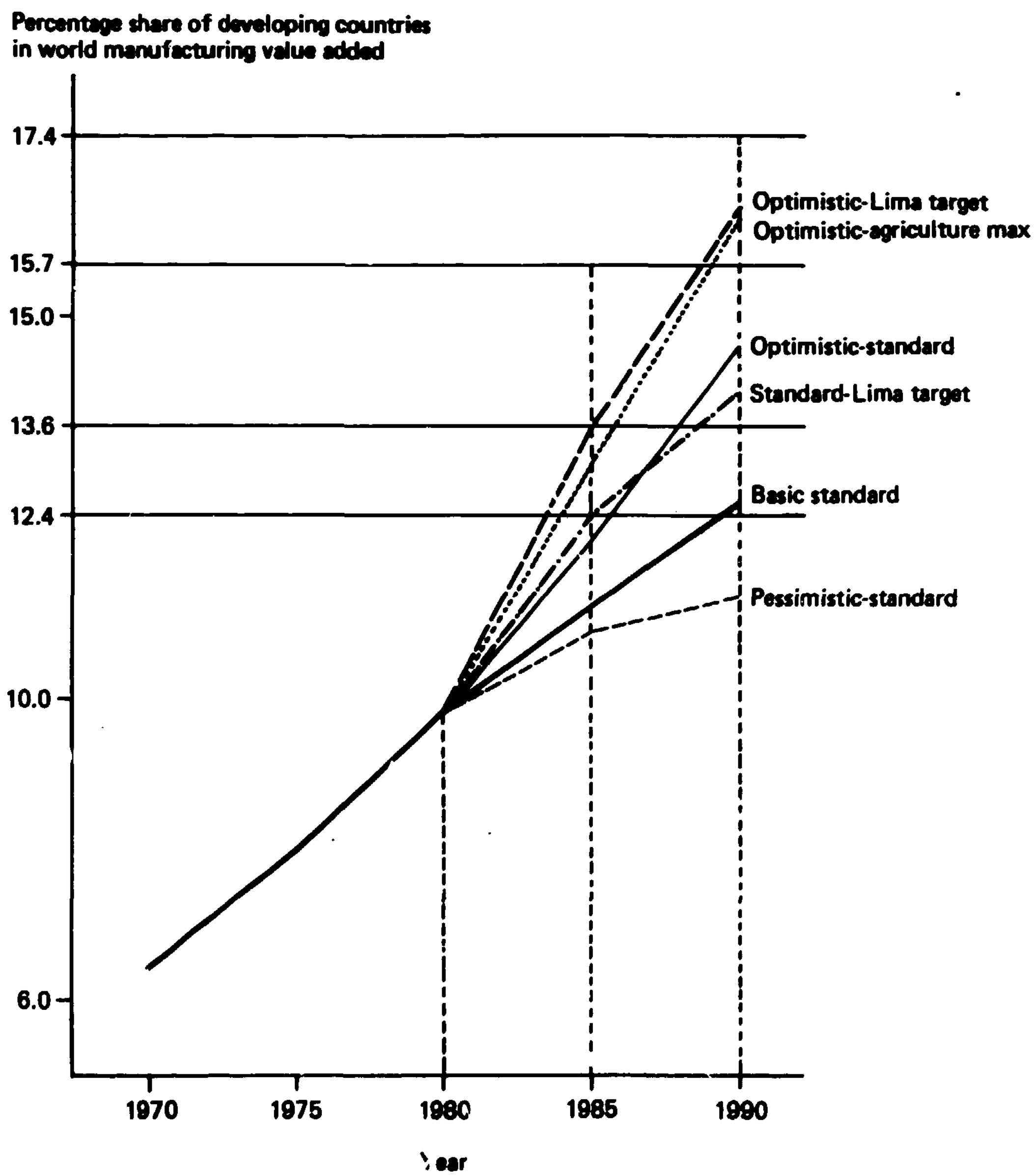
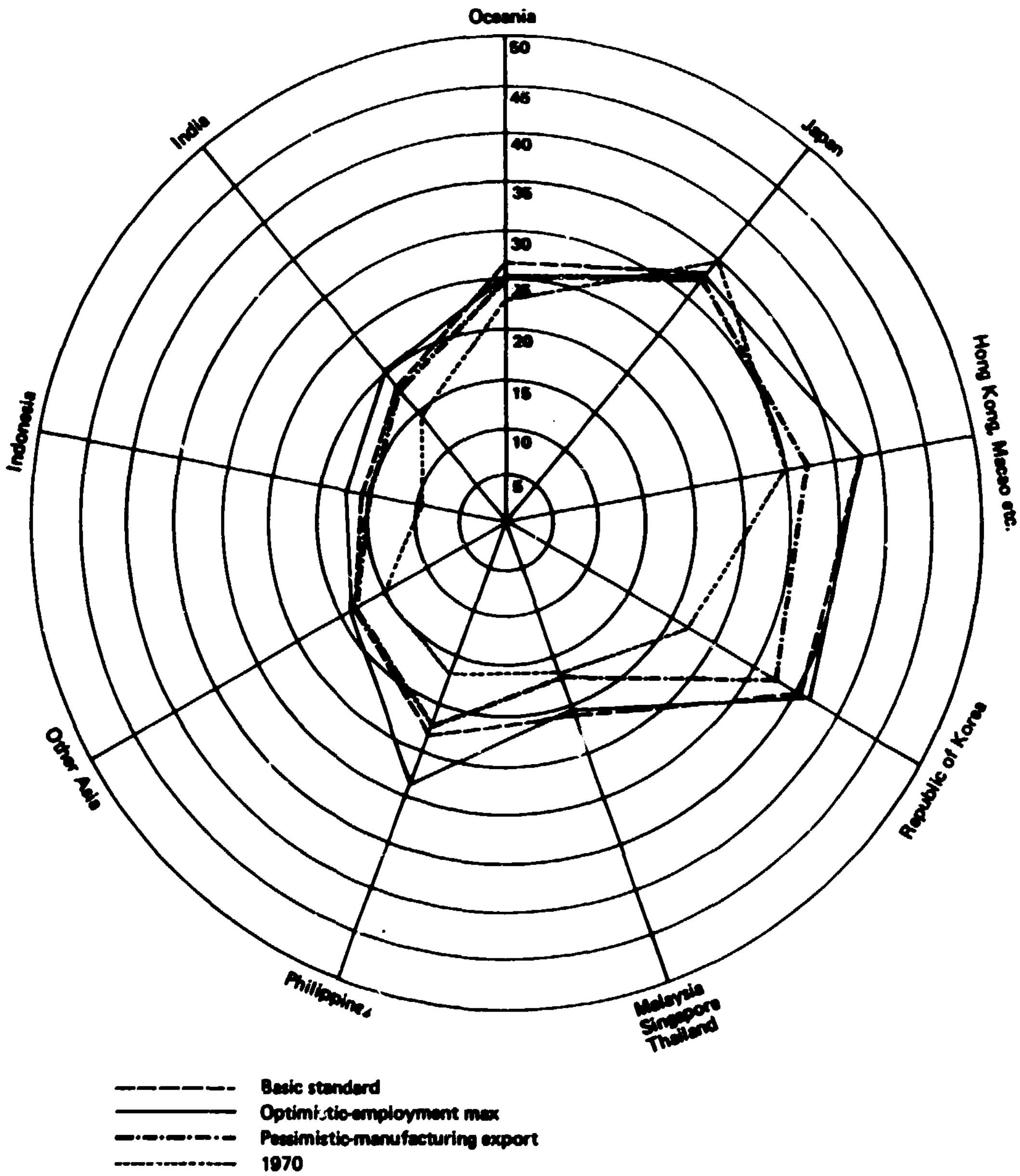


Figure 5. Manufacturing ratio 1985[a]



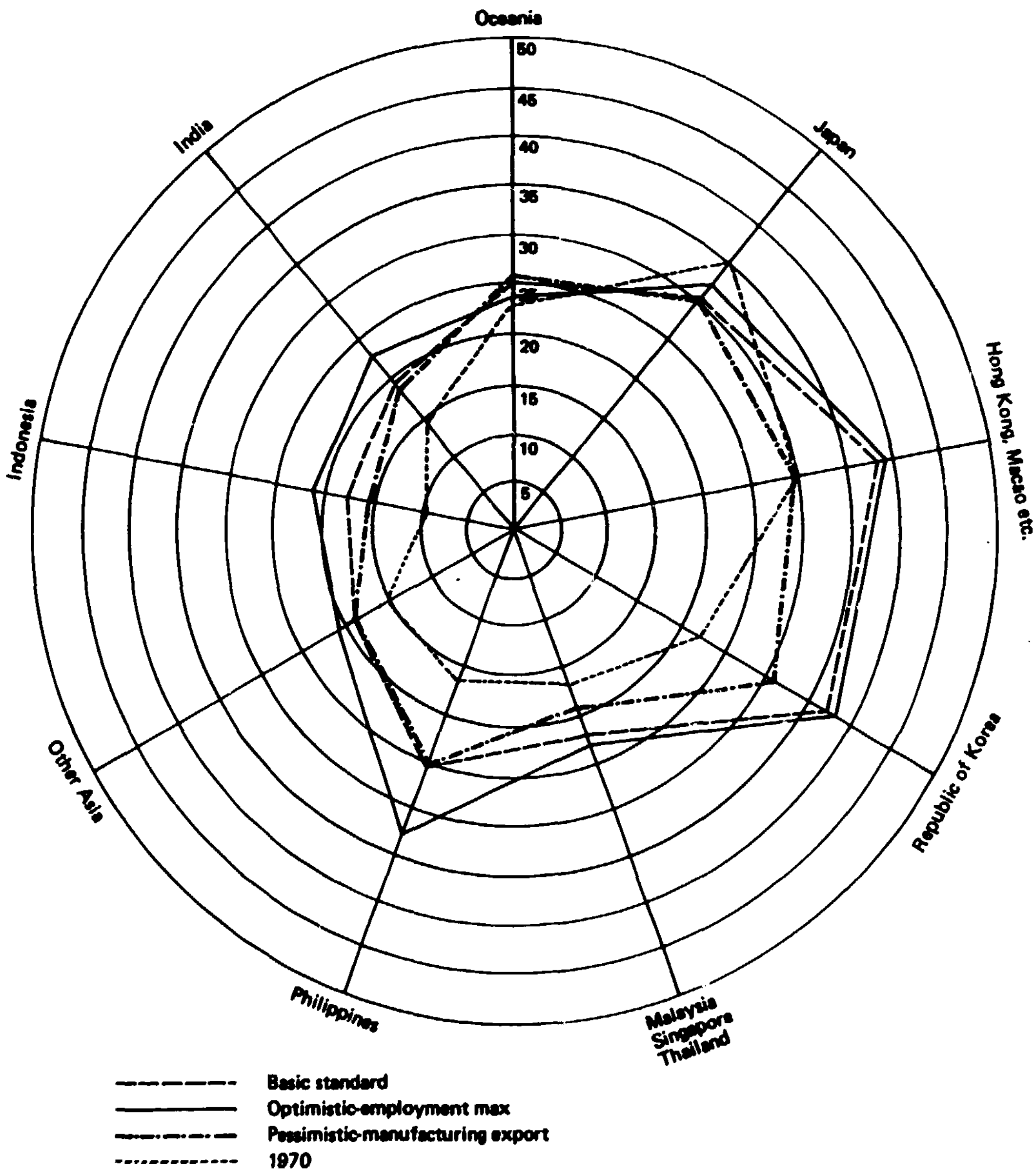
[a]Ratio of value added of industries 3–10 to the total value added.

of the population is in rural areas and consumption demand arises from there. At the same time the I/O structure is such that a significant proportion is based upon agricultural raw materials.

Table 10 indicates the required inputs from agricultural sectors to produce one unit of the manufacturing output in a number of countries. All the developing countries' coefficients are notably higher than those of developed countries—particularly industries 3, 4 and 5.

In the model the technical possibility of increasing agricultural production, for example, by irrigation or by the use of new land cannot be assessed. But the

Figure 6. Manufacturing ratio 1990[a]



[a]Ratio of value added of industries 3–10 to the total value added.

model can envisage the inter-industry relations and can give the maximum possible agricultural production in this connection. Table 11 shows the importance of agriculture in selected countries of the region. Figures in 1985 and 1990 are based upon the I/O standard projections combined with the three macro scenarios.

Employment

Contribution of industry to employment has been modest and will continue to be so according to our projections. Employment in secondary and tertiary industries is compared for various countries in figures 7 and 8. Depending upon

84.12.

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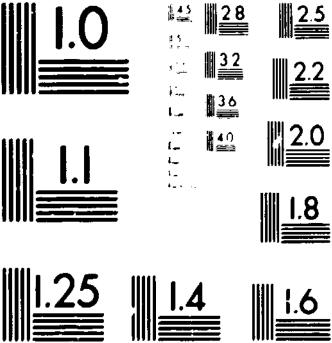

1.0
2.8
2.5
3.2
2.2
3.6
2.0
4.0
1.1
1.8
1.25
1.4
1.6

TABLE 10. COEFFICIENTS OF INPUT FROM AGRICULTURE TO PRODUCE ONE UNIT OF MANUFACTURING OUTPUT, SELECTED COUNTRIES

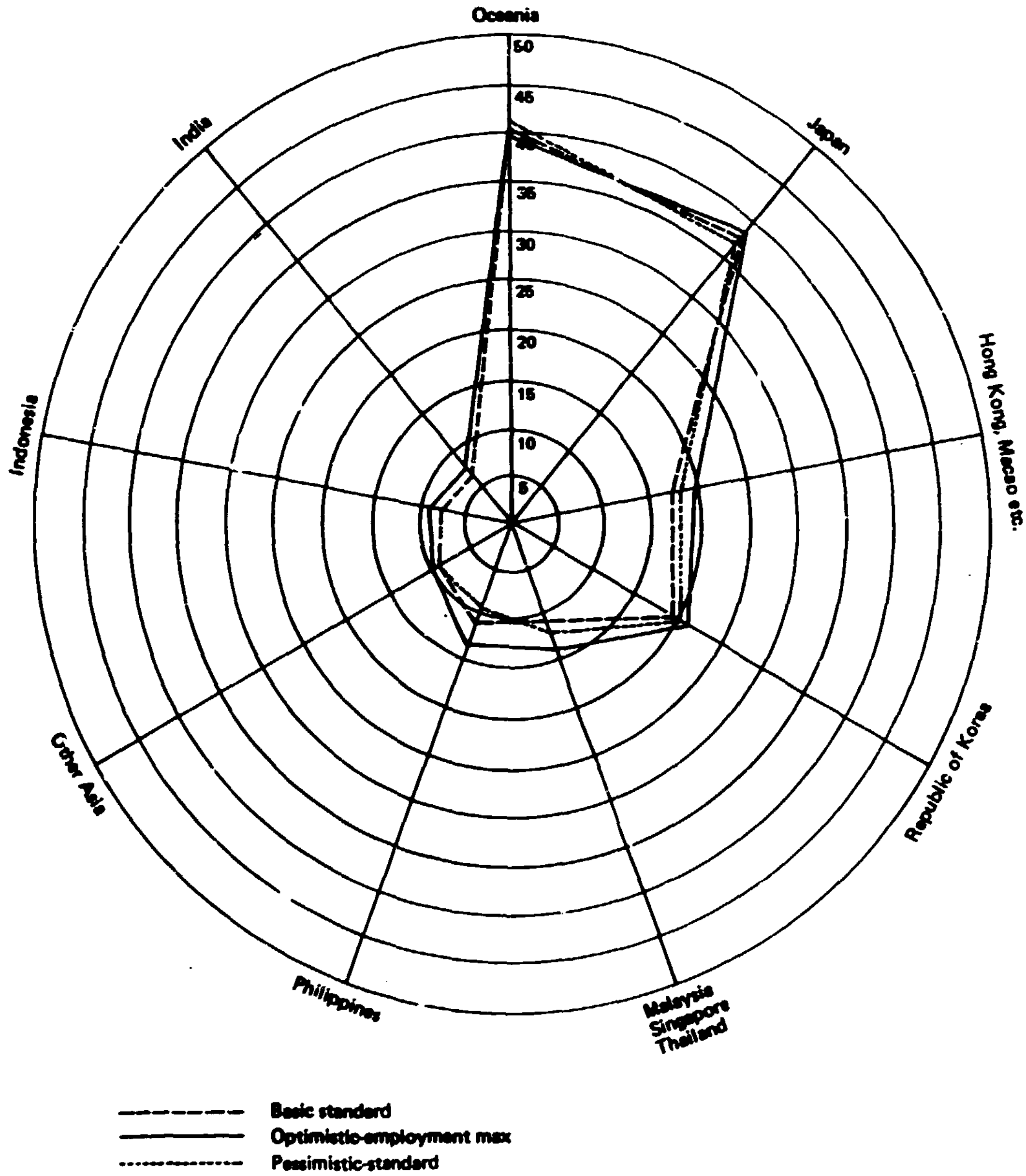
	Industry[a]							
Country grouping	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
Canada and the United States	0.207	0.035	0.013	0.005	0.003	0.003	0.003	0.003
India	0.372	0.189	0.017	0.043	0.004	0.009	0.010	0.007
Indonesia	0.504	0.133	0.028	0.053	0.032	0.024	0.023	0.028
Japan	0.347	0.174	0.032	0.024	0.008	0.009	0.010	0.009
Malaysia, Singapore and Thailand	0.545	0.354	0.088	0.345	0.035	0.044	0.050	0.075
Philippines	0.452	0.136	0.020	0.119	0.024	0.019	0.023	0.014
Republic of Korea	0.296	0.194	0.055	0.022	0.012	0.014	0.013	0.013
Other Asia	0.355	0.179	0.016	0.040	0.003	0.009	0.009	0.006

[a]See annex I for key.

TABLE 11. AGRICULTURAL SHARE IN GDP BY YEAR FOR SELECTED COUNTRIES

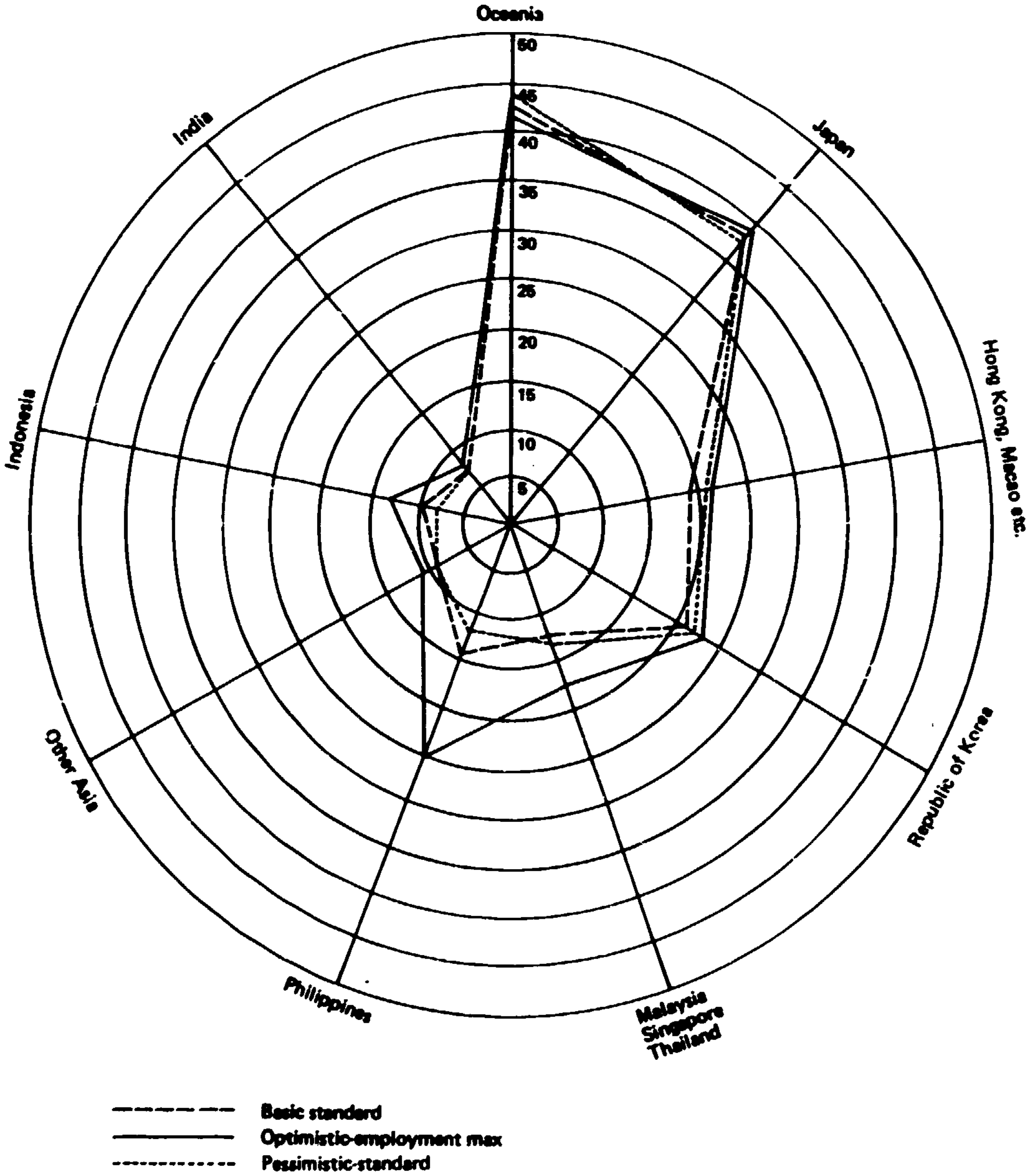
		Share (percentage)				
Country or country group	*Scenario*	*1970*	*1975*	*1980*	*1985*	*1990*
India	Standard	42.9	37.4	33.9	30.9	28.0
	Optimistic				28.9	24.8
	Pessimistic				31.1	28.3
Indonesia	Standard	45.7	37.1	32.6	29.2	25.9
	Optimistic				28.3	24.2
	Pessimistic				30.0	29.1
Japan	Standard	6.2	5.6	5.8	4.3	3.8
	Optimistic				4.2	3.6
	Pessimistic				4.9	4.8
Malaysia, Singapore and Thailand	Standard	28.0	30.4	30.9	31.4	31.6
	Optimistic				32.2	32.9
	Pessimistic				28.6	26.7
Philippines	Standard	27.7	25.9	22.7	20.4	18.6
	Optimistic				21.9	20.3
	Pessimistic				20.0	17.8
Republic of Korea	Standard	28.7	18.6	13.7	10.9	9.1
	Optimistic				10.1	7.9
	Pessimistic				11.0	8.9
United States	Standard	3.1	3.1	2.9	2.7	2.5
	Optimistic				2.7	2.5
	Pessimistic				2.7	2.6
Other Asia	Standard	25.1	22.5	20.6	19.2	17.9
	Optimistic				18.0	15.5
	Pessimistic				19.2	17.8

Figure 7. Ratio of employment to population in manufacturing and tertiary industries in 1985



the types of scenarios, employment in these industries increases or decreases. One very important observation we can make is that for the area Hong Kong and the countries Malaysia, the Republic of Korea, Singapore and Thailand the pessimistic case gives better employment opportunities than the standard case. This is paradoxical but the explanation lies in the following. In these cases the labour value added ratios, which are a decreasing function of GDP, are generally higher in the pessimistic case than in the standard case. This implies that employment creating effects are larger in the pessimistic case than in the standard case.

Figure 8. Ratio of employment to population in manufacturing and tertiary industries in 1990



ANNEX I

CLASSIFICATION OF COUNTRIES, AREAS AND GROUPINGS IN THE MODEL

1. *Asian macro-economic model*

Advanced market economies (AME)

Australia and New Zealand
Canada
France

Germany, Federal Republic of
Italy
Japan
United Kingdom
United States
EEC countries not separately mentioned
Other advanced market economies

Developing market economies (DME)

Africa
Latin America
Asia:

Afghanistan, Bhutan and Nepal
Bangladesh and Pakistan
Hong Kong, Macao etc.
India
Indonesia
Republic of Korea
Malaysia
Philippines
Singapore
Sri Lanka and Maldives
Thailand
Other Asia

Centrally planned economies (CPE)

China and other Asian centrally planned economies
Eastern Europe and USSR

Asian OPEC (OPEC)[a]

Middle East

2. *Asian input-output model*

Australia and New Zealand
Hong Kong, Macao etc.
India
Indonesia
Japan
Philippines
Republic of Korea
Malaysia, Singapore and Thailand
North America
Western Asia and the Islamic Republic of Iran
European CPE countries
European market economies, Israel and South Africa
Latin America and Africa
Other

Asian CPE countries are not yet included in AIOM.

[a]This bloc excludes Indonesia.

ANNEX II

CLASSIFICATION OF MANUFACTURING INDUSTRIES IN THE MODEL

1. Agriculture, forestry and fishery
2. Mining
3. Manufacture of food, beverages and tobacco
4. Textiles, wearing apparel, leather, wood products and furniture
5. Pulp, paper, printing and publishing
6. Chemical products including petroleum and coal products, rubber and glass
7. Metal and metal products
8. Non-electrical machinery
9. Electrical machinery
10. Transport equipment
11. Construction
12. Electricity, gas and water
13. Merchandise and services
14. Transportation and communication

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The Inforum-IIASA international system of input-output models

*Clopper Almon**

An I/O model of one country, no matter how seemingly perfect, is always incomplete, because exports and imports depend on developments in other countries. The economies of countries depend on one another through international trade, and the models of the economies of those countries need to be connected by a trade model. But for national models to be connected to one another, there must be similarities in the output of the models, and the input conventions must be sufficiently alike for one person to be able to perform calculations for all of them. On the other hand, economies differ in what they produce, how they function and, perhaps most of all, in the statistical system used to describe them. The models must be flexible enough to accommodate these differences.

The three bases of a gradually evolving consortium of I/O models and their builders are:

Connection through international trade
Similarity in input and output conventions
Freedom for diversity in internal structure

The Inforum model of the United States of America is the oldest and at present most complete member of the group. Models, varying in size from the 190-industry United States model to the 50-industry Belgian model, are at present available for Belgium, Canada, France, Germany, Federal Republic of, Japan, United Kingdom of Great Britain and Northern Ireland and the United States. Models of Hungary and the Netherlands are under construction, and groups in Austria, Finland, German Democratic Republic, Italy, Republic of Korea and Sweden expect to begin work on members of the family in the near future. The participation of the groups ranges from organization of data and guidance in its use to full determination of the model's equations.

The programming for reading-in data and equations, for imposing scenarios in convenient ways, for performing standard I/O calculations, and for making output displays is the same in all the models. This programming is extensive, tedious, and almost devoid of economic content, except for the I/O accounting scheme it embodies. There is no need for programming to be different for different countries. Indeed, it is precisely the programming uniformity from country to

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country that makes it possible for the basic operating manual of all of the models to be similar enough for one person to run scenarios on all the models.

On the other hand, each model works within the statistical system of its own country. Consequently, it can use the most up-to-date information and the most detailed I/O tables available. The results are in the statistical system familiar to economists and business planners within the country. The international trade model, however, works in a standard, 119-commodity classification based on the Standard International Trade Classification (SITC). The bridge between the country models in their diverse classifications and the SITC-based trade model is explained below where the trade model is described.

Construction of a country model

The construction of a country model of this family begins with the collection and organization of data. An annex to this paper lists the kinds of data needed. With as little as a single I/O table, however, one can produce a functioning model using the basic program. The forecasts this model first makes are extraordinarily uninteresting. All final demands grow at 5 per cent per year, all input coefficients are constant, so that outputs also grow at 5 per cent per year. But without any further programming, one can easily specify the time path of any cell or group of cells in the final demand matrix, or the path of any input coefficient, or of labour productivity or of any component of value added in any industry. Nice tabular displays, clearly labelled in any language, can be easily made. The program that does this is known as Slimforp ("forp" for forecasting program, "Slim" because the program will fatten slightly, probably not more than 5 or 10 per cent as the model is developed).

At this point, the model has only the I/O table as economic content. One then must estimate various sets of behavioural equations and introduce them set by set. For example, one may estimate any sort of consumer demand functions one wishes, write the parameters in a standard format for the program, and add to the Slimforp program, at a clearly indicated spot, the instructions necessary to interpret those parameters properly. The same is true for investment, import, export, input coefficient change, inventory change, labour productivity, wage and profit equations. As each group of equations is added, the model can be run to check the programming and the parameters for that particular addition. One is not required to make all the equations for, say, labour productivity have the same functional form, for one of the instructions can be used to indicate which of several forms a particular equation uses.

All the programming for modifying or overriding the results of the equations remains in place and ready to use after the "fattening" of the program. If, for example, one wants to multiply the 1980 exports of industry 3 by 1.01, the 1985 forecast of this item by 1.15, the 1990 forecast by 1.18, and the intervening years by multipliers obtained from these by linear interpolation, then one needs only the following information:

EXP, M 3, 80,1.01, 85,1.15, 90,1.18

where M is a command which indicates the result is to be multiplied by a time function.

Instead of multiplying the equation's result by a specified function of time, one could change the M in the example to the appropriate command letter. This general procedure may also be applied to groups of cells. One can, for example, control the total consumer expenditure on durables to some prespecified value. This feature has proved especially valuable when using the models to spell out the consequences for industries of macro-economic forecasts made by more aggregated models.

Changes in input coefficients are easily specified. For example, if the base year of the model is 1975 and one wants the A-matrix coefficient for sales from industry 17 to industry 25 to drop to 80 per cent of the 1975 value by 1979, to 60 per cent by 1984 and to 55 per cent by 1990, the only card needed is:

AM, I 17,25, 75,1.0, 79,.80, 84,.60, 90,.55

where AM stands for A-matrix, and the I indicates that the coefficient is being specified as an index.

Each national model has, when finished, two parts, or "sides", the real side and the price-wage-income side (the price side). The real side determines consumption expenditure, exports, imports, industry outputs, investment (all in constant prices) and employment. The price side determines, for various industries, the wage rate and the capital income per unit of output. It permits one to specify the effect on prices of changes in social security taxes, excise taxes or subsidies. The effect of the prices of foreign goods on the prices of domestic products which use the foreign goods is also built into the model. The results of the price side influence the real side through the role of relative prices in the personal consumption functions and the input coefficient change functions. The export and import functions take account of domestic prices relative to prices in other countries. The price side receives influences from the real side primarily through the action of unemployment on wage rates and capital utilization on profit rates.

It would be natural to iterate back and forth between the real side and the price side during each year of the forecast. In fact, because the real side is usually fairly well developed before the price side is started, the two models are separate. Each side is, at present, run separately over the entire forecast period, first the real side, then the price side, then the real side again. This procedure does not correspond to the way the economy works but rather to the evolutionary history of the models. However, on the basis of experience with the models of Japan and the United States, the only ones with well-developed price sides at present, it seems to work reasonably well.

Structure of the real side

The basic logic of the real-side model begins from a target level of employment for future years. A trial projection of disposable income is made. Personal consumption expenditures are calculated and exogenous government expenditures, exports based on foreign demands and domestic-to-foreign price ratios are added. Also added is investment, based, in each year, on replacement requirements and growth in output in that year and previous years. The sum of these components gives final demands except for inventory change and imports. Imports and inventory change are then calculated industry-by-industry along with outputs in a Gauss–Seidel iterative process. Imports generally depend upon domestic use of the product, and inventory change depends on changes in this use since the previous year. From outputs, employment is calculated and compared with the target employment. If it is below the target, the disposable income projection is revised upwards and the calculation repeated to achieve the specified level of employment.

This logic leaves unanswered the question of whether the income generated by this level of employment and output would give the disposable income that was assumed. The full answer to that question is one of the tasks of the price model, to which we shall come shortly. For the moment, however, it is enough to realize that by properly adjusting the tax rates this level of disposable income can certainly be obtained from the before-tax income generated by the forecast.

The flexibility of these models to accommodate various types of equations has been stressed. Nevertheless, in general, equations are used which have already proved successful. For personal consumption expenditure, a system of consumption functions that accommodates both complementarity and substitutability among products, allows strong interdependence among closely related products and weak interdependence among others, yet avoids certain peculiar properties of the other systems is used. Import functions are apt to take the form

$$Y_{it} = (a_i + b_i u_{it})(pf_{it}/pd_{it})^{n_i} \quad (1)$$

where:

Y_{it} = imports of good i in year t;
u_{it} = domestic use of good i in year t;
pf_{it} = foreign prices for i, averaged over several years prior to t and averaged over countries with weights proportional to country's imports from them;
pd_{it} = domestic prices averaged over several years, ending in year t.

Note that although this function has a constant price elasticity, it is a linear function of total use. The parameter b_i gives the asymptotic share of imports in total use, as use grows (without change) in the ratio of foreign and domestic prices.

Labour productivity is apt to follow a simple exponential trend. Many problems may be encountered with more ambitious forms. Investment usually depends on lagged first differences of output, with some provision for replacement investment. Results of the estimation of the behavioural equations are available in Inforum research reports on the various models.

In the United States model, experiments are being made with the Diewert or "square-root" production function for input coefficient change.

Structure of the price side

The price side of the model is based on the equation

$$p = pD + fM + v \tag{2}$$

where:

- p = the row vector of domestic prices;
- f = the row vector of foreign prices;
- v = the row vector of value added;
- D = the I/O matrix of domestically produced inputs;
- M = the I/O matrix of imported inputs.

On the real side, only the total I/O matrix, $A = D + M$, was needed. No attempt was made to use the import matrix, M, to calculate imports by product, because its use would nearly double the core storage requirements of the program at precisely the point of the I/O computations where core already limits the size of the model. In addition, the presence of total use, rather than import-weighted use, makes it easy to keep imports in a reasonable relation to total use.

On the price side of the model, the logic of the equations requires the separation of the D and M matrices. For the United States model, where there is no M matrix available, it was assumed that each row of M was proportional to the corresponding row of A, that is, that for any given.product imports were the same share of each flow in the row. But for most of the European models, the statisticians have made it impossible to make this convenient assumption, since they have provided both D and M matrices, and we feel duty-bound to use them.

But how are D and M forecast separately? It is only the total matrix, A, that has technological meaning and can be forecast from technological or production-function considerations. If from the real-side model there is a forecast of total imports of product i for year t, y_{it}; a forecast of the total I/O flows in each cell in row i in year t, x_{ijt}; and the base-year import shares in each of these cells, m_{ij0}; how should the import shares be revised to be consistent with the total imports? Scaling them all up by the same percentage may drive some of them above 1.0. A rule was needed that would put the highest percentage increases in cells that initially had low import penetration, yet recognize that zero initial penetration probably meant imports were not feasible for those cells. Such a rule is given by the formula:

$$m_{ijt} = m_{ij0} / (m_{ij0} + k_i (1 - m_{ij0}))$$

where:

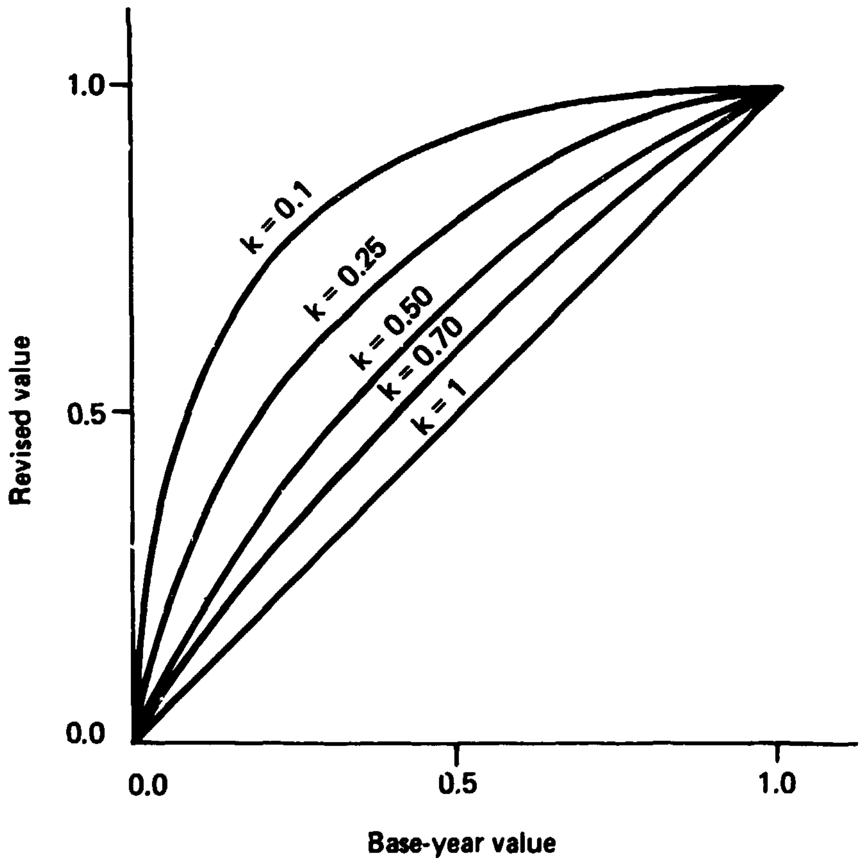
- m_{ijt} = import shares in each cell in year t;
- m_{ij0} = import shares in each cell in the base year.

The value of k_i can be determined using the relationship

$$\sum_j m_{ijt} * x_{ijt} = Y_{it}$$

This non-linear function of k is easily solved by a Newtonian iteration. The relation between the base-year value and the revised value of m_{ij} is shown in the figure for several values of k.

Share revision functions



By careful programming, it is possible to bring into the central processing unit (CPU) the A matrix, convert it row by row to the D matrix, and at the same time to create the f M vector, so that it is never necessary to have both the D and M matrices in core. Consequently, core requirements are kept to the size of the A matrix plus a few vectors.

The calculation just described is part of the standard Slimforp for the price side. The economic substance of the price side is contained in the equations explaining the vector of value added, v. Actually, the model works with a V matrix in which typical rows are:

Wages and salaries of employees
Wage supplements
Proprietors' income
Indirect taxes
Depreciation
Property income (before-tax profits and interest)

Each of these items may be further subdivided. The Slimforp model provides for a number of equations to be calculated and then, guided by an indexing matrix, the model moves forward each element of the base-year V matrix by the percentage indicated by the appropriate equation. The labour-income rows are automatically adjusted for labour productivity. The rows of V are then added together to obtain the v vector that goes into equation (2).

This standard Slimforp for the price side is brand new and is now being tested on the Netherlands model. As with the Slimforp for the real side, it provides easily used, flexible ways to specify scenarios. The other European models do not yet have price sides, and these must be constructed before the price-dependent part of the international linking mechanism can be used.

The computing power required by these models is modest. Every effort has been made to keep the Fortran simple and easy to read. It can probably be run with minor change on practically any system.

The international linking mechanism

Because the country models are not yet all operating, the linking mechanism has not yet been put to work. The necessary data, however, have been gathered, and many of the equations have been estimated.

The trade model, which links the national models, works with 119 commodity groups, defined in SITC terms, and carefully selected to complement the industrial plans of the most detailed national models. Time series back to 1962 on bilateral trade flow in these categories have been organized from OECD tapes obtained from the World Bank. Because of the immense amount of work involved in assembling these time series, and the small extra cost of carrying more product detail, the 119 industries were designed to give the country models "room to grow".

Each country model generates imports in its own classification. These are then expressed in dollars and used as independent variables in regression equations having the country's imports in the 119 industries as the dependent variables. Clearly, imports of some national industries may be used as explanatory variables for more than one of the 119 industries. Less frequently, where one of the 119 overlaps several national industries, the imports of all of these may be used to explain the imports in the one international industry. Note that there is no intention of undertaking a detailed matching of national and SITC classifications. Only general names and the results of the regressions are used.

With a country's imports thus converted into the 119 SITC-based categories, the imports of each category are divided among the source countries. Equations for this purpose have been estimated and described by Douglas Nyhus.[1] They use, as variables to explain changes from the base-year shares, principally relative prices and time trends. Export-push factors, such as investment in the exporting country, have not been considered for lack of data at the time of the estimation. There is now more investment data and such effects could be included. The equation, so far as prices are concerned, is:

$$s_{ijt} = s_{ij0} * (EP_{it} / WP_{jt})^{b_{ij}}$$

where:

s_{ijt} = the share of country i in imports of country j in year t;
EP_{it} = the "effective" price of this product in country i in year t. This price is a weighted average of previous years' prices. No exact matching of the SITC product is attempted. The most appropriate price available from the country model is used;
WP_{jt} = the "world" price of this product as seen from country j.

The world price, WP_{jt}, as seen from country j, is not calculated by any simple averaging process, but is defined implicitly from the requirement that the sum of the shares add up to 1.0:

$$\sum_i s_{ijt} = \sum_i s_{ij0} (EP_{it} / WP_{jt})^{b_{ij}} = 1.0$$

In previous work, the b_{ij} have been the same for all i. The above formulation with the calculation of the world price from its implicit definition adds greatly to the flexibility of the equation, and makes it possible to find quite substantial price effects on trading patterns.

Once the imports of every modelled country have been thus broken down by country of origin, they can be summed over importing countries to give the "exports" of the countries of origin. Of course, these "exports" are not precisely exports as reported by the countries of origin, partly for the usual reasons that make X's exports to Y different from Y's imports from X, and partly because not all countries are included in the system. Nevertheless, when aggregated to match as nearly as possible a country model's classification, exports should prove very good explanatory variables in regression equations for exports in the country models.

As with the iteration between the real side and the price side of the country models, the natural order of iteration of the international system would be to go around all countries each year until equilibrium is reached. But because the country model is the computing unit, that procedure would waste much computer time in shifting from one model to the next. It is planned, therefore, to make a 10-year forecast with each country model and then seek the international equilibrium over all 10 years at the same time. It is hoped to have the first version of this trade linking mechanism working before the end of 1979.

[1]D. E. Nyhus, "The trade model of a dynamic world input-output system". Inforum Research Report No. 14, July 1975.

ANNEX

DATA REQUIREMENTS FOR AN INPUT-OUTPUT MODEL IN THE INFORUM-IIASA FAMILY

An I/O table for one year is indispensable. It should distinguish, within the final demands, between private consumption, government demand, exports, imports, capital investment and inventory change.

For the real-side model

For the estimation of behavioural equations, historical time series are necessary. Probably the most important equations for determining the future growth of the economy are the labour productivity equations. They require, at a minimum, historical series for industry output in constant prices for the I/O industries and employment by I/O industries or aggregates thereof. If historical series cannot be established for the outputs of all the industries in the published I/O tables, perhaps some of the industries can be aggregated to form an industry for which outputs are available. Alternatively, if aggregation would combine a "clean" industry such as "railroads" with a "mess" such as "travel agents, freight forwarders and miscellaneous transportation services", then it would be better to make up some very weak series for the "mess". Dummy industries that have no employment, investment or import need no output series.

The largest final demand component is always personal consumption. To develop equations for it, one needs at least either personal consumption by I/O industry in current and constant prices or personal consumption expenditure by categories found in the national accounts and a bridge matrix showing the composition of each of these categories in terms of the I/O industries for the base year of the table.

To complete a standard real side, one needs: imports, by I/O industry in constant prices; exports, by I/O industry in constant prices; domestic price indexes by I/O industry at least for goods; investment, by purchaser, in I/O industry, or aggregates thereof; an investment flow matrix showing how the purchases of each investing industry were divided among products of the various I/O industries; inventory change (stock building) by I/O industry.

It is useful also to have, for coefficient change studies, time series on individual flows in the I/O table.

For the wage-price-income side

For the wage-price-income side of the model, it is necessary to have the value-added rows of the base-year table. They should include at least a distinction between labour income, capital income, and taxes. Also needed are wage rates, unemployment rates, and any other variables to be used in the wage-rate equations, profitability equations, or relative wage equations. It is advantageous to have a time series of the value-added rows, perhaps at a more aggregate level available from the national accounts. It is not, of course, necessary to have all of these data at the outset. The data bank can grow as the model is built.

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World Bank global modelling research

S. P. Gupta, J. Waelbroeck ***

Since 1974 the World Bank has been conducting annual exercises in global modelling to identify the major constraints affecting growth and development and to explain alternative strategies for development. This paper briefly reviews the SIMLINK model used in the early World Bank exercises, then reviews in greater detail the model used for the 1978 World Development Report. This paper discusses improvements to the 1978 model.

In the early World Bank global modelling work, the problem of describing the interdependence of various parts of the world was simplified by adopting four rather heroic assumptions. The first assumption was that the growth of developing countries was constrained principally by shortage of foreign exchange. The second assumption was that availability of foreign exchange depended largely on developed countries, through the impact of their growth on markets for raw materials and manufactured goods from developing countries, and through the impact of their aid policies. The third assumption was that this causal relation was unilateral; that is, that developed countries influenced the developing world, but there was no reverse effect or feedback. The last assumption was that it was reasonable to aggregate countries into fairly homogeneous regions and thus reflect the Bank's policy interests.

These assumptions made it possible to quickly build a simple model which emphasized the dependence of developing on developed countries as a result of foreign exchange earnings, and emphasized the impact of alternative aid and trade policies of developed countries.

The SIMLINK model (Hicks and others [1]) contained single growth models for individual developing regions, based on the view that regions have to adjust their rates of growth to bring imports into line with the available amount of foreign exchange. These were essentially two-gap models run on the assumption that the trade gap is dominant. The most original feature was a set of commodity market models which provides a coherent view of the impact of demand in developed countries on the prices and quantities of the goods exported by developing countries. Oil prices are of course exogenous; and the model includes equations which allow for the impact of oil price changes on import prices of developing countries, also allow for the aid which developing countries have

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received from OPEC and allow for the foreign exchange earnings earned by exports of goods and services to oil producers.[1]

There was little scope in the SIMLINK framework for independent action by the developing countries to improve their situation, other than their efforts at trade promotion. Such a model was probably an appropriate one when it was built. At that time foreign exchange was the dominant constraint for the developing world, due to the impact of higher oil prices on the cost of imports, and the impact of the recession on the volume and prices of exports. However, as many countries were able to surmount this balance of payments crisis, it became gradually apparent that a more complex model would be needed.

The monitoring of problems in developing countries during 1976 to 1977 by the World Bank clearly suggested that the developing world was moving to a multiple constraint situation. The natural way of allowing for this was to introduce all of the alternative constraints into the model as inequalities and thus to switch from the set of recursive equations of SIMLINK to a representation of the world economy by linked linear-programming models.

The resulting World Development Report (WDR) model is substantially larger than SIMLINK; it is also far more complex, since a system of linked linear programmes is more difficult to manage than a system of recursive equations. The pattern described by the model is that of a world of closely planned developing economies, whose Governments seek the best course of development subject to the balance of payments, savings and capacity constraints confronted by their countries.

The World Development Report Model

A description of the model

The resulting model (which is described in [3]) is made up of three components. The content of the system is a set of linked dynamic linear programming models. The capital flows in that system are generated by a sub-model which consists of an elaborate system of debt equations that embody the Bank's information on capital flows from developed countries and from OPEC to the developing world. A constant-shares trade matrix, adjusted by the RAS procedure to achieve consistency with the trade predictions of the core sub-model, is used to explore the bilateral trade implications of the projection.

The core sub-model combines behavioural and optimizing components. Consumption depends on prices; both imports and exports are likewise price sensitive. These equations are combined with a dynamic linear programme, with

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[1]In practice, a global energy model, SIMRICH, was run in parallel with SIMLINK; SIMRICH served to calculate total imports of oil producers, a figure which was then introduced into SIMLINK as an exogenous variable. See Gunning, Ostereith and Waelbroeck [2].

few degrees of freedom. The system is run to 1990 to produce detailed projections of production and trade in the world economy.

In understanding the model, it is important to be aware of its close dependence on the Bank's system of country and commodity models. About 30 country and 10 commodity models are operated regularly in the World Bank as part of the country and commodity price projections system. These models are a major source of coefficients for the global model, and provide an independent check of the results which it produces. The global model runs are only a part of a broad system of analysis of future development of the world economy.

Another important source of coefficients was the valuable data base embodied in the model built for the United Nations by the Carter–Leontief–Petri team. The coefficients thus borrowed are being gradually replaced by new estimates as the work on the model progresses, but even today the WDR model owes a debt to the empirical work of that team.

Such a model raises three inter-connected types of problems. First is the fundamental question of whether it is appropriate to describe behaviour of developing countries by regional optimizing models. The second is whether the sub-models linkage scheme dovetails the components of the system in a fully correct way. The third is specifying constraints so that the resulting system has a clear interpretation as a description of economic behaviour.

On the first issue, that is whether it is appropriate for the World Bank to represent developments in the developing world by a set of interlocked optimizing models, it should be noted that Governments in these countries surely do try to adopt the best policies. However, an adequate description of optimal policies should also include a careful listing of available policy instruments and of the constraints which limit their use. This is a difficult task in world modelling. A limitation of the model is that it describes regions and yet policies can only be properly defined for countries.

It is tempting to think of using a world model to calculate optimal policies for the world as a whole. This would require specifying some world welfare functions, optimized by selecting the best values of instruments such as aid or tariffs, the values of which can be selected jointly by international co-operation. Because of its apolitical posture, it is, however, not possible for the Bank to select any global objective function.

The optimizing feature of the model, though it is suggestive of what development policy seeks to achieve, should therefore not be taken literally. The real goal is to calculate the impact of good but not truly optimal policies. From this point of view what is required is that the model should simulate behaviour rather than optimize it.

The model as a simulator of market behaviour

It is more attractive to emphasize an alternative interpretation of the model. It is well known that the solution of a properly structured linear programming model can be defined to simulate the behaviour of a competitive world market economy (see [4]). This kind of linkage is somewhat difficult to achieve, as it implies very careful dovetailing of primal and dual variables of the models which are linked together.[2] There should also be a strict specification of equations, excluding any which made no sense in general equilibrium terms, and the tailoring of others to carefully conform to theoretical norms.

Whether the paradigm of general competitive equilibrium should dictate the structure of the Bank's global model is debatable. Few development economists would accept that the development process is properly described by the perfect competition model of mathematical economic theory.[3]

The thinking underlying the WDR model is that its linear programming feature enables it to capture some of the market behaviour which is such an important aspect of the development. The conceptual approach is, however, that the quality of its global model should be judged not by its conformity to some abstract theoretical norm, but by its ability to generate behaviour which is acceptable to the intuition and experience of the user.

General competitive equilibrium versus general equilibrium

It is perhaps useful to distinguish the far more general, almost tautological, concept of general equilibrium from the special case of general competitive equilibrium.

A general competitive equilibrium is a state of the economy in which consumers maximize utility subject to their budget constraints, producers maximize profits subject to production being feasible, and demand exceeds supply for no commodities. Both consumers and producers take prices as given in selecting consumption and production.

This concept dismisses by assumption rigidities, power relations, group behaviour, and monopolist or monopsonist aspects of behaviour which nonetheless are important in studies of development. It is not difficult to encompass these real features by the more general concept of market equilibria, in which all producers and consumers choose the production and consumption levels which are compatible with their behaviour rules, given the information available to them, and demand exceeds supply for no goods.

This is a well-defined concept but one which is so general that it provides little guidance for the practical worker. What does seem worth exploring is a mixed model, in which some features of behaviour correspond to the competitive equilibrium model, while others do not. To give an example, price setting by OPEC may reflect the monopolist power of oil-producers and possibly their non-economic objectives, yet oil-exporting countries could behave competitively as importers on world markets.

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[2]Bénard [5], has made very interesting remarks from this point of view on the present WDR model.

[3]For a discussion of these issues, see [6].

Such an intermediate concept can hopefully provide a realistic picture of the world. Such mixed models should, however, be specified with great care, to ensure that the implied behaviour of each agent is coherent and realistic. Experiments are needed to identify the departures from the competitive equilibrium model which do add to the model's realism, and those which happen to be unimportant.

Model specification

There are two ways of representing a general equilibrium by a model. Under the first approach, illustrated, for example, by the computable general equilibrium (CGE) model of Adelman and Robinson [7], the utility maximization problems of consumers are solved analytically yielding explicit expressions for their demand functions, and the profit maximization problems of producers are solved in the same way to yield explicit supply functions. The behaviour of agents which do not obey the competitive equilibrium model may likewise be represented by appropriate equations. The model thus constructed is a system of equations, which can be solved by an appropriate algorithm. The solution will define the equilibrium prices, consumptions, and productions.

This approach can obviously be used only if the analytical forms of the utility and production functions imply demand and supply curves which have a simple form. Examples of such functions are the Cobb–Douglas and its variants, the CES and the translog functions.

The second approach, illustrated by the Ginsburgh–Waelbroeck GEM model [4], involves jointly solving the optimizing models which represent the utility and profit maximization problems of consumers and producers. The system so constructed cannot be solved either by an optimizing algorithm or by a procedure designed to solve systems of equations. It is necessary to use a special approach combining the two types of algorithms (see [8] and [9]).

The GEM-type approach is more general than the CGE approach in that arbitrary consumption and production functions can be represented. Solving such systems is, however, more expensive than solving models of the CGE type (solving models of the latter type is also not trivial).[4]

The GEM-type solution procedure alternates between optimizing and equation-solving phases. The optimizing phase simulates the decisions which obey the perfect competition model, while the equation solving phase takes care of budget constraints and of behaviour which departs from the competitive norm.

Since computing costs are substantial with both approaches, they have been used up to now only to solve one-period problems (or sequences of one-period problems). This appeared to be a serious drawback given the orientation of World Development Report analysis, which implied a dynamic perspective on the growth process. This has led to considering a perhaps less rigorous but more flexible approach.

[4]We do not discuss the applicability of Scarf-type fixed-point algorithms, which have assured convergence but would be prohibitively expensive for large global models.

The WDR model superposes two types of price determination by embedding in the linear programming framework behavioural relations of the type used, for example, in the CGE models. In practice, production is described by a fixed coefficients I/O scheme, with import, export, and consumption explained by behavioural relations involving relative prices.

This combination of approaches is theoretically unassailable provided that the prices involved in these behavioural relations are consistent with the linear programming dual prices. It has not yet proved possible to fulfil this condition in the present version of the model.[5] To fulfil this condition will require very careful dovetailing of the two price systems which implies the re-specification of a number of equations. Achievement of consistency in this sense is a major goal of present work on the model.

In the present version, the price system involved in the behavioural equations is based on a set of world trade prices of commodities, adjusted by a *tâtonnement* process to balance world exports and imports, subject to appropriate export-import and supply-demand functions. Domestic prices are obtained by adjusting world prices to take account of tariffs; these prices enter linear expenditures systems to determine consumption. The dual prices generated by the linear programme do not coincide with the prices thus calculated, especially for the early years of the simulation. The discrepancy tends to become smaller as the system settles down to a smooth growth path.

Improvements to the model

Work is under way to improve the WDR model in other directions. One is to incorporate into its structure models of a number of large developing countries. This would greatly enhance the operational significance of the model, as it is much more meaningful to discuss policy issues for countries than for the regions of which the model is now composed. An alternative approach, also under consideration, is to keep the country models separate but to adapt both country and regional models to ensure that the results generated are mutually consistent.

Another important but difficult task is the representation of the impact on developed countries of accelerated growth of the developing world. World Bank global models have up to now assumed that the developing world benefited from improved performance of the developed countries, but not vice versa, in spite of awareness that there is a feedback from developing to developed countries and that this is not negligible (see [10]). Representation of this mechanism in the WDR model is an important goal.

The M3 model constructed in Brussels University

The WDR model is the operational model used in Bank projection work, and this tends to make it difficult to experiment with major changes of its structure—

[5]See [5]; also, see [4], particularly chapters 9 and 11, in which such a model and its solution are described.

and to find the time to think through other possible model approaches. This has led the Bank to set up an external research project, carried out by the Centre for Econometrics and Mathematical Economics (CEME) of the Université Libre de Bruxelles. This project involves the construction of the so-called Mark III (M3) model, an experimental general equilibrium model of the world economy (see [9]).

The initial version of M3 does not make use of linear programming, that is, the consumer and producer optimization problems are solved analytically, to yield demand and supply functions which are then introduced into the model. From a technical point of view, it therefore belongs to the CGE class of models (there are, however, plans to introduce at a later stage optimizing components into M3).[6]

There is serious debate within the Bank about the merits of general equilibrium modelling as an approach to the study of development problems. The debate is aimed at the competitive general equilibrium model, as the broader "general equilibrium" concept is flexible enough to accommodate any point of view. M3 will be helpful in testing the relevance of different equilibrium concepts. A first version will feature only obvious departures from perfect competitive behaviour—lack of factor mobility between rural and urban industries, price fixing by oil exporters, the possible lack of rationality of farmers' purchases of inputs, such as fertilizers, and impediments to international trade. Study of the resulting model's properties will suggest which of these distortions have important consequences, and whether other distortions should be recognized to account for how the world economy appears to behave. One candidate is the two-gap hypothesis; another is modelling of the role of increasing returns to scale in the growth process and in trade.

The M3 project also aims to yield improved estimation of key parameters. Three studies are under way. The first is estimation for developing countries of consistent demand equations, such as the Stone–Geary system. The second is analysis of the key relations which characterize the role of agriculture in the growth process. The third is a large-scale investigation of the factors which determine demand for the manufactured exports of developing countries.

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[6]This mixed approach has also been used in the GEM model (see [4], chapter 9).

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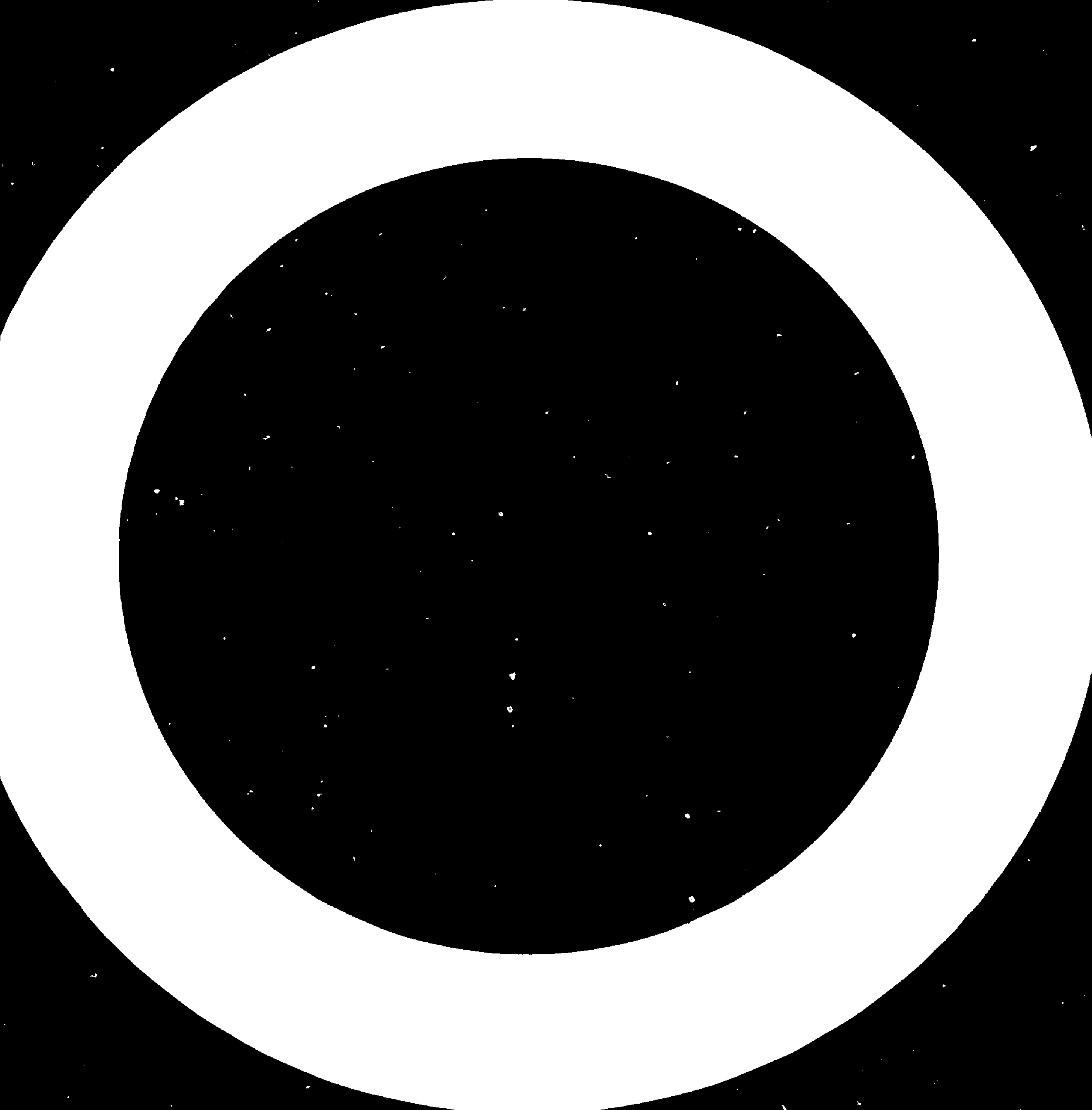
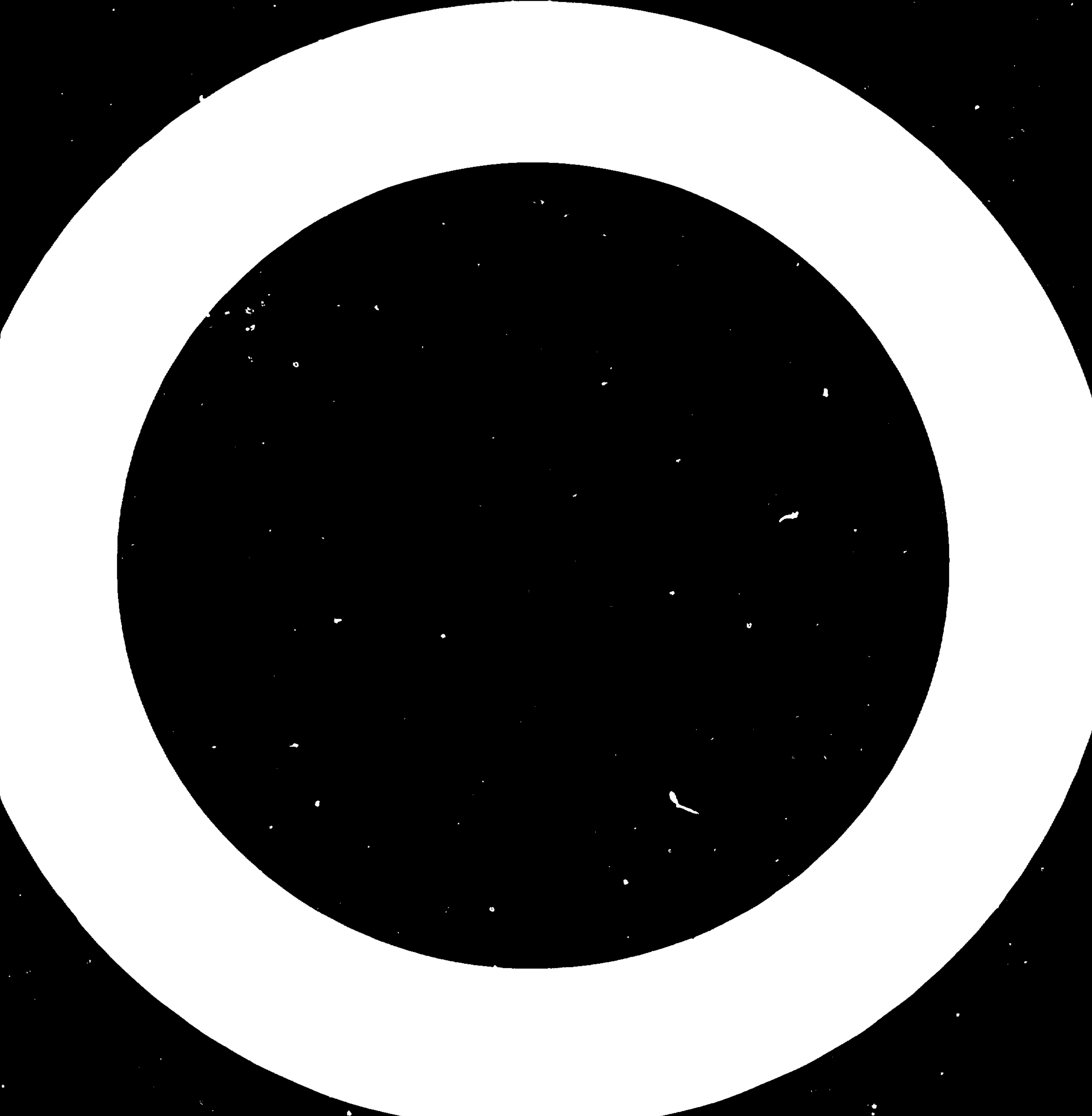


International trade and co-operation



Optimal pattern of international economic co-operation: an application of a two-country turnpike model

Hideto Sato, Shuntaro Shishido* and Jinkichi Tsukui***

Empirical applications of multi-industry optimal growth models for national planning have been done by many economists in recent years. These have yielded various interesting theoretical and policy implications, especially for balanced growth paths or turnpike properties inherent in such models.[1] Few attempts, however, have been made so far towards elucidating international growth issues including trade and aid within a consistent dynamic optimization framework. This is probably due to the enormous data and computational burden required to solve a global model in multi-country, multi-industry and multi-period dimensions.

In view of growing urgent needs for such a global growth model in formulating long-range international policy goals related to a consistent structural adjustment of industrial output, trade and aid, the present paper attempts to build an international multi-industry growth model in which a dynamic optimization procedure is explicitly introduced. Thus the purpose of the present paper is to provide a basis for empirically formulating such a global model, although our model deals only with two regions, Japan and south-eastern Asia. Fairly tentative data had to be used because of the experimental nature of the present model. The major concern is to know the optimal pattern of economic growth and structural changes of the two regions and the desirable aid or investment in south-eastern Asia, thus giving some insight into an optimal division of labour and a pattern of foreign aid in a dynamic context.[2] The turnpike property of the model is also examined as it tends to give a faster growth path favourable for both regions. Special emphasis is placed upon the extent to which such a property can be identified and it is found that the turnpike property, generally, works for stabilizing growth paths in our model.[3]

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[1]For instance, see [1] and [2] for developed, and [1], [3] and [4] for developing countries.

[2]Conventional development models including trade and aid do not seem to have explicitly dealt with this issue. For instance, see [5].

[3]See [2] for a theoretical prototype of the two-country turnpike model.

The prototype model and empirical testing

The prototype model

First, a simplified empirical model is used as a prototype. This model has two regions and ten industrial divisions. This model will be expanded as it includes more realistic constraints as discussed later. The objective function is to maximize average per capita income (μ) for two regions at the terminal year under the following methodology.

Objective function:

$$\text{Maximize } \mu \tag{1}$$

Initial conditions:

$$X^k(1) \leqslant \bar{X}^k \qquad (k = I, II) \tag{2}$$

$$E^k(1) \leqslant \bar{E}^k \qquad (k = I, II) \tag{3}$$

Constraints for productions:

$$[I - A^k - C^k \cdot V^k + B^k] X^k(t) - B^k \cdot X^k(t+1) - [I + T^k] E^k(t) + E^h(t) \geqslant \bar{C}^k$$

$$\begin{bmatrix} k = I, II;\ h = II, I \\ t = 1, 2, \ldots, T-1 \end{bmatrix} \tag{4}$$

Terminal conditions:

$$X^k(T) - w^k \cdot \bar{X}^k_T \cdot \mu = 0 \qquad (k = I, II) \tag{5}$$

$$w^k = \frac{n^k_T}{V^k \cdot \bar{X}^k_T} \cdot d^k \tag{6}$$

Non-negative conditions:

$$X^k(t) \geqslant 0,\ E^k(t) \geqslant 0 \qquad \begin{bmatrix} k = I, II \\ t = 1, 2, \ldots, T-1 \end{bmatrix}$$

where:

A^k = matrix of input coefficients for region k;
B^k = matrix of capital coefficients for region k;
C^k = vector of marginal propensity to consume for region k;
$\bar{C}^k$ = vector of basic consumption for region k;
d^k = relative difference of per capita income at the terminal period T for region k (exogenous);
$E^k(t)$ = export vector for region k;
$\bar{E}^k$ = upper limit of export vector at the initial period;
I = unit matrix;

μ = average level of per capita income at the terminal period;
n_T^k = population at the terminal period for region k;
T^k = matrix of transportation costs for region k;
V^k = row vector of value added ratios for region k;
w^k = weight in terms of output at the terminal period T for region k;
$X^k(t)$ = gross output vector for region k;
$\bar{X}_1^k$ = upper limit of gross output vector at the initial period;
$\bar{X}_T^k$ = component of gross output vector at the terminal period T for region k (exogenous).

Basically, the model is formulated as the "final-state turnpike model", where the level of income is usually maximized at the terminal period for a single country. For multi-regional analysis, however, we introduce an integrated target for two regions in which their per capita incomes are combined by means of the relative difference of d^k as given in (6).

Expansion of a single country optimal growth model to a multi-regional dimension can be formulated as below:

$$(A + B)X(t) - B \cdot X(t + 1) \geqslant 0 \qquad (t = 1, 2, \ldots, T - 1) \tag{7}$$

where:

$$A = \left[\begin{array}{c|c|c|c} I - A^I - C^I \cdot V^I & -I - T^I & I & 0 \\ \hline 0 & I & -I - T^{II} & I - A^{II} - C^{II} \cdot V^{II} \end{array}\right]$$

$$B = \left[\begin{array}{c|c|c|c} B^I & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & B^{II} \end{array}\right]$$

$$X(t) = \begin{bmatrix} X^I(t) \\ E^I(t) \\ E^{II}(t) \\ X^{II}(t) \end{bmatrix}$$

Similarly, (5) can be arranged as:

$$\bar{X}_T \cdot \mu - X(T) = 0 \tag{8}$$

where:

$$\bar{X}_T = \begin{bmatrix} w^I \cdot \bar{X}_T^I \\ 0 \\ 0 \\ w^{II} \cdot \bar{X}_T^{II} \end{bmatrix}$$

For details, see Tsukui [2].

Empirical testing

The prototype model above was tested in order to verify a turnpike property on an international basis. The industry breakdown is indicated in annex I and the data used are explained in annex II. Two types of testing were made in this context with two different assumptions:

(*a*) All goods are internationally tradeable without transportation cost (case A);

(*b*) Only manufactured goods are tradeable with transportation cost (case B).

The turnpike property is supposed to be observed more clearly in case A, since there are few restrictions for division of labour between two regions, and adjustment times to the turnpike at the initial period, and to the target for the terminal periods, are also expected to be shorter because of wider possibilities of choice for the structure of trade and industries.

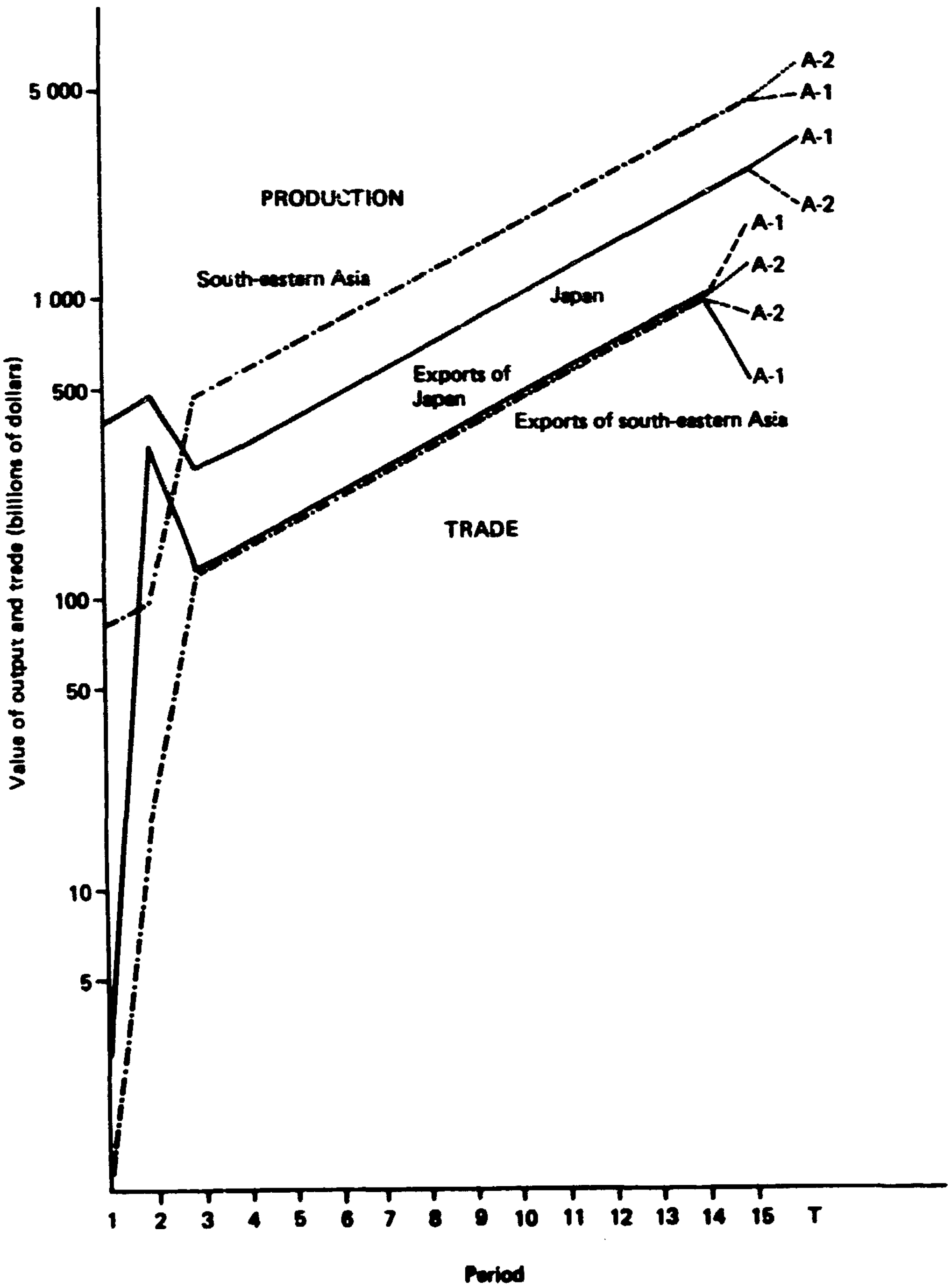
In both cases our simulation covers 15 periods, that is for the 30 years between 1970 and 2000. Each period covers two years.

Case A was divided into two parts, case A-1 and case A-2, according to the difference in terminal condition for the year 2000. Case A-1 assumes that the relative ratio in per capita income between Japan and south-eastern Asia is 1 to 0.5 at the terminal year, while the ratio is 1 to 1 in case A-2. Since Japanese per capita income was $1,892 and that of south-eastern Asia was $175 in 1970, our assumptions for the year 2000 imply a remarkable growth for south-eastern Asia.

Computational results are shown in figure 1 for gross output, exports and imports. Since imports can be represented by exports from other regions, figure 1 indicates only exports from two regions providing the pattern in terms of trade between two regions. As anticipated, figure 1 shows an interestingly similar pattern of initial adjustment and a balanced growth path for the intermediate period even under different assumptions on growth target at the terminal year, implying the existence of turnpike property on a multi-regional case. The growth rate of real GDP is about 10 per cent for both regions and the same rate is observed also for their balanced bilateral trade. It is noted that, in view of a relative shortage in capital in south-eastern Asia, Japan's exports to south-eastern Asia tend to accelerate rapidly during the initial adjustment period and their per capita income differential tends to be reduced sharply during the same period.

Table 1 shows exports from Japan to south-eastern Asia and table 2 shows exports from south-eastern Asia to Japan. The tables show the interesting characteristics of dynamic comparative advantage. Japan's advantage is shown in chemicals, metals, machinery, and construction, and south-eastern Asia's advantage in agriculture, mining, foods, textiles and services. Although somewhat exaggerated because of our simple assumption, the tables seem to provide an important basis for guessing at the optimal pattern of industrial growth in output and trade between the two regions.

Figure 1. Optimal paths of output and trade for no transportation cost



Our next approach towards a more realistic formulation, in case B, is to limit the scope for foreign trade only to manufactured goods and to introduce the cost of transportation between two regions. As before, this approach is divided into two cases according to two different terminal conditions.

Unlike the previous cases one cannot observe a distinct turnpike property in our computational results, as seen in figure 2. It seems that by the introduction of

TABLE 1. EXPORTS FROM JAPAN TO SOUTH-EASTERN ASIA, BY PERIOD,[a] BY INDUSTRY[b]

Period	*A-F-F*	*Mining*	*Food-tex*	*Chemical*	*Metal*	*H-machin*	*L-machin*	*O-M.F.G.*	*Const.*	*Service*	*Total*
Exports from Japan to south-eastern Asia (billions of dollars)											
1	–	–	–	0.670	0.594	1.166	–	–	–	–	2.430
2	–	1.487	–	7.125	–	57.690	17.441	12.299	195.204	20.296	311.542
3	–	–	–	27.895	0.846	25.027	8.633	–	58.844	–	121.245
4	–	–	–	33.721	1.022	30.254	10.436	–	71.134	–	146.568
5	–	–	–	40.764	1.236	36.573	12.616	–	85.991	–	177.179
6	–	–	–	49.277	1.494	44.212	15.250	–	103.951	–	214.184
7	–	–	–	59.569	1.806	53.445	18.435	–	125.661	–	258.916
8	–	–	–	72.010	2.183	64.607	22.286	–	151.905	–	312.990
9	–	–	–	87.049	2.639	78.097	26.941	–	183.627	–	378.353
10	–	–	–	105.228	3.191	94.397	32.570	–	221.962	–	457.346
11	–	–	–	127.198	3.859	114.069	39.379	–	268.257	–	552.762
12	–	–	–	153.736	4.675	137.723	47.628	–	324.042	–	667.804
13	–	–	–	185.737	5.687	165.793	57.653	–	390.694	–	805.564
14	–	–	–	224.153	6.988	198.306	70.188	–	470.512	–	970.157
15	–	–	–	291.748	36.497	163.228	26.908	–	–	–	518.382
Shares (percentage)											
1	–	–	–	27.6	24.4	48.0	–	–	–	–	100.0
2	–	0.5	–	2.3	–	18.5	5.6	3.9	62.7	6.5	100.0
3	–	–	–	23.0	0.7	20.6	7.1	–	48.5	–	100.0
4	–	–	–	23.0	0.7	20.6	7.1	–	48.5	–	100.0
5	–	–	–	23.0	0.7	20.6	7.1	–	48.5	–	100.0
6	–	–	–	23.0	0.7	20.6	7.1	–	48.5	–	100.0

7	–	–	–	23.0	0.7	20.6	7.1	–	48.5	–	100.0
8	–	–	–	23.0	0.7	20.6	7.1	–	48.5	–	100.0
9	–	–	–	23.0	0.7	20.6	7.1	–	48.5	–	100.0
10	–	–	–	23.0	0.7	20.6	7.1	–	48.5	–	100.0
11	–	–	–	23.0	0.7	20.6	7.1	–	48.5	–	100.0
12	–	–	–	23.0	0.7	20.6	7.1	–	48.5	–	100.0
13	–	–	–	23.1	0.7	20.6	7.2	–	48.5	–	100.0
14	–	–	–	23.1	0.7	20.4	7.2	–	48.5	–	100.0
15	–	–	–	56.3	7.0	31.5	5.2	–	–	–	100.0
Change from previous period (percentage)											
2	–	–	–	226.1	–	603.4	–	–	–	–	1 032.3
3	–	–	–	97.9	–	−34.1	−29.6	–	−45.1	–	−37.6
4	–	–	–	9.9	9.9	9.9	9.9	–	9.9	–	9.9
5	–	–	–	9.9	9.9	9.9	9.9	–	9.9	–	9.9
6	–	–	–	9.9	9.9	9.9	9.9	–	9.9	–	9.9
7	–	–	–	9.9	9.9	9.9	9.9	–	9.9	–	9.9
8	–	–	–	9.9	9.9	9.9	9.9	–	9.9	–	9.9
9	–	–	–	9.9	10.0	9.9	9.9	–	9.9	–	9.9
10	–	–	–	9.9	10.0	9.9	10.0	–	9.9	–	9.9
11	–	–	–	9.9	10.0	9.9	10.0	–	9.9	–	9.9
12	–	–	–	9.9	10.1	9.9	10.0	–	9.9	–	9.9
13	–	–	–	9.9	10.3	9.7	10.0	–	9.8	–	9.8
14	–	–	–	9.9	10.9	9.4	10.3	–	9.7	–	9.7
15	–	–	–	14.1	128.5	−9.3	−38.1	–	–	–	−26.9

[a]Periods are two years in duration.
[b]See annex I for industry classifications.

TABLE 2. EXPORTS FROM SOUTH-EASTERN ASIA TO JAPAN, BY PERIOD,[a] BY INDUSTRY[b]

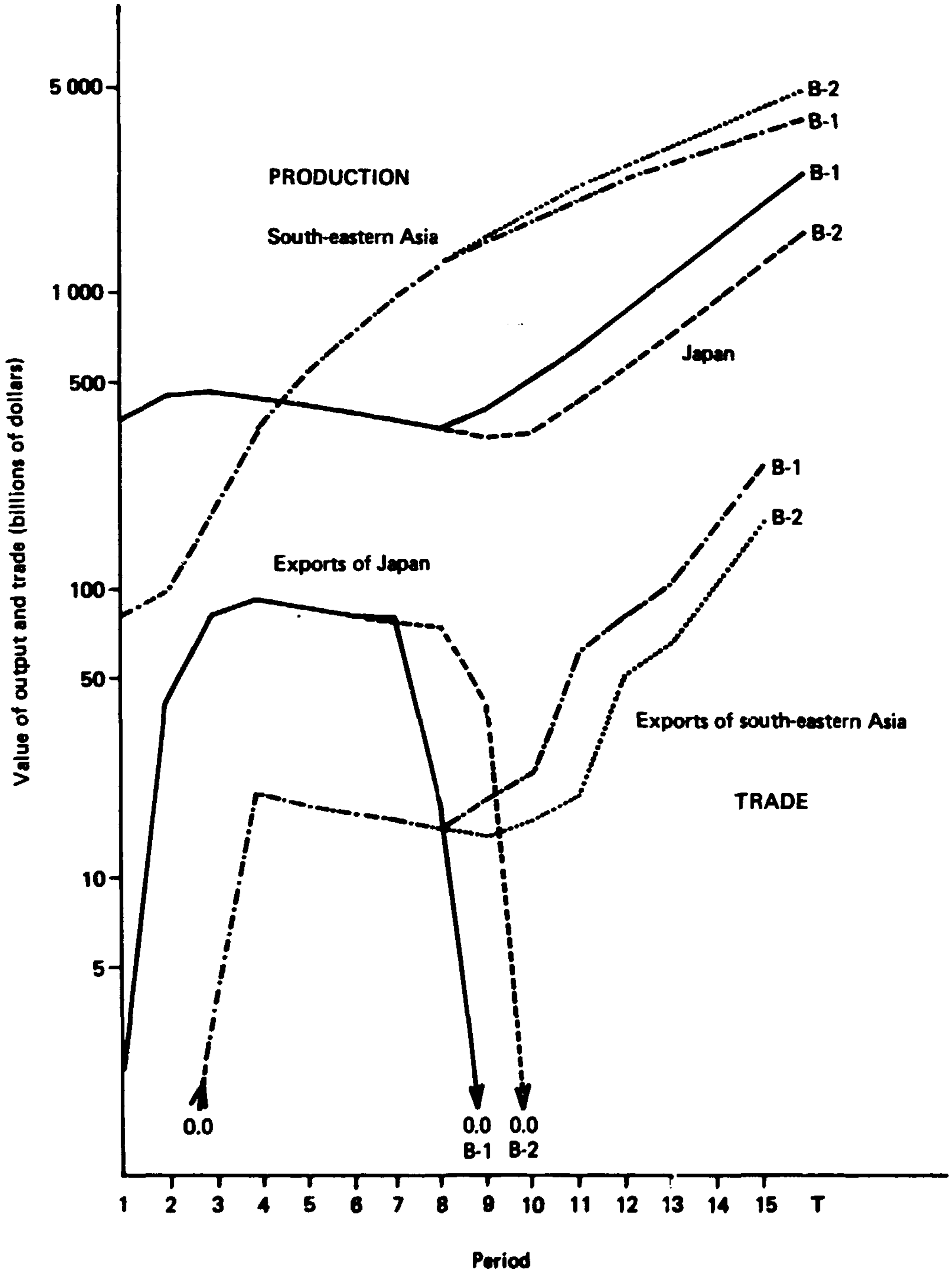
Period	*A-F-F*	*Mining*	*Food-tex*	*Chemical*	*Metal*	*H-machin*	*L-machin*	*O-M.F.G.*	*Const.*	*Service*	*Total*
Exports from south-eastern Asia to Japan (billions of dollars)											
1	–	–	0.345	–	–	–	0.028	0.159	–	–	0.532
2	7.244	–	8.756	–	1.748	–	–	–	–	–	17.748
3	0.017	8.869	6.297	–	–	–	–	17.789	–	83.970	116.942
4	0.020	10.721	7.613	–	–	–	–	21.504	–	101.507	141.366
5	0.025	12.960	9.202	–	–	–	–	25.995	–	122.708	170.891
6	0.030	15.667	11.124	–	–	–	–	31.425	–	148.336	206.582
7	0.036	18.939	13.448	–	–	–	–	37.988	–	179.317	249.728
8	0.044	22.895	16.257	–	–	–	–	45.922	–	216.770	301.887
9	0.053	27.677	19.652	–	–	–	–	55.514	–	262.049	364.946
10	0.064	33.459	23.758	–	–	–	–	67.112	–	316.802	441.195
11	0.076	40.453	28.725	–	–	–	–	81.142	–	383.052	533.448
12	0.090	48.923	34.741	–	–	–	–	98.136	–	463.374	645.263
13	0.097	59.207	42.063	–	–	–	–	118.803	–	561.359	781.528
14	0.052	71.632	51.093	–	–	–	–	144.138	–	682.890	949.805
15	25.123	83.296	63.330	–	–	–	–	226.854	379.514	909.218	1 707.337
Shares (percentage)											
1	–	–	64.8	–	–	–	5.3	29.9	–	–	100.0
2	40.8	–	49.3	–	9.9	–	–	–	–	–	100.0
3	0.0	7.6	5.4	–	–	–	–	15.2	–	71.8	100.0
4	0.0	7.6	5.4	–	–	–	–	15.2	–	71.8	100.0
5	0.0	7.6	5.4	–	–	–	–	15.2	–	71.8	100.0
6	0.0	7.6	5.4	–	–	–	–	15.2	–	71.8	100.0

7	0.0	7.6	5.4	–	–	–
8	0.0	7.6	5.4	–	–	–
9	0.0	7.6	5.4	–	–	–
10	0.0	7.6	5.4	–	–	–
11	0.0	7.6	5.4	–	–	–
12	0.0	7.6	5.4	–	–	–
13	0.0	7.6	5.4	–	–	–
14	0.0	7.6	5.4	–	–	–
15	1.5	4.9	4.9	–	–	–
Change from previous period (percentage)						
2	–	–	403.8	–	–	–
3	–95.2	–	–15.2	–	–	–
4	9.9	9.9	9.9	–	–	–
5	9.9	9.9	9.9	–	–	–
6	9.9	9.9	9.9	–	–	–
7	9.9	9.9	9.9	–	–	–
8	9.9	9.9	9.9	–	–	–
9	9.9	9.9	9.9	–	–	–
10	9.8	10.0	10.0	–	–	–
11	9.5	10.0	10.0	–	–	–
12	8.4	10.0	10.0	–	–	–
13	3.9	10.0	10.0	–	–	–
14	–26.8	10.0	10.2	–	–	–
15	2 100.9	7.3	27.7	–	–	–

[a]Periods are two years in duration.
[b]See annex I for industry classifications.

–	–	15.2	–	71.8	100.0
–	–	15.2	–	71.8	100.0
–	–	15.2	–	71.8	100.0
–	–	15.2	–	71.8	100.0
–	–	15.2	–	71.8	100.0
–	–	15.2	–	71.8	100.0
–	–	15.2	–	71.8	100.0
–	–	15.2	–	71.9	100.0
–	–	13.3	22.2	53.3	100.0
–	–	–	–	–	477.6
–	–	–	–	–	156.7
–	–	9.9	–	9.9	9.9
–	–	9.9	–	9.9	9.9
–	–	9.9	–	9.9	9.9
–	–	9.9	–	9.9	9.9
–	–	9.9	–	9.9	9.9
–	–	9.9	–	9.9	9.9
–	–	10.0	–	10.0	10.0
–	–	10.0	–	10.0	10.0
–	–	10.0	–	10.0	10.0
–	–	10.0	–	10.1	10.1
–	–	10.1	–	10.3	10.2
–	–	25.5	–	15.4	34.1

Figure 2. Optimal paths of output and trade with transportation cost



more restrictions in bilateral trade a longer adjustment period becomes necessary to reach the balanced growth path. The dependence on trade in terms of a ratio to GDP also falls sharply for both regions and Japan's exports surplus or south-eastern Asia's deficit for the earlier period are greatly increased.

Despite the lack of a distinct turnpike property here, it should be noticed that the almost similar growth path observed during the earlier period might be interpreted as a possibility of south-eastern Asia reaching the turnpike path after an adjustment period.

Introduction of Japan's labour constraints

Revision of the model

So far labour constraints have been neglected, as capital is regarded as the most crucial element in determining the growth pattern.

In the revision labour constraints for industrial gross output are introduced, but only for Japan. This treatment may be justified in view of the relatively abundant supply of labour in most countries of south-eastern Asia.

The revised model is slightly changed in view of labour constraints especially in connection with capacity utilization. The rate g of increase in total labour force and the rate θ of its quality improvement by research and development activity are explicitly introduced in the following labour constraint for Japan.

$$(1 + g^I)[L^I \cdot X^I(t) + L^I_a \cdot X^I_a(t) + \theta^I \cdot w^I(t) + u^I(t)] - \\ - L^I \cdot X^I(t+1) - L^I_a \cdot X^I_a(t+1) - u^I(t+1) = 0 \\ (t = 1, 2, \ldots, T-1) \qquad (9)$$

where:

g^I = the rate of increase in total labour force;
L^I = the row vector of labour coefficients for $X^I(t)$;
L^I_a = the row vector of labour coefficients for $X^I_a(t)$;
θ^I = the quality improvement by research and development;
$u^I(t)$ = unemployment;
$w^I(t)$ = the level of labour saving through productivity growth by the research and development activity;
$X^I(t)$ = the gross output vector connected to capital coefficients;
$X^I_a(t)$ = the gross output vector disconnected from capital coefficients (i.e. adjustment factor for capacity utilization).

An additional constraint for the initial condition is given by:

$$L^I \cdot X^I(0) + L^I_a \cdot X^I_a(0) + \theta^I \cdot w^I(0) + u^I(0) = \bar{L}^I \qquad (10)$$

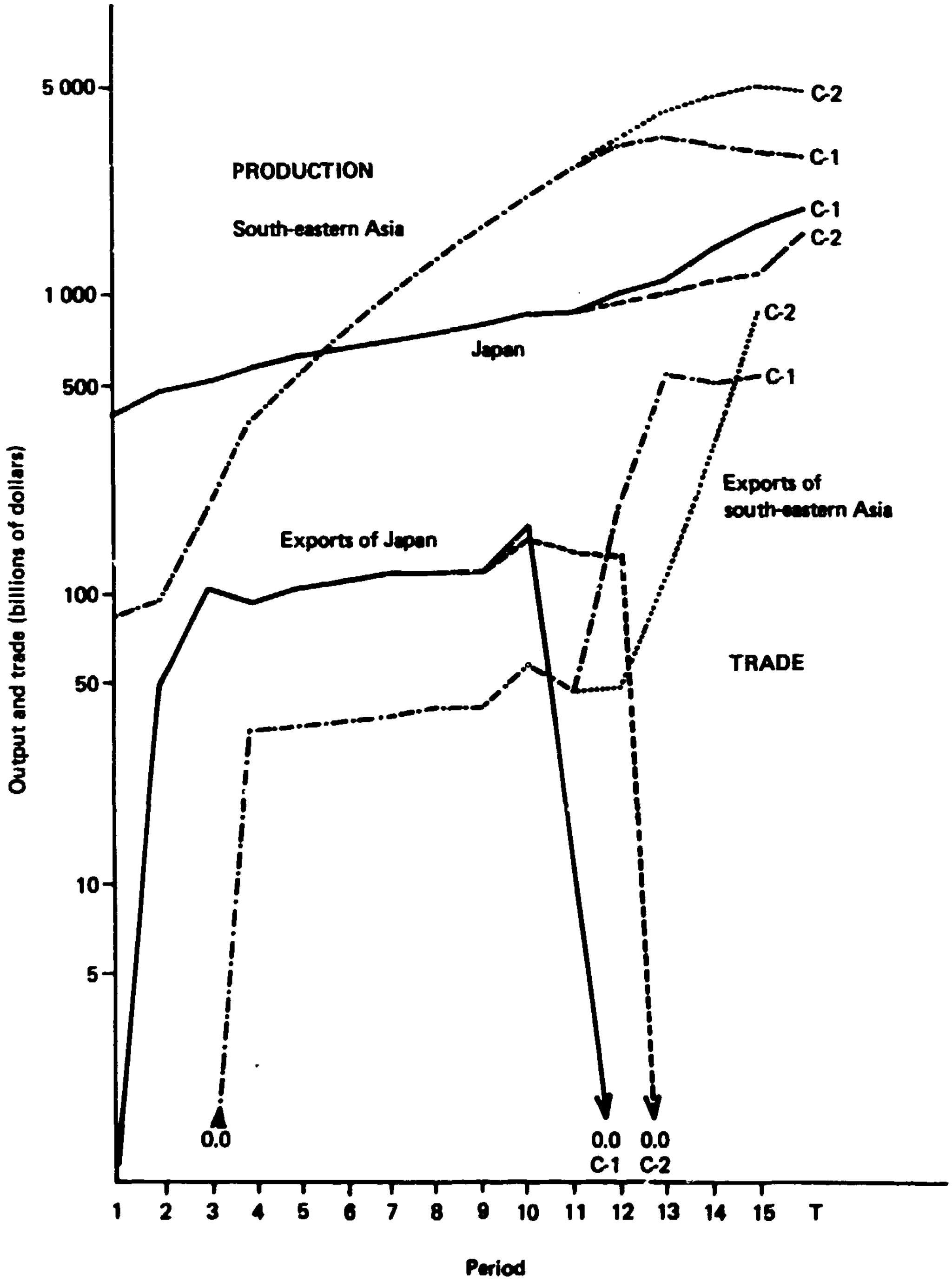
where $\bar{L}^I$ is the total labour force for the initial period.

The production constraint in equation (4) is also revised as below in response to the above revision.

$$[I - A^I - C^I \cdot V^I + B^I]X^I(t) - B^I \cdot X^I(t+1) + [I - A^I - C^I \cdot V^I]X^I_a(t) - \\ - [I + T^I]E^I(t) + E^{II}(t) - H^I \cdot w^I(t) \geqslant \bar{C}^I \qquad (t = 1, 2, \ldots, T-1) \quad (11)$$

where H^I is a vector of input coefficients for research and development activity.

Figure 3. Optimal paths of output and trade under labour constraint for Japan

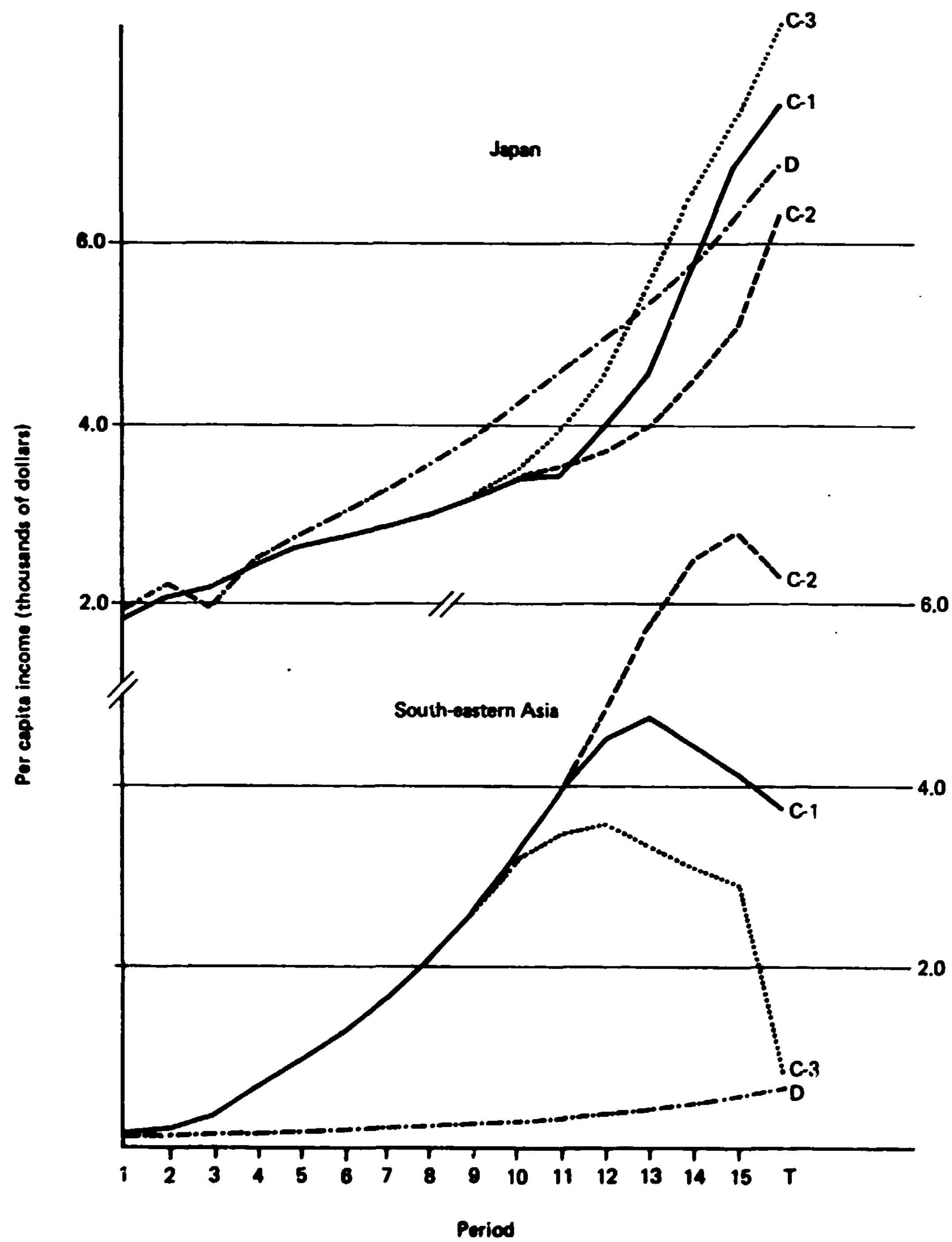


Finally an additional constraint for the upper limit of capacity utilization is added to the system:

$$(\rho - 1)X^{I}(t) - X^{I}_{a}(t) \geqslant 0 \qquad (t = 1, 2, \ldots, T - 1) \qquad (12)$$

where ρ is the ratio of the full utilization level of capital to the normal level, ($\rho = 1$).

Figure 4. Per capita income under labour constraint for Japan



Computational results

The results of si..ulation with the revised model are shown in figure 3 as case C-1 and case C-2 which correspond to the previously defined cases. It is observed that the gross output and trade seem to grow in three different development stages, while there are some similarities between case C and case B (figure 2). The first stage is characterized by a rapid capital import and growth of south-eastern Asia. In the second stage there seems to be a somewhat stable growth pattern for output and trade, with a noticeable division of labour between the two regions (Japan's exports of capital-intensive goods and south-eastern Asia's exports of labour-intensive goods). The third stage has an adjustment period to the terminal conditions. Thus it can be stated that a quasi-turnpike property seems to be observed at least during the second stage as shown by the stability of the growth pattern of gross output and trade in this period.

Another study was attempted to evaluate the efficiency of trade (or opportunity cost of trade) by comparing the above results with case D where there are no trading activities between the two regions. Since it is necessary that the two cases are computed with the same target, case C was revised by maximizing relative per capita income at the terminal year under the assumption of the relative regional difference obtained from case D. The results, given as case C-3, are then compared with case D.

As shown in figure 4, the average per capita income at the terminal period improves by 23 per cent in case C-3 as compared with case D. The advantage of bilateral trade is noticeable especially in south-eastern Asia throughout the whole period, while that in Japan is a little less advantageous for the first half and gives a remarkable improvement in relative income during the second half. The lower growth for Japan for the earlier period in cases C-1, C-2 and C-3 seems to be related to the relative advantage of foreign investment in south-eastern Asia which has a lower capital-output ratio, or higher return to capital.

Since it is unlikely that Japan will continue to accumulate foreign assets and unlikely that south-eastern Asia will accumulate foreign debt without any interest payments, a flow for these payments between the two regions was introduced into the model as a step towards a more realistic formulation. A balance of payments constraint is now explicitly introduced.

Interest on foreign debt

As foreign assets equal foreign debts in value in the bilateral model, the balance of payments and foreign assets are specified as:

$$f(t+1) = (r+1)f(t) + P \cdot E^{I}(t) - P \cdot E^{II}(t) \qquad (t = 1, 2, \ldots, T-1) \quad (13)$$

where:

$f(t)$ = foreign assets of region I;
r = interest rate;
P = row vector of export and the import prices.

The variables r and P are exogenously determined. Usually interest payments affect disposable income and consumption for both regions by the amount $c \cdot r \cdot f(t)$, and equation (11) must be modified to include this.

Similarly, the terminal condition in equation (5) needs to be modified as

$$X^{I}(T) + \frac{\bar{X}^{I}_{T}}{V^{I} \cdot \bar{X}^{I}_{T}} \cdot r \cdot f(T) - w^{I} \cdot \bar{X}^{I}_{T} \cdot \mu = 0 \tag{14}$$

$$X^{II}(T) - \frac{\bar{X}^{II}_{T}}{V^{II} \cdot \bar{X}^{II}_{T}} \cdot r \cdot f(T) - w^{II} \cdot \bar{X}^{II}_{T} \cdot \mu = 0 \tag{15}$$

where $\bar{X}^{I}_{T} / (V^{I} \cdot \bar{X}^{I}_{T})$ is the required gross output per unit of value added at the terminal period.

The amount of foreign assets at the initial period f(1) is defined as given.

As a result of interest payments, south-eastern Asia tends to save imports so as to reduce the trade deficit, and thus Japan's large trade surplus in the previous cases would fall to a certain extent. In order to evaluate these effects in the context of the terminal condition we assumed four different cases as shown in table 3.

TABLE 3. ALTERNATIVE CASES INCLUDING INTEREST ON FOREIGN ASSETS

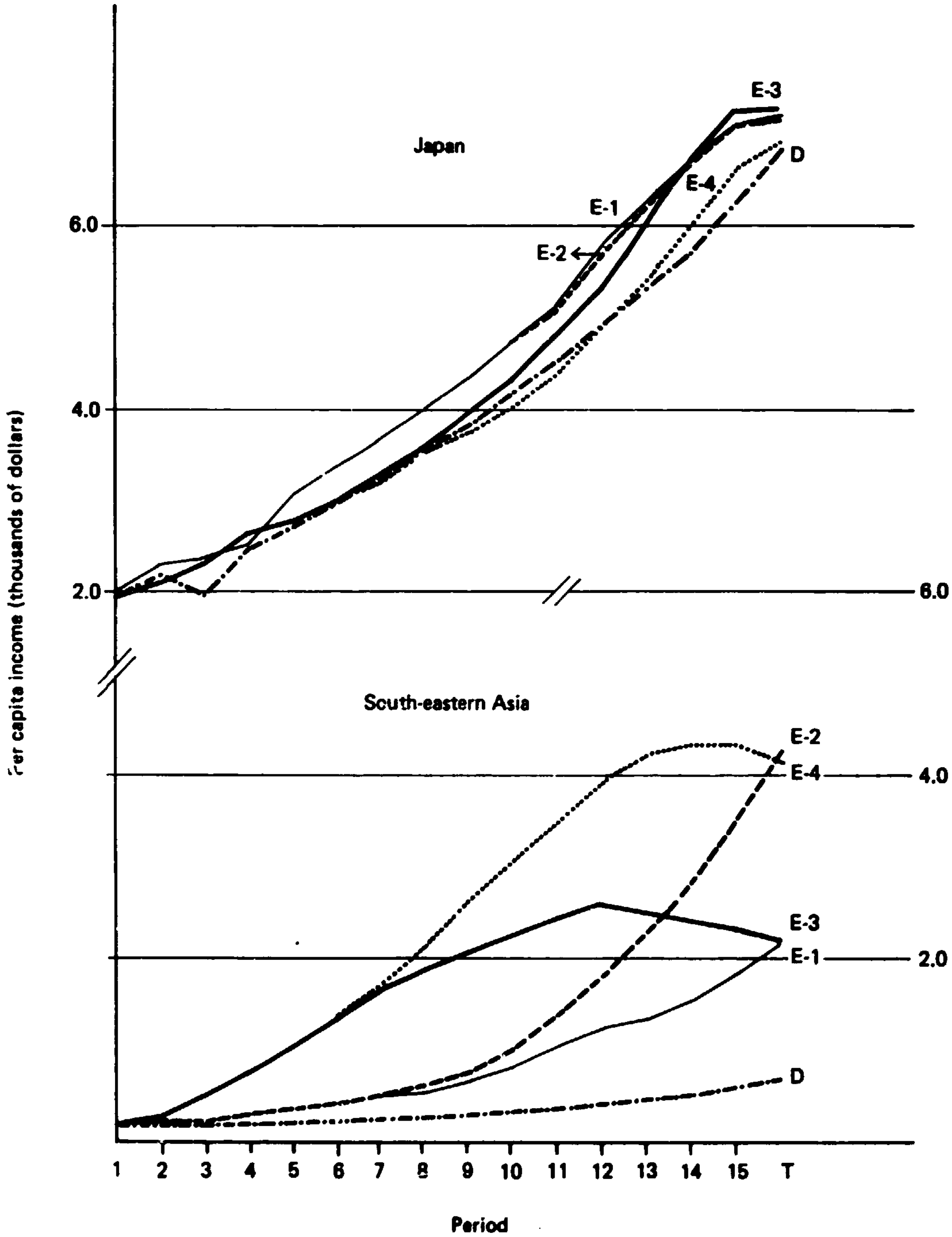
Case	*Rate of interest (%)*	*Terminal condition ratio of per capita income in Japan to that in south-eastern Asia*
E-1	10	1:0.3
E-2	10	1:0.6
E-3	5	1:0.3
E-4	5	1:0.6

As observed in figure 5, most cases, except case E-4 for Japan, indicate higher per capita income compared to case D for which there is no trade. Even for case E-4, which assumes a relatively lower terminal income and a lower rate of interest for Japan, the per capita income exceeds that in case D during the final stage.

Case E-1 with a 10.0 per cent interest rate is depicted in figure 6. The figure indicates an interesting tendency of falling trade imbalance between two regions and a much lower but steady growth for south-eastern Asia as compared with other cases. However, a relatively higher growth rate is still observed. The rates of growth of GDP for Japan and south-eastern Asia in case E-1 are 4.3 and 8.8 per cent respectively.

Case E-3 in figure 7, on the other hand, seems to suggest an alternative possibility inherent in this type of model including interest rate payments. The lower saving ratio to GDP, by increased consumption, tends to reduce investment and growth in Japan, whereas a rapid growth of GDP and trade deficit are observed for south-eastern Asia. The trade patterns tend to become unstable, implying a much longer adjustment period in reaching a turnpike-type growth path.

Figure 5. Per capita income including interest on foreign assets

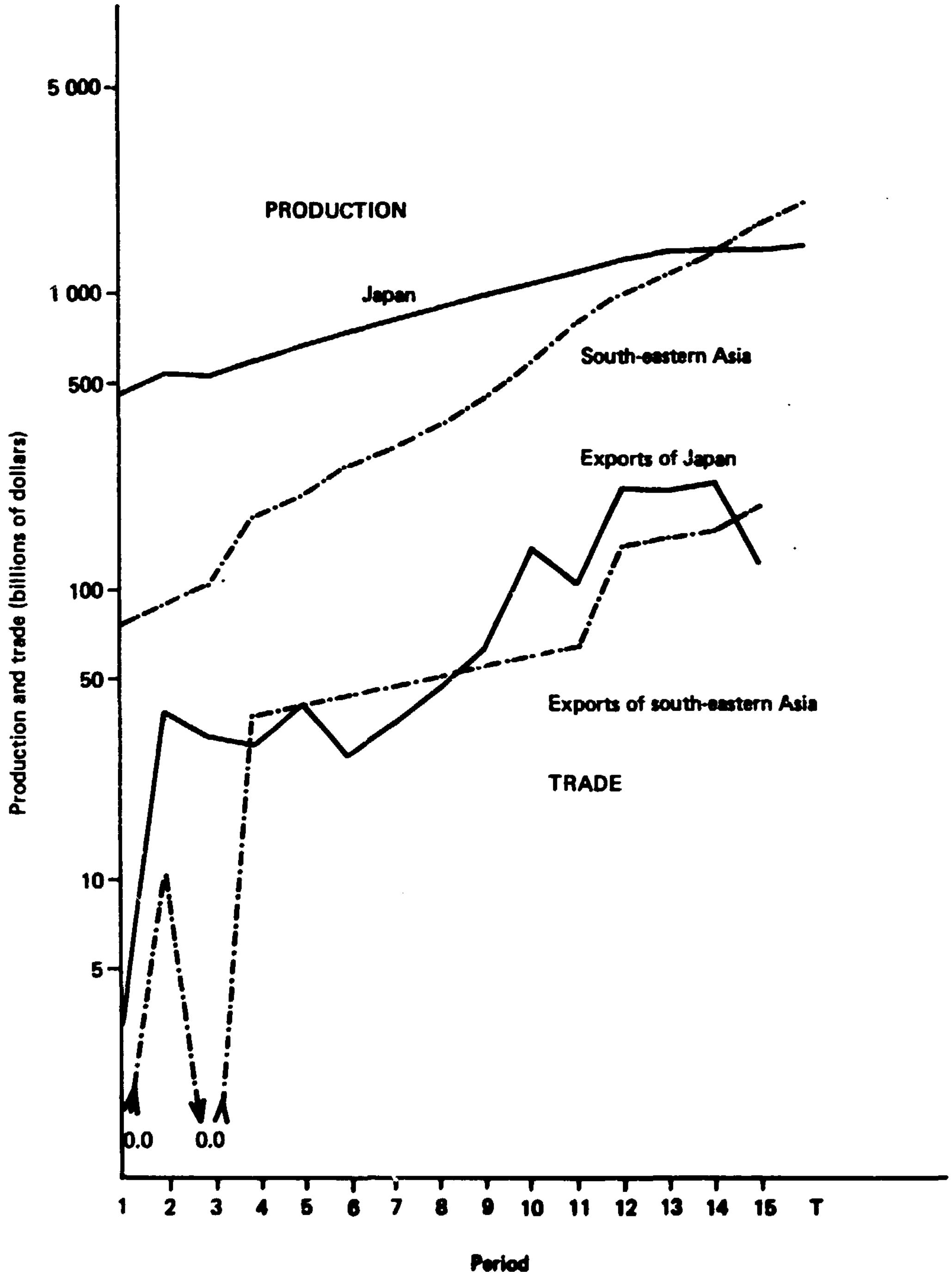


Concluding remarks

Although highly experimental and remaining to be improved in various directions, the present model seems to provide some interesting findings.

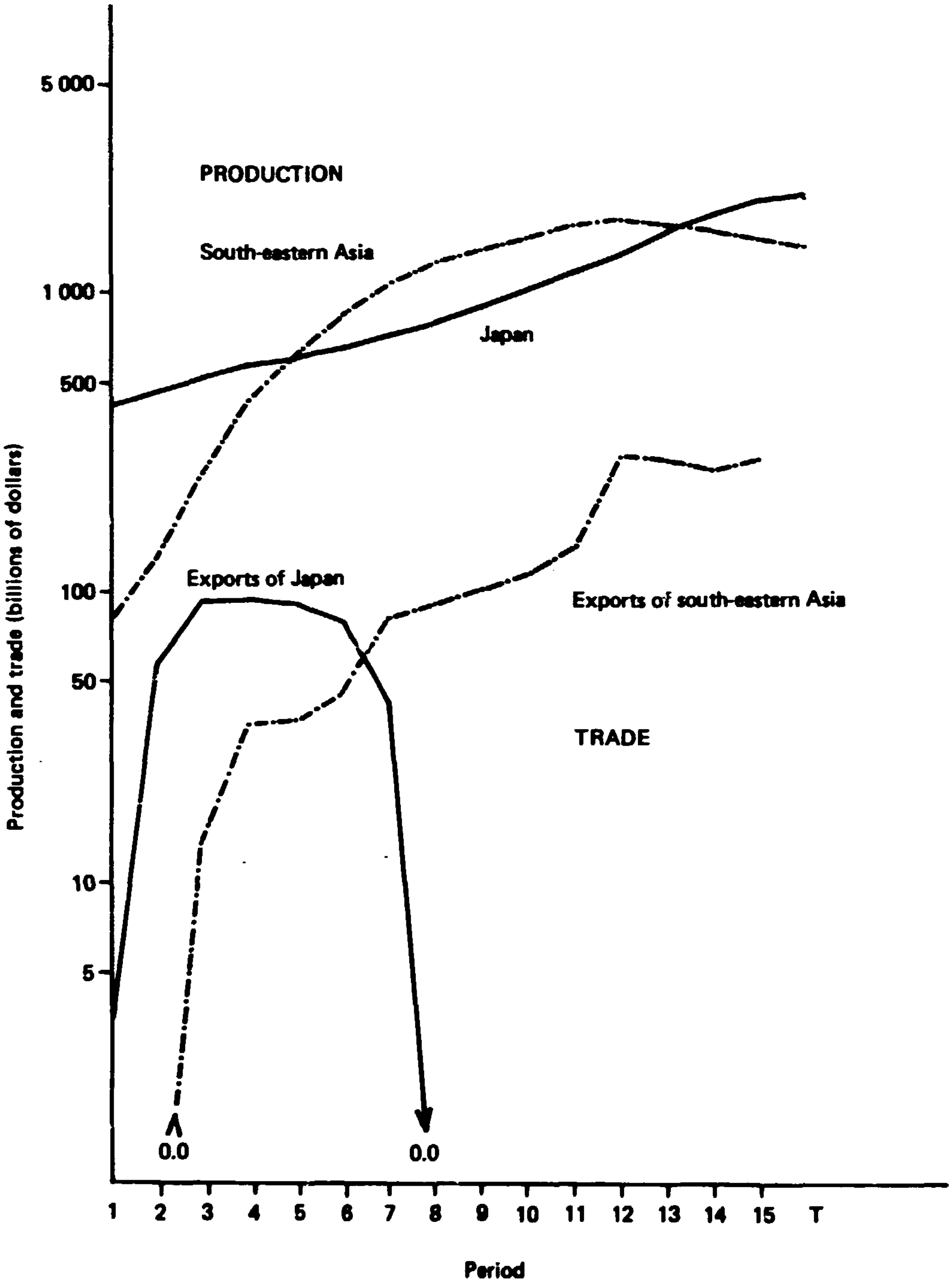
First, the trade between Japan and south-eastern Asia along our optimal path seems to give advantages for both regions in the long run. These advantages increase when active foreign investment from Japan starts earlier.

Figure 6. Optimal paths of output and trade for case E-1 with a rate of interest of 10%



Secondly, the optimal pattern of division of labour in our model is one of exports of heavy industrial and engineering products which are capital-intensive from Japan and exports of light manufactures such as foods, textiles and other light manufactured products which are labour-intensive from south-eastern Asia. A conventional Heckscher–Ohlin type trade pattern is observed for both regions. However, this needs to be modified under a longer and more dynamic perspective

Figure 7. Optimal paths of output and trade for case E-3 with a rate of interest of 5%



under which capital imports play an essential role, especially for south-eastern Asia.

Thirdly, as for the types of economic aid, a grant from Japan seems to lower its economic growth as a result of an excessive trade surplus and capital movement, as compared with a foreign loan with interest payments. The optimal rate of interest on foreign loans as well as the amount of aid remains to be explored with respect to an optimal GDP growth by use of a more sophisticated version of the present model.

ANNEX I

CLASSIFICATION OF INDUSTRIES

Code	*Industry definition*	*Symbol*
1	Agriculture, forestry and fisheries	A-F-F
2	Mining	Mining
3	Food, beverages and textiles	Food-tex
4	Chemicals (Pulp and paper, chemicals, petroleum refinery products and non-metallic mineral products)	Chemical
5	Basic metal	Metal
6	Heavy machinery (Metal products, machinery except electrical machinery and transport equipment)	H-machin
7	Light machinery (Electrical machinery and appliances, medical, scientific and optical instruments, watches and clocks)	L-machin
8	Other manufacturing goods	O-M.F.G.
9	Construction	Const.
10	Services	Service

ANNEX II

DATA REQUIREMENTS

In empirically building a bilateral I/O model, two types of statistical data are needed. These are a bilateral I/O table covering Japan and eight countries of south-eastern Asia and matrices of capital coefficients.

Although both types of data are available for Japan, values for south-eastern Asia are approximate. First, aggregate values in dollars were estimated for gross output, value added, exports and imports according to our industry classification system indicated in annex I. The basic data used were the Statistical Yearbook for Asia and the Pacific of the United Nations, International Financial Statistics of the International Monetary Fund, Yearbook of International Trade Statistics of the United Nations etc. Next, the RAS method was applied by using the Japanese input coefficients as initial values to estimate inter-industry flows of the transaction matrix for south-eastern Asia.

A crude estimation of capital coefficients of south-eastern Asia was made on the basis of the Japanese coefficients and some industrial census data on capital stock and partial estimates of coefficients for some countries of south-eastern Asia.

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Sectoral distribution of development aid and division of labour between developing and developed countries

Dieter Schumacher *

Scope and methodology

The object of the study[1] is to analyse the increase in the efficiency of foreign aid with regard to certain development objectives by taking account of additional division of labour between developed and developing countries and to determine the impact thereof on the pattern of foreign trade and production. The analysis is based on a two-phase linear programming approach, a model for an individual open economy, and a model linking a developing and a developed country via trade flows. Numerical results of a 40-industry version of the model were computed, using empirical data of production and demand functions in the Republic of Korea and the Federal Republic of Germany.

In this paper, after the description of the model and of the statistics used, the empirical results are discussed. The optimal patterns of investment and additional trade between the Republic of Korea and the Federal Republic of Germany are analysed with regard to the implied structural adjustment in the Federal Republic of Germany, the relationship between the alternative objective functions, and the range of substitution opportunities between development aid and additional division of labour which offer the same increase in employment or production. Finally, methods for improvement and extension of the analysis in order to permit a world-wide study of employment and production effects as a result of alternative development aid and foreign trade measure are indicated.

Description of the model

Symbols used

Unless otherwise specified, the variables and coefficients used in this paper refer to the recipient country (RC); reference to the donor country (DC) is indicated by the superscript G.

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[1]The study was financially supported by the Deutsche Forschungsgemeinschaft; a detailed presentation of the results is given in [1].

The mathematics of the model

The economy of the developing country is divided into n industries $j = 1, \ldots, n$, each of which can be represented by a Leontief production function as producing one commodity or a product mix of a constant structure. The constraints described below must be observed.

For each commodity, additional supply via domestic production and (competitive) imports must be greater than or equal to additional intermediate plus additional final demand (commodity balances):

$$\Delta x_i + \Delta M_{Ki} \geqslant \sum_{j=1}^{n} a_{ij} \Delta x_j + \Delta C_i + \Delta G_i + \Delta R_i + \Delta E_i \tag{1}$$

where:

a_{ij} = input of commodity i per unit of gross output value of industry j;
ΔC_i = additional private consumption of commodity i;
ΔE_i = additional exports[2] of commodity i;
ΔG_i = additional public consumption of commodity i;
ΔM_{Ki} = additional competitive imports[2] of commodity i;
ΔR_i = additional fixed capital formation of commodity i;
$\Delta x_j, x_i$ = additional gross output value of industry j and i, respectively; $j, i = 1, \ldots, n$.

The transactions are valued at domestic market prices at factory (producers' prices). The commodity breakdown of additional final demand for public consumption and fixed capital formation is given as

$$\Delta G_i = g_i \Delta G; \qquad \sum_{i=1}^{n} g_i = 1 - m_{NG} - v_G \tag{2}$$

and

$$\Delta R_i = r_i \Delta R; \qquad \sum_{i=1}^{n} r_i = 1 - m_{NR} - v_R \tag{3}$$

where:

ΔG = total additional public consumption;
g_i = demand for commodity i per unit of public consumption;
m_{NG} = non-competitive imports per unit of public consumption;
m_{NR} = non-competitive imports per unit of fixed capital formation;
r_i = demand for commodity i per unit of fixed capital formation;
v_G = GDP per unit of public consumption;
v_R = GDP per unit of fixed capital formation.

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[2]Without the optimized additional division of labour between the RC and the DC.

In order to take account of limited price elasticity, additional private consumption, which is the most important component in final demand, is allowed to vary by 10 per cent around the values resulting from the given commodity structure:

$$0.9 c_i \Delta C \leqslant \Delta C_i \leqslant 1.1 c_i \Delta C;$$

$$\sum_{i=1}^{n} c_i = 1 - m_{NC} - v_C \tag{4}$$

where

ΔC = total additional private consumption;
c_i = demand for commodity i per unit of private consumption;
m_{NC} = non-competitive imports per unit of private consumption;
v_C = GDP per unit of private consumption.

Total induced private consumption (excluding non-competitive imports and other primary inputs) is defined as:

$$(1 - m_{NC} - v_C) \Delta C = \sum_{i=1}^{n} \Delta C_i \tag{5}$$

Additional exports are determined according to the given commodity structure as:

$$\Delta E_i = e_i \Delta E \tag{6}$$

adding up to a total of:

$$\Delta E = \sum_{i=1}^{n} \Delta E_i \tag{7}$$

where:

e_i = the share of commodity i in total exports;[2]
ΔE = total additional exports.[2]

By analogy, competitive imports are estimated according to the relation of imports to domestic production:

$$\Delta M_{Ki} = m_{Ki} \Delta x_i \tag{8}$$

with a total of:

$$\Delta M_K = \sum_{i=1}^{n} \Delta M_{Ki} \tag{9}$$

where:

ΔM_K = total additional competitive imports;[2]
Δm_{Ki} = competitive imports[2] of commodity i per unit of gross output value of industry $j = i$.

Non-competitive imports have also to be considered. These are determined as a function of the increase in intermediate and final demand according to the import coefficients as

$$\Delta M_N = \sum_{j=1}^{n} m_{Nj} \Delta x_j + m_{NC} \Delta C + m_{NG} \Delta G + m_{NR} \Delta R \qquad (10)$$

where:

ΔM_N = total additional non-competitive imports;
m_{Nj} = non-competitive imports per unit of gross output value of industry j.

Total imports add up to

$$\Delta M = \Delta M_K + \Delta M_N \qquad (11)$$

where ΔM is total additional imports.[2]

They require foreign-exchange expenditure of

$$\Delta \tilde{M} = \sum_{i=1}^{n} \pi_i \Delta M_{Ki} + \sum_{j=1}^{n} \tilde{m}_{Nj} \Delta x_j + \tilde{m}_{NC} \Delta C + \tilde{m}_{NG} \Delta G + \tilde{m}_{NR} \Delta R \qquad (12)$$

where:

$\Delta \tilde{M}$ = total foreign-exchange value of additional imports (c.i.f.);[2]
ΔR = total additional fixed capital formation;
$\tilde{m}_{Nj}$ = foreign-exchange value of non-competitive imports per unit of gross output value of industry j;
$\tilde{m}_{NC}$ = foreign-exchange value of non-competitive imports per unit of private consumption;
$\tilde{m}_{NG}$ = foreign-exchange value of non-competitive imports per unit of public consumption;
$\tilde{m}_{NR}$ = foreign-exchange value of non-competitive imports per unit of fixed capital formation.

The first term on the right-hand side represents the foreign-exchange value of competitive imports; the world market price π_i is estimated as domestic price minus tariff rate for the competitive imports of the respective commodity,[3] that is:

$$\pi_i = \frac{1}{1 + t_{Ki}}$$

where t_{Ki} is duty on competitive imports of commodity i per unit of foreign-exchange value.

The foreign-exchange income through exports valued at the same prices is defined by:

$$\Delta \tilde{E} = \sum_{i=1}^{n} \pi_i \Delta E_i \qquad (13)$$

where $\Delta \tilde{E}$ is total foreign-exchange value of additional exports (f.o.b.).[2]

Postulating a zero balance of payments effect of the additional production, such that additional export income equals induced import cost, the foreign-exchange constraint:

[3]This problem is discussed in [2], pp. 71–75.

$$\Delta\tilde{M} = \Delta\tilde{E} \tag{14}$$

must be satisfied. The increase in GDP is given as

$$\Delta Y = \sum_{j=1}^{n} v_j \Delta x_j + v_C \Delta C + v_G \Delta G + v_R \Delta R \tag{15}$$

where:

ΔY = additional GDP (production effect of the development aid);
v_j = GDP per unit of gross output value in industry j.

This comprises the GDP in the various industries and the direct payments of the final demand categories to domestic primary factors of production. Given the propensity to consume, the additional GDP induces additional demand for private and public consumption

$$\Delta C = c \Delta Y \tag{16}$$

and

$$\Delta G = g \Delta Y \tag{17}$$

where:

c = share of private consumption in the total GDP;
g = share of public consumption in the total GDP.

According to the definition of GDP by type of expenditure, the rest:

$$\Delta R = \Delta Y - \Delta C - \Delta G - (\Delta E - \Delta M) \tag{18}$$

is available for additional capital formation. The number of jobs created by the additional production is given by:

$$\Delta L = \sum_{j=1}^{n} l_j \Delta x_j + l_C \Delta C + l_G \Delta G \tag{19}$$

where:

ΔL = the number of additional jobs;
l_j = the number of jobs per unit of gross output value of industry j;
l_C = the number of jobs per unit of private consumption;
l_G = the number of jobs per unit of public consumption.

The requisite investment in the various industries is represented by:

$$\Delta K_j = k_j \Delta x_j \tag{20}$$

where:

Δk_j = investment in industry j (financed by development aid);
k_j = capital stock per unit of gross output value of industry j.

As the study concentrates on the sectoral distribution of investment to be financed by development aid, in this model total investment is not to exceed the development aid available, and the capital constraint must be satisfied. Thus:

$$\sum_{j=1}^{n} \Delta K_j \leqslant \bar{F} \qquad (21)$$

where $\bar{F}$ is development aid.

By maximizing, alternatively, the employment effect ΔL and the production effect ΔY subject to the constraints, an optimal distribution of development aid among industries is obtained upon the integration of the developing country into the world economy as described by constraints (6), (8) and (10).

In order to permit consideration of additional division of labour between recipient and donor country (within the limits of the additional production and beyond the trade relationships already existing), bilateral trade activities for the commodities $i = 1, \ldots, n'(n' \leqslant n)$ are introduced. Their levels Δz_i^{EG} and Δz_i^{GE} represent the additional export of commodity i from the recipient to the donor country and vice versa, valued at world market prices. The new supply and demand alternatives enlarge the commodity balances to

$$\Delta x_i + \Delta M_{Ki} + (1 + t_{Ki}) \Delta z_i^{GE} \geqslant$$

$$\geqslant \sum_{j=1}^{n} a_{ij} \Delta x_j + \Delta C_i + \Delta G_i + \Delta R_i + \Delta E_i + (1 + t_{Ki}) \Delta z_i^{EG} \qquad (1')$$

where:

Δz_i^{EG} = additional exports of commodity i from the RC to the DC (f.o.b. at world market prices);

Δz_i^{GE} = additional exports of commodity i from the DC to the RC (f.o.b. at world market prices).

The levels of the bilateral trade activities are expressed at domestic market prices, using tariff rates. The importing country has to defray the costs of transportation by foreign exchange. Thus, the enlarged foreign-exchange constraint is given by

$$\Delta \tilde{M} + \sum_{i=1}^{n'} d_i^{GE} \Delta z_i^{GE} = \Delta \bar{E} \qquad (14')$$

where d_i^{GE} is equal to d_i^{EG} and is transportation costs of commodity i in the trade between the RC and the DC per unit of f.o.b. value.

The definition of GDP by type of expenditure is:

$$\Delta R = \Delta Y - \Delta C - \Delta G - (\Delta E - \Delta M) - \sum_{i=1}^{n'} (1 + t_{Ki}) (\Delta z_i^{EG} - \Delta z_i^{GE}) \qquad (18')$$

In order to leave the balance of payments situation of the two countries untouched, the additional bilateral trade has to be balanced:

$$\sum_{i=1}^{n'} \Delta z_i^{EG} = \sum_{i=1}^{n'} \Delta z_i^{GE} \qquad (22)$$

The additional trade volume is given by

$$\Delta Z = \sum_{i=1}^{n'} (\Delta z_i^{EG} + \Delta z_i^{GE}) \tag{23}$$

where ΔZ is total value of the additional trade between the RC and the DC (f.o.b. at world market prices).

Important macro-economic aggregates in the developed country are not to be changed by the additional bilateral trade and the implied adjustment of production in an undesired direction. Therefore, restrictions are introduced so as to prevent such changes as against the situation without additional division of labour. The first restriction is that the final demand for the individual commodities also has to be met in the case of gross output values being adjusted to x_i^G:

$$x_i^G + (1 + t_{Ki}^G)\Delta z_i^{EG} - \sum_{j=1}^{n} a_{ij}^G x_j^G - (1 + t_{Ki}^G)\Delta z_i^{GE} - e_i^G \Delta E^G = \bar{N}_i^G \tag{24}$$

where:

x_j^G, x_i^G = adjusted gross output value of industry j and i, respectively, in the DC, taking account of additional trade with the RC;
ΔE^G = total additional exports of the DC;[2]
$\bar{N}_i^G$ = final demand for commodity i in the DC;
t_{Ki}^G = duty in the DC on imports of commodity i per unit of foreign-exchange value;
a_{ij}^G = input of commodity i per unit of gross output value of industry j in the DC;
e_i^G = the share of commodity i in total exports of the DC.[2]

Valuation is at producers' prices in the donor country.[4] The second restriction is that total GDP attain at least the prior value:

$$\sum_{j=1}^{n} v_j^G x_j^G \geqslant \bar{V}^G \tag{25}$$

where:

$\bar{V}^G$ = total GDP of industries in the DC;
v_j^G = GDP per unit of gross output value of industry j in DC.

The third restriction is that foreign-exchange expenditure (including cost of transporting additional imports from the developing country) must be less than or equal to the amount without the additional division of labour, i.e.

[4]In addition to the additional trade with the developing country an increase in exports (ΔE^G at the given commodity structure) is permitted to finance a possible increase in foreign-exchange expenditure; see (26).

$$\sum_{j=1}^{n} m_j^G x_j^G + \sum_{i=1}^{n'} d_i^{EG} \Delta z_i^{EG} - \Delta E^G \leqslant \bar{M}^G \qquad (26)$$

where:

$\bar{M}^G$ = total imports of industries in the DC;[5]
m_j^G = imports[2] per unit of gross output value of industry j in the DC.

The fourth restriction is that, by analogy, labour and capital input are not to exceed their former values, i. e.

$$\sum_{j=1}^{n} l_j^G x_j^G \leqslant \bar{L}^G \qquad (27)$$

and

$$\sum_{j=1}^{n} k_j^G x_j^G \leqslant \bar{K}^G \qquad (28)$$

where:

$\bar{L}^G$ = total number of jobs in the industries in the DC;
$\bar{K}^G$ = total capital stock in the industries in the DC;
l_j^G = number of jobs per unit of gross output value of industry j in the DC;
k_j^G = capital stock per unit of gross output value of industry j in the DC.

Once again, the endogenous variables are to be quantified by maximizing, alternatively, the increase in employment ΔL and in production ΔY in the developing country subject to the enlarged system of constraints (1′), (2) to (13), (14′), (15) to (17), (18′), (19) to (28). The solution now embraces optimal values not only for the sectoral distribution of development aid, but also for the value and structure of the additional trade between the two countries, as well as for the implied adjustment of production in the developed country.

Industry classifications

For the various calculations,[5] 40 industries have been defined according to the International Standard Industrial Classification (ISIS) (see annex), of which 26 are manufacturing industries. An additional dummy sector (41) contains all those transactions that cannot be allocated elsewhere. This residual sector will be ignored in the subsequent analysis. For the additional trade between the Republic of Korea and the Federal Republic of Germany, only manufacturing industry commodities are assumed to be tradeable (except those of "other manufacturing", the product mix of which varies too greatly in the two countries). Thus, trade activities for the 25 commodities i = 6, . . ., 30 are taken into account.

[5]All calculations were carried out by means of the program packages MPSX on an IBM 370-158 and APEX on a CD CYBER 7214.

The objective functions are limited to the industries 1 to 40. This is due to the non-availability of employment figures for the Republic of Korea for both the dummy industry (41) and for public consumption. The persons engaged in private consumption are contained in other private services (40) according to the treatment of their income in the I/O table of the Republic of Korea. Thus l_{41}, l_G, and l_C are equal to zero. By analogy, the discussion of the maximum production effect is based on additional GDP in industries 1 to 40. The tabulated results show the incremental GDP for the whole economy, some 6 per cent of which is due to sector 41 and the public consumption sector (v_C and v_R are equal to zero). Within the final demand induced in the Republic of Korea, rented dwellings (39) are omitted on account of the extremely high capital coefficient. In 1970, about 5 per cent of private consumption was expended on rented dwellings.

All values are expressed in deutsche mark (DM) at 1972 prices. The statistical data are taken from the Federal Republic of Germany and the Republic of Korea [3–10].

Optimal distributions of development aid and foreign trade

The numerical results given in tables 1 and 2 are calculated on a basis of DM 100 million development aid granted to the Republic of Korea for investment purposes.

Without the extra trade between the Republic of Korea and the Federal Republic of Germany, the industrial distribution of development aid maximizing the employment effect differs only slightly—within the assumed limits of flexibility for additional private consumption—from that maximizing the production effect. In both cases, 14.5 per cent of the funds must be invested in agriculture (1), more than 2 per cent in mining (2 to 5), about 20 per cent in manufacturing (6 to 31), 6.5 per cent in energy (32), nearly 2 per cent in construction (33), and more than 55 per cent in services (34 to 40). Within manufacturing, the largest amounts are more than DM 5 million for the textile industry (9) and about DM 2 million each in the chemicals (17), food (6), and petroleum products industries (18). The new production capacities offer about 26,000 jobs and permit a GDP of about DM 70 million in industries 1 to 40. Total additional GDP (including industry 41 as well as the public sector) amounts to DM 74 million, of which just under DM 56 million is spent on private consumption and DM 8 million on public consumption, leaving nearly DM 11 million for additional fixed capital formation. According to the given foreign trade relationships, nearly DM 18 million of imports are induced, of which two thirds are competitive and one third non-competitive imports. Valued at world market prices, they amount to DM 16.5 million; these outlays are financed by exports amounting to the same value of foreign exchange.

Taking account of additional division of labour within manufacturing between the Republic of Korea and the Federal Republic of Germany, aid can result in both higher employment and more production if the pattern of investment and additional bilateral trade are chosen appropriately. Figure 1

TABLE 1. OPTIMAL DISTRIBUTION OF INVESTMENT AND CHANGES IN MACRO-ECONOMIC AGGREGATES

(DM million)[a]

Classification number[b]	Industry	Without additional trade between the Republic of Korea and the Federal Republic of Germany[c]	With additional trade between the Republic of Korea and the Federal Republic of Germany					
			Maximizing the employment effect			Maximizing the production effect		
			Best solutions[d]		Second-best solution[e]	Best solutions[d]		Second-best solution[f]
			Trade volume of DM 30 million[g]	Trade volume of DM 78.2 million[g]	Trade volume of DM 53.9 million[g]	Trade volume of DM 20 million[g]	Trade volume of DM 52.6 million[g]	Trade volume of DM 52.3 million[g]
Distribution of investment by industry								
1	Agriculture, forestry and fishing	14.5	19.2	20.5	16.8	17.0	20.5	16.4
2	Coal mining and products	0.7	0.7	0.7	0.8	0.7	0.7	0.7
3	Petroleum extraction	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	Metal ore, salt, other mining	0.3	0.2	0.2	0.2	0.2	0.2	0.2
5	Quarrying and production of building materials	1.3	1.2	1.0	1.3	1.2	1.2	1.2
2–5	Mining and quarrying	2.2	2.1	1.9	2.2	2.2	2.1	2.1
6	Food	2.3	6.9	8.7	3.4	3.7	6.3	3.3
7	Beverages	0.6	0.6	–	1.4	1.6	2.6	0.6
8	Tobacco	0.4	0.4	–	0.1	1.0	2.0	0.4
9	Textiles	5.3	2.5	–	–	2.5	–	–
10	Wearing apparel (excluding footwear)	0.6	0.6	3.3	0.6	0.6	–	–
11	Leather and leather products (including footwear)	0.1	0.1	–	6.4	0.1	–	5.8
12	Sawmills and wood processing	0.4	0.4	–	1.0	0.4	–	3.0
13	Wood products	0.1	0.1	–	1.5	0.1	–	–
14	Pulp and paper	0.8	–	–	–	–	–	–
15	Paper products	0.1	0.1	–	–	0.1	–	–
16	Printing and publishing	0.4	0.4	–	–	0.4	–	–

17	Chemicals	2.3	–	–	–	2.1	–	–
18	Petroleum products	1.8	–	–	–	–	–	1.9
19	Rubber products	0.5	–	–	–	–	–	–
20	Plastic products	0.5	–	–	–	0.5	–	–
21	Fine ceramic	0.0	0.0	–	–	0.0	–	–
22	Glass and glass products	0.1	–	–	–	0.3	–	–
23	Iron and steel	1.0	–	–	–	–	–	–
24	Non-ferrous metals	0.2	–	–	–	0.2	–	–
25	Metal products	0.5	–	–	–	–	–	–
26	Non-electrical machinery	0.3	0.3	–	–	0.3	–	–
27	Electrical machinery	0.3	0.3	–	–	0.3	–	–
28	Vehicles	0.4	0.4	–	–	0.4	–	–
29	Shipbuilding, aeroplanes	0.1	0.1	–	–	–	–	–
30	Precision engineering, optical goods	0.1	0.1	–	–	0.1	–	–
31	Other manufacturing	0.6	0.6	0.5	0.5	0.6	0.5	0.6
6–31	Manufacturing	20.0	13.8	12.5	14.9	15.3	11.4	15.7
32	Electricity, gas, water	6.5	5.8	5.8	5.3	6.0	5.3	5.4
33	Construction	1.6	1.7	1.4	1.7	1.6	1.7	1.6
34	Trade	8.1	8.7	9.4	9.3	8.5	8.6	8.9
35	Railway transport	7.0	7.2	7.1	7.4	7.3	7.5	7.2
36	Other transport and storage	13.6	13.5	12.7	13.5	13.7	13.1	14.2
37	Communication	2.0	2.1	2.2	2.1	2.1	2.1	2.1
38	Finance and insurance	0.9	0.8	0.8	0.9	0.8	0.8	0.9
39	Rented dwellings	0.1	0.1	0.1	0.1	0.1	0.2	0.1
40	Other (private) services	23.5	25.1	25.5	25.8	25.3	26.8	25.4
34–40	Services	55.2	57.4	57.9	59.2	57.9	59.0	58.8
	Total	100.0	100.0	100.0	100.0	100.0	100.0	100.0
	Changes in macro-economic aggregates							
	Gross output	133.7	143.5	155.2	150.4	141.4	146.3	147.1
	GDP at market prices	74.4	81.7	84.7	83.8	81.8	89.5	81.2
	Private consumption	55.8	61.3	63.6	62.9	61.4	67.2	61.0
	Public consumption	8.0	8.8	9.1	9.0	8.8	9.6	8.8
	Fixed capital formation	10.7	11.3	9.0	11.7	11.1	12.1	10.4
	Exports to Fed. Rep. of Germany[h]	–	16.3	44.0	29.1	11.2	29.4	29.2

TABLE 1 *(continued)*

Classification number[b]	Industry	Without additional trade between the Republic of Korea and the Federal Republic of Germany[c]	With additional trade between the Republic of Korea and the Federal Republic of Germany					
			Maximizing the employment effect			Maximizing the production effect		
			Best solutions[d]		Second-best solution[e]	Best solutions[d]		Second-best solution[f]
			Trade volume of DM 30 million[g]	Trade volume of DM 78.2 million[g]	Trade volume of DM 53.9 million[g]	Trade volume of DM 20 million[g]	Trade volume of DM 52.6 million[g]	Trade volume of DM 52.3 million[g]
	Imports from Fed. Rep. of Germany[h]	–	17.1	42.9	30.4	11.3	30.2	29.3
	Exports[i]	17.7	14.1	10.3	11.8	14.4	9.3	17.9
	Imports[i]	17.9	13.0	8.4	10.3	13.8	7.8	16.6
	Competitive imports[j]	11.6	9.5	6.5	6.1	9.6	6.0	5.7
	Non-competitive imports[j]	6.3	3.5	1.9	4.3	4.2	1.8	10.9
	Foreign-exchange income[i,j]	16.5	13.1	9.5	11.0	13.4	8.6	16.6
	Foreign-exchange expenditure[j]	16.5	12.1	7.8	9.8	12.6	7.3	15.8
	Number of jobs (thousands)	26.1	31.1	34.2	31.0	28.7	31.5	29.2

[a]At prices of the Republic of Korea: owing to rounding errors total values may differ from the sum of the individual values.
[b]See annex.
[c]Maximizing the employment effect, this solution differs only slightly from the solution maximizing the production effect.
[d]Bilateral trade activities for all manufacturing industries (6–30).
[e]Bilateral trade activities for manufacturing industries (6–30) except industries 6 and 10.
[f]Bilateral trade activities for manufacturing industries (6–30) except industries 6 to 8.
[g]Valued at world market prices (f.o.b.).
[h]Additional trade between the Republic of Korea and the Federal Republic of Germany, optimized.
[i]Except the optimized additional trade between the Republic of Korea and the Federal Republic of Germany.
[j]The difference between income and outlays shown here represents the transportation costs of additional Korean imports from the Federal Republic of Germany.

TABLE 2. OPTIMAL SOLUTIONS FOR ADDITIONAL TRADE BETWEEN THE REPUBLIC OF KOREA AND THE FEDERAL REPUBLIC OF GERMANY

(DM million)[a]

Classification number[b]	Industry	*Maximizing the employment effect*			*Maximizing the production effect*		
		Best solutions[c]		*Second-best solution*[d]	*Best solutions*[c]		*Second-best solution*[e]
		Trade volume of DM 30 million	*Trade volume of DM 78.2 million*	*Trade volume of DM 53.9 million*	*Trade volume of DM 20 million*	*Trade volume of DM 52.6 million*	*Trade volume of DM 52.3 million*
	Exports to the Federal Republic of Germany						
6	Food	15.0	21.5	–	3.7	11.5	–
7	Beverages	–	–	3.5	4.0	8.1	–
8	Tobacco	–	–	–	2.3	6.7	–
9	Textiles	–	–	–	–	–	–
10	Wearing apparel (excluding footwear)	–	17.6	–	–	–	–
11	Leather and leather products (including footwear)	–	–	17.2	–	–	15.7
12	Sawmills and wood processing	–	–	1.6	–	–	10.4
13	Wood products	–	–	4.8	–	–	–
14	Pulp and paper	–	–	–	–	–	–
15	Paper products	–	–	–	–	–	–
16	Printing and publishing	–	–	–	–	–	–
17	Chemicals	–	–	–	–	–	–
18	Petroleum products	–	–	–	–	–	–
19	Rubber products	–	–	–	–	–	–
20	Plastic products	–	–	–	–	–	–
21	Fine ceramic	–	–	–	–	–	–
22	Glass and glass products	–	–	–	–	–	–
23	Iron and steel	–	–	–	–	–	–
24	Non-ferrous metals	–	–	–	–	–	–
25	Metal products	–	–	–	–	–	–
26	Non-electrical machinery	–	–	–	–	–	–
27	Electrical machinery	–	–	–	–	–	–
28	Vehicles	–	–	–	–	–	–
29	Shipbuilding, aeroplanes	–	–	–	–	–	–
30	Precision engineering, optical goods	–	–	–	–	–	–

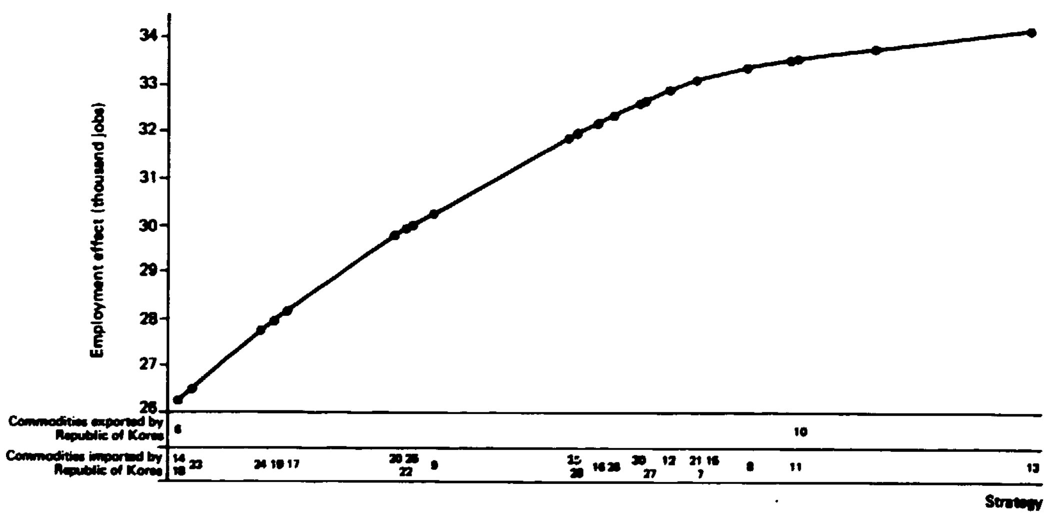
TABLE 2 *(continued)*

Classification number[b]	Industry	*Maximizing the employment effect* — *Best solutions*[c] — *Trade volume of DM 30 million*	*Maximizing the employment effect* — *Best solutions*[c] — *Trade volume of DM 78.2 million*	*Maximizing the employment effect* — *Second-best solution*[d] — *Trade volume of DM 53.9 million*	*Maximizing the production effect* — *Best solutions*[c] — *Trade volume of DM 20 million*	*Maximizing the production effect* — *Best solutions*[c] — *Trade volume of DM 52.6 million*	*Maximizing the production effect* — *Second-best solution*[e] — *Trade volume of DM 52.3 million*
Imports from the Federal Republic of Germany							
6	Food	–	–	–	–	–	–
7	Beverages	–	2.4	–	–	–	–
8	Tobacco	–	2.0	1.5	–	–	–
9	Textiles	3.6	18.0	7.1	3.5	4.1	5.6
10	Wearing apparel (excluding footwear)	–	–	–	–	3.4	4.0
11	Leather and leather products (including footwear)	–	0.5	–	–	0.3	–
12	Sawmills and wood processing	–	1.0	–	–	1.1	–
13	Wood products	–	0.3	–	–	0.5	0.3
14	Pulp and paper	0.7	0.4	0.3	0.9	0.5	0.3
15	Paper products	–	0.2	0.2	–	0.3	0.2
16	Printing and publishing	–	1.0	1.1	–	1.1	1.0
17	Chemicals	4.4	3.7	3.8	–	3.4	4.0
18	Petroleum products	1.8	1.7	1.8	2.0	1.7	–
19	Rubber products	0.7	0.6	1.3	0.7	0.6	1.3
20	Plastic products	0.7	0.8	1.6	–	0.7	1.5
21	Fine ceramic	–	0.1	0.1	–	0.1	0.1
22	Glass and glass products	0.2	0.1	0.4	–	0.6	0.2
23	Iron and steel	1.2	0.7	0.9	1.2	0.9	1.0
24	Non-ferrous metals	0.4	0.1	0.2	–	0.1	0.2
25	Metal products	1.3	1.1	1.5	1.3	1.3	1.4
26	Non-electrical machinery	–	1.4	1.7	–	1.7	1.6
27	Electrical machinery	–	1.4	1.6	–	1.6	1.8
28	Vehicles	–	0.9	1.0	–	1.0	1.0
29	Shipbuilding, aeroplanes	–	0.4	0.5	0.5	0.5	0.5
30	Precision engineering, optical goods	–	0.3	0.3	–	0.3	0.3

[a] At world market prices (f.o.b.).
[b] See annex.
[c] Bilateral trade activities for manufacturing industries (6–30).
[d] Bilateral trade activities for manufacturing industries (6–30) except industries 6 and 10.
[e] Bilateral trade activities for manufacturing industries (6–30) except industries 6 to 8.

shows the employment effect of those trade strategies chosen to maximize employment. Starting with the additional division of labour, the employment effect is increased most by exporting food (6) from the Republic of Korea to the Federal Republic of Germany and pulp and paper (14) as well as petroleum products (19) from the Federal Republic of Germany to the Republic of Korea. Trade extension permits further, albeit diminishing, benefits which are expressed by the shadow price of the trade volume. As the figure shows, the next most significant commodities are iron and steel (23), non-ferrous metals (24), rubber products (19), chemicals (17), plastic products (20), glass (22), and metal products (25), all of which are imported by the Republic of Korea. If trade is extended still further, wearing apparel (10) should also be exported, whereas textiles (9) should be imported. With a volume of DM 78 million, the additional trade comprises all manufactured goods (6 to 30). In the Republic of Korea new manufacturing capacities should be concentrated in the food and wearing apparel sectors (6 and 10). Thus, the employment effect attainable with an investment of DM 100 million achieves its maximum: more than 34,000 jobs or 30 per cent above the effect without additional division of labour.

Figure 1. Employment effect of trade strategies chosen to maximize employment[a]

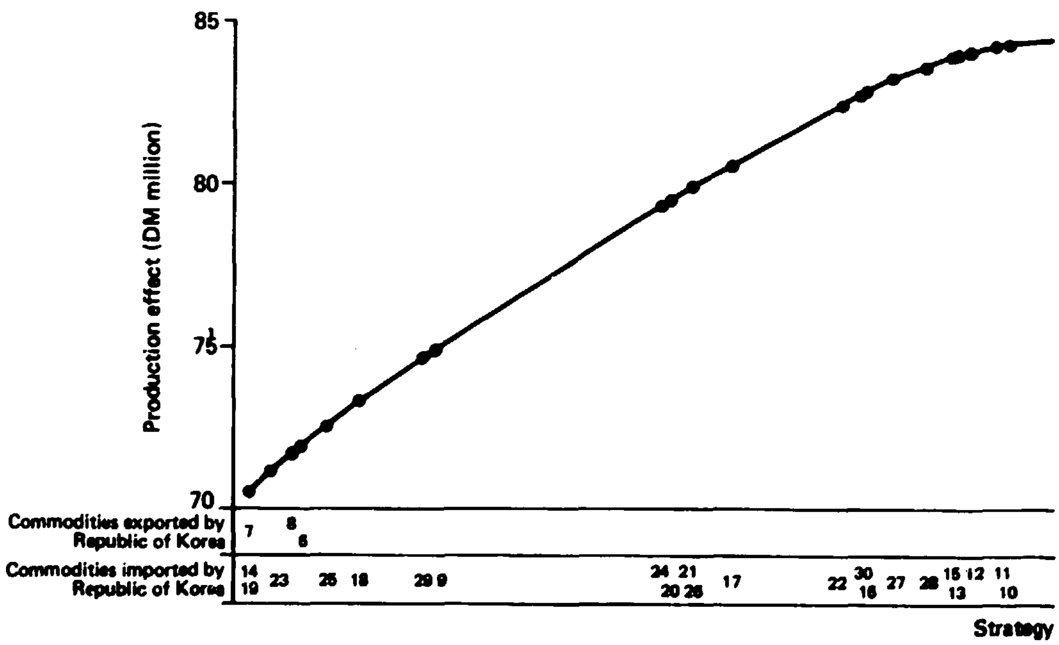


[a]Refer to annex for key.

Because of the linearity of the approach, the solution concentrates on as few export goods as possible for the Republic of Korea. In order to reduce this effect, it is reasonable to identify the "second-best" goods which should be exported so as to maximize the employment effect without taking into account food and wearing apparel within the additional trade between the Republic of Korea and the Federal Republic of Germany. These commodities are leather and leather products (11), wood products (13), beverages (7), and products of sawmills and wood processing (12).[6] The other manufactured goods should be imported. The volume of additional bilateral trade amounts to DM 54 million and 31,000 new jobs are created, that is, the employment effect is about 19 per cent above the original level.

If the production effect is to be maximized (figure 2) the additional bilateral trade should at first comprise beverages (7) as exports from the Republic of Korea, and pulp and paper (14) as well as rubber products (19) as imports. The division of labour should then be extended to include tobacco (8) and food (6) on the export side, and iron and steel (23), metal products (25), petroleum products (18), ships and aeroplanes (29), and textiles (9) on the import side. The maximum production effect is achieved at an additional trade level of DM 53 million and amounts to DM 84 million, more than 20 per cent above the value given the structure of foreign trade of the Republic of Korea. Within manufacturing, investment should now be concentrated in food (6), beverages (7) and tobacco (8). If these sectors are ignored within the additional bilateral trade, the "second-best" export goods of the Republic of Korea with regard to the production effect are leather and leather products (11) and products of sawmills and wood processing (12). This strategy yields a volume of additional bilateral trade amounting to DM 52 million. The production effect is decreased to DM 76.5 million, but it is still more than 9 per cent up on the initial effect.

Figure 2. Production effect of trade strategies chosen to maximize production[a]



[a]Refer to annex for key.

[6]In figure 1 these sectors are placed at the end on the import side. This means that their import is reasonable if exchanged against the "best" export goods (food and wearing apparel), but yields only small net increases in employment.

Structural adjustment in developed countries

An optimal pattern of investment in the developing countries (according to efficient specialization of production) must be supplemented by appropriate trade and structural adjustment measures on the part of the developed countries. The removal of existing trade barriers will impair the competitiveness of several industries in developed countries; on the other hand, other industries will be able to enlarge their production.

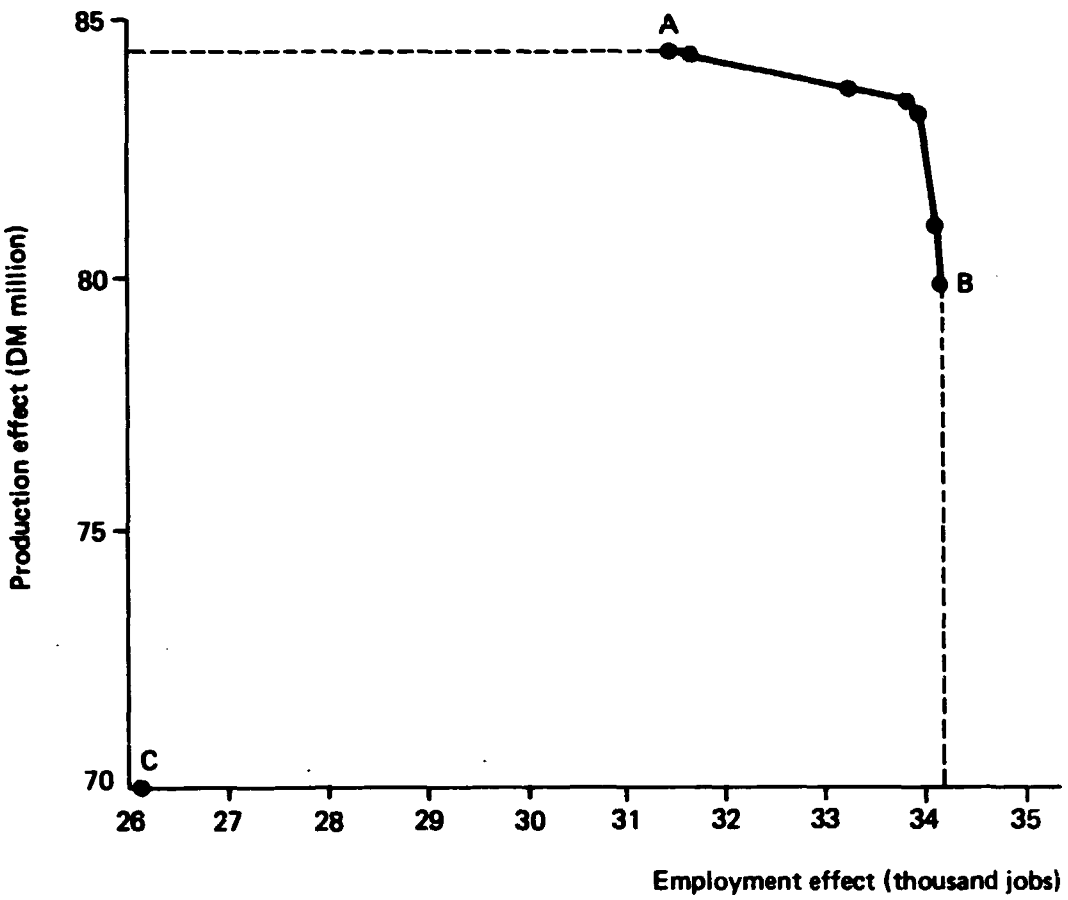
In the model calculations, the implied structural changes in the Federal Republic of Germany are small because of the small amount of investment taken into consideration. Nevertheless, tendencies can be identified. Apart from the industries, the commodities of which are additionally traded between the Republic of Korea and the Federal Republic of Germany, other industries of the Federal Republic of Germany are affected by inter-industry linkages. For example, in all cases both agricultural production (1) and food production (6) decrease even if these commodities are not additionally imported from the Republic of Korea. On the other hand, in all cases the output of mining (2 to 5), energy (32) and the tertiary sectors (34 to 40) increases. The production of non-ferrous metals (24), non-electrical machinery (26), electrical machinery (27), vehicles (28), ships and aeroplanes (29) as well as precision and optical goods (30) increases, even if no additional exports thereof are made to the Republic of Korea. The changes in the production structure in general result in a higher gross production in the Federal Republic of Germany.

Trade-off between production and employment objectives

Choosing the pattern of additional bilateral trade with regard to the production (or employment) objective yields an increase in employment (or respectively production) effect, compared with the situation prior to this model calculation, if only two thirds (or in the case of an employment objective, three quarters) of the maximum possible increase. With appropriate restructuring, about 2,700 new jobs can be created at the expense of DM 4.5 million of production effect. This means a product loss of DM 1.7 million to attain 1,000 extra jobs.

The conflict between the two objectives is illustrated in figure 3 by a transformation curve which shows the trade-off between production and employment effect. Point A at the uppermost end of the curve is that combination of additional production and employment achieved by maximizing the production effect only. Movement along the curve from A to B represents a trade-off between employment and production. Point B at the lower end of the curve is that combination achieved by maximizing the employment effect only. By way of comparison, figure 3 also shows the combination of the employment and GDP effects which result without considering additional division of labour between the Republic of Korea and the Federal Republic of Germany (point C).

Figure 3. Trade-off between production and employment effects



Substitution of trade for development aid

The increase in the positive effects of development aid arising from choosing an appropriate pattern of investment and additional division of labour permits the achievement—within certain limits—of given employment and production targets with different combinations of aid and additional trade. The feasible alternatives can be demonstrated in a system of isoquants. In figure 4 A, the two isoquants show the volume of additional trade between the Republic of Korea and the Federal Republic of Germany necessary at alternative levels of development aid in order to reach 26,200 and 30,000 jobs, respectively. Development aid of DM 100 million creates 26,200 jobs if the additional trade amounts to only DM 0.6 million (point A). Increasing the volume of trade permits a reduction in development aid at an increasing rate of substitution, measured by the increase in additional trade per unit of decrease in development aid. At the most, aid can be reduced by nearly one quarter to DM 77 million, provided additional trade is expanded to DM 60 million (point B). Further substitution is curtailed by constraints. If 30,000 jobs are to be created, the upper and lower points are determined by the combination of development aid of DM 100 million and additional trade of DM 22 million on the one side (point C) and development aid of DM 88 million and additional trade of DM 70 million on the other (point D).

The two curves in figure 4B in respect of production targets of DM 70.2 million and DM 80 million, respectively, can be interpreted in a similar fashion.

If the funds available for development aid are not sufficient to achieve the postulated employment and production targets in a developing country, it is possible to close the gap by trade expansion. At the global level, even if increased considerably, development aid alone will remain too limited, even in the long run, to help create the number of jobs sufficient to eliminate unemployment and to guarantee a per capita product sufficient to meet the basic needs of the bulk of the people. Expansion and the restructuring of trade between developing and developed countries must be added, especially with due consideration given to the more advanced developing countries. Grants should be concentrated more on the least developed countries. In the context of such a strategy, the rate of substitution shown by the isoquants in figure 4 (the slope of the curves) can be taken as a measure for the equivalence of development aid and trade expansion.

Further research

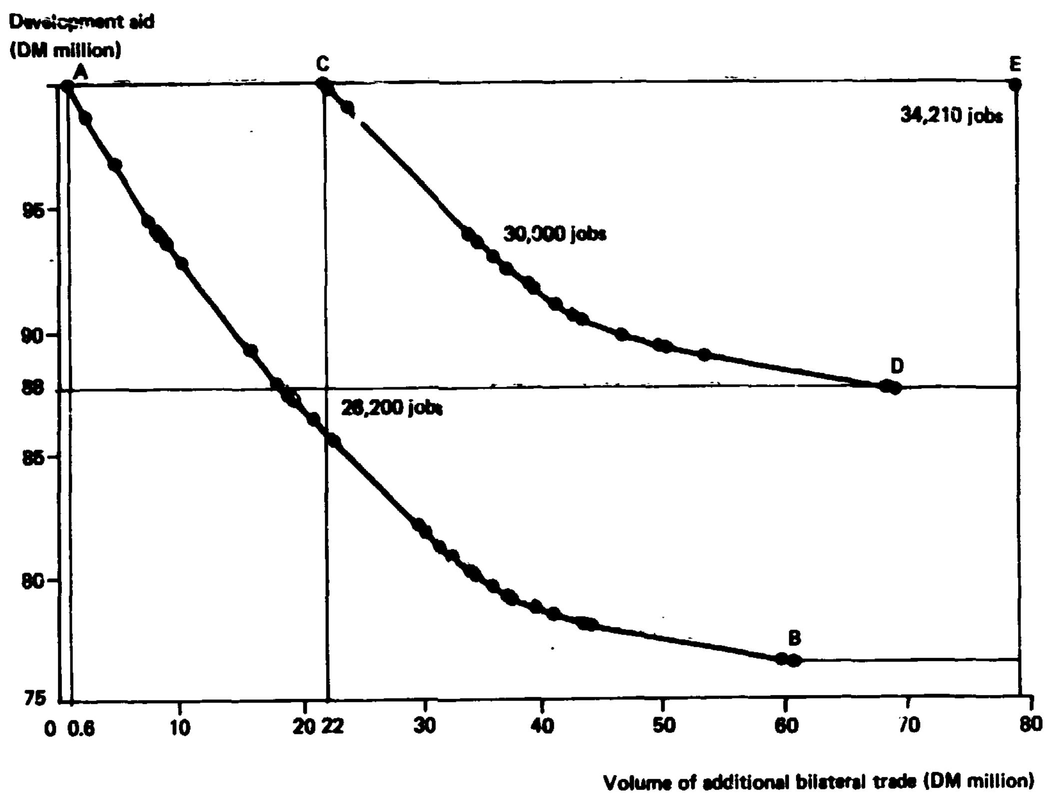
The model calculations presented above show that considerable net benefits can accrue to developing countries from international division of labour, trade and investment patterns being chosen appropriately. Developed countries can, thus, contribute significantly to the achievement of development targets by granting entry to additional imports from developing countries which in turn spend the foreign exchange earnings on imports from developed countries.

In the study, the problem of a favourable integration of developing countries into the world market was treated in a partial analysis from the point of view of an individual country. A policy based on such results is reasonable in so far as it results only in small changes in the world economy. In order to avoid the implied *ceteris paribus* logic, one has to analyse simultaneously a variety of national economies linked by foreign trade flows.

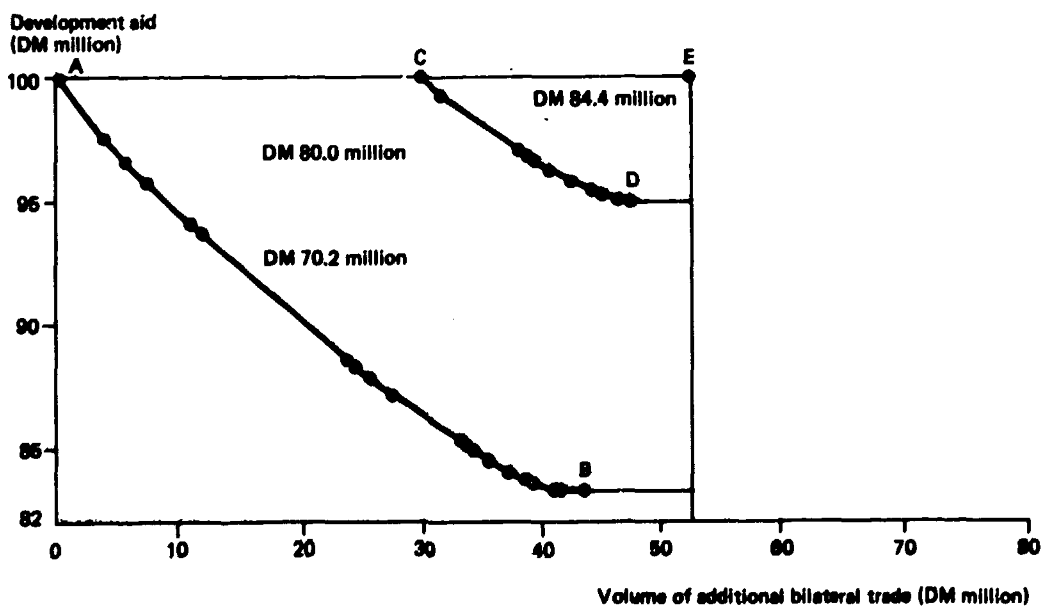
On the one hand, one could consider developing countries of certain regions to determine an efficient pattern of specialization in the framework of reinforced economic integration. Such an analysis should also discuss the distribution of benefits resulting from the increased division of labour among the trade partners. One might also use such an approach for a world-wide study. Thus, magnitudes of the total increase of production and employment in the developing world could be calculated under alternative conditions (volume of the resource transfer, volume and structure of the international division of labour) and the implied structural adjustment in developed countries would be determined simultaneously. The problem is enlarged considerably if many countries are taken into

Figure 4. Substitution of trade for development aid

A. For a given employment effect



B. For a given production effect



account. Moreover, time has to be considered and a certain minimum level of industry disaggregation observed. With regard to the limits arising from the availability of data and their computerized processing, it is advisable first to consider the countries aggregated to a few groups and thereafter to disaggregate on an individual country basis. Further empirical research along these lines is intended.

ANNEX

DEFINITION OF INDUSTRIES

No.	*Industry*	*ISIC No.*[a]
1	Agriculture, forestry and fishing	11, 12, 13
2	Coal mining and products	21, 354
3	Petroleum extraction	22
4	Metal ore, salt and other mining	23, 29 (excl. 2901)
5	Quarrying and production of building materials	2901, 369
6	Food	311, 312
7	Beverages	313
8	Tobacco	314
9	Textiles	321
10	Wearing apparel (excl. footwear)	322
11	Leather and leather products (incl. footwear)	323, 324
12	Sawmills and wood processing	3311
13	Wood products	331 (excl. 3311), 332
14	Pulp and paper	3411
15	Paper products	341 (excl. 3411)
16	Printing and publishing	342
17	Chemicals	351, 352
18	Petroleum products	353
19	Rubber products	355
20	Plastic products	356
21	Fine ceramic	361
22	Glass and glass products	362
23	Iron and steel	371
24	Non-ferrous metals	372
25	Metal products	381, 3842
26	Non-electrical machinery	382
27	Electrical machinery	383
28	Vehicles	3843, 3844, 3849
29	Shipbuilding, aeroplanes	3841, 3845
30	Precision engineering, optical goods	385
31	Other manufacturing	39
32	Electricity, gas, water	41, 42
33	Construction	50
34	Trade	61, 62
35	Railway transport	7111
36	Other transport and storage	71 (excl. 7111)

No.	*Industry*	*ISIC No.*[a]
37	Communication	72
38	Finance and insurance	81, 82
39	Rented dwellings	831
40	Other (private) services	63, 832, 9[b]
41	Not elsewhere classified	0

[a]Of the divisions or groups corresponding to the industry defined. See *International Standard Industrial Classification of All Economic Activities*, Statistical Papers, series M, No. 4, Rev. 2 (United Nations publication, Sales No. E.68.XVII.8).

[b]Except repair services (ISIC major group 951), which are included in the respective production activities, and except public services.

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The size effect on Leontief-Strout gravity coefficient and derivation of quantitative implications of inter-regional co-operation among developing countries

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The importance of space to economic growth is again gaining theoretical recognition. Of course, in the 19th century international trade was considered the main harbinger of economic growth. For instance, Marshall [1] has written "The causes which determine the economic progress of Nations belong to the study of international trade". It was the emergence of "economic development" as a new subject that has shifted the emphasis to capital accumulation and foreign aid lately. The subject of interregional economics mainly bypassed the aspect of capital accumulation and mainly concerned itself with the movement of commodities and services between separate geographic areas.

However, Professor Leontief wrote "The truly comprehensive theory would consider the spatial distribution of economic activities to be an integral aspect of a single system of economic relationships" [2]. There have been quite a few attempts to delineate the growth of internal economic structure starting from a dynamic I/O model; however, hardly any study is concerned with combining it with the changing pattern of interregional trade.

Using the framework of I/O analysis, two types of regional location models have been developed. The first of these depends on the combination of I/O and linear programming models. The location of various industries is determined in such a way that the desired availability of goods is maximized for given factors of production and their spatial distribution. According to this model a commodity can be moved uni-directionally only. However, the intra-industry trade between a given pair of regions (that is those having the same classification in international trade statistics) gives eloquent testimony against universal application of this model. One reason for this is a high degree of aggregation of individual products under a general term. Sometimes differing qualities of commodities may also play their part in this. It is almost impossible to classify commodities in such detail that all qualities will be identifiable. A further consideration is that the other comfortable assumptions of I/O analysis, such as no joint production and consumption, become more and more unrealistic. With existing I/O tables classified by 50 to 200 industries, it is no wonder that such an approach does not lead to a faithful description of spatial distribution of economic activities.

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The second type of model is topographical in character and skirts this difficulty by assuming gravity-type relations, between interregional commodity flows, which may be directly proportional to output or demand of the commodity and inversely proportional to, for example, the total output of the commodity in the economy as a whole. At the Third International Input-Output Conference held in Geneva in 1961, Leontief and Strout [3] presented a form of the gravity-trade model which could be readily applied to a multi-regional I/O analysis. This model has been widely used since then in different parts of the world and has been found to predict interregional trade with sufficient reliability. The gravity coefficient of trade of a particular commodity between two regions is assumed constant. No attempt has so far been made to find out how this gravity coefficient is affected by changes in the conditions of trade or market agreements between different regions and any other such dynamic phenomenon. One cannot use this model in its original form to find out the effect of policy decisions on interregional or international trade, unless the laws affecting the Leontief–Strout gravity coefficients are elucidated. Such models should be very useful under conditions where a commodity is defined as an aggregate of several similar but not strictly identical goods. The two regions will often represent more or less extended areas. Thus the average distance between them might conceal the actual diversity of commodity flows connecting a great many distinct pairs of sending and receiving points [2]. Under conditions of homogeneous commodities and defined points of despatch and destination, the linear programming model may be able to capture the changing economic realities faithfully.

In the following, an attempt is made to analyse the effect of market size on the gravity coefficients. This gives an estimation of the gravity coefficient for any projected combinations of economies in a union. This extended Leontief–Strout gravity model can further be combined with a linear programming framework for agricultural and mining industries to estimate the effects of various international agreements on trade patterns.

Gravity coefficients

Basic gravity coefficients

The Leontief–Strout model can be written as:

$$X_{i \cdot gh} = \frac{X_{i \cdot go} X_{i \cdot oh}}{X_{i \cdot oo}} \cdot Q_{i \cdot gh}$$

$$i = 1 \dots n$$

$$g = 1 \dots m$$

$$h = 1 \dots m \qquad (1)$$

where:

$X_{i \cdot gh}$ = the flow of the i^{th} commodity from region g to region h;
$X_{i \cdot go}$ = the total production of the i^{th} commodity in region g;

$X_{i \cdot oh}$ = total demand of commodity i in the region h;
$X_{i \cdot oo}$ = total production of commodity i in all the regions;
$Q_{i \cdot gh}$ = the gravity constant.

It will be seen that:

$$\sum_g \sum_h X_{i \cdot gh} = \sum_g X_{i \cdot go} = \sum_h X_{i \cdot oh} = X_{i \cdot oo} \tag{2}$$

The flow of the particular commodity i from the region g to any other region h is assumed to be directly proportional to the fraction of the total global output of commodity i produced in region g, as well as to its total demand in region h. The coefficients $Q_{i \cdot gh}$ are empirically determined constants which can be termed the Leontief–Strout gravity constants of the model.

The gravity constant $Q_{i \cdot gh}$ should depend on the distance between the two points, transport peculiarities of the commodity concerned and institutional factors relating to the trade of the commodity in the receiving and despatching regions. Further, the demand for imports of a commodity in a region depends on the extent of industrialization and the size of the market. If the regions are different countries where a trade across a border may sometimes be substantially more difficult than trade within the border, the later considerations may become crucial. Thus the gravity constant $Q_{i \cdot gh}$ would be inversely related to the size of the receiving region h. If the elasticity of $Q_{i \cdot gh}$ with respect to population (indicating the size) is given by $-n$ then:

$$Q_{i \cdot gh} = \mu_{i \cdot gh} p_h^{-n} \tag{3}$$

where:

p_h = the population of region h;
$\mu_{i \cdot gh}$ = the basic gravity coefficient (this is dependent on all the parameters, except size, which affect the flow of i^{th} commodity from region g to region h).

It will be seen below that in certain circumstances, when g represents developed countries as a whole and i manufactured goods, both $Q_{i \cdot gh}$ and $\mu_{i \cdot gh}$ can be taken as inverse indicators of the extent of the industrialization of the country h. This is mainly due to the fact that at present most of the manufactured goods imported by developing countries come from the developed countries. Transport cost and other considerations will affect $Q_{i \cdot gh}$ and $\mu_{i \cdot gh}$ significantly only when different developed countries are distinguished as effective alternative sources of supply.

Let A^h and Y^h be the input coefficient matrix and final demand vector for the country h and let A^{hd} and A^{hm} and Y^{hd} and Y^{hm} be their domestic and imported parts.

Then $A^{hd} + A^{hm} = A^h$

$$Y^{hd} + Y^{hm} = Y^h$$

If each industry consists of a simple homogeneous commodity only, then either that commodity is produced or not produced in h. One can presume that, on the whole, the transport and other anciliary costs of procuring a commodity from within a country are less than those of procuring it from outside. Then it can be said that either a_{ij}^{hd} is zero or a_{ij}^{hm} is zero and that only one of Y_i^{hd} and Y_i^{hm} is positive. In the real world all values are positive. This implies that each industry in I/O analysis consists of myriads of commodities.

Thus viewed, the proportion of a_{ij}^{hm} to a_{ij}^{h} and Y_i^{hm} to Y_i^{h} where i is a manufacturing industry, will describe the inverse of the industrialization of h. The higher these values, the lower is the industrialization of the region or country. The total imports by industry in h will be given by

$$M^h = Y^{hm} + A^{hm}(I - A^{hd})^{-1} Y^{hd} \tag{4}$$

and the value of manufacturing imports by

$$\sum_{i \epsilon s} p_i M_i^h \tag{5}$$

where p_i is the price of the i^{th} commodity.

The value of the Leontief–Strout gravity constant for manufacturing industry as a whole will be

$$Q_{i \cdot gh} = \frac{X_{i \cdot gh}}{X_{i \cdot oh}} \Big/ \frac{X_{i \cdot go}}{X_{i \cdot oo}} = \frac{\sum_{j \epsilon s} p_j M_j^h}{\sum_{j \epsilon s} p_j M_j^h + \sum_{j \epsilon s} p_j X_j^{hd}} \Big/ \frac{X_{i \cdot go}}{X_{i \cdot oo}} \tag{6}$$

where i represents the manufacturing as a whole.

If g represents developed countries, $\frac{X_{i \cdot go}}{X_{i \cdot oo}}$ is the proportion of manufacturing goods produced in developed countries out of the total. This proportion has been almost constant at about 91 per cent from 1960 onwards.

It is clear from the equation (6) that the higher the values of M_j, the higher will be $Q_{i \cdot gh}$ and from equation (4) that the higher the industrialization of the country, the lower will be the values of M_j, as higher industrialization implies a wider range of quality of manufactured goods that could successfully compete with imports with or without government help.

However, the extent of industrialization is affected by the size of the market. Thus, other things being equal, one can say that the larger the extent of industrialization, the lower will be the proportion of a_{ij}^{hm} to a_{ij}^{h} and Y_j^{hm} to Y_j^{h} as a larger market will not only provide the scope of economies of scale and better competitive environments, but also scope for the start of many new lines of production in each industry. In the aggregate, this will allow one to calculate n, the population elasticity of the Leontief–Strout gravity coefficient $Q_{i \cdot gh}$ and the basic gravity coefficient $\mu_{i \cdot gh}$, giving the basic industrial ethos of the country h as well as its transport and other industrial peculiarities.

This formulation enables one to work out the consequences of a projected unification of two or more different economies into one effective integrated market system. The trade pattern of this new unit will be determined by the new Leontief–Strout gravity coefficient of the union. This latter can be calculated by first estimating the "basic gravity coefficient" of the union and then using the total population of the union to estimate the Leontief–Strout coefficient by using equation (3). The "basic gravity coefficient" $\mu_{i\cdot gh}$ for the union may be calculated as a weighted mean of the basic gravity coefficients of the individual countries or regions forming this union, the weights being taken as proportional to the population. This weighting scheme represents an average industrial ethos, average transportation and communication costs, institutional conditions etc., of the union as a whole. However, with time, this joint basic gravity coefficient will converge to the one having the lowest value of all the constituents of the above.

The elasticity of $Q_{i\cdot gh}$ with respect to population is likely to be negative and less than one. This is because increasing market size should stimulate indigenous manufacturing industry. However, ordinarily it is not expected to stimulate so much as to reduce imports from the values before expansion started. As can easily be seen, this will ensure a quantitative gain from the union by reduction of the joint imports from that which would have been the joint imports of the countries separately. Thus, the larger integrated markets would absorb many new types of manufactured commodities which would not have been produced otherwise or would increase the competitiveness of the commodities already being produced. These new goods will be sufficiently similar to the older ones as to come in the same classification, but not sufficiently similar as to be produced before extension of the market. This gives a mechanism to describe quantitatively the effects of economic co-operation on international trade in as much disaggregation as the current statistical system would allow.

Basic gravity coefficients for trade in manufactured goods from developed to developing countries

The extent of imports of all manufactured goods from developed countries as a whole to 60 individual developing countries for which information is readily available is analysed below. With the help of the equation (1) above we have calculated the values of Leontief–Strout gravity coefficients, $Q_{i\cdot gh}$ for the years 1965, 1970 and 1975. For the year 1975, due to data problems, $Q_{i\cdot gh}$ could be calculated for only 43 countries. These are given in table 1. This table also gives the population and GNP for each country for the year 1970 to give an idea of their respective size and level of development.

From these the size elasticity of Leontief–Strout gravity coefficients have been estimated. The three estimated equations are given below:

$$(1965) \qquad \log Q_{i\cdot gh} = -\underset{(0.04)}{0.3978} - \underset{(0.02)}{0.1705} \log p_h$$

$$r = -0.57^{**}$$

TABLE 1. GRAVITY COEFFICIENTS FOR TRADE IN MANUFACTURES BETWEEN DEVELOPED AND DEVELOPING COUNTRIES

Country	*Population, 1970 (million)*	*GNP, 1970 (millions of dollars)*	*Leontief–Strout gravity coefficient $Q_{i \cdot gh}$*			*Basic gravity coefficient $\mu_{i \cdot gh}$*		
			1975	*1970*	*1965*	*1975*	*1970*	*1965*
Group 1 (semi-developed countries)								
Argentina	23.750	18 999.386	0.1784	0.1939	0.1670	0.2993	0.3322	0.2828
Brazil	92.520	9 292.928	0.2755	0.2198	0.1967	0.5820	0.4745	0.4152
Egypt	33.330	7 099.816	0.6332	0.3696	0.2991	1.1295	0.6708	0.5314
India	539.080	50 404.530	0.2465	0.2342	0.2838	0.6831	0.6832	0.8114
Mexico	50.690	33 522.796	–	0.2482	0.3242	–	0.4838	0.6102
Turkey	34.850	9 199.991	0.4033	0.2623	0.3638	0.7253	0.4799	0.6535
Group 2 (other Latin American)								
Barbados	0.240	144.950	0.9393	0.9640	0.9448	0.7524	0.7563	0.7413
Colombia	21.120	6 087.632	0.6390	0.4136	0.1860	0.6165	0.6947	0.3041
Dominican Republic	4.060	1 538.000	–	0.5294	0.4202	–	0.6712	0.5202
El Salvador	3.530	970.000	–	0.5571	0.6137	–	0.6903	0.7362
Guatemala	5.100	1 644.000	–	0.5220	0.5700	–	0.6886	0.7344
Honduras	2.510	668.000	0.7403	0.7623	0.6458	0.8844	0.8914	0.7373
Jamaica	1.870	1 197.648	0.7128	0.8511	0.6217	0.7983	0.9805	0.6844
Panama	1.430	924.000	–	0.7880	0.4848	–	0.8374	0.5022
Peru	13.590	5 738.997	–	0.4147	0.6588	–	0.6462	1.0001
Trinidad and Tobago	1.030	871.000	0.7397	0.6878	0.9436	0.7478	0.6913	0.9377

Group 3 (other Mediterranean — non-African)

Cyprus	0.620	537.317
Greece	8.790	9 599.990
Lebanon	2.470	1 463.385
Spain	33.780	33 390.504
Syrian Arab Republic	6.260	1 684.000
Yugoslavia	20.370	12 560.000

Group 4 (other Asian)

Burma	27.580	2 148.690
Fiji	0.520	200.505
Indonesia	118.800	9 032.353
Iran (Islamic Republic of)	28.660	11 669.949
Iraq	9.440	2 917.950
Jordan	2.300	623.075
Kuwait	0.740	2 080.650
Malaysia	10.390	3 759.546
Pakistan	60.69	9 842.928
Philippines	36.850	9 307.690
Republic of Korea	31.300	7 760.383
Saudi Arabia	7.740	3 022.222
Singapore	2.080	1 878.640
Sri Lanka	12.510	2 053.876
Thailand	36.220	6 009.518

Group 5 (other African)

Ethiopia	24.630	1 769.200
Ghana	8.630	2 034.496
Ivory Coast	4.300	1 398.581
Kenya	11.230	1 544.729
Libyan Arab Jamahiriya	1.990	2 811.537
Malawi	4.440	571.154
Mauritius	0.830	189.919

0.8757	0.8709	0.8772	0.8154	0.8029	0.8019
0.5876	0.5638	0.7060	0.8359	0.8158	1.0168
–	0.7793	0.8295	–	0.9087	0.9448
0.2989	0.4974	0.3895	0.5291	0.9048	0.6987
0.8257	0.5488	0.5830	1.1361	0.7496	0.7748
0.3732	0.4385	0.1894	0.6090	0.7320	0.3136
–	0.4811	0.5134	–	0.8455	0.8857
0.8490	0.7935	0.7511	0.7781	0.7100	0.6582
0.7356	0.5574	0.9237	1.6040	1.2557	1.9828
0.7325	0.4268	0.3073	1.2818	0.7550	0.5305
0.9210	0.6617	0.6291	1.3541	0.9692	0.8968
0.9843	0.8511	0.8526	1.1532	0.9805	0.9517
0.8857	0.9341	0.9081	0.8857	0.8875	0.8016
0.8484	0.7276	0.5792	1.2641	1.0632	0.6426
0.4746	0.4181	0.3312	0.9371	0.9452	0.7372
0.4640	0.4458	0.5299	0.8454	0.8231	0.9450
0.5763	0.5308	0.2858	1.0163	0.9531	0.5047
–	0.6218	0.7137	–	0.8805	0.9874
0.9185	0.9311	0.9929	1.0458	1.0545	1.1064
0.6368	0.6889	0.6100	0.9713	1.0585	0.9282
0.5419	0.5983	0.6801	0.9865	1.1014	1.2105
–	0.5866	0.6957	–	1.0113	1.1829
0.6443	0.6689	0.8300	0.9293	0.9649	1.1783
0.7199	0.7808	0.7336	0.9280	1.0005	0.9221
0.7569	0.7458	0.7753	1.1465	1.1251	1.2140
1.0168	0.9661	0.9389	1.1728	1.0360	1.0191
–	0.6893	0.6265	–	0.8881	0.7899
–	0.7649	0.7437	–	0.7410	0.7098

TABLE 1 *(continued)*

Country	Population, 1970 (million)	GNP, 1970 (millions of dollars)	Leontief–Strout gravity coefficient $Q_{i \cdot gh}$			Basic gravity coefficient $\mu_{i \cdot gh}$		
			1975	1970	1965	1975	1970	1965
Morocco	15.520	2 964.426	0.7279	0.6255	0.6134	1.1487	0.9970	0.9526
Nigeria	55.070	15 978.717	0.8411	0.7261	0.6347	1.6318	1.4156	1.2286
Senegal	3.920	851.000	–	0.7176	0.7155	–	0.9052	0.8849
Somalia	2.790	249.000	–	0.7552	–	–	0.8991	–
Sudan	15.690	2 024.698	0.7573	0.7610	0.6907	1.200	1.2152	1.0782
Togo	1.960	244.860	0.9135	0.7562	0.6993	1.0378	0.8479	0.7653
Tunisia	5.130	1 333.333	0.8626	0.7140	0.5563	1.1418	0.9428	0.7276
Uganda	9.810	1 301.974	–	0.5809	0.7763	–	0.8564	1.1187
United Republic of Cameroon	5.840	1 046.415	0.7963	0.7124	0.5228	1.0717	0.9616	0.6944
United Republic of Tanzania	13.270	1 209.016	0.7279	0.7636	0.7998	1.2250	1.1851	1.2145
Upper Volta	5.380	320.000	0.8394	0.6327	0.8126	1.1190	0.8422	1.0632
Zaire	21.690	1 654.600	0.7359	0.7551	–	1.2309	1.2740	–
Zambia	4.180	1 539.969	0.7166	0.7653	0.9957	0.9241	0.9760	1.2437

$$(1970) \quad \log Q_{i \cdot gh} = -0.725 - 0.1717 \log p_h$$
$$(0.05) \quad (0.045)$$
$$r = -0.70^{**}$$
$$(1975) \quad \log Q_{i \cdot gh} = -0.0285 - 0.1595 \log p_h$$
$$(0.037) \quad (0.03)$$
$$r = -0.63^{**}$$

where p_h is the population of the h^{th} country of the corresponding year.

From the above, size elasticities are found to have remained constant over the period covered by the study. They are also of a negative sign as expected from the hypothesis about the effect of market size stated earlier. This encourages us to take the quantitative magnitude for size elasticities of −0.17 as a norm of the size effect upon imports of manufactures under present techno-economic conditions. It implies that with increasing size of the economy the import matrix (A^{hm}) tends to become a smaller proportion of the matrix of total flows (A) for each element. The immense amount of data required to estimate this matrix element by element is currently not available for most countries. Meanwhile this elasticity coefficient should serve as a guide to changing Leontief–Strout coefficients with respect to population and through that to an estimation of trade.

Using the population elasticity of the Leontief–Strout gravity coefficient the values of the basic gravity coefficients for each country for each year were calculated. The following relationship has been used:

$$\mu_{i \cdot gh} = Q_{i \cdot gh} \, p_h^n \cdots \qquad (7)$$

where:

$\mu_{i \cdot gh}$ = the basic gravity coefficient;
$-n$ = the size elasticity.

The values for $\mu_{i \cdot gh}$ are also given in table 1 for the three years. The annex analyses inter-country and inter-temporal variations in $\mu_{i \cdot gh}$.

Use of basic gravity coefficient for analysing interregional co-operation

Due to complementarity, the rate of growth of developing countries now seems to be generally determined by the rate of growth of the developed world. As developed countries are facing a continuing slow-down of their growth rate, the outlook for the fast economic growth of the developing world seems to be bleak. To overcome this problem the developing countries have been planning a strategy of mutual co-operation so that they can themselves progressively provide the market for the manufacturing goods of each other. Pursuing this line, the world community set a target[1] that the proportion of the world's manufacturing goods

[1] Adopted by the Second General Conference of UNIDO, Lima, Peru, 12–26 March 1976.

produced in developing countries should grow at least to 25 per cent by the end of this century. At a conference held in Mexico in September 1976, it was declared that collective self-reliance for the developing countries was to be achieved by "fostering of greater co-operation amongst themselves, aimed at reinforcing their political and economic independence and their collective work strength in the fulfilment of the objectives in the new international economic order" [4].

The problem addressed here is how to quantify the gains probable from such an economic co-operation. Economic growth of the developing countries is dependent upon the foreign exchange that they can earn in the market of developed countries, which in its turn depends on the rate of growth of those countries themselves. However, if this dependence can be reduced by any means the same foreign exchange earnings will be more useful depending upon the extent of savings of imports of such a policy. Basic gravity coefficients as derived above and the size elasticity of imports give us a means to quantify the possible gains.

Suppose the countries h and h′ decide to co-operate in such a way that for an outsider they are as if one market. Let the joint market be designated by k. Then:

$$p_k = p_h + p_{h'} \tag{8}$$

and to the first approximation,

$$\mu_{i \cdot gk} = (p_h \mu_{i \cdot gh} + p_{h'} \mu_{i \cdot gh'}) \cdot p_k \tag{9}$$

Then:

$$Q_{i \cdot gk} = \mu_{i \cdot gh} p_k^{-n} \tag{10}$$

Giving from equation (1)

$$X_{i \cdot gk} = \frac{X_{i \cdot go} X_{i \cdot ok} Q_{i \cdot gk}}{X_{i \cdot oo}} \tag{11}$$

We can see that for the combined market, the Leontief–Strout gravity coefficient will be less than the weighted average of the coefficients for the two separate markets. This later is given by

$$\bar{Q}_{i \cdot gk} = (p_h Q_{i \cdot gh} + p_{h'} Q_{i \cdot gh'}) \cdot p_k^{-1} = (p_h^{1-n} \mu_{i \cdot gh} + p_{h'}^{1-n} Q_{i \cdot gh'}) \cdot p_k^{-1}$$

while the former can be written as

$$Q_{i \cdot gh} = (p_k^{-n} p_h \mu_{i \cdot gh} + p_k^{-n} p_{h'} \mu_{i \cdot gh'}) \cdot p_k^{-1}$$

As $p_k > p_h, p_{h'}$, it is clear that

$$Q_{i \cdot gk} < Q_{i \cdot gh} \tag{12}$$

This enables us to estimate the reduction in the import dependence of the union. Given the foreign exchange earnings, it will indicate the potentialities of extra-economic growth.

Using the data for the 1970s, the values of basic gravity coefficients for the four continental economic unions of these countries have been calculated. Corresponding values of Leontief–Strout coefficients are then calculated from these. Leontief–Strout coefficients are also calculated as weighted average pertaining to the case of economic self-sufficiency. These are given in table 2.

As a result of economic union the Leontief–Strout gravity coefficient is reduced by 20 to 36 per cent for different regions. If all the developing countries were to join into one big economic union this reduction may be as much as 41 per cent. This would imply that to keep the same level of income and growth the requirement of foreign exchange would be reduced by the corresponding amount by the act of union. If the elasticity of growth with respect to the availability of foreign exchange is α, then with the current earnings of the foreign exchange these regions will be able to grow extra 26 α, 28 α, 36 α and 29 α per cent respectively by the act of the union. For the developing world as a whole this extra percentage will be about 71 α per cent. If the value of α is 1 and this act of economic union of the developing world as envisaged in the Mexico Conference Resolution (77/COOP/CMEX/12; 1976) takes 10 years to implement it would imply a growth rate of 5.5 per cent extra rate of growth of manufacturing over and above what would have been possible even otherwise.

TABLE 2. LEONTIEF-STROUT AND BASIC GRAVITY COEFFICIENTS FOR REGIONS[a]

				Leontief–Strout gravity coefficients		
Regions	*Number of countries*	*Total population (million)*	*Basic gravity coefficients*	*Union (A)*	*Average (B)*	*Reduction due to union in per cent* $\left(\frac{B-A}{B}\right)$
Non-African Mediterranean	6	72.29	0.8311	0.4014	0.5062	21
Asia	17	959.75	0.8148	0.2536	0.3537	28
Africa	21	249.63	1.0828	0.4236	0.6570	36
Latin America	13	221.44	0.5175	0.2066	0.2901	29
Developing countries as a whole	57	1 503.11	0.8163	0.2354	0.4020	41

[a]Countries for which data were not available were not taken into account. Had they been included the result would not have been significantly different.

Conclusion

The method by which Leontief's basic insight into the working of interregional trade can be further used to quantify the effect of union industries has been outlined. For a more complete insight the method may have to be supplemented by a complementary study of location of major industrial plants, mining and agricultural activities. For any success of a model of an economic union a distributive norm between different regions also has to be incorporated into the model. An effort in that direction has been made by the author elsewhere [5–7].

ANNEX

INTER-TEMPORAL AND INTER-COUNTRY VARIATIONS IN BASIC GRAVITY COEFFICIENTS

Basic gravity coefficients are related to overall economic characteristics of the country such as stage of development, natural resources, the rate of growth, transport and communications and other facilities. In a static world, they should remain constant over time. However, in a dynamic world with countries trying to industrialize, we expect the coefficients of manufactured goods from developed countries to become smaller as the countries become more and more developed. During the early phase of development, these coefficients may tend to become larger as the development process will require quite a lot of imports of manufactured capital goods. This aspect is analysed in this annex. The table below gives the analysis of variance for $\log \mu_{i \cdot gh}$. Logs of the functions are used as the basic gravity coefficient is multiplicative rather than additive.

ANALYSIS OF VARIANCE FOR BASIC GRAVITY COEFFICIENTS

$(\log \mu_{i \cdot gh})$

Source of variation	*DF*	*SS*	*MSS*	*F*
Due to years	2	0.1491	0.0746	9.09
Due to countries	61	2.6427	0.0726	8.85
Error	96	0.7858	0.0082	–
Total	159	3.5776		

In making this table, missing observations of table 1 were estimated by the missing plot technique of F. Yates. The assumption is that the country effect and year effect are additive. In this case, they are assumed to be additive in logarithms.

From the table above it will be seen that both the effects due to years as well as due to countries are highly significant. The average values of $\log \mu_{i \cdot g}$ for different years have been found to be 0.1009 in 1965; 0.0735 in 1970; and 0.0328 in 1975. The corresponding average values of the basic gravity coefficient for these years are respectively, 0.7929; 0.8443; and 0.9273.

This shows that the values of basic gravity coefficients have significantly increased over this decade. This may be partly due to the fact that imports of capital equipment have increased to meet extra-developmental requirements but that itself does not seem to be able to explain the whole of it as the rates of growth of the countries have not significantly changed from 1965 to 1975.

On the other hand, it is well known that during this decade the prices of capital goods have been continuously increasing in relation to the prices of other commodities and as the developing countries are shifting their imports more and more to the imports of capital goods, they must have been faced with these higher costs and this should automatically result in this unnatural increase in dependence being reflected in this increasing basic gravity coefficient.

In spite of this we see from table 1 that about 10 countries out of these 57 have managed to reduce their import dependence as reflected by this decline in their basic gravity coefficient. Interestingly this group includes countries in different stages of economic development.

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An econometric model of international trade

*Jean-Jacques Sualla**

There is an increasing demand for econometric models of international trade. In addition to interest in the analysis of trade flows *per se*, there is an obvious need for these tools for the linkage of national models in world models.

Whether or not these models should have a theoretical background depends mainly on the aim of the study. The need for a theoretical justification is not crucial for prediction. A theoretical framework may help to give a coherent economic interpretation of these models and may suggest how to generate families of models with desirable properties.

The methodology is given in section A below. The model is based on the assumption that, for each industry and each country, the structure of preferences of a representative economic agent for importing from the other countries may be represented by a real function of the imported quantities, in the same way as consumer's preferences may be represented by a utility function.

Section B is devoted to the presentation of the operational model. The retained specification can be defined by a committed amount of imports, considered as a function of the total amount of imports, and by two functions of export prices, which may be interpreted as import prices indices.

Some experiments using chronological series of trade flows are reported in section C. The implications of the results with respect to changes in the structure of world trade are discussed. Conclusions are presented in section D.

A. Methodology

In considering a product (or group of products) produced or imported by one or more countries (or groups of countries), the total amount of imports of the j^{th} country M_j is given by:

$$M_j = \sum_{i \neq j} p_i q_{ij}$$

where q_{ij} is the quantity of this product imported by the j^{th} country from the i^{th} country during a given period $i \neq j$; p_i is the price of this product imported from the i^{th} country.

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These prices are assumed to be expressed in a common currency.

The share of the i^{th} country in the internal market of the j^{th} country w_{ij} is:

$$w_{ij} = \frac{p_i q_{ij}}{M_j}, \; i \neq j \tag{1}$$

Following Armington's [1] approach, the model has an index of preferences of the importing agents of each country. This index may be formalized as:

$$u_j = u_j(q^{(j)}) \tag{2}$$

where $q^{(j)}$ is the vector of quantities imported by the j^{th} country, and is given by: $q^{(j)} = [q_{ij}], \; i \neq j, \; i = 1, 2, \ldots, n$.

We assume that the product demand functions may be derived from the maximization of (2), or of any Paretian transform of (2), subject to the budget constraint:

$$M_j = p^{(j)\prime} q^{(j)} \ldots = \sum_{i \neq j} P_i q_{ij} \tag{3}$$

where $p^{(j)} = [p_i], \; i \neq j, \; i = 1, 2, \ldots, n$.

Alternatively, in the classical dual approach, these product demand functions are assumed to be derived from the minimization of an indirect index of preferences:

$$v_j = v_j(p^{(j)} M_j^{-1}) \tag{4}$$

subject to the same budget constraint.

A difference from Armington's theory is that no preference index is assumed to exist at the macro-economic level. For each country, the distribution of total imports among industries is taken as predetermined. The existence of a preference index at a semi-aggregate level only, that is to say, for buyers of the same group of products or of the same industry is assumed in this case. However, the question of the compatibility of these preference indices with a hypothetical global index will be discussed in the case of the particular operational specifications retained.

Another difference is that the assumption that the internal demand of the product is excluded for each country. This assumption is made for convenience and has its origin in the fact that no compatible data were available for these internal demands.

Writing:

$$q^{(j)} = q^{(j)}(M_j, p^{(j)}), \; j = 1, 2, \ldots, n \tag{5}$$

for the product demand functions, the total amount of exports of the i^{th} country E_i is an endogenous variable of our model and can be expressed as:

$$E_i = p_i \sum_{j \neq i} q_{ij} = p_i \sum_{j \neq i} q_i^{(j)}(M_j, p^{(j)}), \; i = 1, 2, \ldots n \tag{6}$$

The product demand functions derived from the optimizing behaviour of the representative buyer satisfy the well-known properties of additivity:

$$p^{(j)\prime} q^{(j)}(M_j, p^{(j)}) = M_j$$

and of homogeneity of degree zero:[1]

$$\forall \lambda \neq 0, \; q^{(j)}(\lambda M_j, \lambda p^{(j)}) = q^{(j)}(M_j, p^{(j)})$$

The consequences are that the exports add up to world demand D:

$$\begin{aligned}\sum_i E_i &= \sum_i p_i \sum_{j \neq i} q_i^{(j)}(M_j, p^{(j)}) \\ &= \sum_j \sum_{i \neq j} p_i q_i^{(j)}(M_j, p^{(j)}) \\ &= \sum_j p^{(j)\prime} q^{(j)}(M_j, p^{(j)}) = \sum_j M_j = D\end{aligned}$$

and that the structure of commercial shares in the world trade:

$$\frac{E_i}{\sum_k E_k} = \frac{E_i}{D}, \; i = 1, 2, \ldots, n$$

are invariant with respect to homothetic changes in the structure of the international prices and demand.

Writing:

$$E_i = E_i(p, M^{(i)}), \; i = 1, 2, \ldots, n$$

where $p = \| p_i \|, i = 1, 2, \ldots, n, \; M^{(i)} = \| M_j \|, j \neq i, j = 1, 2, \ldots, n$

we have for the endogenous exports of the i^{th} country:

$$\begin{aligned}\forall \lambda \neq 0, \; E_i(\lambda p, \lambda M^{(i)}) &= \lambda p_i \sum_{j \neq i} q_i^{(j)}(\lambda M_j, \lambda p^{(j)}) \\ &= \lambda p_i \sum_{j \neq i} q_i^{(j)}(M_j, p^{(j)}) \\ &= \lambda E_i(p, M^{(i)})\end{aligned}$$

and therefore:

$$\frac{E_i(\lambda p, \lambda M^{(i)})}{\sum_k E_k(\lambda p, \lambda M^{(k)})} = \frac{\lambda E_i}{\lambda \sum_k E_k} = \frac{E_i}{D}$$

This invariance property is of crucial importance here since it justifies the arbitrary choice for the common currency in which the prices are expressed.

B. The operational model

In order to specify a model for operational use, we now introduce a particular choice for the direct indices of preferences (4):

[1]These functions naturally satisfy the Slutsky symmetry conditions too.

$$v_j = v_j(p^{(j)}, M_j^{-1}) = \varphi_j\left(\frac{M_j}{A_j}\right)\frac{A_j}{B_j},\ j = 1, 2, \ldots, n \tag{7}$$

where:

φ_j = a continuously derivable, but otherwise arbitrary function;
A_j and B_j = two functions of the vector of prices $p^{(j)}$, homogeneous of degree one and normalized such that $A_j(\iota) = B_j(\iota) = 1$;
ι = the unit vector.

The index (7) above has been introduced in a different context[2] and makes it possible to derive a very rich family of production demand functions. Using Roy's theorem, these functions may be easily seen to be of the following form:

$$q^{(j)} = \frac{\partial A_j}{\partial p^{(j)}} C_j(\tilde{M}_j) + \frac{\partial B_j}{\partial p^{(j)}} \frac{A_j}{B_j} (\tilde{M}_j - C_j(\tilde{M}_j)) \tag{8}$$

where:

$\tilde{M}_j = M_j/A_j$;
C_j = a function defined by $\varphi_j(\tilde{M}_j) = \frac{\partial}{\partial \tilde{M}_j} [\varphi_j(\tilde{M}_j)] [\tilde{M}_j - C_j(\tilde{M}_j)]$.

Interpreting A_j as a price index, $\tilde{M}_j$ may be interpreted as the total amount of imports of the j^{th} country in real terms. Under the hypothesis:

$$\tilde{M}_j \geqslant C_j(\tilde{M}_j) \geqslant 0 \tag{9}$$

the function C_j may be interpreted as a committed amount of imports, in real terms, whereas $\tilde{M}_j - C_j(\tilde{M}_j)$ represents the real excess amount of imports. As C_j may be freely chosen, the product demand functions may be nonlinear functions of the total imports. This is an important possibility for our model. Writing down the model (8) at the level of the market shares (1), it becomes

$$w_{ij} = E(A_j/p_i) S_j(\tilde{M}_j) + E(B_j/p_i)[1 - S_j(\tilde{M}_j)], \tag{10}$$

where:

E = the elasticity operator;
$S_j(\tilde{M}_j)$ = the ratio of committed to total imports, both deflated by the function of prices A_j.

Then:

$$S_j(\tilde{M}_j) = \frac{C_j(\tilde{M}_j)}{\tilde{M}_j},\ \text{with } 0 \leqslant S_j(\tilde{M}_j) \leqslant 1. \tag{11}$$

Thus, the share of the i^{th} country in the market of the j^{th} country is expressed as a linear convex combination of the elasticities of the two price indices A_j and B_j, with respect to its own export price p_i. The coefficient of the convex combination is simply the proportion of committed imports in the total amount of imports.

[2]See Snella [2].

The committed part of imports $C_j(\tilde{M}_j)$ summarizes all the commercial habits, political links, geographical vicinities and technological affinities of the j[th] country, in the particular industry considered. Thus the ratio $S_j(\tilde{M}_j)$ may be interpreted as the "degree of integration" of the economic activity of this country in the world economy of this industry.

Another interesting possibility for our model can be seen by looking at the expression (10) for market shares. One may take more or less complex specifications for the functions A_j and B_j, allowing for more or less price sensitivity. Unfortunately, one should not be too optimistic about this possibility at a semi-aggregate level, where experience shows that significant price effects can rarely be observed. If S_j is taken as a monotonic function of $\tilde{M}_j$, the model is able to reproduce a shift of the pattern of imports towards a more desirable situation which is characterized by one of the "import" price indices A_j or B_j, depending on whether S_j is an increasing or decreasing function.

In the case where some of the elasticities $E(A/p_j)$ or $E(B/p_j)$ are non-positive, the range of variation of the real total amount of imports should naturally be limited to an interval such that the market shares (10) remain positive in this interval.

For completeness, the compatibility of the particular family of indirect preference functions, retained for a given group of products, with the existence of a global preference index (for all the products) should be verified. This, it is hoped, can be very quickly investigated by using Gorman's result [3] on perfect aggregation.

Dropping the subscript j for convenience, we take a monotonic transform of the chosen index:

$$v = \varphi\left(\frac{M}{A}\right)\frac{A}{B}$$

By taking the log of both sides:

$$\log v = \log\varphi\left(\frac{M}{A}\right) + \log\frac{A}{B}$$

$$= G\left(\frac{M}{A}\right) + Q\left(\frac{A}{B}\right)$$

$$= G\left(\frac{M}{A(p)}\right) + Q(p) = u$$

where Q is homogeneous of degree zero in p.

The above expression is precisely of Gorman's polar form. Thus, if a similar index were chosen for every product, the global preference index for the j[th] country could be expressed as:

$$u = u^{(1)} + u^{(2)} + \ldots + u^{(k)}$$

where $u^{(k)}$ is the index chosen for the k[th] product.

These $u^{(k)}$ indices should have, up to a monotonic transform, the same formal expression as (7), but might differ with respect to the choice of A, B and C functions. The specification above for the global index u is compatible with a two-stage allocation of total imports of a given country. In a first step, the total amount of imports is broken down into imports for each product or group of products. In a second step, the imports for each product are distributed among the exporting countries.

C. Experiments with the world trade of equipment goods (1965–1974)

Data

From the available data[3] at a semi-aggregate level the chronological series of trade flows, at current and constant 1970 prices, for a group of products corresponding to equipment goods have been selected. This group includes:

Electrical and non-electrical machinery
Office machinery and computers, precision instruments
Motor vehicles, cycles and motorcycles
Railway vehicles
Aircraft
Ships and boats

The 11 world trade regions and countries which were retained are the following: Belgium-Luxembourg, Canada, France, Germany, Federal Republic of, Italy, Japan, Netherlands, Spain, United Kingdom, United States, rest of the world.

All the trade flows are expressed in United States dollars.

Econometric method

Denoting by t the period of time, the following econometric specification was retained for the system of product demand functions of each country:

$$\hat{p}_t^{(j)} q_t^{(j)} = \hat{p}_t^{(j)} q_t^{(j)} (M_{jt}, p_t^{(j)}; \beta_j) + u_t^{(j)}, \ t = 1,2,\ldots,T, \ j = 1,2,\ldots,n \qquad (12)$$

where:

β_j = a vector of parameters;

$u_t^{(j)}$ = a stochastic term, with the classical assumptions, $E\,u_t^{(j)} = 0$, $E\,u_t^{(j)} u_\tau^{(j)\prime} = \delta_{t\tau}\Omega_j$, where Ω_j is a singular matrix with $\iota\Omega_j = 0'$.

[3] These data were gathered by the Battelle Institute, Geneva Research Centre, for the Explor Multitrade Model.

Each demand system (12) may be viewed as a nonlinear system of seemingly unrelated relations, and the parameters involved were estimated by the ordinary least squares method, using the GCM program for nonlinear multivariate regression.[4]

Results

The flexibility of the model regarding the measurement of price effects cannot be fully exploited here, because of the high level of aggregation and the short time series of observations available. The specifications retained for the price indices A and B were the linear and the geometric form:

$$A = \sum_i a_i p_i, \quad B = \prod_i p_i^{b_i}, \quad \sum_i a_i = \sum_i b_i = 1 \tag{13}$$

and a combination of these two specifications was tried for A:

$$A = \sum_i a_i^0 p_i + \sum_i p_i^{a_i^1}, \quad \sum_i a_i^0 = 0, \quad \sum_i a_i^1 = 1 \tag{14}$$

This last specification generally performed very badly,[5] with the exception of the Netherlands and Spain, the reason being that the information contained in the sample on the evolution of prices was insufficient to estimate all parameters appearing in (14). As for the C function, four different specifications were tried, namely:

(a) $C(\tilde{M}_t) \equiv \gamma$;
(b) $C(\tilde{M}_t) = (1+\gamma)^t C_0$;
(c) $C(\tilde{M}_t)/\tilde{M}_t = S(\tilde{M}_t) = \exp\{-\gamma/\tilde{M}_t\}$;
(d) $C(\tilde{M}_t)/\tilde{M}_t = S(\tilde{M}_t) = 1 - \exp\{-\tilde{M}_t/\gamma\}$.

With specification *(a)*, where the committed amount of imports is a constant, and using specification (13) for the prices indices A and B, the model can be shown to reduce to a simple linear expenditure system which is often used in the analysis of consumer behaviour.

Dropping the j subscript for the importing country:

$$q_{it} = a_i \gamma + \frac{b_i}{p_{it}} B_t \frac{A_t}{B_t}\left(\frac{M_t}{A_t} - \gamma\right)$$

$$= a_i^* + \frac{b^i}{p_{it}}\left(M_t - \sum_k a_k^* p_{kt}\right)$$

where γ is the rate of increase of committed imports and $a_i^* = a_i \gamma$.

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[4]See Snella [4]. Computations were performed on a Univac 1108 at the University of Geneva.

[5]The numerical method failed to converge or estimates for the parameters a^0 and a^1 were unreliable.

Specification *(b)* introduces a trend in committed imports for each country, with rather good results in the case of France and the Netherlands, where γ was estimated to be 20 per cent.

Specifications *(c)* and *(d)* are closely related. For $\gamma > 0$, we have in both cases:

$$\lim_{\tilde{M} \to 0} S(\tilde{M}) = 0, \quad \lim_{\tilde{M} \to \infty} S(\tilde{M}) = 1, \quad \frac{dS(\tilde{M})}{d\tilde{M}} > 0$$

The difference in this case is that we have an inflexion point at $\tilde{M} = \gamma/2$ in the case of specification *(c)*, whereas S is a strictly concave function in the case of specification *(d)*.

For a positive value of γ, we note that the condition (9) for interpreting C as a committed amount of imports is automatically fulfilled. For specification *(d)*:

$$\begin{aligned} \tilde{M} - C(\tilde{M}) &= \tilde{M} - \tilde{M}[1 - \exp\{-\tilde{M}/\gamma\}] \\ &= \exp\{-\tilde{M}/\gamma\} \geqslant 0 \end{aligned}$$

and $C(\tilde{M}) \geqslant 0$.

The two last specifications performed very well, and often substantially better than the constant specification *(a)* and slightly better than specification *(d)*. The table presents the parameter estimates for all countries obtained with this specification. The row corresponds to the importing country and the column to the exporting country, except the last column which shows the degree of integration at the beginning and end of the period $S_j(\tilde{M}_{jt})$ for each importing country. The upper figure is the value of S_j evaluated at $\tilde{M}_{j\,1965}$ and the lower figure is the value of S_j evaluated at $\tilde{M}_{j\,1974}$.[6]

The parameter γ has been estimated to be positive for all countries, so that the degree of integration of each country is increasing. In each row of the table the estimates for the parameters of the product demand functions of each importing country can be found. The upper figure corresponds to the parameters b_i and the lower figure to the parameters a_i[7] involved in the definition of the two price indices B and A, respectively. As the degree of integration increases, the a_i parameters characterize the more desirable pattern of trade, towards which the system of product demand functions tends, whereas the b_i parameters are related to the past situation. The table enables one to have a general view of the changes in the structure of the world trade flows. The entries in the last column, that is to say the degrees of integration, indicate which proportion of the new structure has been reached during the period of analysis. The estimates obtained show the predominant role played by the Federal Republic of Germany, which, as an exporting country, is characterized by very high b_i and a_i parameters in almost all the product demand systems of the importing countries. The challenging role of France and Japan is also displayed. The corresponding a_i parameters are generally higher than the b_i parameters of these two countries in most of the product

[6] For instance, the degree of integration of Spain has grown from 0.548 to 0.800 during the period 1965–1974.

[7] The values have been multiplied by 10^3.

EQUIPMENT GOODS (1965–1974)

(Parameter estimates and degree of integration, by country)

Importing country	*Exporting country*[a]											
	Belgium	*France*	*Germany, Federal Republic of*	*Italy*	*Netherlands*	*United Kingdom*	*Spain*	*Canada*	*United States of America*	*Japan*	*Rest of the world*	*Degree of integration*[b]
Belgium		109	430	73	109	113	−8	0.7	139	−15	49	0.108
		416	308	−44	6	−83	49	11	−10	227	120	0.236
France	34		398	156	44	127	−42	5	267	−47	57	0.168
	190		345	68	69	8	132	4	−154	170	167	0.378
Germany, Federal	113	138		192	73	143	−6	6	247	−34	129	0.337
Republic of	95	218		89	107	35	17	5	107	125	202	0.619
Italy	25	29	495		41	139	−9	7	268	−25	29	0.333
	57	256	341		39	39	16	2	69	48	133	0.713
Netherlands	387	−112	723	122		147	−77	−3	−37	−157	7	0.829
	134	98	315	47		97	12	9	135	55	99	0.970
United	30	4	213	53	44		−16	30	425	−9	225	0.159
Kingdom	37	255	185	64	75		55	−6	−148	283	198	0.421
Spain	14	119	328	107	29	253		−4	140	−48	61	0.548
	24	149	215	125	32	27		11	233	64	119	0.800
Canada	3	−3	40	16	−0	73	−3		911	−48	10	0.051
	−15	126	−68	−87	41	−25	30		−29	102	233	0.101
United States	8	33	278	58	23	192	5	374		76	−47	0.318
of America	15	18	109	16	6	22	2	315		296	201	0.775
Japan	−6	18	200	41	16	114	2	10	569		35	0.447
	10	25	115	15	4	38	1	8	616		167	0.855
Rest of the	3	26	314	59	13	470	−13	16	192	−81		0.501
world	28	106	281	78	35	−4	21	12	233	210		0.733

[a]The upper figure is the estimated value of the b_i parameter multiplied by 10^3, and the lower figure is the estimated value of the a_i parameter, multiplied by 10^3.
[b]The upper value is the estimated degree of integration for the year 1965, the lower value is for 1974.

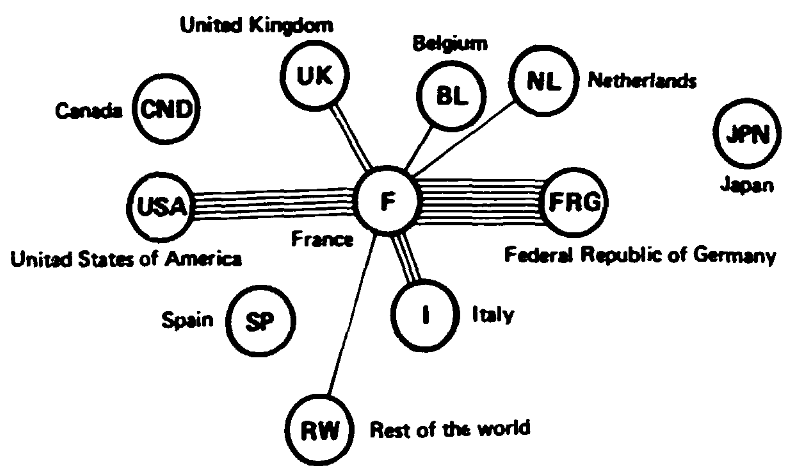
demand systems. On the other hand, the declining role of United Kingdom in the sector of equipment goods should be noticed.

The strong integration of Canada with the United States economy is shown by the value of 0.911 obtained for the b_i parameter corresponding to United States in the product demand system. Canada appears to be only slightly integrated in the world trade of equipment goods, since its degree of integration reaches only the value 0.101 at the end of the period.

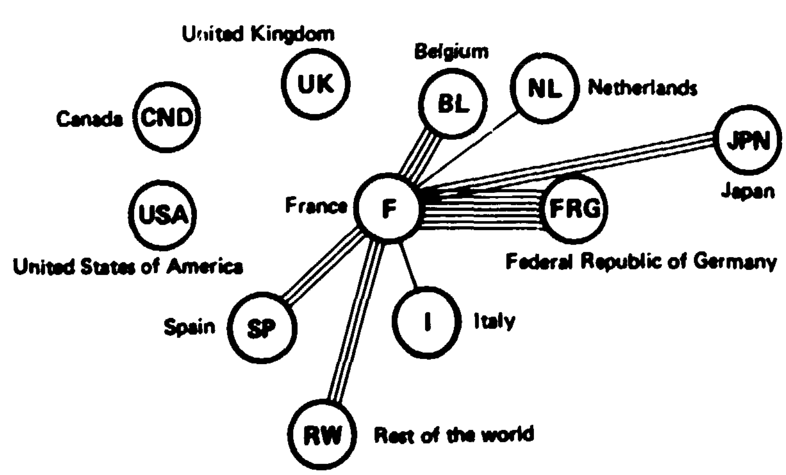
The system of preferences which induces the changes in the structure of the world trade is illustrated for one country, France, in the figure. The number of straight lines between each exporting country and the importing country in part A is proportional to the value of the b_i parameters. No line was drawn for the very

Estimated structure of preferences for equipment goods, France (1965–1974)

A. Structure corresponding to the index b



B. Structure corresponding to the index a



small value of b_i. In part B, the number of straight lines is proportional to the values of a_i. Remembering that b_i can be interpreted as the market share associated with a null level of total imports, whereas the a_i parameters are associated with an infinite level of total imports, the figure indicates the shift in the commercial preferential links of France as far as the industry of equipment goods is concerned. In 1965 only 16.8 per cent of the structure associated with index a (part B of the figure) was reached. This proportion grew to 37.8 per cent in 1974, as indicated in the last column of the table in the row for France.

D. Conclusion

The very good fit of the model to the data is rather encouraging. Global determination coefficients were high (from 0.9833 to 0.9972) for all the systems and the comparison of predicted exports with observed exports showed the good overall performance of the model. Due to the small number of parameters involved in each system of import functions, reliable estimates were usually obtained. The non-linearity of the functional relationship between the imports from each country and the total amount of imports is an important feature of the model, and is supported by the experimentation.

Refinement of the model (more detailed analysis of price effects, dynamic specifications) does not seem possible with the information available. The explanatory power of prices is questionable in the context of the chosen industry, and the introduction of new specific explanatory variables is possible without altering the general methodology of the model.

The concordance of the estimates with the growth of exports of the different countries is rather comforting. The Federal Republic of Germany and Japan appear as leaders in the equipment goods industry and this fact is clearly displayed by the model.

If longer time series were available, dynamic extensions of the model could be easily considered and these extensions would certainly bring an improvement.

The model is a simple tool for the medium-term analysis of the impact of economic growth and price evolutions (including exchange rates evolution) on the structure of world trade flows at a highly aggregated level.

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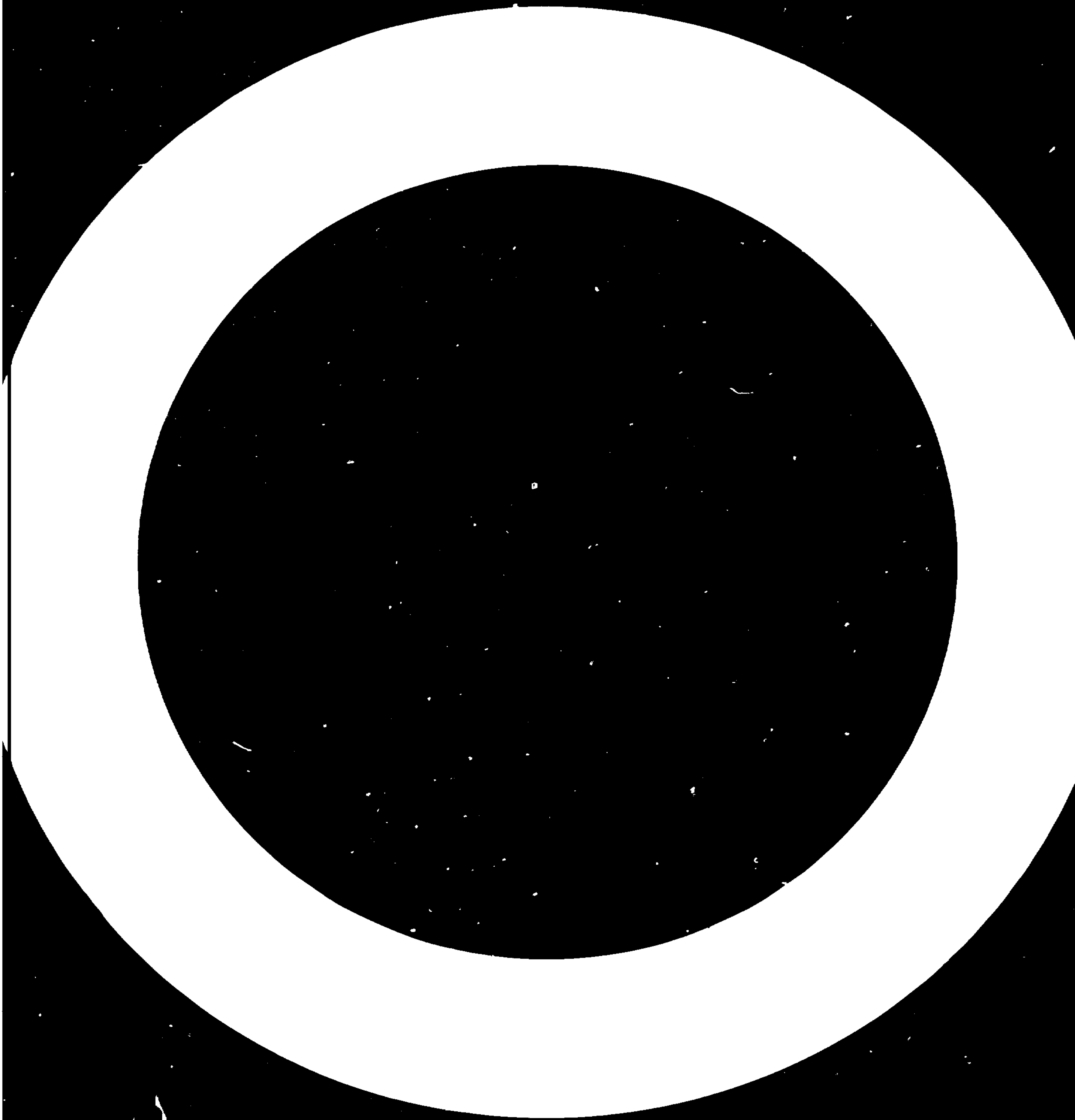
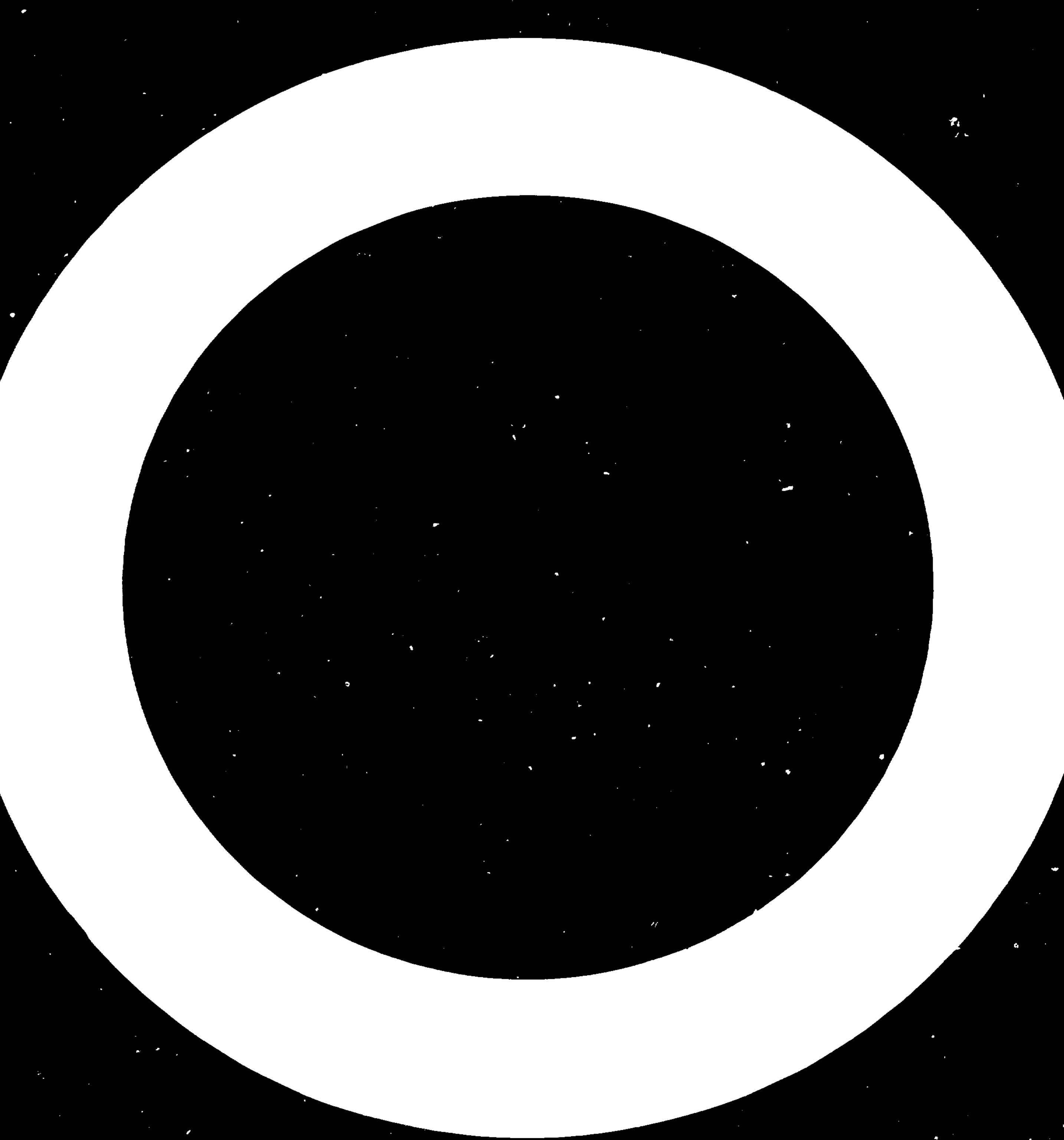
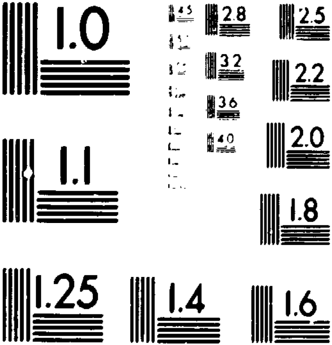
Part two

National input-output



National models





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IDIOM: an international dynamic input-output model

Terry Barker, William Peterson and L. Alan Winters *

The International Dynamic Input-Output Model (IDIOM), which is a computer package developed by members of the Cambridge Growth Project[1] is not, as its title might suggest, a model of the world economy along the lines pioneered by Leontief [1]. Instead it is an attempt, based on the experience gained from constructing and simulating the Project's dynamic I/O model of the British economy (MDM) (Barker [2], Barker, Peterson and Winters [3]), to generalize the accounting framework and modelling approach so that it can be extended without difficulty and applied to a wide range of national or regional economies.[2]

Such economies may be characterized by widely varying levels of development and institutional structures, as well as by differences in the availability and reliability of data series and the national accounting conventions used to construct them. Further, the resources available for building an I/O model, and the purposes to which such a model will be put, will not be the same in all countries. Thus in designing IDIOM an attempt was made to build in as much structural flexibility as is consistent with providing a computer package which is easy to use and efficient in its demands upon machine resources. In particular it was made possible for the model-builder to choose not only the extent to which individual "blocks" of the model are to be disaggregated, but also the division between endogenous and exogenous variables and the specification of many of the behavioural and technical relationships involved. The level of generality attained makes it possible to modify the program so that the user can supply commands in his native language and can introduce variable names which are specific to the country being modelled.

So far IDIOM has been employed only to construct a pilot model of the British economy. This is a small I/O model with only four productive sectors which has been designed as an aid to program development rather than for forecasting purposes. It is hoped that it will be possible to extend the package to cover two other types of models of interest, national models which incorporate a

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[1]This work is being carried out partly for UNIDO in order to assist in national linkage aspects of the UNIDO World Industry Co-operation Model.

[2]The authors have been helped, in the development of IDIOM, by Cecil Bloch who worked on the programming and by other members of the Cambridge Growth Project, including John Beath, Vani Borooah, Tony Lawson, David Vines and Roger Witcomb.

spatial or regional disaggregation as well as the conventional industrial breakdown of I/O analysis, and "mixed-mode" dynamic models in which the solution period can vary in length (from three months for short-term forecasts to a five-year span for long-term analysis) as the simulation evolves.

General structure of the model

The philosophy underlying the design of IDIOM is that the operations necessary for the solution of a highly complex dynamic econometric model can be broken down into three groups:

(a) "Housekeeping" operations, including the provision of input and output facilities, the maintenance of accounting balance throughout the model, and the incorporation of diagnostic checks for error conditions. The techniques adopted to provide these are mainly of interest to the computer specialist;

(b) "Linear" operations, such as the computation of intermediate demands corresponding to a trial gross output vector, or the passage from one functional classification (such as consumers' expenditure) to another (such as commodities). Such operations form the basis of the conventional I/O model;

(c) "Non-linear" or "functional" operations, defined as the solution of a behavioural or technical relationship which explains either a macro-economic aggregate or the components of such an aggregate (such as specific types of consumption). The functional form, explanatory variables and parameters of such a relationship will normally result from an earlier econometric investigation.

This threefold breakdown of operations has been reflected in the structure of the IDIOM program. The "housekeeping" operations represent an area where the model-builder requires a wide range of facilities for input and output of data and for diagnostic monitoring of the flow of control and of computation, invoked by a simple and comprehensible set of control instructions. This is achieved by providing an interpreted "command" language containing approximately 20 straightforward keywords, each of which instructs the computer to perform a particular action such as reading a set of data values or printing a series of tables. Since facilities for repeated and conditional execution of commands are built into this language, it possesses much of the range and flexibility of the high-level programming languages, such as Fortran, in which conventional unique econometric models are usually written. However, the freedom offered to the model-builder (for example, there is no need for instructions to be punched in any particular format) together with the power of individual commands means that the language is both easier to learn and less tedious to write.

It would have been possible to extend this command language to include the linear operations required by an econometric model based on an I/O framework. There already exist some commercial high-level programming languages, such as APL (Iverson [4]), which incorporate matrix and vector operations. Although such languages are not yet standardized or universally available there is no reason why the development of a general-purpose program to interpret and evaluate statements in linear algebra should pose exceptional difficulties. An early example of such a program, GEM (Slater [5]), has been available in Cambridge for several years. The great advantage of such a language is that it allows the model-builder complete control over the specification of the model, which runs by interpreting the model statements provided, and relieves the model-builder from the considerable task of writing a highly specific program in a compiled high-level programming language which does not provide the matrix-manipulation, diagnostic or data-handling facilities which he requires.

Nevertheless, there were two important reasons why in designing IDIOM this approach of implementing a fully interpretive modelling language was rejected in favour of an alternative, and in some ways less sophisticated, solution. In the first place many of the "linear" operations, for which the advantages of such an approach are most obvious, are also operations which, because they are concerned with the maintenance of accounting balance, should form part of the structure of any I/O model. Thus if, as in the United Kingdom of Great Britain and Northern Ireland, data on consumers' expenditure are published on a different classification from data on commodity demand and supply, the predicted vector of consumers' expenditure must undergo a linear transformation, via a bridge matrix or classification converter A_{qc}, in order to obtain the resulting commodity demand vector q_c

$$q_c = A_{qc}c \qquad (1)$$

Similarly the dual linear transformation

$$p_c = A'_{qc}p \qquad (2)$$

is needed to generate the vector of consumer prices p_c given commodity prices p. These two transformations will be necessary in any model where the relevant classifications differ. In addition, in cases where classifications are the same it is convenient to retain the concept of a notional transformation by specifying that A_{qc} is an identity matrix. These transformations are therefore incorporated in the language. Such a procedure reduces the model-builder's control over the model, but also releases the model-builder from the responsibility of ensuring that the model specification always includes the statements (1) and (2).

The second reason for not basing the design of IDIOM on a fully interpretive matrix language, along the lines suggested above, is the need to incorporate a wide range of non-linear operations. The development of the Cambridge Growth Project model, from its first operational version (Stone and Brown [6]) to the latest manifestation (Barker, Peterson and Winters [3]), has been marked by the progressive inclusion of behavioural relationships based on the econometric analysis of time series data and designed to forecast variables which had previously either been regarded as exogenous or projected using simple linear relationships extrapolated from cross-section data for a particular year. It is possible to observe a similar pattern of development for the Canadian model CANDIDE (Economic Council of Canada [7]) and the INFORUM group of models (Almon and others [8]). The specification of such behavioural relationships, in terms of the functional form or parameterization selected, the explanatory variables employed and the error structure assumed, is dictated by the need to find a satisfactory representation of the data and not by the need to preserve linearity of the forecasting model. Such linearity is no longer necessary once the matrix inversion procedure of the static I/O model has been replaced by an iterative procedure such as the Gauss–Seidel algorithm.

However, this need to incorporate a wide range of non-linear relationships makes it difficult to implement a general dynamic forecasting model by straightforwardly interpreting a sequence of commands supplied by the user specifying the relationships involved. The introduction of non-linear functional forms means that both the syntax of the modelling language and the appropriate interpreter program become excessively complex. Furthermore, there is now a difficult choice between specifying the model in terms of vector-valued or scalar-valued functions. If a concise vector notation is adopted (as is done in the model specification of MDM provided by Barker [2]) it is difficult to deal with the infuriating anomalies which inevitably occur. Thus for example, in the model of the United Kingdom, consumption expenditure by foreign tourists is treated for accounting purposes as part of consumers' expenditure, but it is not a function of domestic real disposable income and requires a special behavioural equation in which relative prices at home and abroad are the principal explanatory variable. Such exceptions can of course be ignored in setting out the specification of a model, but they must be incorporated into the computer program which solves it.

The alternative approach is to require that each endogenous variable in the model be projected by an individually specified behavioural equation. In other words the model is defined as a set of scalar-valued functions which is interpreted and evaluated in accordance with a user-determined sequence. This approach does provide the model-builder with the maximum of flexibility, but at considerable expense. Unless the interpretive language is highly sophisticated, with well developed facilities for repetitive and conditional execution of commands, the number of program statements to be written by the user is immense and it is not clear that the method provides advantages over using a compiled high-level language in which such facilities are provided as a matter of course. Furthermore, interpreted programs, since they require substantially more computer time anyway, limit the extent to which it is possible to use the resulting model for sensitivity analysis or policy prescription and optimization, tasks for which repeated solution of the model is necessary. Nor is it possible to avoid these problems entirely by adopting the approach of the builders of CANDIDE and pre-processing the model language into Fortran for subsequent compilation, for the resulting model language is verbose and the extra steps involved reduce computational efficiency.

Programming language

IDIOM represents an attempt to avoid these difficulties at the expense of imposing on the model-builder a well-defined accounting framework dictated by the structure of a dynamic I/O model of a national economy. Thus by abandoning the quest for total generality, implicit in the idea of a fully interpretive model language, it becomes possible both to simplify the model-builder's task considerably by providing a concise syntax for interaction with the model and by automatically ensuring that accounting relationships are maintained, and also to design a model which is both capable of representing a wide range of economic behaviour and computationally efficient. The two basic concepts which make this flexibility possible, and around which IDIOM is built, are the classification and the function-set.

A classification is a functional disaggregation of one of the aggregate variables usually included in a conventional macro-economic model. The table gives a list of the 16 basic classifications around which version 1 of IDIOM has been constructed, together with the corresponding macro-variable and, as an illustration of the magnitudes likely to be encountered in implementing the model for a complex industrialized economy, the number of sectors distinguished in the Cambridge Growth Project model of the United Kingdom. It is worth noting that in most practical applications the full range of classifications will not be needed. Thus, for example, the pilot version of IDIOM neglects all financial effects on real behaviour other than those which occur as a result of changes in exogenously determined interest rates, so that it is possible to ignore the breakdown of financial assets (classification 9, denoted by F). In a more sophisticated model this classification would form the basis for a set of equations explaining the portfolio behaviour of wealth-holding institutions in the economy.

Many of the computations needed to solve a national dynamic I/O model can now be seen as linear transformations from one of these 16 classifications to another. Thus, for example, if q_y is the vector of commodity requirements required as intermediate inputs into industries, y the vector of industry outputs and A_{qy} the matrix of input coefficients (assuming an industry technology), then:

$$q_y = A_{qy}\, y \qquad (3)$$

The matrix A_{qy} is a generalized form of classification converter or bridge matrix whose columns will not sum to unity because of the existence of primary inputs. It is clear that many of the potential classification conversions which could be derived from the table will not be required for any conceivable model structure. In practice the important conversions will be those which transform the various components of final expenditure (such as consumption, government spending and investment) into demand for commodities since it is the equalization of demand and supply for commodities which characterizes the solution of the model.

The generality which this method of representing the operations making up an I/O model allows can be illustrated by considering the example of a model in which each industry produces only one product, so that there is no distinction between commodities and industries. Then all that needs to be done to fit such a model into the framework of IDIOM is to specify that the "make" matrix A_{yq}, which determines the transformation from commodity to industry demands:

$$y = A_{yq}\, q \qquad (4)$$

IDIOM FUNCTIONAL CLASSIFICATIONS

Classification				
Symbol	*Number*	*Description*	*Macro-variable*	*Sectors in the model*
Q	1	Commodities	Gross output	57
Y	2	Industries	Gross output	40
C	3	Consumers' expenditure categories	Consumers' expenditure	42
G	4	Government expenditure categories	Government expenditure	5
R	5	Types of receipt and payment	National income	49
H	6	Income-receiving sectors	National income	8
S	7	Stocks and inventories	Total stockbuilding	2
V	8	Durable capital assets	Gross fixed capital formation	9
F	9	Financial assets	Net acquisition financial assets	1
X	10	Exports	Exports	16
M	11	Imports	Imports	57
E	12	Employment	Total employment	1
D	13	Direct tax brackets (income distribution)	Household income	25
T	14	Indirect taxes	Net indirect taxes	8
A	15	World areas		10
K	16	Social capital formation	Social capital formation	5
O	17	Overall classification (dummy: used to hold data relating to classifications 1–11)		

is an identity matrix. A special command to do this efficiently is provided in the IDIOM language. Similarly the model-builder can, by specifying that a classification converter is a unit vector, deal with the case where one of the macro-variables of the model is not to be disaggregated. As an example, projecting the breakdown of the labour force by sex or skill level is an optional facility. If suitable data are not available the demand and supply for labour can be analysed in aggregate.

The concept of a function-set provides the mechanism by which IDIOM can handle the more complex non-linear relationships which are introduced into the model as the result of econometric analysis, and which are perhaps the distinctive feature of dynamic forecasting models. Each of the principal endogenous variables in IDIOM has associated with it up to eight different explanatory equations or functional forms, which have been programmed in advance, together with the eight corresponding parameter vectors resulting from an earlier estimation procedure. If the endogenous variable in question is a vector (for example budget shares) or a matrix (for example investment by industry and asset) rather than a scalar then the explanatory equations apply to each element independently and the corresponding parameter array becomes a two-dimensional or three-dimensional matrix. Each time that IDIOM is simulated the model-builder can select for each scalar variable or element of an array the functional form and parameters which on the basis of analysis and judgement are considered to be most suitable. One can also specify that any endogenous variable is to be regarded as exogenous.

An example will make it clear how this procedure works. One of the principal endogenous variables in IDIOM is the matrix of industrial investment, disaggregated both by the purchasing industry, since the process of investment may lead to higher productivity levels, and by the asset purchased, since this information is needed for projecting the composition of commodity demand. The pilot version of IDIOM allows each element of this matrix to be explained independently by one of the following:

(a) A neoclassical investment function with the user cost of capital as an explanatory variable;

(b) A simple accelerator mechanism based on past output changes;

(c) An equation relating investment to real profitability in the industry concerned;

(d) As an exogenously fixed value in real terms.

This range of alternative explanations is needed because it is unlikely in practice that any one of them would be suitable for all industries and assets. Thus, for example, although the model-builder might wish to treat private investment as endogenous, the short-run investment behaviour of state-owned industries may be fixed as part of government expenditure plans. Such a structure can easily be modelled by choosing the appropriate combination of functional forms from the investment function-set.

It is a simple task to use the IDIOM language to construct a wide range of models, of varying degrees of complexity, which share a common underlying accounting methodology and data base. Thus at one extreme by specifying that all components of final demand and foreign trade are to be exogenous, and that labour productivity (the inverse of the conventional employment coefficient) is also fixed, it would be possible to construct a basic I/O model which computed the gross output and employment necessary to provide a given bill of goods. Such a model would, of course, be dynamic only in as much as this bill of goods or the technical coefficients evolved through time. Alternatively the most complex model conceivable using the current version of IDIOM would permit all components of final demand (except government expenditure) and of foreign trade to be endogenous, would incorporate econometric relationships to explain earnings in different sectors of the economy and the structure of relative prices, and would possess a consistent set of income and expenditure accounts linking the earnings and profits generated in production with the level of consumption expenditure and savings. A model of this type can exhibit complex patterns of dynamic behaviour as a result of the effects of lagged endogenous variables. Thus past changes in output will affect current levels of investment and stockbuilding and hence feed back into the determination of current levels of output. Further significant dynamic responses are likely to occur as a result of stock-flow relationships (for example that between private wealth and the level of consumption) and "vintage" effects which make the short-term rate of productivity growth dependent on the level of investment.

The figure shows the commands needed to solve the pilot four-sector model of the United Kingdom which have been built to test the IDIOM package. The logical structure involved is simple to follow. The user first defines the specification and dimensions of his model, then reads a suitable set of starting values and parameters and then, for each year of the projection period, reads new values for exogenous variables and computes the model solution. The figure also indicates the brevity and power of the IDIOM command language, since only 90 lines are needed for all these tasks. This compares very favourably with the 8,000 lines of Fortran required for the Cambridge MDM model, or the 4,000 lines of GEM (an interpretive matrix language) needed for a model of similar size.

The analysis of dynamic responses and their mutual interactions within the framework of a disaggregated I/O model is a highly complex process. Indeed much of the Cambridge project's research since the construction of MDM has centred around efforts to explain the conflict between the view of the economic system as a self-stabilizing mechanism inherent in most conventional economic theories and the patterns of cumulative expansion and decline to be found in the numerical results of our model. In this effort the complexity of the model has made it difficult to disentangle the dynamic interactions involved. For example, how far is the foreseen decline of British manufacturing industries a result of a shift in taste towards greater variety in consumption (which as Barker [9] argues can only be met by imports), and how far is it due to a deteriorating performance in terms of cost and quality which results from the dynamic diseconomies of scale experienced in a slow-growing economy? By using the structural flexibility of IDIOM as a tool for eliminating or modifying some of the linkages which lead to

IDIOM commands to run a pilot model of the United Kingdom economy

```
SWITCH ABCHECK ; SWITCH NO ECHO ; SWITCH ABORT -1 ; SWITCH DUMP
NAME USER ANAP RUN 1 TITLE 'Idiom version 1 final test run'
SETUP TABLES TITLES DUMP ; # O 11
# Q 4 ; # Y 4 ; # C 2 ; # G 1 ; # R 5 ; # H 4 ; # S 1 ; # V 2
# F 1 ; # X 2 ; # M 2 ; # E 1 ; # D 2 ; # T 3 ; # A 3 ; # K 1
FUNCTION QMO FIXM ? ; FUNCTION PQM LLIN ?
FUNCTION PY LMAT ? ; FUNCTION YV NCL ?:1 FIX ?:2
FUNCTION V INDL 1 LFEE 2 ; FUNCTION T ADVA 1 SPEC 2 SUBS 3
FUNCTION YEO RECU 1,2,3 LLIN 4 ; FUNCTION YS FIX ?
FUNCTION C LES ? ; FUNCTION SC LNNW ?
FUNCTION R WAGE 1 ITAX 2 PROF 3 IMP 4 GOOD 5 ; FUNCTION XA LLIN ?
FUNCTION PX LLUM ? ; FUNCTION PE EXGW ?
FUNCTION YULC ACTU 1,3,4 STAN 2 ; FUNCTION Q IDEN ?
FUNCTION UE SIMP ? ; FUNCTION HRP NULL ?
FUNCTION H HOUS 1 CORP 2 CGOV 3 ROW 4
CONVERT R Y FULL ; CONVERT R C FULL ; CONVERT R G FULL
CONVERT Q Y FULL ; CONVERT Q C FULL ; CONVERT M Q FULL
CONVERT Q G FULL ; CONVERT Q X FULL ; CONVERT Q S FULL
CONVERT Q V FULL ; CONVERT V K FULL ; CONVERT Q YP FULL
CONVERT E YE ADD ; CONVERT E CE ADD ; CONVERT E GE ADD
CONVERT H R FULL ; CONVERT D R FULL ; CONVERT Q YS IDENTITY
INPUT
REAL SPX,DFE,WPOP,FW 25000,50000,26000,50000
REAL SPYM,SPCM,SPGM 100,500,100 ; REAL WVOL 100 WPRI 1
REAL EMPY,EMPC,EMPG 20000,100,4500
REAL WAGY,WAGC,WAGG,WAGE 26000,130,6000,32130
REAL GDP 55000. SPC,SC,SX 34000,34000,100; RPDI,PDI 40000,40000
REAL PSYM,PSCM,PSGM 1.,1.,1. SYM,SCM,SGM 100.,500.,100.
REAL EX,HUC 1.,1. ; REAL AW,AWY,AWC,AWG 1300,1300,1300,1300
INTEGER YA72,YA70 1,3 ; REAL SPM,SM 75000.,25000. PUP 1.
SELECT INPUT 2 CARDS
READ Q,QCO,QGC,QMO,QSO,QVC,QXC,QYO,PQ,PQH,PQM,PQX,QMTO,QMTZ,QMQ
READ QXTO,QXTZ,Y,YL1,YL2,YL3,YEO,YEL1,YP
SELECT INPUT 2 GEM ; READ YV
SELECT INPUT 2 CARDS
READ YS,PY,PYVP,PVA,PYV1,PYV2,PYE,YRO,YTO,YTZ,C,PC,CRO,CTO
READ CTZ,G,PG,GRO,GTO,GTZ
SELECT INPUT 2 GEM ; READ HRR,HRP
SELECT INPUT 2 CARDS
READ H,RYO,RCO,RGO,S,V,VKO,PV,VTO,VTZ
SELECT INPUT 2 GEM ; READ XA
SELECT INPUT 2 CARDS
READ PX,M,PFM,E,EYO,ECO,EGO,D,TQMO,TYO,TCO,TGO,TVO,K,TQMB,TYB,TCB
READ TGB,TVB,TQXB
SELECT INPUT 2 GEM ; READ RYB,RCB,RGB,QYC,MQC
SELECT INPUT 2 CARDS ; READ YQC
SELECT INPUT 2 GEM ; READ QCC,QGC,QSC,QVC,QXC
SELECT INPUT 2 CARDS ; READ VKC
SELECT INPUT 2 GEM ; READ QYPC
SELECT INPUT 2 CARDS
READ TITLES Q,Y,C,G,R,H,S,V,F,X,M.E,D,T,A,K 2
READ PARS QMO LLIN 2 FIXQ 2 FIXM 2 QMQ 2
READ PARS PQM LLIN 2 FIX 2 PFM 2
READ PARS YV NCL 2 FIX 2 ; READ PARS V LFEE 2
READ PARS PY LMAT 2 LKEY 2 LLM 2 LLK 2 PVA 2 PRS 2
READ YH,YHL1,R,PM,T,TO,YULC,YEXP,CEO,CEOB,GEO,GEOB,HPD,VRO,RVO,UE
READ LF,PE,PEL1
SELECT INPUT 2 CARDS ; READ RVB
READ PARS C LES 2 LLIN 2 FIX 2 DUR 2
READ PARS PE PHIL 2 SARG 2 EXCI 2 EXGW 2
READ PARS SC LEXN 2 LNNW 2 LRIW 2 LLXW 2 LLNW 2 LLRW 2
READ PARS UE VARP 2 ; READ PARS YEO RECU 2 LLIN 2
READ DN,DBT,H
SELECT INPUT 2 GEM ; READ DAB,DDB,DBC,HRC
SELECT INPUT 2 CARDS ; READ TQXO,PQHH,PYS,YSA
READ PARS YS FIX 2 ; READ AD1,AD2,DI1,DI2
READ PARS XA LLIN 2 ; READ PARS PX LLUM 2
READ PCL0,PCL1,PCL2,PCL3,PWL0,PWL1,PWL2,PWL3
READ PQH1,PQH2,XTO,XTO1,XTO2,PXL1,PXL2
REAL EXL1,EXL2 1.,1. TXPO,TXPI,PCET 1.,1.,1.
REAL PCE,PCEL 1.,1. RET .3 POP 55.765 UNEM 500. EMPL 25000.
SWITCH PRINT ; SWITCH PAGE
SOLVE
FOR YEAR = 1 TO 5
CRITERION C,Q,PY 10.,10.,0.1
UPDATE ALL
SELECT INPUT 2 CARDS
READ PCL0,PWL0,AD1,AD2,DI1,DI2
FOR ITER = 1 TO 40
COMPUTE ALL
LOOP ITER
SELECT OUTPUT 3 PRINTER
WRITE Q,Y,QMO,QXO,XA,YV,YEO,PY,PQH,C ; PAGE
SELECT OUTPUT 15 DUMP
PUT ALL
LOOP YEAR
FINISH
```

this pattern of decline one may be able to go some way towards quantifying the magnitudes involved.

A second, more strictly technical, reason for building a high degree of structural flexibility into IDIOM is that such flexibility can enormously speed up the process of model development and testing. The dominant feature of this process is the search for errors in the computer program and in the associated data. Equations which fit well may forecast poorly, and boundary conditions (such as non-negative unemployment) which could be ignored in the estimation procedure may suddenly become significant. The high degree of interdependence in a complex model means that the "rogue" equation or coefficient which is normally the cause of such problems is hard to isolate; even when it is found, further testing and development of the model cannot take place until a satisfactory replacement equation or coefficient is available. Thus the critical path for developing a new version of the model is long and follows an unpredictable course.

It is true that many of these problems can be eliminated by carefully testing each block of the model before it is fitted into place. One purpose of IDIOM is in fact to provide the software necessary for this to be done with the minimum of effort on the part of the model-builder. But such a procedure cannot detect the problems which arise from the interaction between the individual blocks. To take a simple example, while it may be both feasible and sensible to estimate a model in which wages, prices and the exchange rate are simultaneously interdependent, simulating the resulting dynamic time path may lead to unexpected and unacceptable results. The design of IDIOM is such that it is possible to introduce new relationships individually into the framework of a consistent if initially oversimplified model, and to modify or suppress temporarily those interactions which complicate the process of model testing and development. It is hoped that these features will, by reducing the human and mechanical drudgery needed to construct a large-scale model, provide more opportunity for the econometric analysis and simulation exercises which are the essential purpose of the exercise.

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A long-term planning model at work

*Ildiko Krekó**

Drafting the 15-year plan for the 1970–1985 period began in 1968. The first cycle of the plan was completed in 1972. The second cycle took place in 1978 and outlined long-term concepts for the next (1975–1990) 15 years.

Economic planning is going on at two levels. Sectoral experts design the possible alternatives of the parts: branches, foreign trade, consumption, income distribution etc. The planning centre's task is to evaluate and co-ordinate the variants of the parts, within the framework of the whole system of macro-economic interrelations, taking into account the socio-economic objectives as well as exogenous constraints of the development.

Application of mathematical models, made possible by large computers and speedy development of computer techniques, has become necessary owing to the requirements for handling the large quantity of data and the need to reveal interrelations and possible contradictions.

Models developed are partly decision-models and partly analytical models. Decision-models are used to produce consistent and optimal alternative paths of the economy, that is "macrovariants". Analytical models help to explore these many-sided macrovariants. Among the decision-models a macro-economic, multi-industrial, multi-periodic linear programming model HOVA has become the central co-ordinating model. This paper will discuss that model.

Variants of the parts

It is fundamentally important to explore the internal links and specific features of a given part, and to assess the effect of these links and features on future development. Quantification of the development paths for the centre is the final phase of the process. It is necessary to describe the partial field mainly through external relations. This is why the parts have to be characterized with the aid of a standard indicator system.

In the two long-term planning cycles carried out up to now, sectoral, living standards and foreign trade variants were worked out. The standard indicator system includes the industry's production, the inputs necessary for production, material input, investment, inventory accumulation and labour requirements. It also includes the uses of production, the changes in fixed capital, and export and

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import by markets. Foreign trade is characterized by the constraints on export and import, by the possible level of the stock of foreign exchange, and by predicted changes in the world-market prices.

Data on living standards include level of consumption and its structure by industry. All these indicators are given by periods. The 15 years are broken down into three five-year sub-periods. The input data of the HOVA model are mostly data worked out in the framework of industrial planning.

The structure of the model follows.

The original model

The model represents a linear programming problem which includes integer variables. Each partial variant is represented in the model by two variables, an integer and a continuous variable. If the model selects the variants best corresponding to the given constraints and objective, its level may change continuously in a predetermined and relatively narrow zone. The production, investment and foreign trade activities of the individual five-year periods are represented for the quantities added up over the five years. The equilibrium conditions have to be met for the whole of the five-year period.

Variables

The variables of the model can be divided into two basic groups: variables with fixed dynamics, interpreted for the whole 15-year period, and variables interpreted for the sub-periods of five years. Fixed dynamics of the development paths of the partial variants is an idea essential to the modelling of the long-term plan. If parts are planned in variants, an important criterion of forming variants is its dynamics. Therefore, different variants can be assigned to different development paths. On the other hand, one of the most important questions determining development, the relationship between investment and production, can be solved if the laws of movement of the given partial field are known; that is, within the framework of industry planning.

In the model for industry, living standards and foreign trade variant variables are defined. The industry variants are characterized by indicators which refer to the three periods and represent the sums of the five-year period. These indicators are:

(a) Output of industry;

(b) Material input; use for investment and inventory accumulation purposes necessary for the given output from the output of the other industries;

(c) Total labour requirements of the industry.

If the value of an industry variant variable is exactly 1 in a given solution of the model, then the quantities of production, material input, investment, stockbuilding and the labour used, agree in all three periods with the quantities originally planned. And if this level of realization is 0.95 or 1.05, then the values of all these quantities are 5 per cent lesser or greater.

The living standards variant variables comprise the industry breakdown of consumption and distinguish material consumption and consumption of services, for all three periods. If the value of the variable is smaller or larger than 1, the same dynamics of pattern of consumption are implied, merely on a modified level. The variables of foreign trade variants contain the lower and upper constraints to trade grouped for main groups of commodities by currency areas for exports and imports. These variables also contain the constraints on the changes in the stocks of foreign exchange.

The foreign trade variant variables are variables with fixed dynamics for every five years, representing the whole 15-year period. This means that the constraints appearing in the model constitute a system, and their individual elements cannot be changed or separated from the other elements, during any period.

With this formulation the values usually appearing on the right-hand side of the model were "introduced" into the model. Thus, the selection of the system of constraints best fitting the objective sought, such as, for example, selection of the most suitable foreign trade strategy, takes place within the model.

The second large group of variables of the model is defined by periods. The dynamics of these activities are not fixed, but develop within the model. These variables represent foreign trade activities and include the value of exports and imports by industries and by currency areas, and the variables representing the changes in the stock of foreign exchange for all three periods.

The system of constraints

The constraints of the model are equilibrium conditions, conditions regulating the dynamics of foreign trade and logical conditions. For equilibrium conditions the equilibrium of the industries, labour, and foreign exchange is established within the model for five-year periods. The conditions are separately interpreted for each period.

The industry balances prescribe that output and complementary imports must cover the demands for material, investment, stock-building, export and consumption. Thus there can be no shortage in any of the industrial balances. However, surpluses were allowed.

The labour constraint assures that the labour used by the industries does not exceed the labour resources available in the economy. The balance of trade was controlled indirectly, by constraining the changes in the stock of foreign exchange, by socialist and non-socialist countries. In a given period the changes in foreign exchange are obtained as the sum of the balance of trade, the balance of items other than trade, and the balance of the interests paid and received during the previous period. The balance of items other than trade was considered to be constant, thus the constraints relate only to the sum of the other two elements. The possible extent of the balance of a given period also depends on the previous period. It is thus possible to reschedule the need to resort to foreign sources to a certain extent. This is true if these are not fully used before. Later, by saving interest payments, additional resources become available. Thus, the conditions relating to changes in the stock of foreign exchange are intertemporal conditions linking the periods.

The level and dynamics of foreign trade were controlled with the aid of several groups of conditions.

The level of exports and imports by periods and groups of countries and by main commodity groups is constrained by the foreign trade variables. The main commodity groups are too highly aggregated to reflect special requirements related to the foreign trade level of certain industries. Therefore, these requirements were introduced by individual limits. The dynamics of exports and imports are regulated in the model with dynamic constraints. These prescribe that trade by industries cannot grow, or diminish, from one period to the next faster than a predetermined rate. Thus the smoothness of transition between periods is assured.

The possibility of introducing an unrealistically large foreign trade was excluded from the model by requiring that total exports of a given industry not exceed a predetermined percentage of output. Other conditions assure that one and only one variant for each industry could be selected.

The conditions reviewed constitute the "backbone" of the model. In addition, many conditions were applied which "held" only for a single computation or some series of computations, according to the requirements of analysis.

In summary, the model selects from among the industrial data and foreign trade variants, and generates consistent macrovariants, which satisfy the equilibrium conditions and the selected system of foreign trade constraints by period; the solutions produced are such as to result in the highest level of consumption in the structure formulated in the selected living standards variant.

The "cone"

Unfortunately, apart from a few experimental computations, the model was not operated in this original, selecting form. This was because there was no successful solution for a set of variants which would qualitatively describe different real alternatives related to the development of the parts of either of the cycles. The "variants" produced were either not true variants (that is, they differed only in the level of output, but not in respect of input pattern) or were judged by the central "jury" to produce an unacceptable development path.

Thus, the central computations were finally performed using the structural characteristics of a single "variant" for each part. In this form the model does not select and consistency is produced by continuously changing the level of the variables. In the original model the level of the variables may change over a predetermined, very narrow interval. But in the case when the parts are represented by a single variant it is almost impossible to find feasible solutions using variables representing 15-year development paths with fixed dynamics and which move in a narrow interval.

For the first cycle the problem was solved with the simplest though still unsatisfactory technique. The possible domain for the realization level of the variables was expanded, and the level of the variants initially moving between narrow limits was allowed to deviate considerably from the original. This solution is problematic for two reasons. Firstly, for the case of most industries the assumption that the structure of inputs belonging to greatly differing production levels is identical is not justified. This is particularly true for investment inputs. Secondly, the quantities represented by the variable may change by the same percentage in all three periods. If some variable has either a higher or much lower value than originally planned, the transition from the base period to the first plan period may become distorted and a smooth transition from the base period to the plan period will not be certain.

The problems were somewhat eased by expanding the zone of movement of the industrial variants by a "cone" procedure. In essence the "cone" procedure provides that the nearer one is to the base period, the lower the degree of freedom of movement of the industrial variants. This procedure made fitting to the base period smoother. Within the zone of movement it was assumed that investment demands change in a non-linear manner.

The domain of movement for variants solutions widens as a function of time and was produced in such a way that to each variant a lower and an upper constraint-variant was assigned. The domain limited by the constraint-variants was called "cone". It was assured that the model would determine the relative level of the variants in such a manner as to remain within this domain (see figure).

This means that the model contains the lower and upper constraint-variants instead of the original variants of each industry. The level of realization of the industry is the result of their weighted contributions.

The assumptions used to produce the constraint-variants were:

(a) That the variants developing in the "cone" should retain the dynamic trend of the original variant. This implies that if in the original variant an accelerating or a decelerating growth was assumed, such a trend does not change after the introduction of the "cone". This was attained by stipulating that the relative ratios of the increments for each period to each other agree with the corresponding ratios of the original variant in the lower and the upper constraint-variants;

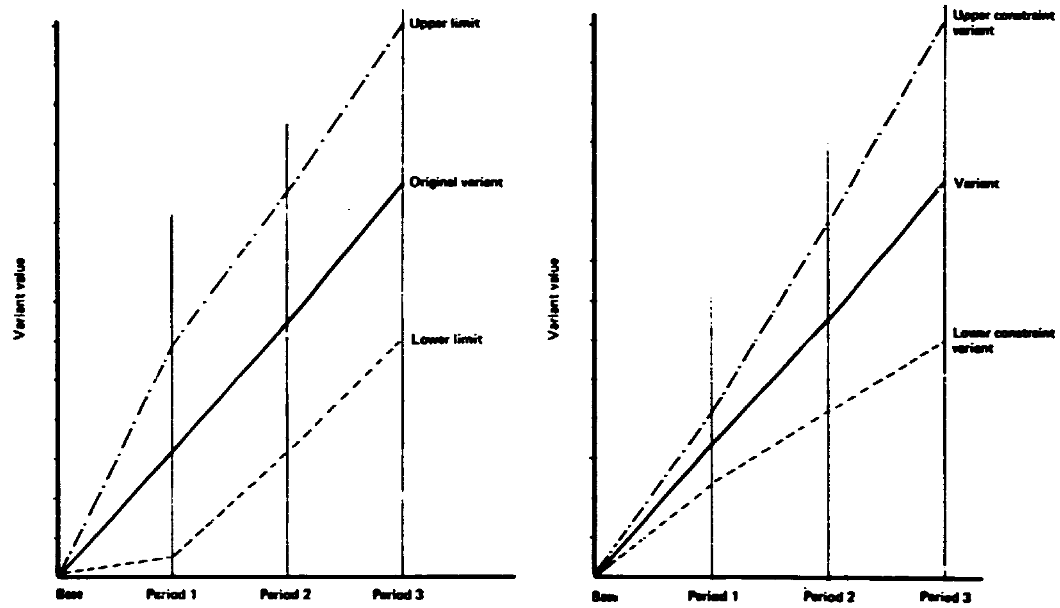
(b) That the original variant be expressible as a convex linear combination of the constraint-variants, and for every indicator of the variants the relationship, $0.5\,V_{\text{lower}} + 0.5\,V_{\text{upper}} = V_{\text{original}}$ must hold, where V denotes a given indicator of the variant;

(c) That the width of the "cone", that is, the distance between the lower and upper constraint-variants differ by industry; this was determined by economic considerations and after consultations with the industry planners.

If the constraint-variants are produced so that these assumptions are applied equally to all indicators of the variant (that is, to its output and inputs) then,

Effect of the "cone" procedure

A. Before "cone" B. After "cone"



within the domain limited by the constraint-variants, the ratio of output to inputs is constant and agrees with the ratios of the original variant.

If, instead, different input structures are assigned to the production levels of the lower and upper constraint-variants, the specific inputs within the "cone" will be no longer constant. The relationship between output and the inputs will be non-linear. With this method, it becomes possible to change the input structure within the model as a function of the volume of production. In practice this possibility has been used only for investment inputs. It was assumed that with growing production the investment-intensity rises, that is, that production according to the upper constraint-variant requires relatively more investment, and that according to the lower relatively less investment than planned in the original variant.

Application of the model

In principle, the model computation can be started with the partial variants as the model relies basically on the data of the partial variants. It is not expedient, however, to feed this huge mass of data into a model whose behaviour is not well known. Such a procedure would make it very difficult and time-consuming to explore to what extent and at what points the unavoidable "nonsense" found in the first results of the model were to be attributed either to the basic data or to the technical characteristics of the model. Therefore, a preliminary analysis of the variants obtained is necessary. To this end, prior to starting the model computations, national economic combinations were produced. These involve a simple summation of a variant from each of the partial fields, without any correction. The variants are naturally non-consistent.

Analysis and study of the combinations greatly promotes the analysis of the partial variants and helps to find contradictions in relations between industries. One also gets a picture of the main national economic indicators on the basis of a simple summation of the parts, including their dynamics, the extent of the structural changes planned, the extent of inconsistency, and the fields to which deviations can be attributed. It is worth while to start the model computations only after getting acquainted with the basic data. From experience, it still takes rather long to "run in" the model, before the first interpretable solution of the model is obtained.

In the first stage of the model computations, economically meaningful "basic" solutions were sought. In the case of the HOVA this meant that on the basis of pre-selected partial variants consistent macrovariants resulting in maximum consumption which deviated from the corresponding combination only to an extent justified by consistency were produced. But a single solution cannot be evaluated in itself. What we aim to do is first to explore the zones of movement, then to examine tendencies, and then to compare the effect of the different conditions or systems of conditions.

Therefore, in the second stage of the model computations a series of computations were performed to examine different economic problems.

Characteristic results

Some interesting series of computations include the following.

(a) In Hungary, manpower resources have not significantly increased from the middle of the 1970s. This trend will continue until about the year 2000. Therefore, parameterizing the labour force constraint was examined, investigating how scarce labour resources influence the dynamics, structure and foreign trade of the economy;

(b) In most of the material industries greatly increasing capital output ratios were planned. This restrained the dynamics of consumption in the model. Therefore, the efficiency requirements for a satisfactory dynamics of consumption were explored while changing the industry capital-output ratios;

(c) The computations were examined by alternative systems of foreign trade constraints, which represent different foreign trade strategies, how the structure of exports and imports changes, and how these changes in foreign trade structures influence the development of the whole economy;

(d) The effects of various alternatives of world-market prices were investigated by parameterizing a few components of the prognosticated world-market prices;

(e) By combining the two components of the objective function, material consumption and consumption of services with different weights, the role of the structure of consumption in the growth of the economy was analysed; as was the "price" of infrastructural development.

The series of computations surveyed above were the most important. In addition, many other solutions for the two cycles answering different questions arising during the course planning were produced.

A non-linear input-output model in physical units and its application in China

Chen Xikang and Sun Shizeng *

In this paper an I/O table of the national economy of China for 1973, which was compiled in recent years in physical units, is briefly described. It is the first attempt to make such a balanced computation in China. When this table was compiled and used in economic analysis and planning, some theoretical and procedural problems, such as the estimation of the other intermediate input, were met. For this there were three kinds of good regression equations and a non-linear I/O model in physical units was thus constructed. This model is useful in planning. The solution of the model may be obtained with an iterative method or with a method of the sum of least squares.

The use of the method of incremental coefficients in economic analysis is also discussed. Finally, the concepts of incremental total input coefficients and differential incremental total input coefficients are proposed.

Input-output tables expressed in value and in physical units

There are two kinds of I/O table according to the units of measurement. One kind is expressed in physical units, and the other is expressed in value units. When this report was compiled, an initial compilation of the table in physical units had the following advantages:

(a) The economic meaning of the table was clear, and was useful in working out the general plan of the national economy and checking its results;

(b) There was much less trouble in collecting statistical data than there would have been in compiling the tables in value units;

(c) Many difficulties in estimating values in accounting and in solving the problems of "net" industry (without secondary products), and others that would otherwise certainly have been met in compiling the tables in value units, could be avoided.

In China, an I/O table of the national economy of China in physical units was compiled for 1973. The first quadrant of this table contained 61 industries, which included 6 agricultural industries, 53 manufacturing industries, construction and freight railway transport. The second quadrant of this table contained 8 categories

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of final demands. The coefficients of direct and total inputs of the individual products on the basis of this I/O table have been computed. The applications of the table in economic analysis and planning have been investigated.

A non-linear input-output model in physical units and its applications

The basic equation of the I/O model in physical units is:

$$AX + W + Y = X \tag{1}$$

where:

A = the square matrix of direct input coefficients;
X = the column vector of outputs;
Y = the column vector of final demands;
W = the column vector of the "other intermediate inputs".

Intermediate inputs are divided into two parts. One part is intermediate inputs to commodities listed in the industry classification of the I/O table (61 industries). The other part is intermediate inputs to other than the 61 industries ("other intermediate inputs").

Because many commodities have not been listed in the industry classification of the I/O table, one of the main characteristics of the I/O table in physical units is that the weights of the other intermediate inputs on the outputs are by no means negligible. For example, in the table the weights of W_i in X_i are 7.89% in grains production, 4.52% in cotton, 13.41% in salt, 24.71% in coal, 31.64% in electricity, 13.03% in iron, 5.34% in steel. The weights depend on the technical level of production, the size of the I/O table, the aggregation of the industries etc. It is well known that the coverage of the physical table in material production is much less than that of the table in value units, so that the weights of W_i in X_i for some widely used products such as electricity and coal are significant, even if the size of the table in physical units is very large.

Therefore, when the I/O model is applied in physical units in planning, the estimation of W_i will strongly affect the reliability of the model solution. Sometimes W_i is treated as an exogenous variable such as final demand. Thus, when Y and W are given, X will be computed from:

$$X = (I - A)^{-1}(W + Y) \tag{2}$$

This method does not represent the underlying relations between W and X. The statistical data for 14 consecutive years were analysed, and it was found that there were linear or non-linear relations between the other intermediate input W_i and the intermediate inputs listed in the industry classification of the I/O table. There are three kinds of regression equations that represented these relations.

(a) For industries such as coal and tyres the other intermediate inputs grew more slowly than the intermediate inputs listed in the industry classification of the I/O table. Some good regression relations of power functions for them were

obtained. Their correlation coefficient r was usually near 0.99 (the number of observations K = 14). The functions are, for the i^{th} industry, in general:

$$W_i = h_i^{(1)} \left(\sum_{j=1}^{61} a_{ij} X_j\right)^{h_i^{(2)}} + h_i^{(3)} \quad (0 < h_i^{(2)} < 1) \tag{3}$$

where the h_i are parameters to be estimated.

To quote an example, for the coal industry we obtained:

$$W_{coal} = 62.3338 \left(\sum_{j=1}^{61} a_{coal\,j} X_j\right)^{0.3652} - 6000 \tag{3a}$$

Coal is measured in units of 10^4 tons.

(b) For some industries (such as electricity) the other intermediate input grew more quickly than the intermediate inputs listed in the industry classification of the I/O table. Some good regression relations of exponential functions were obtained. Their correlation coefficients were also near 0.99 (K = 14). The functions are in general:

$$W_i = h_i^{(1)} e^{h_i^{(2)}} \left(\sum_{j=1}^{61} a_{ij} X_j\right) + h_i^{(3)} \quad (0 < h_i^{(2)} < 1) \tag{4}$$

For the electricity industry we obtained:

$$W_{el} = 72.6866\, e^{0.0023} \left(\sum_{j=1}^{61} a_{elect.\,j} X_j\right) \quad (r = 0.9944) \tag{4a}$$

Electricity is measured in units of 10^2 GWh.

(c) For industries such as rubber and soda-ash the other intermediate input was linearly related to the intermediate inputs listed in the industry classification of the I/O table. Their correlation coefficients were usually more than 0.97 (K = 14). To quote an example, the regression equation for soda-ash was:

$$W_{soda} = 2.1394 + 6.3484 \left(\sum_{j=1}^{61} a_{soda\,j} X_j\right) \quad (r = 0.9497) \tag{5}$$

Soda is measured in units of 10^4 tons.

The statistical data of the past, and the future probable changes in the next period of planning, were used to select the final fitting curves in order to select the better function forms for the next period. It should be noted that the curves fitted are good only during a limited interval of the independent variables, and sometimes they are completely unreliable outside the interval.

The use of the model in planning

Generally the other intermediate inputs W_i are linear or non-linear functions of outputs X. So we have:

$$W_i = W_i(X) \tag{6}$$

It is known that some direct input coefficients a_{ij} are linear or non-linear functions of outputs X. Therefore, one has a non-linear I/O model. This model is useful in planning in three ways, as discussed below.

Checking the equilibrium of the plan

If the plan in physical units has already been made by the traditional method, one can check its equilibrium with this model. It must be made clear that the definition in the plan must be consistent with those in the previous I/O table. For example, the coke industry in the previous I/O table includes the coke made by mechanized coke ovens, and simple and indigenous ovens, but the coke industry in the usual plan for the next plan period generally includes only the coke made by mechanized ovens. The coke-production plan is thus modified to make it consistent with the previous I/O table. For this a linear transformation R is applied to the indicators in the next plan made by the traditional method. Let Z be the vector of production indicators in the plan made by the traditional method, R be the matrix of linear transformation, then:

$$X = RZ \tag{7}$$

After making the indicators consistent with the previous I/O table, the parameters and the exogenous variables in equation (1) are determined by the appropriate computations. Then we can make the following computation:

$$\begin{aligned} \delta &= X - AX - W - Y \\ &= RZ - ARZ - W(RZ) - Y \end{aligned} \tag{8}$$

where δ is a column vector.

Here, R, Z, A, Y are given; δ is a column vector; and $\delta = (\delta_1, \delta_2, \ldots \delta_n)^T$. In this case δ_i represents the difference between the planned output and the demand in the i^{th} industry. If $\delta_i > 0$, then the supply (domestic production plus import) is more than the demand, and vice versa. Let

$$H_i = \delta_i / X_i \tag{9}$$

where H_i is a measure of imbalance in the i^{th} industry.

This measure represents the ratio of difference between supply and demand to the output of the industry. If some of the measures of imbalance are too large, then the plan needs to be restudied and revised.

Providing reference schemes

Providing reference schemes for planning is complex. Firstly, the final demand must be determined. There are usually several hypotheses. Secondly, the direct input coefficients for the next period or their functional expressions must be determined. Finally, the functional expressions for W must be found. Then a reference scheme for the plan of the next period can be worked out from the solution of the non-linear I/O model. The solution of this model may be obtained with an iterative method:

$$\text{Let } X_i^{(k)} = \sum_{j=1}^{61} a_{ij}(X^{(k-1)})X_j^{(k-1)} + W_i(X^{(k-1)}) + Y_i \qquad (10)$$

$$(i = 1, 2, \ldots, 61)$$

where $a_{ij}(X^{(k-1)})$ is a function of $X^{(k-1)}$ and $X^{(k-1)}$ represents the k^{th} step's iterative value of X.

To decrease the number of iterative steps, $X_i^{(0)}$ can be taken as the real output of the last year for which statistical data are available. The computation showed that the convergence rate of this iteration was very fast. It needed only about 3 minutes on a computer which performed approximately 50,000 arithmetic operations per second. However, this iteration may not converge in all cases.

The solution may also be obtained with a method of the sum of least squares:

$$\text{Let } Q(X) = \sum_{i=1}^{61} \delta_i^2(X) \qquad (11)$$

where

$$\delta_i(X) = X_i - \sum_{j=1}^{61} a_{ij}(X)X_j - W_i(X) - Y_i \qquad (11a)$$

$$(i = 1, 2, \ldots, 61)$$

Then, let $Q(X) \rightarrow$ minimum. This is a problem of seeking a minimum without constraints, which can be solved by several non-linear programming methods. And if $Q(X^*) = 0$, then X^* is the required solution. Then the inverse transformation R^{-1} is applied to X^*, and we have:

$$Z^* = R^{-1}X^* \qquad (12)$$

where Z^* is the reference scheme.

Naturally one can work out a number of such schemes by changing the Y, W(X) etc. according to the different cases.

Estimating demand for manpower

The demand for manpower (HR) can be computed from:

$$HR = L(l_1X_1^* + l_2X_2^* + \ldots + l_{61}X_{61}^*) \qquad (13)$$

Where l_i represents the labour coefficient per unit of product of the i^{th} industry in the next plan period. l_i can be determined by some revision, with the probable changes in mind, of the real labour coefficient of the previous table. Some l_i will decrease and others will increase. L represents the ratio of the total manpower in national production to the manpower in the 61 industries.

The method of incremental coefficients

The direct input coefficients are the basis of I/O analysis. In short-term planning these coefficients can be seen as constants, and allow one to compute the plans with the coefficients of the previous I/O table. With most industries this procedure is acceptable, but there are some for which it is unsatisfactory. In these industries the input coefficients have changed in a short period, and thus unfavourably affected solutions. There are two main situations. One is that in some industries technical progress and mechanization developed fast, and thus the electricity, fuel and equipment used by them increased while some materials used by them decreased. The other one is that there was compositional shift in some industries. For example, in chemical fertilizer production, the rate of growth of use of nitrogen fertilizer was different from the rate of growth of use of phosphate fertilizer and potassium, and the volumes of the materials used by these three fertilizers were very different from each other. Thus the direct input coefficients of the chemical fertilizer industry greatly changed. If there are no better methods to estimate the changes of these input coefficients, one can use the following simple method of incremental coefficients.

To formulate a general description, the method of incremental coefficients is applied to all industries. Letting $\tilde{a}_{ij}$ represent the incremental direct input coefficients, we have:

$$\tilde{a}_{ij} = \frac{\Delta X_{ij}}{\Delta X_i} \tag{14}$$

Here ΔX_{ij} represents the incremental intermediate inputs to the j^{th} industry when this industry's production grows by ΔX_i. Let $\tilde{A} = (\tilde{a}_{ij})$, an $n \times n$ matrix. Then:

$$AX^0 + \tilde{A}(X - X^0) + W + Y = X \tag{15}$$

and

$$X = (I - \tilde{A})^{-1}(Y + W + (A - \tilde{A})X^0) \tag{16}$$

where X is the column vector of the outputs of the next plan period. X^0 is the column vector of the outputs of the past period and the $\tilde{a}_{ij}$ of the next plan period can be determined by making some revisions on the real $\tilde{a}_{ij}$ of the previous table. One can assume that these incremental coefficients are constant. The following equations can then be derived:

$$\Delta Y = (I - \tilde{A})(\Delta X - \Delta W) \tag{17}$$

$$\Delta X = (I - \tilde{A})^{-1}(\Delta Y + \Delta W) \tag{18}$$

and

$$X = X^0 + \Delta X$$
$$= X^0 + (I - \tilde{A})^{-1}(\Delta Y + \Delta W) \tag{19}$$

Using (16) or (19), we can compute the outputs of the next plan period. The set of equations (16) is identical with (19). From (16) we have

$$\begin{aligned} X &= (I - \tilde{A})^{-1}(Y + W) + (I - \tilde{A})^{-1}(A - \tilde{A})X^0 \\ &= (I - \tilde{A})^{-1}(Y + W) - (I - \tilde{A})^{-1}(I - A)X^0 + X^0 \\ &= (I - \tilde{A})^{-1}(Y + W - Y^0 - W^0) + X^0 \\ &= X^0 + (I - \tilde{A})^{-1}(\Delta Y + \Delta W) \end{aligned} \tag{20}$$

In many industries the direct input coefficients and the incremental input coefficients differed greatly. For example, in agriculture, some years' grain production per hectare was 3 t, and the chemical fertilizer, electricity and fuel oil expended per hectare were 0.3 t, 150 kWh and 75 kg, respectively. When the production per hectare grew to 3.4 t, the chemical fertilizer, electricity and fuel oil expended per hectare grew to 0.4 t, 190 kWh and 95 kg, respectively. The original input coefficients were 0.1, 50 and 25, but the incremental input coefficients were 0.25, 100 and 50 and thus larger than the original coefficients. Therefore, if the production per hectare is planned to grow from 3.4 t to 3.5 t, it is best to compute how much the incremental chemical fertilizer, electricity and fuel oil will be increased from the incremental input coefficients rather than from the original direct input coefficients. The average direct input coefficients of the next plan period will be affected by the volumes of the increment of the next plan period, and so are the functions of outputs.

The incremental total input coefficients and the differential incremental total input coefficients

The total input equals the sum of the direct input and the indirect input. It is well known that the matrix of the coefficients of the total inputs B equals the inverse Leontief matrix minus the identity matrix, so we have:

$$B = (I - A)^{-1} - I \tag{21}$$

Clearly, B is dependent on A. B and A show the average level of intermediate inputs when the outputs are X^0.

If the outputs X^0 grow by ΔX, one can compute the incremental direct input coefficient from ΔX, and one can also compute the incremental total input coefficients. Let $\tilde{B}(X^0)$ represent the matrix of the incremental total input coefficients under the condition that the outputs are X^0, then we have:

$$\tilde{B}(X^0) = [I - A(X^0)]^{-1} - I \tag{22}$$

and

$$X = X^0 + (\tilde{B} + I)(\Delta Y + \Delta W) \tag{23}$$

The matrix of the incremental direct input coefficients and the matrix of the incremental total input coefficients are dependent on X^0, and on ΔX. When ΔX approaches zero, the limits of the incremental direct and total input coefficients are called the differential incremental direct and total input coefficients.

Let $a^*_{ij}(X^0_j)$ represent the differential incremental direct input coefficients, then:

$$a^*_{ij}(X^0_j) = \frac{dX_{ij}}{dX_j} \tag{24}$$

Let $A^*(X^0)$ and $B^*(X^0)$ represent the matrix of the differential incremental direct input coefficients and the matrix of the differential incremental total input coefficients, respectively; then:

$$B^*(X^0) = [I - A^*(X^0)]^{-1} - I \tag{25}$$

If the direct input coefficients are constant when the outputs grow, then these three kinds of total input coefficients are equal to each other.

The economic meanings of these three kinds of total input coefficients are different, and the ways in which they are used differ also. $A(X^0)$ and $B(X^0)$ are to be used to show the average level of intermediate inputs during some period. $\tilde{B}(X^0)$ is to be used to show the changes of total inputs for the increments of final demands or to show the increments of the outputs for the increments of final demands, while $B^*(X^0)$ can be used to compute these approximately. For example, if the export of cotton cloth and of rice is planned to grow by 50 million metres and 500,000 t, respectively, and the final demand of the other industries will not be changed during the same period, then the planned outputs of rice, cotton, cotton cloth, electricity, coal, chemical fertilizer and steel are to be raised to a new level. The increments needed can be computed by $\tilde{B}(X^0)$:

$$\Delta X = (\tilde{B}(X^0) + I)(\Delta Y + \Delta W) \tag{26}$$

or by $B^*(X^0)$ approximately:

$$\Delta X \approx (\tilde{B}^*(X^0) + I)(\Delta Y + \Delta W) \tag{27}$$

Because other intermediate inputs W_i are linear or non-linear functions of X, their increments ΔX_i are also linear or non-linear functions of X^0 and ΔX.

$$\Delta W_i = \Delta W_i(X^0, \Delta X) \tag{28}$$

Thus

$$\Delta X = (\tilde{B}(X^0) + I)(\Delta Y + \Delta W(X^0, \Delta X)) \tag{29}$$

and

$$\Delta X \approx (B^*(X^0) + I)(\Delta Y + \Delta W(X^0, \Delta X)) \tag{30}$$

Using an iterative method and other methods, we can obtain the solution of ΔX.

The investment requirements needed for the growth of final demand, can also be computed by $\tilde{B}(X^0)$ or $B^*(X^0)$.

Summary

In this paper an I/O table of the national economy of China, which was compiled in recent years in physical units, is briefly described. It is the first attempt to make such a balanced computation in China. When this table was compiled and

used in economic analysis and planning, some theoretical and procedural problems, such as the estimation of the other intermediate input, were met. For this there were three kinds of good regression equations and a non-linear I/O model in physical units was thus constructed. This model is useful in planning. The solution of the model may be obtained with an iterative method or with a least-squares method.

The use of the method of incremental coefficients in economic analysis is also discussed. Finally, the concepts of incremental total input coefficients and differential incremental total input coefficients are proposed.

A social accounting matrix for Egypt

O. M. Osman *

This paper sketches the procedures by which a social accounting matrix (SAM) can organize I/O data in a way that is useful for analysis and policy planning, using the 1976 I/O table for Egypt as a starting point. The structure of the SAM for Egypt is then described, and, finally, a general equilibrium model (GEM) is constructed.

The GEM models and the data which they contain, can potentially provide a major advance in economic policymaking in Egypt. Some of the data have never been estimated for Egypt, and there has never been an organized consistent SAM; solutions for the model have thus never been computed. It is hoped that further improvement and expansion of this work will be made and will prove fruitful.

Updating the I/O table for 1976

The 12 × 12 industry I/O table for 1976 was based on the 1970–1971 27 × 27 industry I/O table and represents an aggregation of the latter table. The table for 1976 was transformed to fit the 1976 final demand vector. The 1970–1971 I/O table was constructed according to the rows method, and not by columns. The rows method was adopted because estimation of the I/O table depends on the data available from estimates of commodity balances of physical production.

In order to make the 27 × 27 industry 1970–1971 I/O consistent with the 12 industry social accounting matrix and the model used, three operations were carried out. These were:

(a) Restructuring the 1970–1971 I/O table to transform it into a 12 × 12 industry table. This restructuring required disaggregation of certain industries and the aggregation of others to fit the new industrial classification system. It was necessary to take into account production activities which had been created after 1970–1971, such as the new steel complex, the aluminium complex and the reopening of the Suez Canal;

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(b) Transformation of values of the flows in the 1970–1971 table to 1976 prices. The transformation was carried out in two operations. The rows were multiplied by index numbers of prices and the columns by index numbers of quantities, and the consistency of the results was checked;

(c) Adjustment of the new 12 × 12 industry table using 1976 prices so as to make the table consistent with the 1976 final demand vector derived from national income accounts. This was carried out through the RAS method.

Structure of the SAM

The matrix is composed of 41 rows and 41 columns (see table). The interindustry flow matrix for the 12 industries which represent an aggregation of the original 27 × 27 industry I/O table is in the north-western block of the matrix (block 1). These 12 industries are: staple food; non-staple food; cotton; other agriculture; food processing industries; textile industries; other industries; construction; crude oil and products; transport and communication; housing services; other services.

The private consumption demand in block II (columns 15–20 and 21) is assumed to come from six socio-economic classes. There are three such classes each for both "urban" and "rural" consumers. Both the "urban" and "rural" consumers are divided into classes of the bottom 60 per cent, the middle 30 per cent, and the top 10 per cent of income. In this classification, "rural" means only the agricultural population or agricultural incomes.

The final demand of the government was subdivided into three categories (columns 22–24). These are "the public sector", "conventional government", (including traditional activities of the government: health, education, public administration and defence. The category therefore comprises the traditional sources of revenues which are tariffs and taxes, and current expenditures) and lastly, the "government trading" sector. These categories made it possible to classify quite different types of activities. The special emphasis on the government trading sector is justified because through that sector the activities of the general supply authority provide basic consumer necessities to the household sector in the ration shops (consumer co-operatives and other places where subsidies are incurred).

The final demand also includes exports (columns 25, 26 and 27), which are comprised of government trade exports, a price differential realized on government trade exports and other exports. The next component of final demand is fixed capital formation (columns 24–38). This component is divided into private investment; the investment corresponds to the three sectors of the government and foreign investment. However, in the present matrix, due to the unavailability of the necessary data, all investment is aggregated into a single category of total investment (column 40).

The last final demand vector is changes in stocks (column 30).

Columns 28 to 33 represent import tariffs, indirect taxes, direct taxes, and transfers. There are four rows that correspond to these columns. The last column, 40, is gross production of the sectors concerned.

After the rows for each of the first 12 producing industries, row 13 of block I, the inter-industry-flow matrix, represents the total inputs to the 12 industries.

Row 14 represents gross value added for the 12 industries concerned. It is disaggregated in rows 21 and 22. Row 21, though "household value added", actually represents the gross income of the household sector. Thus, this row includes both value added in the private sector and wages generated in the government and the public sectors. Row 22 then represents just the surplus of public sector enterprise. Part of the surplus is retained by the public sector, the rest is transferred to the government. However, the surplus is treated as one unit in row 22 and is in column 22 of the SAM.

Rows 15 to 21 represent the gross income of the households from each industry distributed among the six socio-economic groups and correspond to columns 15 to 20 which represent consumption of the six socio-economic classifications. Remittances of Egyptians abroad are treated here as a household export and added to gross income. These remittances appear in row 21, column 26. The remittances are distinguished by amount for each of five income classes and added to the gross income. This income appears in column 41, corresponding to rows 15 to 20.

Rows 23 and 24 correspond to columns 23 and 24. Row 23 is "conventional government revenue" derived from import tariffs, indirect taxes, direct taxes and other revenues. The government trade row, 24, represents revenue from "sales of intermediate goods" by the government to producing sectors and sales directly to the household sector, in row 24, column 21. This amount is distinguished for each of six socio-economic classifications. Row 24 also includes "government direct sales" abroad, that is, government exports.

Row 23 and column 23 must balance and the balancing item is "government deficit" which appears in row 36, column 23. An analogous balancing must occur with "government trade purchases" in column 24 and "government trade sales" in row 24. The balancing item in this case is "government trade deficit", in the box in column 37 and row 24.

Row 27 represents "total imports" which are subdivided into rows 25 and 26. Row 25 represents "government trade imports" which are imports directly by the government. Imports in this row corresponding to columns 1 to 12 represent intermediate imports for the 12 industries. They are treated here as non-competitive imports, that is as if they are domestically not producible and so are not competitive with domestic production. There are also government trade imports directly to the household sector in row 24, column 21, which is again divided among the six income categories.

The second class of imports in row 26 is "other imports". These include:

(a) Public sector and household imports; imports in the row corresponding to columns 1 to 12 represent intermediate goods;

(b) Direct imports to the household sector at the intersection of row 26 and

A SOCIAL ACCOUNTING MAT

(Millions of Egypti

Delivering industries and services	1. Staple food	2. Non-staple food	3. Cotton	4. Other agri-culture	5. Food process-ing	6. Textile	7. Other industries	8. Construc-tion	9. Crude oil	10. Transpor-tation	11. Housing	12. Other services	13. Total inter-mediate	14. Total final demand	15. Urban, lowest 60%	16. Urban, middle 30%	17. Urban, top 10%	18. Rural, lowest 60%	19. Rural, middle 30%
1. Staple food	46				179					1		10	238	217	22	14	5	23	7
2. Non-staple food	1	37	0	5	350	1				0		17	416	852	179	187	111	114	66
3. Cotton			0		1	73	0						75	160	2	0	0	2	0
4. Other agriculture	54	282	9	2	5	34	1			0		0	392	72	14	16	12	7	4
5. Food processing		39			144	3				2	0	93	290	1 232	324	275	140	234	100
6. Textile	2	0	3	0	1	310	1	1	0	6	0	53	373	512	80	97	55	38	31
7. Other industries		0	2	0	1	13	164	11	9	8	0	70	292	1 100	96	99	60	45	23
8. Construction		1	0		0	1	0	0	2	9	5	14	37	599					
9. Crude oil	12	305	3	2	3	2	11	3	132	35		80	355	254	21	18	9	12	5
10. Transportation	0	0	0	0	0	2	0	2	0	2	0	31	40	537	30	45	171	8	6
11. Housing	0	0	0	0	1	0	0	0	0	0		5	10	132	17	39	55	4	4
12. Other services	2		7	0	31	53	10	335	12	28	2	488	968	2 150	208	223	152	78	63
13. Total inputs	130	408	29	11	719	487	201	302	157	87	8	867	3 486	7 817	995	1 014	772	565	312
14. Value added	317	806	202	438	164	330	569	254	324	454	134	2 049	6 041						
15. Urban, lowest 60%	0	0	0	0	40	91	86	36	0	52	34	631							
16. Urban, middle 30%	0	0	0	0	45	90	87	35	28	61	40	647							
17. Urban, top 10%	0	0	0	0	64	93	98	95	45	79	50	683							
18. Rural, lowest 60%	135	289	90	180	0	0	0	0	0	0	0	0							
19. Rural, middle 30%	87	240	58	128	0	0	0	0	0	0	0	0							
20. Rural, top 10%	79	225	53	129	0	0	0	0	0	0	0	0							
21. Household, value added	302	755			150	277	274	170	75	193	125	1 962	4 923						
22. Government and public value added	15	51			14	53	295	84	249	261	9	87	1 118						
23. Government, conventional																			
24. Government, trade					25								25		20	15	7	20	11
25. Government, trade imports	12	37	10	8	391							94	552		50	45	32	44	29
26. Other imports	7	22	4	15	9	47	435		66	51			656		15	30	89	12	20
27. Total imports	19	59	14	23	425	47	435		66	51		94	1 208						
28. Import tariffs	1	5			166	12	100		42				326						
29. Indirect taxes					205	65	35		15			20	340		38	39	31	19	15
30. Subsidies	−12	−10	−10	−8	−182	−86	−3		−8	−15		−12	−346		−63	−48	−23	−13	−10
31. Government transfers															79	94	123		
32. Direct taxes					25	30	55		13			100	223		6	33	70	4	15
33. Sum of items 28 to 30																			
34. Private savings															−97	−98	202	70	143
35. Government and public saving																			
36. Government other deficit																			
37. Government trade deficit																			
38. Foreign finance																			
39. Change in stocks																			
40. Total savings																			
41. Gross production	455	1 268	235	464	1 522	885	1 392	636	609	577	142	3 118	11 303		1 043	1 126	1 305	721	535

SECTION 1

G MATRIX FOR EGYPT, 1976

Egyptian pounds)

	Receiving sectors																					
…al, …blic	20. Rural top 10%	21. Total house-hold	22. Govern-ment and public purchases	23. Govern-ment conven-tional	24. Govern-ment trade	25. Price difference	26. Exports	27. Total exports	28. Import tariffs	29. Indirect taxes	30. Subsidies	31. Govern-ment transfers	32. Direct taxes	33. Sum	34. Private invest-ment	35. Govern-ment public invest-ment	36. Other	37. Govern-ment trade invest-ment	38. Foreign invest-ment	39. Change in stocks	40. Total invest-ment	41. Sum of gross produc-tion
7	4	75		14	95	17		17												16	16	455
66	53	710		60	28	4	49	53													1	1 268
0	0	5			100	55		55														235
4	2	56					16	16														464
00	71	1 146		50			26	26												10	10	1 522
31	26	327		27			109	109									3			46	49	885
23	20	343		268			89	89									360			40	400	1 392
																	599				599	636
5	3	69		35		100	49	149												1	1	609
6	5	265		100			172	172														577
4	8	128		4																		142
63	76	802		1 013	42		171	171									122				122	3 118
312	245	3 926		1 571	265	176	681	857									1 085			113	1 198	11 303
								310				296										6 647
							65															1 043
							90															1 126
							93															1 305
							25															721
							20															535
							18															503
							310	310														5 233
												296										1 414
									477	469			378									1 324
11	4	79			768		119	119		21												1 012
29	4	216																				768
20	19	125															391				391	1 172
		341															391				391	1 940
		60															91				91	477
15	7	150																				490
10	− 10	− 168		146	368																	
		296																				296
15	26	155																				378
143	172	394																				394
			1 414																			1 414
				− 393																		− 398
					− 389																	− 389
								654														654
		394	1 414	− 393	− 389			659														1 680
535	503	5 233	1 414	1 324	1 012	176	1 110	1 940	477	490		296	378				1 567			113	1 680	

SECTION 2

column 21, which is again divided among the six income classes as part of their expenditure.

Rows 28 to 32 represent types of government revenues. Thus, row 28 shows "import tariffs" paid by each of the 12 production sectors and tariffs paid by households on their own imports. Row 29 contains "indirect taxes" considered as part of government revenues. These are also divided among the 12 industries. Column 21 in row 29 shows indirect taxes paid by the households sector on its own expenditure. This also has been divided among the six income categories.

Row 30 contains "government subsidies". These go either to the 12 industries which appear in the row corresponding to columns 1 to 12,[1] or are paid directly to the household sector. These affect the prices of the commodities sold directly to the household. The effect appears in row 30, column 21. This last item is again distributed among the six socio-economic categories in row 30, columns 15 to 20.

Row 31 represents "government transfers" considered from the revenue side and not the expenditure side. These transfers include the net contribution by households to the social security and pension funds as shown in row 31, column 21. This is again distributed among the six income classes in row 31, columns 15 to 20. Row 32 represents direct taxes as revenue to the government. These include direct taxes paid by the 12 industries (for example, the corporate tax); direct taxes paid directly by the household sector (such as wage tax and land tax). The direct taxes appear in line 32, column 21, and are distributed among the six income categories corresponding to columns 15 to 29 in row 32.

Row 34 shows "private savings" which appear as figure corresponding to that row under column 21. This is distributed among the six income categories, and is a balancing item between household income and household expenditure. Thus, the totals of rows 15 to 20 which represent the income accruing to the six income categories must balance with the totals of columns 15 to 20 which represent their expenditure. The balancing item is private savings whether positive or negative.

Row 35 represents "government and public sector savings" which is the surplus realized in government and public sector enterprises. Row 36 shows the "deficit of conventional government" which is again a balancing item. Row 37 shows "government trade deficit" which is again the balancing item between sales and purchases. Row 38 shows "foreign savings" which represents the balancing item between total exports in column 27, and total imports, row 27.

Row 39 shows "total savings" which balances with the sum of column 40 of total investment. Total savings equals private savings plus government and public sector savings minus government conventional deficit minus government trade deficit plus foreign finance.

Row 41 shows "gross production" which corresponds to column 41 for the 12 industries. Data in row 41, from columns 1–12, could be arrived at in this way. Gross production equals total inputs plus value added plus imports plus import tariffs plus indirect taxes minus subsidies.

[1]Subsidies for the first four industries are subsidies on fertilizers and pesticides; in the fifth industry subsidies are on wheat flour and other items.

Structure of the planning model and policy options

Given the data layout in the SAM, it is straightforward to construct a general equilibrium model around the data. The annex gives the equations and explains the variables and parameter names.

The model in its various versions is a simplified multi-industry static GEM. In the different versions it achieves macroeconomic consistency (GEM 1); identifies discrepancies in resource demands and availabilities (GEM 2); adjusts resource prices and uses to resource availabilities (GEM 3). The GEM is useful for simple macroeconomic effective demand analysis and for the study of the macroeconomic and some limited microeconomic effects of expenditure, export and import and tax and subsidy policies affecting prices and factor returns and distributional issues.

The various GEMs are not well suited to investigate the growth implications of alternative resource allocations and investment policies. Additional models will be developed for these purposes.

Testing alternative policies and comparison and analysis of the resulting solutions is as essential an application of the models as are the actual calculations. The Egyptian economy is widely characterized by a variety of goods and factor price distortions created by market imperfections and government intervention through taxes, subsidies and direct regulation. These distortions, in turn, are often thought to be major barriers to the efficient use of intermediate and final products and primary resources. At the same time, the interventions may offset market distortions and help achieve distributive goals. In fact there is a high priority for analysis which can deal with this set of issues. This is the basic virtue of the SAM used. One of the distinguishing features of the SAM is a relatively detailed accounting of government taxes and subsidies, tariffs and government trading activities. On the other hand, to a considerable extent the model structure follows from the accounting presented in the SAM and consists largely of the identities in that matrix.

ANNEX

EQUATIONS FOR THE EGYPTIAN GENERAL EQUILIBRIUM MODELS

The relationship for I/O is:

$$X_i \equiv \sum_{j=1}^{12} a_{ij} X_j + \sum_{k=1}^{6} QH_{ik} + QG1_i + QG2_i + QG3_i + PD_i + E_i + INV_i + DST_i \qquad i = 1, \ldots, 12$$

where

X_i = gross output of industry i;
X_j = gross output of industry j;
a_{ij} = input coefficients of the i^{th} row and j^{th} column;

QH_{ik} = consumption of i^{th} industry output by k^{th} income class;
PD_i = price differentials on exports imposed by the government trading sector;
$QG1_i$ = exogenously specified use of i^{th} industry output by the first government sector (the public enterprises);
$QG2_i$ = exogenously specified use of i^{th} industry output by the second government sector (conventional government);
$QG3_i$ = exogenously specified use of i^{th} industry output by the third government sector (government trading);
DST_i = deliveries of goods for inventory accumulation by the i^{th} sector;
E_i = exports of the i^{th} industry;
INV_i = deliveries of fixed investment goods by the i^{th} industry.

The relationships for household consumption are:

$$\hat{S}_k = \sum_{i=1}^{12} \theta_{ik} B_i \qquad k = 1,\ldots,6 \quad i = 1,\ldots,12$$

$$QH_{ik} = \theta_{ik} + \frac{\alpha_{ik}}{B_i}[S_k - \hat{S}_k] \qquad k = 1,\ldots,6 \quad i = 1,\ldots,12$$

where:

S_k = total consumption expenditure by k^{th} income class = YE_k;
$\hat{S}_k$ = value of subsistence consumption expenditure by k^{th} income class;
B_i = price of output of the i^{th} industry;
θ_{ik} = quantity of the i^{th} industry goods consumed by classification k at subsistence income, $\hat{S}_k$;
α_{ik} = marginal expenditure on industry i goods by income class k.

The relationship for price determination is:

$$B_j = \sum_{i=1}^{12} a_{ij} B_i + HP_j + HG_j + a_{24j} B_j + a_{25j} BI_j + a_{26j} BI_j + a_{28j} BI_j + a_{29j} B_j + a_{30j} B_j + a_{32j} B_j \qquad j = 1,\ldots,12$$

where:

B_j = price of output of the j^{th} industry—see above;
BI_j = price of imports of type j;
HG_j = factor cost of government value added per physical unit of output of industry j;
HP_j = factor cost of household value added per physical unit of output of industry j.

The relationships for determination of value added by industry are:

$$VHP_j = WAGE_j^{SPL_j} KAP_j^{SPK_j} RENT_j^{SPT_j} \qquad j = 1,\ldots,12$$

$$SPL_j + SPK_j + SPT_j = 1 \qquad j = 1,\ldots,12$$

$$HP_j = CH_j \cdot VHP_j \qquad j = 1,\ldots,12$$

$$HM_j = HP_j \cdot X_j \qquad j = 1,\ldots,12$$

$$VHG_j = WAGEG_j^{SGL_j} KAPG_j^{SBX_j} \qquad j = 1, \ldots, 12$$

$$SGL_j + SBK_j = 1 \qquad j = 1, \ldots, 12$$

$$HG_j = CG_j \cdot VHG_j \qquad j = 1, \ldots, 12$$

$$GM_j = HG_j \cdot X_j \qquad j = 1, \ldots, 12$$

where:

- HM_j = factor cost of private household value added in industry j;
- VHP_j = factor cost of a unit of private value added;
- SPL_j = share of labour in private value added in j^{th} industry;
- SPK_j = share of capital in private value added in j^{th} industry;
- SPT_j = share of land in private value added in j^{th} industry;
- $WAGE_j$ = wages in private activity in industry j, exogenously specified in GEM-1 and GEM-2 and endogenous in GEM-3;
- $RENT_j$ = returns to land in private activity in industry j, exogenously specified in GEM-1 and GEM-2 and endogenous in GEM-3;
- KAP_j = returns to capital in private activity in industry j, exogenously specified in GEM-1 and GEM-2 and endogenous in GEM-3;
- CH_j = amount of private value added accruing to households per unit of output of industry j;
- GM_j = factor cost of government value added in industry j.

The relationships for income generation are:

$$YH_k = \sum_{j=1}^{12} HM_j[SPL_j \cdot SSPL_{jk} + SPK_j \cdot SSPK_{jk} + SPT_j \cdot SSPT_{jk}] + \sum_{j=1}^{12} GM_j[SGL_j \cdot SSGL_k] + RM \cdot SRM_k \qquad k = 1, \ldots, 6$$

$$YD_k = [1 - a_{30k} - a_{31k} - a_{32k}] YH_k \qquad k = 1, \ldots, 6$$

$$YET_k = [1 - a_{34k} YH_k/YD_k] YD_k \qquad k = 1, \ldots, 6$$

$$YE_k = [1 - (a_{24k} B_I + a_{25k} B_I + a_{29k}) YH_k/YET_k] YET_k \qquad k = 1, \ldots, 6$$

where:

- YD_k = gross income of k^{th} income class adjusted for subsidies, transfers and taxes;
- YE_k = YET_k adjusted for purchases from government trade sector and its imports and indirect taxes;
- YET_k = YD_k minus private saving of k^{th} income class = S_k;
- YH_k = gross income of k^{th} income class;
- RM = remittances by migrants;
- SRM_k = share of k^{th} income class in remittances by migrants;
- $SSGL_j$ = share of k^{th} income class in value added by labour in j^{th} government sector;
- $SSPL_{jk}$ = share of k^{th} income class in private value added by labour in j^{th} industry;
- $SSPK_{jk}$ = share of k^{th} income class in private value added by capital in j^{th} industry;
- $SSPT_{jk}$ = share of k^{th} income class in private value added by land in j^{th} industry.

The relationships for government expenditure and revenue are:

$$G1R = \sum_{j=1}^{12} GM_j \cdot SGK_j + \sum_{k=1}^{6} a_{31k} YH_k$$

$$G2E = \sum_{i=1}^{12} Z_{i23} B_i - 0.284 \left[\sum_{j=1}^{12} a_{30j} B_j X_j + \sum_{k=1}^{6} a_{30k} YH_k\right]$$

$$\begin{aligned} G2R = & \sum_{j=1}^{12} a_{28j} BI_j X_j + 0.324 \sum_{k=1}^{6} QH_{26k} BI_{26} + \\ & + 0.2327 Z_{26,40} BI_{26} + \\ & + 0.957 \left[\sum_{j=1}^{12} a_{29j} B_j X_j + \sum_{k=1}^{6} a_{29k} YH_k\right] + \\ & + \sum_{j=1}^{12} a_{32j} B_j X_j + \sum_{k=1}^{6} a_{32k} YH_k \end{aligned}$$

$$\begin{aligned} G3E = & \sum_{i=1}^{12} Z_{i24} B_i + \sum_{j=1}^{12} a_{25j} BI_j X_j + \\ & + \sum_{k=1}^{6} (a_{24k} + a_{25k}) YH_k - \\ & - 0.716 \left[\sum_{j=1}^{12} a_{30j} B_j X_j + \sum_{k=1}^{6} a_{30k} YH_k\right] + \sum_{i=1}^{12} Z_{i24'} \cdot B_i \end{aligned}$$

$$\begin{aligned} G3R = & \sum_{j=1}^{12} a_{25j} BI_j X_j + \sum_{k=1}^{6} a_{25k} YH_k + \sum_{k=1}^{6} a_{24k} YH_k + \\ & + a_{24,5} BI_5 X_5 + \sum_{i=1}^{12} Z_{i25} BE_i + \\ & + 0.043 \left[\sum_{j=1}^{12} a_{29j} B_j X_j + \sum_{k=1}^{6} a_{29k} YH_k\right] \sum_{i=1}^{12} Z_{i24'} \cdot BI \end{aligned}$$

$$\begin{aligned} PD = & - Z_{1,25}(BE_1 - B_1) - Z_{2,25}(BE_2 - B_2) - Z_{3,25}(BE_3 - B_3) - \\ & - Z_{9,25}(BE_9 - B_9) \end{aligned}$$

where:

- $G1R$ = revenues of public sector enterprise and net receipts from social security system;
- $G2E$ = total expenditures of conventional government sector;
- $G2R$ = total revenues of conventional government sector;
- $G3E$ = total expenditures of government trading sector;
- $G3R$ = total revenues of government trading sector;
- Z_{i23} = expenditure by conventional government sector in outputs of various industries;
- Z_{i24} = government trade sector purchases for domestic use;
- $Z_{i24'}$ = government trade sector imports.

The relationships for exports and imports are:

$$EXP = \sum_{i=1}^{12} Z_{i26} BE_i + RM\,BE_{21} + Z_{1,25} BE_1 + Z_{2,25} BE_2 + Z_{3,25} BE_3 + Z_{9,25} BE_9$$

$$IMP = \sum_{j=1}^{12} a_{25j} BI_j X_j + \sum_{k=1}^{6} a_{25k} YH_k +$$

$$+ \sum_{j=1}^{12} a_{26j} BI_j X_j + 0.676 \sum_{k=1}^{6} QH_{26k} BI_{26} + Z_{26,36} BI_{36}$$

The relationships for savings and investment are:

$$SP = \sum_{k=1}^{6} a_{34k} YH_k$$

$$SG1 \equiv G1R - G1E$$

$$SG2 \equiv G2R - G2E$$

$$SGE \equiv G3R - G3E$$

$$SF \equiv IMP - EXP$$

$$SSS = SP + SG1 + SG2 + SG3 + SF$$

$$INV = \sum_{i=1}^{12} INV_i B_i + Z_{26,36} BI_{36} (1.232)$$

$$DST = \sum_{i=1}^{12} Z_{i39} B_i$$

$$INVT \equiv INV + DST$$

$$SSS = INVT$$

where:

INV = total value of fixed investment;
INVT = value of total investment;
SF = foreign savings;
SG 1 = savings of first government sector;
SG 2 = savings of second government sector;
SG 3 = savings of third government sector;
SP = private savings;
SSS = total savings;
Z_{i39} = deliveries by i^{th} industry for inventory accumulation;
$Z_{26,36}$ = imports of investment goods.

The relationships for resource demands are:

$$LP_i = SPL_i HM_i / WAGE_i \qquad i = 1, \ldots, 12$$

$$KP_i = SPK_i HM_i / KAP_i \qquad i = 1, \ldots, 12$$

$$TP_i = SPT_i HM_i / RENT_i \qquad i = 1, \ldots, 12$$

$$LG_i = SGL_i GM_i / WAGEG_i \qquad i = 1, \ldots, 12$$

$$KG_i = SGK_i GM_i / KAPG_i \qquad i = 1, \ldots, 12$$

where:

TP_i = private demand for land in the i^{th} industry;
KG_i = government demand for capital in i^{th} industry;
KP_i = private demand for capital in i^{th} industry;
LG_i = government demand for labour in i^{th} industry;
LP_i = private demand for labour in i^{th} industry.

The relationships for resource constraints are:

$$EEL_i = LP_i - BL_i \qquad i = 1, \ldots, 12$$
$$EEK_i = LK_i - BK_i \qquad i - 1, \ldots, 12$$
$$EET = \sum_{i=1}^{4} LT_i - \sum_{i=1}^{4} BT_i$$

where:

EEK_i = excess demand for capital in i^{th} industry;
EEL_i = excess demand for labour in i^{th} industry;
EET_i = excess demand for land;
BK_i = supply of capital in i^{th} industry;
BL_i = supply of labour in i^{th} industry;
BT_i = supply of land in i^{th} industry.

Multi-stage optimization of long-term economic development

U. F. Pugachev *

A national economy is an extremely complex object for optimization. Ignoring social and other aspects the total number of items produced and used in a country like the USSR amounts to 2×10^7. If one assumes that every product can be produced on the average by 10 activities in 50 different regions and transported to 50 different regions of consumption, then the total number of variables for 15 years is $2 \cdot 10^7 \cdot 10 \cdot 50^2 \cdot 15$, or of the order of 10^{11}. It is plain that optimization of such an array as a whole is entirely out of the question. It is thus necessary to use the principle of multi-stepness, which implies hierarchical systems of models and optimization techniques.

Long-term national economic planning allows the use of multi-stage optimization systems. Long-term plans maximize the efficiency of optimization approaches. Further, the size of the long-range planning problems appears to be manageable and requires no more than two or three optimization steps.

This paper is devoted to a description of the system which was developed and partially used in connection with working out the Soviet long-term plan of economic development for 1976 to 1990 [1]. This system was elaborated in accordance with the approximative scheme of multi-stage optimization and embraces three levels: the "industry level", the multi-industrial complex and the national economy as a whole. The most complete presentation of the studies carried out is contained in [2].

The elaborated system was designed for direct adoption into planning practice. The system corresponds to the existing pattern of national economic planning, was based on well-known, already tested models, and makes no excessive demands upon computing facilities and mathematical optimization techniques.

Basic principles of multi-stage optimization

Co-ordination of the economic-mathematical models into the united multi-stage optimization system of long-term planning is possible provided some conditions are met. First one must ensure co-ordination between selected indices

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and the possibility of transition from aggregated values to disaggregated ones and vice versa.

A degree of choice is the second condition in co-ordination of calculations. Local plan elaborations cannot arrive by themselves at the plans that are best from the national economy point of view because in carrying them out it is not possible to account for the interests and capacities of the whole economy.

Partial calculations produce "brackets", which are sets of variables containing the correct solutions within themselves. The "brackets" obtained with the help of optimizing calculations must be promptly arrayed in the course of adjusted calculations of the models and lead to the end of the process at the univalent solution, that is, the final variant of the plan. Therefore the third condition of co-ordination of the models is the capacity to organize the cyclical process of sequential optimal calculations. In this case the number of adjusted calculations of the models must be small, otherwise the complex of models ceases to be a genuine planning tool. Thus the iterative process must rapidly converge.

The experimental elaboration of the optimal national economic plan by means of the complex of economic-mathematical models was described earlier [3, chapter 3]. The plan elaboration was constructed on the basis of an approximating scheme of multi-stage optimization providing the use of methods of aggregation and of alternative local optimization. The experiments indicated that if the multi-stage complex of models as well as the functional characteristics of the models were correctly constructed, then the solution for the optimum would be arrived at with few iterations.

Working out the optimal national economic plan is impossible without a specific optimality criterion. The problem of construction of the national economy criterion has always attracted great attention (see for example [2], chapter III). The criterion and its ability to express the requirement of maximal satisfaction of societal needs is now established. However, the studies revealed a number of principal and practical difficulties in exact quantitative accounting of and measure of consistency of societal needs of all types. The best way out may be found in carrying out alternative optimal calculations with the help of a simplified optimality criterion. This criterion is not an unchangeable one but on the other hand contains some variable, adjustable indices which allow testing of different hypotheses about economic development.

With such an approach the criterion becomes the tool of preliminary selection of those variants of the national economic plan which deserve further consideration. The final choice of the optimal plan must be made on the basis of analysis and comparison of calculated variants.

The desired maximal final product given its maximized requirements may serve as a simplified national-economic criterion. The requirements of the final product composition cannot be formulated prior to compilation of the plan. The plan must include alternatives based on different mixes in the final product. The final choice of the product composition and the appropriate alternative plan can be made after clarification of the real national economy capacities and needs in the course of the iterative process of the national economic planning. Here the role of social, political and other considerations is of great importance.

If the national economic criterion is decisive in optimization of the aggregate plan, then the same role is played by a local optimality criterion for levels of local planning. The local optimality criterion cannot be chosen arbitrarily as local proposals for the plan would then not be in accord with the broader interests of the national economy. A local criterion must correspond to the national economic one to effectively serve as a tool for economic management.

However, it is not possible to obtain a local criterion which fully corresponds to the interests of the whole economy in one operation. A simplified model is successively made more precise until it is acceptable.

In theory the best local criterion is the maximization of the national economic effectiveness [2, 4, 5] of the economic activity involved. The national economic effectiveness can be calculated by integrating the societal benefits of all of its specific components. In practice it is necessary to use simplified forms of the local criterion such as the maximum of the required product complex at fixed cost, minimal full discounted costs or a quadratic form. In this case it is necessary to assign either the composition, or the volume and composition of output, or alternatively the relationships of prices to output volumes. Initially it is possible to approximate these relationships and later to make them more accurate in the course of calculations based on the results of balances of production and distribution.

The most widespread form is the criterion of the minimum of full discounted costs of planned output. This criterion can be considered a special case of the general local criterion mentioned above because with fixed volumes of all outputs the general criterion becomes that of minimization of costs.

This local criterion may naturally correspond to the national economic one if all volumes of output, prices, and discount coefficients are correctly chosen from the optimal national economic plan. Such selection is only possible at the end of the multi-stage optimization process. At the beginning of the process, it is necessary to vary the criterion parameters in order to get the required "brackets" of indices of industrial plans.

The volumes of output and the discount coefficient "E" may serve as variable parameters. Output variations are necessary to embrace different levels of national economic needs in a sector output to ensure solutions for the balanced aggregate plan. Variations of E are needed to get alternative industrial plans with different labour to capital ratios. These plans are necessary for consequent balancing of the consolidated national economic plan with respect to labour and capital.

The quantitative determination of initial values of E has certain difficulties. Prior to carrying out all the optimization calculations E may be approximated as a "bracket". From experience the discount coefficient has a value from 0.12 to 0.25. It was reasonable to carry out calculations within these limits at the first stage, realizing that the consequent course of the iterative multi-stage optimization process would yield a precise value of E for the whole economy.

Methodology for optimization of the long-term national economic plans

At the preliminary stage of working out the long-range plan, variant calculations must be carried out by means of demographic models, macromodels of growth and utilization of national income, and models of demand structure determination. These with other models make it possible to outline alternative levels of the final product feasible under assumed hypotheses of socio-economic development of the country.

Further preliminary variant calculations of the inter-industry balance make it possible to determine feasible ranges of the national economic needs of output of every industry. Then the needs are described and classified by industry and location with appropriate consideration of the problems of industrial planning. The detailing is to be elaborated by industrial research institutes on the basis of analyses of development dynamics of the principal users of the industrial output. Interregional mathematical models can be used for this purpose.

After the demands for output of various industries are calculated, calculation of industrial plans can be made on the basis of optimization models of development and location. Industrial calculations must be carried out in the form of variants within the selected "brackets" with respect to the rate of output and the rate of discount, E. At least two values of output must be taken. One value is the minimum below which the industry output must not drop and another value is a maximum which can not realistically be exceeded. At least two values of the discount coefficient must be chosen. Thus the total number of the calculated industrial plan variants is four or more. If it later becomes desirable to look at other situations the total number of the alternatives will have to be increased.

In those cases where, in addition to industrial optimization calculations, studies of multi-industrial complexes optimization are also conducted, the results of alternative industrial studies are combined into complex development variants. The construction of the variants can also be done as for separate industries [6].

After having carried out alternative calculations for industries and multi-industry complexes, the consolidated optimization calculations must be done with the aid of the inter-industry optimization model. Each industry is represented by several variants of its development which had been calculated in the process of model optimization.

The basic economic constraints on capital investment and labour are explicitly introduced into the model. The model's optimality criterion is the maximization of final product given requirements for its composition.

By means of the inter-industry optimization model variants of the national economic plan are calculated. These variants are balanced in composition and attempt to secure the best use of labour and capital resources so as to maximize output. (Calculations may cover the basic indices and sections of the national economic plan, that is, volume of output and consumption, capital investments and labour resources, labour productivity, finance, balance of population incomes and expenditures, volume of sales, and foreign trade.)

Analysis of the calculated aggregate plan alternatives is done next. The choice of one variant (or some combination of variants) implies the direction of the long-term economic development plan. At the same time the value of the discount coefficient E is determined for the aggregate balance of capital investment and labour resources. Aggregate and industrial optimization calculations also give information to improve the price system with regard to capital stock payments, rents, output deficits, product substitution in use and so on.

The first iteration (the stage of principal directions of the long-term plan) thus comes to an end. Further calculations are needed for analysing the approved national economic plan to the level of specific industrial decisions. These calculations are done on the detailed long-term national economic plan.

It is worthwhile to begin the second iteration with a new solution for industrial problems. It is necessary to assign industries output volumes and the common discount coefficient E, which come from the multi-industrial problem. After that the industries must be subjected to repeat optimization calculations either as one variant or as very narrow "brackets" around the fixed outputs and around the common discount coefficient E. Newly obtained industrial plans are calculated with an accuracy requisite for five-year planning.

New variants of industrial plans can be also given to the multi-industrial optimization model which allows a further definition of the aggregate indices. This completes the second cycle of calculations.

Iterations can be carried on further, but are not necessary. Further iterations are needed only for balancing the national economic plan with respect to detailed available resources. This task will have been better carried out while working out one-year plans.

Unified industrial model of production development and location

Industrial optimization models are the basis of the multi-stage system of calculations. It was necessary to work out about 100 industrial optimal plans in variant form and again according to unified procedure and technology.

A unified industrial model of production development and location, and the mathematical software for industrial problems [3, chapter V] was constructed. The general notation of the unified model is:

$$\sum_{r \in R_l} B_{lr} \Lambda^{lr} = \sum_{s \in L} u^{ls}$$

$$\mathring{a}^l + \sum_{r \in R_l} A_{lr} \Lambda^{lr} = \sum_{s \in L} u^{ls}$$

$$F_{lr} \Lambda^{lr} \leqslant \delta_{lr} f^{lr}$$

$$\sum_{r \in R_{\mu l}} \delta_{lr} \leqslant 1; \quad \delta_{lr} \in \{0, 1\}$$

$$\Lambda^{lr} \geqslant 0; \quad u^{ls} \geqslant 0$$

$$\min \{ \sum_{l, r \in R_l} (c^{lr} \Lambda^{lr}) + \sum_{l,s} T^{ls} u^{ls} + E \cdot \sum_{l,r} K_{lr} \delta_{lr} \}$$

where:

- l, s = indices of possible locations of production and consumption;
- L = the number of these locations;
- r = the index of a development variant;
- R_l = the number of all such alternatives at location l;
- $R_{\mu l}$ = the number of incompatible variants of the μ^{th} group at the l^{th} location;
- B_{lr} = matrices of normative outputs;
- A_{lr} = matrices of normative inputs of transportable resources;
- F_{lr} = matrices of normative inputs of non-transportable (local) resources;
- $\mathring{a}^l$ = vectors of fixed requirements not depending on production development at l^{th} location;
- f^{lr} = vectors of available local resources;
- Λ^{lr} = vectors of production intensities;
- u^{ls} = vectors of transportation volumes from l^{th} location to s^{th} one;
- δ_{lr} = employment intensities of production development variants;
- c^{lr} = vectors of current inputs;
- T^{ls} = vectors of transportation costs;
- K_{lr} = variant investments;
- E = the discount normative.

The unified model embraces most of the possible modifications of the models of production development and location. The mathematical software was able to reduce greatly the labour involved in the preparation of initial information and output of calculated results.

Aggregate inter-sectoral optimization model

The upper level of multi-stage system of calculations is the inter-industrial optimization model. Each industry is represented by four or more development alternatives. The model contains explicitly written basic economic constraints on investments and labour.

The model's optimality criterion is maximization of final product under the condition that composition requirements are met.

The mathematical form of the model is:

$$x^s \sum_{\nu,j} a^{s\nu j} \xi^s_{\nu j}; \quad \sum_{\nu,j} \xi^s_{\nu j} = 1; \quad \xi^s_{\nu j} \geqslant 0$$

$$\xi^s_{\nu_1 j_1} \cdot \xi^s_{\nu_2 j_2} = 0;$$

$$\text{either } |\nu_1 - \nu_2| > 1; \quad \text{or} \quad |j_1 - j_2| > 1$$

$$\sum_{\nu} E^s_{\nu} \sum_{j} \xi^s_{\nu j} = \text{const. (s)}$$

$$\sum_{s} x^s - y \geqslant 0; \quad \sum_{s} x^s \leqslant r$$

$$y = \theta \bar{y}; \quad \theta \rightarrow \max.$$

where:

s = the industrial index;
ν = the index for E;
j = the index of output variants;
x^s = the vector of industrial outputs and inputs;
$a^{s\nu j}$ = the vector of the industrial plan basic variant;
$\xi^s_{\nu j}$ = intensities of basic plans x^s;
r = the vector of limited resources;
y = the vector of final demand;
$\bar{y}$ = the given configuration of final product;
θ = the maximized parameter.

With the help of this inter-industrial optimization model the balanced national economic plan variants are calculated so as to secure the best use of all labour and capital resources for a maximized final product.

System improvement

The first stage of improvement of the multi-stage long-term economic-planning optimization system will be the transition from static models to dynamic ones. This transition involves computer and information problems.

The first of these problems is caused by the large scale of the problems to be solved and their qualitative complications.

The second problem lies in the dynamization of all initial information, and in the transition from a static to a dynamic description of all variants of plant reconstruction and constructions, all industrial, regional and inter-industrial constraints, and all elements of the national economic and local optimality criteria.

Definition of the national economic optimality criterion leads to more comprehensive and accurate representation of societal needs. The extension of computing capacities will promote transition to more complicated forms of the national economic criterion; from linear to non-linear, from static to dynamic. For example, societal needs may be represented rather comprehensively with the aid of the national economic criterion described in [2]. In that case the non-linear objective function allows the interdependence between consumption volumes and prices to be accounted for. Weighting the static objective functions with respect to time (discounting) makes future interests commensurate with present ones.

Direct sources of information for determination of the national economic optimality criterion parameters are not always available. Some parameters must be determined by other methods.

Thus volumes of consumption and prices of final product components may be approximately known, prior to the plan compilation, from forecasts of national socio-economic development. Then coefficients of the linear terms may be taken as equal to predicted prices and the coefficients of the quadratic terms as "penalties" for deviations from desired consumption volumes. Sample surveys of family budgets, commerce statistics and other studies may yield values of price elasticities with respect to consumption volumes for some final product components. In these cases more valid coefficients of quadratic terms may be computed.

The weighting function for the national economic optimality criterion may be chosen on the basis of results of preliminary aggregative calculations [2, chapter II]. One may estimate the post-planning period on the basis of its asymptotic description or extrapolation of trends.

It is clear that the national economic criterion constructed in this fashion will not yield a comprehensive and absolutely accurate representation of societal needs. It is thus necessary to vary criterion parameters within certain bounds, successively defining them more accurately in the course of calculation.

Therefore even in the future the multi-stage optimization system must not be oriented towards working out the single final optimal long-term plan but rather towards alternative calculations of several projections which may deserve further examination. The degree of elaboration of the projections, their number and effectiveness will depend directly on the level of multi-stage system development reached.

Definition of the local optimality criterion will contribute to more comprehensive and precise representation of the national economy interests at the industrial and multi-industrial level. The extensions of computer facilities will make it possible to proceed from linear forms of the local criterion currently used to modifications more corresponding to its general form [4, 5].

The quadratic local criterion satisfies the theoretical requirements and can be calculated. The initial choice of parameters can be made on the basis of projected output volumes and prices in the same way as for the national economic criterion. Further parameter specification is possible in the course of the multi-stage optimization iterative process. At the system's local levels the principles of alternative determination of criterion parameters and of variant elaboration of plan projections may also be applied. It is necessary for the approximate description of industrial and multi-industrial development possibilities.

The improvement paths of different level models in the multi-stage optimization system will depend on computer facilities. At the industrial level the unified dynamic multi-product optimization model of production development and location with linear constraints and a quadratic criterion with continuous and integer variables will be used. The model (or system of models) of similar type with emphasis on the industrial structure will be used on the level of multi-industrial complexes. The aggregate optimization model will be a dynamic one with linear constraints and a quadratic criterion. Unified mathematical software of the same type as for industrial optimization is needed for practical application of the above models. This will minimize the work of preparation, computation and obtaining of calculated results.

Accordingly modified methods of mutual adjustments of model calculations will be used. The approximation scheme of the multi-stage optimization must be used in its dynamic form.

It is necessary to represent local goals as "dynamic polyhedrons" the basic points of which are characteristic paths of their development for the planning period. After aggregation of outputs (done by approximation by hyperplane) these paths will be included in the aggregate model as dynamic industrial and multi-industrial development alternatives.

In the course of aggregation, however, the need may arise to return to industries to calculate additional development alternatives. Dynamic adjustment of industrial plans requires the fulfilment of many more balances than static adjustment. The dynamic multi-stage optimization system, unlike the static system, requires more than independent calculations made on separate models. Hence new and more exacting demands are placed upon the organization of the whole optimization calculations system. The static scheme is more or less indifferent to when and how the industrial alternatives are to be calculated. The dynamic scheme must be carried out with a system of interrelated computer centres. All initial information must be continually stored in the computer's memory in readiness for additional calculations.

In the course of iterative multi-stage optimization of the system of dynamic models all calculations necessary for working out the detailed long-term economic development plan must be carried out with the necessary precision and inclusion of detail. If the plan embraces a period of 15 to 20 years, then the first five-year plan must be calculated with precision especially for the first one to two years. Thus the harmonic adjustment of the long-term, five-year and current plans may be accomplished.

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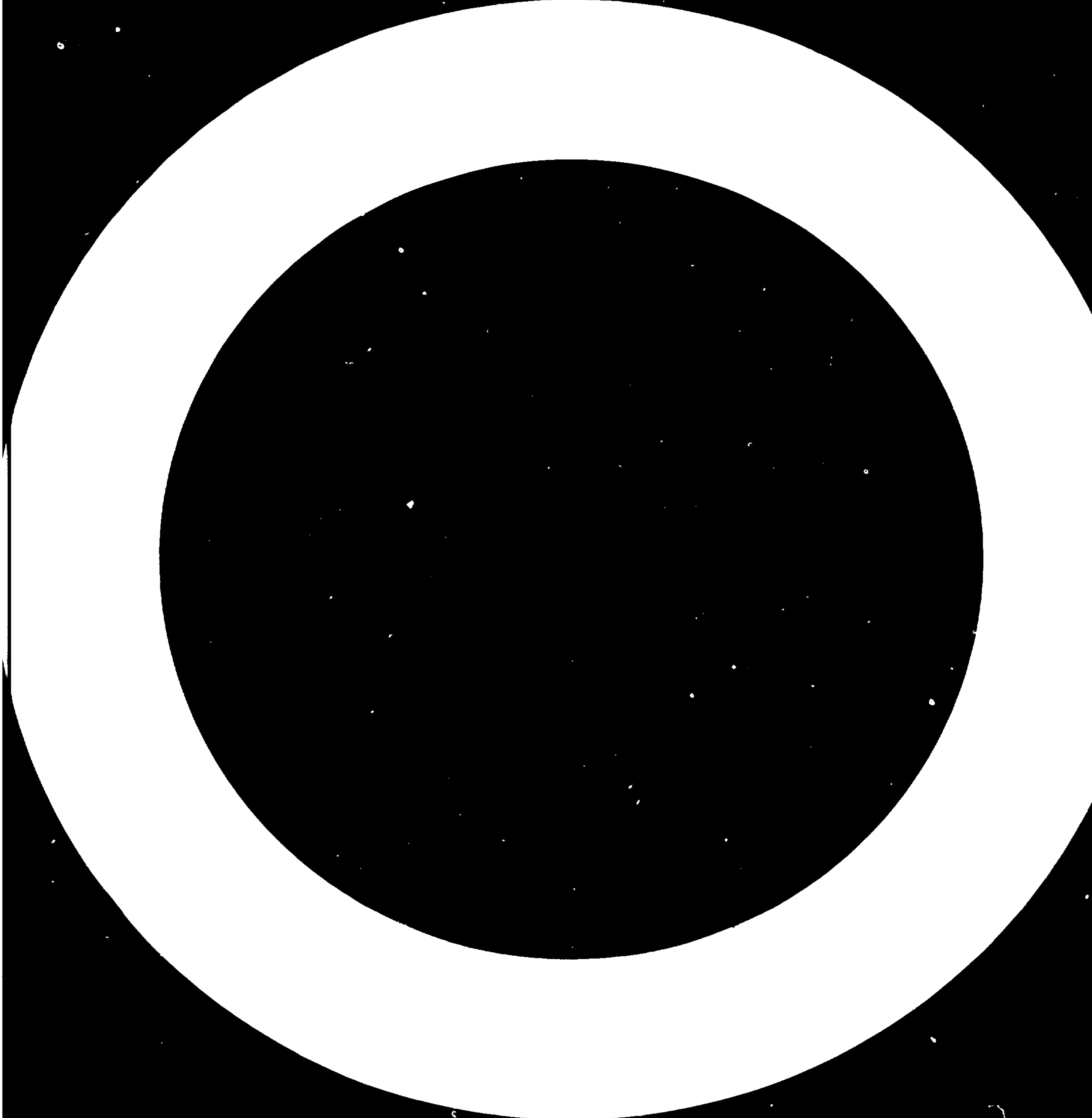
Economic growth and structural change



Sources of industrial growth and structural change: a comparative analysis of eight economies

Yuji Kubo and Sherman Robinson *

There have been a number of studies comparing the aggregate growth experience of different developing economies, but there have been no studies to date that have attempted to make such comparisons in a multi-industry framework. Such a comparison must be based on comparable I/O data for different economies and for the different time periods. Such data have not been available until recently. As part of a research project at the World Bank, the necessary I/O data have been compiled for eight economies—Colombia, Israel, Japan, Mexico, Norway, the Republic of Korea, Turkey and the Taiwan Province of China. For each economy, at least three mutually consistent I/O tables covering 15 or more years have been compiled.[1] The tables for the different economies have been aggregated into a comparable classification which is given in the annex.

The I/O data base allows comparison of the growth and structural changes of the eight economies. The methodological approach described in more detail below is to decompose the sources of industry and aggregate growth into four groups:

(a) Domestic demand expansion;

(b) Export expansion;

(c) Import substitution;

(d) Change in intermediate input use (that is, input coefficients).

The approach views from the point of view of sources of demand rather than of changes in factor supply. A more complete analysis would seek to integrate the two approaches, but much can be learned from an analysis from the demand side alone.

In this paper, the results from such a decomposition analysis of growth for the eight economies are presented. The results indicate that there are distinctive variations among economies and over time in the important sources of growth and that these variations are related to the different development strategies that have been pursued. Thus the analysis provides some insight into the relationship between an economy's development strategy and the resulting changes in growth and economic structure, even though the I/O methodology does not explicitly include policy instruments and the associated causal links.

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[1]The exception is Israel, where the coverage is the years between 1958 and 1968.

First, the methodology is discussed. Next, the important characteristics of the eight economies, including their policy regimes, are briefly described. Then, there is a discussion of the decomposition of growth both at the aggregate level and at the individual industry level, comparing the experience in the eight economies. This is followed by a brief conclusion.

Methodology

The methodology for the present analysis of industrial growth patterns is based on the work of Chenery [1], and Chenery, Shishido and Watanabe [2]. Those approaches start from an accounting identity of demand and supply and attempt to explain differential growth of disaggregated industrial production by such causal factors as non-proportional expansion of domestic demand and exports, import substitution and changes in intermediate input requirements. However, apart from the explicit use of I/O relationships in Chenery, Shishido and Watanabe [2], there is a difference in the treatment of import substitution in the two approaches. This difference has been discussed extensively in the literature.[2] Recently, Syrquin [3] proposed an alternative measure of sources of industrial growth and structural change which extends and refines the original method. The present analysis is based on the Syrquin extension of the methodology.[3]

In an open Leontief system, the basic material balance between demand and supply for a given commodity can be written as:

$$X_i = u_i (D_i + W_i) + E_i \tag{1}$$

where:

X_i = domestic production;
D_i = domestic final demand;
W_i = intermediate demand;
E_i = exports;
u_i = the domestic supply ratio.

Intermediate demand is determined by production levels and input coefficients (a_{ij}) as $W_i = \sum_j a_{ij} X_j$. System (1) can be solved as

$$X_i = \sum_j \bar{r}_{ij} (u_j D_j + E_j) \tag{2}$$

where the $\bar{r}_{ij}$ are elements of the Leontief domestic inverse $(I - \hat{u} A)^{-1}$, in which $\hat{u}$ is the diagonal matrix of the u; and A is the input coefficient matrix.

Equation (2) shows that the change in gross output over time can be traced back to changes in domestic demand, exports, domestic supply ratios, and input coefficients. The difference of (2) between two time periods is:

[2]See, for example, Desai [4], Eysenbach [5], Fane [6, 7] and Morley and Smith [9].
[3]The proposed methodology is also discussed in Chenery and Syrquin [8].

$$\Delta X_i = \sum_j r_{ij}^2 u_j^2 \Delta D_j + \sum_j \bar{r}_{ij}^2 \Delta E_j + \sum_j \bar{r}_{ij}^2 \Delta u_j (D_j^1 + W_j^1) + \\ + \sum_j \bar{r}_{ij}^2 u_j^2 \sum_k \Delta a_{jk} X_k^1 \tag{3}$$

where superscripts 1 and 2 denote the initial and terminal year, respectively. The first two terms on the right-hand side are the changes in gross output induced by expansion of domestic demand and exports. The third term measures the import substitution effect on production, as expressed by the changes in domestic supply ratios. The last term shows the effect of changes in input coefficients. This effect represents widening and deepening of interindustry relationships over time brought about by the changing mix of intermediate inputs. The effect is caused by the changes in production technology as well as by substitution among various inputs (perhaps produced by changes in relative prices), although one cannot separate these two causes.

The above equation decomposes the first difference in output into various causal factors. Alternatively, one can derive, also from (2), an equation which decomposes the deviation of output from balanced growth into similar expressions; balanced growth is defined by a path in which all elements grow proportionally at the growth rate of national income without changing the initial structure.[4] The first difference formulation is useful in identifying the "engines" of growth of industries, and the deviation formulation is more suitable for analysing structural change associated with rising income. Both formulations will be used in the empirical analysis below.

In the decomposition equation, import substitution is defined as arising from changes in the ratio of imports to total demand. This implicitly assumes that imports are imperfect substitutes for domestic goods, since the source of supply constitutes an integral part of the economic structure. It differs from the treatment of imports by Chenery, Shishido and Watanabe [2] as perfect substitutes for domestic goods. In their treatment, domestic and imported goods are lumped together without distinction in final and intermediate demand and hence in the I/O material balance equation. Note that in equation (1) imports are not included in the material balance equation and can be treated independently of the domestic production balances.

Morley and Smith [9, p. 729] proposed a more extreme definition of import substitution which measures "the domestic production necessary to substitute completely for imports" including the intermediate effects of replacing a unit of

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[4]Letting λ denote the ratio of national income of the terminal to the initial year, the resulting equation has the form

$$\delta X_i = \sum_j \bar{r}_{ij}^2 u_j^2 \delta D_j + \sum_j \bar{r}_{ij}^2 \delta E_j + \sum_j \bar{r}_{ij}^2 \Delta u_j \lambda (D_j^1 + W_j^1) + \sum_j \bar{r}_{ij}^2 u_j^2 \sum_k \Delta a_{jk} \lambda X_k^1$$

where δ is deviation from balanced growth (for example $\delta X_i = X_i^2 - \lambda X_i^1$). The methodology presented in Chenery [1], Chenery, Shishido and Watanabe [2] and Syrquin [3] are in deviation forms. Syrquin provides a discussion of relationships between the first difference and deviation measures.

imports. They count both the direct and indirect domestic production required to produce formerly imported commodities as import substitutions. In addition to assuming that domestic and imported goods are perfect substitutes, their approach has the unusual property that import substitution can be observed in industries in which there are no imports.

The aggregate contribution of import substitution to growth, as defined here, is sensitive to the level of industry disaggregation. For example, it is possible to have positive import substitution in every industry but have the ratio of total imports to total demand increase because of changes in the industry composition of demand. In the analysis below, the sources of growth estimates for the aggregate sectors will be obtained from measurements carried out at the industry level.

The treatment of imports in equation (1) such that a single domestic supply ratio is assumed for both intermediate and final demand can easily be extended to include different ratios for different categories of demand. For example, the matrix of domestic input coefficients can have different domestic supply ratios cell-by-cell and be given by a matrix A^d instead of $\hat{u}A$. The resulting decomposition equation is similar to equation (3) except that the import substitution for intermediate goods must be separated from that for final demand using different import ratios by destination, and that the domestic supply ratios in the first and fourth terms must be replaced by those for final and cell-by-cell intermediate demand, respectively.[5]

The effect of changes in input coefficients includes changes in the total coefficients and does not separately distinguish between imported and domestically produced goods. Thus, the input coefficients may remain constant ($\Delta a_{ij} = 0$) and hence the last term in (3) will be zero even though there are changes in domestic supply ratios (which result in changes in $\hat{u}A$ or A^d). Changes in technology are defined as changes in the total coefficients while any changes in the intermediate domestic supply ratios are included in the import substitution term.

Each term in the decomposition is multiplied by elements of the Leontief domestic inverse. The terms therefore capture both the direct and indirect impact of each causal expression on gross output taking into account the linkages through induced intermediate demand.

There is an index number problem implicit in the decomposition equation because the decomposition can be defined either using the terminal-year structural coefficients and initial-year volume weights as in equation (3) or using the initial-year structural coefficients and the terminal-year volume weights, which are analogous to Paasche and Laspeyres price indices. In the analysis below, both indices have been computed separately for the decomposition in each period and the averages of the two are presented.[6]

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[5]See Syrquin [3] for a complete discussion of this approach.

[6]Another approach suggested by Fane [6, 7] is to define continuous growth paths and compute appropriate Divisia indices. Such indices would probably not differ significantly from the average of Paasche and Laspeyres indices.

Aggregate growth and structural change

Table 1 presents comparative economic indicators for the eight economies. The economies varied widely in population and level of income. Israel had the smallest population, 2.7 million in 1968; Japan had the largest population, 104.3 million in 1970. Per capita gross national product in the sample varied from a low of $131 (the Republic of Korea in 1955) to a high of $2,769 (Norway in 1969) in 1970 dollars. The economies also varied widely in trade orientation. Turkey was the most self-sufficient, with ratios of total exports to gross domestic products not exceeding 10 per cent in any period. Colombia, Mexico and Turkey had quite low ratios of manufacturing exports to gross domestic products throughout the period while Japan had steadily high ratios. Israel, Norway, the Republic of Korea and Taiwan Province had increasing shares of manufactured exports, with the Republic of Korea and Taiwan Province showing the most dramatic change.

Table 2 presents information regarding the development and foreign trade policies of the eight economies. Column three distinguished between import-substitution and export-promotion strategies. Israel, Japan, the Republic of Korea and Taiwan Province have pursued export promotion strategies coupled with varying degrees of control over imports. Israel is something of a special case, having pursued both strong import substitution and export promotion policies simultaneously. Colombia and Mexico seem representative of the import-substitution development strategies pursued in much of Latin America. Turkey in the first two sub-periods followed a largely self-sufficient development strategy, with both strong import-substitution and policies that discriminated against exports. In the 1968–1973 period, Turkey devalued and received windfall foreign exchange receipts in the form of workers' remittances from abroad. During that period, both exports and imports expanded.

Column four gives the trade and exchange-rate regime according to a classification scheme developed for the series of country studies under the general editorship of Bhagwati [11] and Krueger [12]. The economies have varied widely in their reliance on quantitative restrictions on imports and in the degree of convertibility of their currencies. The last column indicates the level of foreign capital inflow. Only Israel, the Republic of Korea and Taiwan Province have had periods of medium and high levels of capital inflow, as a ratio to gross domestic product.

Aggregate structural change and its causes

The model will now be used to analyse the causes of industrial growth and structural change in the eight economies. The sources of growth measurements were carried out at a comparable level of about 20 industries and the results were further aggregated into a smaller number of sectors to simplify exposition, except where growth of individual industries is discussed.

During the period under study, all eight economies underwent significant changes in the structure of production. The standard pattern of rapid expansion in

TABLE 1. COMPARATIVE ECONOMIC INDICATORS

Economy	Year	Population		Per capita GNP		Ratios to GDP in current prices (%)					
		Number (million)	Average annual growth rate (%)	Level ($ 1970)	Average annual growth rate (%)	Primary exports	Manufactured exports	Total exports[a]	Gross domestic investment	Primary value added	Manufacturing value added
Colombia	1953	12.5		274		14.7	0.2	15.6	15.3	40.4	18.0
	1966	18.4	3.0	330	1.4	8.6	1.0	12.1	20.4	32.4	22.4
	1970	20.6	2.9	369	2.8	9.9	1.2	14.2	21.5	30.7	23.0
Israel	1958	2.0		1 067		4.1	4.7	11.5	28.1	13.0	31.7
	1965	2.6	3.6	1 587	5.8	4.4	10.6	18.9	28.1	8.2	36.3
	1968	2.7	2.3	1 759	3.5	5.1	14.6	27.3	22.6	7.9	33.4
Japan	1955	89.0		500		1.3	9.1	10.7	24.7	25.0	26.5
	1960	94.1	1.1	753	8.5	1.0	8.4	11.1	33.7	15.1	38.2
	1965	98.8	1.0	1 159	9.0	0.5	8.9	10.8	32.9	10.7	37.9
	1970	104.3	1.1	1 897	10.4	0.4	9.4	11.2	39.4	7.2	43.2
Mexico	1950	26.3		380		n. a.	n. a.	14.1	13.5	20.3	24.6
	1960	36.0	3.2	479	2.9	4.7	1.6	11.3	20.1	17.5	26.7
	1970	50.4	3.4	670	3.4	2.1	1.6	8.1	19.6	12.7	30.9

Norway	1953	3.4		1 171		9.9	9.1	38.1	29.5	15.5	35.0
	1961	3.6	0.9	2 028	7.1	6.4	12.3	39.7	31.1	9.9	32.6
	1969	3.9	0.8	2 769	4.0	5.0	17.2	41.2	25.0	6.2	34.0
Republic of Korea	1955	21.4		131		0.7	0.2	1.7	12.0	48.1	13.1
	1963	27.0	2.8	149	1.6	1.3	1.2	4.8	18.6	46.8	16.9
	1970	31.4	2.2	250	7.7	2.4	8.7	14.8	27.3	32.4	25.5
	1973	32.9	1.6	323	8.9	4.1	24.3	31.7	26.0	28.8	29.7
Turkey	1953	22.8		239		6.9	0.3	7.8	12.4	51.1	12.1
	1963	29.7	2.7	319	2.9	5.1	0.3	5.5	15.4	40.7	19.0
	1968	33.5	2.4	377	3.3	4.1	0.3	5.3	18.0	32.3	24.4
	1973	38.3	2.7	461	4.1	5.6	1.5	7.6	19.0	31.0	24.5
Taiwan Province	1956	9.2		203		8.9	0.9	9.0	15.9	29.8	24.3
	1961	11.0	3.6	231	2.6	6.0	5.1	12.8	19.8	29.3	25.4
	1966	12.8	3.1	305	5.7	8.2	8.7	20.6	23.1	24.2	29.8
	1971	14.8	2.9	426	6.9	6.0	24.6	36.8	26.1	14.8	39.3

Source: Comparative data compiled for "Patterns of industrial development" research project, World Bank; *World Tables 1976*, World Bank; background data for Chenery and Syrquin [10].

[a]Merchandise exports plus exports of non-factor services.

TABLE 2. DEVELOPMENT AND FOREIGN TRADE POLICIES

Economy	*Years*	*Development strategy*[a]	*Trade and exchange rate regime*[b] *(Bhagwati-Krueger Scheme)*	*Level of foreign capital inflow*[c]
Colombia	1953–1966	IS–MC	III, IV, II, III, IV	Low
	1966–1970	IS–MC	IV	Low
Israel	1958–1965	IS–EP–MC	IV	High
	1965–1968	IS–EP–MC	IV	High
Japan	1955–1960	EP–MC	II	Low
	1960–1965	EP–MC	III, IV	Low
	1965–1970	EP–MC	IV	Low
Mexico	1950–1960	IS–MC	IV, V	Low
	1960–1970	IS–MC	V	Low
	1970–1975	IS–MC	V, I	Low
Norway	1953–1961	Open	V	Low
	1961–1969	Open	V	Low
Republic of Korea	1955–1963	IS–MC	II, III, II	High
	1963–1970	EP–MC	III, IV	High
	1970–1973	EP–MC	IV	Middle
Turkey	1953–1963	Semi-closed	I, II, III, IV	Low
	1963–1968	Semi-closed	II	Low
	1968–1973	Semi-open	II, III, IV	Low
Taiwan Province	1956–1961	IS–MC	II, III	Middle
	1961–1966	EP–MC	IV	High
	1966–1971	EP–MC	IV, V	Middle

[a]IS is import substitution. EP is export promotion. MC is import controls, quantity restriction or protective tariffs. Semi-closed is encouraging import substitution and discriminating against exports. Semi-open is few quantity restrictions and no discrimination against exports. Open is export promotion with little import controls.

[b]I is intense quantity restrictions (QR). II is intense QR with corrective measures. III is formal devaluation or reduced reliance on QR. IV is further reduction of QR. V is liberalized exchange-rate regimes and no QR. See Bhagwati (1978) [11] for a detailed discussion. Trade and exchange-rate regimes for Colombia, Israel, the Republic of Korea and Turkey are taken from formal country studies in the Bhagwati-Krueger series. Those for other areas are our estimates. Different phases are shown for each period in the order of their emergence.

[c]Foreign capital inflow as percentage of GDP is classified as high (greater than 10 per cent), middle (between 5 and 10 per cent) and low (less than 5 per cent).

manufacturing production and relative decline in primary production in the course of development is shown in table 3 by the differential growth rates of output as well as by the output deviations from balanced growth shown in column 1 (as percentages of the change in aggregate gross output). The output deviations indicate the changes in the composition of gross production in each economy and do not depend on currency units. The figures are thus comparable across economies and show that structural change in production during the period under study proceeded most rapidly in the Republic of Korea and Taiwan Province and least rapidly in Norway and Israel, with other economies falling in between.

The sources of differential output growth in the three sectors are shown in table 3, which are also expressed as percentages of the change in aggregate gross

TABLE 3. COMPONENTS OF OUTPUT DEVIATION BY SOURCE FROM BALANCED GROWTH[a]

(Percentage)

Production category by economy	*Average output growth rate*	*Output deviation*	*Sources of output deviation: Domestic demand expansion*	*Export expansion*	*Import substitution*	*Change in input coefficients*
Colombia (1953–1970)						
Primary	4.5	−3.7	−7.9	3.8	0.3	0.1
Manufacturing	8.1	16.1	1.4	1.4	7.8	5.6
Services	5.5	1.9	−0.8	1.2	0.4	1.0
Israel (1958–1968)						
Primary	6.8	−4.4	−4.7	1.0	0.0	−0.7
Manufacturing	12.3	6.8	−2.7	7.5	−0.6	2.5
Services	10.2	−3.2	0.2	4.9	−4.2	−4.1
Japan (1955–1970)						
Primary	2.2	−7.6	−3.2	−0.2	−1.9	−2.4
Manufacturing	13.3	12.6	5.0	2.8	−1.2	5.9
Services	11.4	1.0	−1.0	1.2	−1.0	1.7
Republic of Korea (1955–1973)						
Primary	5.7	−11.3	−7.7	1.8	−2.3	−3.1
Manufacturing	15.8	27.5	4.7	21.1	1.4	0.2
Services	10.3	4.6	0.9	4.3	0.2	−0.7
Mexico (1950–1975)						
Primary	4.8	−5.8	−2.3	−2.6	−0.2	−0.7
Manufacturing	7.7	10.1	3.5	−0.2	4.3	2.5
Services	6.4	1.0	0.5	−0.5	0.6	0.5
Norway (1953–1969)						
Primary	2.5	−4.7	−3.1	0.0	−2.0	0.4
Manufacturing	5.2	7.8	−1.6	12.6	−9.2	6.0
Services	4.8	6.2	−0.8	9.3	−3.1	0.8
Turkey (1953–1973)						
Primary	2.5	−18.1	−11.5	−2.1	0.2	−4.7
Manufacturing	8.1	17.2	4.6	4.5	2.4	5.8
Services	6.7	9.0	3.6	2.1	0.2	3.1
Taiwan Province (1956–1971)						
Primary	7.1	−7.6	−4.2	1.5	−2.7	−2.2
Manufacturing	16.2	28.2	0.7	20.1	3.5	3.9
Services	9.7	−2.0	−4.5	3.5	0.1	−1.2

[a]Results for Colombia, Israel and Mexico are preliminary.

output and add up to column 1 for each sector. In virtually all economies the relative decline in primary production was mainly caused by the compositional shift of domestic demand, presumably resulting from the low income elasticity of demand for primary products and high elasticity for most manufacturing products. The changes in input coefficients also resulted in reducing the demand for, and hence production of, primary products in almost all economies at the same time increasing the demand for manufactured goods. The effect of changes in input coefficients and in the composition of domestic demand together account for over 75 per cent of the relative decline in primary production in all economies except Mexico and Norway (about 50 per cent).

The sources of the disproportional expansion of manufacturing production reveal an interesting variation among the eight economies. In the Republic of Korea and Taiwan Province, the significant deviation of manufacturing output from balanced growth is largely due to the rapid expansion of exports, which accounts for over 70 per cent of the deviation, with all other considerations playing a minor role. Thus, the rapid structural change experienced in these two economies is due to their export-led, outward looking industrial development strategies adopted since the early 1960s. Japan also adopted various export promotion schemes in the postwar period with fairly restrictive import controls. The fact that the export expansion effect was not as pronounced as in the Republic of Korea and Taiwan Province can be explained by the much larger size of Japan's domestic market relative to exports, which also accounts for the importance of domestic demand expansion in its industrial growth.

In contrast, the manufacturing expansion in Colombia and Mexico was little dependent on export expansion. Instead, import substitution played the most important role (about 45 per cent) in the overall expansion of manufacturing production, which is reinforced by the effect of the changes in input coefficients. Thus, the development experience in these economies reflect their import-substitution oriented, inward-looking development strategies. Turkey, with the effects of import substitution and changes in input coefficients accounting for half the manufacturing output deviation, may be classified with this group, although import substitution played a more limited role than in the other two countries. It is not surprising to observe a sizable effect of changes in input coefficients in these economies as import substitution often requires introduction of new technologies, although this is not specific to the economies that adopted an import substitution strategy.

Israel and Norway have a different pattern of manufacturing growth, with export expansion and change in input coefficients more than offsetting the inadequate domestic demand growth and increased dependence on imports (negative import substitution). In particular, in Norway, rapid export expansion was associated with a significant increase in import dependence, resulting from a liberal import policy in that country.[7] In turn, as their main export items are manufactured products, the significant effect of changes in input coefficients for the manufacturing sector can be viewed as reflecting technological changes required to meet international competition. An important difference of this growth pattern from that of the Republic of Korea and Taiwan Province is the relatively slow rate of structural change within the economy. Considering the large difference in per capita incomes between the two groups, Israel and Norway may reflect the growth pattern of more mature industrial economies.

[7]Norway's growth pattern and trade structure in relation to changes in comparative advantage is discussed in detail in Balassa [13, 14].

Growth of manufacturing production over time

The above analysis of structural change for the whole period in each economy revealed different growth patterns, especially for manufacturing, among the eight economies which reflect differences in their development strategies. However, the analysis for the whole period conceals a considerable variation in the relative importance of causal factors over time. In order to bring out this variation, the decomposition was carried out for each sub-period for each economy. Since the major feature of the structural changes observed above is the rapid expansion of manufacturing production, attention will be focused on the sources of growth of the manufacturing sector and the variation over time. For this purpose, the sources of growth were based on the first differences in output rather than in output deviations, and the results presented in table 4 are for the manufacturing sector only. Additional sub-periods, 1914–1935 and 1935–1955, were added for Japan in this table to incorporate the earlier stage of its transition to development. As no I/O tables are available for 1914 and 1935, the 1955 table was applied for the calculations involving these periods. Hence, the estimates for these periods, especially for 1914–1935, are subject to unknown error and are not comparable to those for other periods, and economies. The last column shows the average output growth rates in the respective sub-periods, and columns one to four show the percentage contributions of each causal factor to the change in gross manufacturing output, which add up to 100 per cent for each sub-period.

In all economies, a large proportion of the increase in manufacturing output is induced by the growth of domestic final demand. Its relative magnitude is generally larger in large economies such as Japan, Mexico and Turkey, and smaller in Israel, Norway and Taiwan Province, reflecting the differences in the size of the domestic market.

In most economies there is a period which is marked by a sizable effect of import substitution on manufacturing output growth. In the Republic of Korea and Taiwan Province, this period corresponds to their early phase of industrialization, from the mid-1950s to early 1960s. In Japan, the period was dominated by postwar reconstruction. However, after these particular sub-periods, import substitution was insignificant as a source of manufacturing output growth. In Colombia, Mexico and Turkey, the import substitution phase continued for an extended period of 13 to 20 years. Although the choice of benchmark years precludes exact estimates, the period from the early 1950s to the late 1960s was one of important import substitution in these economies. The importance of import substitution declined dramatically thereafter.

The period marked by significant import substitution is associated with a comparable effect of changes in input coefficients in many cases. However, the changes in input coefficients continue to be important to the growth of manufacturing production even after import substitution becomes insignificant. This reflects the continuing nature of technological change in production as the economies move towards the more intermediate-intensive production structure of developed economies.

TABLE 4. SOURCES OF CHANGE IN MANUFACTURING PRODUCTION[a]

(Percentage)

Period for each economy	*Average annual growth rate*	*Sources of change*			
		Domestic demand expansion	*Export expansion*	*Import substitution*	*Changes in input coefficients*
Colombia					
1953–1966	8.3	60.3	6.8	22.1	10.8
1966–1970	7.4	75.5	4.7	4.3	15.5
Israel					
1958–1965	13.6	58.9	26.2	9.8	5.2
1965–1968	9.4	68.7	54.8	−27.7	4.2
Japan					
1914–1935	5.5	69.9	33.6	4.7	−8.2
1935–1955	2.8	70.9	−7.1	15.5	20.7
1955–1960	12.6	76.2	11.9	−3.4	15.2
1960–1965	10.8	82.3	21.7	−0.3	−3.7
1965–1970	16.5	74.2	17.6	−1.4	9.6
Mexico					
1950–1960	7.0	71.8	3.0	10.9	14.4
1960–1970	8.6	86.1	4.0	11.0	−1.0
1970–1975	7.2	81.5	7.7	2.6	8.2
Norway					
1953–1961	5.1	64.3	37.3	−15.9	14.4
1961–1969	5.3	50.6	58.8	−19.4	10.0
Republic of Korea					
1955–1963	10.4	57.3	11.5	42.2	−11.0
1963–1970	18.9	70.1	30.4	−0.6	0.1
1970–1973	23.8	39.0	61.6	−2.5	1.8
Turkey					
1953–1963	6.4	81.0	2.2	9.1	7.7
1963–1968	9.9	75.2	4.5	10.4	9.9
1968–1973	9.7	68.2	21.0	−1.6	12.3
Taiwan Province					
1956–1961	11.2	34.8	27.5	25.4	12.3
1961–1966	16.6	49.2	44.5	1.7	4.6
1966–1971	21.1	34.9	57.0	3.8	4.3

[a]Results for Colombia, Israel and Mexico are preliminary.

In the three Asian economies, the import substitution period was followed by periods where export expansion became very important. In particular, in the Republic of Korea and Taiwan Province, import substitution accounted for more than half of the growth of manufacturing in the final sub-period, and in Japan for 18 to 22 per cent between 1960 and 1970. The increased importance to manufacturing growth of export expansion in these economies reflects the shift in emphasis of the governments' trade and industrial policies away from encouraging domestic industries to import-substitute through restrictive import measures to promoting exports through various incentive policies. These policies were very successful as shown in the sources of growth decompositions. Behind these successes, however, there was a period of extensive import substitution and technological improvements during which domestic industries developed and strengthened international competitiveness. Without this preparatory stage, the successful industrialization led by export expansion in these economies might not have been possible.

In Colombia and Mexico, the relative importance of exports to manufacturing growth did not increase much after the import substitution phase. In Turkey, the export expansion effect accounted for 21 per cent of the growth of manufacturing output between 1968 and 1973, due mainly to the currency devaluation and favorable export market conditions during this period. The growth of manufacturing production in these countries, however, was generally much slower than that of the Asian economies during their export promoting phases. The failure of export expansion to contribute more to the growth of manufacturing may be attributed to the prolonged protection of domestic industries which left them unable to compete in international markets or to the absence of export incentives.

Israel and Norway show a different pattern from the above two groups. In both economies, export expansion played an important role in manufacturing output growth, but the dependence on imports increased substantially at the same time (except for 1958–1965 in Israel). The sizable effect of the changes in input coefficients in these economies may reflect technological change in response to international competition. The significant effect of export expansion and the increased dependence on imports show that these two economies, especially Norway, are rapidly liberalizing trade and moving towards more open development strategies.

Growth patterns of individual industries

The analysis at the aggregate level showed that variations over time in the sources of manufacturing growth roughly match the differences and changes in development strategies adopted in the eight economies. However, the industry pattern of growth and the relative importance of different industries vary from economy to economy not only because of differences in policies, but also because of differences in stage of development and resource availability. The selected industries are: textiles and clothing, chemicals, basic metals and machinery. The first two are typical of the industries that achieve prominence at early and middle stages of development, respectively, and the latter two usually become important at a later stage.[8]

The sources of growth of these four industries are shown in tables 5 to 8 for the eight economies and for each sub-period. The measurements are based on the

[8] See Chenery and Taylor [15].

first difference in gross output. In each table, column six shows the average annual growth rate of industry output and columns one to five give the results of the sources of growth decomposition expressed as percentages of the change in aggregate gross output over all industries in each sub-period. Thus, columns two to five add up to column one (except for rounding).

Textiles and clothing

As shown in column six of table 5, the growth of textiles and clothing production has been quite diverse among the eight economies. Growth was most rapid in the Republic of Korea and Taiwan Province, and almost stagnant in Norway, with the other economies falling between these two extremes.

In all economies, a substantial part of this industry's output growth was brought about by domestic demand expansion, coupled in some cases with significant export expansion. The changes in input coefficients played very little role in all economies and significant import substitution was observed only in the first sub-period in the Republic of Korea.

The experiences of Japan, the Republic of Korea and Taiwan Province are similar and represent an interesting pattern. In the Republic of Korea, apart from the significant effect of domestic demand expansion, this industry's growth benefited substantially first from import substitution and then later from rapid growth of exports. Taiwan Province's textiles and clothing sector experienced the same pattern, although the major import substitution phase in this sector (1950 to 1956) falls outside the period of present analysis. This can be inferred from the Government's decision in 1950 to develop the textiles industry because "at that time textiles constituted the largest item of import" (K. Y. Yin [16]). The growth pattern in the Republic of Korea and Taiwan Province closely resembles the development of the textiles and clothing industry in Japan prior to 1941, where the import substitution phase (1890 to 1913) was followed by rapid export expansion during the 1914 to 1935 period. The development of the textiles and clothing sector in Japan prior to 1941 is described in detail in Lockwood [17]. All these economies have a very limited natural resource base, and the early development of textiles was largely aimed at reducing imports and providing an additional source of foreign exchange. The sector exhibits a similar pattern in Israel. However, as the more recent experience of Japan suggests, the relative importance of the textiles and clothing sector to the foreign exchange earnings and to the economy's growth seems to decline gradually as development proceeds.

In contrast, the industry's growth in Colombia, Mexico and Turkey was almost entirely due to domestic demand expansion. The fact that export expansion played a very small role except for the last sub-period in Turkey seems to reflect the inward-looking nature of these economies' development.

Chemicals

The sources of growth of the chemical industry are shown in table 6. Unlike textiles and clothing, the importance of the chemical industry to the economy's

TABLE 5. SOURCES OF CHANGE IN TEXTILES AND CLOTHING PRODUCTION[a]

(Percentage change in aggregate gross output)

Period and economy	*Change in sectoral output*	*Sources: Domestic demand growth*	*Export growth*	*Import substitution*	*Changes in input coefficients*	*Average annual growth rate*
Colombia						
1953–1966	6.5	5.0	0.4	0.8	0.3	6.1
1966–1970	8.5	7.6	1.0	0.4	−0.6	7.8
Israel						
1958–1965	5.7	3.0	2.7	−0.1	0.2	11.7
1965–1968	5.9	2.9	5.9	−2.7	−0.2	7.9
Japan						
1914–1935	12.5	4.5	5.8	0.3	1.9	7.7
1935–1955	0.8	5.7	−2.3	0.3	−2.9	0.2
1955–1960	6.0	3.9	1.0	−0.1	1.3	9.7
1960–1965	3.6	4.0	0.4	−0.1	−0.7	6.2
1965–1970	2.4	2.5	0.5	−0.3	−0.3	7.5
Mexico						
1950–1960	3.7	2.9	−0.1	0.4	0.5	3.5
1960–1970	6.5	6.8	0.1	0.0	−0.4	7.9
1970–1975	4.2	3.8	0.4	0.1	−0.1	4.3
Norway						
1953–1961	1.7	3.6	0.8	−2.4	−0.3	1.6
1961–1969	0.7	2.3	0.8	−2.6	0.2	1.0
Republic of Korea						
1955–1963	12.6	10.1	1.7	5.6	−4.8	9.5
1963–1970	10.3	6.2	5.8	−0.7	−1.1	17.4
1970–1973	15.5	5.0	12.9	−1.4	−1.2	23.6
Turkey						
1953–1963	5.6	5.2	0.2	0.4	−0.3	5.6
1963–1968	9.7	7.8	0.0	0.3	1.5	11.0
1968–1973	8.5	3.8	6.8	−0.1	−2.0	8.6
Taiwan Province						
1956–1961	9.1	4.1	3.5	1.1	0.4	32.8
1961–1966	8.8	3.0	5.4	−0.2	0.6	23.9
1966–1971	20.6	5.3	13.5	0.3	1.4	22.9

[a]Results for Colombia, Israel and Mexico are preliminary.

output growth continuously increased in all economies except Colombia, despite large differences in growth rates and levels of development.

The causes of the chemical industry's growth were quite varied, but the economies can be classified into two groups according to the role of export expansion. In Colombia, Mexico and Turkey, there were no sub-periods in which

TABLE 6. SOURCES OF CHANGE IN CHEMICALS PRODUCTION[a,b]

(Percentage change in aggregate gross output)

Period and economy	*Change in sectoral output*	Sources: *Domestic demand growth*	*Export growth*	*Import substitution*	*Changes in input coefficients*	*Average annual growth rate*
Colombia						
1953–1966	7.4	4.9	0.5	1.0	1.0	13.8
1966–1970	2.4	1.4	0.3	0.3	0.4	3.1
Israel						
1958–1965	2.6	1.4	0.7	0.3	0.2	15.0
1965–1968	5.4	2.0	3.0	−1.5	1.9	17.2
Japan						
1914–1935	1.7	0.9	0.6	0.7	−0.6	5.9
1935–1955	6.1	1.8	0.0	1.8	2.5	5.[illegible]
1955–1960	3.9	2.2	0.4	−10.6	1.9	13.1
1960–1965	6.1	3.4	1.2	0.1	1.4	16.3
1965–1970	6.1	3.4	1.1	−0.2	1.7	18.7
Mexico						
1950–1960	4.7	2.8	0.2	0.5	1.2	10.7
1960–1970	5.8	4.2	0.4	0.9	0.3	10.7
1970–1975	8.6	6.1	0.5	0.7	1.4	10.6
Norway						
1953–1961	6.1	1.8	2.7	0.2	1.4	7.6
1961–1969	7.4	2.4	5.1	−1.3	1.2	8.0
Republic of Korea						
1955–1963	5.4	1.5	0.2	3.9	−0.2	26.2
1963–1970	4.8	2.3	1.0	0.4	1.1	24.7
1970–1973	8.0	2.6	3.6	0.5	1.3	29.0
Turkey						
1953–1963	1.4	1.5	0.0	−0.4	0.4	6.6
1963–1968	3.5	2.1	0.1	−0.1	1.4	15.7
1968–1973	4.7	3.4	0.3	−0.3	1.3	14.9
Taiwan Province						
1956–1961	2.5	−0.5	0.3	2.5	0.2	13.4
1961–1966	5.9	2.5	1.9	0.6	0.9	31.0
1966–1971	7.3	2.6	3.9	0.3	0.7	24.3

[a]Results for Colombia, Israel and Mexico are preliminary.
[b]Includes rubber products for Colombia.

export expansion played a significant role, and the effect of import substitution was also quite limited. Thus, the growth of the chemical industry in these countries was largely dependent on domestic demand expansion (60 to 72 per cent) and changes in input coefficient (15 to 40 per cent). In all other economies export expansion played an increasingly important role. In the most recent sub-periods in these countries, except Japan, export expansion alone accounted for 50 per cent or more of the industry's output growth. In Japan, the Republic of Korea and

Taiwan Province growth was first facilitated by a significant effect of import substitution (and changes in input coefficients in Japan). Subsequent growth was jointly brought about by domestic demand expansion, export expansion, and changes in input coefficients, as was the case in Norway and the final sub-period in Israel.

The changes in input coefficients almost always affected growth positively and often quite significantly in all economies. Because of the diverse nature of the products included, the reasons for the positive effect of coefficient change probably vary economy to economy. For example, the positive effect of coefficient change may represent increased use of fertilizer in agriculture, more use of chemical fibres in textiles, or substitution of wood and metals by synthetic resin. However, the effect clearly reflects an important aspect of structural change in production, in which increasing amounts of chemical products are being used as intermediate inputs, whether due to changes in production technology or in production structure.

Basic metals and machinery

These industries will be discussed together. As shown in tables 7 and 8, these two industries grow roughly in parallel with the machinery industry generally growing at a slightly higher rate. Exceptional cases are Taiwan Province and the final sub-period in the Republic of Korea, where machinery production grew twice as fast as basic metals. In terms of their contributions to the economy's growth, the machinery industry has become important in postwar Japan, Norway, and in the final sub-period in the Republic of Korea and Taiwan Province, while a substantial contribution by the basic metals industry is observed only in Japan.

The growth patterns of the two industries appear quite similar in most economies. As for chemicals, it is useful to divide the economies into two groups according to the importance of export expansion. In Colombia, Mexico and Turkey, the major sources of growth for these sectors were domestic demand expansion and import substitution, with export expansion playing only a minor role. Furthermore, changes in input coefficients had a generally negligible effect on these industries, in contrast to the chemical industry. Instead, import substitution played a more significant role than it did for chemicals.

In the other economies export expansion became increasingly significant for basic metals and machinery over time. The effects of export expansion and domestic demand growth accounted for almost the entire growth of these industries in the most recent sub-period in each economy. In Japan, the Republic of Korea and Taiwan Province the earlier growth was facilitated by a significant effect of import substitution.

Except for the early periods in Japan and the Republic of Korea the contribution of changes in input coefficients to growth in basic metals was quite small. It would seem that the technology of those industries which draw on basic metals (metal products and machinery) is set quite early in the development process and, unlike chemicals, does not change much thereafter.

TABLE 7. SOURCES OF CHANGE IN BASIC METALS PRODUCTION[a]
(Percentage change in aggregate gross output)

Period and economy	Change in sectoral output	Sources: Domestic demand growth	Sources: Export growth	Sources: Import substitution	Sources: Changes in input coefficients	Average annual growth rate
Colombia						
1953–1966	3.5	1.4	0.1	2.1	−0.1	15.0
1966–1970	2.2	1.6	0.3	0.0	0.3	6.1
Israel						
1958–1965	4.3	2.7	0.7	0.6	0.3	16.5
1965–1968	5.6	4.1	1.7	−0.5	0.4	12.1
Japan						
1914–1935	5.1	4.2	1.4	0.7	−1.2	8.0
1935–1955	9.2	0.5	1.5	2.9	4.3	4.0
1955–1960	11.8	8.9	1.4	−0.9	2.4	18.8
1960–1965	11.0	8.1	4.5	0.8	−2.4	13.0
1965–1970	13.0	9.8	2.9	0.1	0.2	18.8
Mexico						
1950–1960	4.6	2.6	0.1	1.3	0.6	11.3
1960–1970	4.9	3.8	0.4	1.0	−0.2	9.8
1970–1975	4.1	4.5	0.8	−1.4	0.1	6.1
Norway						
1953–1961	6.3	1.2	5.0	0.2	−0.1	9.4
1961–1969	6.5	0.3	7.7	−1.2	−0.3	7.9
Republic of Korea						
1955–1963	4.5	1.5	1.3	0.2	1.5	17.8
1963–1970	3.3	2.7	0.7	0.4	−0.4	20.3
1970–1973	7.3	1.4	5.5	−1.0	1.3	33.8
Turkey						
1953–1963	3.0	2.6	−0.1	0.4	0.1	4.6
1963–1968	8.7	6.4	0.2	1.5	0.6	14.7
1968–1973	6.2	6.4	0.5	−1.3	0.6	8.6
Taiwan Province						
1956–1961	3.0	0.0	0.8	1.9	0.4	13.2
1961–1966	3.8	1.8	2.4	−0.3	0.0	20.3
1966–1971	4.5	1.9	2.7	0.6	−0.7	20.4

[a]Results for Colombia, Israel and Mexico are preliminary.

Finally, the rapid expansion of the machinery industry in the Republic of Korea and Taiwan Province requires further analysis. In the mid-1950s, the machinery industry for these cases consisted of two-thirds non-electrical and one-third electrical machinery. By the early 1970s, however, the electrical machinery industry had far outstripped the non-electrical machinery industry. The electrical machinery industry then comprised nearly 80 per cent of machinery output. In fact, these areas have developed rapidly in the field of electronics, and such

TABLE 8. SOURCES OF CHANGE IN MACHINERY PRODUCTION[a]
(Percentage change in aggregate gross output)

Period and economy	*Change in sectoral output*	*Sources* *Domestic demand growth*	*Export growth*	*Import substitution*	*Changes in input coefficients*	*Average annual growth rate*
Colombia						
1953–1966	2.3	0.6	0.1	1.6	0.0	18.4
1966–1970	2.0	0.7	0.1	0.1	1.1	8.4
Israel						
1958–1965	3.2	1.7	0.2	1.2	0.2	20.4
1965–1968	4.2	3.6	1.2	−1.6	1.0	13.1
Japan						
1914–1935	5.0	4.8	0.9	0.3	−1.0	9.0
1935–1955	4.2	1.2	0.1	0.7	2.1	2.3
1955–1960	12.0	9.1	1.1	0.1	1.7	25.1
1960–1965	8.2	6.3	2.0	−0.1	−0.1	11.4
1965–1970	15.4	12.2	2.4	0.0	0.8	24.6
Mexico						
1950–1960	1.9	1.2	0.0	0.4	0.3	10.3
1960–1970	3.5	2.1	0.5	0.8	0.1	13.6
1970–1975	6.9	5.5	1.0	1.1	−0.7	14.6
Norway						
1953–1961	9.9	9.2	2.9	−2.7	0.5	6.1
1961–1969	12.1	5.5	4.8	0.9	0.9	7.0
Republic of Korea						
1955–1963	4.3	1.6	0.2	2.2	0.2	21.2
1963–1970	2.5	2.4	0.9	−0.7	−0.1	18.4
1970–1973	12.2	2.2	8.4	0.3	1.3	58.4
Turkey						
1953–1963	2.1	1.3	0.0	1.4	−0.6	5.8
1963–1968	3.3	2.6	0.0	0.6	0.1	10.7
1968–1973	4.3	4.8	0.1	0.4	−0.1	11.5
Taiwan Province						
1956–1961	1.8	1.0	0.3	0.4	0.0	26.0
1961–1966	6.0	1.9	1.9	2.0	0.3	48.0
1966–1971	12.4	3.2	8.1	0.5	0.6	36.8

[a]Results for Colombia, Israel and Mexico are preliminary.

products have become one of the most important export items in recent years. The rapid expansion of electronics exports explains the overwhelming importance of export expansion to this industry's growth in the early 1970s. However, the non-electrical machinery industry is still not well developed in these economies, and hence the machinery industry's rise to prominence in the economy's growth should be interpreted differently from that in, say, Japan where electrical and non-electrical machinery achieved a more balanced growth.

Conclusion

The decomposition of the sources of growth over time within economies and the comparison of economies shows that differences in trade regimes and development strategies had a strong effect on the pattern of growth at both the sectoral and industry levels. In the economies where import-substitution oriented, inward-looking policies were adopted, industries' growth tended to depend heavily on domestic demand expansion, with import substitution or changes in input coefficients mostly accounting for the balance. In the economies where export-promoting, outward-looking strategies are emphasized, industries' growth is largely brought about by export expansion and domestic demand growth. However, even these economies had early periods when import substitution and changes in input coefficients were important. One might view these early periods as phases where the economies built the industrial base and technological structure necessary to support a more open development strategy.

Every economy but the Republic of Korea had at least one period in which changes in input coefficients were an important source of total manufacturing growth. Since a deepening in the structure of the use of intermediate inputs is almost a defining characteristic of development, it is not surprising to find such periods. Indeed, the lack of such a period in the Republic of Korea is very surprising, especially considering that it embarked on a prolonged period of export-led growth. There may in fact be a problem in the Republic of Korea with the choice of 1955 as the base year, since it follows so closely after the war conditions of the early 1950s.

The decomposition methodology used in this paper has proved to be a useful tool for sorting out in a multi-industry framework the various factors affecting growth and structural change in an economy. It has also proved to be useful in relating those changes to the development strategies and policy regimes undertaken in different economies. The explicit comparison of the experience of different economies has been especially important to the analysis of the interactions between development strategy, growth, and structural change. However, the methodological framework does not explicitly incorporate policy variables and so cannot trace out the sequence of causal links between policies and effects. Such an analysis requires more detailed models of individual economies that can build on the simpler methodology.

ANNEX

BASIC SECTOR AND INDUSTRY CLASSIFICATION CODE

Sector	*Industry*
Primary	Agriculture, forestry and fishery
	Crude oil
	Coal
	Other mining

Sector	*Industry*
Manufacturing	Food processing
	Textiles
	Clothing and leather products
	Lumber and wood products
	Paper and pulp products
	Printing and publishing
	Miscellaneous manufacturing
	Unallocated
	Rubber products
	Chemicals
	Petroleum and coal products
	Non-metallic products
	Basic metals
	Machinery
	Transport equipment (including shipbuilding)
Services	Construction
	Public utilities
	Transport and communication
	Trade
	Other services (including real estate)

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Technological change and the pattern of economic development

*I. Ozaki and M. Shimizu**

In a Leontief dynamic model, the equilibrium path is usually calculated using constant structural parameters. However, in Professor Leontief's "dynamic inverse" model, analysis is focused on the effects of changes over time in these structural parameters. In this study, the effects of technological changes on the pattern of economic development are investigated. First, a multi-industry capital accumulation model that incorporates technical changes is introduced and then, on the basis of that model, a comprehensive description is provided of the interrelations among conditions that sustained the economic growth and economic policies of the Japanese economy during the period 1951–1968. The model building is based on the following two elements: one is Leontief's "dynamic inverse" model [1] and the other is the introduction of the effects of economies of scale on changes over time in the capital input coefficients matrix B^{t+1}. The latter is based on the empirical study of production functions presented by one of the authors in the Sixth International Conference of Input-Output Techniques (Ozaki [2]).

Leontief [1] described a dynamic economic system that involves technical changes by the following formula:

$$A^t X^t + B^{t+1}(X^{t+1} - X^t) + C^t = X^t \tag{1}$$

where:

X^t = the gross output vector;
C^t = the final demand vector;
t = the time period;
A^t = the input-output coefficient matrix;
B^{t+1} = the capital input coefficient matrix, whose elements are b_{ij}^{t+1}, $B^{t+1} = [b_{ij}^{t+1}]$.

In the equation system (1), the suffix (t + 1) attached to the capital input coefficient matrix B has an important meaning for the analysis of structural change, because it indicates an interrelationship between the pattern of production processes and the changes in technology embodied in capital investment. In other words, it implies that production in period t, shown in the equation system (1), produces new technology embodied in new capital goods that is expected to operate in the next period, t + 1.

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However, how the capital input coefficient matrix B^{t+1} changes over time in the process of economic growth was not specified in Leontief's "dynamic inverse" model. In this respect, an attempt is made to introduce the effect of economies of scale on changes over time in the capital input coefficient matrix B^{t+1}.

The model

Turning now to the Leontief dynamic system, an attempt is made to build a dynamic model that incorporates the effect of technical change. First, let us focus on the changes over time in the capital input coefficient matrix, $B^{t+1} = [b_{ij}^{t+1}]$, in equation (1).

The basic assumption is that the capital input coefficient, b_{ij}, in the Leontief dynamic model is a function of production capacity, X_j. This means that the assumption of indivisibility of equipment is introduced. Then, the capital input function will be represented by the following equation:

$$S_{ij} = f_{ij}(X_j) \qquad i,j = 1,2,\ldots,n \qquad (2)$$

where S_{ij} stands for the stock of a commodity produced by industry i and used by industry j as a capital stock at period t, and X_j is defined as the production capacity of capital stock for the j^{th} sector in terms of the input functions (2).

Let us specify equation (2) in the following log-linear form:

$$S_{ij} = \alpha_{ij} X_j^{\beta_{ij}} \qquad i,j = 1,2\ldots,n \qquad (3)$$

where α_{ij} and β_{ij} are parameters to be estimated.

From equation (3), we obtain:

$$b_{ij} = \frac{S_{ij}}{X_j} = \alpha_{ij} X_j^{\beta_{ij}-1}, \qquad i,j\ldots,n \qquad (4)$$

Now, we define:

$$b_{ij}^{*} = \frac{dS_{ij}}{dX_j} = \alpha_{ij}(\beta_{ij} - 1) X_j^{\beta_{ij}-2} \qquad (5)$$

where b_{ij}^{*} is the marginal capital coefficient which is a function of production capacity and

$$B^{*} = [b_{ij}^{*}] \qquad (6)$$

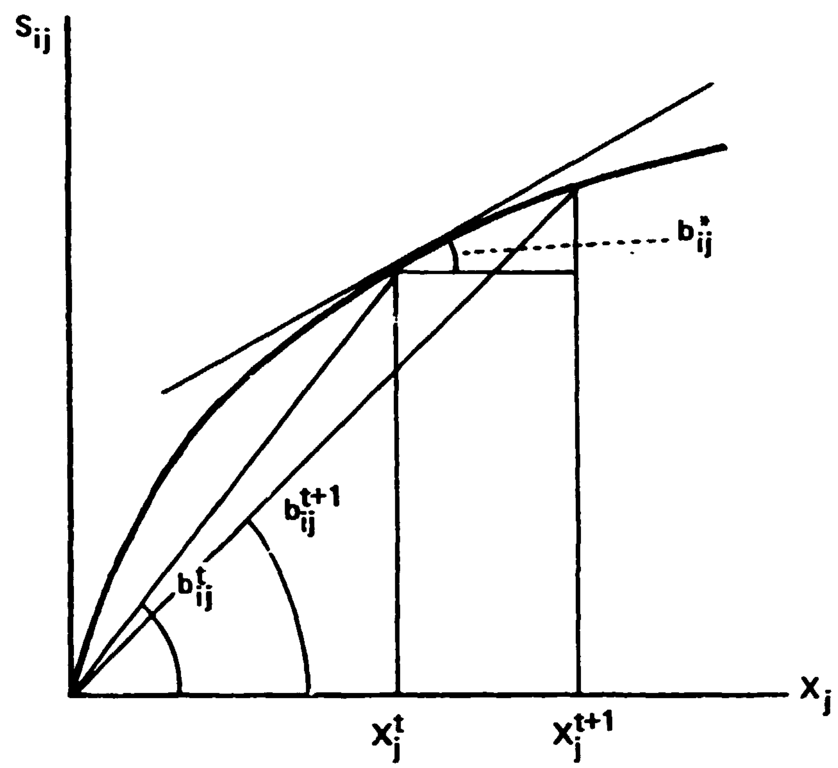
The basic commodity balance equation in the dynamic inverse model can be written as follows, using the notation B^{*} instead of B^{t+1}:

$$A^t X^t + B^{*}(X^{t+1} - X^t) + C^t = X^t \qquad (7)$$

where $B^{*} = [b_{ij}^{*}]$ and $b_{ij}^{*} = \dfrac{dS_{ij}}{dXj}$.

The way we formulate the relationship between B^{*} and b^{t+1} is illustrated graphically in figure 1.

Figure 1. S_{ij} vs X_j, showing the relationship between B* and b^{t+1}



Now, corresponding to the commodity balance equation (2) or (7), the price determination equation can be written following Leontief's [1] formulation:

$$p^t = A'^t P^t + [(1 + r^{t-1}) B'^t P^{t-1} - B'^{t+1} P^t] + v^t \tag{8}$$

where:

p^t = a column vector of the price of goods and services;
r^{t+1} = the annual money rate of interest;
A'^t = the transpose of the input-output coefficient matrix A^t;
B'^t = the transpose of the capital coefficients stock matrix B^t;
v^t = the value-added column vector.

In Leontief's price equation, "for purpose of proper cost accounting, the stocks of capital goods are assumed to be acquired by each industry, in accordance with technological requirements, one year before the delivery of the output they produce and then sold off together with that output" [1]. Thus, the capital goods newly purchased in each industry at time t will push the price up or down at time t + 1. This is the meaning of $[- B'^{t+1} P^t]$ in equation (8).

In the earlier papers by Ozaki [2, 3], the following factor limitational type of production function was statistically estimated:

$$L_j = a_{Lj} X_j^{\beta_{Lj}} \tag{9}$$

$$K_j = \alpha_{Kj} X^{\beta_{Kj}} \tag{10}$$

where

L_j = the labour required for production level X_j in the j^{th} industry;
K_j = the capital required for production level X_j in the j^{th} industry.

In this case, the capital stock K_j^t can be related to the previous stock variable S_{ij}^t in the following equation:

$$P_w^{t-1} K_j^t = \sum_{i=1}^{n} P_i^{t-1} S_{ij} \qquad (11)$$

where P_w is the wholesale price index for capital goods.

The technology type for each industry was then statistically determined based on the estimation of production function (2). These determinations are summarized in table 1. All industries were grouped into the following six technology types:

Large-quantity processing technology (type K (I-B))
Large-scale assembly production technology (type K (I-M))
Capital-intensive technology (type K (II))
Cobb-Douglas (constant returns to scale) technology (type L-K)
Labour-intensive technology (type $L_I, \gamma_L < 1$)
Labour-intensive technology (type $L_{II}, \gamma_L > 1$)

In table 1, we observed that (type K (I-B)) industries include:

(a) Electric power supply;
(b) Gas and water supply;
(c) Petroleum refining products;
(d) Basic organic chemicals;
(e) Artificial fibre materials;
(f) Iron and steel;
(g) Non-ferrous primary products.

Large-scale assembly production technology industries are:

(a) Ships and ship repairing;
(b) Motor vehicles;
(c) Machinery;
(d) Electrical machinery;
(e) Precision instruments;
(f) Fibre spinning;
(g) Beverages and alcoholic drinks.

Thus our basic model is summarized as follows:

In terms of commodity balance,

$$A^t X^t + \underline{B}^* (X^{t+1} - X^t) + C^t = X^t \qquad (12)$$

where $\underline{B}^* = [b_{ij}^*]$ and $b_{ij}^* = \frac{dS_{ij}}{dX_j} = d_{ij}(\beta_{ij} - 1) X^{\beta_{ij}^{-1}}$, $\exists \beta_{ij} \neq 1$, for some β_{ij}.

TABLE 1. PRODUCTION FUNCTION PARAMETRIC CHARACTERISTICS OF VARIOUS INDUSTRIES, BY TECHNOLOGY TYPE

Large-quantity processing technology	*Production function parameters* β_L[a]	β_K[b]	$\left(\frac{\bar{K}}{L}\right)_j$ *1951–1968 average*
Electric power supply	0.12	0.80	17.43
Gas and water supply	0.68	0.73	2.59
Petroleum refining products	0.27	0.65	14.76
Basic organic chemicals	0.33	0.72	5.70
Artificial fibre materials	0.10	0.84	3.89
Iron and steel	0.30	0.80	3.86
Non-ferrous primary products	0.38	0.73	3.84

Large-scale assembly production technology	*Production function parameters* β_L[a]	β_K[b]	$\left(\frac{\bar{K}}{L}\right)_j$ *1951–1968 average*
Ships and ship repairing	0.07	0.80	1.19
Motor vehicles	0.46	0.70	2.12
Machinery	0.52	0.88	0.62
Electrical machinery	0.55	0.91	1.00
Precision instruments	0.53	0.97	0.59
Fibre spinning	0.26	0.59	2.07
Beverages and alcoholic drinks	0.33	0.79	2.26

Capital-intensive technology	*Production function parameters* β_L[a]	β_K[b]	$\left(\frac{\bar{K}}{L}\right)_j$ *1951–1968 average*
Paper	0.13	1.03	3.07
Pulp	−0.29	1.23	3.94
Cement	0.08	1.03	9.07
Basic inorganic chemicals	0.04	1.01	2.71
Chemical fertilizer	−0.71	1.71	4.97
Miscellaneous coal products	−0.09	1.67	1.50
Tobacco	0.18	2.30	1.83

Cobb-Douglas (constant returns to scale) technology	β_o[c]	$\left(\frac{\bar{K}}{L}\right)_j$ *1951–1968 average*
Agriculture, forestry and fisheries	0.67	0.46
Coal and lignite	0.56	0.90
Mining	0.64	0.56
Silk reeling and spinning	0.70	0.59
Vegetable and animal oil and fat	0.69	1.91
Wood milling	0.78	0.68

TABLE 1 *(continued)*

Labour-intensive technology (increasing returns to scale)	γ_L^d	γ_K^d	$\left(\frac{\bar{K}}{L}\right)_j$ *1951–1968 average*
Type $L_I(\gamma_L < 1)$			
Building and Construction	0.75	0.45	0.25
Meat	0.44	0.61	1.52
Seafood, preserved	0.90	0.48	0.59
Transport services	0.70	0.67	1.04
Paints	0.58	0.73	1.51
Rubber products	0.99	0.63	0.99
Glass products	0.44	0.88	1.46
Miscellaneous industrial products	0.83	0.93	0.78
Type $L_{II}(\gamma_L > 1)$			
Other transport equipment	1.31	0.54	1.01
Metal products	1.35	0.30	0.49
Leather products	2.21	−0.07	0.40
Furniture and fixtures	1.82	0.44	0.40
Other wood products	2.33	0.68	0.26
Paper articles	1.29	0.56	0.72
Pottery, china and earthenware	1.39	0.55	0.51
Structural clay products	1.59	0.96	0.57
Other non-metallic mineral products	1.87	0.19	1.15
Medicine	1.20	0.80	1.25
Weaving and other fibre products	1.75	0.63	0.79
Footwear and wearing apparel	1.93	0.28	0.31
Printing and publishing	1.43	0.27	0.57
Other food, prepared	1.26	0.35	0.65
Trading	1.95	0.84	0.65
Finance and insurance	1.60	0.22	0.70
Communicatiou services	3.38	0.08	0.17

[a] $L = \alpha_L X^{\beta_L}$ [c] $\frac{X}{L} = \alpha_0 \left(\frac{K}{L}\right)^{\beta_0}$

[b] $K = \alpha_K X^{\beta_K}$ [d] $X = \alpha L^{\gamma_L} K^{\gamma_K}$

Corresponding to equation (12), in terms of price determination,

$$p^t = A'^t X^t + [(1 + r^{t-1}) B'^t P^{t-1} - B'^{t+1} P^t] + V^t \qquad (13)$$

In both models,

$$(b_{ij}^{t+1}/b_{ij}^t) = (X_j^{t+1}/X_j^t)^{\beta_{ij}^{-1}} \qquad i, j = 1, 2 \ldots, n \qquad (14)$$

Depending on whether $\beta_{ij} < 1$, $\beta_{ij} = 1$ or $\beta_{ij} > 1$, the capital input coefficients b_{ij}^{t+1} $(i, j = 1, 2 \ldots, n)$ change over time in accordance with either economies of scale, constant returns to scale or diminishing returns to scale, respectively. Using this model, it is expected that in allocating resources for economic development industries with smaller values of parameter β_{ij} will receive a relatively greater amount of capital investment. It is hypothesized that the heavy industrialization in the Japanese economy can be explained by the unbalanced economic growth described in the basic equation system (12)–(14).

Statistical test for the effectiveness of the model

During the 1950s and 1960s, the wholesale price index remained surprisingly stable while the industrial production index increased rapidly in the Japanese economy as shown in figure 2.

Behind this stable movement of the price index, however, different patterns of price changes were observed for different industries. Figures 3 to 9 show the different patterns of the changes over time in the producers' prices for the types of technology mentioned above. To summarize the observations:

(a) Figures 3 and 5 show the rapidly decreasing pattern of producers' price of each commodity with large-quantity processing technology (type K (I-B)) and capital intensive technology (type K (II)) during the period 1951–1968, except for petroleum refining products and non-ferrous primary products which had been heavily dependent on the amount and prices of imported materials;

(b) On the other hand, the prices in the traditional industries with Cobb-Douglas (constant returns to scale) technology (type L-K), labour-intensive technology (type $L_I, \gamma_L < 1$) and labour-intensive technology (type $L_{II}, \gamma_L > 1$) had been steadily increasing during the same period, as shown in figures 6 to 9;

(c) As shown in figure 4, the movement of the producers' price of capital good with large-scale assembly production technology (type K (I-M)) was rather moderate during these years. These different patterns of prices by technology type are represented by figure 3, which shows each pattern of price movement. The theoretical model presented above consistently explains these observations.

In Leontief's dynamic inverse model the price equations were described as follows:

$$P^t = A'^t P^t + [(1 + r^{t-1})B'^t P^{t-1} - B'^{t+1} P^t] + v^t \qquad (15)$$

In equation (15), given the value of r and B'^{t+1} and v^t for each period, the theoretical value of $P^t(t = 0, 1, \ldots, n)$ can be successively calcuiated by the method of iteration.

In order to perform this calculation, the 10×10 industry I/O table (see annex) for Japan for 1965 was used. In this table the capital input coefficient matrix B^{1965} as well as the input-output coefficient matrix A^{1965} are available as a data set.

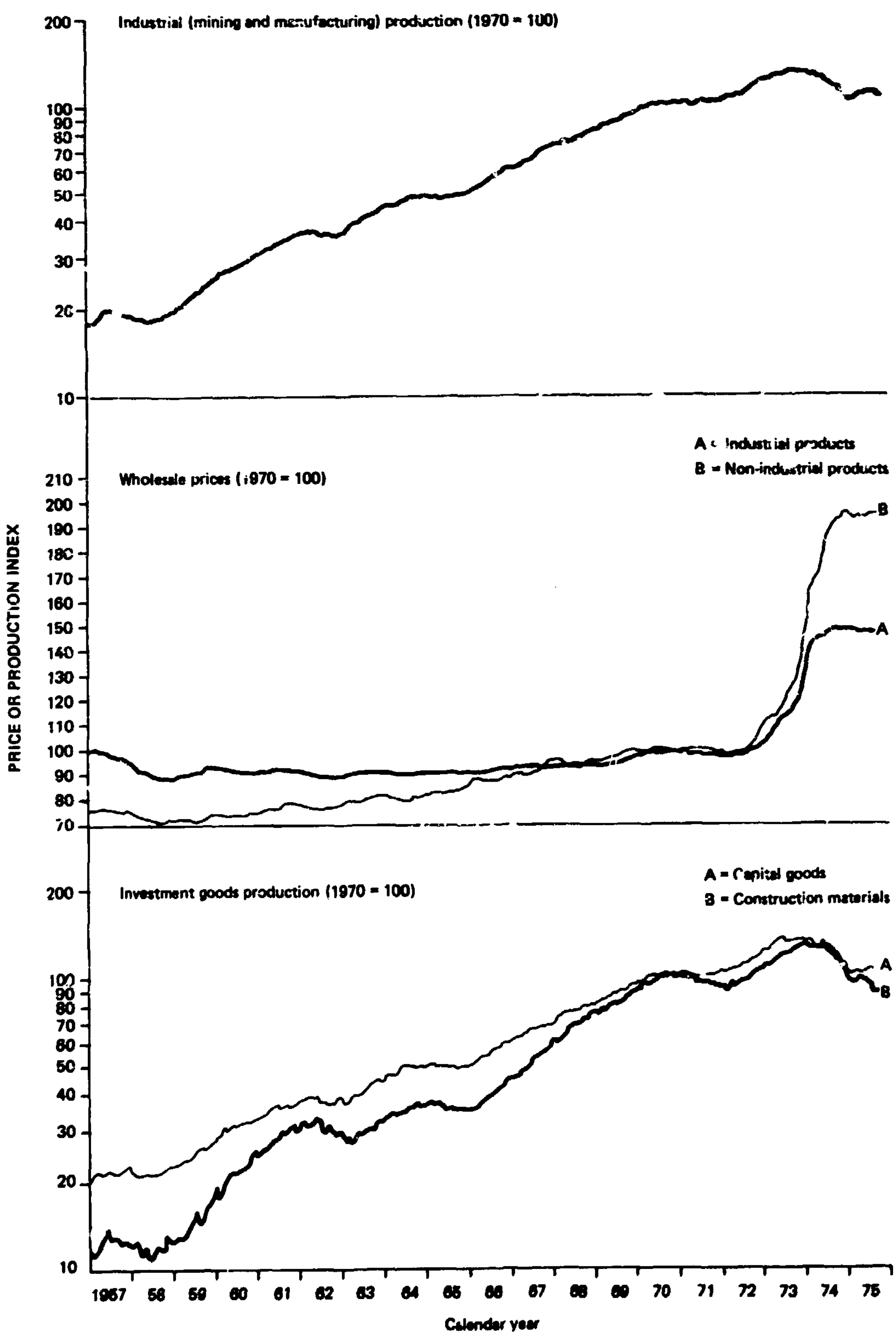
Next, the series of B^t matrices ($t = 1, 2, \ldots, n$) were calculated using the following equation:

$$(b_{ij}^{t+1} / b_{ij}^t) = (X_{ij}^{t+1} / X_j^t)^{\beta_{Kj} - 1} \qquad (16)$$

where β_{Kj} is the estimated value of the parameter in the production function equation (10). The value of β_{Kj} instead of β_{ij} is used in the basic model II as an approximation. Thirdly, with respect to the estimation of v^t series, we obtained the calculated value of v^t from the next equation:

$$\left(\frac{v^{t+1}}{v^t}\right) = \left(\frac{w^{t+1}}{w^t} \frac{l^{t+1} + R^{t+1}}{l^t + R^t}\right) \Big/ \left(\frac{X_j^{t+1}}{X_j^t}\right)^{\beta_{Lj} - 1} \qquad (17)$$

Figure 2. Wholesale price index and industrial production, 1957-1975



Source: Japanese economic indicators, Economic Planning Agency, No. 283 (Japan, 1975).

Figure 3. Producers' price by industry for large-quantity processing technology (type K (I-B))

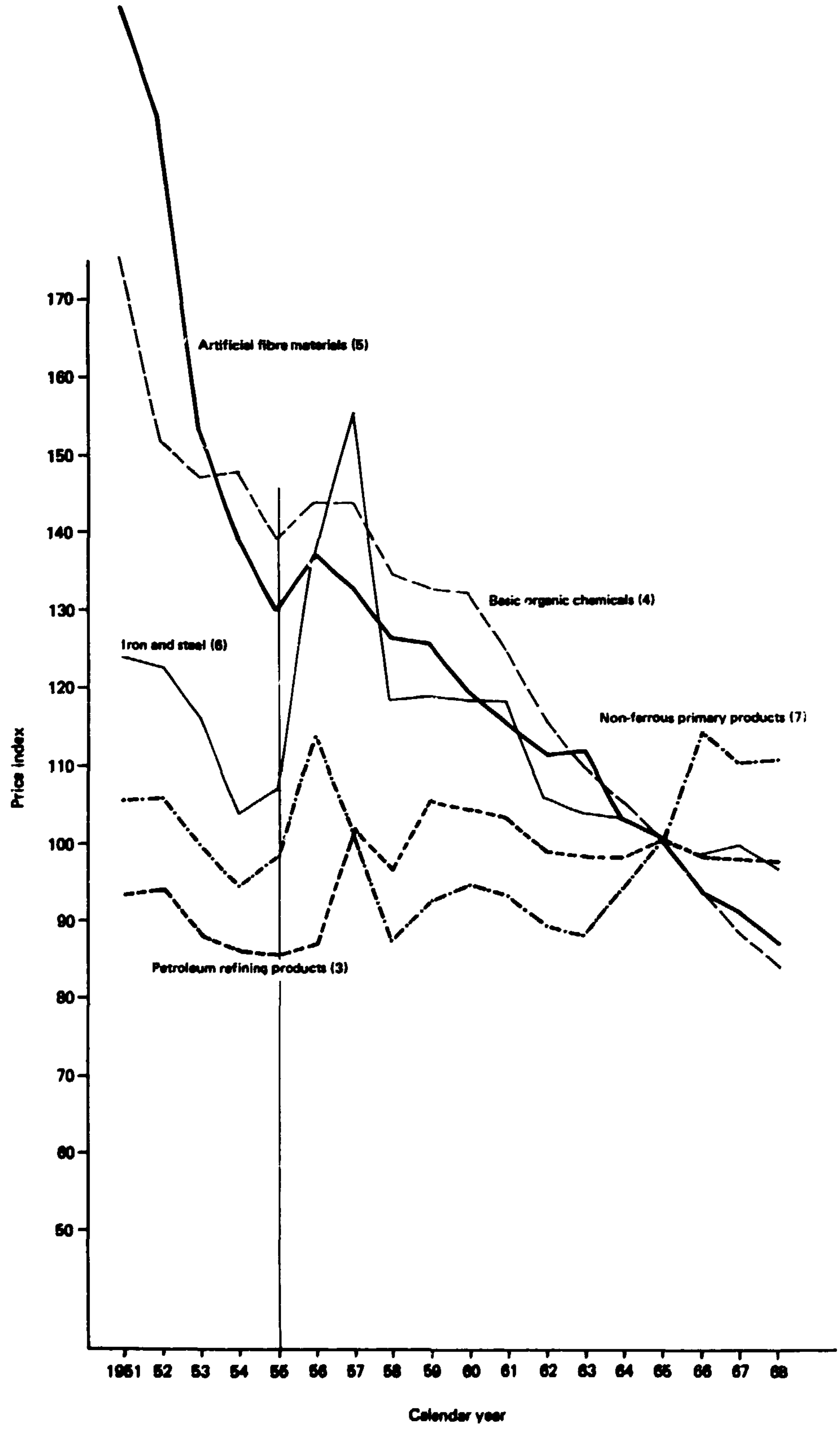
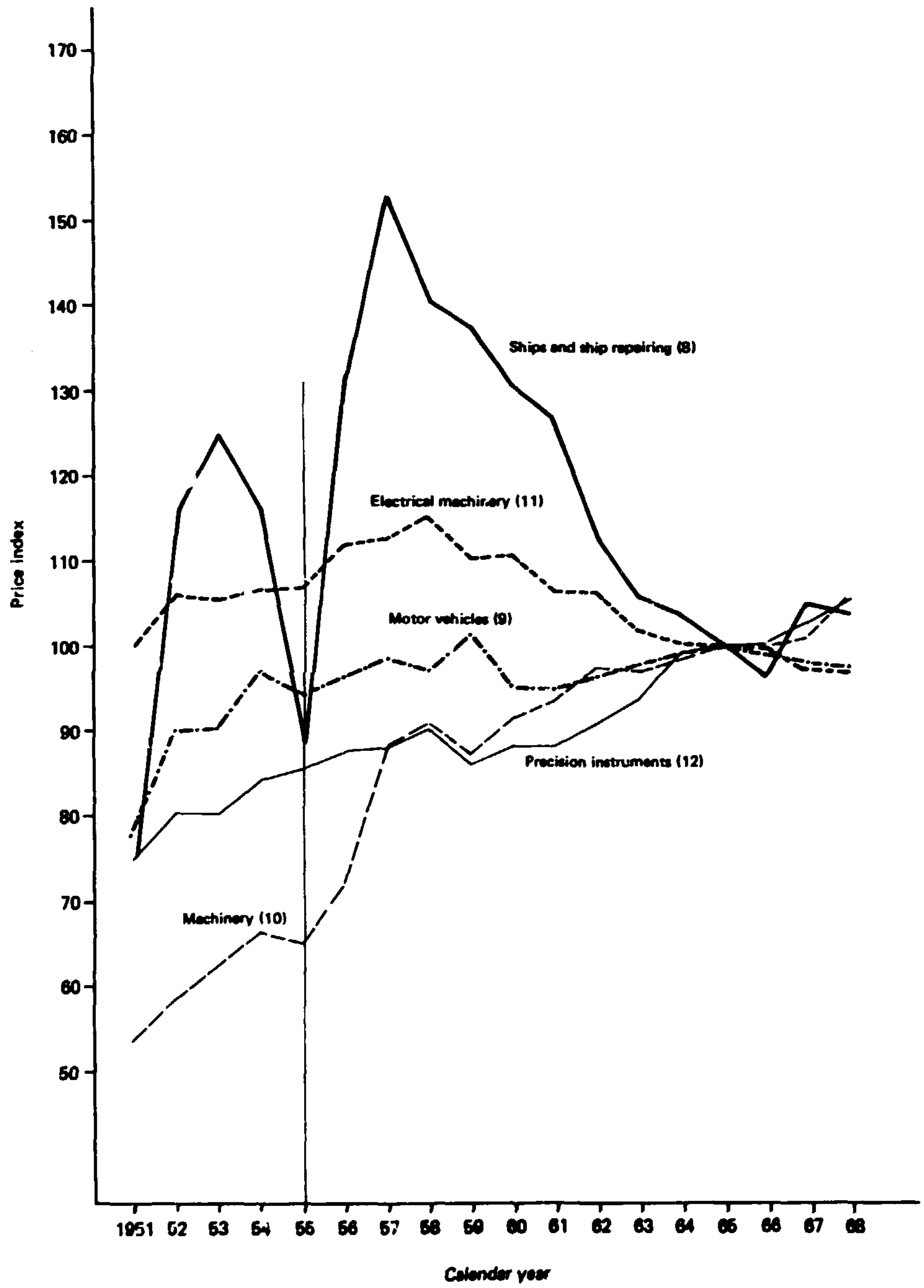


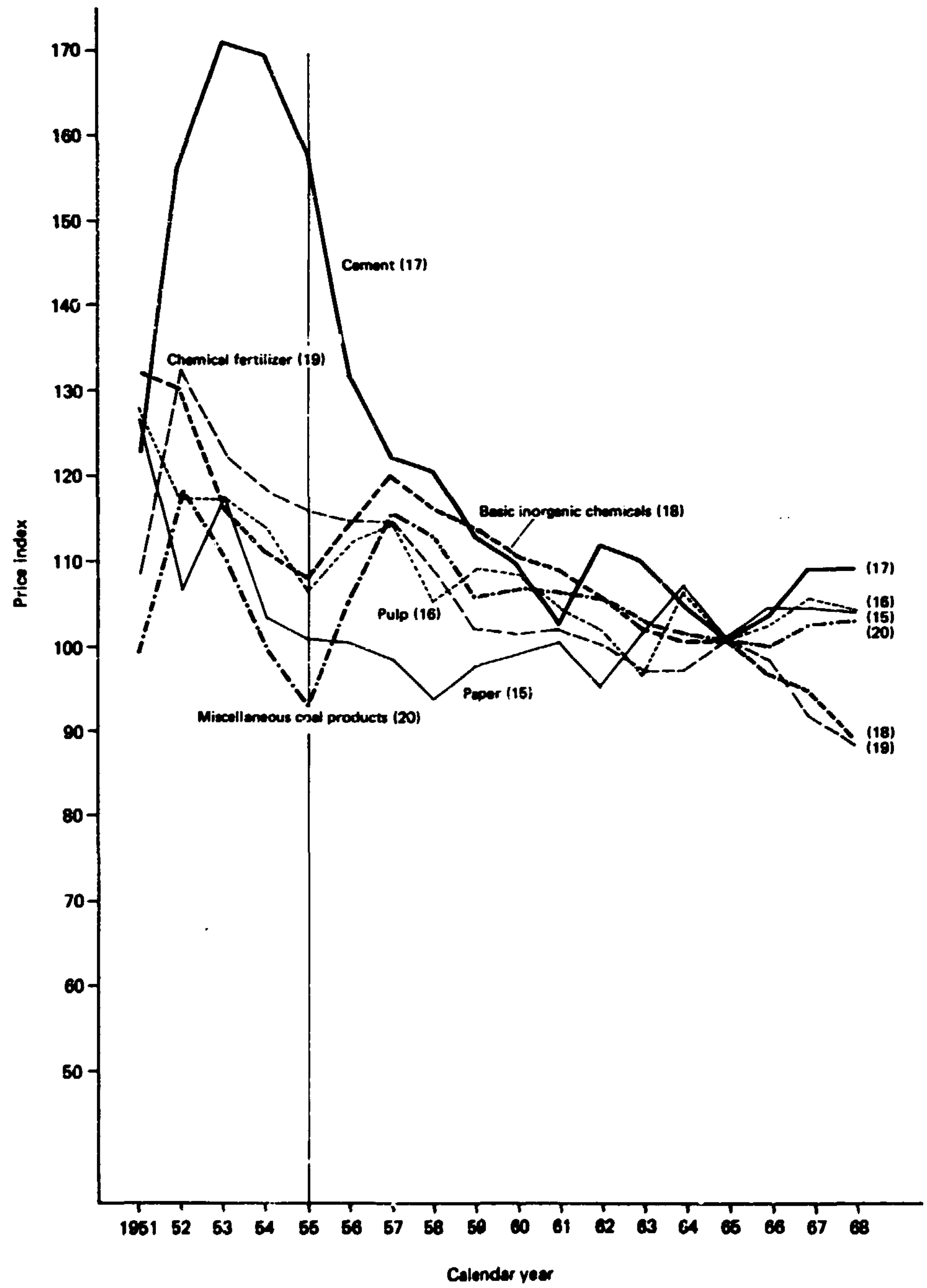
Figure 4. Producers' price by industry for large-scale assembly production (1965 = 100)



where $\hat{\beta}_{Lj}$ is the elasticity parameter of $Lj = \alpha_{Lj} X_j^{\beta_{Lj}}$, w is the wage rate and R is the normal profit rate. Equation (17) represents the assumption that the change over time in value added is proportional to the change in production capacity.

In the simulation experiment, the following values were given on the basis of the results obtained from the estimation of production functions: $\beta_{Kj} = 0.8$ and $\beta_{Lj} = 0.4$ for j = 4, 5, 6, while for the other industries, $\beta_{Kj} = 1.0$, $\beta_{Lj} = 1.0$. We

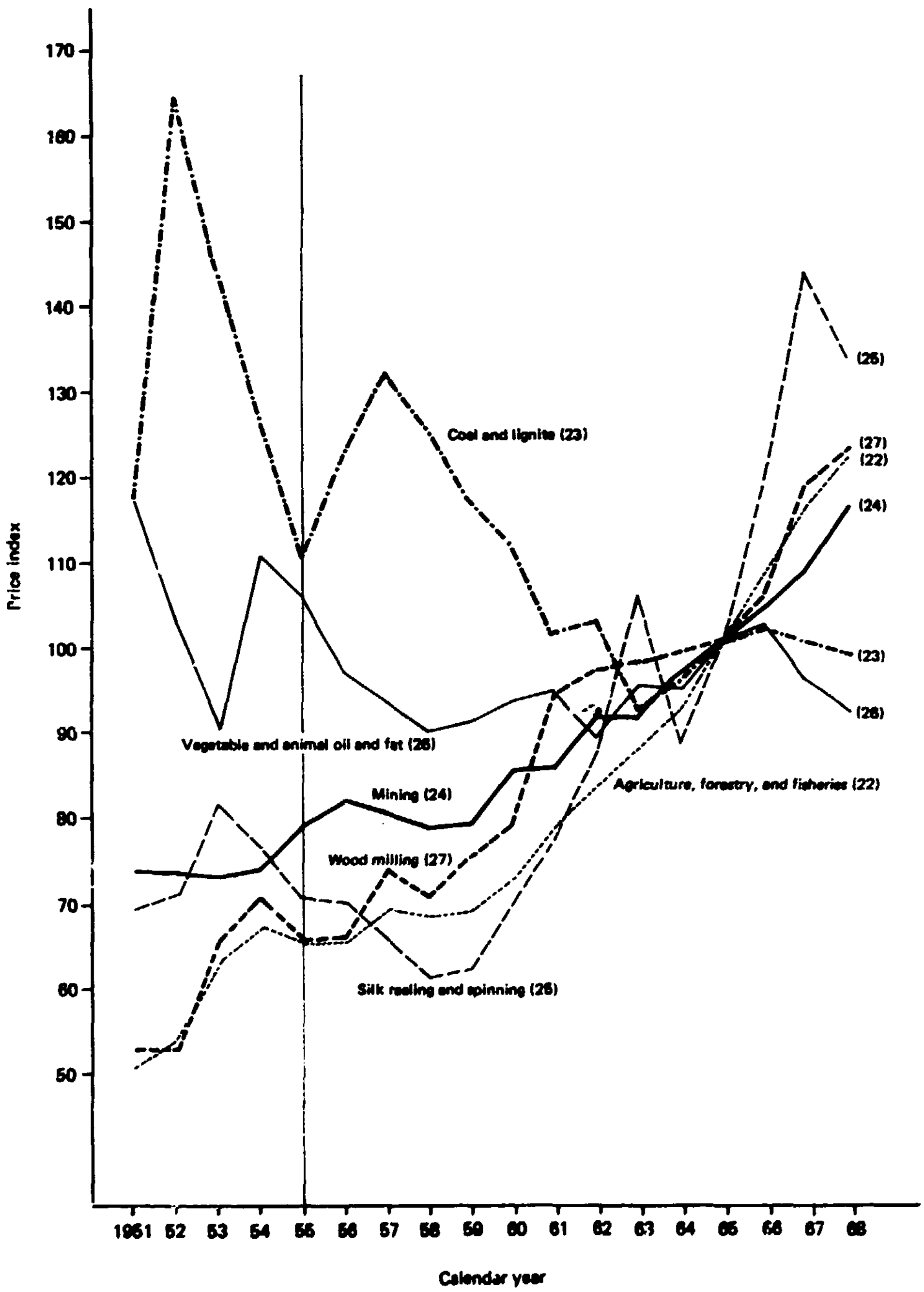
Figure 5. Producers' price by industry for capital-intensive technology (type K (II)) (1965 = 100)



also assumed R = 0.1. The movement of the calculated value of prices by each industry grouped by technology type is shown in figure 10. In figure 10 a pattern develops in which industries 4, 5, 6 (see figure key) have a value of $\beta_x = 0.8$ and $\beta_L = 0.4$. Industries 1, 2, 3, 7, 8, 9, 10 (see key) have $\beta_x = 1.0$ and $\beta_L = 1.0$.

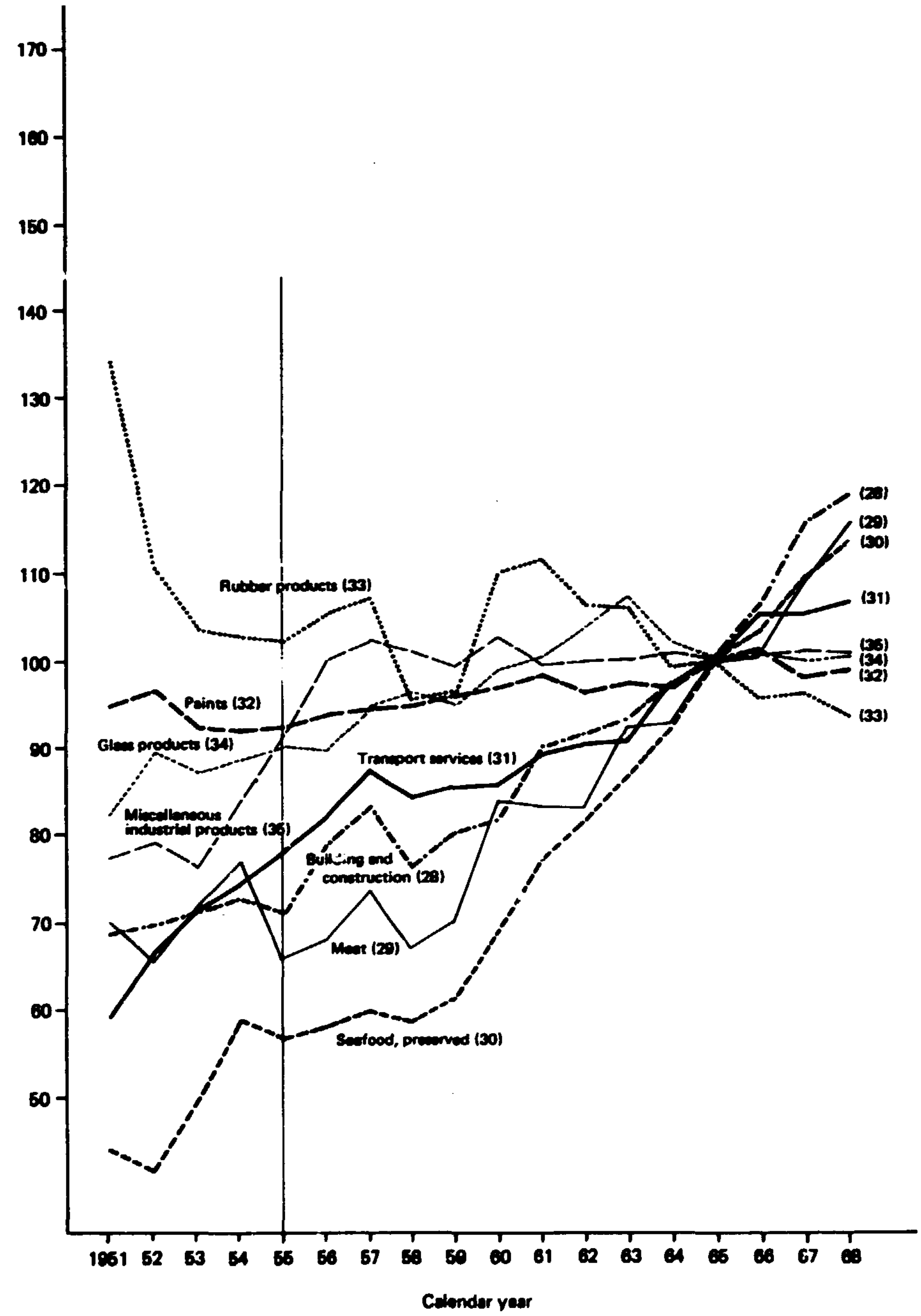
In figure 10, the theoretical movements of the prices of the commodities with large-quantity processing technology (type K (I-B)) show a downward tendency,

Figure 6. Producers' price by industry for Cobb-Douglas (constant returns to scale) technology (type L-K) (1965 = 100)



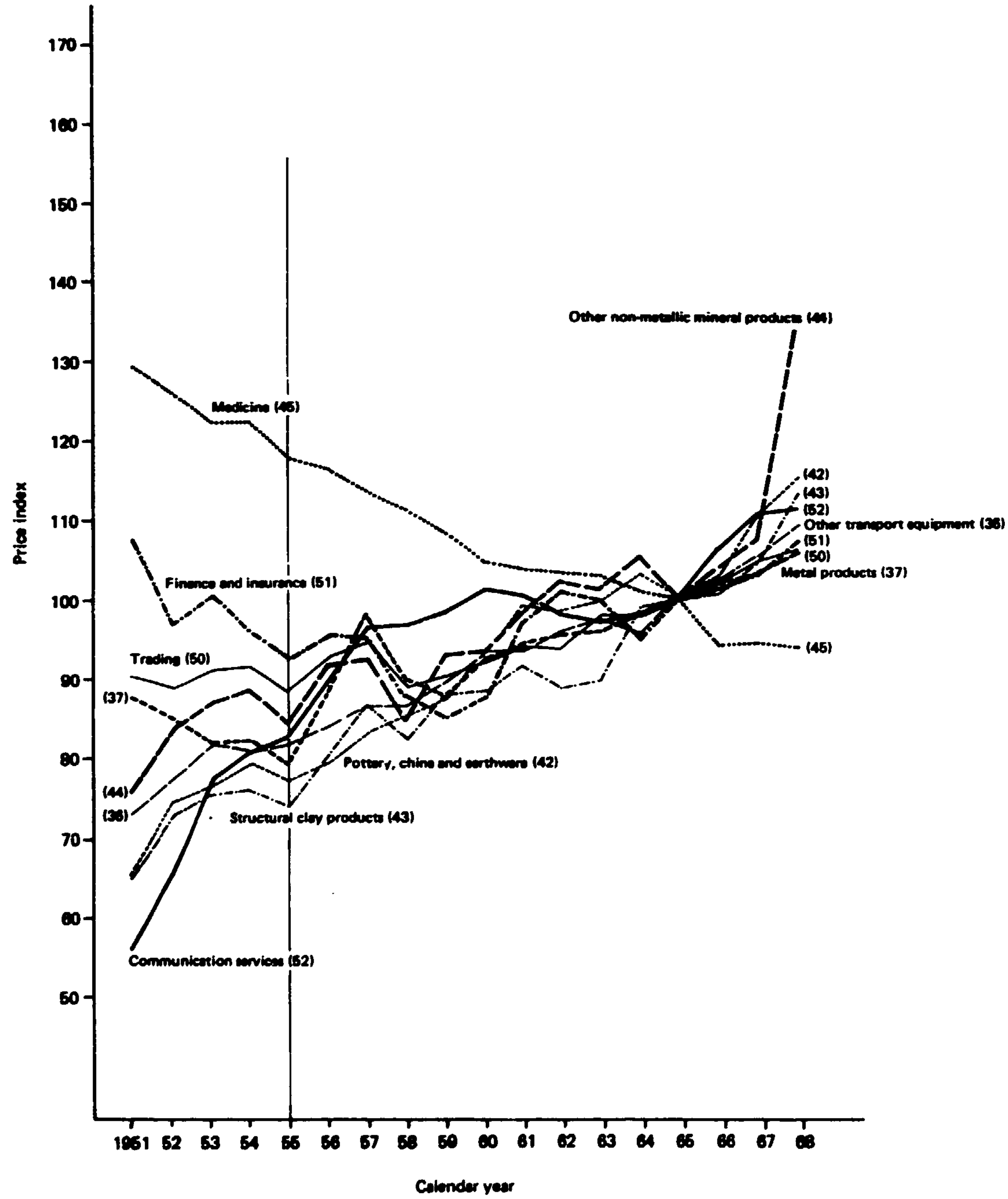
while the prices of the commodities with labour-intensive technology (type $L_I, \gamma_L < 1$) and labour-intensive technology (type $L_{II}, \gamma_L > 1$) are uniformly increasing. The prices of capital good sectors with large-scale assembly production technology (type K (I-M)) are relatively constant between $P_{K(I-B)}$ and P_L. From the results, a strong similarity seems to exist between the movement of price in theoretical values and that of actual values. The conclusion is therefore, that

Figure 7. Producers' price by industry for labour-intensive technology (type $L_1, \gamma_I < 1$) (1965 = 100)



technological change is dominant in the process of development of the Japanese economy; and technological changes are caused by the effect of economies of scale in large-scale industries having large-quantity processing technology (type K (I-B)). The different patterns of price movements calculated by the model are illustrated in figure 11.

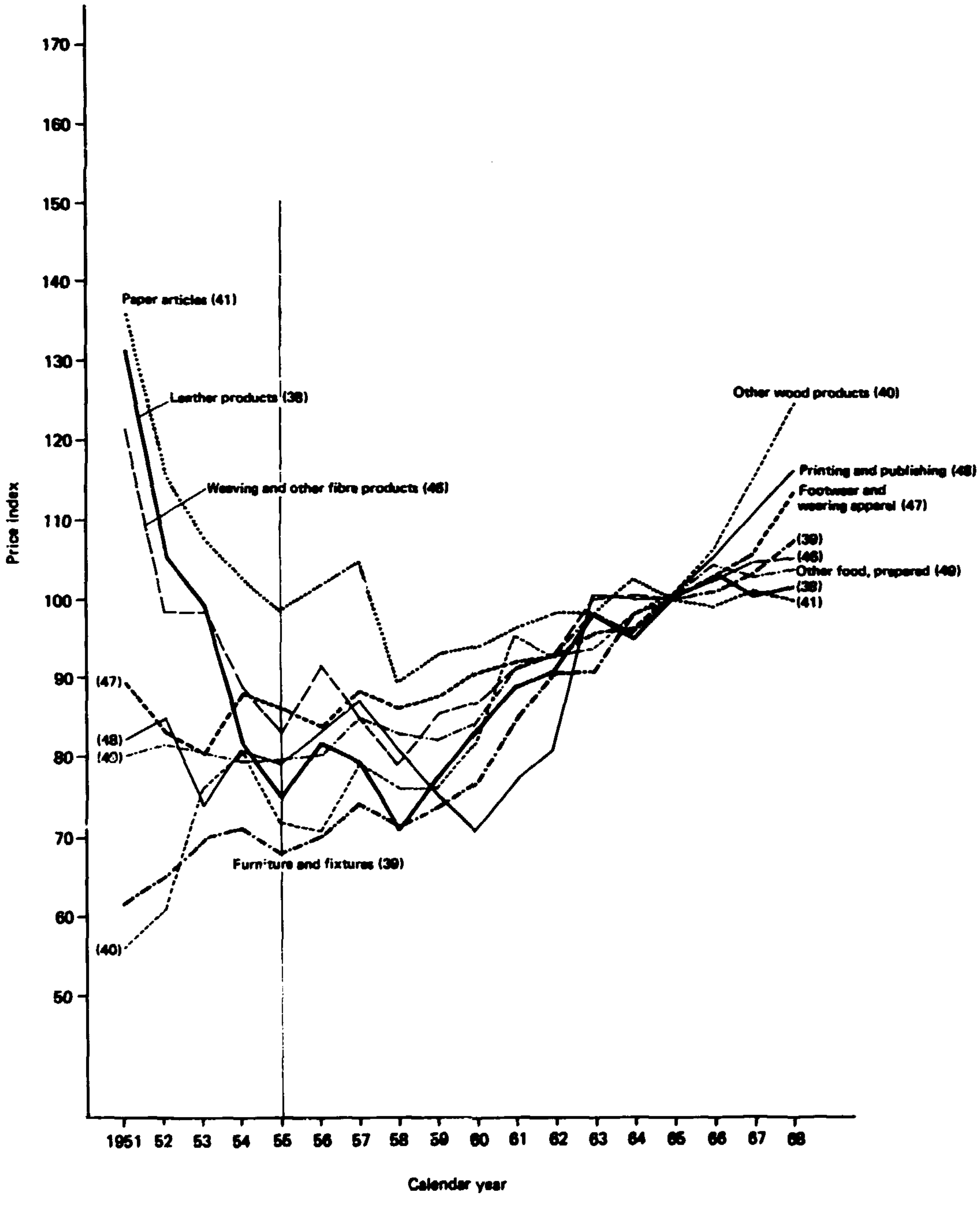
Figure 8. Producers' price by industry for labour-intensive technology (type $L_{II}, \gamma_L > 1$) (1965 = 100)



Pattern of development

Since the end of the Second World War, the Japanese economy achieved rapid industrialization with a real rate of economic growth of approximately 10 per cent per year. During this period ending with the year of the first oil crisis, 1973, the economy experienced much structural change. The size of the

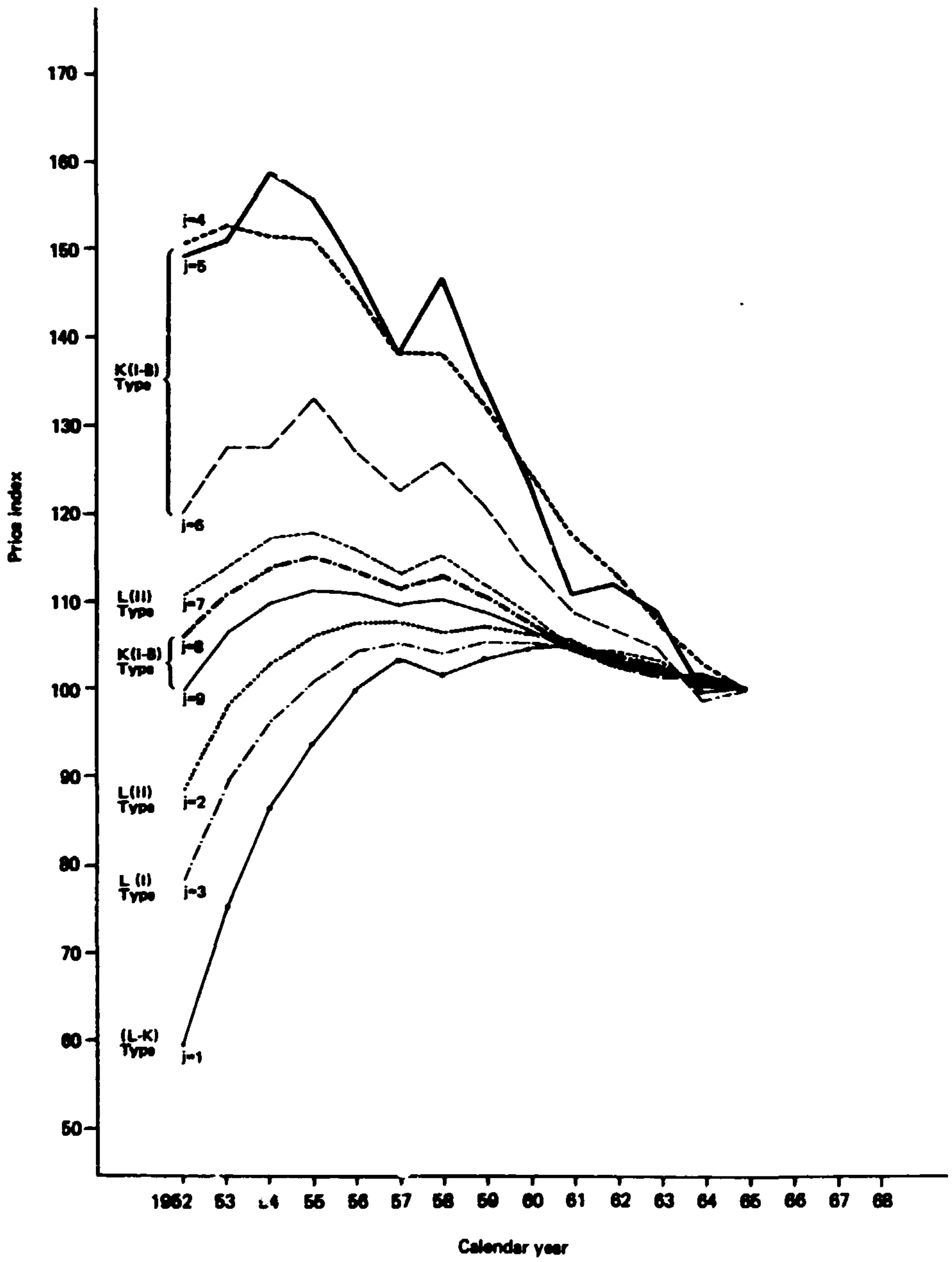
Figure 9. Producers' price by industry for labour-intensive technology (type $L_{II}, \gamma_L > 1$) (1965 = 100)



agricultural labour force rapidly decreased, while the size of the labour force in manufacturing and services increased. Urbanization proceeded at high speed. The percentage of employees in the total labour force steadily increased, and the wage rate in real terms constantly rose.

In the previous section, the industrialization process was explained as resulting from the pursuit of gains from large-scale production technology.

Figure 10. Calculated producers' price by industry (1965 = 100)

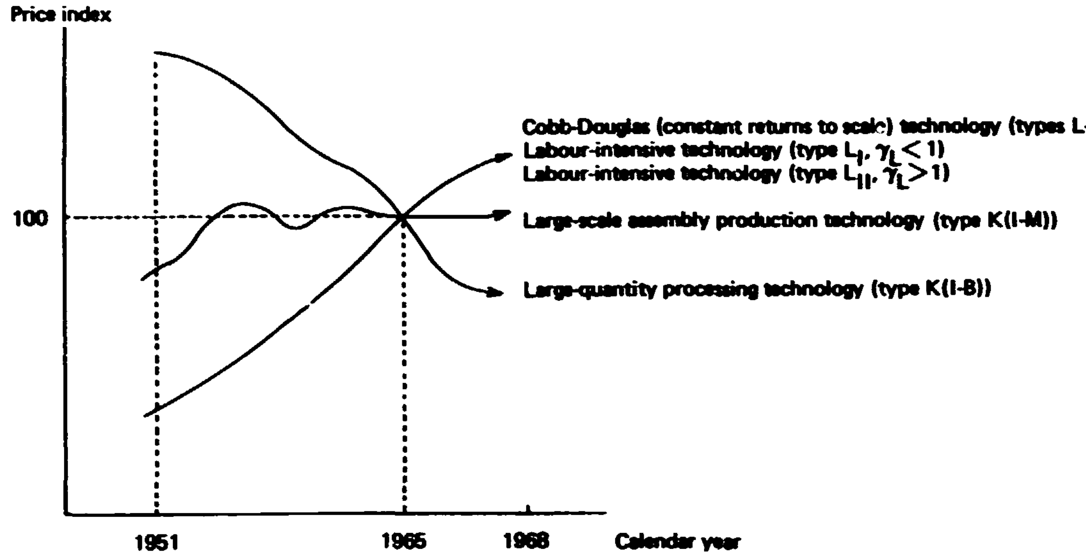


Key:

1. Agriculture and mining
2. Textile
3. Other industrial products
4. Chemicals
5. Iron and steel products
6. Non-ferrous primary products
7. Metal products
8. Machinery and electrical machinery
9. Transport equipment and precision instruments
10. Construction and services

However, the sustained economic growth accompanied by rapid structural change over these twenty years cannot be achieved solely by supply conditions; that is, the utilization of large-scale production methods. There must have been a corresponding change in the demand structure during that period. Thus, in this

Figure 11. Price index by calendar year for various industry types



section, we focus on the effects of changes in the demand structure on economic growth and especially we emphasize the role of demand created by government policies on economic development during this period.

Let us begin, firstly, by observing the changes during the years 1955–1968, in the distribution of labour and capital, accompanied by the changes in the gross output and export, according to the type of technology. These changes are shown in table 2.

This table shows how capital formation was concentrated during this period in industry groups with large-quantity processing technology (type K (I-B)) and

TABLE 2. INDUSTRY TYPE AND PERCENTAGE COMPOSITION OF LABOUR FORCE, CAPITAL STOCK, GROSS OUTPUT AND EXPORTS[a]

	Labour force (%)		*Capital stock*		*Gross output*		*Export*	
Industry type	*1955*	*1968*	*1955*	*1968*	*1955*	*1968*	*1955*	*1968*
A and B	7.1	13.1	27.9	35.4	17.9	34.0	28.5	63.4
C	1.0	0.8	3.1	4.2	4.5	3.6	3.9	3.4
D	52.2	27.3	30.9	19.7	27.0	8.6	11.8	2.4
E and F	39.7	58.3	38.1	38.2	44.3	49.9	55.9	30.9
Unclassified	–	0.5	–	2.5	5.8	3.9	–	–

[a]The total for each column shows the total amount of all industries, except for industry 53, real estate and rents, and industry 54, miscellaneous manufacturing and other business and personal services. For these two industries, it was impossible to determine statistically the type of technology, owing to the lack of capital stock data.

Key:

A Large-quantity processing technology (type K (I–B))
B Large-scale assembly production technology (type K (I-M))
C Capital-intensive technology (type K (II))
D Cobb-Douglas (constant returns to scale) technology (type L-K)
E Labour-intensive technology (type $L_I, \gamma_L < 1$)
F Labour-intensive technology (type $L_{II}, \gamma_L > 1$)

large-scale assembly production technology (type K (I-M)) having 27.9 per cent in 1955, 35.4 per cent in 1968. By 1968 the labour force was sharply reduced in the industries centred on agriculture of Cobb-Douglas (constant returns to scale) technology (type L-K) having 52.2 per cent in 1955, 27.3 per cent in 1968. In contrast, a large absorption of the labour force took place in industries with labour-intensive technology (type $L_I, \gamma_L < 1$) and labour-intensive technology (type $L_{II}, \gamma_L > 1$) having 39.7 per cent in 1955, 58.3 per cent in 1968.

As a result of these changes in the distribution of resources, gross output in terms of 1965 constant prices grew quickly in large-quantity processing technology (type K (I-B)), large-scale assembly production technology (type K (I-M)) and capital-intensive technology (type K (II)) from 17.9 per cent in 1955 to 34.0 per cent in 1968; while there were comparative decreases in Cobb-Douglas (constant returns to scale) technology (type L-K) from 27.0 per cent in 1955 to 8.6 per cent in 1968. At the same time, there were large increases in percentage shares of exports by large-quantity processing technology (type K (I-B)) and large-scale assembly production technology (type K (I-M)) industries (steel, petrochemical products, shipbuilding, automobiles, electrical machinery, etc.).

These changes indicate that, in this period, the pattern of resource allocation in the Japanese economy showed a concentration of capital in the large-quantity processing technology (type K (I-B)) and large-scale assembly production technology (type K (I-M)) type industries. As a result, the production capacity in these industries has increased with the pursuit of the merits of large-scale production, and this in turn has strengthened international competitiveness.

As already mentioned, however, the pursuit of economies of scale does not always correspond with the demand of each commodity that is needed for the expansion of production capacity. To depict changes in the demand structure that respond to the pursuit of economies of scale, table 3 was constructed, in which a clear correspondence between the technology types estimated and commodity categories classified by end-use can be observed.

In this table, with few exceptions, most consumer goods categories rely on labour-intensive industries of labour-intensive technology (type $L_I, \gamma_L < 1$) and labour-intensive technology (type $L_{II}, \gamma_L > 1$), while the capital goods categories are dominated by large-scale assembly production technology (type K (I-M)) industry type.

The intermediate goods sector, especially that for basic intermediate goods, corresponds to large-quantity processing technology, K (I-B) type, and capital-intensive technolgy, K (II) type. The raw materials sector employs largely (L-K) type technology, whereas agriculture, mining and energy supply sector, consisting of electric power, gas, and water supply, use K (I) type technology.

Turning now to the capital investment, table 4 shows changes in the composition of investment (in constant 1965 prices) that occurred from 1952 to 1969.

With respect to manufacturing, during the first period, 1952–1954, cotton spinning was the leading industry, followed by steel, coal, electrical machinery, weaving and other textiles, and pulp and paper.

TABLE 3. RELATIONSHIP BETWEEN COMMODITY CLASSIFICATION (BY END-USE) AND INDUSTRY TYPE[a]

	Industry type[b]			
Commodity classification[a]	*E and F*	*A and B*	*C*	*D*
Consumer goods (including tertiary industries)	Meat (29) Seafood, preserved (30) Transport services (31) Paints (32) Rubber products (33) Glass products (34) Miscellaneous industrial products (35) Metal products (37) Leather products (38) Furniture and fixtures (39) Other wood products (40) Paper articles (41) Pottery, china and earthenware (42) Medicine (45) Weaving and other fibre products (46) Footwear and wearing apparel (47) Printing and publishing (48) Other food, prepared (49) Trading (50) Finance and insurance (51) Communication services (52)	Beverage and alcoholic drinks (14)	Tobacco (21)	

TABLE 3 *(continued)*

	Industry type[b]			
Commodity classification[a]	*E and F*	*A and B*	*C*	*D*
Capital goods (including housing construction)	Building and construction (28) Other transport equipment (36)	Ships and ship repairing (8) Motor vehicles (9) Machinery (10) Electrical machinery (11) Precision instruments (12)	Heavy industry and chemical industry	
Intermediate goods				
Other intermediate products	Structural clay products (43)	Fibre spinning (13)		Silk reeling and spinning (25) Vegetable oil, animal oil and fat (26) Wood milling (27)
	Other non-metallic mineral products (44)			
Basic intermediate goods		Petroleum refining products (3) Basic organic chemicals (4) Artificial fibre materials (5) Iron and steel (6) Non-ferrous metal products (7)	Paper (15) Pulp (16) Cement (17) Basic inorganic chemicals (18) Chemical manure (19) Miscellaneous coal products (20)	
Raw materials				Agriculture, forestry and fisheries (22) Coal and lignite (23) Mining (24)
Energy supply		Electric power supply (1) Gas and water supply (2)		

[a]Commodity groups are classified according to the ratio of intermediate demand in the Japanese input-output tables. Automobiles and electrical machinery are included in capital goods.
[b]See table 2 for key to industry type.

TABLE 4. COMPONENTS OF AVERAGE INVESTMENT FOR FOUR PERIODS, 1952–1969[a]

(In constant 1965 prices)

	1952–1954		*1955–1959*		*1960–1964*		*1965–1969*	
Rank	*Industry*	*Share (%)*	*Industry*	*Share (%)*	*Industry*	*Share (%)*	*Industry*	*Share (%)*
1	(Transportation and communications)	(12.75)	(Transportation and communications)	(19.80)	(Electric power)	(9.04)	(Commerce)	(10.50)
2	(Electric power)	(11.45)	(Electric power)	(9.00)	(Commerce)	(8.45)	(Transportation and communications)	(8.41)
3	(Commerce)	(6.55)	Iron and steel	7.11	Iron and steel	8.42	(Electric power)	(6.53)
4	Cotton spinning	3.79	(Commerce)	(6.76)	(Transportation and communications)	(7.95)	Iron and steel	6.33
5	(Banking and insurance)	(3.72)	Electrical machinery	3.75	Electrical machinery	4.49	Automobiles	4.92
6	Iron and steel	3.46	Inorganic chemistry	2.97	Automobiles	4.33	Construction	4.53
7	Coal and lignite	2.94	Pulp and paper	2.40	Construction	3.75	Electrical machinery	3.74
8	Electrical machinery	2.74	Ceramics and quarrying	2.27	Organic chemistry	3.51	Organic chemistry	3.49
9	Weaving	2.41	Automobiles	2.23	(Banking and insurance)	(2.98)	Non-ferrous primary goods	3.30
10	Pulp and paper	2.33	Cotton spinning	2.11	General machinery	2.76	General machinery	2.74
11	Inorganic chemistry	2.28	Organic chemistry	1.93	(Real estate)	(2.61)	(Banking and insurance)	(2.43)
12	Ceramics and quarrying	2.25	(Banking and insurance)	(1.84)	Ceramics and quarrying	2.55	Ceramics and quarrying	2.30
13	Non-ferrous primary goods	1.64	Synthetic fibre material	1.83	Pulp and paper	2.33	Metal products	2.03

[a]Gross investment data based on chronological data for forty industries from the Japan Data Development Center. Items within parentheses do not belong to the manufacturing industry.

In the second, third and fourth periods, iron and steel took the lead in the share of growing investment, followed by the electrical machinery, automobile and organic chemistry industries, while investments in cotton spinning and textiles decreased rapidly.

Cotton spinning, weaving and textiles, pulp and paper, and inorganic chemistry were important industries in the first period, but do not appear in the top 10 of the fourth period. In contrast, iron and steel maintained the top position from the second period on. It is clear that a large part of capital investment was absorbed by large-scale industries such as steel, machinery, organic chemistry, cement and pulp.

The rapid increase of production capacity in these industries must have been accompanied by changes in demand. Thus, it is necessary to outline the *causes* of these demand changes.

Demand created by government policy

As is shown in table 4, all industries with large-quantity processing technology (type K (I-B)) produce basic intermediate goods. Theoretically, the production size of these industries should be limited to a size determined by the increase of consumption and investment expenditure in the economic system. Nevertheless, in this period, the rate of expansion of investment in these industries was very high and the high rate of economic growth in Japan has been brought out by this concentration of investment in these basic intermediate goods industries. The question arises why it was possible for large-quantity processing technology (type K (I-B)) type industries to maintain expanding demand over such a long period.

In this respect, the basic model emphasizes the role of the demand created by the Government's capital formation. The equation

$$AX^t + B^{*t-1}(X^{t+1} - X^t) + C^t + G^t + E^t = X^t + M^t \qquad (18)$$

indicates that the balance between demand and supply should be maintained by the repercussion effects of the capital formation in the government sector, G^t, given the level of exports and imports. In fact, the policies between 1955 and 1965 emphasize land development measures such as the construction of roads, harbours, railways and industrial cities and made long-term expansion of demand possible.

A brief chronology showing government industrial plans and enactment of the Economic Development Law from the mid-1950s to the mid-1960s is given in table 5. The Comprehensive Development of National Territory Law was enacted in 1950 and a 10-year plan based on it was announced the following year, with an emphasis on reconstruction programmes.

The situation changed from 1955 onwards with the initiation of many land development projects. These were projects requiring large-scale investment in urban planning, area development, coastal industrial zones, development of

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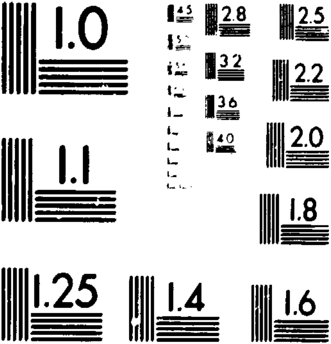

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TABLE 5. CHRONOLOGY OF INDUSTRIAL AND ECONOMIC POLICIES AND EVENTS RELATED TO LAND DEVELOPMENT

Calendar year and month		*Laws and Plans*	*Events related to land development*
1954	April	Basic Plan for the Development of Sources of Electric Power	
	April	5-year Shipbuilding Plan	
	October	3-year Plan for New Cement Plants	
1955	June	5-year Plan for the Development of the Petrochemical Industry	
	December	5-year Plan for Economic Independence	
1956	January	10-year Plan for the Modernization of Electric Power Facilities	Sakuma Dam completed
	March	Japan Highway Corporation Law	
	April	Capital Zone Equipment Law	
	June	Industrial Water Supply Law	
1957	March	Law on Multipurpose Dams	Meishin highway construction started
	March	Tōhoku Development Promotion Law	Hachirōgata reclamation started
	April	Highspeed National Motorway Construction Law	Capital highway construction started
	April	Law for the Construction of Motor Throughways for Land Development	
	June	Aichi water supply project	Aichi water supply project started
	August	5-year Plan for Increased Transport Capacity by Private Railways	
	September	Housing Building Plan	
1958	March	Law for Emergency Measures for Road Improvement	Kammon highway tunnel started
	April	Basic Plan for the Equipment of the Capital Zone	
	September	5-year Harbour Plan	
	December	Plan for the Construction of the New Tokaido Line	
1959	April	Capital Highway Corporation Law	New Tokaido line construction started
	April	Kyūshū Development Law	

TABLE 5 *(continued)*

Calendar year and month		*Laws and Plans*	*Events related to land development*
1960	March	Industrial Zoning Law	
	December	Income Doubling Plan	
		Hokuriku Development Promotion Law	
		Chūgoku Development Promotion Law	
		Shikoku Development Promotion Law	
1961	November	Law of the Industrialization of Low Developed Areas	Construction of Senriyama new town started
	November	Water Resources Development Law	Niigata pipeline installed
1962	April	Law for the Development of Mining Districts	
	May	New Industrial Cities Promotion Law	
	May	Comprehensive National Development Law	
1963		"New industrial cities" designated	Kurobe Dam completed
		"Special Districts for Industrial Development" designated	
		Second Stage of Hokkaidō Comprehensive Plan put into effect	
	July	Kinki Equipment Law	
		Law for the Development of New Residential Cities	
1964			Meishin highway opened
			New Tokaido line begins operation
			First stage of Equipment Plan for Tokyo International Airport completed
			Tokyo Olympics held
			Work begun on Kashima new industrial equipment special district
1965		Mid-term Economic Programme	

large-scale transportation networks such as expressways and high-speed railroad lines, such as the New Tokaido line, and in large-scale public works to improve roads and harbours.

In sum, the series of projects for land development concentrated in the period 1955–1966, rapidly expanded demand for products from large-quantity processing technology (type K (I-B)) and large-scale assembly production technology (type K (I-M)) industries, particularly from the iron and steel industries.

Summary

In the 1950s and 1960s:

(a) The peculiarities of the Japanese industrial structure can be found in the extraordinary expansion of the basic intermediate goods industries;

(b) The technology of the basic intermediate goods industries was of the large-quantity processing technology (type K (I-B)) type. In these industries, the effects of economies of scale in terms of labour and capital inputs that were enjoyed during a period of the continued expansion of demand, brought about an increase in productivity through increased capacity. These effects enhanced the comparative advantages of these industries. As a result, the share of large-quantity processing technology (type K (I-B)) industries in export increased rapidly;

(c) The steady expansion of demand in the basic intermediate goods industries was guaranteed by the Government's industrial planning that took place from the latter half of the 1950s until the end of the 1960s;

(d) An increase in the production and exports of the large-quantity processing technology (type K (I-B)) type industries implies a decrease in the employment coefficient per unit of production. From the point of view of production the basic intermediate goods industries, which played a leading role in the growth, expanded excessively. Since these leading industries did not have the ability to absorb manpower directly, labour was absorbed by other industries. This caused uneven progress in production and employment with the result that the present structure of the Japanese economy largely relies on the growth of heavy industry;

(e) The economic development in a specific country is a dynamic system accompanied by structural change, which results from the interrelations between technical progress and changes in demand structure. These are largely dependent on the objectives of a Government and its willingness to use various policy instruments for industrialization especially in the developing process.

The pattern of economic development described above can be well explained by a multi-industry growth model that involves structural change.

ANNEX

THE INDUSTRY CLASSIFICATION OF THE 10 × 10 INPUT-OUTPUT TABLE

1. Agriculture and mining
2. Textiles
3. Other industrial products
4. Chemicals
5. Iron and steel products
6. Non-ferrous primary products
7. Metal products
8. Machinery and electrical machinery
9. Transport equipment and precision instruments
10. Construction and services

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An assessment of the RAS method for updating input-output tables*

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The United Kingdom Central Statistical Office has published official I/O studies for the years 1954, 1963 and 1968, based upon comprehensive censuses of production. These inquiries provided detailed data on industry inputs and outputs analysed by commodity, allowing firmly based I/O tables to be constructed. Tables have also been produced for 1970, 1971 and 1972, years for which no detailed information on industry inputs was available. For these years, the industry input structures as shown by the absorption matrix in the published tables were estimated by updating the 1968 I/O structure using a modified version of a mechanical method known as the RAS method. This is a technique in which the base year absorption matrix is adjusted to sum to given row and column totals for the update year, by successive prorating of the rows and columns until consistency is achieved.

Since being developed in the Department of Applied Economics, Cambridge (Lecomber [1]), the RAS method has been examined in various studies (Paelink and Waelbroeck [2]; Schneider [3]; Tilanus [4]; Allen [5]; Barker [6]; Lecomber [7]; Allen and Lecomber [8]). These studies have increasingly emphasized the potential inaccuracies arising out of the RAS method. This paper examines an updating of the 1963 United Kingdom absorption flow matrix to 1968 by the RAS method, and compares the updated matrix and derived statistics with the corresponding ones for the firmly based 1968 tables, at a 69 industry level of disaggregation.

The RAS method is described and then normal United Kingdom practice in producing I/O tables for years for which there is no comprehensive purchases information is explained. The basic data are described and the results of the study are set out in three sections. A straightforward RAS update is compared with the firmly based 1968 absorption matrix and large differences are revealed, even with inclusion of a limited amount of exogenous information. A comparison of derived tables from the updated and firmly based absorption matrices shows that the RAS update is only a slight improvement on the original base year matrix for 1963 as a means of estimating various aspects of the economic structure of the United

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Kingdom in 1968. Finally sensitivity tests show that high levels of aggregation and exogenous information are necessary for appreciable gains in accuracy.

The RAS method

The basic method is described in the Programme for Growth series, volume 3 ([1], pp. 27–30), and in a publication of the United Nations on tables and analyses ([9], pp. 65–74). In terms of the input coefficient matrix A obtained from the absorption flow matrix by dividing each cell value by the total corresponding industrial input (identically equal to industrial output), each a_{ij} can be subject to two effects over time:

(a) The effect of substitution, measured by the extent to which commodity i has been replaced by, or used as a substitute for, other commodities in industrial production;

(b) The effect of fabrication, measured by the extent of which industry j has come to absorb a greater or smaller ratio of intermediate to total inputs in its production.

It is assumed that each effect works uniformly, that is, that commodity i is increasing or decreasing as an input into all industries at the same rate, and that any change in the ratio of intermediate to total inputs into an industry has the same effect on all commodities used as inputs. The substitution multipliers which operate along the rows are denoted as the vector r and the fabrication multiplier operating on the columns as the vector s. Each cell in the base matrix A_0 will be subject to these two effects and the new coefficient matrix A_1 can thus be written as $A_1 = r A_0 s$, r and s being matrices with the vectors r and s in the diagonals and zeros elsewhere.

In terms of the absorption flow matrix, the RAS method consists of finding a set of multipliers to adjust the rows of the existing matrix and a set of multipliers to adjust the columns so that the cells in the adjusted matrix will sum to the required row and column totals relating to the update year.

In order to find the updated absorption matrix X_1, knowing the row and column totals and the base matrix X_0, the estimation process of obtaining X_1 from X_0 by the RAS method is the proportional adjustment of the base matrix successively along its rows and columns until convergence is reached, and $X_1 = r X_0 s$. In this case the r and s are no longer simple measures of substitution and fabrication. A description of the mathematical properties of the method can be found in Bacharach [10].

This method can be modified to allow the fixing of selected rows and columns and individual cells to known values in the update year. This is accomplished by setting the value of the original cells corresponding to the fixed ones to zero and then applying the normal RAS procedure to this modified matrix so that the new row and column totals less the fixed cell values are satisfied, and finally inserting the fixed cell values for the update year. A fuller description is given in [9].

Methodology for United Kingdom input-output tables

For United Kingdom I/O studies, three basic tables were constructed. They are:

(a) An analysis of commodities made by industries; this is the "make matrix";

(b) An analysis of domestically produced commodities purchased by industries and by final demand; this is the "absorption matrix";

(c) An analysis of the imported commodities purchased by industries and by final demand; this is the "imports matrix".

I/O tables for 1968 have been published based upon the comprehensive Census of Production for 1968 which provided data on industry inputs and industry outputs analysed by commodity. These formed a firm base for the detailed analysis of commodity outputs and commodity inputs (items *(a)* and *(b)* above) and also provided some basis for the detailed analysis of the purchases of final demand by commodity.

Tables for 1970, 1971 and 1972 have also been published and their construction is described in articles in *Economic Trends* (May 1974, April 1975). The major difference between the 1968 and later I/O tables is that no detailed information on industry inputs was available for these later years. The annual Census of Production only provided figures of industry total outputs and purchases of materials and fuels, wages and salaries and stocks held by census industries. Hence the industry input structures for 1970, 1971 and 1972 were based upon an updating of the 1968 absorption matrix using the RAS method. This requires that the intermediate row and column totals of the absorption matrix for the update years be calculated.

The first step is to derive the overall row and column totals of the absorption matrix, that is, the total commodity and industry outputs respectively. This is done via the make matrix where the industries are considered in two groups. The first of these consists of all manufacturing industries plus construction. For these industries the commodity outputs (rows) are first revalued at the update year prices. This revaluation leads to a new set of industry outputs (columns) which are adjusted *pro rata* to equal the actual industry outputs taken from the annual Census of Production. This in turn gives revised row totals which are now estimates of update year commodity outputs at update year prices. The entries in the make matrix for the remaining industries are estimated directly from other sources.

The overall row and column totals thus estimated for the make matrix provide the overall row and column totals for the absorption matrix. The intermediate row totals are calculated by subtracting final demand from total commodity outputs and the intermediate column totals are similarly obtained by subtracting primary inputs from total industry outputs. Much of the data for both final demand and primary inputs is available directly, although 1968 patterns are used for some items. The estimation of the elements of the intermediate transactions matrix can then be undertaken. Six rows and columns of the 1970 absorption matrix were estimated from data provided by government departments (agriculture, forestry and fishing) or from data provided by the Digest of United Kingdom Energy Statistics (coal mining, mineral oil refining, gas and electricity). There remained 84 rows and columns of the absorption matrix for which no firmly based information was available. To provide estimates of purchases of commodities by these domestic industries, the RAS method was used. A similar procedure was carried out for 1971 and 1972, when only 59 industries were identified compared with 90 for 1970.

Data used for this study

This study was made possible by the existence of firmly based I/O tables for the years 1963 and 1968. The 1963 tables were originally compiled on the 1958 version of the International Standard Industrial Classification (ISIC) and had first to be reclassified to the 1968 version. In order to produce comparable tables, it was necessary to reduce the 70 industry tables of 1963 to 69 industry tables, and so the 90 industry tables of 1968 were also reduced to 69 industry tables. It was then possible to assess RAS updates of the 1963 absorption matrix to 1968 by comparison with the firmly based 1968 matrix. It was found helpful in assessing the magnitude of errors in the updated tables to compare them with the errors which occur using the most recent firmly based tables, without updating—in this case the 1963 tables.

In the early stages of this study, several discrepancies between the updated and firmly based 1968 absorption matrices had obviously arisen because of improvements in the allocation of unidentified purchases in 1968. Although it is a perfectly valid exercise to assess the updating procedure without allowing for such improvements, specifically in order to test the RAS method the large discrepancies were resolved as far as possible where justification existed for changes in allocations in the 1963 table. This was done by adjusting the particular cells which could be reasonably changed in 1963 by a comparison of 1963 and 1968 procedures, and then consistency obtained by a series of row and column *pro rata* adjustments. This is similar to the method used by Allen [5] in an assessment of the RAS method using 1954 and 1963 I/O tables.

It was therefore possible to produce make, absorption, primary inputs and final demand matrices for 1963 on the 1968 ISIC basis and as far as possible adjusted for differences in procedures for the allocation of unidentified purchases by an industry.

Updating the absorption matrix

This exercise is a test of the RAS method under near ideal conditions of data availability. The row and column intermediate totals from the firmly based 1968 absorption matrix were used as the constraints in updating the 1963 matrix to 1968. Table 1 shows in absolute and percentage terms the number and total value of elements in the absorption matrix for 1968 above various lower bounds.

TABLE 1. NUMBER AND TOTAL VALUE OF ELEMENTS IN THE 1968 ABSORPTION MATRIX

Category	*Lower bound for this category*	*Number of elements*	*Percentage of total*	*Value of elements (millions of £)*	*Percentage of total*
A	non-zero	2578	54	27738	100
B	£10 million	471	10	23578	85
C	One per cent of intermediate input of individual industry	1028	22	25609	92

In the following results zero elements occurring in both base and update matrices are not included in the error measures since the RAS method ensures that a zero element in the base year matrix will be zero in the update. The percentage error is defined as $100(a_u - a_f)/a_f$ for $a_f \neq 0$, where a_u is an element of the updated matrix and a_f is an element of the firmly based matrix. This conforms with the natural idea that if £75 million is the update estimate for a firmly based element of £100 million, the percentage error is −25%. This definition yields a skew distribution, the largest negative error possible being −100% with no limit to the size of the positive errors.

The updating procedure was performed both with and without exogenous information, that is by the modified and standard RAS methods. The same six rows and columns were used as in the update of 1968 to 1970 to produce the 1970 I/O tables: agriculture, forestry and fishing, coal mining, mineral oil refining, gas and electricity. The differences between the updated and firmly based 1968 absorption matrices are shown in table 2 for the standard and modified RAS, both including and excluding the fixed elements in the error measure.

TABLE 2. ROOT MEAN SQUARE ERROR IN THE ABSORPTION MATRIX FOR 1968 BY STANDARD AND MODIFIED RAS TECHNIQUES

(Millions of pounds)

Category[a]	*Standard RAS*	*Modified RAS excluding fixed elements*	*Modified RAS including fixed elements*
A	9.0	8.9	8.2
B	20.6	20.3	18.6
C	14.0	13.8	12.7

[a]See table 1 for definitions of categories by lower bounds.

The distribution of percentage errors as percentages of their own population is shown in table 3 for categories B and C of the lower bounds. The distribution for category A is similar in pattern, apart from a bias to high percentage errors due to small elements of the 1968 absorption matrix occurring in the denominator of the error measure.

TABLE 3. DISTRIBUTION OF PERCENTAGE ERRORS IN THE UPDATED ABSORPTION MATRIX OF THE STANDARD AND MODIFIED RAS PROCEDURE

(Percentage)

Method	Lower bound category	Population grouping								
		−100 to −75	−75 to −50	−50 to −25	−25 to 0	0 to 25	25 to 50	50 to 75	75 to 100	Over 100
Standard RAS	B	4.9	9.2	17.3	24.8	23.5	10.0	5.11	2.4	2.8
	C	6.0	9.1	15.6	22.0	22.0	11.2	4.9	2.4	5.8
Modified RAS										
Excluding fixed elements	B	4.5	8.5	16.3	25.5	24.8	11.3	5.0	1.0	3.1
	C	6.0	8.9	15.8	22.4	20.6	12.1	6.0	2.7	5.5
Including fixed elements	B	3.8	7.3	13.9	29.1	28.4	9.6	4.3	0.8	2.8
	C	4.6	7.9	12.8	27.4	26.8	8.6	5.5	1.7	4.7

These results suggest that updating by the RAS method yields errors in the absorption matrix estimate for the later year which are disturbingly large. For entries in the 1968 absorption matrix greater than £10 million (85% of total by value), in the standard RAS case approximately 50% of the update elements are in error by more than 25%, and 25% of the update elements are in error by more than 50%. The inclusion of exogenous information improves the overall accuracy slightly, but has surprisingly little effect on the accuracy with which the non-fixed elements are estimated.

It should be borne in mind that the size of the errors shown is due in part to three considerations:

(a) The length of time (five years) over which the update is made;

(b) No reclassification exercise such as that carried out on the 1963 absorption matrix to change from the 1958 ISIC to the 1968 ISIC can be wholly satisfactory;

(c) A general improvement in coverage of data-sources, quality of data and techniques of data processing occurs with time, resulting in differences between the allocation of purchases of commodities amongst industries for 1963 and 1968.

Updated input/output tables as aids in economic analysis

Despite the considerable errors in the updated absorption matrix, the tables derived from this matrix could still show a significant improvement over those obtained from the original 1963 matrix. To test this possibility, comparisons were made of derived tables for 1968 obtained by three different means:

(a) By using the firmly based 1968 absorption matrix and final demand;

(b) By using the updated matrix and the 1968 final demand;

(c) By using the 1963 matrix and the 1968 final demand.

For many analytical purposes, the fundamental I/O matrix is the industry by industry input coefficient matrix A in the equation

$$X = AX + F \quad (1)$$

where X is the gross industry output vector and F is the total final demand vector.

It is this matrix A which has been most subject to tests in previous assessments of the RAS method. It was necessary therefore to construct the matrix A for 1963, 1968 and the update to 1968 from the corresponding absorption matrices using the Central Statistical Office's normal procedures as described in the 1968 I/O study.

The 1963 coefficient matrix A was changed to flow form by multiplying each element by the appropriate 1968 gross industry output. This allowed all the matrices to be compared in flow form, at the same time allowing the lower bound criteria to be applied although the percentage errors apply equally well to the coefficient form. Both the 1963 version and the 1968 updated version (shown in the tables as "1963–68") were compared with the firmly based 1968 one, and the results are shown in table 4. The figures in brackets are the corresponding errors for the 34 industry version of the 69 industry tables (the summary tables published regularly in *Economic Trends* are 34 industry). The 34 industry version was obtained by reducing the 69 industry flow matrices as a last step.

TABLE 4. ERRORS IN THE ESTIMATES OF COEFFICIENT MATRIX A FOR 1968[a]

Lower bound estimate (millions of £)	*Version*	*Root mean square error for flow form (millions of £)*	*Mean absolute error (%)*
0	1963	7.9 (17.8)	456.0 (369.8)
	1963–68	6.6 (15.4)	496.8 (294.0)
10	1963	22.0 (28.9)	45.4 (46.4)
	1963–1968	17.7 (24.7)	40.6 (41.4)
20	1963	28.4 (33.0)	38.9 (37.9)
	1963–68	23.0 (28.5)	34.3 (33.3)

[a]Figures for the 34 industry versions are shown in brackets.

Using the three different versions of A, the inverses and associated derived matrices of the 1968 published tables were then calculated using the same methodology as for the 1968 study. At first glance it might seem that a good test of the adequacy of the Leontief inverse $(I - A)^{-1}$ (table A of the 1968 publication) would be how successfully total output for 1968 could be predicted using

$$X = (I - A)^{-1} F \quad (2)$$

However, the RAS method explicitly uses the relationship between gross output and final demand to derive intermediate outputs and so this equation is satisfied identically by an RAS updated matrix A. Another way of measuring the adequacy of the inverse is to consider how well each element of the inverse is estimated. In order to apply a lower bound criterion to exclude smaller elements, each element of the inverse was weighted by the appropriate final demand for 1968. As each element a_{ij} of A represents the amount of industry i gross output required to produce one unit of output going to final demand for industry j, the flow version is the allocation of industry gross output to final demand both directly and indirectly. The results are shown in table 5 and, as before, the percentage figures apply equally to the coefficient form of the inverses.

TABLE 5. ERRORS IN THE ESTIMATES OF THE INVERSE MATRIX $(I-A)^{-1}$ FOR 1968[a]

Lower bound estimate (millions of £)	*Version*	*Root mean square error for flow form (millions of £)*	*Mean absolute error (%)*
5	1963	13.3 (19.6)	33.4 (31.5)
	1963–68	9.5 (13.5)	28.0 (24.2)
10	1963	17.0 (22.9)	26.7 (27.1)
	1963–68	12.0 (15.7)	21.0 (20.6)

[a]Figures for the 34 industry versions are shown in brackets.

The usual practice of the Central Statistical Office is to calculate the inverse, excluding intra-industry transactions, i. e. the diagonal elements of A. However, because of differences in the size of the intra-industry transactions between the 1968 flow matrix AX and the RAS update, this results in the measure of total domestic output net of intra-industry transactions being different for the 1968 firmly based and updated versions of the matrices, as reflected in

$$X_n = (I - A_n)^{-1} F \tag{3}$$

where n is net of intra-industry transactions.

Similarly this measure of domestic gross output will change as the level of aggregation changes and will complicate comparisons of updated and firmly based versions of the 1968 matrices at different levels of aggregation. For these reasons the intra-industry transactions are included when calculating the inverse for the purpose of this study.

Although a test on the adequacy of the updated matrix A in its role in predicting gross output from final demand for the update year is rendered trivial by the nature of the RAS method, it is nevertheless of interest to compare the allocations of gross or net output amongst the various sectors of final demand such as consumers' expenditure or exports for individual industries. Taking F to be the matrix of final demand, equation (2) becomes

$$X_f = (I - A)^{-1} F \tag{4}$$

where X_f is a matrix showing the allocation of gross output of final demand by industry.

It is of course possible to premultiply rows of this matrix by the ratio of net to gross output for each industry and so express it in net output terms, but as percentage errors are perhaps the most illuminating this was not felt necessary.

Table 6 summarizes the results of the comparison between the 1968 flow version of X_f in gross output terms and the alternative estimates. The 1963 version is obtained by applying the 1963 inverse to the 1968 final demand matrix on an industry basis, and similarly the 1963–68 update inverse provides the updated versions. It must be remembered that the RAS method assures that the total final demand allocation of an industry's output in the updated version is identical with the 1968 total. It is straightforward to adjust the rows of the 1963 matrix X_f to conform to these industry output totals also by simple pro-rating. Thus a more rigorous test of the RAS method in as far as it accomplishes more than just consistency with 1968 totals, is to constrain the 1963 estimates of X_f to 1968 totals and compare the errors in these matrices of final demand in flow form, as shown in table 6. This is equivalent to considering the estimated matrices as a means of indicating the proportional allocation of total output between the sectors of final demand by industry, the form of table I of the 1968 I/O study. The 1963 estimates constrained to 1968 totals are denoted by 1963(c) in tables 6 and 7.

TABLE 6. ERRORS IN THE ESTIMATES OF THE 1968 MATRIX SHOWING ALLOCATION OF TOTAL OUTPUT TO FINAL DEMAND[a,b]

Lower bound estimate (millions of £)	*Version*	*Root mean square error for flow form (millions of £)*	*Mean absolute error (%)*
5	1963	14.6 (20.3)	11.7 (10.8)
	1963(c)	8.4 (11.3)	7.0 (6.6)
	1963–68	5.0 (6.4)	5.6 (4.4)
10	1963	15.5 (21.4)	11.4 (10.8)
	1963(c)	8.9 (11.9)	6.6 (6.1)
	1963–68	5.3 (6.7)	5.2 (4.0)
20	1963	17.0 (22.5)	10.2 (9.0)
	1963(c)	9.7 (12.6)	5.2 (4.4)
	1963–68	5.7 (7.1)	4.6 (3.9)

[a]Data from 1968 study, table H.
[b]Figures for the 34 industry version are shown in brackets.

As each estimate of the flow matrix X_f includes the same direct components of final demand for 1968, a truer assessment of the adequacy of the inverse estimates is achieved by considering only the indirect flow part of X_f. The differences are shown in table 7.

TABLE 7. ERRORS IN THE ESTIMATES OF THE 1968 MATRIX SHOWING INDIRECT ALLOCATION OF OUTPUTS TO FINAL DEMAND[a, b]

Lower bound estimate (millions of £)	*Version*	*Root mean square error for flow form (millions of £)*	*Mean absolute error (%)*
5	1963	15.5 (21.3)	22.0 (21.3)
	1963(c)	6.2 (8.1)	10.0 (8.1)
	1963–68	5.3 (6.6)	9.5 (7.5)
10	1963	17.1 (22.5)	21.4 (20.2)
	1963(c)	6.8 (8.7)	9.4 (7.8)
	1963–68	5.9 (7.1)	8.6 (6.9)
20	1963	19.6 (24.5)	19.7 (17.3)
	1963(c)	7.7 (9.5)	8.2 (6.4)
	1963–68	6.6 (7.8)	7.3 (6.0)

[a]Data from 1968 study, table I.
[b]Figures for the 34 industry version are shown in brackets.

The figures shown in tables 5, 6 and 7 prompt the following deductions:

(a) The updated versions of A, $(I - A)^{-1}$ and X_f are better estimates of the 1968 equivalents than the 1963 versions, although the improvement is relatively modest;

(b) The results of the constrained 1963 table comparison in tables 6 and 7 do, however, suggest that simple pro-rating procedures yield almost as good results as the standard RAS method in determining allocations of output to final demand, an important use of I/O methods;

(c) The aggregation of the tables from 69 to 34 industries as a last step does little to improve the percentage errors occurring in the basic and derived I/O tables.

This last, rather surprising result can be rationalized as follows. The aggregation of 69 industries to 34 is not the simple 2 : 1 reduction that it seems. Of the 34 industries created by the aggregation, 20 are unchanged from the 69 industry classification and so many errors revealed in the 69 industry comparison are unaffected by aggregation. Also if errors in cells at the 69 industry level which are aggregated are positively correlated, then percentage errors will not decrease to the extent that would be expected if the errors were independent. Where a cell value at the 69 industry level constitutes most of the value of a 34 industry aggregated cell then positively correlated errors can result in an increase of the average absolute percentage error on aggregation.

The effect of different aggregation and exogenous information levels on the adequacy of RAS updated input-output tables

It is trivially true that either increasing the exogenous information level to 100% or reducing the level of aggregation to 1 will produce updated tables with no estimation errors. These tables will have the disadvantage of either costing much more as more exogenous information is required or of revealing less of the economic structure in the update year due to the high level of aggregation. The question naturally arises whether updated tables can be produced using an RAS with exogenous information which are significantly less costly to produce in terms of time and resources (including the resources required to conduct the Census of Production and purchases inquiry) than firmly based tables, in which the errors do not exceed acceptable levels and yet the level of aggregation allows meaningful deductions and analytical exercises highlighting economic structure to take place.

Four different levels of aggregation were chosen, 69, 34, 13 and 6. In the progressive aggregation from 69 to 6, homogeneous industries were chosen to be combined in as far as this was possible. Four different levels of exogenous information were chosen which in terms of fixed rows and columns at the 69 industry level were 0, 6, 10 and 20. In choosing the rows and columns to be determined exogenously, the criterion used was the likelihood of such information becoming available in the United Kingdom in the foreseeable future short of a full scale purchases inquiry, e.g. industries with a large content of nationalized enterprises were chosen first and industries such as distribution were not determined exogenously.

Complete rows and columns were chosen at the 69 industry level for convenience. It would have been possible to choose individual cells instead. Further the criterion used is not the only one possible for selecting exogenous information. An equally valid criterion is whether the elements in question are likely to be badly estimated by RAS (such as a heterogeneous commodity group). Another question is whether the elements to be fixed are important in terms of uses to which the I/O table can be put (Allen [5]). Methods of incorporating not fully reliable exogenous information and the results of tests on these methods are given in a paper by Allen and Lecomber [8].

The details of the method used in this study are as follows. Cells in the 1963 absorption matrix corresponding to the fixed rows and columns were set to zero. The RAS procedure was then applied to this modified absorption matrix and the exogenous information added to produce a 1968 update. At higher levels of aggregation, the endogenous and exogenous matrices were treated separately. The 1963 absorption matrix at the 69 industry level with cells for which information existed set to zero was aggregated and then the RAS procedure applied, and then the aggregated form of the matrix of exogenous information was added back to produce the 1968 update. This allowed the level of exogenous information to be held constant in terms of value and ease of procurement whilst the aggregation level was varied.

The absorption matrices were then transformed by the Central Statistical Office's usual procedures to industry by industry matrices. To provide a 1963 estimate of 1968, the 1963 flow matrix was put into coefficient form by dividing intermediate industry inputs by the appropriate gross industry output for 1963, and then converted to 1968 flow form by multiplying by the corresponding 1968 gross industry outputs. This allowed the matrices to be compared using lower bound criteria for inclusion, but yielded percentage errors which applied equally well to the coefficient forms of this industry by industry matrix. Table 8 shows the results of this exercise, for a lower bound of £5 million. The percentage errors shown in the table are the mean absolute percentage errors.

TABLE 8. ERRORS IN THE ESTIMATES OF THE 1968 SYMMETRIC FLOW ABSORPTION MATRIX[a, b]

		1963–68 exogenous information levels (in terms of fixed rows and columns)			
	1963	*0*	*6*	*10*	*20*
Aggregation levels	*root mean square error (millions of £)*	*Root mean square error (millions of £)*			
69	17.6 (51.9)	14.2 (48.3)	12.3 (40.3)	11.0 (32.1)	9.3 (23.8)
34	25.0 (51.9)	21.6 (47.0)	19.3 (36.1)	15.2 (26.2)	12.6 (19.9)
13	48.3 (31.3)	37.6 (28.9)	25.4 (18.9)	20.6 (11.7)	13.8 (9.6)
6	75.8 (19.9)	55.8 (16.9)	23.6 (7.8)	28.1 (6.9)	27.0 (6.5)

[a]Data from 1968 study, table D.
[b]Percentage errors are in brackets.

The exogenous information levels denoted by the number of fixed rows and columns in table 8 can be represented as a percentage of non-zero cells fixed or of value fixed as is illustrated in table 9.

TABLE 9. LEVEL OF EXOGENOUS INFORMATION AVAILABLE

	Exogenous information levels (in terms of fixed rows and columns)			
Number of fixed rows and columns at 69 industry level	0	6	10	20
Percentage of non-zero cells fixed	0	17.1	30.8	52.6
Percentage of value fixed	0	19.4	38.6	59.7

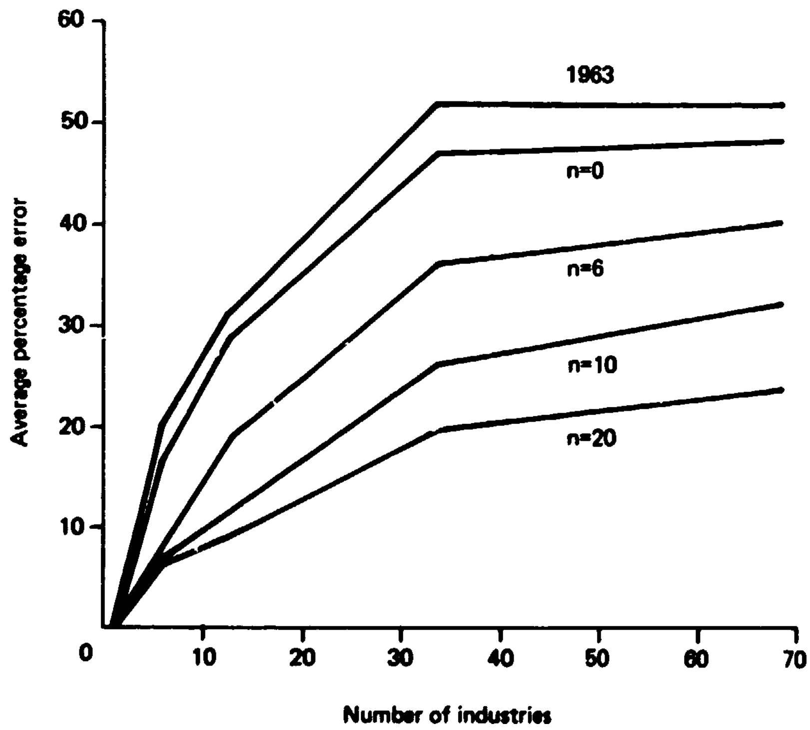
Table 10 shows the percentage coverage of the £5 million lower bound for different levels of aggregation.

TABLE 10. PERCENTAGE OF NON-ZERO CELLS IN THE SAMPLE USING £5 MILLION LOWER BOUND FOR DIFFERENT AGGREGATION LEVELS

Aggregation level	*Non-zero cells counted by number (%)*	*Non-zero cells counted by value (%)*
69	21	92
34	51	98
13	87	99.8
6	100	100

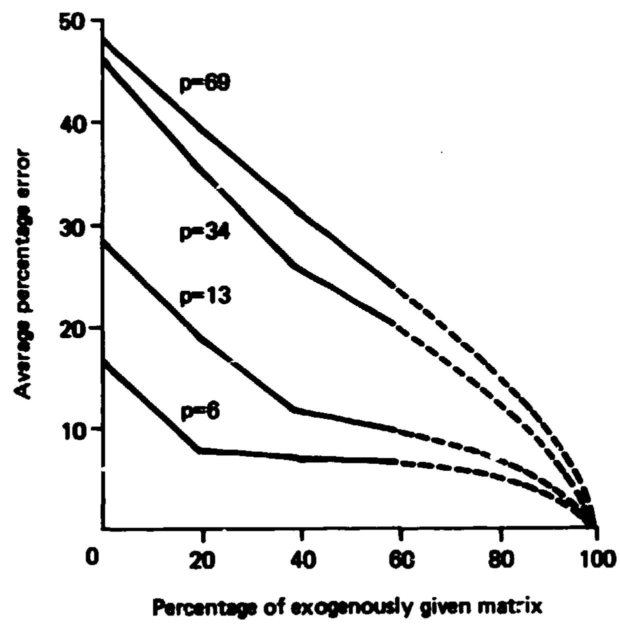
The error measures of table 8 are of the updated matrix including the exogenous information. Thus the errors are a measure of the RAS method with extra information. The results of table 8 are illustrated in figures 1 and 2. Figure 1 demonstrates that increasing the number of rows and columns increases the mean absolute percentage error for the various levels of exogenous information, but at a decreasing rate. Figure 2 shows that increasing the level of exogenous information reduces the average absolute percentage error as would be expected, but the effect is greater at higher levels of disaggregation.

Figure 1. Average percentage error, for different numbers of rows and columns, with respect to number of industries[a]



[a]The number of rows and columns fixed is given by "n". The 1963 estimate is shown as a reference.

Figure 2. Average percentage error, for different levels of disaggregation, with respect to the percentage of an exogenously given matrix value[a]



[a]The number of industries identified is given by "p".

Conclusions

This study has shown that large percentage errors exist in an update of the 1963 absorption flow matrix to 1968 using the RAS method. The estimate is improved by the inclusion of six fixed rows and columns but the errors remain large. It has also demonstrated that the 1963 firmly based absorption matrix can be used to examine the economic structure in 1968 by operating on a firmly based 1968 final demand matrix, almost as effectively as an updated version of the absorption matrix. In order to bring percentage errors down to the order of 10% in the updated absorption matrix, either the number of industries must be reduced to six or lower, or the exogenous information by either value or percentage of non-zero cells must be about 60%, that is, about 18 rows and columns in terms of this study of 69 industries.

There can be no doubt that the reclassification exercise and improvements in data sources and methodology between 1963 and 1968 are responsible to some extent for the differences between the updated and firmly based tables for 1968. These characteristics are of course difficult to measure in quantitative terms. However, it is unlikely that differences arising from these characteristics significantly affect either the assessment of the RAS method described in this paper, or the conclusions drawn.

It would seem, therefore, that the RAS method can be used as a convenient means of constraining matrices to given row and column totals, but that little should be expected of it as a means of forecasting absorption matrices over a period as long as five years at the 69 industry level. This in turn suggests that publishing I/O absorption matrices for years between full-scale censuses is of dubious merit unless a considerable amount of exogenous information is available. In terms of illustrating the contemporary economic structure of the United Kingdom, the last available firmly based table will do almost as well.

The Central Statistical Office is very much aware of these concepts, and efforts are being made to incorporate as much exogenous information as possible into the I/O tables for years for which there was no purchases inquiry.

Summary

The United Kingdom Central Statistical Office produces I/O tables at approximately five year intervals based upon comprehensive data on industries' sales and purchases. For the intervening years when little information is available on purchases, it is possible to estimate the input structures by updating the base year absorption matrix using the RAS method. The method consists of successive pro-rating of rows and columns to known totals for the update year until consistency is achieved. This paper examined an updating of the 1963 United Kingdom absorption flow matrix to 1968 by the RAS method, as there were firmly based tables for both years. Sensitivity test results were presented which demonstrated the effect of aggregation of rows and columns, and different levels of exogenous information, on the measures of accuracy of the updated tables.

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Structural change in the Canadian economy 1961–1971

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The purpose of this paper is to describe the methodology and to present some of the results of a descriptive analytical study of structural change in the Canadian economy, undertaken at Statistics Canada.

The study examines the growth in the real output of domestically produced and imported goods and services, and the changes in the structures of demand, supply and production between the years 1961 and 1971 as reflected in the constant dollar I/O tables for Canada for those years. For analytical purposes a distinction is made between the demand for goods and services by final purchasers and the demand for goods and services by industries for intermediate use as inputs in the process of production.

The sectors which are the final purchasers, namely "consumers" and the personal sector, "government current expenditure", "business and government investment" and "exports", are examined in detail to identify their contribution to change due to changes in the level of their aggregate expenditure and in the composition of goods and services demanded.

Changes in the structure of industrial interdependence, and in the pattern of inputs used in production, are assessed using a number of different techniques and measures.

The study is descriptive in that it indicates the areas, magnitude and direction of change. However, the changes observed are the combined result of changes in classification and statistical definition, in the representation of the structure of the economy in the I/O framework, as well as real changes in the composition and pattern of final demand and in the product mix and input patterns of industries. The real changes are, in turn, due to changes in the structure of industry, the availability of resources, relative prices and the technology of production.

The study does not attempt to identify the particular underlying causes of the changes which are observed.

Methodology

The data which have been used to analyse the changes in the Canadian economy that occurred between 1961 and 1971 are those of the Statistics Canada I/O tables. This structural representation of the supply and disposition of the

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goods and services imported or produced in the Canadian economy, forms the basis of the study. The tables which have been used are those for 1961 to 1971 in constant 1961 prices. These data also form the structural parameters of the I/O models which are used in the analysis.

The I/O tables

The "accounting framework" of the Canadian I/O tables consists primarily of three matrices, "Make (V)", "Use (U)" and "Final Demand (FD)", of rectangular format, with a larger number of commodities (595) than industries (191) and final demand sectors (136). Each industry produces one or more commodities and each commodity is produced by one or more industries. The Use and Final Demand matrices each contain an associated primary input matrix.

The method of construction of the tables is that the Make matrix, which shows the value of domestic output of commodities by industries, is compiled in producer's prices. The Use and Final Demand matrices, which show the disposition of supply among industries and final demand sectors, domestic and foreign, are first constructed in purchaser's prices. Total imports are shown as a sector of Final Demand, as negative entries against each commodity. At the second stage, the margins that make up the difference between producer's and purchaser's prices are deducted. These margins, which relate to the cost of transportation, retailing and wholesaling and indirect commodity taxes, are reassigned to service or dummy commodities and to taxes. Finally, these revalued tables and the Make matrix are deflated to constant 1961 prices by the method of double deflation, each row of the tables being deflated by one price index for each commodity group, taxes being re-estimated and value added being derived by difference.

The I/O model

The underlying assumptions of the I/O model used for this study are fixed market shares and industry technology. The latter assumption is expressed in the matrix B, whose $(i,j)^{th}$ element represents the value of intermediate commodity i used per dollar of output of the j^{th} industry (B is calculated from the Use matrix for a given year); the fixed market shares assumption is represented by the matrix D, whose $(j,i)^{th}$ element is the proportion of the domestic output of commodity i produced by the j^{th} industry (D is calculated from the Make matrix). The market share matrix D serves to allocate commodity demands among producing industries. Multiplying these matrices gives DB, an industry-by-industry matrix which is the analogue of the traditional Leontief technology matrix A.

In this study two variants of the I/O model are used; their difference is in the treatment of imports.

In the first variant total imports are deducted from final demand. Thus for a final expenditure Y and vectors of industry and commodity outputs X_g and X_q respectively, the model is:

$$X_g = (I - DB)^{-1} DY \quad (1)$$

where $Y = F + E - M$.

(Total imports M are deducted from the sum of domestic final demand F and foreign demand for exports E, to give final expenditure on goods and services Y.)

$$X_g = DX_q \quad (2)$$

$$X_q = BX_q + F + E - M \quad (3)$$

Substituting (3) into (2) and solving

$$X_g = (I - DB)^{-1} D(F + E - M) \quad (4)$$

$$\text{or } (I - DB)^{-1} DY$$

In the second variant, imports are endogenized using the import share assumption. If a fixed share μ_i of the domestic disappearance of each commodity i is assumed to be imported

$$M = \hat{\mu}(BX_g + F) \quad (5)$$

Substituting (5) into (3)

$$X_g = [I - D(I - \hat{\mu})B]^{-1} D[(I - \hat{\mu})F + E] \quad (6)$$

The relationship between the two variants is given by (4) and (6)

$$X_g = (I - DB)^{-1} D(F + E - M) = [I - D(I - \hat{\mu})B]^{-1} D[(I - \hat{\mu})F + E].$$

The calculations of the model are all performed at the large aggregation of 191 industries and 595 commodities and the results are aggregated for presentation to categories shown in annex I which correspond to the medium aggregation (except for the final demand sectors) of the published version of the tables.[1] For reasons of confidentiality the I/O tables are not published at the large aggregation.

Analysis of changes in output and demand

The study begins with an overview of aggregate supply, output and demand in the Canadian economy for domestically produced and imported goods and services (values in 1961 constant dollars) as well as of real GDP at factor cost and employment, between the years 1961 and 1971.

Relative effect of changes in final demand and intermediate demand

The variation in domestic output between two years can be attributed to growth, changes in the composition of final demand and changes in the structure of intermediate demand. The analysis in this section uses the I/O model (variant 1)

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[1] *The Input-Output Structure of the Canadian Economy in Constant 1961 Prices 1961–1971*; Catalogue 15-507E Occasional (Ottawa, Statistics Canada, 1971).

to examine the extent to which the variation in aggregate domestic output between two years is attributable to changes in the level and composition of final expenditure and to changes in intermediate demand as a consequence of changes in the input structures of industry.

The "final demand effect" is estimated as the difference in direct and indirect output required to produce the actual final expenditure of two different years, with constant "structure and technology" or the input coefficients of one year.

The "intermediate demand effect" is estimated as the difference in direct and indirect output required to produce one year's final expenditure with changing "structure and technology" or the input coefficients of the two different years.

The actual methodology used is:

$$\text{Total change} = X_t - X_{t-1} = [(I - DB)^{-1} D]_t Y_t - (I - DB)^{-1} D_{t-1} Y_{t-1}$$

$$\text{Final demand effect} = [(I - DB)^{-1} D]_{t-1} Y_t - [(I - DB)^{-1} D]_{t-1} Y_{t-1}$$

$$\text{Intermediate demand effect} = [(I - DB)^{-1} D]_t Y_t - [(I - DB)^{-1} D]_{t-1} Y_t$$

where:

Y = final expenditure and excludes total imports;
$t - 1$ = a reference to a previous time period.

Growth by industry

A more detailed analysis by industry, of the growth in output between 1961 and 1971 is undertaken next, in which the final demand effect is sub-divided between the extent to which growth in the level of final expenditure (scale effect) and change in the commodity composition of final expenditure (pattern effect) may be regarded as having contributed to the estimated change.

The actual methodology used is:

$$\text{Final demand effect} = [(I - DB)^{-1} D]_{t-1} Y_t - [(I - DB)^{-1} D]_{t-1} Y_{t-1}$$

$$\text{Scale effect} = [(I - DB)^{-1} D]_{t-1} (Y_{t-1} \lambda) - [(I - DB)^{-1} D]_{t-1} Y_{t-1}$$

$$\text{Pattern effect} = [(I - DB)^{-1} D]_{t-1} Y_t - [(I - DB)^{-1} D]_{t-1} (Y_{t-1} \lambda)$$

for a scalar $\lambda = (\sum_i (Y_i)_t [\sum_i (Y_i)_{t-1}]^{-1})$.

Analysis of final demand

The methodology described above can be extended to isolate the contribution to change of as many components as one may wish to distinguish, by changing each component in turn in combination with all the others, in as many permutations and combinations as there are components. In view of the fact that the choice of order biases the results, ideally each component's contribution should be estimated as an average of the results of all the possible combinations.

This approach could therefore have been extended to estimate the contribution of the major components of final demand, that is to say consumers' expenditure, government current expenditure, fixed capital formation, exports and imports and the change in the commodity composition of each, to the change in commodity or industrial output. However, for this study, the analysis has been confined to an examination of the contribution of the various categories of final purchasers or sectors of final demand, to changes in the level and composition of final expenditure. The approach adopted is that of direct comparison of the final demand matrices for 1961 and 1971.

The structure of production

The analysis in this section of the paper concentrates on an examination of "interdependence" between industries and of industrial input structures, as represented in the matrix of technological coefficients.

Forward linkages in intermediate production

The right dominant eigenvector of the technology matrix DB can serve as a key structural indicator. Its derivation and interpretation is given in annex II where it is shown that the ranking of industries in the dominant eigenvector is an intrinsic (that is demand-independent) measure of the relative importance of industries as producers of intermediate goods, and therefore serves to measure forward linkage. This indicator should be interpreted with care, however, since it is sensitive both to relative price changes and to the level of industry aggregation.

The dominant industries in terms of forward linkage possess an inherent importance to the economy. Their dominance as intermediate producers in response to any stochastic increase in final demand means that capacity bottlenecks in these industries are in general detrimental to smooth short-term performance. Conversely, the short-term resilience of the system in terms of its ability to meet stochastic increases in final demand would vary directly with the spare capacity in industries with dominant forward linkages.

As the linkage patterns between domestic industries have been modified both by technological change and by varying trade patterns, the analysis separates these effects using variant two of the model which endogenizes imports (see equation (6)). First the changes over time in the actual systems are compared and then the changes which are purely attributable to changing trade patterns are measured, by comparing the forward linkage eigenvectors of the following matrices:

(*a*) $D_{71}(I - \hat{\mu})_{71} B_{71}$ is compared to $D_{61}(I - \hat{\mu})_{61} B_{61}$;

(*b*) $D_{71}(I - \hat{\mu})_{71} B_{71}$ is compared to $D_{71}(I - \hat{\mu})_{61} B_{71}$.

The eigenvectors are normalized to one million constant 1961 dollars (with "dummy" sectors eliminated), and changes in forward linkage are measured simply as differences between the eigenvectors corresponding to (*a*) and (*b*). The vectors were calculated at the large industry aggregation (191 sectors, corresponding roughly to the 3-digit ISIC) and aggregated to the medium (43 sector) level to mask some of the "noise" in the system.

Industrial input structures

The input structures of industries are examined from various points of view.

Direct demand for commodity inputs

To begin with, the extent to which each industry changed its direct commodity input structure, whether as a consequence of a change in its output mix or as a consequence of changes in the technology of production, is examined by analysing the input coefficient matrix B.

The summary measure used is:

$$C_j = K(1/2\sum_i |B_{ij(1971)} - B_{ij(1961)}|)$$

where:

i = commodities;
j = industries;
k = a scaling factor which produces indices of a convenient magnitude.

C is thus an index of the amount of share exchanged among commodities in the input structure of industry j between 1961 and 1971. It measures the extent to which commodity shares of industry input structures (of domestically produced and imported commodities taken together) changed.

Direct and indirect demand for inputs

Industries draw upon domestic production and imports for both their direct and their indirect input requirements. Therefore their demand is analysed by making the import share assumption, and estimating the direct and indirect requirement from each source needed to produce a unit of industry output. Changes in the direct and indirect requirements for amount of labour per unit of industry output are also examined.

Using variant two of the model, the direct and indirect requirements are calculated as shown below.

Domestically produced goods and services $= i_r'(I - \hat{\mu})B[I - D(I - \hat{\mu})B]^{-1}$

Imports $= \hat{\mu}'B[I - D(I - \hat{\mu})B]^{-1}$

Man years of employment $= l'[I - D(I - \hat{\mu})B]^{-1}$

where:

i_r' = inputs of goods and services;
$\hat{\mu}'$ = the proportion of demand met from imports;
$l' = L_g'\hat{X}_g^{-1}$
L_g' = man years of labour by industry;
X_g' = output by industry.

The calculations are performed at the large aggregation and the results are weighted by industry outputs for aggregation to the industry groups shown in the tables. The changes in direct and indirect requirements is derived by subtracting the results for 1961 from those for 1971.

Input substitution

Lastly, the change in the input requirements of industries is examined by comparing the pattern of goods and services that would have been required by industry to produce the final expenditure (that is final demand net of total imports) of 1971, with the changing structures and technologies of production reflected in the coefficient matrices of different years.

$$X_{q(INT)t} = [(I - DB)_t^{-1} Y_{1971}] - Y_{1971}$$

where:

$X_{q(INT)}$ = the demand for goods and services by intermediate industry;
t = the year 1961, 1966 or 1971.

Results

Overview

The period 1961 to 1971 was a period of relatively steady annual growth in real terms for the Canadian economy as compared to the more erratic growth of the 1950s. The total supply of goods and services, whether domestically produced or imported, valued in 1961 prices, grew from $75.0 billion in 1961 to $131.9 billion in 1971, at an average growth rate of 5.8% per annum. The final demand which this total of goods and services met, either directly or by being embodied in production, grew from $40.7 billion to $71.8 billion, also at an average rate of 5.8% per annum. The proportion of aggregate goods and services which were directly purchased by final demand remained an almost constant proportion of total supply, between 53.4% and 54.7%. The income generated by domestic output (GDP at factor cost) was $35.5 billion in 1961 and $60.1 billion in 1971. Employment grew at 3.3% per annum from 4.999 million man-years in 1961 to 6.924 million man-years in 1971.

The economy became more open to trade (particularly after the signing of the automotive agreement with the United States in 1966). The share of imports as a proportion of total supply which had been 10.6% in 1961 was 12.7% in 1971, the average growth of domestic output being 5.6% per annum and of imports, 7.7% per annum. Exports grew at 9.1% per annum. Their share of domestic output grew from 10.6% in 1961 to 14.8% in 1971. Data for the years 1961 to 1971 are presented in table 1.

The year to year changes in total domestic output between 1961 and 1971 as well as the overall change between the two years is shown in table 2. These changes are attributed to what can be described as the "final demand effect", or the change due to changes in the level and composition of final expenditure, and the "intermediate demand effect" of the change due to changes in industrial input structures. The contribution of each effect to the total change is also shown.

TABLE 1. SUPPLY, OUTPUT, AND DEMAND, BY YEAR, 1961–1971

Item	*Period* 1961	1962	1963	1964	1965	1966	1967	1968	1969	1970	1971	1961–1971
Domestic output X $(10^{9}\$)$	67.0	71.4	75.9	82.1	88.5	94.6	97.0	102.1	107.3	109.0	115.1	
Annual growth rate (%)		6.6	6.2	8.3	7.7	6.9	2.5	5.3	5.1	1.5	5.6	5.6
Imports M $(10^{9}\$)$	8.0	8.5	8.6	9.6	10.8	12.3	12.9	14.2	16.0	15.6	16.8	
Annual growth rate (%)		6.4	0.9	12.6	12.1	13.9	4.9	9.7	13.2	−2.6	7.5	7.7
Total supply X + M $(10^{9}\$)$	75.0	79.9	84.4	91.8	99.3	106.9	109.9	116.3	123.4	124.6	131.9	
Annual growth rate (%)		6.6	5.6	8.7	8.2	7.7	2.8	5.9	6.1	1.0	5.9	5.8
Domestic output (shares)	89.4	89.4	89.9	89.5	89.1	88.5	88.3	87.8	87.0	87.5	87.3	
Imports (shares)	10.6	10.6	10.1	10.5	10.9	11.5	11.7	12.2	13.0	12.5	12.7	
Total supply (shares)	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	
Final demand $(10^{9}\$)$	40.7	43.5	45.4	49.1	53.3	57.5	59.1	62.8	67.5	67.8	71.8	
Annual growth rate A (%)		6.8	4.5	8.0	8.7	7.8	2.8	6.3	7.5	0.3	6.0	5.8
Intermediate demand $(10^{9}\$)$	34.3	36.5	39.0	42.7	45.9	49.4	50.8	53.5	55.8	56.8	60.1	
Annual growth rate A(%)		6.3	7.0	9.6	7.5	7.5	2.8	5.4	4.4	1.7	5.7	5.8
Total demand X + M $(10^{9}\$)$	75.0	79.9	84.4	91.8	99.3	106.9	109.9	116.3	123.4	124.6	131.9	
Annual growth rate A (%)		6.6	5.6	8.7	8.2	7.7	2.8	5.9	6.1	1.0	5.9	5.8
Final demand (shares)	54.3	54.4	53.8	53.4	53.7	53.8	53.8	54.0	54.7	54.4	54.5	
Intermediate demand (shares)	45.7	45.6	46.2	46.6	46.3	46.2	46.2	46.0	45.3	45.6	45.5	
Total demand (shares)	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	
GDP at factor cost $(10^{9}\$)$	35.5	38.0	40.0	42.7	45.8	48.7	50.1	52.9	56.1	57.0	60.1	
Annual growth rate (%)		6.9	5.2	6.8	7.3	6.3	2.8	5.6	6.1	1.6	5.4	5.4
Estimated employment (10^{6} man-years)	4.999	5.260	5.333	5.656	5.857	6.131	6.311	6.436	6.680	6.770	6.924	
Annual growth rate (%)		5.2	1.4	6.1	3.6	4.7	2.9	2.0	3.8	1.4	2.3	3.3

Note: All dollar amounts are in constant 1961 dollars.

TABLE 2. RELATIVE EFFECT OF CHANGES IN FINAL DEMAND AND INTERMEDIATE DEMAND, 1961–1971[a]

	Output (millions of constant 1961 dollars)		Change								
			Partition of change (at constant 1961 dollars)			Contribution to change (%)		Per annum change (%)		
Period	Total output X_t	Total output X_{t-1}	FD[b] effect	INTD[c] effect (millions of dollars)	Total change	FD[b] effect	INTD[c] effect	FD^b/X_{t-1}	$INTD^c/X_{t-1}$	TC^d/X_{t-1}
1961–1971	115 107	67 003	46 346	1 758	48 104	96	4	5.4	0.3	5.6
1961–1962	71 427	67 003	4 706	−282	4 424	95	5	7.0	−0.4	6.6
1962–1963	75 863	71 427	3 965	471	4 436	89	11	5.6	0.6	6.2
1963–1964	82 146	75 863	5 590	693	6 283	89	11	7.3	0.9	8.2
1964–1965	88 458	82 146	6 860	−548	6 312	93	7	8.3	−0.7	7.7
1965–1966	94 571	88 458	5 700	413	6 113	93	7	6.4	0.5	6.9
1966–1967	96 966	94 571	1 790	605	2 395	75	25	1.9	0.6	2.5
1967–1968	102 148	96 966	5 319	−137	5 182	98	2	5.4	−0.1	5.3
1968–1969	107 333	162 148	5 626	−441	5 185	93	7	5.4	−0.4	5.0
1969–1970	108 955	107 333	921	701	1 622	57	43	0.9	0.6	1.5
1970–1971	115 107	108 955	6 269	−117	6 152	99	1	5.7	−0.1	5.6

[a] Totals may not be exact sums of components due to rounding.
[b] Final demand.
[c] Intermediate demand.
[d] Total change.

As a rule the contribution of the intermediate demand effect is much smaller than that of the final demand effect. However, in five of the ten years positive changes in the demand for goods and services on account of the two effects reinforced one another, but in the other five, a positive change in the final demand effect was offset by a negative change in the intermediate demand effect. In the two years of very low growth, 1967 and 1970, the contribution of the final demand effect to the total change fell considerably.

Growth by industry

During the period 1961–1971 real gross domestic output grew by 5.6%. Detailed data of the growth in output by industry is shown in table 3. The table shows that the fastest growing industries between 1961 and 1971 were transportation equipment, which became the largest manufacturing industry by 1971, rubber and plastics products, and mineral fuels, as well as the two service industries — services to business management and communication. Among the very slow growing industries were leather, forestry, fishing, hunting and trapping and other personal services, particularly during the second half of the period, that is between 1966 and 1971, when the first three industries experienced negative rates of growth of output.

Many industries such as machinery, electrical products and metal fabricating experienced very high rates of growth in the first half of the period but grew very slowly during 1966–1971. Though not as extreme, the picture is similar for many other industries. For output as a whole the period 1961 to 1966 is one of relatively more rapid per annum growth, at 7%, than the period 1966 to 1971 at 4%.

The change in output by industry between 1961 and 1971 is analysed further in terms of how much of the change in the output of industries can be attributed to the growth in the level of final demand (scale effect), how much to the change in the composition of final demand (pattern effect) and how much to the change in the pattern of industrial demand (intermediate demand effect).

In table 3 the column entitled final demand (scale effect), shows the increase in output that would have been required from each industry if there had been a uniform increase (of 69%) in final expenditure, without any change in its composition or in the input per unit of output requirements of industry.

Of the industries whose output grew faster than the average increase (of 69%) in final expenditure between 1961 and 1971, there were some for which the scale effect was enhanced either by a positive change in the composition of final expenditure (for example for transportation equipment, mineral fuels) or by a positive change in the composition of intermediate demand (services to business management). For some, such as rubber and plastic products, the positive effect due to intermediate demand was slightly modified by an adverse effect due to the change in the pattern of final expenditure. For the industries which grew more slowly, table 3 shows whether and to what extent it was the change in the composition of final expenditure or of intermediate demand that offset the scale effect and reduced the growth of output of the industry.

TABLE 3. OUTPUT, GROWTH RATES, AND ANALYSIS OF CHANGE, BY INDUSTRY, 1961–1971

Classification number (see annex I.B)	Industry	Total output (millions of constant 1961 dollars)			Average per annum growth rates (%)			Partition of change (millions of constant 1961 dollars)			
		1961	1966	1971	1961–1966	1966–1971	1961–1971	Total change 1961–1971	Final demand scale effect	Final demand pattern effect	Intermediate effect
22	Transportation equipment	1 945	4 207	6 290	16.7	8.4	12.5	4 345	1 351	2 535	459
10	Rubber and plastics products	451	831	1 170	13.0	7.1	10.0	719	313	-18	424
5	Mineral fuels	688	1 066	1 722	9.1	10.1	9.6	1 034	478	556	0
38	Services to business management	992	1 557	2 363	9.4	8.7	9.1	1 371	689	4	678
30	Communications	1 099	1 619	2 293	8.1	7.2	7.7	1 198	763	329	106
21	Machinery	765	1 573	1 584	15.5	0.1	7.5	819	532	244	43
12	Textiles	847	1 297	1 708	8.9	5.7	7.3	861	588	13	260
23	Electrical products	1 290	2 259	2 597	11.9	2.8	7.2	1 307	896	294	117
31	Electrical power, gas, other utilities	1 033	1 508	2 035	7.9	6.2	7.0	1 002	717	148	137
13	Knitting mills	221	316	430	7.4	6.4	6.9	209	154	-53	109
42	Operating, office, lab and food	2 599	4 142	5 050	9.8	4.0	6.9	2 451	1 805	391	255
6	Non-metal mines and quarries	273	440	530	10.0	3.8	6.9	257	190	-42	109
20	Metal fabricating	1 554	2 696	2 999	11.6	2.2	6.8	1 445	1 079	260	105
32	Wholesale trade	2 529	3 872	4 856	8.9	4.6	6.7	2 327	1 757	547	23
29	Transportation and storage	3 507	5 093	6 623	7.7	5.4	6.6	3 117	2 435	357	325
41	Transportation margins	1 688	2 420	3 145	7.5	5.4	6.4	1 456	1 173	239	44
36	Education and health services	699	935	1 273	6.0	6.4	6.2	574	486	86	3
27	Miscellaneous manufacturing	574	807	1 037	7.0	5.1	6.1	462	399	4	59
26	Chemical and chemical products	1 498	2 241	2 699	8.4	3.8	6.1	1 201	1 040	-16	178
16	Furniture and fixtures	366	575	646	9.4	2.4	5.8	280	254	-18	7
24	Non-metallic mineral products	697	1 057	1 193	8.7	2.5	5.5	496	484	66	-54
37	Amusement and recreation services	261	354	440	6.3	4.5	5.4	179	181	11	-13
35	Other finance, insurance and real estate	4 313	5 636	7 260	5.5	5.2	5.3	2 947	2 996	-129	80
19	Primary metal	2 462	3 490	4 086	7.2	3.2	5.2	1 624	1 710	-262	176
43	Travel and advertising, promotion	1 409	1 946	2 315	6.7	3.5	5.1	906	979	113	-186
7	Services incidental to mining	184	263	295	7.4	2.3	4.8	111	128	-39	22
25	Petroleum and coal products	1 242	1 578	1 935	4.9	4.2	4.5	693	863	-35	-135

TABLE 3 *(continued)*

Classification number (see annex I.B)	Industry	Total output (millions of constant 1961 dollars)			Average per annum growth rates (%)			Partition of change (millions of constant 1961 dollars)			
		1961	1966	1971	1961–1966	1966–1971	1961–1971	Total change 1961–1971	Final demand scale effect	Final demand pattern effect	Intermediate effect
17	Paper and allied	2 229	3 003	3 441	6.1	2.8	4.4	1 212	1 548	−310	−25
33	Retail trade	4 318	5610	6 657	5.4	3.5	4.4	2 339	2 999	−460	−200
15	Wood	1 060	1 383	1 626	5.5	3.3	4.4	566	736	−39	−130
28	Construction	7 084	9 429	10 859	5.9	2.9	4.4	3 775	4 920	−777	−368
39	Accommodation and food services	1 652	1 866	2 422	2.5	5.3	3.9	770	1 147	−306	−72
8	Food and beverages	5 157	6 439	7 528	4.5	3.2	3.9	2 371	3 581	−1 398	187
4	Metal mines	1 093	1 381	1 591	4.8	2.9	3.8	499	759	−197	−63
1	Agriculture	2 844	4 099	4 107	7.6	0.0	3.7	1 263	1 975	−378	−334
14	Clothing	817	1 038	1 177	4.9	2.5	3.7	360	567	−231	23
18	Printing and publishing	875	1 067	1 212	4.1	2.6	3.3	337	608	12	−282
9	Tobacco products	335	384	447	2.7	3.1	2.9	112	233	−123	2
34	Owner-occupied dwellings	2 472	2 889	3 263	3.2	2.5	2.8	791	1 717	−925	0
40	Other personal and misc. services	651	797	804	4.1	0.2	2.1	153	452	−270	−30
3	Fishing, hunting and trapping	124	158	146	5.1	−1.6	1.7	23	86	−43	−21
2	Forestry	808	922	918	2.7	−0.1	1.3	110	561	−176	−276
11	Leather	295	331	328	2.3	−0.2	1.1	33	205	−187	15
	Total	67 003	94 571	115 106	7.1	4.0	5.6	48 105	46 534	−187	1 757

Though, in general, the intermediate demand effect contributes less to total change than the final demand effect, this analysis shows that for certain industries, changes in the pattern of industrial demand contributed substantially to their change in output. These industries were, in the order of their rates of growth of output, rubber and plastic products, services to business management, textiles and knitting mills, non-metal mines, printing and publishing and forestry.

Table 3 also shows that the structure of production and input pattern of industries between 1961 and 1971 changed in the direction of reducing demand for the outputs of construction, agriculture, printing and publishing, forestry (that is, those industries for which the intermediate demand effect is negative), and of increasing demand for the output of rubber and plastics products, services to business management, non-metal mines etc. (that is, those for which intermediate demand effect is positive).

Analysis of final demand

The period 1961 to 1971 was one of "export led" growth. Table 4 shows the relative rates of growth in the demand for goods and services by the various categories of final purchasers.

Consumers' expenditure accounts for about half of final demand but at 4.4% its per annum growth rate was the slowest of the various categories, resulting in a reduction in its share of final demand from 56% to 49% between 1961 and 1971. This decline in share was almost entirely taken up by exports whose share increased from 17% to 23%. Exports grew by 9.1%, followed by government current expenditure at 7.4% and fixed capital formation at 5.8%.

Fixed capital formation grew almost as fast as exports between 1961 and 1966 at 9.3% per annum but between 1966 and 1971 investment growth fell back to 2.4% accounting for the slow growth in the output of capital goods manufacturing industries during this period, which has been observed earlier.

The level of final demand for the output of the various industries as well as total imports of the products of those industries in 1961 and 1971 is shown in table 5. The change between 1961 and 1971 is analysed in table 6 in which the relative contribution of the different categories of final demand and of imports to the total change in final expenditure for the output of industries, between those years, is also shown. Looking at the major industries in the order of their growth rates between 1961 and 1971, it is of some interest to note that exports played the dominant role in the contribution of final demand to the growth in transportation equipment and the following major primary, extractive and resource based industries: mineral fuels, metal and non-metal mines and agriculture, primary metal and paper and wood industries. Consumers' expenditure was dominant for communication, textiles and the utilities and in the cases of rubber and plastics products and transportation, both consumers' expenditure and exports made an almost equal contribution.

During this period imports increased and moderated the growth in final demand. The increase in imports was important in the case of transportation

TABLE 4. FINAL DEMAND FOR GOODS AND SERVICES, VALUE, AVERAGE GROWTH RATE AND SHARES

Item	Final demand (millions of constant 1961 dollars)			Average per annum growth rates (%)			Shares (%)	
	1961	1966	1971	1961–1966	1966–1971	1961–1971	1961	1971
Consumers' expenditure	23 311	29 709	35 950	5.0	3.9	4.4	56.6	49.4
Fixed capital formation	8 246	12 861	14 453	9.3	2.4	5.8	20.0	19.8
Inventories	57	1 039	362	79.0	−19.0	20.4	0.1	0.5
Government current expenditure	2 295	3 064	4 670	6.0	8.8	7.4	5.6	6.4
Exports	7 128	11 253	17 074	9.6	8.7	9.1	17.3	23.4
Re-exports	135	224	333	10.7	8.3	9.5	0.3	0.5
Total final demand	41 172	58 151	72 842	7.1	4.6	5.9	100.0	100.0
Imports	−7 964	−12 300	−16 778	9.1	6.4	7.7	−19.3	−23.0
Government revenue (sale of goods and service)	−481	−651	−1 019	6.2	9.4	7.8	−1.2	−1.4
Total final expenditure	32 726	45 199	55 046	6.7	4.0	5.3	79.5	75.6

TABLE 5. FINAL EXPENDITURE, 1961 AND 1971

(In millions of constant 1961 dollars)

Classification number (see annex I.B)	Industry	1961						1971					
		CE[a]	FCF[b]	INV[c]	GOVT[d]	E[e]	M	CE[a]	FCF[b]	INV[c]	GOVT[d]	E[e]	M
1	Agriculture	528	0	−368	2	780	−246	623	0	−23	4	1 107	−321
2	Forestry	35	1	2	1	39	−14	14	1	−21	0	30	−22
3	Fishing, hunting and trapping	3	0	0	0	33	−18	8	0	0	0	36	−25
4	Metal mines	4	11	39	0	434	−69	8	13	37	1	698	−86
5	Mineral fuels	58	9	10	12	196	−504	76	6	−8	5	879	−742
6	Non-metal mines and quarries	5	3	7	17	149	−48	16	2	9	23	296	−100
7	Services incidental to mining	0	84	0	0	1	−6	0	91	−1	0	2	−5
8	Food and beverages	3 419	11	101	2	417	−444	4 855	21	36	3	718	−653
9	Tobacco products	226	0	11	0	28	−9	308	0	−5	0	47	−13
10	Rubber and plastics products	168	9	6	2	18	−121	284	20	17	1	128	−506
11	Leather	237	0	5	0	16	−46	300	1	11	0	16	−112
12	Textiles	216	9	2	8	46	−372	508	11	32	24	116	−727
13	Knitting mills	203	0	3	1	2	−40	364	0	15	4	14	−154
14	Clothing	766	1	44	4	9	−67	1 098	1	49	10	50	−139
15	Wood	30	13	8	0	391	−77	37	21	−37	−3	665	−105
16	Furniture and fixtures	254	76	13	3	5	−41	406	145	21	5	40	−67
17	Paper and allied	118	9	4	−12	1 132	−124	[illegible]92	19	20	2	1 633	−197
18	Printing and publishing	244	5	4	11	11	−137	308	3	3	55	31	−209
19	Primary metal	12	4	−1	10	1 056	−355	18	13	43	0	1 537	−796
20	Metal fabricating	120	135	9	13	67	−446	190	296	−48	12	295	−806
21	Machinery	49	681	10	20	158	−732	106	1 471	−79	37	557	−1 653
22	Transportation equipment	897	489	36	291	210	−890	2 036	1 158	230	146	3 762	−3 686
23	Electrical products	445	339	28	43	87	−445	838	726	−6	46	397	−1 010
24	Non-metallic mineral products	57	14	16	1	40	−154	63	19	9	1	96	−201
25	Petroleum and coal products	522	1	21	61	10	−140	781	3	44	96	67	−199
26	Chemical and chemical products	374	8	20	60	211	−400	665	19	26	113	430	−863
27	Miscellaneous manufacturing	313	94	16	41	54	−322	554	233	9	63	188	−773
28	Construction	21	5 463	0	475	0	0	40	8 958	0	429	0	0
29	Transportation and storage	514	1	0	71	351	−81	834	5	0	152	694	−144

TABLE 5 *(continued)*

Classification number (see annex I.B)	Industry	1961 CE[a]	1961 FCF[b]	1961 INV[c]	1961 GOVT[d]	1961 E[e]	1961 M	1971 CE[a]	1971 FCF[b]	1971 INV[c]	1971 GOVT[d]	1971 E[e]	1971 M
30	Communication	350	33	1	57	25	−13	830	71	−4	166	41	−41
31	Electrical power, gas, other utilities	534	0	1	−30	16	−7	990	1	0	49	34	−7
32	Wholesale trade	750	250	1	38	114	−20	1 357	481	−9	67	509	−86
33	Retail trade	3 483	78	3	17	9	−19	5 236	157	1	22	12	−11
34	Owner-occupied dwellings	2 472	0	0	0	0	0	3 263	0	0	0	0	0
35	Other finance, insurance and real estate	2 165	331	0	36	35	−103	3 825	338	0	48	51	−228
36	Education and health services	821	0	0	−122	0	0	679	0	0	590	0	0
37	Amusement and recreation services	205	0	0	−14	0	0	386	1	0	−41	0	0
38	Services to business management	151	1	0	121	36	−146	293	4	0	279	106	−309
39	Accommodation and food services	1 347	1	0	−2	0	0	1 976	1	0	−12	0	−1
40	Other personal and miscellaneous services	557	0	0	21	0	0	672	0	0	12	0	0
41	Transportation margins	335	80	0	19	329	0	494	141	0	31	846	0
42	Operating, office, lab and food	42	0	0	478	0	0	165	0	0	1 078	0	0
43	Travel and advertising, promotion	15	0	0	132	0	0	57	0	0	258	0	0
	Industry total	23 066	8 246	54	1 883	6 514	−6 659	35 753	14 453	371	3 776	16 178	−14 997

[a]Consumers' expenditure.
[b]Fixed capital formation.
[c]Inventories
[d]Government current expenditure minus government revenue.
[e]Exports plus re-exports.

TABLE 6. ANALYSIS OF FINAL EXPENDITURE CHANGE, 1961–1971

Classification number (see annex I.B)	*Industry*	*Rate of growth of industry output (%)*	*Change in final expenditure (10^7 dollars)*						*Contribution to change (%)*					
			CE[a]	*FCF*[b]	*INV*[c]	*GOVT*[d]	*E*[e]	*M*	*CE*[a]	*FCF*[b]	*INV*[c]	*GOVT*[d]	*E*[e]	*M*
22	Transportation equipment	12.5	114	67	19	−14	355	−280	13	8	2	2	42	33
10	Rubber and plastics products	10.0	12	1	1	0	11	−39	18	2	2	0	17	61
5	Mineral fuels	9.6	2	0	−2	−1	68	−24	2	0	2	1	71	25
38	Services to business management	9.1	14	0	0	16	7	−16	27	1	0	29	13	30
30	Communication	7.7	48	4	0	11	2	−3	71	6	1	16	2	4
21	Machinery	7.5	6	79	−9	2	40	−92	3	35	4	1	18	40
12	Textiles	7.3	29	0	3	2	7	−36	38	0	4	2	9	46
23	Electrical products	7.2	39	39	−3	0	31	−57	23	23	2	0	18	33
31	Electrical power, gas, other utilities	7.0	46	0	0	8	2	0	82	0	0	14	3	0
13	Knitting mills	6.9	16	0	1	0	1	−11	53	0	4	1	4	38
42	Operating, office, lab and food	6.9	12	0	0	60	0	0	17	0	0	83	0	0
6	Non-metal mines and quarries	6.9	1	0	0	1	15	−5	5	1	1	5	65	23
20	Metal fabricating	6.8	7	16	−6	0	23	−36	8	18	7	0	26	41
32	Wholesale trade	6.7	61	23	−1	3	39	−7	45	17	1	2	29	5
29	Transportation and storage	6.6	32	0	0	8	34	−6	39	0	0	10	42	8
41	Transportation margins	6.4	16	6	0	1	52	0	21	8	0	2	69	0
36	Education and health services	6.2	−14	0	0	71	0	0	17	0	0	83	0	0
27	Miscellaneous manufacturing	6.1	24	14	−1	2	13	−45	24	14	1	2	13	45
26	Chemical and chemical products	6.1	29	1	1	5	22	−46	28	1	1	5	21	44
16	Furniture and fixtures	5.8	15	7	1	0	4	−3	52	24	3	1	12	9
24	Non-metallic mineral products	5.5	1	0	−1	0	6	−5	5	4	5	1	47	38
37	Amusement and recreation services	5.4	18	0	0	−3	0	0	87	0	0	13	0	0
35	Other finance, insurance and real estate	5.3	166	1	0	1	2	−13	91	0	0	1	1	7
19	Primary metal	5.2	1	1	4	−1	53	−44	0	1	4	1	51	42
43	Travel and advertising, promotion	5.1	4	0	0	13	0	0	25	0	0	75	0	0
7	Services incidental to mining	4.8	0	1	0	0	0	0	0	73	9	1	8	8
25	Petroleum and coal products	4.5	26	0	2	4	6	−6	59	0	5	8	13	14
17	Paper and allied	4.4	7	1	2	1	50	−7	11	1	2	2	73	11
33	Retail trade	4.4	175	8	0	1	0	1	95	4	0	0	0	0

TABLE 6 *(continued)*

Classification number (see annex I.B)	Industry	Rate of growth of industry output (%)	Change in final expenditure (10^7 dollars)						Contribution to change (%)					
			CE[a]	FCF[b]	INV[c]	GOVT[d]	E[e]	M	CE[a]	FCF[b]	INV[c]	GOVT[d]	E[e]	M
15	Wood	4.4	1	1	−5	0	27	−3	2	2	12	1	75	8
28	Construction	4.4	2	350	0	−5	0	0	1	98	0	1	0	0
39	Accommodation and food services	3.9	63	0	0	−1	0	0	98	0	0	2	0	0
8	Food and beverages	3.9	144	1	−7	0	30	−21	71	1	3	0	15	10
4	Metal mines	3.8	0	0	0	0	26	−2	1	1	1	0	91	6
1	Agriculture	3.7	9	0	35	0	33	−7	11	0	41	0	39	9
14	Clothing	3.7	33	0	0	1	4	−7	73	0	1	1	9	16
18	Printing and publishing	3.3	6	0	0	4	2	−7	32	1	1	21	10	36
9	Tobacco products	2.9	8	0	−2	0	2	0	68	0	14	0	16	3
34	Owner-occupied dwellings	2.8	79	0	0	0	0	0	100	0	0	0	0	0
40	Other personal and miscellaneous services	2.1	12	0	0	−1	0	0	93	0	0	7	0	0
3	Fishing, hunting and trapping	1.7	1	0	0	0	0	−1	35	0	0	1	19	44
2	Forestry	1.3	−2	0	−2	0	−1	−1	33	0	38	0	15	13
11	Leather	1.1	6	0	1	0	0	−7	46	0	5	0	0	49
	Industry total	5.6	1 269	621	32	189	966	−834	32	16	1	5	25	21

Note: Negative entry denotes positive change in imports.
[a]Consumers' expenditure.
[b]Fixed capital formation.
[c]Inventories.
[d]Current expenditure net of government revenue.
[e]Exports plus re-exports.

equipment (vehicles and parts) and in the case of capital goods of all categories including machinery and electrical products. Imports of industrial inputs such as primary and fabricated metals, chemicals and mineral fuels also increased as well as those of manufactures such as plastic products, food and beverages, textiles and miscellaneous manufactures.

The structure of production

Forward linkage and intermediate production

Table 7 shows the forward linkage vectors (with imports endogenized) for 1961, for 1971 as well as the hypothetical linkages corresponding to a Canadian economy with 1971 technology and 1961 propensities to import (as embodied in the 1961 import share coefficients) in column (2). In 1961 the most strongly forward-linked sectors were transport, finance and real estate, and wholesale trade, followed by primary metals, and pulp and paper products. This reflects the dependence of the economy on "infrastructure" in the case of the first three, and a reliance on two basic materials in the latter cases. Not surprisingly, column (3) shows the same ranking of the first four sectors in 1971, but by then chemicals had displaced paper as the fifth most strongly forward-linked sector.

Columns (4) and (5) of table 7 show changes in forward linkages measured as changes in shares of the one million constant 1961 dollars to which each eigenvector is normalized. Recalling that these vectors are aggregates of a much more detailed industrial break-down, an indication of a significant uniformity of the linkage trend in elements of the aggregate is shown by an asterisk. This indicates that at least 75% of the total absolute value of changes in shares of the elements of the aggregate are of the same sign.

The changes in both technology and propensities to import from 1961 to 1971 resulted in the changes in forward linkage shown in column (4). A variety of sectors loosely based on "renewable" resources declined in forward linkage: agriculture, forestry, food and beverages, wood, paper, and printing. Additional significant declines may be noted for petroleum and coal products, construction (which is entirely repair construction), and accommodation and food services. The decline in repair construction is probably related to the younger capital stock in 1971 which resulted from the large investments of the early 1960s.

Column (4) shows major increases in forward linkage for a variety of infrastructure sectors: transport, wholesale and retail trade, communications, and services to mining and to business management. Electricity and mineral fuels increased in forward linkage, as did rubber and plastics, textiles, primary metals, metal fabricating, and machinery.

The changes in forward linkage due to changes in import propensities alone, as shown in column (5), are generally smaller. Transport equipment, chemicals, and miscellaneous manufacturing declined as a result of changing import shares, and finance and real estate increased, whereas there was no significant trend for these sectors in column (4). There was increasing forward linkage both as a result

TABLE 7. EFFECTS OF CHANGING TECHNOLOGY AND IMPORT SHARES ON FORWARD LINKAGES[a,b]

(Shares normalized to 1 million constant 1961 dollars)

Classification number (see annex I.B)	Industry	(1) 1961 technology; 1961 import coefficients	(2) 1971 technology; 1961 import coefficients	(3) 1971 technology; 1971 import coefficients	(4) Changing technologies and import coefficients (change in shares)[c,d]	(5) Changing import coefficients (change in shares)[c,e]
1	Agriculture	39 382	27 103	30 899	−8 483*	3 796*
2	Forestry	30 622	20 944	21 493	−11 129*	549*
3	Fishing, hunting and trapping	1 431	977	916	−515*	−61*
4	Metal mines	18 663	18 812	19 427	764	615
5	Mineral fuels	24 312	24 364	26 397	2 085*	2 033*
6	Non-metal mines and quarries	3 009	4 221	3 722	714*	−499*
7	Services incidental to mining	2 467	4 758	4 879	2 413*	121*
8	Food and beverages	42 097	33 668	36 214	−5 884*	2 546*
9	Tobacco products	101	75	76	−24*	2*
10	Rubber and plastics products	12 993	20 603	19 245	6 252*	−1 358*
11	Leather	694	855	816	122*	−39
12	Textiles	15 713	20 515	16 741	1 027	−3 775*
13	Knitting mills	284	945	869	585*	−76*
14	Clothing	1 219	1 337	1 243	24*	−94*
15	Wood	13 867	11 152	11 457	−2 410*	305*
16	Furniture and fixtures	1 217	1 044	1 032	−185*	−12*
17	Paper and allied	50 183	45 769	45 313	−4 369*	45
18	Printing and publishing	46 740	36 559	37 479	−9 261*	920*
19	Primary metal	52 711	60 667	54 592	1 881	−6 075*

20	Metal fabricating	34 869	35 889	36 179	1 311	290
21	Machinery	13 489	14 785	15 673	2 184	883*
22	Transportation equipment	17 798	23 713	17 550	− 248	− 6 163*
23	Electrical products	19 773	20 531	20 533	760	3
24	Non-metallic mineral products	9 026	8 424	8 680	− 346	256
25	Petroleum and coal products	31 727	29 118	30 185	− 1 542*	1 067*
26	Chemical and chemical products	47 372	49 224	48 116	745	− 1 108*
27	Miscellaneous manufacturing	9 159	10 882	9 849	690	− 1 033*
28	Construction	39 026	26 379	27 005	− 12 021*	626*
29	Transportation and storage	134 015	146 669	149 307	15 292*	2 638*
30	Communication	30 809	34 322	34 525	3 716*	203*
31	Electrical power, gas, other utilities	21 241	24 209	24 132	2 890*	− 77*
32	Wholesale trade	56 807	63 710	63 403	6 596*	− 307*
33	Retail trade	40 491	42 123	42 904	2 413*	780*
35	Other finance, insurance and real estate	73 886	72 615	74 112	226	1 497*
36	Education and health services	28	86	87	58*	0*
37	Amusement and recreation services	5 271	4 013	4 074	− 1 197	62*
38	Services to business management	28 842	37 476	38 562	9 720*	1 086
39	Accommodation and food services	22 678	17 838	18 149	− 4 529*	310*
40	Other personal and miscellaneous services	3 989	3 625	3 664	− 325*	39*

[a] Imports are endogenized in all columns.
[b] Values are normalized.
[c] An asterisk (*) indicates a significant linkage trend.
[d] Given as (1971 technology, 1971 import coefficients) minus (1961 technology, 1961 import coefficients).
[e] Given as (1971 technology, 1971 import coefficients) minus (1971 technology, 1961 import coefficients).

of changing import shares and technology in the case of mineral fuels and transport. Finally, there are a number of instances of significant but opposite trends in columns (4) and (5). The agriculture, food and beverage, and petroleum products sectors exhibited increased forward linkage as a result of changing import shares, whereas their total forward linkage declined. Sectors such as rubber and plastics, textiles, and primary metals displayed decreasing forward linkage due to changing import shares, whereas their total forward linkage increased.

The general picture given by this analysis is one of a Canadian economy less dependent on intermediate consumption of domestic renewable resource-based goods, and increasingly dependent on intermediate use of domestic infrastructure items, domestic energy, plastics, metals, and metal goods. It would appear that changes in the structures of imports have moderated the trend due purely to technology.

Industrial input structures

Change in industrial input patterns

Industries change their input structures on account of changes in the commodity mix of their output and more importantly as a consequence of changes in the techniques of production.

A measure of the extent to which industries changed the commodity shares of their input structures between 1961 and 1971 is shown in table 8. Industries are ranked by their rate of growth of output. As this analysis is concerned with changes in industrial input structures it does not distinguish between domestically produced and imported inputs. The knitting, clothing and textile industries were among the industries which underwent maximum change in input patterns and the tobacco products industry was the manufacturing industry with the least change in input structure. No particular relationship between the rate of growth of output of an industry and changes in its input structure is discernible.

Direct and indirect input requirements by industries

Industrial input structures can also be analysed in terms of the direct and indirect requirement of industries for domestically produced and imported goods and services and for labour. The main picture that emerges from such an analysis, also shown in table 8, is that whether as a consequence of increased specialization, increased intra-industry transactions, or changes in input requirements, most industries marginally increased their direct and indirect use of domestic goods and services per thousand dollars of output, between 1961 and 1971. However, some service industries such as services to mining, communication, retail and wholesale trade and education and health reduced their demand for all categories of goods and services. Other industries which substantially decreased their direct and indirect demand for domestic goods and services were transportation equipment and leather, but this decline was in part offset by an increase in their direct and

TABLE 8. CHANGES IN INPUT PATTERNS, 1961–1971

Classification number (see annex I.B)	*Industry*	*Index of change of input pattern*	*Rate of growth of industry output (%)*	*Changes in direct and indirect input requirements per thousand dollars of output, 1961–1971*		
				Goods and services		*Employment (man-years)*
				Domestic	*Imports*	
22	Transportation equipment	14 269	12.5	−138.6	96.2	−0.0469
10	Rubber and plastics products	10 222	10.0	−37.5	8.8	−0.0437
5	Mineral fuels	8 211	9.6	25.0	5.0	−0.0169
38	Services to business management	6 838	9.1	63.7	11.6	−0.0185
30	Communication	7 049	7.7	−80.5	−0.7	−0.0478
21	Machinery	13 935	7.5	82.5	48.6	−0.0289
12	Textiles	18 391	7.3	64.8	−22.6	−0.0479
23	Electrical products	7 150	7.2	−28.0	7.9	−0.0343
31	Electrical power, gas, other utilities	6 630	7.0	11.7	29.6	−0.0130
13	Knitting mills	22 470	6.9	29.1	69.2	−0.0766
6	Non-metal mines and quarries	6 949	6.9	−64.1	−3.0	−0.0330
20	Metal fabricating	6 730	6.8	−6.5	−2.7	−0.0351
32	Wholesale trade	6 180	6.7	−69.2	−3.1	−0.0458
29	Transportation and storage	5 328	6.6	−4.0	8.3	−0.0456
36	Education and health services	3 277	6.2	−21.0	5.2	−0.0175
27	Miscellaneous manufacturing	8 335	6.1	−38.7	26.0	−0.0420
26	Chemical and chemical products	10 037	6.1	3.8	−3.7	−0.0278
16	Furniture and fixtures	7 957	5.8	−19.7	29.9	−0.0461
24	Non-metallic mineral products	6 297	5.5	37.1	−10.1	−0.0325
37	Amusement and recreation services	13 923	5.4	24.7	49.4	−0.0007

TABLE 8 *(continued)*

Classification number (see annex I.B)	Industry	Index of change of input pattern	Rate of growth of industry output (%)	Changes in direct and indirect input requirements per thousand dollars of output, 1961–1971		
				Goods and services		Employment (man-years)
				Domestic	Imports	
35	Other finance, insurance and real estate	4775	5.3	37.2	8.9	−0.0066
19	Primary metal	9815	5.2	52.2	5.0	−0.0152
7	Services incidental to mining	16738	4.8	−189.1	−46.1	−0.0076
25	Petroleum and coal products	8421	4.5	63.9	6.3	−0.0083
17	Paper and allied	10511	4.4	77.7	18.5	−0.0237
33	Retail trade	6585	4.4	−74.2	−3.1	−0.0353
15	Wood	12782	4.4	21.3	20.8	−0.0464
28	Construction	9628	4.4	22.9	6.5	−0.0312
39	Accommodation and food services	9003	3.9	81.3	18.4	−0.0004
8	Food and beverages	8513	3.9	−16.2	−8.7	−0.0690
4	Metal mines	16682	3.8	188.6	37.3	−0.0072
1	Agriculture	6088	3.7	2.5	−0.7	−0.1334
14	Clothing	19325	3.7	140.7	19.8	−0.0417
18	Printing and publishing	6075	3.3	81.3	13.4	−0.0235
9	Tobacco products	4717	2.9	33.7	2.1	−0.0761
34	Owner-occupied dwellings	632	2.8	5.4	2.9	−0.0064
40	Other personal and miscellaneous services	6899	2.1	51.4	29.8	0.0381
3	Fishing, hunting and trapping	10734	1.7	88.4	47.4	0.0053
2	Forestry	12248	1.3	25.9	13.4	−0.0404
11	Leather	9235	1.1	−121.5	28.5	−0.0545

indirect use of imports, as was also the case for other manufacturing industries showing reduced demand for domestic production.

In general, industries increased their import requirements, indicating that on average there was a relative decline in the share of domestically produced goods and services in total intermediate demand. All but two industries, fishing, hunting and trapping and personal services, showed an increase in labour productivity, and showed a decline in the man-years of employment directly and indirectly required to produce a dollar of industrial output.

Input substitution

The analysis of changes in input requirements is extended by the technique of examining changes in the commodity mix of the intermediate demand that would have been required to produce the goods and services demanded by final purchasers in 1971 (less total imports) if the output mix and input structures had been those of 1961, 1966, and 1971. Table 9 shows the resultant demand for each commodity as a proportion of total intermediate demand and the average annual rate at which their shares changed between 1961 and 1971. The table shows that commodities for which the rate of change is positive increased their share at the expense of commodities for which the rate of change is negative.

Although it is not possible to identify exactly which commodities were substituted, one for the other, a broad picture does emerge. Industry appears to become more energy efficient, with an observed shift away from coal and fuel oil towards natural gas and electricity. Between 1961 and 1971 the share of industrial chemicals and plastic fabricated products grew sharply, whereas the share of wood and wood products and paper and paper products all declined. Within the metals group it is evident that the share of iron and steel and aluminium products increased, whereas the share of nickel, other non-ferrous metal products, copper and other metal fabricated products declined. Among the services, a relative decline in the use of postal services and an increase in the use of telephone and telegraph services can be observed.

Conclusion

The paper has attempted to highlight some of the changes that took place during the 1960s in Canada's industrial structure. The period between 1961 and 1971 was one of relatively high average growth in the output of goods and services The economy became more open to trade. Exports gained in importance. Imports grew faster than domestic output, though not as fast as exports. Rates of growth in investment which were high in the first half of the period, slowed down noticeably in the second half.

Notable in the country's industrial structure was the marked growth of the transportation equipment industry, which was a consequence of both a sharp increase in exports and an increase in consumers' expenditure, as well as an increase in intermediate demand, on the part of the industry itself, for its own

TABLE 9. THE CHANGING PATTERN OF INDUSTRIAL USE

Classification number (see annex I.C)	*Commodities*	*Industrial use (normalized to one million constant 1961 dollars)*			*Average annual rate at which shares changed by period*		
		1961	*1966*	*1971*	*1961–1966*	*1966–1971*	*1961–1971*
	Agriculture, forestry, fishing						
6	Hunting and trapping products	338	326	236	−2	−18	−10
5	Fish landings	2 205	1 945	1 880	−52	−13	−33
1	Grains	6 504	6 137	5 580	−73	−111	−92
2	Live animals	24 523	23 564	21 695	−192	−374	−283
3	Other agricultural products	20 859	18 193	17 934	−533	−52	−293
4	Forestry products	20 371	16 961	15 990	−682	−194	−438
	Minerals						
13	Services incidental to mining	3 185	3 867	3 813	136	−11	63
12	Non-metallic minerals	4 814	5 158	4 842	69	−63	3
7	Iron ores and concentrates	2 595	2 594	2 596	0	0	0
8	Other metallic ores and concentrates	16 386	16 165	14 692	−44	−295	−169
	Energy						
11	Natural gas	720	1 080	1 261	72	36	54
78	Electric power	12 520	13 435	12 813	183	−124	29
63	Other petroleum and coal products	6 048	5 954	5 824	−19	−26	−22
10	Crude mineral oils	20 942	18 532	20 206	−482	335	−74
9	Coal	3 728	4 069	2 815	68	−251	−91
62	Gasoline and fuel oil	16 759	15 405	13 959	−271	−289	−280
	Food, beverages, and tobacco products						
15	Dairy products	4 075	4 882	4 534	161	−70	46
18	Feeds	9 246	7 989	9 562	−251	315	32
24	Alcoholic beverages	834	885	975	10	18	14
25	Tobacco	1 750	1 663	1 779	−17	23	3

26	Cigarettes and tobacco manufacturing	6	27	3	4	−5	0
19	Flour, wheat, meal and other cereals	1 839	1 837	1 815	0	−4	−2
17	Fruits and vegetables preparations	2 156	2 110	2 062	−9	−10	−9
21	Sugar	1 696	1 681	1 585	−3	−19	−11
16	Fish products	1 095	1 113	981	4	−26	−11
20	Breakfast cereal and bakery	1 661	1 482	1 444	−36	−8	−22
23	Soft drinks	1 071	990	836	−16	−31	−23
22	Miscellaneous food	5 765	5 619	5 053	−29	−113	−71
14	Meat	14 259	12 320	12 679	−388	72	−158
	Manufactured items						
64	Industrial chemicals	17 103	19 981	23 064	576	617	596
29	Plastic fabricated	3 613	5 677	8 333	413	531	472
31	Yarns and man-made fibres	7 119	8 334	10 277	243	389	316
33	Other textile	5 941	6 474	6 574	107	20	63
28	Other rubber	3 131	3 362	3 698	46	67	57
35	Clothing and accessories	2 042	2 105	2 577	13	94	54
27	Tyres and tubes	4 659	4 893	5 152	47	52	49
40	Pulp	1 569	1 990	1 890	84	−20	32
60	Cement and concrete	9 010	10 081	9 328	214	−151	32
32	Fabrics	13 901	14 035	14 192	27	31	29
30	Leather and leather products	1 585	1 663	1 841	16	36	26
66	Pharmaceuticals	1 536	1 711	1 746	35	7	21
69	Other manufactured	6 345	5 662	6 530	−137	174	19
42	Paper products	15 829	16 215	16 001	77	−43	17
39	Furniture and fixtures	829	803	807	−5	1	−2
38	Other wood fabricated materials	8 538	8 704	8 492	33	−42	−5
37	Veneer and plywood	3 482	3 791	3 430	62	−72	−5
34	Hosiery and knitted wear	177	80	38	−19	−8	−14
67	Other chemical	14 326	13 291	13 494	−207	41	−83
41	Newsprint and other paper stock	14 529	13 526	13 162	−201	−73	−137
65	Fertilizers	2 728	1 787	1 029	−188	−152	−170
61	Other non-metallic mineral	12 311	11 380	10 271	−186	−222	−204
36	Lumber and timber	7 955	6 642	5 152	−263	−298	−280

TABLE 9 *(continued)*

Classification number (see annex I.C)	*Commodities*	*Industrial use (normalized to one million constant 1961 dollars)* 1961	1966	1971	*Average annual rate at which shares changed by period* 1961–1966	1966–1971	1961–1971
	Metal products						
45	Iron and steel	31 582	33 554	33 999	394	89	242
46	Aluminium	4 326	5 364	5 868	208	101	154
50	Boilers, tanks and plates	2 842	3 112	3 792	54	136	95
51	Fabricated structural metal	12 694	13 457	12 746	153	−142	5
48	Nickel	695	476	431	−44	−9	−26
49	Other non-ferrous metal	6 209	5 638	5 434	−114	−41	−77
47	Copper and copper alloy	6 662	6 734	5 725	14	−202	−94
52	Other metal fabricated	34 466	33 330	32 559	−227	−154	−191
56	Motor vehicle parts	34 437	41 361	40 026	1 385	−267	559
54	Other industrial	16 993	17 860	19 156	173	259	216
59	Other electrical	21 132	22 806	22 930	335	25	180
58	Appliances and receivers, household	1 775	2 134	2 314	72	36	54
68	Scientific	4 428	4 506	4 879	16	75	45
55	Motor vehicles	1 123	1 535	1 181	82	−71	6
53	Agricultural	1 340	1 047	1 041	−58	−1	−30
57	Other transport	4 067	3 902	3 584	−33	−64	−48
	Services						
84	Business	21 617	25 301	29 100	737	760	748
76	Telephone and telegraph	10 953	11 248	13 723	59	495	277
80	Wholesale margins	44 461	45 081	47 180	124	420	272
91	Operating, office, laboratory and food	61 419	66 924	63 889	1 101	−607	247
74	Transportation and storage	77 607	77 726	79 719	24	399	211
79	Other utilities	1 985	2 562	3 343	115	156	136
89	Other personal and miscellaneous	21 345	23 077	22 430	347	−129	109

73	Pipeline transportation	4 139
75	Radio and television broadcasting	2 795
86	Health	16
82	Imputed rent owner-occupied dwellings	0
85	Education services	0
87	Amusement and recreation	1 592
90	Transportation margins	27 479
81	Retail margins	11 393
83	Other finance, insurance, real estate	58 569
77	Postal services	4 505
88	Accommodation and food	9 362
43	Printing and publishing	13 511
44	Advertising, print media	8 379
92	Travel, advertising and promotion	37 819
72	Repair construction	30 072

3966	4734	−35	154	59
3360	3094	113	−53	30
15	51	0	7	3
0	0	0	0	0
0	0	0	0	0
1511	1528	−16	3	−6
26621	27412	−172	158	−7
10795	11129	−119	67	−26
54740	57798	−766	612	−77
4022	3120	−97	−180	−138
8393	7437	−194	−191	−192
11607	11056	−381	−110	−246
6381	5462	−400	−184	−292
35986	33579	−367	−481	−424
25696	22027	−875	−734	−805

products. The increase in transportation equipment dominated the growth in exports. Other industries which made an important contribution to the increase in exports were mineral fuels, paper and wood, primary metals and machinery.

With regard to the interdependence of the economy, the analysis of "forward linkages" showed that the broad picture was of a production system less structurally dependent on the domestic production of renewable resource based industries, on petroleum and coal products and on repair construction, while becoming in general more structurally dependent on the domestic production of energy goods, plastics, infrastructure services, services to business management, metals, and metal goods.

The analysis of "input substitution" came to the same conclusion and showed, for example, that the use of domestically produced industrial chemicals, plastic fabricated products and man-made fibres, iron and steel and aluminium products, and business services increased. During this period, industry appeared to become more energy efficient while at the same time the use of domestically produced mineral fuels increased. A small shift away from coal and fuel oil towards natural gas and electricity also occurred.

By and large industries marginally increased their total direct and indirect requirements for goods and services except for certain service industries such as services incidental to mining, communications, trade, education and health, and some manufacturing industries, whose requirements declined.

With respect to imports, the majority of industries increased their dependence on imported inputs. However, in general, changes in the pattern of imports paralleled those of domestic demand so that the structural change in industrial use that took place during the period can be attributed more to changes in the input requirements for production and the output-mix of industries than to the pattern of imports.

ANNEX I

DEFINITIONS OF AGGREGATIONS

A. *Definition of final demand aggregation in terms of sequential numbers*

Final demand title	*Sequential number*
Consumer expenditure	1–40
Fixed capital formation	41–119
Inventories	120–121
Government current expenditure	122–127
Exports	128
Re-exports	129
Imports	130
Government revenue from the sale of goods and services	131–136

B. Definition of industry aggregation in terms of sequential numbers and United Nations International Standard Industrial Classification (ISIC) major group

Industry number	*Industry title*	*Sequential number*	*ISIC major group*
1	Agriculture	1	111, 112
2	Forestry	2	121, 122
3	Fishing, hunting and trapping	3	113, 130
4	Metal mines	4–7	230
5	Mineral fuels	8–9	210, 220
6	Non-metal mines and quarries	10–14	290
7	Services incidental to mining	15	
8	Food and beverage industries	16–32	311–313
9	Tobacco products industries	33–34	314
10	Rubber and plastic products industries	35–38	355, 356
11	Leather industries	39–42	323, 324
12	Textile industries	43–55	321
13	Knitting mills	56–57	321
14	Clothing industries	58	322
15	Wood industries	59–64	331
16	Furniture and fixture industries	65–68	332
17	Paper and allied industries	69–72	341
18	Printing and publishing	73–74	342
19	Primary metal industries	75–82	371, 372
20	Metal fabricating industries	83–91	381
21	Machinery industries	92–95	382
22	Transportation equipment industries	96–102	384
23	Electrical products industries	103–110	383
24	Non-metallic mineral products industries	111–120	369
25	Petroleum and coal products industries	121–122	353, 354
26	Chemical and chemical products industries	123–130	351, 352
27	Miscellaneous manufacturing industries	131–137	361, 362, 385, 390
28	Construction industries	138–146	500
29	Transportation and storage	147–157	711–713, 719
30	Communication	158–160	720
31	Electrical power, gas, other utilities	161–163	410, 420
32	Wholesale trade	164	610
33	Retail trade	165	620
34	Owner-occupied dwellings	166	
35	Other finance, insurance and real estate	167–170	810, 820, 821
36	Education and health services	171–173	931–933
37	Amusement and recreation services	174–175	941, 942, 949
38	Services to business management	176–177, 183	832, 833

Industry number	Industry title	Sequential number	ISIC major group
39	Accommodation and food services	179	631, 632
40	Other personal and miscellaneous services	178, 180–182	951–953, 959
41	Transportation margins	187	Dummy industries
42	Operating, office, lab. and food	184–186, 188, 191	Dummy industries
43	Travel and advertising, promotion	189–190	Dummy industries

C. Definition of commodity aggregation in terms of sequential numbers

Commodity number	Commodity title	Sequential number
1	Grains	6–8
2	Live animals	1–5
3	Other agricultural products	9–23
4	Forestry products	24–28
5	Fish landings	29
6	Hunting and trapping products	30
7	Iron ores and concentrates	34
8	Other metal ores and concentrates	31–33, 35–36
9	Coal	37, 40
10	Crude mineral oils	38
11	Natural gas	39
12	Non-metallic minerals	41–50
13	Services incidental to mining	51
14	Meat products	52–66
15	Dairy products	67–74
16	Fish products	75
17	Fruits and vegetables preparations	76–84
18	Feeds	85–89, 100, 103, 118
19	Flour, wheat, meal and other cereals	90–91
20	Breakfast cereal and bakery products	92–95
21	Sugar	101
22	Miscellaneous food products	96–99, 102, 104, 106–113
23	Soft drinks	114–115
24	Alcoholic beverages	116, 119–120
25	Tobacco processed unmanufactured	121
26	Cigarettes and tobacco manufacturing	122–123
27	Tyres and tubes	125–128
28	Other rubber products	124, 129–134
29	Plastic fabricated products	135–138
30	Leather and leather products	139–144

Commodity number	*Commodity title*	*Sequential number*
31	Yarns and man-made fibres	145–146, 151, 154–157, 161, 164
32	Fabrics	147–148, 152, 158–159, 167–168, 181–182
33	Other textile products	149–150, 153, 160, 162–163, 165–166, 169–179
34	Hosiery and knitted wear	180, 183
35	Clothing and accessories	184–189
36	Lumber and timber	191
37	Veneer and plywood	195
38	Other wood fabricated materials	190, 192–194, 196–203
39	Furniture and fixtures	204–208
40	Pulp	209
41	Newsprint and other paper stock	210–216
42	Paper products	217–227
43	Printing and publishing	228–231, 233–234
44	Advertising, print media	232
45	Iron and steel products	235–244, 247–252
46	Aluminium products	257, 264
47	Copper and copper alloy products	254, 265–266
48	Nickel products	253, 268
49	Other non-ferrous metal products	246, 255–256, 258–263, 267, 269–271
50	Boilers, tanks and plates	272–275, 300
51	Fabricated structural metal products	276–279
52	Other metal fabricated products	280–298, 301–313
53	Agricultural machinery	314–315
54	Other industrial machinery	316–329
55	Motor vehicles	334–339
56	Motor vehicle parts	340–344
57	Other transport equipment	330–333, 345, 352
58	Appliances and receivers, household	299, 353–357
59	Other electrical products	358–374
60	Cement and concrete products	375, 377–379
61	Other non-metallic mineral products	376, 380–393
62	Gasoline and fuel oil	394, 396
63	Other petroleum and coal products	245, 397–402, 548
64	Industrial chemicals	117, 404–407, 411, 416–470, 473–474, 479–480
65	Fertilizers	403
66	Pharmaceuticals	408

Commodity number	*Commodity title*	*Sequential number*
67	Other chemical products	105, 409–410, 412–415, 471–472, 475–478, 481, 496
68	Scientific equipment	497–503
69	Other manufactured products	504–521
70	Residential construction	523
71	Non-residential construction	524–529
72	Repair construction	522
73	Pipeline transportation	540
74	Transportation and storage	530, 539, 541–542
75	Radio and television broadcasting	543
76	Telephone and telegraph	544
77	Postal services	545
78	Electric power	546
79	Other utilities	547, 549
80	Wholesale margins	550
81	Retail margins	553
82	Imputed rent owner-occupied dwellings	557
83	Other finance, insurance, real estate	554–556, 558–560
84	Business services	566–567, 575–576
85	Education services	561
86	Health services	562–563
87	Amusement and recreation services	564–565
88	Accommodation and food services	569–571
89	Other personal and miscellaneous services	551–552, 568, 572–574, 577–579, 595
90	Transportation margins	583
91	Operating office, lab. and food	580–582, 584, 587
92	Travel, advertising and promotion	585–586
93	Non-competing imports	588–593
94	Unallocated imports and exports	594
95	Indirect taxes	596, 598
96	Subsidies	597
97	Wages and salaries	599
98	Supplementary labour income	600
99	Net income, unincorporated business	601
100	Other operating surplus	602

ANNEX II

ANALYSIS OF STRUCTURE—THE DOMINANT EIGENVECTOR

There is an extensive literature on the importance and interpretation of the dominant eigenvector, that is the eigenvector corresponding to the dominant eigenvalue, for dynamic Leontief systems which typically include a capital matrix. "Structure" can, however, also be viewed as a static concept. This section will attempt to show how the right dominant eigenvector of a static I/O system may be interpreted as an indicator of structure in the sense of indicating the relative strength of the forward linkages associated with each industry. Comparing changes in the eigenvector over time therefore provides another measure of structural change in the Canadian economy.

The right eigenvector has been described as representing the general structure of linkages of the technology matrix by Levitt in the Statistics Canada Input-Output Study of the Atlantic Provinces.[a] The following development is a clarification and extension of this methodology.

We begin with the usual power series expression of the solution to a static I/O system for a semi-positive final demand Y:

$$X = \sum_{n=0}^{\infty} X^n$$

where $X^n = (DB)^n Y$, in which DB is the technology matrix and Y is the final demand.

It is straightforward to show that there exists an integer N such that for any Y:

$$X^N \simeq K_Y r^N X_r$$

where:

K_y = a scalar constant;
r = the dominant eigenvalue;
X_r = the corresponding eigenvector of the technology matrix DB.

Although the eigenvector (which is real and semi-positive) is unique only up to a scalar multiple, there is information contained in the relative ranking of its elements. The last expression suggests that independent of the final demand the industry corresponding to the largest element of the eigenvector has the largest N^{th} round gross output, and will continue to do so for all succeeding iterations.

It is essential to avoid the fallacy of attributing any implicit time dimension to the power series solution. There exists an integer N such that for any demand Y the solution may be written as

$$X \simeq Y + \sum_{n=1}^{N-1} (DB)^n Y + K_y \frac{(r^N) X_r}{1 - r}$$

Numerically, N may be determined by specifying a desired criterion and accuracy of convergence. If Z_r is the dominant eigenvector of $(DB)'$, and is therefore real and semi-positive, the following can be proved:

$$K_Y = Z_r' Y$$

for Z_r' such that

$$Z_r' X_r = 1 .$$

With these facts it is clear that in general gross production in a Leontief system is the sum of three terms: the net production Y, a portion of intermediate production which is linear in Y and exhibits a pattern dependent on Y, and a portion of intermediate production which, although also linear in Y, exhibits the pattern of the dominant eigenvector of DB for any arbitrary Y.

[a]Kari Levitt, *Input-Output Study of the Atlantic Provinces*, vol. 1, Catalogue 15-503 Occasional (Ottawa, Statistics Canada, 1975).

The dominant eigenvector is therefore of fundamental structural importance in the composition of intermediate production. The relative size of the value corresponding to a given industry in the dominant eigenvector is an intrinsic measure of that industry's importance as a producer of intermediate goods, which is determined by the structure of the production system. In this sense we can refer to the dominant eigenvector as an indicator of the forward linkage of industries.

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Economic development and changes in linkage structure: an input-output analysis of the Republic of Korea and Japan

Yasuhiko Torii and Kiichiro Fukasaku *

One of the major subjects in development economics is the analysis of structural changes that manifest themselves in the process of economic development. There is, however, no universal definition of structural change. Kuznets [1], for example, indicated industrialization, urbanization, changes in income distribution, changes in the composition of final demands and changes in the patterns of trade as major structural changes. Chenery [2] and his co-authors [3, 4, 5, 6] have analysed the patterns of growth emphasizing the changes in industrial and trade structures, whereas Leontief has pursued changes in input coefficients as a major characteristic of structural change [7, 8, 9, 10, 11, 12, 13, 14]. In this paper, attention is directed to the analysis of changes in industrial structure using I/O tables as a major analytical device.

There are three methods of classifying and measuring the changes in industrial structure. The first method is the three-fold basic classification of industries into primary, secondary and tertiary sectors. This method is the most widely used. Fisher [15], Clark [16], Kuznets [17] and others used the method to analyse long-term structural changes in the relative sectoral shares of value-added and labour force. The second method typically used in the cross-section studies on growth patterns by Chenery [2] and others traces changes in composition of manufacturing industries (two-digit ISIC classification). The third method uses I/O tables to measure the varying effects on industrial output of changes in domestic final demands, exports, and import-and-input coefficients.

This paper defines the industrial structure in terms of "combination", "composition" and "linkage structure" of the industry complexes that constitute a national economy. The concept of "combination" of industries reflects what industries are actively existing in an economy. The concept of "composition" implies the relative share of industries in total output of the economy. The term "linkage structure" implies the existence of interdependence among industries through intermediate transactions.

Economic development can be viewed as a process where the changes in the combination, composition and linkage structure of the industries cause the increases in per capita national income. The studies of composition were for long the central issue in the analysis of economic development. Now linkage structure

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is receiving full attention.[1] However, the question of combination has seldom been discussed in proper perspective.[2]

The most visible phenomenon in the developing economies is the changing combination as shown in the emergence of new industries and the disappearance of traditional industries. This process often results in the alteration of the linkage structure. The emergence of new industries is mainly due to changes in the structure of final demand in accordance with rising income; to increased demands for related industries caused by the expansion of production scale of a given industry; to changes in the available technology mix; and to improved competitiveness of industries by changes in relative prices. As new industries emerge, the linkage structure becomes more complex and densely related. This is an "evolution process" of industrial structure. Generally speaking, the linkage structure among industries is slight and loose at the early stage of economic development. The evolution process proceeds with industrialization and gradually saturates as the economy reaches an advanced stage of development with a mature linkage structure as in developed countries.[3]

The purpose of the present stumy to clarify the idea of the evolution process of industrial structure by observing the changes in linkage structure. The general hypothesis proposed here is that the emergence of some specific leading industries might lead to rapid growth of other industries which would have closer linkage structures. An attempt to test this hypothesis has been made using 1965, 1970, 1975 I/O tables for Japan and 1966, 1970, 1975 I/O tables for the Republic of Korea.

Basic observation on changes in industrial structure

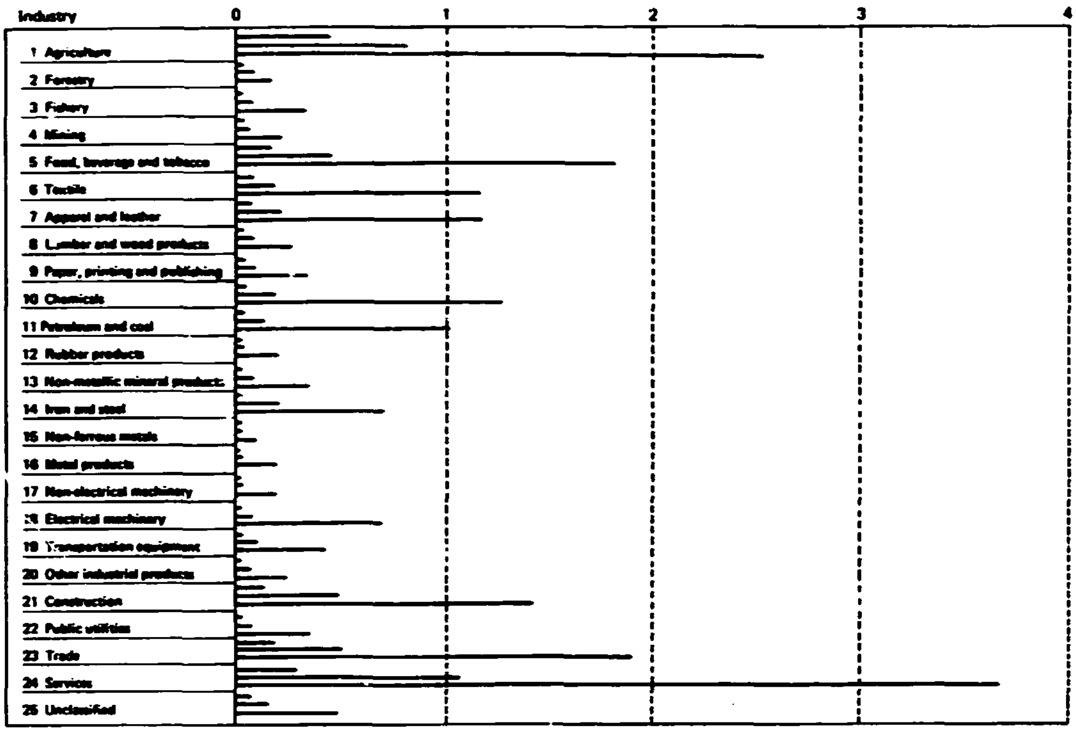
Industrial structure of the Republic of Korea has shown a rapid change since the mid 1960s as shown in figure 1. In 1966, the last year of the first five-year development plan of the Republic of Korea, the industrial output came mainly from agriculture and related industries, and manufacturing production was limited to those of food (5), textile (6) and apparel (7) industries, with very little development of the machinery (17, 18, 19), metal fabrication (14, 15, 16) and petrochemical (10, 11) industries. By 1970, substantial expansion was attained for the petrochemical (10, 11), iron and steel (14), construction (21), trade (23) and services (24) as well as food (5), textile (6) and apparel (7) industries. In 1975, the fourth year of the third five-year plan, the most significant growth was seen in electrical machinery (18), transportation equipment (19), iron and steel (14) and petrochemicals (10, 11). The growth of these industries triggered the evolution process of the industrial structure of the Republic of Korea. In figure 1, such

[1]For the analyses of linkage structures, see such works as Chenery and Watanabe [18], Leontief [19], Simpson and Tsukui [20], Ozaki and Ishida [21], Santhanam and Patil [22], Laumas [23], Song [24], and Torii and Fukasaku [26].

[2]For the study of combination, for example, see Hirschman [25].

[3]Torii and Fukasaku [26] attempted to make a study on the "evolution process" of industrial structure in Asia.

Figure 1. Changes in industrial output in the Republic of Korea



UNIT: 1 Billion won

Key: 1966 —
70 —
75 —

traditional industries as agriculture (1) and food (5), as well as some early grown modern industries such as textile (6) and apparel (7), continued to grow rapidly and to act as leading industries, even during the rapid industrialization of 1970–1975, as also seen in table 1.

Table 1 indicates the shares of increases in industrial output and exports in the Republic of Korea. The shares of increases in industrial output of textile (6), apparel (7), petrochemicals (10, 11), iron and steel (14), machinery (17, 18) increased greatly during 1970 to 1975. During the same period the share of increases in manufactured exports expanded; especially in iron and steel (14), electric machinery (18), transportation equipment (19) and chemicals (10).

From the above, the evolution process of industrial structure in the Republic of Korea should be analysed by two approaches.

To explain growth factors of each industry, the factor decomposition model will be used. In addition, the forward linkage effect of industries with substantial linkage on the growth of modern manufacturing industries should be analysed in detail, and the backward linkage effect of these industries should be analysed with regard to overall industrial structure. For this purpose, the linkage structure model will be used.

TABLE 1. SHARES OF INCREASES IN INDUSTRIAL OUTPUT AND IN MANUFACTURED EXPORTS IN THE REPUBLIC OF KOREA, BY INDUSTRY

Industry		*Shares of increases in industrial output*		*Shares of increases in manufactured exports*	
No.	*Name*	*1966–1970 (%)*	*1970–1975 (%)*	*1966–1976 (%)*	*1970–1975 (%)*
1	Agriculture	11.2	10.7		
2	Forestry	1.2	0.6		
3	Fishery	1.3	1.6		
4	Mining	0.9	1.0		
5	Food, beverage and tobacco	9.1	8.6	4.3	7.4
6	Textile	2.9	6.1	10.6	12.5
7	Apparel and leather	4.5	6.0	33.8	28.1
8	Lumber and wood products	1.5	1.2	11.5	4.1
9	Paper, print and publishing	1.4	1.6	0.6	0.9
10	Chemicals	4.0	6.8	2.7	5.2
11	Petroleum and coal products	2.8	5.5	4.0	2.6
12	Rubber products	0.4	1.0	2.1	4.9
13	Non-metallic mineral products	1.6	1.6	0.9	2.3
14	Iron and steel	2.2	3.7	1.2	5.5
15	Non-ferrous metals	0.3	0.5	0.6	0.2
16	Metal products	0.6	1.0	1.7	2.8
17	Non-electrical machinery	0.4	1.0	–0.1	0.9
18	Electrical machinery	1.5	3.9	7.3	13.1
19	Transportation equipment	2.0	2.1	0.8	4.4
20	Other industrial products	1.7	1.0	18.0	5.1
21	Construction	10.6	5.9		
22	Public utilities	1.3	1.7		
23	Trade	9.8	8.7		
24	Services	24.0	16.3		
25	Unclassified	2.8	1.9		
	Total	100.0	100.0	100.0	100.0
	Shares of manufactured to total exports			67.0	76.2

Analytical models

Factor decomposition model of industrial growth

The I/O analysis on factors of industrial growth was originated by Chenery [2] and followed by Chenery, Shishido and Watanabe [3], and Chenery and Syrquin [5]. These authors defined changes in industrial structure as the "deviations from the proportional growth". This paper attempts to present the following "factor decomposition model" of industrial growth, which is different from the Chenery-type approach with regard to the direct decomposition of increases in industrial output in contrast to that of the deviations from the proportional industrial growth by Chenery; to the definition of import coefficients of intermediate and final goods and to the decomposition of changes in I/O coefficients in Leontief inverse matrix. The following are balance equations:

$$X = AX - M_m + F_d - M_f + E \qquad (1)$$

$$M_m = \bar{M}_m AX \qquad (2)$$

$$M_f = \bar{M}_f F_d \qquad (3)$$

where:

X = the industrial gross output vector;
A = the input coefficient matrix;
M_m = the import vector of intermediate demand;
M_f = the import vector of final demand;
F_d = the domestic final demand vector;
E = the export vector;
$\bar{M}_m$ = the diagonal matrix of import coefficients of intermediate demand;
$\bar{M}_f$ = the diagonal matrix of import coefficients of final demand.

Substituting (2) and (3) in (1) and rearranging (1) with regard to X, we obtain:

$$X = [I - (I - \bar{M}_m)A]^{-1}[(I - \bar{M}_f)F_d + E] \qquad (4)$$

The increase in industrial output between the base year with superscript 0, and the current year with superscript 1, is denoted as the nowing:

$$\Delta X = X^1 - X^0 \qquad (5)$$

Substituting (4), equation (5) is decomposed as follows:

$$\begin{aligned}\Delta X &= [I - (I - \bar{M}_m^1)A^1]^{-1} \cdot [(I - \bar{M}_f^1)F_d^1 + E^1] - \\ &\quad - [I - (I - \bar{M}_m^0)A^0]^{-1} \cdot [(I - \bar{M}_f^0)F_d^0 + E^0] \\ &= B^1 \cdot G^1 - B^0 \cdot G^0 = (B^0 + \Delta B)(G^0 + \Delta G) - B^0 \cdot G^0 \\ &= B^0 \Delta G + \Delta B \cdot G^0 + \Delta B \Delta G = B^1 \cdot \Delta G + \Delta B \cdot G^0 \qquad (6)\end{aligned}$$

where:

$$B^1 = [I - (I - \bar{M}_m^1)A^1]^{-1};$$
$$B^0 = [I - (I - M_m^0)A^0]^{-1};$$
$$G^1 = [I - (I\bar{M}_f^1)F_d^1 + E^1];$$
$$G^0 = [I - (I - \bar{M}_f^0)F_d^0 + E^0].$$

Decomposing B in (6), we obtain the following:

$$\begin{aligned}\Delta B &= B^1 - B^0 \\ &= [I - (I - \bar{M}_m^1)A^1]^{-1} - [I - (I - \bar{M}_m^0)A^0]^{-1} \\ &= \{[I - (I - \bar{M}_m^1)A^1]^{-1} - [I - (I - \bar{M}_m^0)A^1]^{-1}\} + \\ &\quad + \{[I - (I - \bar{M}_m^0)A^1]^{-1} - [I - (I - \bar{M}_m^0)A^0]^{-1}\} \\ &= (B^1 - B^*) + (B^* - B^0) \end{aligned} \tag{7}$$

where:

$$B^1 = [I - (I - \bar{M}_m^1)A^1]^{-1};$$
$$B^0 = [I - (I - \bar{M}_m^0)A^0]^{-1};$$
$$B^* = [I - (I - \bar{M}_m^0)A^1]^{-1}.$$

Thus, $B^1 - B^*$ denotes the effects of changes in $\bar{M}_m$ with A constant, while $B^* - B^0$ denotes the effects of changes in A with $\bar{M}_m$ constant. On the other hand, G in (6) can be decomposed in the similar way.

$$\begin{aligned}\Delta G &= G^1 - G^0 \\ &= [(I - \bar{M}_f^1)F_d^1 + E^1] - [(I - \bar{M}_f^0)F_d^0 + E^0] \\ &= \{[(I - \bar{M}_f^1)F_d^1 + E^1] - [(I - \bar{M}_f^0)F_d^1 + E^1]\} + \\ &\quad + \{[(I - \bar{M}_f^0)F_d^1 + E^1] - [(I - \bar{M}_f^0)F_d^0 + E^0]\} \\ &= (G^1 - G^*) + (G^* - G^0) \end{aligned} \tag{8}$$

where:

$$G^1 = [(I - \bar{M}_f^1)F_d^1 + E^1];$$
$$G^0 = [(I - \bar{M}_f^0)F_d^0 + E^0];$$
$$G^* = [(I - \bar{M}_f^0)F_d^1 + E^1].$$

Thus $G^1 - G^*$ denotes the effects of changes in $\bar{M}_f$ with F_d and E constant, and $G^* - G^0$ denotes the effects of changes in F_d and E with $\bar{M}_f$ constant. Substituting (7) and (8) in (6) and rearranging it, we obtain

$$\begin{aligned}\Delta X = B^1[(I - \bar{M}_f^0)F_d^1 - (I - \bar{M}_f^0)F_d^0] + &\qquad \text{(a)} \\ + B^1 \cdot [E^1 - E^0] + &\qquad \text{(b)} \\ + B^1 \cdot [(I - \bar{M}_f^1)F_d^1 - (I - \bar{M}_f^0)F_d^1] + &\qquad \text{(c)} \end{aligned}$$

$$+ (B^1 - B^*) \cdot [(I - \bar{M}_f^0) F_d^0 + E^0 + \qquad (d)$$

$$+ (B^* - B^0) \cdot [(I - \bar{M}_f^0) F_d^0 + E^0] \qquad (e) \qquad (9)$$

where:

(a) = effects of changes in domestic final demand (FD-effect);
(b) = effects of changes in export (E-effect);
(c) = effects of changes in import coefficients of domestic final demand (M_f-effect);
(d) = effects of changes in import coefficients of intermediate demand (M_m-effect);
(e) = effects of changes in input coefficients (A-effect).

Linkage structure model

Another task of the study is to trace and indicate to what extent and through what channels of industries the linkage effects extend. The first attempt to introduce the concepts of "linkage effects" into development economics was done by Hirschman, with the two types of now-familiar linkage effects; namely, "Input-provision effect" (or backward linkage effect) and "Output-utilization effect" (or forward linkage effect).[4] Attempts to measure the Hirschman linkage effects have been made so far utilizing I/O tables. As indices of backward and forward linkage effects, the intermediate input and demand ratio, the column-sum and row-sum of Leontief inverse matrix, the indices of power of dispersion and sensitivity of dispersion, have been used. These indices, however, are insufficient, because they do not indicate to what extent and through what channels the linkage effects extend.[5]

The following linkage structure model for the I/O analysis may be used to trace the dispersion of linkage effects. The balance equation is:

$$X_i = \sum_{j}\sum_{k} a_{ij} b_{jk} F_k + F_i \qquad (10)$$

$$i = 1, 2, \ldots, n$$
$$j = 1, 2, \ldots, n$$
$$k = 1, 2, \ldots, n$$

where:

X_i = the total output (control total) of i^{th} industry;
a_{ij} = the input coefficient of j^{th} from i^{th} industry;
b_{jk} = the Leontief inverse coefficient of j^{th} from k^{th} industry;
F_k, F_i = the total final demand of either the k^{th} or i^{th} industry;
n = the number of industries.

[4] Hirschman [25], chapter 6, p. 100.

[5] The "variability index" of Rasmussen [27] indicates the extent of dispersion of the linkage effects, but it does not trace the channels of dispersion.

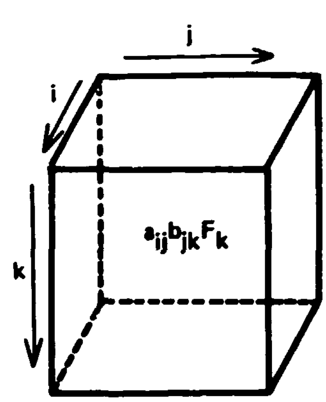
The matrix $[a_{ij} b_{jk}]$ which appears in the right-hand side of equation (10) plays an important role in our linkage structure model. The function of product matrix $[a_{ij} b_{jk}]$ is to isolate the induced intermediate input demands from the total induced demaNds. This can be proved in terms of the following calculation:

$$
\begin{aligned}
AB &= A(I - A)^{-1} \\
&= (I - A)^{-1} - (I - A)(I - A)^{-1} \\
&= B - I \qquad (11)
\end{aligned}
$$

Thus, the total output of i^{th} industry X_i can be decomposed, by utilizing the matrix AB in equation (10), into two separate categories, namely, induced intermediate input demands, $\sum_j \sum_k a_{ij} b_{jk} F_k$, and final demands F_i, both of which appear in equation (10).

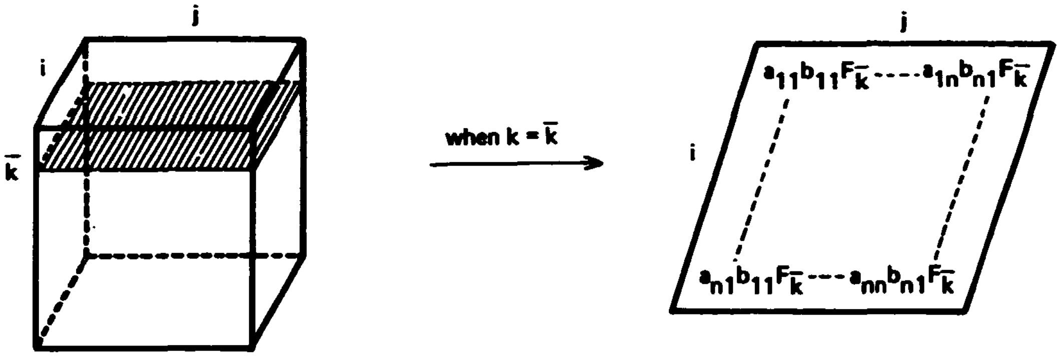
In addition to the ability to decompose total output into intermediate transactions and final demands, the product matrix $[a_{ij} b_{jk}]$ has another advantage of indicating to what extent and through what channels the linkage effects disperse. The matrix $[a_{ij} b_{jk}]$ has three dimensions of i, j and k, and forms a cube $(n \times n \times n)$ as is shown in figure 2. The economic implication of each cell of the cubic matrix, $a_{ij} b_{jk}$, can be understood in the following "two step" interpretation. The first step is that a unit increase of final demand in the k^{th} industry ($F_k = 1$) gives rise to b_{jk} of induced demands in the j^{th} industry, which includes both intermediate and final demands. As the second step, in order to separate the intermediate demands out of b_{jk}, the input coefficient a_{ij} can be multiplied to make $a_{ij} b_{jk}$, which tells how much of intermediate input is to be supplied from industry i to maintain the induced production of b_{jk}.

Figure 2. Image of the linkage structure matrix [AB]



The cubic matrix $[a_{ij} b_{jk}]$ is a "linkage structure matrix"; by extracting some two-dimensioned plane matrix or one-dimensioned vector from it, one can trace the channels through which the linkage effects disperse. The $(i \times j)$ plane with given $k (k = \bar{k})$ in figure 3, for example, indicates the channels through which a unit increase in final demand of $\bar{k}^{th}$ industry induces the expansion of production of each i^{th} industry to provide for the increases in intermediate demand necessary for the production of each j^{th} industry. This interpretation of the extracted plane matrix gives a detailed observation of the backward linkage effects of $\bar{k}^{th}$ industry, so that it is a "backward linkage matrix".

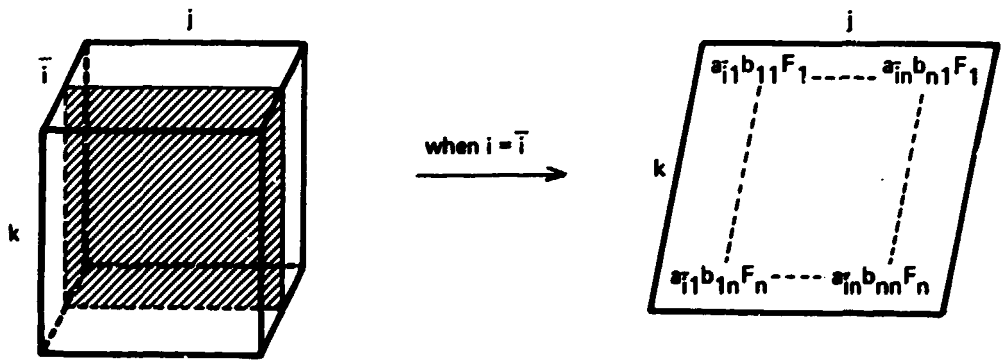
Figure 3. Backward linkage matrix



The (j × k) plane matrix with given i ($i = \bar{i}$) in figure 4, for example, shows the channels through which the production of $\bar{i}^{th}$ industry is induced by the increases in intermediate demand necessary for the expansion of each j^{th} industry to satisfy the increases in final demand of each k^{th} industry. In other words, it gives a detailed observation of forward linkage effects of the $\bar{i}^{th}$ industry, so that it is a "forward linkage matrix".

Some other types of plane matrices and vectors can be extracted from the linkage structure matrix, $[A_{ij} B_{jk}]$. This paper limits analysis to the two types above.

Figure 4. Forward linkage matrix



Empirical decomposition of factors of industrial growth

This chapter describes the application of the factor decomposition model to the I/O tables of Japan and the Republic of Korea. Results of the decomposing calculations are given in tables 2 to 5. Table 2.A (1966–1970) and table 3.A (1970–1975) for Republic of Korea and table 4.A (1965–1970) and table 5.A (1970–1975) for Japan present the percentage contribution of each factor to the total growth of industries, while table 2.B (1966–1970) and table 3.B (1970–1975) for Republic of Korea and table 4.B (1965–1970) and table 5.B (1970–1975) for Japan present the sectoral shares of each growth factor.[6]

Industrial growth of the Republic of Korea

As shown in table 4.A, industrial growth of the Republic of Korea during 1966–1970, before its rapid growth, was mainly led by the final demands, F-effects, although textile (6), other industrial products (20), apparels (7) and wood products (8) had already come into the stage of export-oriented growth. During the period 1966–1970 in the Republic of Korea as shown in table 4.B, sectoral shares of growth factors of agriculture (1), food (5), construction (21), trade (23) and services (24) to the total growth induced by final demands were so large that they amounted to over 70% (FD-effect in table 4.B). The sectoral shares of textile (6), apparel (7), other industrial products (20), trade (23) and services (24) to the total growth induced by changes in exports were more than 54% (E-effect in table 2.B). Food (5), iron and steel (14), transportation equipment (19) and services (24) had large percentages of output-expansion effects induced by import substitution of final demand products (M_f-effect in table 4.A).

During the period of rapid growth of the economy of the Republic of Korea, 1970–1975, the industrial outputs expanded remarkably, a phenomenon induced mainly by exports. Textile (6), apparel (7), rubber products (12), iron and steel (14), non-ferrous metals (15), electrical machinery (18) and other industrial products (20) found expansion in export to be the largest factor of industrial growth (E-effect in table 3.A). In the other industries changes in final demand were the largest factors of industrial growth (FD-effect in table 5.A). A remarkable increase in production through import substitution of final demand products was attained in food (5), non-electrical machinery (17) and electrical machinery (18) (M_f-effect in table 5.A). Non-electrical machinery (17) and electrical machinery (18) had large expansion effects induced by import substitution of final demand products (M_f-effect in table 5.A). Import substitution of intermediate demands were dominant in textile (6), chemicals (10), iron and steel (14) and metal products (16) (M_m-effect in table 5.A). The output-expansion effects induced by changes in input coefficients rose markedly during this period, (A-effect in table 5.A), especially in mining (4), food (5), chemicals (10), petroleum and coal products (11), non-metallic mineral products (13), iron and steel (14) and electrical machinery (18) (A-effect in table 5.B).

[6]From table 2.A to 5.B, "M-effect" stands for the sum of M_f-effect and M_m-effect, or "total induced changes in import coefficients".

Characteristics of Japanese industrial growth

The most important characteristics of Japanese industrial growth, during the period of maturity of industrialization, 1965–1970, were expansion of final demands with its attendant changing composition (FD-effects in table 2.A), especially in food (5), chemicals (10), iron and steel (14), non-electrical machinery (17), electrical machinery (18), transportation equipment (19), trade (23) and services (24) industries. The shares of contribution of these industries amounted to about 68% (FD-effect in table 2.B). Expansion of exports was the second characteristic of industrial growth (E-effect in table 2.A), especially in chemicals (10), iron and steel (14), non-electrical machinery (17), electrical machinery (18), transportation equipment (19), trade (23) and services (24), the shares of which amounted to over 68% (E-effect in table 2.B). Import substitution of final demands of iron and steel (14) was induced by import substitution of intermediate demand products (M_m-effect in table 2.B). Changes in input coefficients induced growth of fishery (3), apparel (7), non-metallic mineral products (13), metal products (16), non-electrical machinery (17), electrical machinery (18), other

TABLE 2. INDUSTRIES' CONTRIBUTION TO GROWTH FOR JAPAN, 1965 TO 1970[a]
(Percentage)

Industry	*FD-Effect*	*E-Effect*	*M_f-Effect*	*M_m-Effect*	*M-Effect*	*A-Effect*
			A. Contribution by industry			
1	114.62	2.10	−0.78	0.75	−0.02	−23.70
2	220.22	15.06	10.59	−72.10	−61.50	−73.78
3	106.03	−1.15	−5.44	−0.74	−6.19	1.30
4	106.60	18.69	−2.34	−18.73	−21.02	−4.21
5	96.27	4.39	1.59	−0.17	1.42	−2.08
6	107.44	30.42	−5.12	−2.59	−7.72	−20.15
7	93.07	7.36	−1.70	−0.11	−1.81	1.38
8	102.52	5.52	−0.26	−2.11	−2.37	−5.67
9	95.88	13.36	−1.48	−0.44	−1.93	−7.31
10	85.00	23.07	−2.52	−0.62	−3.14	−4.93
11	92.53	13.32	−2.19	0.59	−1.59	−4.26
12	84.98	33.69	−0.88	−0.22	−1.10	−17.57
13	88.08	7.72	−0.16	−0.03	−0.19	4.30
14	73.32	27.71	−0.10	0.69	0.59	−1.63
15	82.41	24.83	−1.52	−3.14	−4.67	−2.57
16	88.64	10.51	−0.14	−0.04	−0.19	−1.[illegible]3
17	83.14	14.70	0.73	0.23	0.96	1.18
18	81.84	18.83	−0.95	0.01	−0.93	0.26
19	77.90	24.95	−0.66	0.00	−0.66	−2.19
20	80.00	21.82	−4.19	0.14	−4.04	2.21
21	99.09	0.71	−0.08	−0.02	0.07	0.26
22	100.25	11.92	−0.64	−0.37	−1.02	−11.15
23	90.32	8.24	−0.26	−0.01	−0.27	1.70
24	95.02	8.06	−0.69	−0.20	−0.90	−2.18
25	99.37	14.88	−1.29	−0.71	−2.00	−12.25

TABLE 2 *(continued)*

Industry	*FD-Effect*	*E-Effect*	*M_f-Effect*	*M_m-Effect*	*M-Effect*	*A-Effect*
			B. Share by industry			
1	2.26	1.31	−2.17	3.00	−0.05	−15.55
2	0.64	0.32	4.29	−41.99	−14.70	−7.10
3	0.50	−0.04	−3.60	−0.71	−2.42	0.21
4	0.58	0.75	−1.80	−20.64	−9.53	−0.77
5	4.48	1.49	10.43	−1.63	5.48	−3.22
6	2.07	2.87	−13.85	−10.06	−12.30	−12.92
7	1.14	0.66	−2.94	−0.28	−1.85	0.57
8	2.35	0.92	−0.85	−9.73	−4.49	−4.32
9	2.14	2.99	−5.96	−2.56	−4.56	−6.95
10	3.12	6.19	−12.98	−4.63	−9.55	−6.02
11	1.83	1.92	−6.06	2.37	−2.60	−2.80
12	0.35	1.01	−0.51	−0.19	−0.37	−2.39
13	1.73	1.18	−0.45	−0.12	−0.31	2.82
14	6.16	16.99	−1.29	11.82	4.10	−4.57
15	1.16	2.55	−3.01	−8.93	−5.44	−1.21
16	2.48	2.15	−0.58	−0.27	−0.45	0.97
17	5.57	7.19	6.90	3.14	5.36	2.64
18	5.17	8.69	−8.64	0.17	−4.91	−0.65
19	4.20	9.82	−5.02	0.01	−2.95	−3.94
20	1.37	3.72	−13.71	0.67	−7.81	1.72
21	11.46	0.61	−0.84	−0.61	−0.75	1.03
22	1.51	1.31	−1.36	−1.14	−1.27	−5.59
23	9.17	6.12	−3.72	−0.35	−2.33	5.75
24	22.07	13.69	−22.70	−9.64	−17.34	−16.89
25	5.37	2.97	−9.80	−7.74	−8.95	−22.00
Total	100.00	100.00	−100.00	−100.00	−100.00	−100.00

[a]Industry classification shown in annex.

TABLE 3. INDUSTRIES' CONTRIBUTION TO GROWTH FOR JAPAN, 1970 TO 1975[a]
(Percentage)

Industry	*FD-Effect*	*E-Effect*	*M_f-Effect*	*M_m-Effect*	*M-Effect*	*A-Effect*
			A. Contribution by industry			
1	108.70	2.52	−13.08	4.30	−8.78	−2.45
2	216.36	13.72	−38.96	1.03	−37.93	−92.15
3	99.18	2.35	−5.03	−4.19	−9.22	7.68
4	88.35	15.70	10.36	−125.86	115.49	91.44
5	107.31	2.46	−9.71	1.37	−8.33	−1.44
6	105.04	18.96	−4.14	−6.24	−10.38	−13.61
7	118.63	−0.17	−6.60	−0.99	−7.60	−10.84
8	125.08	7.70	−2.69	−2.17	−4.87	−27.91
9	91.98	15.97	−0.82	−1.09	−1.92	−6.03
10	73.16	34.92	−1.26	0.77	−0.48	−7.61
11	58.49	18.37	−0.67	0.26	−0.40	23.53
12	59.80	43.92	−1.00	−2.94	−3.95	0.22
13	96.52	14.14	−0.44	−0.38	−0.82	−9.84

Industry	*FD-Effect*	*E-Effect*	M_f*-Effect*	M_m*-Effect*	*M-Effect*	*A-Effect*
14	52.21	72.43	0.55	1.66	2.21	−26.86
15	76.54	47.24	2.38	1.55	3.94	−26.71
16	94.74	20.69	−0.65	−1.17	−1.82	−13.60
17	48.71	42.83	3.83	−2.02	1.80	6.65
18	76.59	52.54	1.98	−3.32	−1.33	−26.79
19	39.50	54.78	0.21	−1.00	−0.79	6.50
20	75.43	23.39	5.73	−4.18	1.54	−0.37
21	0.73	0.73	−0.12	−0.01	−0.14	−1.32
22	76.96	12.54	−0.57	−0.55	−1.13	11.62
23	90.95	8.84	−1.60	0.02	−1.57	1.72
24	84.24	7.07	−1.64	−0.03	−1.54	10.22
25	91.25	24.12	−1.21	−3.35	−4.57	−10.80
			B. Share by industry			
1	3.47	0.42	−21.85	17.35	−10.37	−3.86
2	0.50	0.16	−4.74	0.30	−3.26	−10.58
3	0.60	0.07	−1.60	−3.22	−2.07	2.30
4	0.33	0.69	2.04	−59.78	−16.06	16.96
5	6.88	0.81	−32.52	11.12	−19.74	−4.57
6	1.43	1.38	−3.06	−11.13	−5.42	−9.47
7	1.10	−0.00	−3.20	−1.16	−2.60	−4.95
8	1.67	0.53	−1.88	−3.67	−2.40	−18.34
9	2.53	2.26	−1.19	−3.79	−1.95	−8.18
10	2.48	6.08	−2.23	3.31	−0.61	−12.69
11	2.64	4.27	−1.59	1.52	−0.68	52.34
12	0.26	0.97	−0.23	−1.59	−0.63	0.03
13	1.46	1.10	−0.35	−0.73	−0.46	−7.34
14	2.81	20.03	1.35	11.30	4.40	−71.14
15	0.63	2.04	1.05	1.65	1.22	−11.07
16	1.60	1.80	−0.58	−2.50	−1.14	−11.32
17	1.76	7.98	7.25	−9.25	2.41	11.86
18	1.52	5.44	2.08	−8.43	−0.99	−26.56
19	1.98	14.10	0.57	6.36	−1.46	16.02
20	1.37	2.18	5.42	−9.57	1.02	−0.34
21	12.23	0.16	−0.80	−0.23	−0.64	−7.95
22	2.11	1.77	−0.83	−1.92	−1.15	15.62
23	9.77	4.89	−9.00	0.39	−6.23	9.41
24	34.72	14.99	−31.45	−4.54	−23.57	207.70
25	4.10	5.35	−2.56	−19.07	−7.60	−23.46
Total	100.00	100.00	−100.00	−100.00	−100.00	−100.00

[a]Industry classification shown in annex.

industrial products (20), construction (21) and trade (23) (A-effect in table 2.A), although their contribution shares were smaller than other growth characteristics (A effect in table 2.B).

The output-expansion effects of exports were outstanding among Japanese industries during 1970–1975. With chemicals (10), rubber products (13), iron and steel (14), non-ferrous metals (15), non-electrical machinery (17), electrical machinery (18) and transportation equipment (19) as leading sectors, export promotion was very important to industrial growth of Japan. In particular, iron and steel (14) and transportation equipment (19), received the largest inducement from export expansion (E-effect in table 3.A). During the period, the sectoral shares of construction (21), trade (23) and service (24) to the FD-effect increased so markedly that they amounted to over 56% in comparison with about 43% during the previous period (FD-effect in table 2.B and 3.B). The output-expansion effect of import substitution was insignificant, but only iron and steel (14) and non-electrical machinery (17), was worth notice (M-effect in table 3.B). On the other hand, the output-expansion effect of changes in input coefficients during the period was larger than those during the previous one. The following industries had such an especially strong tendency: mining (4), petroleum and coal products (11), non-electrical machinery (17), transportation equipment (19), public utilities (22) and services (24) (A-effect in table 5.B).

TABLE 4. INDUSTRIES' CONTRIBUTION TO GROWTH FOR THE REPUBLIC OF KOREA, 1966 TO 1970[a]

(Percentage)

Industry	*FD-Effect*	*E-Effect*	*M_f-Effect*	*M_m-Effect*	*M-Effect*	*A-Effect*
			A. Contribution by industry			
1	110.96	5.85	−11.33	−0.98	−12.32	−4.48
2	93.23	18.61	−2.79	−7.88	−10.68	−1.16
3	74.41	34.12	0.15	−0.26	−0.11	−8.42
4	90.69	34.41	−1.44	−14.96	−16.41	−8.69
5	89.73	5.27	2.95	−1.61	1.33	3.65
6	53.92	57.30	0.77	−6.58	−5.81	−5.41
7	57.25	43.87	0.26	−0.01	0.24	−1.39
8	60.97	47.69	0.40	−0.61	−0.20	−8.46
9	96.21	13.91	−0.02	−3.59	−3.62	−6.50
10	69.29	17.99	−0.16	8.77	8.60	4.10
11	76.51	15.99	0.01	3.88	3.90	3.58
12	69.45	40.43	5.16	−3.10	2.05	−11.95
13	93.72	7.20	−0.08	0.80	0.71	−1.65
14	73.44	10.70	10.58	2.87	13.45	2.39
15	79.03	40.58	1.50	−2.72	−1.22	−18.39
16	82.32	71.11	7.76	−6.64	1.12	−4.56
17	119.84	6.22	−24.49	−2.91	−27.41	1.34
18	69.55	30.49	−1.81	−3.53	−5.34	5.30
19	58.75	4.27	39.25	−9.89	29.36	7.61
20	45.53	59.93	−6.42	−0.20	−6.63	1.15
21	99.66	1.79	0.04	0.00	0.04	−1.51
22	81.51	18.56	1.14	0.08	1.22	−1.30
23	86.14	11.38	0.66	−0.23	0.43	2.03
24	88.15	7.95	0.82	0.14	0.97	2.92
25	78.47	17.13	0.36	0.60	0.96	3.47
			B. Share by industry			
1	14.52	4.66	−997.43	−21.83	−369.31	−113.41
2	1.27	1.55	−25.60	−18.14	−33.34	−3.08
3	1.10	3.06	1.49	−0.66	−0.38	−23.95
4	1.00	2.31	−10.71	−27.89	−41.50	−18.54

Industry	*FD-Effect*	*E-Effect*	*M_f-Effect*	*M_m-Effect*	*M-Effect*	*A-Effect*
5	9.49	3.39	209.70	−28.73	32.42	75.59
6	1.82	11.81	17.60	−37.61	−45.05	−35.37
7	2.97	13.86	9.37	−0.17	2.96	−13.83
8	1.05	5.00	4.73	−1.77	−0.80	−28.14
9	1.54	1.35	0.31	−8.68	−13.25	−20.04
10	3.20	5.06	−5.26	68.40	91.05	36.68
11	2.53	3.22	0.31	21.66	29.51	22.84
12	0.30	1.08	15.15	−2.29	2.06	−10.08
13	1.78	0.83	−1.13	2.59	3.12	−6.04
14	1.85	1.65	177.59	12.23	77.75	11.66
15	0.25	0.79	3.04	−1.47	−0.87	−11.35
16	0.57	0.89	36.00	−7.73	1.77	−6.07
17	0.56	0.18	−76.40	−2.28	−29.12	1.21
18	1.25	3.33	−21.85	−10.61	−29.93	−18.35
19	1.38	0.61	621.06	−39.09	158.20	34.39
20	0.90	7.05	−85.81	−0.69	−30.16	4.44
21	12.27	1.35	3.89	0.05	1.33	−36.03
22	1.27	1.76	11.94	0.22	4.36	−3.82
23	9.82	7.91	51.15	−4.56	11.24	44.47
24	21.60	13.52	155.64	6.79	62.23	157.68
25	2.71	3.61	8.60	3.52	2.67	23.16
Total	100.00	100.00	100.00	100.00	100.00	100.00

[a]Industry classification shown in annex.

TABLE 5. INDUSTRIES' CONTRIBUTION TO GROWTH FOR THE REPUBLIC OF KOREA, 1970 TO 1975[a]

(Percentage)

Industry	*FD-Effect*	*E-Effect*	*M_f-Effect*	*M_m-Effect*	*M-Effect*	*A-Effect*
			A. Contribution by industry			
1	95.95	8.84	−1.23	−2.16	−3.40	−1.39
2	83.12	31.13	4.65	−12.22	−7.56	−6.69
3	54.54	44.53	−0.17	−0.37	−0.55	1.47
4	83.69	33.43	−11.88	−41.01	−52.89	35.76
5	83.21	15.71	0.73	−1.59	−0.86	1.93
6	35.16	64.04	−1.37	1.28	−0.09	0.88
7	41.87	59.50	−1.40	−0.72	−2.12	0.74
8	51.27	50.33	0.17	0.23	0.40	−2.02
9	69.10	28.91	0.23	−0.24	−0.01	1.99
10	60.24	35.64	−2.35	3.20	0.84	3.25
11	62.97	24.69	−0.08	−1.97	−2.06	14.39
12	33.25	67.50	−0.92	0.00	−0.92	0.16
13	69.26	24.33	1.61	−0.51	1.09	5.30
14	46.87	47.69	−1.79	4.53	2.74	2.65
15	38.63	50.99	7.63	0.63	8.27	2.10
16	51.77	45.15	−2.97	7.02	4.04	−0.97
17	56.59	21.10	20.36	−0.24	20.11	2.18
18	36.24	49.44	13.36	−1.26	12.10	2.20
19	97.21	30.92	−27.04	2.29	−24.74	−3.39
20	42.06	65.88	−4.63	−3.74	−8.38	0.42
21	99.81	1.06	−0.06	−0.04	−0.11	−0.76

TABLE 5 (continued)

Industry	FD-Effect	E-Effect	M_f-Effect	M_w-Effect	M-Effect	A-Effect
22	71.89	24.94	−0.46	−0.32	−0.78	3.95
23	74.78	26.58	−0.24	−0.10	−0.35	−1.01
24	83.82	16.72	−0.00	−0.58	−0.59	0.03
25	74.27	32.96	−0.21	−0.81	−1.03	−6.19
			B. Share by industry			
1	14.54	3.27	−28.56	−36.57	−33.19	−9.91
2	0.71	0.65	6.04	−11.58	−4.14	−2.66
3	1.21	2.41	−0.60	−0.90	−0.78	1.53
4	1.13	1.10	−24.47	−61.66	−45.97	22.60
5	10.10	4.65	13.62	−21.59	−6.33	10.89
6	3.04	13.52	−18.18	12.36	−0.53	3.59
7	3.54	12.26	−24.13	−6.84	−11.58	2.95
8	0.84	2.01	0.44	0.40	0.43	−1.55
9	1.52	1.55	0.78	−0.60	−0.02	2.05
10	5.80	8.36	−34.55	34.30	5.25	14.64
11	4.94	4.72	−1.00	−17.78	−10.41	52.87
12	0.46	2.30	−1.98	0.01	−0.82	0.10
13	1.61	1.38	5.73	−1.34	1.64	5.78
14	2.47	8.12	−14.44	26.61	9.29	6.62
15	0.26	0.84	7.87	0.47	3.60	0.66
16	0.74	1.58	−6.49	11.20	3.73	−0.65
17	0.77	0.70	49.19	−0.37	17.57	1.39
18	1.98	6.52	111.28	−7.67	42.52	5.84
19	2.87	2.22	−121.19	7.50	−46.80	−9.66
20	0.60	2.30	−10.03	−5.92	−7.65	0.28
21	8.27	0.21	−0.80	−0.45	−0.60	−2.87
22	1.77	1.49	−1.73	−0.87	−1.24	4.14
23	9.05	8.02	−4.65	−1.45	−2.81	5.83
24	19.37	9.13	−0.21	−15.07	−8.80	0.43
25	2.16	2.34	−0.87	−2.64	−1.94	8.43
Total	100.00	100.00	−100.00	−100.00	−100.00	−100.00

[a] Industry classification shown in annex.

Empirical study on linkage dispersion

The empirical study on linkage dispersion attempts to shed light on the characteristics of linkage structure that cannot be clarified by the factor decomposition model. The linkage structure model is applied to the I/O tables of Japan and the Republic of Korea. Changes in final demands and exports were very important factors of industrial growth in both economies. The following industries serve as examples of this phenomenon: construction (21) for analysing the backward linkage effect of its FD-effect; chemicals (10) and petroleum and coal products (11) for the forward linkage effects of their FD-effects; iron and steel (14), and electrical machinery (18) for the forward linkage effects of their E-effects. Channels of dispersion of linkage effects are traced through examination of these industries. Tables 6 through 10 show linkage-effect comparisons between the two countries in absolute monetary terms.

Backward-linkage effects of construction (21)

In both economies the construction industry (21) made a large contribution to the increases in production of other industries through its backward-linkage effects. Table 6 shows through what channels and how much of the backward-linkage effects of the construction industry (21) dispersed, by observing the largest three major channel industries. The first step of dispersion is captured by the direct effects ($\sum_i a_{ij} b_{jk} F_k; k = 21$) for j^{th} industries and the second step by the indirect effects ($a_{ij} b_{jk} F_k; k = 21$) for i^{th} industries. In table 6 for the Republic of Korea, the largest direct backward-linkage effect induced by construction (21) went to non-metallic mineral products (13). Iron and steel (14) was the second largest recipient and lumber and wood products (8) the third largest. For the Republic of Korea, in 1966 indirect backward-linkages, or intermediate input demands which were induced by the above mentioned directly linked industries, can be observed: the largest major direct effects of non-metallic minerals (14) were in the intermediate input production of mining (4), services (24) and trade (23).

The channels and dispersion of backward-linkage effect of the construction industry of the Republic of Korea changed from 1970 to 1975, as shown in table 6. Lumber and wood products (8), which received the third largest direct effect from the construction industry in 1966 was, by 1970, no longer among the three industries which received the largest direct effects. Services (24) took its place as the third major recipient of the direct effect. Observing the indirect backward-linkage effects in the similar way, petroleum and coal products (11), paper and publishing (9) and public utilities (22) became major intermediate inputs, or indirect backward-linked industries of construction in the Republic of Korea after 1970.

Comparing the effects for Japan and for the Republic of Korea in table 6, the largest indirect backward effect was for petroleum and coal products (11) in Japan, while it was a rather new entry for the Republic of Korea. Forestry (2) has supplied a certain amount of intermediate inputs to construction (21) via lumber and wood products (8) in Japan, that did not appear for the Republic of Korea, due to the lack of domestic forestry resources.

Forward-linkage effects of the chemicals industry (10)

The chemicals industry (10) is one that typically has the widest range of dispersion of forward-linkage effects, since modern technologies in any industry must use chemical products as raw materials. As shown in table 7, the chemicals industry (10) of the Republic of Korea, in 1966 supplied its products to agriculture (1) through channels of chemicals (10), food (5), and services (24), to services (24) through chemicals (10), paper and publishing (9) and construction (21), and to construction through mining (4), chemicals (10) and non-metallic minerals (13).

TABLE 6. BACKWARD-LINKAGE EFFECTS OF THE CONSTRUCTION INDUSTRY (21) BY YEAR[a,b]

A. Effects for the Republic of Korea (millions of won)

1966				*1970*				*1975*			
(j)	*Direct effects*	*(i)*	*Indirect effects*	*(j)*	*Direct effects*	*(i)*	*Indirect effects*	*(j)*	*Direct effects*	*(i)*	*Indirect effects*
(13)	6 983.6	(4)	1 644.2	(14)	21 536.2	(23)	2 118.7	(14)	132 587.7	(23)	9 329.8
		(24)	1 010.9			(24)	1 846.6			(22)	6 971.0
		(23)	689.7			(22)	1 694.6			(24)	6 469.5
(14)	5 069.1	(23)	705.3	(13)	30 033.5	(24)	6 186.9	(13)	120 053.7	(11)	30 333.9
		(24)	337.4			(4)	6 041.0			(4)	20 776.6
		(4)	283.9			(22)	3 369.1			(23)	13 057.8
(8)	3 500.9	(23)	1 450.8	(24)	11 534.1	(11)	1 768.6	(24)	41 188.9	(11)	10 757.4
		(24)	314.1			(21)	1 209.4			(23)	3 562.2
		(2)	187.0			(23)	1 194.2			(9)	2 136.2

B. Effects for Japan (100 million yen)

1965				*1970*				*1975*			
(j)	*Direct effects*	*(i)*	*Indirect effects*	*(j)*	*Direct effects*	*(i)*	*Indirect effects*	*(j)*	*Direct effects*	*(i)*	*Indirect effects*
(14)	9 574.4	(11)	550.8	(14)	22 189.1	(11)	1 461.9	(24)	730 479.6	(11)	73 488.0
		(24)	439.5			(24)	843.9			(23)	52 107.6
		(23)	356.4			(23)	764.8			(21)	42 585.3
(8)	5 134.6	(2)	2 052.6	(8)	9 965.5	(2)	2 688.0	(14)	259 386.1	(11)	25 887.1
		(23)	398.6			(23)	824.2			(24)	13 138.8
		(24)	381.8			(24)	784.1			(22)	9 580.4
(24)	3 516.3	(23)	869.7	(13)	9 615.3	(4)	1 793.9	(23)	211 424.6	(24)	131 307.4
		(5)	368.8			(24)	1 485.3			(9)	8 283.3
		(21)	266.9			(23)	767.7			(22)	5 733.3

[a]Industrial classifications are given in the annex.
[b]Effect is given as $a_{ij} b_{j21} F_{21}$.

In 1970, the destinations of forward linkages of the chemicals industry (10) of the Republic of Korea shifted to agriculture (1), apparels (7) and services (24), and in 1975, the forward linkage was typical of the stage of industrialization with the top three destinations, apparel (7), textile (6) and agriculture (1).

On the contrary, as shown in table 7, in Japan, such a traditional and early-stage industry as textile (6) which remained among three major destinations of forward linkage of chemicals (10) in 1965 was no longer a major destination after 1970, and construction (21) became one of the major destinations of chemicals. In Japan, the services industry (24) always has received the largest forward-linkage effects from chemicals. In other words the chemicals industry (10) which has a less labour absorption effect has indirectly been supported by the very labour absorbing service industries (24) through the channels as shown in table 7.

Forward-linkage effects of the petroleum and coal industry (11)

The petroleum and coal products industry (11) is commonly believed to have a wide range of dispersion. Comparing the effects for Japan and the Republic of Korea in table 8, it may be seen that the channels of forward-linkage dispersion are quite different. The Japanese iron and steel industry (14) played a dominant role as a channel of dispersion of petroleum forward linkages to other industries, while in the Republic of Korea the effect was very small. In the Republic of Korea, petroleum forward linkages dispersed through public utilities (22), chemicals (10) and non-metallic mineral products (13).

Forward-linkage effects of iron and steel (14)

The iron and steel industry (14) includes transactions between iron smelting and steel milling. Thus, the largest forward-linkage effect of iron and steel (14), in table 9, both for Japan and the Republic of Korea, is seemingly dispersed to itself, that is, to iron and steel (14). The forward linkages of the iron and steel industry (14) of Japan dispersed to transportation equipment (19) and non-electrical machinery (17); while in the Republic of Korea dispersed to metal products (16), electrical machinery (18) and construction (21). Such difference of direct and indirect end-use structure of the iron and steel industry between two countries reflects the different approaches to their post-war industrialization. Japanese post-war industrialization has been mainly led by transportation equipment and other machinery industries, while the Republic of Korea started its industrialization with textile, apparel and food industries together with some metal products, construction and electrical machinery industries.

Forward-linkage effects of the electrical machinery industry (18)

Similar to the case of the iron and steel industry (14), the electrical machinery industry (18) supplies the largest amount of its products to electrical machinery itself, as shown in table 10. Thus one observes the largest dispersion of the

TABLE 7. FORWARD-LINKAGE EFFECTS OF CHEMICALS INDUSTRY (10), BY YEAR[a,b]

A. Effects for the Republic of Korea (millions of won)

1966				1970				1975			
(k)	*Forward-linkage effects to (k)*	*(j)*	*Channels of dispersion*	*(k)*	*Forward-linkage effects to (k)*	*(j)*	*Channels of dispersion*	*(k)*	*Forward-linkage effects to (k)*	*(j)*	*Channels of dispersion*
(1)	6 096.8	(10)	325.1	(1)	33 372.9	(10)	1 851.2	(7)	178 978.0	(6)	48 174.8
		(5)	165.4			(5)	381.3			(10)	18 046.7
		(24)	75.4			(24)	295.2			(1)	812.0
(24)	4 279.2	(10)	222.2	(7)	17 393.1	(6)	5 458.3	(6)	134 138.7	(10)	9 077.1
		(9)	164.1			(10)	613.8			(1)	347.0
		(21)	158.3			(1)	217.0			(24)	110.0
(21)	3 536.9	(4)	206.8	(24)	13 890.5	(10)	731.2	(1)	132 916.8	(10)	29 896.9
		(10)	190.6			(1)	455.6			(5)	1 578.3
		(13)	130.4			(9)	404.0			(8)	1 294.6

B. Effects for Japan (100 million yen)

1965				1970				1975			
(k)	*Forward-linkage effects to (k)*	*(j)*	*Channels of dispersion*	*(k)*	*Forward-linkage effects to (k)*	*(j)*	*Channels of dispersion*	*(k)*	*Forward-linkage effects to (k)*	*(j)*	*Channels of dispersion*
(24)	3 956.3	(10)	1 059.3	(24)	8 300.7	(10)	2 216.8	(24)	301 451.2	(10)	128 368.4
		(20)	181.5			(20)	402.7			(20)	17 149.5
		(1)	155.4			(5)	269.7			(1)	15 796.1
(5)	3 905.5	(10)	1 045.7	(5)	5 502.8	(10)	1 469.6	(21)	144 811.0	(10)	61 665.6
		(1)	990.1			(1)	1 213.3			(24)	31 755.8
		(6)	85.3			(20)	145.6			(20)	9 955.8
(6)	2 386.3	(10)	638.9	(21)	5 289.4	(10)	1 412.6	(5)	74 618.0	(10)	31 774.9
		(1)	32.4			(20)	578.6			(1)	14 096.6
		(24)	21.0			(24)	460.4			(24)	11 929.4

[a]Industrial classifications are given in the annex.
[b]Effect is given as $a_{10j} b_{jk} F_k$.

TABLE 8. FORWARD-LINKAGE EFFECTS OF THE PETROLEUM AND COAL PRODUCTS INDUSTRY (11)[a,b]

A. Effects for the Republic of Korea (millions of won)

1966				1970				1975			
(k)	*Forward-linkage effects to (k)*	*(j)*	*Channels of dispersion*	*(k)*	*Forward-linkage effects to (k)*	*(j)*	*Channels of dispersion*	*(k)*	*Forward-linkage effects to (k)*	*(j)*	*Channels of dispersion*
(24)	8 209.9	(22)	194.8	(24)	34 387.7	(22)	1 024.6	(24)	248 699.0	(22)	14 936.0
		(11)	132.9			(10)	494.4			(11)	8 528.1
		(21)	114.3			(11)	415.6			(10)	5 287.6
(21)	3 098.8	(13)	629.4	(21)	14 411.0	(13)	2 547.9	(21)	110 036.6	(13)	30 333.9
		(24)	324.5			(24)	1 768.6			(22)	13 047.1
		(14)	151.1			(22)	1 082.4			(24)	10 757.4
(5)	1 297.8	(3)	239.1	(5)	5 529.9	(24)	786.5	(7)	66 038.5	(10)	6 853.4
		(22)	196.3			(22)	696.0			(22)	5 488.0
		(24)	184.2			(3)	479.4			(6)	5 037.0

B. Effects for Japan (100 million yen)

1965				1970				1975			
(k)	*Forward-linkage effects to (k)*	*(j)*	*Channels of dispersion*	*(k)*	*Forward-linkage effects to (k)*	*(j)*	*Channels of dispersion*	*(k)*	*Forward-linkage effects to (k)*	*(j)*	*Channels of dispersion*
(20)	2 945.3	(14)	550.8	(21)	6 765.4	(14)	1 461.9	(24)	358 514.3	(22)	47 326.1
		(13)	283.8			(4)	571.7			(14)	28 190.5
		(24)	204.2			(13)	519.9			(10)	26 901.8
(24)	2 618.8	(10)	120.0	(24)	4 554.4	(10)	333.5	(21)	176 546.0	(24)	73 488.0
		(14)	89.9			(22)	275.3			(14)	25 887.1
		(22)	78.5			(14)	201.5			(22)	25 703.7
(5)	931.5	(1)	171.4	(23)	1 985.8	(24)	199.4	(23)	103 319.9	(24)	52 290.4
		(10)	118.4			(22)	105.0			(22)	15 106.7
		(24)	83.0			(14)	85.3			(14)	8 571.3

[a] Industrial classifications are given in the annex.
[b] Effect is given as $a_{11j} b_{jk} F_k$.

TABLE 9. FORWARD-LINKAGE EFFECTS OF THE IRON AND STEEL INDUSTRY (14), BY YEAR[a,b]

A. Effects for the Republic of Korea (millions of won)

1966				*1970*				*1975*			
(k)	*Forward-linkage effects to (k)*	*(j)*	*Channels of dispersion*	*(k)*	*Forward-linkage effects to (k)*	*(j)*	*Channels of dispersion*	*(k)*	*Forward-linkage effects to (k)*	*(j)*	*Channels of dispersion*
(14)	982.1	(13)	16.5	(14)	1 668.1	(16)	4.9	(14)	78 984.3	(16)	167.6
		(16)	5.6			(17)	2.7			(17)	66.1
		(4)	3.1			(19)	1.6			(4)	46.4
(21)	297.1	(14)	91.1	(16)	1 500.5	(14)	418.1	(16)	28 487.6	(14)	12 028.7
		(16)	12.6			(17)	4.9			(17)	49.5
		(13)	5.2			(19)	1.8			(19)	19.5
(16)	293.8	(14)	90.1	(21)	827.6	(14)	230.6	(18)	18 200.7	(14)	7 685.1
		(17)	1.4			(16)	39.0			(16)	856.6
		(4)	0.7			(13)	17.3			(17)	250.9

B. Effects for Japan (100 million yen)

1965				1970				1975			
(k)	*Forward-linkage effects to (k)*	*(j)*	*Channels of dispersion*	*(k)*	*Forward-linkage effects to (k)*	*(j)*	*Channels of dispersion*	*(k)*	*Forward-linkage effects to (k)*	*(j)*	*Channels of dispersion*
(14)	673.1	(17)	18.4	(14)	11 989.8	(17)	39.5	(14)	63 106.4	(16)	875.5
		(16)	10.0			(16)	20.2			(19)	699.0
		(19)	5.8			(19)	7.3			(17)	695.3
(19)	1 528.1	(14)	870.0	(19)	4 135.9	(14)	2 253.9	(19)	45 872.8	(14)	29 938.8
		(17)	76.2			(17)	244.6			(17)	3 149.0
		(16)	37.6			(16)	114.0			(16)	2 417.6
(17)	849.6	(14)	483.7	(17)	2 906.5	(14)	1 583.9	(17)	23 924.6	(14)	15 614.3
		(16)	17.5			(16)	52.9			(16)	1 102.6
		(18)	11.2			(18)	30.7			(19)	663.4

[a]Industrial classifications are given in the annex.
[b]Effect is given as $a_{14j} b_{jk} F_k$.

TABLE 10. FORWARD-LINKAGE EFFECTS OF ELECTRICAL MACHINERY INDUSTRY (18), BY YEAR[a,b]

A. Effects for the Republic of Korea (millions of won)

1966				*1970*				*1975*			
(k)	*Forward-linkage effects to (k)*	*(j)*	*Channels of dispersion*	*(k)*	*Forward-linkage effects to (k)*	*(j)*	*Channels of dispersion*	*(k)*	*Forward-linkage effects to (k)*	*(j)*	*Channels of dispersion*
(18)	131.5	(15)	0.6	(18)	980.9	(24)	3.6	(18)	39 576.1	(24)	73.3
		(24)	0.3			(22)	2.8			(22)	43.1
		(4)	0.3			(21)	1.7			(15)	32.7
(21)	107.8	(18)	8.2	(21)	278.6	(18)	17.5	(24)	2 233.1	(18)	292.9
		(24)	0.7			(24)	2.5			(21)	241.9
		(4)	0.7			(4)	1.3			(19)	57.1
(24)	63.3	(21)	17.5	(24)	256.4	(21)	44.8	(7)	2 021.8	(18)	265.2
		(18)	4.8			(18)	16.1			(6)	209.7
		(19)	3.1			(19)	15.8			(19)	211.5

B. Effects for Japan (100 million yen)

1965				*1970*				*1975*			
(k)	*Forward-linkage effects to (k)*	*(j)*	*Channels of dispersion*	*(k)*	*Forward-linkage effects to (k)*	*(j)*	*Channels of dispersion*	*(k)*	*Forward-linkage effects to (k)*	*(j)*	*Channels of dispersion*
(18)	826.6	(17)	5.8	(18)	3 106.3	(17)	19.6	(18)	11 355.9	(17)	356.4
		(22)	2.6			(22)	5.3			(19)	295.8
		(15)	2.4			(21)	3.6			(24)	182.3
(19)	265.9	(18)	56.2	(19)	909.7	(18)	223.3	(19)	10 622.6	(18)	3 539.5
		(17)	31.8			(17)	102.7			(17)	1 471.4
		(22)	4.2			(14)	9.1			(24)	416.5
(17)	193.2	(18)	40.9	(17)	711.9	(18)	174.8	(17)	5 566.7	(18)	1 854.8
		(22)	2.0			(19)	8.1			(19)	306.8
		(14)	1.8			(14)	6.4			(24)	187.4

[a] Industrial classifications are given in the annex.
[b] Effect is given as $a_{18j} b_{jk} F_k$.

forward-linkage effect of electrical machinery (18) is to itself. Comparing Japan and the Republic of Korea in table 10, the destination industries to which the forward-linkage effects of electrical machinery (18) disperse, are again different for the two countries. In Japan dispersion was mainly to transportation equipment (19) and non-electrical machinery (17). It was remarkable that the major channels have been machinery industries (17, 18, 19) as well as services (24). On the other hand, in the Republic of Korea dispersions were to services (24), construction (21) and apparel (7) industries through channels of public utility (22), non-ferrous metal (15), construction (21), transportation equipment (19), electrical machinery (18) and textile (6) industries.

Summary and conclusion

In this study, the authors have proposed that "the industrial structure" should be analysed in terms of the following three aspects: "combination", "composition" and "linkage structure" of industries that constitute a national economy. In the present paper, the authors have applied the factor decomposition model to identify factors of industrial growth and also the linkage structure model to shed light on the channels through which the backward- and forward-linkage effect of some major leading industries disperse.

The remarkable feature of industrialization in developing countries is the emergence and rapid growth of new modern industries and the decline and disappearance of traditional ones. This makes linkage structure complex and densely related. The authors have called this phenomenon "the evolution process of industrial structure", and it proceeds rapidly with the progress of industrialization and gradually approaches saturation when a national economy reaches an advanced stage of industrial growth with matured linkage structure. In order to clarify the concept of the evolution process of industrial structure, an empirical study was made of both Japan and the Republic of Korea.

The empirical results show basic features of changes in composition and linkage structure of industries of the two economies.

In the mid-1960s, the Republic of Korea had an industrial structure typical of economies at an early stage of industrialization. In the first half of the 1970s, the Republic of Korea showed rapid growth in the machinery, iron and steel, textile and apparel, and petrochemical industries. The major cause of industrial growth during the 1966–1970 period which led to the drastic changes in industrial structure was the increase in output induced by changes in final demands. During the 1970–1975 period, the increases in output induced by changes in export were much more outstanding.

During the whole period, the linkage structure of Japan showed a different pattern from that of the Republic of Korea. This would be mainly attributed to a different growth stage of machinery, iron and steel, metal fabrication, and petrochemical industries. In Japan, those industries reached a more advanced stage of industrial growth with matured linkage structure. The linkage structure of Japan thus remained relatively stable during 1965 to 1975. This stability of linkage structure suggests that the evolution process of industrial structure in Japan has reached saturation. On the other hand, the linkage structure of the Republic of Korea showed remarkable changes in the 1970s. This phenomenon indicates that the rapid growth has started in the industry groups of machinery, iron and steel, textile and apparel, and petrochemical industries.

The changes in linkage structure imply not only changes in magnitude of linkage effects, but also the changes in channels through which the linkage effects extend. The detailed analysis of the linkage structure of both Japan and the Republic of Korea indicated two different patterns in the two countries:

(*a*) A similarity of linkage structure exists between Japan and the Republic of Korea in construction and petroleum and coal products.

(*b*) A large difference of linkage structure exists between Japan and the Republic of Korea in chemicals, iron and steel, and electrical machinery.

ANNEX

INDUSTRY CLASSIFICATION CODES

(1) Agriculture
(2) Forestry
(3) Fishery
(4) Mining
(5) Food, beverage and tobacco
(6) Textile
(7) Apparel and leather
(8) Lumber and wood products
(9) Paper, printing and publishing
(10) Chemicals
(11) Petroleum and coal products
(12) Rubber products
(13) Non-metallic mineral products
(14) Iron and steel
(15) Non-ferrous metals
(16) Metal products
(17) Non-electrical machinery
(18) Electrical machinery
(19) Transportation equipment
(20) Other industrial products
(21) Construction
(22) Public utilities
(23) Trade
(24) Services

DATA SOURCE

Korea*

1966 Input-Output Tables (Bank of Korea, 1968).
1970 Input-Output Tables (Bank of Korea, 1973).
1975 Input-Output Tables (Bank of Korea, 1978).

Japan

1965 Input-Output Tables (Administrative Management Agency, Government of Japan, 1969).
1970 Input-Output Tables (Administrative Management Agency, Government of Japan, 1974).
1975 Input-Output Tables (Administrative Management Agency, Government of Japan, 1978).

*Concerning rearrangement of input-output tables of the Republic of Korea, see Torii and Fukasaku [26].

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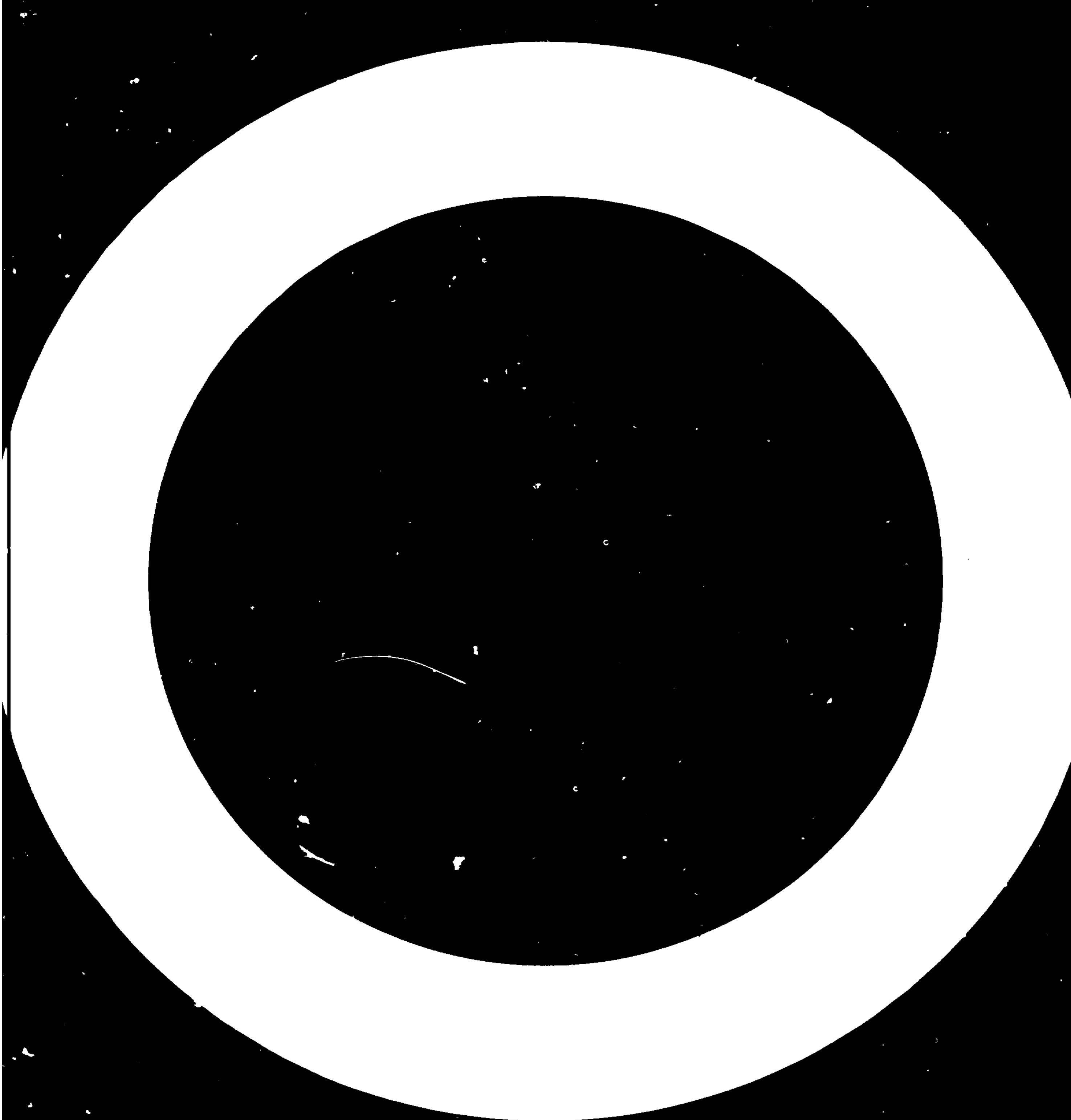

Sectoral prices and income distribution

Simultaneous determination of quantities and prices in a dynamic input-output model, demonstrated for the Federal Republic of Germany

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In I/O models of the conventional type either industrial quantities or industrial (shadow) prices are the variable to be determined. In contrast to standard economic theory, quantities do not depend on prices and prices do not depend on quantities in such models. In I/O research there are efforts to take into account dependence of quantities on prices or interdependence of both. In one type of approach variable input proportions, such that input coefficients and therefore input quantities depend on prices, are used. Prices are, in turn, either shadow prices or actual prices; they are taken as exogenously given (see, for example, de Boer [1], Sevaldson [2]). In a second type of approach, I/O models are part of larger econometric models having quantities and prices as endogenous variables (e. g. Almon and others [3], Preston [4], Krelle, Frerichs and Kübler [5]). In this case interdependence of both types of variables is achieved, via price-determined input coefficients and quantity-determined price setting. In most of the econometric models the I/O section is not fully integrated in the sense of a simultaneous determination of all variables. There are, instead, macro variables determined in a global model, which are then fed into the I/O model. Backward effects from I/O to macro variables are not fully taken into account.

The approach used in this paper is of the second variety described above. It is a fully integrated model. Output quantities delivered to final demand by an industry are not assumed to be exogenously given, but are explained by a consumption function, an investment function, and an export function. Prices are among the explanatory variables of these functions. The home price and the export price of every industry is, in turn, explained by a price function. According to the price functions cost-push and demand-pull elements determine the price setting behaviour of the industries. Prices would influence input proportions, if substitutional production functions were admitted and industrial cost minimization assumed. It was decided, however, to use Leontief linear limitational production functions and, therefore, price independent input coefficients. Interdependence of quantities and prices is achieved by the quantity functions and the price functions mentioned above.

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For the model's empirical implementation, a time series of 14 industry I/O tables for the Federal Republic of Germany compiled by DIW (Deutsches Institut für Wirtschaftsforschung, Berlin), is available. Up to now, the series of tables 1954 to 1967 has, together with complementary data, been used to estimate parameters of a version of the model adapted to the economy of the Federal Republic of Germany during that time period. A special characteristic of this version, supported by statistical tests, is that consumption, investment and export quantities react to price variables of the same year, whereas prices react to cost-push and demand-pull variables of the last year. In this version, the generally non-linear interdependent model is block recursive and consists of two blocks. The first block is itself recursive, the second is interdependent and linear in its variables.

Results of an *ex post* simulation of industrial quantities and prices are presented which demonstrate the workability of the empirically implemented model. Differences between simulated and observed values of variables are small in most cases, in particular for quantities. The model correctly simulates turning points in the development of quantities and prices.

The empirically implemented model is applied to test how quantity and price development in the Federal Republic of Germany depends on the development abroad for the period under consideration. One of the results contrary to those of more naive models is that higher growth of quantities abroad (possibly connected with higher growth of prices abroad and of import prices) will in the first periods generally raise growth rates of quantities, but may in the following periods set in motion a wage-price spiral and lead to reduced growth of, for example, investment due to the fact that cost push outweighs demand pull.

Outline of the model

The general model consists of definitional and behavioural equations represented in tables 1 and 2. Table 1 needs only short comments. (1) are the Leontief balance equations in which the output of industry (or sector) i used as the current input of industry j, (i, j = 1, . . ., n) is expressed by the Leontief production function $a_{ij}x_j$. Final demand components x_i^C, x_i^I, and x_i^E are endogenous. (4) indicates that, in contrast to labour income, the non-labour income originating in an industry is a residual that results by subtracting cost for current inputs, imports, labour, depreciation and indirect taxes from receipts on home and foreign markets. According to (5), (6) and (7), price indices for consumption, investment and exports are weighted by the last period's quantities, indicated by index -1, for computational reasons. In (8), (9) and (10) global foreign indices relevant for exported quantities and export price setting contain as weights the shares $X_a^E/\Sigma_a X_a^E$ of the main trading partners a = 1, 2, . . ., in national exports.

Table 2 requires more interpretation. In the brackets of the functions in column B variables are stated which are thought to exert a major influence on the variable on the left hand side of the equation. Column C indicates, by the sign of the partial derivative, the expected direction of a variable's influence. Column D gives alternative specifications of some of the variables in terms of magnitudes contained in the model. In all of the behavioural equations (11) to (15) explaining a quantity or a price, the influence of the same variable in the last period is taken into account in order to consider habits and continuity of behaviour. In the consumption functions (11) an income variable is introduced which may be specified as labour or labour plus non-labour income, either nominal or deflated by a consumption price index (indicating existence or non-existence of money illusion). Furthermore, an influence of price relations is admitted by p_i/P^C, in the negative direction. Investment functions (12) represent the whole economy's gross investment demand for an industry's output. An income variable influencing this kind of demand may be specified as nominal or real (deflated by investment price index) non-labour income. The symbol p_i/P^I stands for a negative influence of price relations. Instead of the income variable either total production X or change of total production $X - X_{-1}$ may be taken into account. If disaggregated data become available, it should prove better to use, in place of (12), functions representing investment demand of single industries allowing consideration of the industry specific capacity and profit situation and explanation of industrial capacity building using investment coefficients. In the export functions (13), foreign demand for an industry's products is dependent on the foreign industrial production index X^A and, in the negative direction, on the ratio of the sector's export price to the foreign cost of living or the wholesale price index.

A home price function (14) is meant to describe price setting behaviour of an industry i on home markets. Cost push influences on price setting are industrial unit cost of production k_i and industrial import price index p_i^M. Unit cost are specified as either the unit wage cost k_i^l or the unit wage plus current input cost k_i^{l+v}. Demand pull influences on price setting, indicated by dd_i, are specified either by the actual industrial production x_i and capacity utilization x_i^k or by the difference between potential and actual industrial output, $x_i^{pot} - x_i$.[1] In the first of these two cases the direction of demand-pull influence should be positive, in the second case it should be negative. There may be (e. g. computational) reasons to take the unit-cost and demand-pull variables of the last period (indicated by index -1). An export price function (15) explains an industry's price setting on its foreign markets. It is supposed to be influenced by the home price of the same sector, by the foreign industrial production index X^A, and by the foreign cost of living or wholesale price index P^{AC} or P^{AG}. If disaggregated data and more research results become available, it might be better to use variables pertaining to individual foreign countries and sectoral foreign markets instead of global indexes of production and prices.

The model contains neither coefficients determining primary input quantities nor functions explaining primary input prices. For labour the model uses wage bill

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[1]Capacity utilization (expressed as a percentage) and potential output are related by $x_i^{pot} = 100\, x_i/x_i^K$. With investment functions of the type (12), as used here, it is not possible to explain industrial capacity building endogenously. Information on industrial capacity utilization and potential output must, therefore, be taken from exogenous sources.

TABLE 1. DEFINITIONAL EQUATIONS OF THE MODEL[a]

Item	*Equation*
(a) Definitions of quantities and incomes	
(1) Sectoral output and its use, industry i = 1, ..., n	$x_i = \sum_j a_{ij} x_j + x_i^C + x_i^I + x_i^E + x_i^F$
(2) Total quantities	$X = \sum_i x_i;\ X^C = \sum_i x_i^C;\ X^I = \sum_i x_i^I;\ X^E = \sum_i x_i^E;\ X^F = \sum_i x_i^F$
(3) Total labour and non-labour income	$L = \sum_j l_j + l_s;\ Q = \sum_j q_j.$
(4) Non labour income from industry j = 1, ..., n	$q_j = (x_j - x_j^E) p_j/100 + x_j^E p_j^E/100 - \sum_i a_{ij} x_j p_i/100 - a_{mj} x_j p_j^M/100 - l_j - d_j - s_j$
(b) Definitions of price and quantity indexes	
(5) Consumption price index[b]	$P^C = \sum_i p_i x_{i,-1}^C / \sum_i x_{i,-1}.$
(6) Investment price index[b]	$P^I = \sum_i p_i x_{i,-1}^I / \sum_i x_{i,-1}^I.$
(7) Export price index[b]	$P^E = \sum_i p_i^E x_{i,-1}^E / \sum_i x_{i,-1}^E.$
(8) Foreign cost of living index	$P^{AC} = \sum_a P_a^{AC} X_a^E / \sum_a X_a^E.$
(9) Foreign wholesale price index	$P^{AG} = \sum_a P_a^{AG} X_a^E / \sum_a X_a^E.$
(10) Foreign net production index	$X^A = \sum_a X_a X_a^E / \sum_a X_a^E.$

[a]A list of all symbols is given in the annex.
[b]Price indices are weighed by last period's quantities as indicated by the index −1.

TABLE 2. BEHAVIOURAL EQUATIONS OF THE MODEL (INDUSTRY $i = 1, \ldots, n$)

A *Name of function*	*B* *General formulation*	*C* *Sign of partial derivative with respect to 1st, 2nd, ... explaining variable*	*D* *Alternative specifications of explaining variables*
(a) Functions explaining components of final demand			
(11) Consumption functions	$x_i^C = f(x_{i,-1}^C; Y^C; p_i/P^C)$	$f_1, f_2 > 0; f_3 < 0$	Y^C: L or $L + Q$ or L/P^C or $(L + Q)/P^C$
(12) Investment functions	$x_i^I = f(x_{i,-1}^I; Y^I; p_i/P^I; X; X - X_{-1})$	$f_1, f_2, f_4, f_5 > 0; f_3 < 0$	Y^I: Q or Q/P^I
(13) Export functions	$x_i^E = f(x_{i,-1}^E; X^A; p_i^E/P^A)$	$f_1, f_2 > 0; f_3 < 0$	P^A: P^{AC} or P^{AG}
(b) Functions explaining prices			
(14) Home price functions	$p_i = f(p_{i,-1}; k_i; dd_i; p_i^M)$	$f_1, f_2, f_4 > 0; f_3 \gtrless 0$	k_i: $k_i^l = (l_i/x_i)_r$ or $k^{l+v} = \{(l_i/x_i) + \sum_j a_{ji} p_j\}_r$ dd_i: $(x_i; x_i^K)_r$ or $(x_i^{pot} - x_i)_r$ r = zero (no time index) or -1
(15) Export price functions	$p_i^E = f(p_{i,-1}^E; p_i; X^A; P^A)$	$f_1, f_2, f_3, f_4 > 0$	P^A: P^{AC} or P^{AG}
(c) Functions explaining wage bill and taxation			
(16) Wage bill functions	$l_i = f(l_{i,-1}; P^C; x_i; t)$	$f_1, f_2, f_3 > 0; f_4 < 0$	
(17) Tax functions	$s_i = f(x_i p_i)$	$f_1 > 0$	

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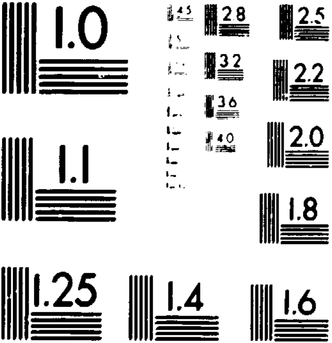

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functions (16) instead, to describe determination of an industry's wage bill or labour income. As in (11) to (15), the last period's value of that variable is supposed to influence this period's value. The consumption price index P^C and the industry's output x_i are included in the function, because they represent major arguments in wage negotiation. A time trend t takes care of technical progress, which may exert a negative influence on the wage bill. Non-labour income originating in an industry is, as mentioned above, a residual defined by (4). In tax functions (17), indirect taxes (i.e. taxes paid by an industry) are explained by industry's total receipts valued at home price, $x_i p_i$.

The model in the present state of research does not include monetary variables. There is no money-supply function, no money-demand function, no interest rate equating supply of and demand for money. There are no interest rate or real-balance effects considered in the behavioural equations.

Specification and empirical implementation of the model

In this chapter the model is specified and implemented for the economy of the Federal Republic of Germany on the basis of a time series of 14 industry I/O tables for 1954 to 1967 (see Krengel and others [6, 7]). Complementary data were capacity utilization indexes (see Görzig [8] supplemented by other calculations using Krengel and others [9]); indexes of industrial production, cost of living and wholesale prices of the most important trade partner countries of the Federal Republic of Germany (OECD [10, 11]) and percentages of these countries in total exports of the Federal Republic of Germany (cf. Statistisches Bundesamt [12]).

An I/O model treating quantities and prices as endogenous variables usually is a general non-linear system of interdependent variables. As a computer program for such a model was not readily available, it was necessary to start with a hypothesis giving the model a recursive structure favourable for computation and plausible for that time of development in the Federal Republic of Germany. Price variables in consumption, investment and export functions (11), (12), (13) are non-lagged, whereas cost-push and demand-pull variables k_i and dd_i in home price functions (14) are specified a priori in the $r = -1$ (i.e., the one period lagged) version. Together with the common practice of assuming linear behavioural functions and with the convention that weights of consumption, investment and export price indexes also are one period lagged variables (see (5), (6), (7) in table 1), this hypothesis ensures that the model is a block recursive system consisting of two blocks, the first of which itself is recursive and the second of which is interdependent and linear in its variables. The hypothesis says that consumption, investment and export production react more quickly on price changes than home prices react on demand-pull and cost-push variables. During the period in question, immigrating labour prevented bottle-necks and made possible a short-term reaction of production. The average price rise, on the other hand, still was moderate, such that quick and frequent sectoral price reactions were not typical. The hypothesis thus may be considered plausible for 1954 to 1967. It certainly has to be modified for the time period thereafter, when bottlenecks came up and inflationary pressure favoured immediate price reactions.

Two of the 14 industries, namely "13, public sector" and "14, private households and non-profit organizations", are to be treated in a separate manner. Inputs are included in exogenous final demand $x_1^F, \ldots, x_{12}^F$. Outputs, taken as a sum x_s, are an exogenous variable. Indexes i and j in tables 1 and 2 generally run from 1 to 12 only. Because industries 13 and 14 substantially contribute to labour income, there is, however, a wage bill function (16) for explanation of labour income l_s, and this is part of L in (3).

Tables 3, 4 and 5 give results of estimating coefficients of the behavioural functions. Estimation was carried out by ordinary least squares. Alternative specification of variables according to column D of table 2 led to a number of alternative formulations and estimates of every function, from which, according to the coefficient of determination R^2, the Durbin Watson statistic DW and the t-values, the one seeming most appropriate was chosen and entered in tables 3, 4 or 5. Most of the consumption functions (11) show the influence of the last period's consumption (habit persistence). All formulations indicate dependence of consumption on either labour or labour plus non-labour income; in the majority of cases it is real income that matters, indicating absence of money illusion. In eight of the 12 functions the price relation is relevant, in the (correct) negative direction. In the investment functions (12) one finds four cases of habit peristence, which may be due to replacement requirements. Real non-labour income or, alternatively, total output or change of total output, exert influence on the economy's investment demand for every industry's output. In seven of the 12 functions again the price relation turns out to be important. The export functions (13) show the influence of the last period's exports in five industries, of the price relation in seven industries, and of foreign industrial production in 11 industries. (In the indexes of foreign wholesale prices and of foreign industrial production, countries of the European Economic Community, Austria, Canada, Japan, Sweden, Switzerland and the United States are represented, their percentages in exports of the Federal Republic of Germany serving as weights.) For industries 2 (energy and mining) and 9 (construction) statistical results are bad. As these industries' exports are very small, this does not violate the workability of the model.

In the price functions of table 4 in only a few cases the last period's price is of importance. For home price functions (14) this is plausible, since, according to the hypothesis stated above, cost-push and demand-pull variables of the last period determine this period's price setting. In ten industries cost push, represented either by unit wage cost or by unit wage plus current input cost, shows significant influence on home prices. Demand pull was relevant for an equal number of industries, exerting its influence on price setting either in the positive direction by the last period's output and percentage of capacity utilization or in the negative direction by difference of potential and actual output. In six of the industries prices of imported inputs proved to be of some weight for the industry's home price. Export price functions (15) show that export price setting in all industries depends on the respective home price (in industry 9 (construction) it did not make sense to differentiate between home price and export price). In most of the industries, there was an influence of the foreign global index of price, represented by wholesale price index.

TABLE 3. COEFFICIENTS OF FUNCTIONS EXPLAINING COMPONENTS OF FINAL DEMAND

A. Consumption functions (11)

Dependent variable	Constant	Explanatory variables[a] $x^C_{i,t-1}$	L	L/P^C	$\frac{L+Q}{P^C}$	$100p_i/P^C$	Coefficient of determination, R^2	Durbin Watson value, W
x^C_1	9 481.406	0.204 (1.400)	0.00588 (5.593)			−5 351.964	0.978	2.002
x^C_2	4 821.363	0.494 (2.464)		1.005 (2.188)		−3 595.128 (−2.169)	0.981	2.223
x^C_3	12 409.047	0.433 (3.173)	0.02037 (1.882)			−11 080.171 (−2.277)	0.998	2.369
x^C_4	34.938	0.322 (1.094)		0.234 (2.242)			0.974	1.122
x^C_5	−4 593.755			7.403 (30.158)			0.988	1.169
x^C_6	8 127.092			4.855 (11.304)		−7 528.226 (−1.050)	0.994	0.977
x^C_7	40 784.271			10.539 (24.886)		−31 630.025 (−2.786)	0.994	1.171
x^C_8	43 427.406	0.455 (4.934)	0.04643 (4.823)			−28 582.078 (−2.583)	0.998	1.747
x^C_9	168.381	0.489 (2.938)	0.00060 (2.605)				0.963	1.584
x^C_{10}	951.429	0.278 (1.339)			12.980 (3.482)		0.996	1.116
x^C_{11}	9 725.926	0.557 (1.977)			1.015 (1.176)	−8 252.09 (−2.815)	0.984	2.037
x^C_{12}	29 368.607	0.736 (4.688)	0.09051 (2.937)			−33 556.133 (−1.648)	0.993	2.616

B. Investment functions (12)

Dependent variable	Constant	Explanatory variables[a]					R^2	DW
		$x^I_{i,-1}$	$\frac{Q}{P^I}$	$100p_i/P^I$	X	$X-X_{-1}$		
x^I_1	425.936		0.295 (3.045)	−428.076 (−2.018)			0.930	1.768
x^I_2	1 223.906		1.136 (4.713)	−1 124.963 (−2.493)			0.945	1.525
x^I_3	−550.304	0.357 (1.849)			0.00186 (3.346)		0.961	1.546
x^I_4	−255.932	0.600 (3.617)	0.938 (2.149)				0.986	1.010
x^I_5	58 452.144	0.541 (2.229)		−49 744.121 (−1.613)		0.0473 (2.395)	0.979	1.559
x^I_6	−1 945.549	0.565 (3.515)	5.501 (2.378)				0.974	1.204
x^I_7	−975.833	0.186 (1.847)			0.00511 (8.199)		0.997	1.512
x^I_8	68.563		0.052 (1.656)	−74.623 (−1.336)			0.908	1.875
x^I_9	4 584.749				0.03980 (5.093)		0.981	2.026
x^I_{10}	23 686.355			−18 861.289 (−39.140)		0.0048 (2.570)	0.994	2.029
x^I_{11}	3 389.336			−2 945.879 (−3.281)	0.00136 (3.323)		0.922	2.612
x^I_{12}	2 715.956			−4 121.569 (−4.223)	0.00462 (14.387)		0.981	2.007

TABLE 3 *(continued)*

C. Export functions (13)

Dependent variable	*Constant*	*Explanatory variables*[a] $x^E_{i,-1}$	$100p^E_i/p^{AG}$	X^A	R^2	*DW*
x^E_1	644.776		−8.451 (−5.058)	6.997 (9.877)	0.980	2.345
x^E_2	992.925	0.611 (2.477)			0.359	1.500
x^E_3	−4935.426	0.641 (3.323)		84.921 (2.386)	0.993	1.680
x^E_4	−548.944	0.672 (2.918)		27.015 (1.565)	0.935	1.729
x^E_5	64321.878		−661.950 (−2.519)	157.260 (2.951)	0.986	0.912
x^E_6	19428.592		−176.077 (−1.846)	60.798 (2.219)	0.961	1.156
x^E_7	−2171.775			55.941 (20.165)	0.974	0.855
x^E_8	899.174		−18.928 (−1.550)	22.661 (4.918)	0.968	1.281
x^E_9	307.464		−2.042 (−1.017)	1.373 (2.162)	0.375	2.075
x^E_{10}	−207.478	0.670 (3.569)		11.786 (2.196)	0.960	1.111
x^E_{11}	1720.869		−19.190 (−2.475)	35.471 (9.238)	0.972	0.919
x^E_{12}	8336.237	0.518 (3.850)	−95.849 (−2.384)	19.600 (2.812)	0.898	2.300

[a]See annex for definition.

TABLE 4. COEFFICIENTS OF FUNCTIONS EXPLAINING PRICES

A. Home price functions (14)

Dependent variable	Constant	Explanatory variables[a] $p_{i,-1}$	$k^I_{i,-1}$	$x^{I+V}_{i,-1}$	$x_{i,-1}$	$x^K_{i,-1}$	$(x_i^{pot} - x_i)_{-1}$	p_i^M	R^2	DW
p_1	−3.569			151.312 (9.129)			−0.002449 (−4.210)	0.419 (2.764)	0.909	1.808
p_2	−5.919	0.533 (2.627)		88.464 (1.828)			−0.001599 (−2.896)		0.935	1.927
p_3	46.725				0.0000501 (8.104)	0.15844 (2.352)		0.339 (6.245)	0.871	2.654
p_4	77.095				0.0003121 (4.719)			0.104 (1.579)	0.690	1.552
p_5	44.683		260.062 (7.235)				−0.000926 (−2.080)		0.930	0.575
p_6	12.974		188.478 (7.627)			0.411 (2.830)			0.880	1.440
p_7	−6.887		240.145 (13.309)				−0.000802 (−3.101)	0.554 (9.976)	0.971	2.374
p_8	18.573	0.556 (2.847)	122.783 (3.060)					0.153 (1.881)	0.963	2.351
p_9	15.272		255.920 (14.806)				−0.001011 (−1.427)		0.963	0.713
p_{10}	72.551		133.906 (18.263)				−0.000717 (−3.703)		0.972	1.221
p_{11}	4.515		109.625 (3.391)		0.000653 (9.629)			0.376 (2.460)	0.947	2.277
p_{12}	−21.653			222.346 (26.592)					0.985	1.308

TABLE 4 *(continued)*

B. Export price functions (15)

Dependent variable	Constant	Explanatory variables[a]: $P^E_{i,t-1}$	p_i	X^A	p^{AG}	R^2	DW
P^E_1	−89.503		0.868 (3.736)		0.984 (2.577)	0.739	1.791
P^E_2	−49.966		1.109 (6.839)		0.388 (1.773)	0.827	0.986
P^E_3	−27.286	0.756 (9.059)	0.516 (1.833)			0.941	1.470
P^E_4	−70.742		0.444 (0.973)	0.216 (3.766)	1.506 (5.538)	0.833	1.891
P^E_5	−12.529	0.501 (5.428)	0.477 (5.821)		1.157 (4.023)	0.997	1.450
P^E_6	18.691		0.692 (3.967)	0.113 (3.483)		0.988	1.162
P^E_7	35.969		0.410 (12.733)		0.232 (4.220)	0.961	2.300
P^E_8	−21.291		1.240 (10.530)			0.910	1.300
P^E_9			1.000				
P^E_{10}	30.271		0.610 (36.168)		0.087 (3.712)	0.993	2.317
P^E_{11}	−76.894		0.391 (4.170)		1.365 (5.524)	0.906	1.849
P^E_{12}	5.200		0.943 (123.386)			0.999	1.043

[a]See annex for definition.

TABLE 5. COEFFICIENTS OF FUNCTIONS EXPLAINING WAGE BILLS AND TAXATION

A. Wage bill functions (16)

Dependent variable	Constant	Explanatory variables[a] $l_{i,-1}$	x_i	p^C	t	R^2	DW
l_1	−5652.394	0.538 (3.763)		81.627 (4.175)	−134.384 (−4.151)	0.804	1.241
l_2	1447.602	0.396 (1.962)	0.095 (2.128)			0.911	1.816
l_3	−14443.140		0.142 (5.425)	172.339 (1.898)		0.996	1.045
l_4	−17257.001		0.226 (5.238)	171.433 (3.905)		0.975	1.218
l_5	−83867.009		0.181 (4.782)	941.468 (7.833)	−558.444 (−2.113)	0.997	2.042
l_6	−22349.589		0.223 (9.800)	247.234 (5.503)		0.997	1.821
l_7	−25443.192		0.221 (6.278)	257.181 (4.505)		0.994	1.600
l_8	−1395.054	0.6342 (2.774)	0.057 (1.633)			0.994	1.162
l_9	−43890.993		0.559 (5.280)	376.081 (2.364)	450.439 (−1.484)	0.993	1.652
l_{10}	−35221.306	0.411 (2.220)	0.072 (2.557)	402.736 (2.198)		0.998	1.849
l_{11}	−21509.633	0.316 (2.233)	0.088 (4.038)	274.472 (3.767)		0.998	2.606
l_{12}	−32814.877		0.133 (4.307)	346.770 (4.572)		0.998	1.889
l_g	−1633.581	0.792 (5.358)	0.162 (2.027)			0.996	1.870

TABLE 5 *(continued)*

B. Tax functions (17)				
Dependent variable	*Constant*	*Explanatory variable:* $x_i p_i$	R^2	*DW*
s_1	−17.853	0.0445 (9.378)	0.889	1.543
s_2	1023.195	0.0300 (4.776)	0.675	1.028
s_3	−2699.716	0.1510 (34.133)	0.991	1.025
s_4	31.548	0.0487 (10.279)	0.906	1.009
s_5	273.353	0.0379 (36.732)	0.992	1.476
s_6	170.505	0.0419 (31.107)	0.989	1.066
s_7	−218.653	0.0550 (43.705)	0.994	1.269
s_8	−1159.570	0.1697 (33.796)	0.991	0.879
s_9	−302.107	0.0684 (45.065)	0.995	1.087
s_{10}	−1103.084	0.1755 (35.006)	0.991	1.731
s_{11}	−175.116	0.0560 (14.921)	0.953	0.980
s_{12}	955.347	0.0603 (40.552)	0.993	1.136

[a]See annex for definition.

For the wage bill functions (16) of table 5, in about half of the cases the last period's wage bill is important. In all industries but industry 1 (agriculture, forestry and fishing) the volume of output turns out to be decisive for the wage bill paid. For most of the industries the consumption price index played its role in wage bill determination. A negative influence of a time trend due to rising labour productivity is discernible only for industries 1, 5 (constructional steel, machinery, vehicles), and 9 (construction). For the construction industry, the negative influence is not statistically significant. Results for tax functions (17) need no comment.

With the behavioural functions specified according to tables 3, 4 and 5 for the Federal Republic of Germany from 1954 to 1967, one may now evaluate the block recursive structure of the model. Let y_t be the vector of endogenous variables in period t; and z_t the vector of exogenous variables in period t, arranged in the following way (omitting time index t):

$$y_t = \begin{pmatrix} p \\ p^E \\ x^E \\ l_s \\ x \\ x^C \\ x^I \\ l \\ q \end{pmatrix}, \qquad z_t = \begin{pmatrix} x^F \\ v \\ x^K \\ x^{pot} \\ x_s \\ p^M \\ p^{AG} \\ X^A \\ a \end{pmatrix} \tag{18}$$

The model may be put in the form

$$y_t = f(y_t, y_{t-1}, z_t, z_{t-1}) \tag{19}$$

Inspection of tables 3, 4 and 5 reveals that y_t can be partitioned into a vector y_t^1 of recursive and a vector y_t^2 of interdependent variables,

$$y_t^1 = \begin{pmatrix} p \\ p^E \\ x^E \\ l_s \end{pmatrix}, \qquad y_t^2 = \begin{pmatrix} x \\ x^C \\ x^I \\ l \\ q \end{pmatrix} \tag{20}$$

such that:

$$\begin{aligned} y_t^1 &= f^1(y_t^1, y_{t-1}, z_t, z_{t-1}) \\ y_t^2 &= f^2(y_t^1, y_t^2, y_{t-1}, z_t) \end{aligned} \tag{21}$$

The two blocks of the structure come out more clearly in the matrix diagram below, where a star means that the variable in the head column depends on the variable in the head row (the time index t is not omitted here). The first block is itself recursive, the second is interdependent. When solving (19) for y_t, one can, therefore, in a first step solve successively for the recursive variables of y_t^1, and in a second step solve simultaneously the linear relations for y_t^2.

	p_t	p_t^E	x_t^E	$l_{s,t}$	x_t	x_t^C	x_t^I	l_t	q_t	y_{t-1}	z_t	z_{t-1}
p_t										*	*	*
p_t^E	*									*	*	
x_t^E		*								*	*	
$l_{s,t}$										*	*	
x_t			*		*	*	*				*	
x_t^C	*			*				*	*	*		
x_t^I	*			*				*	*	*		
l_t					*					*	*	
q_t	*	*	*		*						*	

Ex post simulation of industrial outputs and prices

In order to demonstrate the general workability of our approach, an *ex post* simulation of industrial outputs and prices for the data period was carried out. In the first step projected endogenous variables, y_{1955}^{pr}, were computed by inserting into the model non-lagged and lagged observed exogenous variables z_{1955} and z_{1954} and lagged observed endogenous variables y_{1954}. In the second step we computed y_{1956}^{pr} by inserting observed z_{1956}, z_{1955} and projected y_{1955}^{pr}, and so on. Thus exogenous variables (including coefficients) are taken, step by step, with their observed values. For endogenous variables observed values are used only in the first step (lagged endogenous variables y_{1954}); in all further steps projected values of endogenous variables are inserted.

Use of observed values of exogenous variables in all steps of the computation is possible only in *ex post* simulations. It prevents deviations of projected from observed values of endogenous variables which are due, in *ex ante* simulations, solely to errors in projections of exogenous variables. All deviations in an *ex post* simulation of the type discussed here are, therefore, attributable to imperfections of the model and its statistical estimation (neglecting errors of observation).

Development of every industry's projected output and price is represented in figure 1 (dotted lines). Different scales are used on the vertical axes to bring out characteristics more clearly. For comparative reasons, the development of observed values of variables also is given (solid lines). What frequently is considered to have been a period of a smooth overall development and of only mild growth of prices, comes out as a period of noteworthy divergences in the

Figure 1. Output and price index for the years 1955–1967

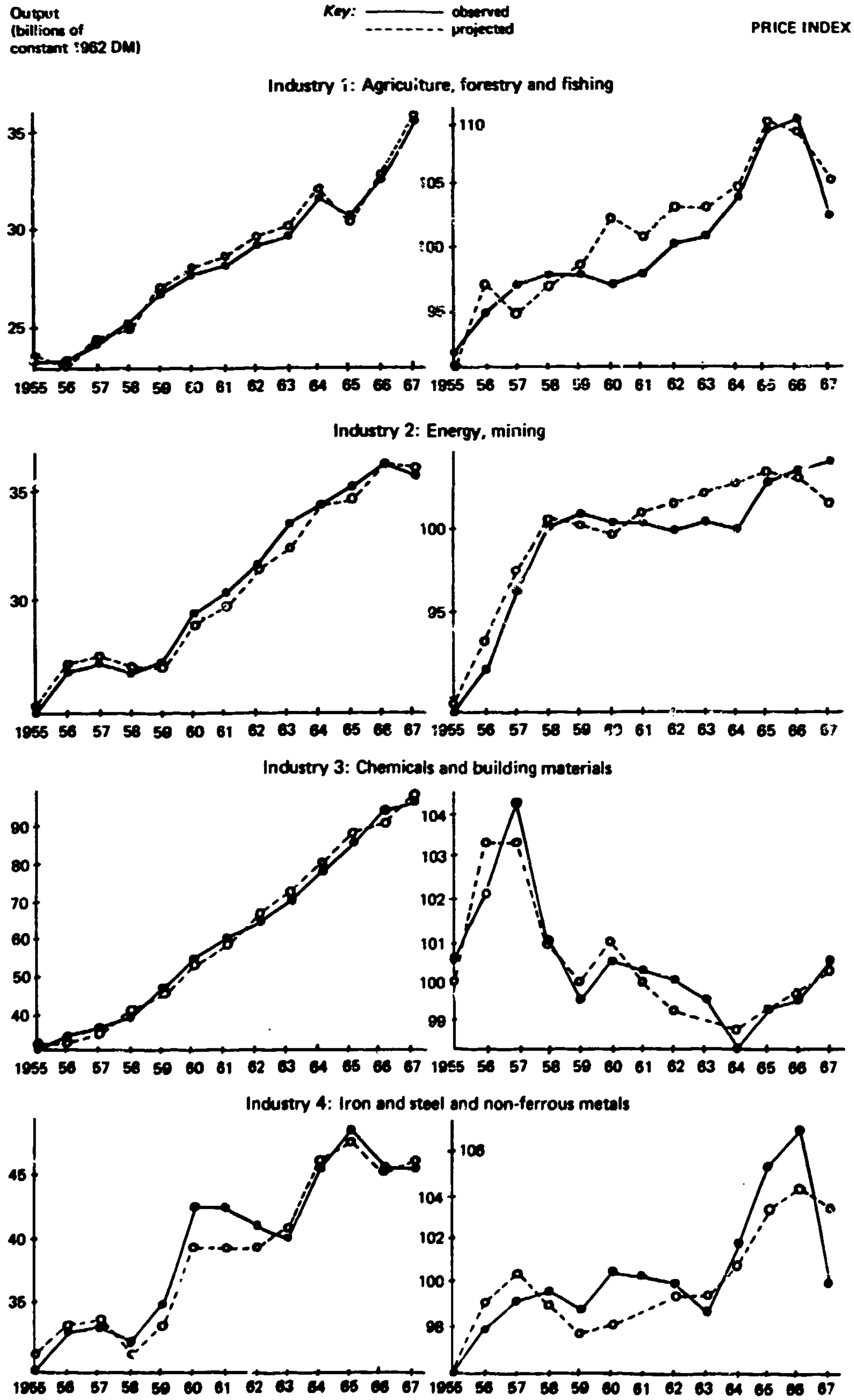


Figure 1 *(continued)*

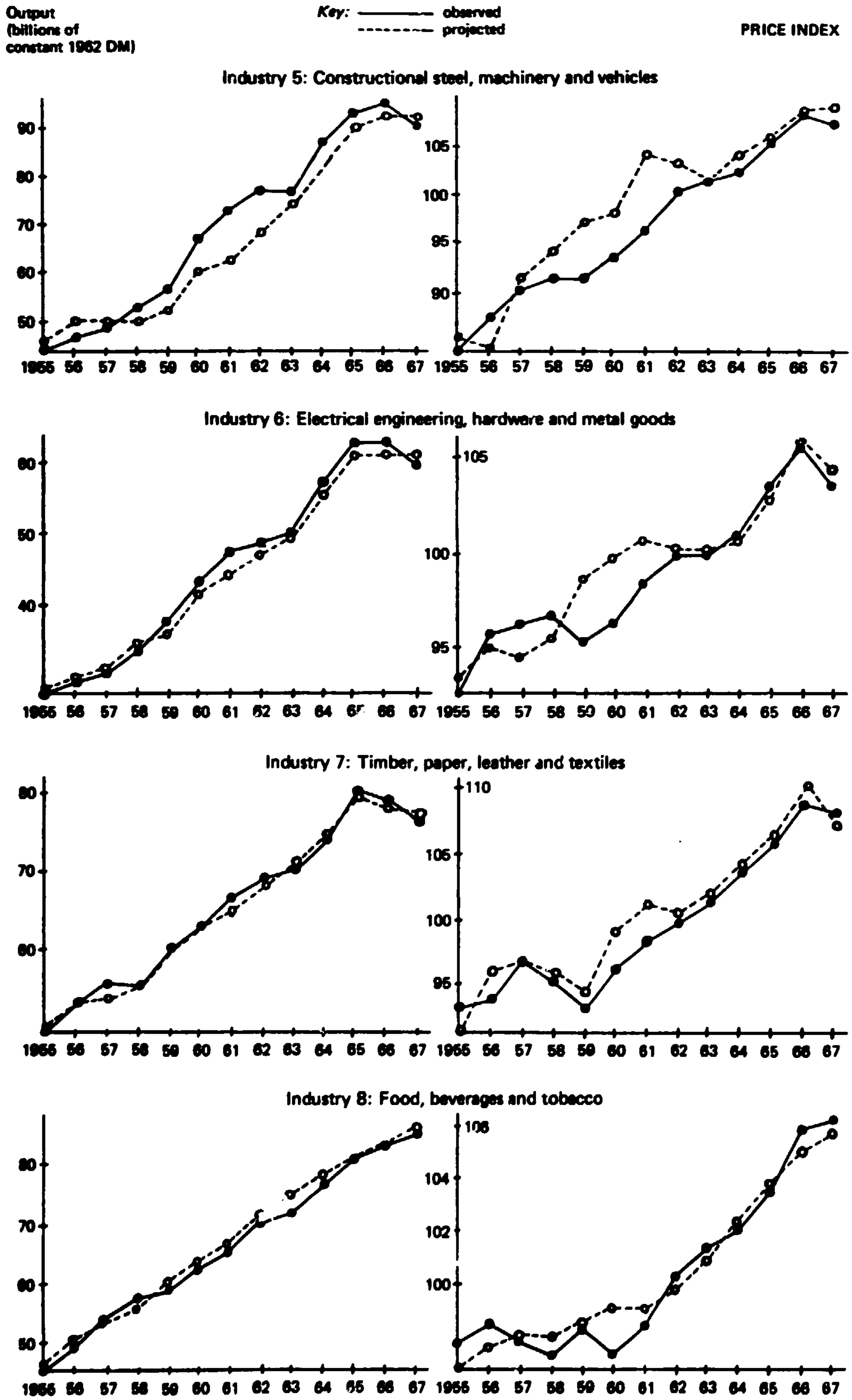
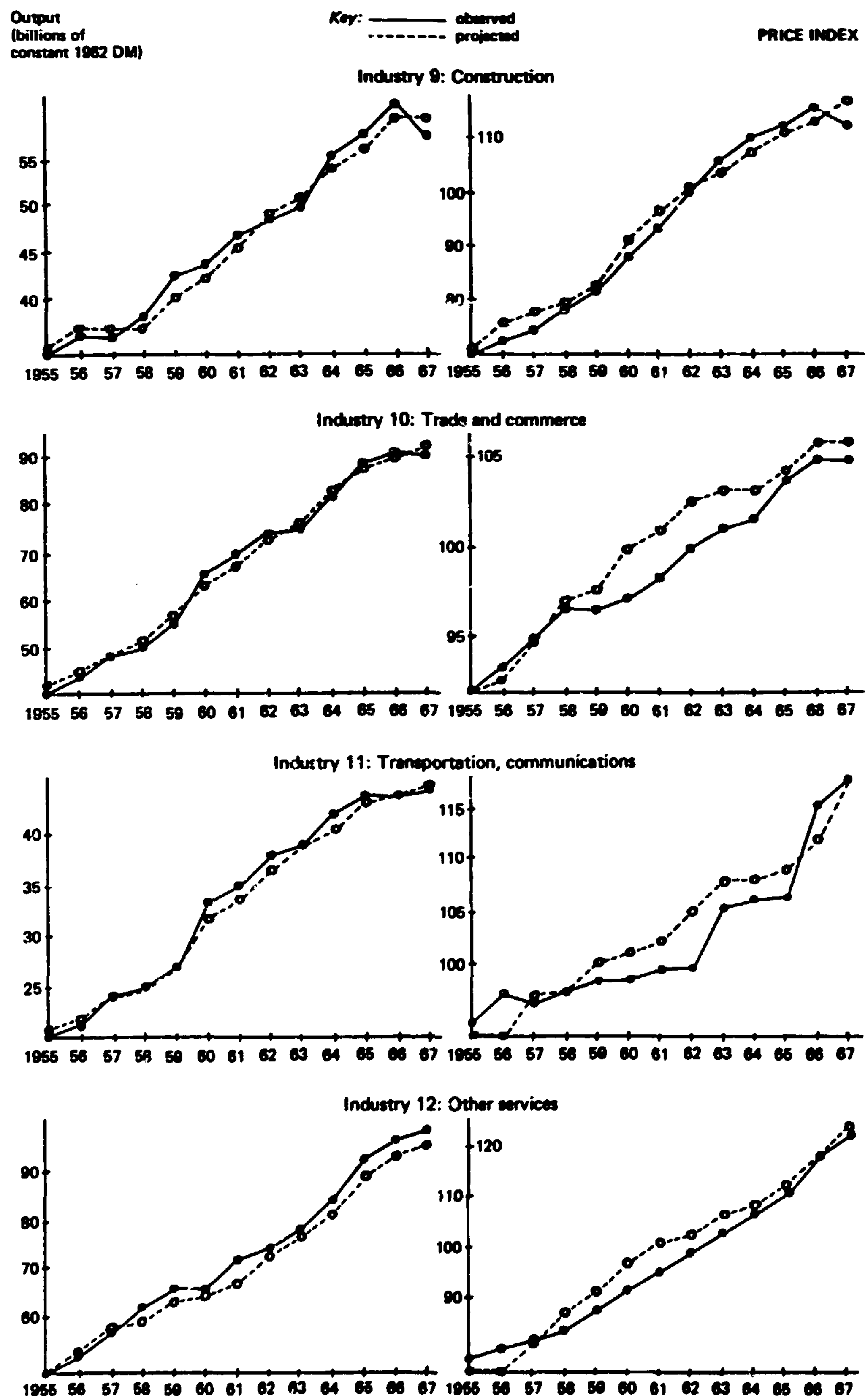


Figure 1 *(continued)*



development of industries' outputs and of prices. Comparison of projected with observed development of industrial output shows that there is some deviation for industries 4 (iron and steel, non-ferrous metals) and 5 (constructional steel, machinery, vehicles) for 1960 to 1963, but that turning points of industry 4's cyclical development and most industries' recessive development towards the end of the period are well simulated by the model. As industrial divergences in price development are more severe than those in output, it is not too surprising that the model's performance is less good for prices. Price variation on the average seems to be understated by projected price variation. The model is, however, able to simulate such opposite industrial tendencies as price decrease in some industries and price increase in others during 1957 to 1960 and 1966 to 1967. Another example of good workability is the model's reproduction of downward cyclical price development in industry 3 (chemicals, building materials) and of upward cyclical price development in industry 4 (iron and steel, non-ferrous metals).

It would be possible to simulate development of other endogenous variables of the model, for example of global variables and indexes resulting from aggregation according to equations (1) to (10) of table 1.

Though some characteristics of the model in its present specification and implementation, such as investment functions and the hypothesis of lagged price reaction as well as the ordinary least squares estimation procedure, are preliminary, the general workability of our type of approach should have become evident.

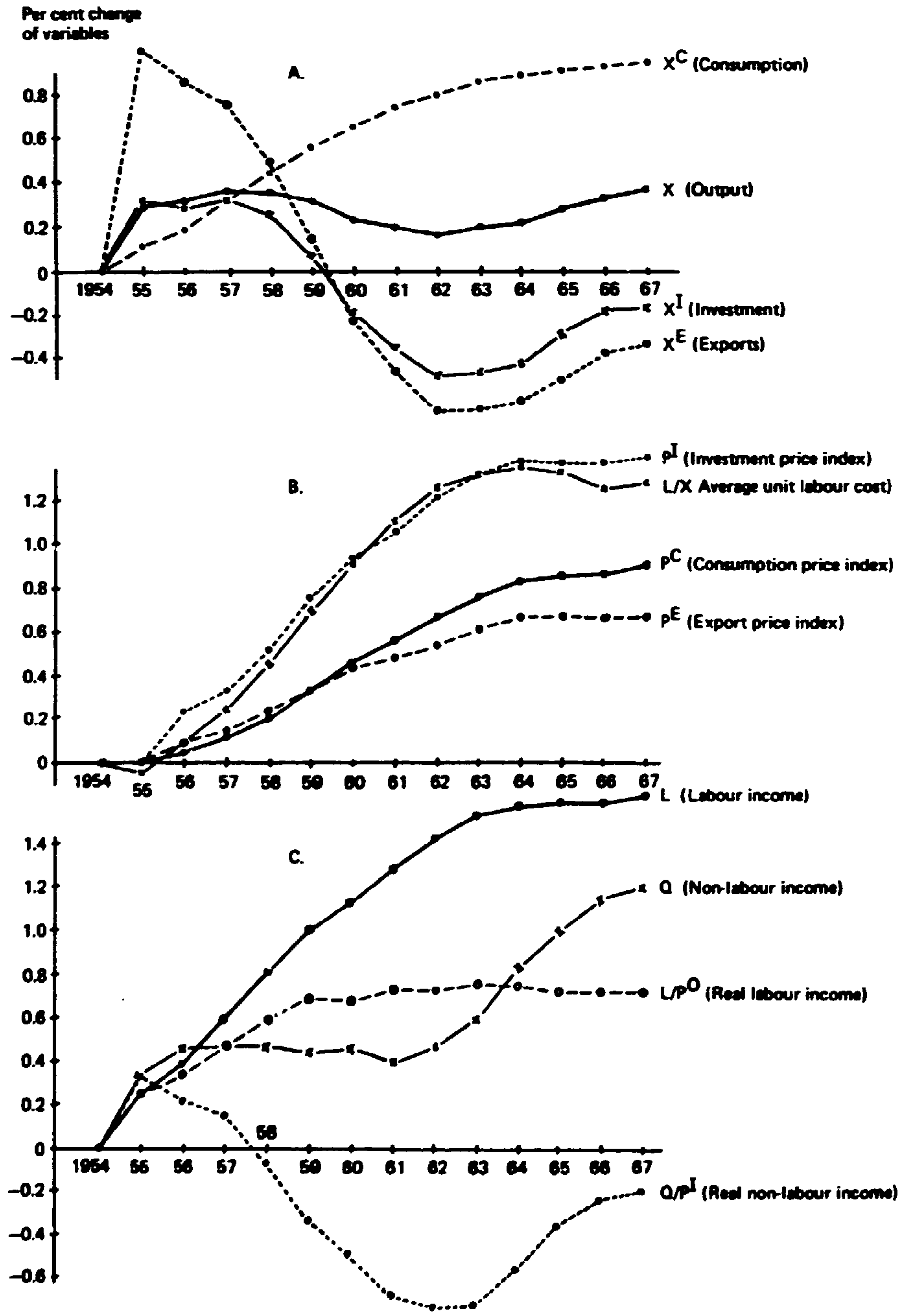
Dependence of global and sectoral variables on foreign economic development

The problem now to be studied by a simulation technique is that of how enhanced growth of aggregate foreign production would have influenced global and industrial variables of the economy of the Federal Republic of Germany during the period of investigation. This is a problem of dynamic-multiplier theory, relating changes of endogenous variables over time to changes of exogenous variables over time. It is assumed that the exogenous variable X^A is, from 1955 to 1967, one per cent higher than actual. What change does this induce in, for example, total consumption, total investment, total exports, total wage bill, total non-labour income or in the respective variables of an industry? The change of a variable is expressed in per cent of its projected value in the simulation. Results give information on the dependence of global and industrial variables on foreign economic development.

In figure 2, results are given for global variables. On the vertical axis percentage change of variables is measured. Because this simulation clearly shows the importance of taking into account interdependence of all industries' quantity and price variables and because some of the results of the simulation might not have been expected at first sight, the process of transmission of the positive impulse from abroad will be traced in some detail in the following.

Aggregate foreign production X^A is one of the explanatory variables in 11 of the 12 export functions (13) and in two of the 12 export price functions (15) (see

Figure 2. Dependence of global variables of the Federal Republic of Germany on foreign production development

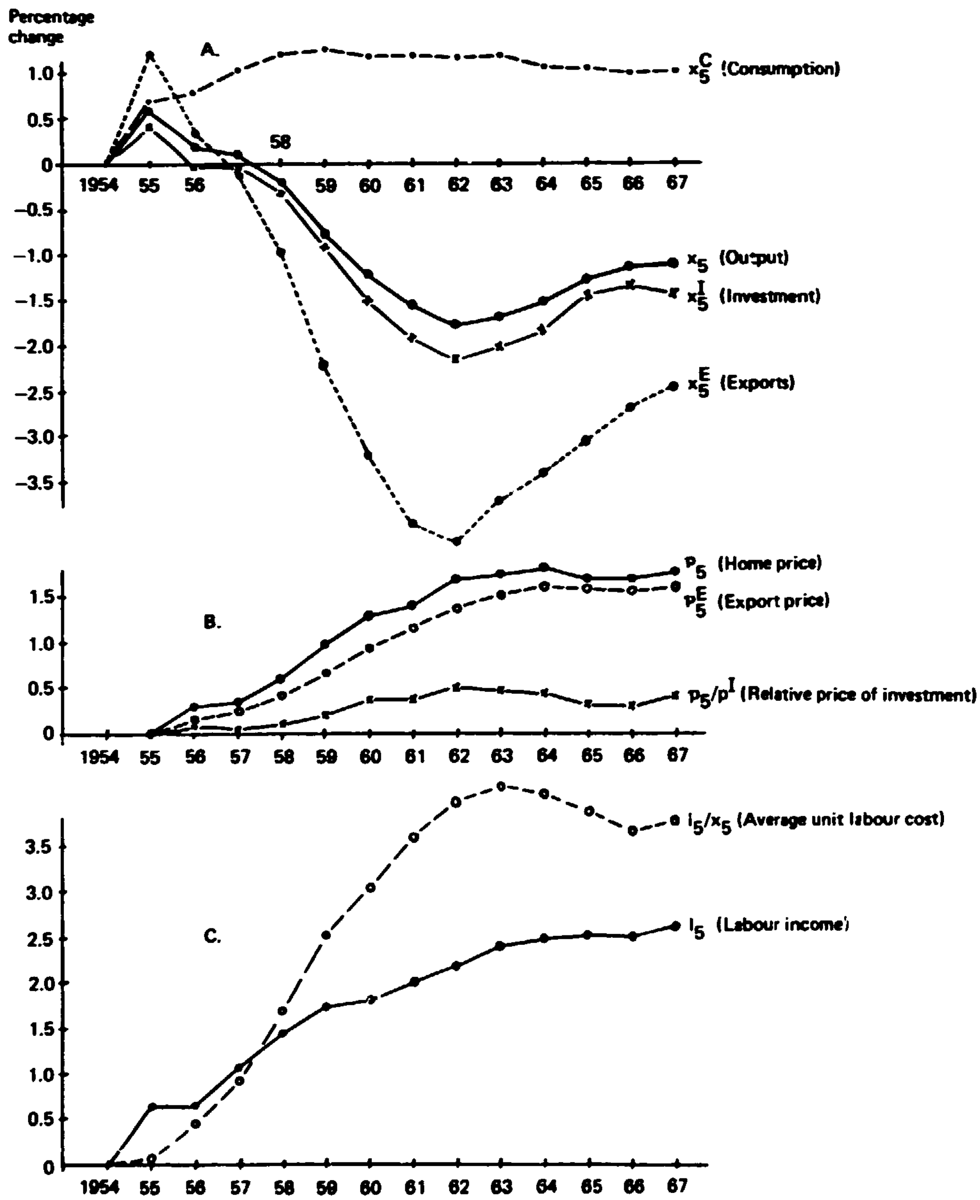


tables 3 and 4). A 1 per cent rise of X^A, beginning 1954, at first exerts its main influence, therefore, on exports of the Federal Republic of Germany. Figure 2 A shows a 1 per cent increase of total exports X^E in the first of the simulation periods, reflecting an average of partly higher and partly lower export increases in the different industries. This in the same period induces higher output X, higher labour and non-labour income L and Q (see figure 2 C) all of about 0.3 per cent. Because of lagged reaction prices do not yet rise, such that real incomes L/P^C and Q/P^I go up. This results in higher investment and in (due to habit persistence, only slightly) higher consumption. In the second period price reactions begin. Global indexes P^C, P^I and P^E (see figure 2 B) rise, due partly to industrial price changes, partly to changes in the composition of the respective final demand component serving as a weight for index construction. Industrial price change mainly is attributable to demand pull resulting from first period's output growth. Cost push in the second period still is negligible, since there is only a minor change of average unit labour cost L/X.

In the following periods the situation drastically alters. The influences of rising P^C and of last period's labour income on wage bill determination suffice to expand labour income more than output, such that L/X rises substantially. This major cost push component causes home prices to rise. In this way, a wage price spiral is set in motion, exerting unfavourable effects on exports and on real non-labour income. Rising prices p_i induce rising prices p_i^E. This turns the development of percentage change of exports, X^E, in the negative direction, in spite of 1 per cent higher growth of foreign production. The price rise, P^I, is larger than the rise of Q. The development of percentage change of real non-labour income Q/P^I therefore also changes in the negative direction. Up to the fifth and sixth period, respectively, the percentage changes of exports and non-labour income remain positive, but then become negative. Real non-labour income being an important determinant of investment demand for 5 of the 12 industries, the change of X^I also becomes negative in the seventh period. Due to the permanent rise of L and L/P^C, the change of consumption demand X^C goes on in the positive direction. This does not prevent a temporary change of total production in the negative direction; the change of X remains, however, positive. The development of percentage change of X^I, X^C and X in figure 2 A indicates that investment is partly crowded out by consumption.

Having demonstrated the effects of a 1 per cent rise of foreign production on global variables, industrial variables may be examined. Industry 5 (constructional steel, machinery, vehicles) is focused on because, compared to other industries, exports of this industry are higher, both absolutely and in relation to industry's output, and because this industry contributes substantially to the wage price spiral. Figure 3 A gives percentage changes of variables of industry 5 corresponding immediately to global variables of figure 2 A. Figure 3 B shows percentage changes of home price and export price and of the relation of the price of investment goods from industry 5 to the price index of all goods devoted to investment. Figure 3 C presents percentage change of labour income and of unit labour cost in industry 5. The type of development of the industrial variables in general is similar to that of corresponding global variables. Highly relevant is the

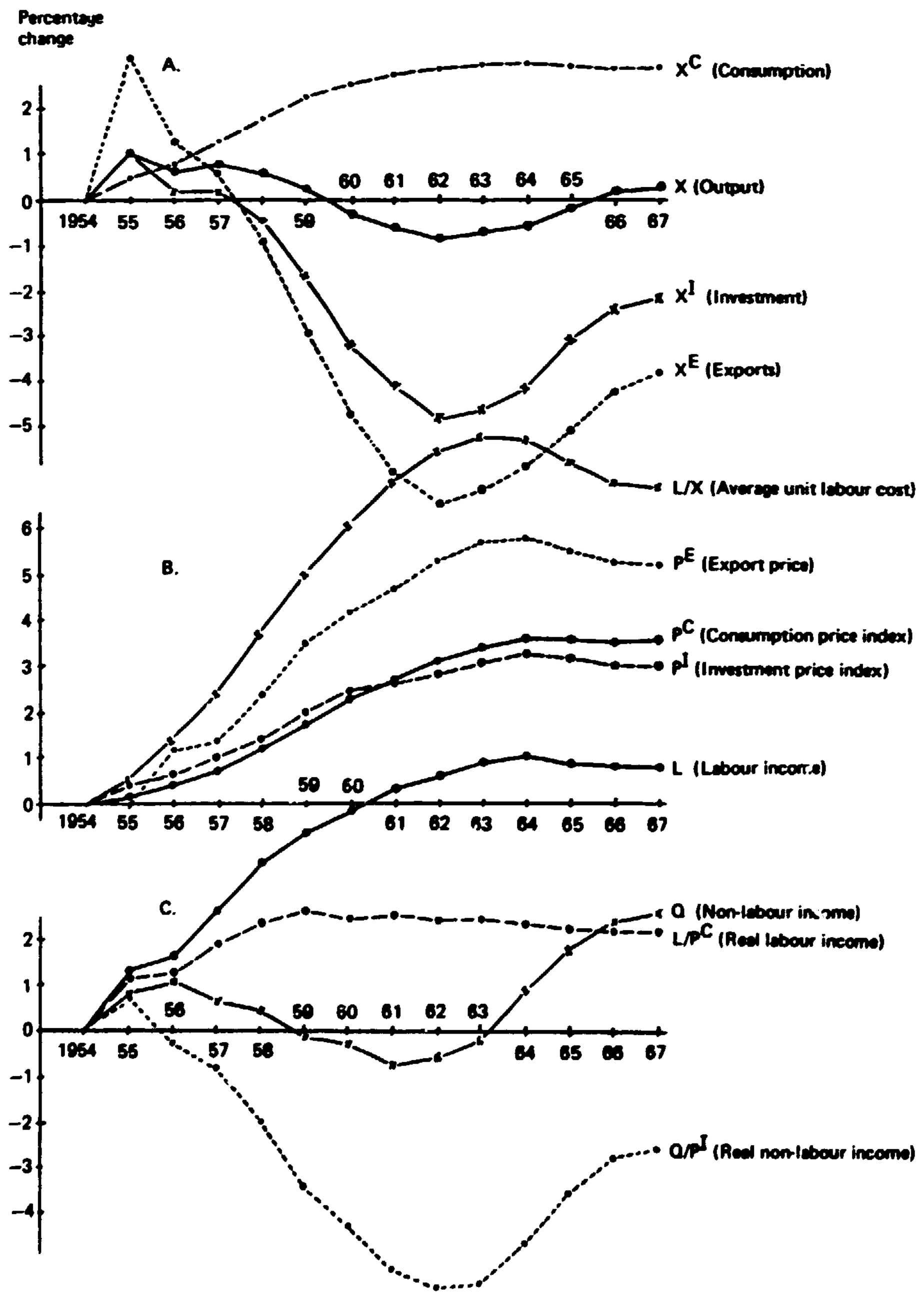
Figure 3. Dependence of industrial variables of the Federal Republic of Germany (industry 5: constructional steel, machinery, vehicles) on foreign production development



fact that reactions of this industry's variables come earlier and are stronger than those of global variables. Percentage change of exports shifts into the negative direction for industry 5 in the third period, for total exports in the sixth period. Percentage change of investment demand begins to be negative for products of industry 5 in the second period, that of total investment demand in the sixth period. Maximal distances from the time axis of curves for industrial variables l_5/x_5, x_5^I and x_5^E are three to seven times higher than maximal distances of curves for corresponding global variables. Early and sharp reactions of industry 5 have two main causes. The first cause is that in its wage bill function the price index, P^C, and in its home price function, the unit wage costs play a decisive role (see tables 4 and 5). The second cause is that a rising home price leads to rising export price. Both causes quickly and substantially reduce investment and export demand for products of industry 5.

Figure 4 presents results of a further simulation of global variables. In this simulation it is assumed that a 1 per cent higher growth of aggregate foreign production is accompanied by a 1 per cent higher rise of aggregate foreign price,

Figure 4. Dependence of Federal Republic of Germany's global variables on foreign production and price development



P^{AG}. Because of the sudden price advantage, exports, X^E, in the first period rise much more strongly than in the previous simulation. Demand pull causes rise of prices. This initiates a growth of labour income substantially higher than growth of output. The cost push component, L/X, strongly comes into action. The wage price spiral and its negative effects on exports, real non-labour income and investment are of greater dimension than in the previous investigation. Since consumption is changed in the positive direction, there again is crowding out of investment by consumption.

Results of simulations of dependence on foreign economic development may be summarized preliminarily by the observation that the higher growth of foreign production and prices have a spectrum of different effects. Due to the rules of wage negotiation and price formation embodied in the wage bill functions and the price functions, a wage price spiral will come into action so that initial demand pull is outweighed by cost push, and negative effects on exports and, most importantly, on investment are the consequence. Positive change of consumption and negative change of investment imply a crowding out effect, which in the long run reduces the growth potential of the economy. There are industries, such as industry 5, that can be identified as playing a leading role in the transmission process.

One can mention some feasible amendments of the model and the expected direction of their influence on results. In the first periods of simulation exports expand more than imports. A positive change of the trade balance may, in a system of fixed exchange rates, induce an additional inflow of foreign currency and, therefore, an expansion of the quantity of money. This may reduce interest rates and stimulate investment or, via real balance effects, expand spending in general. These positive influences may, however, be countervailed by the wage price spiral.

A rise in the unit labour cost will induce substitution of labour by capital, with existing techniques, and initiate technical progress of the labour saving type. Such processes will be connected with downward shifts of wage bill functions of the type identified already by the negative time trend in wage bill functions for industries 1, 5 and 9 (see table 5). This may contribute to a dampening of the wage price spiral. A policy of restricting substitution and labour-saving progress clearly would not alleviate the negative effects of the wage price spiral.

There may be repercussions on foreign countries. The rise of production and of prices may induce processes abroad similar to those projected in the simulations. Such processes again may be transmitted back to the economy of the Federal Republic of Germany.

It is, of course, impossible to estimate *a priori* the full effects of monetary expansion, of substitution and technical progress, and of foreign repercussions.

Summary

An I/O model is enlarged in such a way that an econometric model disaggregated to the industry level results. Every industry's output quantities delivered to final demand are explained by a consumption function, an investment

function, and an export function. In these functions, the respective quantities are determined, *inter alia*, by price variables. The home price and the export price of every industry are, in turn, explained by price functions. According to the price functions, cost-push and demand-pull elements determine the price setting behaviour of the industry. Leontief input coefficients are applied for current inputs, but there is no explicit production function for primary inputs. Labour income generation of an industry is explained by a wage bill function. Industrial non-labour income generation is a residual.

The model is implemented for 1954 to 1967 on the basis of a time series of 14 industry I/O tables for the Federal Republic of Germany. Assuming that final demand components react on prices of the same year, and that prices react on cost push and demand pull of last year, the generally non-linear interdependent model is a block recursive model consisting of two blocks, the first itself being recursive, the second being interdependent and linear in its variables.

An *ex post* simulation of industrial production and prices is sketched to demonstrate the workability of the approach.

As an example to what type of problems this model can be applied, effects of increased growth of foreign production and prices on domestic global and industrial variables are simulated. The results obtained are not derivable by more conventional models, but seem to be characteristic for present day economies. Wage and price formation, as reflected in wage bill functions and price functions, may set into motion a wage price spiral such that initial demand pull is outweighed by cost push. By this process positive impulses are transformed into negative effects on exports and on investment. Consumption crowding out investment reduces the growth potential of the economy.

ANNEX

LIST OF SYMBOLS AND INDUSTRIAL CODES

Scalar	*Column vector*	*Meaning*
		Endogenous variables
x_i	x	Industrial output
x_i^C	x^C	Industrial consumption
x_i^I	x^I	Industrial (gross)investment
x_i^E	x^E	Industrial export
p_i	p	Industrial home price
p_i^E	p^E	Industrial export price
l_i	l	Industrial labour income (wage bill)
q_i	q	Industrial non-labour income (profit)
s_i	s	Industrial (indirect) taxes
k_i		Industrial unit cost
k_i	k^l	Industrial unit wage cost

Scalar	*Column vector*	*Meaning*
k_i^{l+v}	k^{l+v}	Industrial unit wage plus current input cost
l_s		Labour income from industries 13 and 14
X		Total output
X^C		Total consumption
X^I		Total (gross) investment
X^E		Total exports
L		Total labour income (wage bill)
Q		Total non-labour income (profit)
P^C		Consumption price index
P^I		Investment price index
P^E		Export price index
Y^C		Income relevant for consumption
Y^I		Incomr relevant for investment
dd_i		Industrial demand pull
		Exogenous variables and coefficients
x_i^F	x^F	Industrial exogenous final demand
x_i^K	x^K	Industrial capacity utilization in per cent
x_i^{pot}	x^{pot}	Industrial potential output
d_i	d	Industrial depreciation minus public subventions
p_i^M	p^M	Industrial import price
a_{ij}		Input coefficient
a_{mj}		Import coefficient
	a	Input coefficients and import coefficients (length of vector: $(n+1)n$; a consists of $a_{ij}(i,j=1,\ldots,n)$ and $a_{mj}=(j=1,\ldots,n)$
x_s		Output of industries 13 and 14
X_a		Net industrial production index of country a
X^A		Foreign industrial production index
X_a^E		Exports to country a
P^A		Foreign price index
P_a^{AC}		Cost of living index of country a
P^{AC}		Foreign cost of living index
P_a^{AG}		Wholesale price index of country a
P^{AG}		Foreign wholesale price index

Industries[a]

1. Agriculture, forestry, fishing
2. Energy, mining
3. Chemicals, building materials

[a]For the precise demarcation of industries see [8], p. 7.

4. Iron and steel, non-ferrous metals
5. Constructional steel, machinery, vehicles
6. Electrical engineering, hardware and metal goods
7. Timber, paper, leather, textiles
8. Food, beverages and tobacco
9. Construction
10. Trade and commerce
11. Transportation, communications
12. Other services
13. Public sector
14. Private households and non-profit organizations

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Income distribution and the structure of production in an input-output framework

*A. Ghosh and A. Sengupta**

Income distribution is usually treated as an extraneous component rather than as an integral part of the economic system which interacts with other components. The tendency has been to treat income distribution in terms of two highly aggregated macro-economic categories, wage earners and profit earners. Today such a highly simplistic treatment is inadequate, and suitable expansion of the classical Leontief model gives a methodology for a system which integrates income distribution with the structure of production. This expansion of the Leontief matrix elucidates interaction processes which have so far been treated as irrelevant, but which by their sheer magnitude and their direct impact change the course of economic processes.

Macro-economic considerations

To facilitate the subsequent discussion of the models, a section on the different types of elaboration of the national accounts on which these models are based follows. The discussion of this accounting frame may be initiated with the Keynesian framework which considers a single producing sector and a single consuming sector.[1] Kaldor [2] however first directly examined the Keynesian model and elaborated it in developing a theory of income distribution and growth. The Keynesian methodology of Kaldor was further elaborated by Pasinetti [3], who considered a single production sector, but two final consumption sectors, "workers" and "capitalists". This methodology has been extended to include two production sectors and two consumption sectors. The Leontief model dealing with many production sectors but only one final sector is extended to a model involving many production sectors and many final sectors. The taxonomy of these accounting systems makes it easier to appreciate the implications of these extensions with reference to the mechanics of the overall system itself.

Figure 1 shows the arrangements of the national accounts in the Keynesian income generation process.

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[1]Kalecki [1] discussed the relationship between income distribution and the market structure in a paper as early as 1939. His main theme however was the degree of monopoly and its effect on income distribution between wages and profits.

Figure 1. Arrangements of national accounts in the Keynesian model

Producing sector \ Consuming sector	Production	Final: Consumption	Final: Investment
Production	x^i_{pp}	x^c_{pf}	x^k_{pf}
Final (primary)	x^i_{fp}		

The Keynesian balance equation cancels out the intermediate flows x^i_{pp} and is reduced to the following:

$$\text{National income (Y)} = x^c_{pf} + x^k_{pf}$$
$$= x^i_{fp}$$

where x^c_{pf} and x^k_{pf} are final deliveries to consumption and investment from production, x^i_{fp} is the delivery from final (primary) sector in the form of value added to production. Income distribution among different classes was not considered as being very relevant to deciding the level of total employment or output. The interaction between production sectors, or between final sectors was just cancelled out by aggregation.

The extension of Kaldor and Pasinetti

Kaldor extended the simple Keynesian model by considering the final sector as consisting of two functional categories, the "workers" and the "capitalists". Pasinetti also uses this division, but the primary sector is a mixed sector with both capitalists and workers included, as shown in figure 2.

Figure 2. Arrangements of national accounts in the model of Pasinetti

Producing sector \ Consuming sector		Production	Final: Workers (1): Consumption	Final: Workers (1): Investment	Final: Capitalists (2): Consumption	Final: Capitalists (2): Investment
Production		x^i_{pp}	$x^c_{pf(1)}$	$x^k_{pf(1)}$	$x^c_{pf(2)}$	$x^k_{pf(2)}$
Primary	Workers (1)	$x^i_{f(1)p}$				
	Capitalists (2)	$x^i_{f(2)p}(c) +$ $x^i_{f(2)p}(w)$				

Ignoring again the intermediate output x^i_{pp} one gets,

$$Y = x^c_{pf(1)} + x^k_{pf(1)} + x^c_{pf(2)} + x^k_{pf(2)}$$
$$= x^i_{f(1)p} + x^i_{f(2)p}(c) + x^i_{f(2)p}(w)$$

In his later work Pasinetti extended this to a multi-sector tableau making sectoral expansion of "final demand" and "value added" by eliminating intermediate demand. Profit was treated as a residual. Pasinetti obtained the condition of static and dynamic equilibrium under this elaboration but as far as the role of income distribution is concerned the treatment was incomplete.

The Walras-Leontief tableau

The standard Leontief flow matrix has many production sectors but only one final sector (figure 3). This gives rise to the following balance relations:

$$x_i = \sum_j x_{jj} + x_{p(i)f}$$

$$Y\,(\text{value added}) = \sum_{p(i)} x_{p(i)f} \qquad i,j = 1,2\ldots n$$

$$= \sum_{f(j)} x_{f(j)}$$

It is evident from these treatments that income distribution, as an active variable, does not play a part in these models. It was not felt that elaboration of income distribution processes was essential to the problems of economic equilibrium.

In a complex economy, however, it is equally important to elaborate the distribution of income, not simply as a description of the nature of social welfare, but in its own right as an important component having an impact on the working of the production system. To do this, an extension of the Leontief matrix with many production sectors and many final sectors must be considered, as in figure 4.

This elaboration leads to the following balance equation:

$$_{p}x_i = \sum_j {}_{pp}x_{ij} + \sum_j {}_{pf}x_{ij}$$

$$_{f}x_i = \sum_j {}_{fp}x_{ij} + \sum_j {}_{ff}x_{ij} \qquad i,j = 1,2\ldots n$$

where:

$_{pp}x_{ij}$ = delivery from production sector to other production sector;
$_{pf}x_{ij}$ = delivery from production sector to final sector;
$_{fp}x_{ij}$ = summary input into production;
$_{ff}x_{ij}$ = delivery from final sector to the end sectors.

The balance relation of the above type from the cost aspect is given under equilibrium conditions by:

$$_{p}x_j = \sum_i {}_{pp}x_{ij} + \sum_i {}_{fp}x_{ij}$$

$$_{f}x_j = \sum_i {}_{pf}x_{ij} + \sum_i {}_{ff}x_{ij}$$

Figure 3. Arrangements of national accounts in the Walras-Leontief model

	i \ j	Production sector 1	2	....	n	Final sector
Production sector	1	x_{11}	x_{12}	....	x_{1n}	$x_{p(1)f}$
	2	x_{21}	x_{22}	....	x_{2n}	$x_{p(2)f}$
	.	.	.	....	.	.
	.	.	.	....	.	.
	.	.	.	....	.	.
	n	x_{n1}	x_{n2}	....	x_{nn}	$x_{p(n)f}$
Value added		$x_{f(1)}$	$x_{f(2)}$	....	$x_{f(n)}$	

Figure 4. Arrangements of national accounts in an extension of the Leontief matrix

Consuming sector \ Producing sector	Production	Final
Production	$_{pp}x_{ll}$ .... $_{pp}x_{ln}$ $_{pp}x_{nl}$ .... $_{pp}x_{nn}$	$_{pf}x_{ll}$ .... $_{pf}x_{ln}$ $_{pf}x_{nl}$ .... $_{pf}x_{nn}$
Final	$_{fp}x_{nll}$ .... $_{fp}x_{ln}$ $_{fp}x_{nl}$ .... $_{fp}x_{nn}$	$_{ff}x_{ll}$ .... $_{ff}x_{ln}$ $_{ff}x_{nl}$ .... $_{ff}x_{nn}$

Skolka and Paukert [4] discussed the problem of income distribution in several sectors but in their case the final sector was kept exogenous and naturally therefore the integration of production, consumption and income distribution within the model was not in their scheme.

Analytical results derived from the models

Analytical results derived from the models based on the arrangements of national accounts presented in the previous section with reference to the pattern of income distribution, are discussed below.

Two sector extension of Kaldor's models

The Kaldor relationship is:

$$Y = W + P$$
$$I = S$$
$$S = S_w + S_p$$
$$= s_w W + s_p P$$

where:

Y = total values added consisting of wages (W) and profits (P);
I = investment;
S = savings (worker's saving plus capitalist's savings);
s_w, s_p = constant propensities to save of the two groups, workers and profit earners.

The following relationships then hold:

$$\frac{I}{Y} = (s_p - s_w)\frac{P}{Y} + s_w$$

$$\frac{P}{Y} = \frac{1}{s_p - s_w} \cdot \frac{I}{Y} - \frac{s_w}{s_p - s_w}$$

There is thus a relationship established between the share (profit) of the capitalist and the rate of investment in the economy. The Kaldor approach ignores the effect of a sectoral distribution of I, the total investment, on the distribution of income by taking total investment rather than investment in different sectors.

Assuming that there are just two production sectors in the economy, it follows that:

$$W = W_1 + W_2$$

$$P = P_1 + P_2$$

We assume that the wages and profits respond according to the Leontief rule:

$$W = a_{w(1)}x_1 + a_{w(2)}x_2$$

$$P = a_{p(1)}x_1 + a_{p(2)}x_2$$

We also assume the I/O rules for a two sector aggregation, with I_1 the investment (autonomous) supply from the first sector and I_2 the investment supply from the second sector and with consumption related to income proportionately as in the consumption function of the Keynesian type. The marginal propensities to consume the products of a sector are β_1 and β_2, respectively. Then:

$$x_1 = x_{11} + x_{12} + I_1 + C_1$$

$$x_2 = x_{21} + x_{22} + I_2 + C_2$$

$$Y = (a_{w(1)} + a_{p(1)})x_1 + (a_{w(2)} + a_{p(2)})x_2$$

$$= \alpha_1(I_1 + C_1) + \alpha_2(I_2 + C_2)$$

$$= \alpha_1 I_1 + \alpha_1\beta_1 Y + \alpha_2 \cdot I_2 + \alpha_2\beta_2 Y$$

or,

$$Y = (1 - \alpha_1\beta_1 - \alpha_2\beta_2)^{-1}(\alpha_1 I_1 + \alpha_2 I_2)$$

where α_1 and α_2 are the corresponding multipliers relating the outputs to final demands such that:

$$\frac{P}{Y} = \frac{1}{(s_p - s_w)\beta^*} \cdot \frac{I_{1/I_2} + 1}{\left(\alpha_1 \cdot \dfrac{I_1}{I_2} + \alpha_2\right)} - \frac{s_w}{s_p - s_w}$$

where $\beta^* = (1 - \alpha_1 \beta_1 - \alpha_2 \beta_2)^{-1}$ and C_1 and C_2 are eliminated as functions of Y.

It may be concluded from the two-sector extension that sectoral composition of the investment also affects the determination of the share of wages and profits, given the size of total investment, since α_1 and α_2 are functions of all the coefficients of the system, a conclusion totally submerged by the process of aggregation.

Extension of the Pasinetti model

Pasinetti considered the possibility of workers also saving and contributing to capital formation in an elaboration of the model on the basis of the tableau discussed earlier.

In this formulation we have:

$$\frac{P_c}{Y} = \frac{1}{s_c - s_w} \cdot \frac{I}{Y} - \frac{s_w}{s_c - s_w}$$

$$\frac{P_c}{K} = \frac{1}{s_c - s_w} \cdot \frac{I}{Y} - \frac{s_w}{s_c - s_w} \cdot \frac{Y}{K}$$

On the lines of our earlier approach, by including intermediate production we have:

$$\frac{P_c}{K} = \frac{I}{s_c - s_w} \cdot \left[\left(1 - \alpha_1 \cdot s_w \beta^* \frac{I_1}{K}\right) + \left(1 - \alpha_2 \cdot s_w \beta^* \frac{I_2}{K}\right)\right]$$

This emphasizes the effect of the sectoral composition of investment on P_c/K, which is blurred by the Pasinetti approach.

Pasinetti further considered a multi-sector extension of this model as follows, eliminating, however, intermediate sectors, as in figure 5.

Figure 5. Arrangement of national accounts in a multi-sector extension of the Pasinetti model

		Production sector		Household sector	Total
		1	2		
Production sector	1	—	—	$a_{11}x_1$	$= x_1$
	2	—	—	$a_{21}x_1$	$= x_2$
Value added	3	$a_{31} \cdot x_1 + a_{32}x_2$		X	$= V$

In the Pasinetti model, it should be noted that the labour coefficients a_{31} and a_{32} are not the same as in the Leontief model since they include both direct and indirect labour used in the product.[2]

This enables discussion of the economy in terms of the labour used directly and indirectly in each sector. Pasinetti considered only one final sector and, therefore, the distribution aspect and its interactions did not enter the matrix but are outside the model.

The condition of equilibrium of the model is given by:

$$\begin{vmatrix} -1 & 0 & a_{1f} \\ 0 & -1 & a_{2f} \\ a_{31} & a_{32} & -1 \end{vmatrix} = 0$$

The solution is given by:

$$X_1 = a_{1f} \cdot \bar{X}$$

$$X_2 = a_{2f} \cdot \bar{X}$$

where $\bar{X}$ is the numeraire.

The corresponding dual in prices with $\bar{P}$ as the numeraire is given by:

$$P_1 = a_{31} \cdot \bar{P}$$

$$P_2 = a_{32} \cdot \bar{P}$$

It may be noted that by omitting the income classes here, the important interaction between income distribution and sectoral composition was missed.

Generalized versions and extensions of models

A generalized version of the Leontief model

From the foregoing it is clear that all the previous models have tried to focus attention on either sectoral interactions of intermediate outputs or final outputs, or have cancelled the interactions out of the economic system. None of these approaches goes into the problem of distribution except in the broadest way. The inter-relationship of sectoral relations and structure of income classes is not given any significance. This point of view may be accepted in an economy where income earning groups are fairly homogeneous, and can be designated as workers or

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[2]Generally the coefficients $a'_{n,i}$ of Pasinetti are given by:

$$\begin{pmatrix} a'_{n,1} \\ \vdots \\ a'_{n,n-1} \end{pmatrix} = (I - A_{n-1})^{-1} \begin{pmatrix} a_{n,1} \\ \vdots \\ a_{n,n-1} \end{pmatrix}$$

where the $a'n,i$ are the direct coefficients and $(I - A_{n-1})^{-1}$ is the Leontief inverse.

capitalists, and the behaviour of the group reasonably described by one or two broad classifications. In that case, the question of interaction between demands of different income classes and production sectors can be ignored.

But modern complex economies present a different picture. The income earning groups are no longer divisible into two homogeneous groups with similar behaviour patterns and an overall theory has to be developed relating to structure of production and structure of income distribution together.

It seems that the I/O format may be conveniently used for a more generalized approach embracing distribution aspects among social groups and growth or equilibrium aspects between economic sectors.

The traditional Leontief approach concentrates on intermediate sectors leaving the final demands aggregated. But, on the face of it, there is no specific reason why final output (value-added) and final consumption may not be distributed into homogeneous groups and coefficients defined for them as in the Leontief matrix, with the consumption matrices as functions of value added, that is, income generated by each group.

The I/O approach can then be used for a simultaneous study of sectoral distribution of output and also of income in one consistent system.

Assume as a simple case that the production is carried out in two sectors and there are three income classes, the workers, salary earners and profit earners. Then the balance equations may be described as follows:

$$\begin{pmatrix} x_{11} + x_{12} \\ x_{21} \quad x_{22} \end{pmatrix} + \begin{pmatrix} x_{1w} + x_{1s} + x_{1p} \\ x_{2w} \quad x_{2s} \quad x_{2p} \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\begin{pmatrix} x_{w1} + x_{w2} \\ x_{s1} + x_{s2} \\ x_{p1} + x_{p2} \end{pmatrix} + \begin{pmatrix} x_{ww} + x_{ws} + x_{wp} \\ x_{sw} + x_{ss} + x_{sp} \\ x_{pw} + x_{ps} + x_{pp} \end{pmatrix} = \begin{pmatrix} x_w \\ x_s \\ x_p \end{pmatrix}$$

We now define several sets of coefficients:

$$a_{ij} = \frac{x_{ij}}{x_j},\ a_{wj} = \frac{x_{wj}}{x_j},\ a_{sj} = \frac{x_{sj}}{x_j},\ a_{pj} = \frac{x_{pj}}{x_j}$$

$$\text{and} \quad a_{iw} = \frac{x_{iw}}{x_w}$$

$$a_{is} = \frac{x_{is}}{x_s}$$

$$a_{ip} = \frac{x_{ip}}{x_p}$$

These are the coefficients relating input to production (a_{ij}); coefficients of value-added by different earning groups to production (a_{wj}); and the coefficients relating consumption to income of different earning groups (a_{iw}, a_{is}, a_{ip}).

The last two sets of coefficients are not traditionally used in the Leontief model but if one assumes short-period stability within homogeneous groups with respect to their consumption habits and a similar stability of the ratio of value-added to output, the coefficient matrix is considerably enlarged. One then has in partitioned form, the coefficient matrix:

$$\left(\begin{array}{c|c} A_{pp} & A_{ph} \\ \hline A_{hp} & A_{hh} \end{array}\right)$$

where:

A_{pp} = the sub-matrix of coefficients of the intermediate flows;
A_{ph} = the sub-matrix of coefficients of the consumption to income by sectors;
A_{hp} = the matrix of coefficients relating value added to production;
A_{hh} = the sub-matrix of coefficients for flows from household to household.

The equations of the Leontief model with only the household sector considered exogenous are:

$$(A_{pp})\,x_p + a_{ph} \cdot x_h = x_p$$
$$a'_{hp} \cdot x_p + 0 = x_h$$

where

(A_{pp}) = a matrix;
a_{ph} = a column vector;
a'_{hp} = a row vector.

An elaboration of the above with three endogenous household sectors gives the following equations:

$$(A_{pp})_{2\times 2}\,x_p + (A_{ph})_{2\times 3}\,x_p = x_p$$
$$(A_{hp})_{3\times 2}\,x_h + (A_{hh})_{3\times 3}\,x_h = x_h$$

where all the bracketed expressions are matrices.

Introducing the structure of income into the model one may now examine the interaction missed by aggregation. Consider the matrix in partitioned form given earlier. The inverse (of the given partitioned matrix) may be written as:

$$\begin{pmatrix} (A_{pp} - A_{ph}A_{hh}^{-1}A_{hp})^{-1} & -A_{pp}^{-1}A_{ph}(-A_{hp}A_{pp}^{-1}A_{ph} + A_{hh})^{-1} \\ -A_{hh}^{-1}A_{hp}(A_{pp} - A_{ph}A_{hh}^{-1}A_{hp})^{-1} & (-A_{hp}A_{pp}^{-1}A_{ph} + A_{hh})^{-1} \end{pmatrix}$$

The inverse demonstrates how all the sub-matrices enter into the interaction process in the generation of output and income. A numerical assessment of each of these may reveal the interaction terms we miss by aggregating out various portions of the matrix.

An extension of the Pasinetti model

The following sets of equations give the Pasinetti model elaborated in three household sectors. Here the system is only a semi-closed system insofar as investments and exports (denoted by f) are considered exogenous:

$$0 + (a_{ph})x_p + f_p = x_p$$

$$(a_{hp})x_h + 0 + f_h = x_h$$

The analogue of the Leontief inverse for the Pasinetti matrix as elaborated for different income earning groups is then given by the expression:

$$\begin{pmatrix} (I - a_{ph}\cdot a_{hp})^{-1} & + a_{ph}(I - a_{ph}\cdot a_{hp})^{-1} \\ + a_{hp}(I - a_{ph}a_{hp})^{-1} & (I - a_{ph}a_{hp})^{-1} \end{pmatrix}$$

It may be seen that this Pasinetti inverse misses the interactions of the matrices, A_{ph} and A_{hp} and considers only the interactions of the vectors formed by aggregating A_{ph} and A_{hp}. The quantitative effect of these missing interactions may be measured from a computation of the three inverses, the Leontief inverse, the Pasinetti inverse and the present inverse, which has been done in subsequent sections.

It is interesting to note however, that the extensions of the Pasinetti model by considering three household sectors give considerably more information, as now vectors a_p' and a_h are replaced by matrices, so that we have:

$$\begin{pmatrix} (I - A_{ph}\cdot A_{hh}^{-1}\cdot A_{hp})^{-1} & A_{ph}(-A_{hp}\cdot A_{ph} + A_{hh})^{-1} \\ + A_{hh}^{-1}\cdot A_{hp}(I - A_{ph}\cdot A_{hh}^{-1}\cdot A_{hp})^{-1} & (-A_{hp}\cdot A_{ph} + A_{hh})^{-1} \end{pmatrix}$$

The form retains all the interactions A_{ph}, A_{hp}, A_{hh} and misses only the inter-industrial effects of A_{pp}.

The extended Leontief model: a demonstration

In this section, the extended Leontief model is developed by including several classes of income-earners as endogenous sectors.

It is assumed that in the economy there are n processing industries and m household sectors. The coefficient matrix of all economic transactions in partitioned form is given as the expanded I/O matrix:

$$B = \left[\begin{array}{c|c} A_{pp} & A_{ph} \\ \hline A_{hp} & 0 \end{array}\right]$$

The income and consumption of the household sectors are treated endogenously.

$$a_{ij} = \frac{x_{ij}}{x_j},$$

where

x_{ij} = flow of input from the i^{th} sector to the j^{th} sector;
x_j = output in the j^{th} sector;
A_{pp} = (A_{ij}), the production coefficient matrix.

$$_{hp}a_{kj} = \frac{W_{kj}}{x_j},$$

where

W_{kj} = income earned by the k^{th} household sector in the j^{th} processing sector;
A_{hp} = $(_{hp}a_{kj})$, the income coefficient matrix.

$$_{ph}a_{ik} = \frac{C_{ik}}{W_k},$$

where

C_{ik} = consumption of the output of the i^{th} sector by the k^{th} household sector;
W_k = income of the k^{th} household sector (in all processing sectors);
A_{ph} = (C_{ik}), the consumption coefficient submatrix;
0 = (0), a null matrix expressing the absence of any I/O link within the household sector.

Income generated to the k^{th} household sector in the processing of direct and indirect output requirements of unit exogenous demand in the i^{th} sector may be expressed by the elements of the inverse of $(I - A)$. The formula for the "simple" multiplier of the j^{th} processing sector with respect to the k^{th} household sector may be written as $\Sigma_i a_{ij}^{-1} v_{ki}$ where a_{ij}^{-1} refers to the inverse of the matrix $(I - A)$ and v_{ki} denominates the primary input of the k^{th} household in the j^{th} sector $(= {}_{np}a_{ki})$.

Let $(I - B)^{-1} = (\alpha^{ij})$ then the elements of the last m rows of the $(I - B)^{-1}$ matrix give the income flows to the m household sectors in the processing of direct and indirect output requirement and also the output consumption due to unit exogenous demand in various processing sectors. $\alpha^{j,n+k}$ gives the income flow to the k^{th} processing sector in the processing of direct and indirect output requirement and also the output requirement due to the feedback effect of induced consumption.

So the "total" multiplier for the j^{th} processing sector with respect to the k^{th} household sector may be expressed as the income generated in the direct and indirect output requirement and also the output requirement resulting from the feedback effect of induced consumption stimulated by unit exogenous demand in the j^{th} sector.

Sectoral multipliers for the Indian economy

The expanded matrix for the Indian economy was developed in 19 production sectors and 3 household sectors. The $(I - A)^{-1}$ and the $(I - B)^{-1}$

matrices have been obtained. The values of sectoral simple and total multipliers were computed utilizing these matrices. The values of the multipliers or selected sectors are shown in table 1 for workers and profit earners. In generating this series of outputs, actual estimates of the investment, government consumption, export and stock changes for the period were used.

The derived results reveal that income induced by household consumption for the household group (workers) and the household group (profit earners) is more than four or six times the original income generated by direct and indirect output requirements. The consumption multipliers may appear too large to be true. An investigation of the underlying causes reveals that the high values of the consumption multipliers for the household sector workers and the household sector profit earners are not unrealistic for the Indian economy. For the Indian economy, if household consumption is treated as a separate category of final demand, it becomes the largest component of final demand. Again, if household consumption is treated as an intermediate sector, household consumption, as a part of an intermediate input requirement to meet any category of exogenous demand, is much larger than the total values of other input requirements. Income generated in the processing of direct and indirect output stimulated by exogenous demand, causes a demand for household consumption, which again stimulates further output requirement and consequent generation of income. A substantial portion of income generation in the processing of direct and indirect output requirements is spent on consumption. Also, in a country such as India, direct income coefficients in several sectors (especially agriculture) are sufficiently high in comparison to the input coefficients. Thus the multiplier effect generated in the feedback process of induced consumption is much larger than the multiplier effect generated simply in the processing of direct and indirect output requirement. This is particularly true for the agriculture sector. In the processing of output in the agriculture sector the direct and indirect intermediate input requirement is small. Hence, almost the entire multiplier effect operates through consumption requirements.

Slightly more than half of national income in India is generated in the agriculture sector. Obviously, the income multiplier operating in this sector determines the basic pattern of the income multiplier process operating in the Indian economy.

Comparison of results using various models

A comparison of the results using the Leontief model with one household sector, the Pasinetti model with one household sector and the Pasinetti model with three household sectors shows that in income generation with the same final demand, both the Leontief and the Pasinetti model with one household sector treated as exogenous miss a large segment of the generation of income due to interaction.

The expanded version of the Leontief matrix shows the largest volume of actual income generation, and even the Leontief model with one endogenous

TABLE 1. MULTIPLIER VALUES FOR INCOME

Sector	*Direct income per unit of exogenous demand in the processing sector*		*Income generated in the processing of direct and indirect output requirement (simple multiplier)*		*Income generated in the processing of direct and indirect output requirement and the output requirement caused by feedback effect of induced consumption (total multiplier)*	
	Worker	*Profit earner*	*Worker*	*Profit earner*	*Worker*	*Profit earner*
Construction	0.1210	0.2343	0.2598	0.4519	1.6717	1.2372
Electrical equipment	0.0960	0.2177	0.1858	0.3327	1.2836	0.9553
Transport equipment	0.0899	0.1384	0.2135	0.2839	1.4471	0.9806
Non-electrical equipment	0.1200	0.1901	0.2518	0.3611	1.6671	1.1615
Iron and steel	0.1022	0.2607	0.1230	0.4463	1.5544	1.2307
Other metal and non-metal	0.0797	0.1238	0.1634	0.2507	1.0240	0.7379
Cement	0.0652	0.2121	0.2438	0.2318	1.8719	1.1118
Leather products	0.0822	0.1380	0.3643	0.3362	1.8719	1.1876
Food	0.0379	0.1750	0.4497	0.2248	2.0242	1.2597
Textile	0.1867	0.0860	0.5193	0.2654	2.1441	1.1738
Agriculture	0.6050	0.2636	0.6745	0.2915	2.4180	1.2671
Fertilizer	0.0860	0.1240	0.2248	0.2891	1.5080	1.0111
Chemicals	0.0575	0.2400	0.1614	0.3576	1.2731	1.0145
Wood and glasswares	0.1616	0.1930	0.2387	0.2635	1.3486	0.8898
Fuel	0.0430	0.4170	0.1070	0.5629	2.8630	1.2314
Transport	0.2623	0.1904	0.2968	0.2463	1.7136	1.0980
Rubber products	0.0617	0.3489	0.2095	0.5406	1.7090	1.3720
Trade and service	0.2500	0.1400	0.2625	0.1547	1.9776	1.1119
Electricity	0.0809	0.1978	0.2004	0.3503	1.4861	1.0790

household generates larger income than the simple Leontief model. Thus when considering the income generation process aggregating the household sector (or leaving out either the intermediate sectors) does not give a realistic picture of the income generation process. In this sense the final output approach has a serious omission in that by ignoring the interaction process in income generation it misses out very important components of the income generation from the total form.

Redistribution of income and the structure of production

In this section, an attempt is made to analyse the relation between production and income distribution by maximizing the income of certain groups of earners, or by maximizing certain specific functions of final demand.

The implication of this type of approach is quite obvious. Income redistribution leads to inescapable changes in the pattern of production and final demand.

Several types of objective functions were used along with the I/O equations as constraints, leaving the target variables and other variables of the system to be decided by optimization (using linear programming). In these models y_1, y_2, and y_3 denote the value added by household sectors 1, 2 and 3; f_1 through f_5 denote the final demands (in this case investment and net foreign trade). The model considered is an aggregation of the original matrix of 22 × 22 into one of 8 × 8.

MODELS

Objective function	*Constraints*
1. Maximize $3y_1 + y_2 + y_3$	$x_i(t) \leqslant x_i(63\text{-}64)$ $\sum_j a_{ij}x_j + f_i = x_i, i = 1, 7$
2. Maximize $10y_1 + y_2 + y_3$	$x_i \leqslant x_i(63\text{-}64)$ $\sum_j a_{ij}x_j + f_i = x_i$
3. Maximize $f_1 + 5f_2 + 4f_3 + 3f_4$	$x_i \leqslant x_i(63\text{-}64)$ $\sum_j a_{ij}x_j + f_i = x_i$
4. Maximize $(1 - a_6)y_1 + (1 - a_7)y_2 + (1 - a_8)y_3$	$x_i \leqslant x_i(63\text{-}64)$ $\sum_j a_{ij}x_j + f_i = x_i$
5. Maximize $2f_1 + 3f_2 + 5f_3 + f_4 + 4f_5$	$x_i \leqslant x_i(63\text{-}64)$ $\sum_j a_{ij}x_j + f_i = x_i$
6. Maximize $y_1 + 15y_2 + y_3$	$\sum_j a_{ij}x_j + f_i = x_i$ $x_i \leqslant x_i(63\text{-}64)$ $f_i \leqslant f_i(63\text{-}64)$
7. Maximize $y_1 + 15y_2 + y_3$	$x_i \leqslant x_i(63\text{-}64)$ $\sum_j a_{ij}x_j + f_i = x_i$

8. Maximize y_1

$$x_i \leqslant x_i\,(63\text{-}64)$$
$$\sum_j a_{ij} x_j + f_i = x_i$$
$$\sum_i (1 - \sum_j a_{ij}) y_i = \sum_i f_i$$

9. Maximize y_2

$$x_i \leqslant x_i\,(63\text{-}64)$$
$$\sum_j a_{ij} x_j + f_i = x_i$$
$$\sum_i (1 - \sum_j a_{ij}) y_i = \sum_i f_i$$

10. Maximize y_3

$$x_i \leqslant x_i\,(63\text{-}64)$$
$$\sum_j a_{ji} x_j + f_i = x_i$$
$$\sum_i (1 - \sum_j a_{ij}) y_i = \sum_i f_i$$

11. Maximize $3y_1 + y_2 + y_3$

$$x_i \leqslant x_i\,(63\text{-}64)$$
$$\sum_i a_{ij} x_j + f_i = x_i$$
$$\sum_i (1 - \sum_j a_{ij}) y_i = \sum_i f_i$$

Models (1), (2), (6) and (7) aim to maximize the national income for one or the other income group. In (4) attempt is made to maximize savings in the economy; (8), (9), (10) aim at maximizing y_1 or y_2 or y_3 and (11) attempts to maximize a weighted function of the y_1, y_2 and y_3. The constraint system is the open Leontief system of equations in an expanded form. Table 2 shows the output for models by production sector or household income sector. Table 3 shows sectoral final demand.

Models (8), (9) and (10) are used to analyse the relation of different types of household income with final demand and output. Income of household type (1) is related to final demand of sector (1), that of household type (2) is related to final demand of sector (4) and that of household (3) to final demand of sector (3).

Model (8) expands output of sector (1), model (9) of sector (4), model (10) expands sector (3) and sector (4). This analysis thus lays bare the structural implications of different "income" expansion programmes. Models having preference functions of the three income groups together give performance not much unlike the separate models in the sense that the income group with the heaviest weighting predominates.

Another interesting observation to this is that models (1) and (4) maximizing different functions of income with no restraint on final demand, have a zero vector for final demand consisting of investment and net foreign trade. Functions giving varying importance to different types of final demand emphasize the role of different final demand components on different types of income. Model (4) generates the largest total income. Thus direct "income" targets may not be the best way of expanding income. Care must be taken to find out the links of different income groups with production and final demand structure and any attempt at

TABLE 2. SOLUTIONS MAXIMA OF OBJECTIVE FUNCTIONS FOR OUTPUT BY MODEL

(Hundreds of millions of rupees)

	Model										
Sector	*(1)*	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*	*(9)*	*(10)*	*(11)*
Production											
1.	14635.4	12130.7	10942.7	14635.4	12130.0	10643.2	12130.7	17147.7	13108.9	11950.8	17147.7
2.	882.0	2129.4	2129.4	882.0	2129.4	2129.4	2129.4	586.1	644.9	812.4	586.1
3.	1081.5	1427.2	1427.5	1091.5	1427.2	1427.2	1427.2	869.0	890.6	5131.0	869.0
4.	8101.1	7326.2	7326.2	8101.0	5701.4	6543.8	7326.2	16097.7	10056.1	6795.3	6097.7
5.	2.8	1689.3	3.7	2.8	1689.3	762.8	1689.3	2.4	2.3	13.3	2.3
Income											
6.	8599.4	7665.8	6934.7	8599.4	7293.9	6711.7	7665.8	9210.25	8320.3	7544.0	9210.2
7.	3585.0	3300.6	3309.7	3585.0	2681.8	2999.3	3360.6	2743.8	4370.4	3179.5	2743.8
8.	11672.2	5092.8	2875.4	11672.0	3143.5	6408.1	5092.8	6559.47	6495.3	7334.4	6559.5

TABLE 3. FINAL DEMAND SOLUTIONS BY MODEL

(Hundreds of millions of rupees)

	Model										
Sector	*(1)*	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*	*(9)*	*(10)*	*(11)*
Production											
1.	0	619.4	864.37	0	1784.4	0	619.0	3352.2			3352.2
2.	0	950.2	1306.6	0	1050.9	1064.2	950.2				
3.	0	114.8	548.0	0	211.8	309.8	114.8			3500.6	
4.	0	1118.4	2398.8	0	552.5	510.0	1118.4		3426.6		
5.	0	1332.4		0	1332.4	600.0	1332.4				

changing the income structure must be studied for its impact on production and final demand.

The exercise thus demonstrates clearly that every income plan has its impact on the structure of the final demand and outputs and any final demand plan has its impact on the structure of the income distribution. Therefore discussions of redistribution of income without reference to the structure of the final demand and output are meaningless as they are likely to give results quite different from the one foreseen. This is due to the interactions of the type discussed which may not be perceived without such an exercise.

ANNEX I

SECTOR CLASSIFICATION

The sector classification is as follows:

Production sector

1 Construction
2 Electrical equipment
3 Transport equipment
4 Non-electrical equipment
5 Iron and steel
6 Other metal and non-metals
7 Cement
8 Leather products
9 Food industries
10 Textiles
11 Agriculture
12 Fertilizer
13 Chemicals
14 Wooden products and glasswares
15 Fuel
16 Transport
17 Rubber products
18 Trade and services
19 Electricity

Household sector

1 Workers
2 Other employees
3 Profit earners

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Analysis of sectoral price movements in a developing economy: effects of movement in agricultural prices and production on industrial prices, demand pattern and income distribution

*R. Radhakrishna and A. Sarma**

Price structure is set in the long run by production techniques. Demand does not play a significant role in price formation. In other words, price structure is related to technology. Such long-term price structure can be examined with the Leontief I/O model. The observed prices are the result of long-term and short-term causes, namely demand patterns and short-term fluctuations in industrial output.

Demand patterns and supply bottle-necks in a developing economy play a crucial role in short term price formation. It has been observed that in countries such as India the fluctuations in agricultural output not only alter industrial terms of trade but also income distribution. It is also observed that during the period of rising prices, the burden of the price rise falls more heavily on the classes at the bottom of the income scale.[1] Thus, the demand pattern cannot be approximated with simplistic demand models in which prices and income distribution are treated as independent of each other. For example, variations in the prices of items such as food grains affect the rural income distribution because of the skewed distribution of marketed surplus. In the I/O model, agriculture should be treated as exogenous and the cost-based prices of the remaining industries may be approximated with the I/O model while the demand shifters may be examined in relation to agricultural output and price movements.

This paper is presented in the context of the importance of the effects of agricultural production on demand patterns and income distribution in India. An attempt is made to develop a methodology to analyse short term industrial price movement in India. The following section presents the analytical model for analysing price behaviour. In the next section, the demand patterns and income distribution that play an important role in near-term price movement is discussed. The impact of fluctuations in agricultural output and of prices on the demand pattern through effects on income distribution is examined in the final part of this paper.

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[1]See R. Radhakrishna and A. Sarma [1, 2].

Methodology

Outline of the model

The long term prices in a simple Leontief system can be described by a simultaneous linear equation:[2]

$$P_i = \sum_{j=1}^{n} a_{ji} P_j + v_i \qquad (1)$$

where:

P = $[p_i]$, the column vector of prices, where the i^{th} element denotes the price of the i^{th} commodity;
V = $[v_i]$, the column vector of value added per unit of output of the i^{th} commodity;

$$\text{or } P_i = \sum_{j=1}^{n} a_{ji} P_j + w\, l_i + \pi_i \qquad (2)$$

where:

L = $[l_i]$ the column vector of labour per unit of the i^{th} commodity;
π = $[\pi_i]$ the column vector of profit per unit of output of the i^{th} commodity;
w = a scalar of a uniform wage rate.

Equations (1) and (2) can be written in vector form:

$$P = A'P + V \qquad (3)$$

$$\text{or} \quad P = A'P + wL + \pi \qquad (4)$$

where A is $[a_{ij}]$, the matrix of input coefficients.
The solution for P is

$$P = (I - A')^{-1}[wL + \pi] \qquad (5)$$

Assuming that each producer retains a profit margin and introducing m_i on commodity i, the basic price equation can be written as:

$$P_i = \sum_{j=1}^{n} a_{ji} P_j + w\, l_i + m_i P_i$$

The profit margin matrix M is diagonal:

$$\begin{matrix} m_1 & \cdot & 0 \\ \cdot & \cdot & \cdot \\ 0 & \cdot & m_n \end{matrix} \qquad (6)$$

Solving for P in vector form:

$$P = [I - A' - M']^{-1}[wL] \qquad (7)$$

In the above formulations profit is not linked to capital. This can be done by introducing the capital matrix as:

[2]For a similar formulation for developed countries see P. N. Mathur [3] and A. Dramais [4].

$$P = [I - A' - RB]^{-1}[wL] \tag{8}$$

where B is $[b_{ij}]$, the matrix of capital coefficients, where b_{ij} is the requirement of the commodity i to increase the capacity of commodity j by one unit.

If profit is at the uniform rate r, $R = rI$ is the diagonal matrix of rate of profit.

However this model may not be able to capture the price behaviour of certain types of commodities. For example in India, the behaviour of agricultural prices depends to a large extent on exogenous factors such as weather. Other examples include cases where the prices of certain commodities are regulated and cases where commodities are mainly imported. The analysis of the behaviour of such commodities needs a separate treatment. Keeping this consideration in view, the model can be further refined as follows: let us treat commodities for which the model does not capture price behaviour as exogenous and keep their prices outside the model. The model will be used for the prices of the rest of the commodities. Then the price relationship becomes:

$$P = (I - A)^{-1}[wL + \pi + \bar{A}'\bar{P}] \tag{9}$$

where:

$\bar{A}$ = $[\bar{a}_{ij}]$, the matrix of input coefficients exogenous to the system;
$\bar{P}$ = the column vector of commodity prices exogenous to the system.

The equations (7) and (8) can be respectively written as:

$$P = [I - A' - M]^{-1}[wL + \bar{A}'\bar{P}] \tag{10}$$

$$P = [I - A' - RB]^{-1}[wL + \bar{A}'\bar{P}] \tag{11}$$

Technological decomposition

There is evidence[3] to show that the A matrix for India can be decomposed into a block-diagonal matrix. The property of decomposibility of the A matrix permits the distinction of different sets of commodities and the examination of price relationship of each set separately. This approach ensures computational ease. The method that has been developed and applied to the oilseed-based industries to demonstrate its suitability for analysing the price behaviour follows in a later section.

Let A_r be the block diagonal matrix for the r^{th} subset of interrelated commodities and P_r be its price vector. It can be shown that (10) becomes:

$$P_r = [I - A_r]^{-1}[wL_r + \pi_r + \bar{A}_r' \cdot \bar{P}_r] \tag{12}$$

The model closing for the rate of profit using the capital matrix as in (8) cannot be simplified unless the capital matrix is decomposible. Three variants of the model can be considered:

[3] D. T. Lakdawala, Yoginder K. Alagh and A. Sarma [5] and Desai Rohit [6].

(a) For some block, A, there may not be any common commodities between them or between the corresponding capital sub-matrices. Suppose A_r and B_r are such technological sub-matrices having no commodites for the r^{th} block, and P_r and $\bar{P}_r$ the prices facing A_r and B_r respectively. Then it can be shown that:

$$P = [I - A_r'] [wLr + \bar{A}_r \bar{P}_r + R_r B_r' P_r] \quad (13)$$

In this case the rate of profit has to be adjusted for the changes in the prices of commodities entering the capital block. If a capital matrix is not readily available, the profit rate may be adjusted for the changes in the machinery price index;

(b) All the commodities are common between A_r and B_r then:

$$P_r = [I - A_r' - R_r B_r']^{-1} [wL_r + \bar{A}_r' \bar{P}_r] \quad (14)$$

(c) Some commodities are common between A_r and B_r such that the prices of the commodities of B_r not entering A_r may be considered exogenous and those entering A_r as endogenous.

These models dealing with the long-term situation do not consider demand because in the long run, supply adjusts itself to demand. But what one observes in reality is that it is not a long-term price situation but a short-term situation which is influenced by demand and other short-term forces. In the short run the rigidity of production gives rise to price fluctuation particularly of those commodities whose demand is price inelastic. One can visualize the following types of price behaviour in the short-term due to demand shifts. If all capacities are fully utilized, the industries are likely to react to upward change in demand by raising the prices above the cost-based prices of the commodities in whose favour demand has shifted in the short run and there will be a corresponding rise in profit. Thus, in this case, the prices are mostly guided by demand consideration. On the other hand, the industries operating below the full capacity level will be guided by cost consideration when they confront the situation of demand shift. Thus one expects the short-term price to be influenced by its cost-based price until capacities are fully utilized and thereafter by the demand pull.[4] Therefore it follows that profit in the short run consists of two elements: one consistent with the technology and the other due to demand shift. To put it another way, the difference between observed and cost-based price is due to changes in profit rate. After discussing the short-term forces such as demand pattern and income distribution and also the impact of fluctuations in agricultural prices and output on demand pattern, the empirical results of the study of the short-term price movement in the oilseed-based industry will be presented. The model has been applied empirically to explain the price movements in the oilseed-based industry in India. For economy of space, the results, though included in the original version of the paper, have not been presented here.

[4]For fuller exposition see R. Radhakrishna [7].

Income distribution and demand patterns

Demand functions

In this section it is argued and empirically shown that growth implications in terms of income generation and distribution have differential impact on the demand for agricultural and industrial goods depending on in whose favour—top or bottom income classes—income shifts. It is also shown that such forces result, in the ultimate analysis, in a variation between the cost-based long-term prices and short-term prices.

In an earlier study an attempt has been made to estimate a demand model to handle the income redistribution effects in a realistic manner.[5] We present below the methodology and some of the relevant results.

The National Sample Survey (NSS) furnishes per capita monthly consumer expenditure data for 12 or 13 expenditure classes. The expenditure classes have been stratified separately into five groups for rural and urban areas on the basis of the seventeenth round (1961/62) per capita monthly expenditure classes.[6] Categories with expenditures of 0-8 rupees form the first group; 8-11 and 11-13 rupees the second; 13-15, 15-18 and 18-21 rupees the third; 21-24, 24-28 and 28-34 rupees the fourth; and 34-43, 43-55, 55-75 and more than 75 rupees the fifth. For each group, the Linear Expenditure System is given by

$$p_{it} q_{it} = C_i p_i + b_i [\mu_t - \sum_t C_j p_j] + (\varepsilon_{it}) \tag{15}$$

$$0 < b_i < 1 \;\; \forall i \sum_{i=1}^{n} b_i = 1 \qquad \begin{aligned} E(\varepsilon_{it}) &= 0 \\ E(\varepsilon_t \varepsilon_t') &= \Omega \,\forall t \\ E(\varepsilon_t \varepsilon_s') &= 0 \,\forall t \neq s \end{aligned}$$

where:

- q_i = the quantity consumed by the i^{th} commodity;
- p_i = the price of the i^{th} commodity;
- μ = total expenditure;
- b = the marginal budget shares;
- C = committed quantities.[7]

This interpretation is only suggestive and it is not always possible, particularly when C_i is negative. A negative C_i is still consistent with theory.

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[5]The model has been developed as part of the Agricultural Policy Model of India being developed at the International Institute for Applied Systems Analysis (IIASA). See for details R. Radhakrishna and K. N. Murthy [8].

[6]Intertemporal variation in the prices have been taken into consideration; see R. Radhakrishna and K. N. Murthy [8]. Stratification has been resorted to as no single demand system could adequately describe the consumption patterns over the entire expenditure range.

[7]Since the pioneering study of R. Stone [9] a number of studies have been reported. See A. S. Goldberger and T. Gameletsos [10], J. A. C. Brown and A. S. Deaton [11], A. S. Deaton [12] etc.

The Linear Expenditure System has been estimated for each group by employing the maximum likelihood method, taking 1961/62 as base for prices utilizing two specifications for the co-variance matrix:

(*a*) $\Omega = \sigma^2 I$, and the non-diagonal elements of Ω are zero;
(*b*) non-diagonal elements are zero.

The results under specification (*a*) called Model 1, and (*b*) called Model II are presented in the annex. The approximate standard errors of the parameter estimates are shown below the parameter estimates. In order to examine the goodness of fit, the value of the square of correlation coefficient (R^2) and Thiel's average information inaccuracy parameter (T) are also given in the annex. On the whole, the fit is satisfactory. The value of R^2 exceeds 0.80 in 146 out of 180 equations. The parameter estimates also possess the right signs: all the b's are positive. It can also be seen that Models I and II give almost identical estimates.

Marginal budget shares

It is obvious that there are sizeable variations in marginal budget shares both across income groups and between rural and urban areas. Nevertheless, there are some visible patterns. The cereal group takes a major share of the marginal budgets of the lower groups and its weight declines with the total expenditure level. For example, as one moves from lower to higher expenditure groups, the marginal budget share of cereals declines from 0.58 to 0.08 in rural areas and from 0.49 to 0.01 in urban areas. One notices that the fall in marginal budget share of cereals is compensated by other non-food items. Rural-urban dichotomy is also seen. Nevertheless, these differences are of lower magnitude than the differences across the expenditure groups.

The above results imply that the expansion of demand for individual items very much depends on which income group or groups are favoured by economic growth. It is the lower income group which exerts a significantly greater influence on the expansion or contraction of agricultural products such as cereals. Any policy of income transfers to these groups will result in demand pressure in the food grains market. Any growth of supply of cereals has to be absorbed by the growth of income of this group. On the other hand, the demand for non-food items, which are mostly industrial products, expands with the incomes of the higher income groups.

The commodities have been grouped into agricultural and industrial sectors and the respective marginal budget shares are obtained by adding the b's of the respective components.[8] These are presented in table 1. The table brings out clearly the dominance of agricultural commodities in the marginal budgets of the lower expenditure groups.

[8]Cereals; milk and milk products; meat, eggs and fish; and some items included under "other food" fall in the agricultural sector. The other food group has been split up on the basis of the data on the detailed items available for the seventeenth round.

TABLE 1. MARGINAL BUDGET SHARES
(Percentage)

	Rural areas		*Urban areas*	
Group	*Agricultural commodities*	*Industrial commodities*	*Agricultural commodities*	*Industrial commodities*
I	0.77	0.23	0.69	0.31
II	0.72	0.28	0.61	0.39
III	0.56	0.44	0.49	0.51
IV	0.47	0.53	0.39	0.61
V	0.21	0.79	0.19	0.81

Expenditure and price elasticities

Tables 2 and 3 show the expenditure elasticities (η_{i0}) and own price elasticities (η_{ii}) computed at the sample mean for each group. It has been observed that among the cross price elasticities cereal cross price elasticity (η_{ij}) is large in magnitude and the other cross price elasticities are of low order. Therefore only the cereal cross price elasticities are shown in table 4. The expenditure elasticities more or less reinforce the findings based on marginal budget shares. For food items, there is a tendency for the absolute value of the own price elasticity to fall as one moves from lower to higher expenditure groups. This implies that with development, defined as a shift of population from lower to higher expenditure groups, price flexibility of food items increases.

Demand potential for cereals

If one assumes that the consumption of each group is a fixed proportion of its income, the following specification results:

$$\mu_t^r = (1 - \theta^r)\, y_t^r \qquad (16)$$

where:

y_t = the per capita income in period t;
θ^r = the saving proportion;
r = an expenditure group.

The above specification implies the saving rate remains the same for each group and varies across the groups. It may be noted that the same assumption is retained in part 4 too. The per capita demand for cereals (q_1) of each group expressed in time derivatives and dropping the group subscript for group (r) can be written as:

$$\dot{q}_i = \eta_{10}\, \dot{y} + \sum \eta_{ij}\, \dot{p}_j$$

TABLE 2. EXPENDITURE ELASTICITIES FOR NINE COMMODITY GROUPS

Sector number	*Commodity group*	*Rural group I*	*Rural group II*	*Rural group III*	*Rural group IV*	*Rural group V*	*Urban group I*	*Urban group II*	*Urban group III*	*Urban group IV*	*Urban group V*
1	Cereals	0.954	0.827	0.583	0.460	0.343	0.971	0.758	0.461	0.140	0.094
2	Milk and milk products	1.962	2.245	2.222	1.701	0.728	1.712	2.030	2.053	1.547	0.886
3	Edible oil	1.527	1.247	0.968	0.783	0.985	1.244	0.997	1.067	0.980	0.410
4	Meat, fish and eggs	1.546	1.693	1.569	1.149	0.606	1.129	1.247	1.589	1.512	0.798
5	Sugar and gur	1.363	1.655	1.537	1.379	0.803	1.092	0.966	1.302	0.828	0.445
6	Other food	1.115	1.008	1.121	0.871	0.674	1.043	0.984	1.069	1.290	0.808
7	Clothing	0.823	0.644	1.468	1.541	1.044	1.589	1.075	0.979	1.088	1.178
8	Fuel and light	0.589	0.963	0.814	0.587	0.508	0.825	0.947	0.792	0.646	0.550
9	Other non-food	1.072	1.370	1.763	1.816	1.781	0.852	1.494	1.601	1.593	1.573

Note: Computed at the mean level from Model 1.

TABLE 3. OWN PRICE ELASTICITIES

Sector number	*Commodity group*	*Rural group I*	*Rural group II*	*Rural group III*	*Rural group IV*	*Rural group V*	*Urban group I*	*Urban group II*	*Urban group III*	*Urban group IV*	*Urban group V*
1	Cereals	−0.881	−0.760	−0.528	−0.254	−0.157	−0.900	−0.694	−0.348	−0.089	−0.016
2	Milk and milk products	−1.459	−1.459	−1.234	−0.442	−0.238	−1.399	−1.389	−0.951	−0.658	−0.123
3	Edible oil	−1.143	−0.839	−0.568	−0.175	−0.262	−1.029	−0.715	−0.515	−0.405	−0.028
4	Meat, fish and eggs	−1.159	−1.125	−0.902	−0.258	−0.159	−0.937	−0.882	−0.744	−0.605	−0.054
5	Sugar and gur	−1.025	−1.101	−0.885	−0.308	−0.219	−0.907	−0.691	−0.616	−0.338	−0.028
6	Other food	−0.863	−0.718	−0.693	−0.264	−0.225	−0.888	−0.745	−0.584	−0.613	−0.171
7	Clothing	−0.625	−0.442	−0.853	−0.402	−0.346	−1.310	−0.761	−0.470	−0.457	−0.140
8	Fuel and light	−0.473	−0.669	−0.495	−0.147	−0.143	−0.707	−0.691	−0.401	−0.281	−0.047
9	Other non-food	−0.821	−0.924	−1.007	−0.544	−0.754	−0.734	−1.042	−0.804	−0.751	−0.602

TABLE 4. CROSS-PRICE ELASTICITY WITH RESPECT TO CEREAL PRICES

Sector number	*Commodity group*	*Rural group I*	*Rural group II*	*Rural group III*	*Rural group IV*	*Rural group V*	*Urban group I*	*Urban group II*	*Urban group III*	*Urban group IV*	*Urban group V*
1	Milk and milk products	−0.335	−0.567	−0.743	−0.602	−0.156	−0.168	−0.430	−0.604	−0.389	−0.367
2	Edible oil	−0.261	−0.315	−0.324	−0.277	−0.211	−0.122	−0.211	−0.314	−0.247	−0.215
3	Meat, fish and eggs	−0.264	−0.427	−0.524	−0.406	−0.130	−0.111	−0.264	−0.467	−0.381	−0.272
4	Sugar and gur	−0.233	−0.417	−0.514	−0.488	−0.172	−0.107	−0.205	−0.383	−0.208	−0.192
5	Other food	−0.190	−0.254	−0.375	−0.308	−0.144	−0.102	−0.209	−0.314	−0.324	−0.303
6	Clothing	−0.140	−0.162	−0.491	−0.545	−0.224	−0.156	−0.228	−0.288	−0.274	−0.360
7	Fuel and light	−0.101	−0.243	−0.272	−0.208	−0.109	−0.081	−0.201	−0.233	−0.162	−0.177
8	Other non-food	−0.183	−0.346	−0.589	−0.642	−0.382	−0.084	−0.317	−0.471	−0.401	−0.360

where the dot on the variables denotes the rate of change over time. Denoting for each group Q_l as its aggregate food grains demand, Y as its aggregate total expenditure (income) and r as its population growth rate, we can write the aggregate food grains demand for each group as:

$$\dot{Q}_l = r(1 - \eta_{l0}) + \eta_{l0}\dot{y} + \sum_{j=1}^{n} \eta_{lj}\dot{p}_j \qquad (17)$$

In the above expression we can ignore the cross-price elasticities as they are minimal. As the expenditure elasticities for the lowest rural and urban expenditure groups are close to unity, their cereals demand can be expressed as:

$$\dot{Q}_l \simeq \eta_{l0}\dot{y} + \eta_{l0}\dot{p}_j \qquad (18)$$

For higher expenditure groups (rural and urban), as expenditure elasticities are low, their cereal demand can be expressed as:

$$\dot{Q}_l \simeq r \qquad (19)$$

It is clear from the above expressions that the demand for cereals by the lowest rural and urban expenditure groups depends on their income while for higher expenditure groups, on their population growth. The demand relationships of other groups lie in between the above two cases. These results imply that in the short run it is the income of the lower strata that influence the demand for cereals. Incidentally, it may be noted that any substantial strides in grains production, if at all possible due to the green revolution, will be absorbed without any fall in its price only if the prosperity is widespread and augments the incomes of the lower strata.

Impact of fluctuations in agricultural prices and production

As a prelude to the discussion on effect of fluctuations in cereal prices and production, it is necessary to establish the correspondence between the expenditure groups adopted for Linear Expenditure Systems and economic groups in rural India. Cereal production in the Indian context constitutes an overwhelming proportion (60 per cent in terms of area) of the total agricultural production.

The economic groups are: agricultural labourers, small farmers, medium farmers and large farmers. According to the available estimates, agricultural labour households account for 15 per cent (National Sample Survey, twenty-fifth round, 1970/71) and 13 per cent (Labour Enquiry Report, 1974/75) of the total rural households. Small farmer households account for about 5 per cent of the rural households (National Sample Survey, twenty-fifth round, 1970/75). On the basis of this information the proportion of persons in the expenditure groups I and II roughly correspond to agricultural labour households and the group III with small farmers; on the basis of an a priori assumption, groups IV and V have been associated with medium and large farmers respectively.

In order to examine the impact of fluctuations of cereal production and prices on income distribution, it is necessary to examine the share of each of these economic groups in cereal production. Small farmers derive around 40 per cent of their income from farm sector and 60 per cent by selling labour (National Sample Survey, twenty-fifth round, and farm management reports). As small farmers are likely to mainly produce cereals the entire farm income has been attributed to cereal production. Informed guesses are that medium farmers derive 60 per cent of their income from cereals and 40 per cent from other crops and the non-agricultural sector while large farmers derive 80 per cent from cereal production and 20 per cent from other crops.

Aggregate income and aggregate cereal production relation for the r^th^ group can be expressed as:

$$Y^r = P_1(1 - a^r)Q_s^r + Y_o^r \qquad (20)$$

where:

- Y = aggregate income;
- Q_s = aggregate supply of cereals;
- P_1 = price of cereals;
- a = I/O ratio for cereal crop;
- Y_o = other income.

The rate of change can be derived from (20) as:

$$\dot{Y}^r = S^r(\dot{P}_1 + \dot{Q}_s^r) + (1 - S^r)\dot{Y}_o^r \qquad (21)$$

where the dot denotes rate of change, that is, $\dot{Y} = \frac{dY}{Y}$, etc.; and S is the share of income from cereal crop in total.

In (21) if $\dot{Y}_o^r = 0$, the expression becomes:

$$\dot{Y}_r = S^r(\dot{P}_1 + \dot{Q}_s^r) \qquad (22)$$

This expression has been used to arrive at the effect of cereal price and production changes on the economic groups. For classes such as agricultural labourers and urban population, S is zero.

The effect of cereal price change, given cereal production and other prices, on the aggregate of the i^{th} commodity after incorporating the shift in the income distribution is given by:

$$\sum_r \omega_i^r \dot{Q}_i^r = \sum_r \omega_i^r (\eta_{i1}^r + S^r \eta_{io}^r)\dot{P}_1 \qquad (23)$$

where:

- ω_i^r = the share of the r^{th} group in the aggregate consumption of the i^{th} item;
- η = the elasticities given in Section III;
- subscript 1 = cereals;
- r = group.

The shares of each group in the consumption of various items have been worked out on the basis of the National Sample Survey (seventeenth round

1961/62), consumption data and the census of population for 1961. The effect of cereal price rise has been computed and presented in table 5. Commodities have been aggregated in terms of agriculture and industry. These results have also been presented in table 5.

TABLE 5. EFFECT OF A ONE PER CENT RISE IN CEREAL PRICES

(After incorporation income shift)

Sector number	*Item*	*Percentage change in aggregate demand*
1	Cereals	−0.202
2	Milk and milk products	−0.095
3	Edible oil	0.082
4	Meat, fish and eggs	0.046
5	Sugar and gur	0.151
6	Other food	0.027
7	Clothing	0.265
8	Fuel and light	0.028
9	Other non-food	0.286
Aggregate sectors		
A	Agriculture	−0.086
B	Industry	0.199

An important conclusion that emerges is that with a one per cent rise in cereal prices the aggregate demand for agricultural goods declines by 0.09 per cent while the aggregate demand for industrial goods rises by 0.20 per cent. Examining at the disaggregated level, it is found that the demand for cereals declines by 0.20 per cent while that for items such as clothing, other non-food, and sugar and gur, which are industrial goods, expand by 0.29 to 0.15 per cent. This is understandable. The poorer segment of the population in whose budget cereals occupy a predominant place is affected by the price rise in cereals while with the increase in farm income of the farmers, the demand for industrial goods expands.

Now the effect of higher production of cereals allowing for its price change can be examined. Suppose the production of cereals goes up and $\dot{Q}_s = \alpha$ in (22) for each of the farming groups as specified earlier. Then the new price of cereals is set by the interaction between its aggregate demand and supply. The rate of change in cereal price ($\dot{P}_1$) can be expressed as:

$$\dot{P}_1 = \frac{[1 - \sum_r \omega_1^r s^r \eta_{10}^r]\alpha}{\sum_r \omega^r [s^r \eta_{10}^r + \eta_{11}^r]} \tag{24}$$

where

η_{10} = cereal expenditure elasticity;
η_{11} = cereal own price elasticity.

Using the earlier mentioned information, P has been computed as -4.14α. It can be inferred that the income of the small farmers consequent upon a one per cent rise in cereal production decreases by about 1.3 per cent, medium farmers by 1.9 per cent and large farmers by 2.5 per cent.

The overall effect of simultaneous change in production and price changes on the i^{th} commodity is given by:

$$\sum_r \omega_i^r \dot{Q}_i^r = \sum_r \omega_i^r [S^r(\dot{P}_1 + \alpha) + \eta_{i1}, \dot{P}_1] \tag{25}$$

Using (25), the effect of a one per cent rise in cereal production allowing for cereal price change on demand is presented in table 6. The effect on aggregate demand separately for agricultural and industrial goods have also been presented in the same table. Table 6 indicates that a one per cent rise in cereal production results in a fall of 0.15 per cent demand for industrial goods while the demand for agricultural goods goes up by 0.70 per cent. The explanation for such a phenomenon is just the opposite to the one given above. In greater detail, it is observed that the demand for food items expands while the demand for non-food items declines considerably. However, a fall in prices of cereals following its higher production is likely to be moderated by government policies such as minimum price support, buffer stock etc. It should be added that the effects quantified above are primary effects and ignore the secondary effects due to changes in other prices likely to be induced by the change in demand pattern. However there is reason to believe that the incorporation of this effect does not drastically change the basic results.

TABLE 6. EFFECTS OF ONE PER CENT RISE IN CEREAL PRODUCTION

(With cereal price as endogenous)

Sector number	*Item*	*Percentage change in aggregate demand*
1	Milk and milk products	0.093
2	Edible oil	0.069
3	Meat, fish and eggs	0.269
4	Sugar and gur	−0.111
5	Other food	0.235
6	Clothing	−0.376
7	Fuel and light	0.193
8	Other non-food	−0.227
Aggregate sectors		
A	Agriculture[a]	0.700
B	Industry	−0.147

[a]Includes cereals.

Concluding observations

The major thrust of the paper is to show how the general equilibrium approach can be adopted by using the decomposibility property of technological matrices to empirically study the movement of sectoral prices. It is argued that while the long-term prices are set by technology, the short-term prices are set by cost-based prices and short-term forces such as demand pattern, and fluctuations in agricultural prices and production. The analysis of the demand pattern shows that an increase in the size of population of the higher income classes, and an increase in the purchasing power and net population for the case of lower income classes result in the expansion of the demand for agricultural goods.

With an increase in agricultural production, the farm income declines because of a more than proportionate fall in agricultural prices, resulting in a shrinkage in the demand for industrial goods. In contrast, the rise in agricultural prices is accompanied by an increase in the purchasing power of the bottom classes resulting in an expansion of the demand for industrial goods and shrinkage of the demand for agricultural goods.

By introducing some of the short-term forces such as demand consideration in the case study of oilseed-based industries, the deviation of observed prices from the cost-based prices is narrowed down. This suggests that the short-term forces play a significant role in short-term sectoral price behaviour.

ANNEX

LINEAR EXPENDITURE SYSTEM PARAMETER ESTIMATES

Sector number	Commodity group	Model I			Model II		
		b	*C*	R^2	*b*	*C*	R^2
		All India Rural: Group I					
1	Cereals	0.5774 (0.0051)	1.0365 (0.2047)	0.9932	0.5789 (0.0111)	1.6475 (0.1275)	0.9924
2	Milk and milk products	0.0361 (0.0050)	−0.0539 (0.0274)	0.7215	0.0333 (0.0034)	−0.0041 (0.0141)	0.7140
3	Edible oil	0.0405 (0.0047)	−0.0241 (0.0260)	0.9485	0.0388 (0.0014)	0.0263 (0.0071)	0.9667
4	Meat, fish and eggs	0.0271 (0.0047)	−0.0161 (0.0221)	0.8864	0.0238 (0.0014)	0.0222 (0.0063)	0.8964
5	Sugar and gur	0.0247 (0.0039)	−0.0027 (0.0173)	0.9164	0.0272 (0.0016	0.0130 (0.0043)	0.9278
6	Other food items	0.1495 (0.0049)	0.1205 (0.0512)	0.9763	0.1497 (0.0045)	0.2652 (0.0280)	0.9785
7	Clothing	0.0152 (0.0050)	0.0579 (0.0339)	0.0002	0.0222 (0.0075)	0.0423 (0.0421)	0.0182
8	Fuel and light	0.0537 (0.0050)	0.3800 (0.0384)	0.9696	0.0516 (0.0028)	0.4630 (0.0160)	0.9645
9	Other non-food items	0.0758 (0.0051)	0.1066 (0.0469)	0.9035	0.0746	0.2200 (0.0307)	0.9004
			T = 0.0147			T = 0.0130	

1	Cereals	0.4653 (0.0133)
2	Milk and milk products	0.0775 (0.0148)
3	Edible oil	0.0395 (0.0106)
4	Meat, fish and eggs	0.0369 (0.0108)
5	Sugar and gur	0.0379 (0.0077)
6	Other food items	0.1388 (0.0128)
7	Clothing	0.0208 (0.0105)
8	Fuel and light	0.0727 (0.0109)
9	Other non-food items	0.1106 (0.0119)

All India Rural: Group II

2.7093 (0.2547)	0.9880	0.5154 (0.0211)	1.7926 (0.2970)	0.9864
−0.1892 (0.1238)	0.8731	0.0624 (0.0071)	−0.1196 (0.0522)	0.8770
0.0575 (0.0788)	0.9653	0.0295 (0.0025)	0.1091 (0.0170)	0.9804
−0.0297 (0.0759)	0.9333	0.0272 (0.0022)	0.0186 (0.0166)	0.9403
−0.0248 (0.0522)	0.9214	0.0231 (0.0016)	0.0631 (0.0074)	0.9655
0.4574 (0.1132)	0.9928	0.1372 (0.0062)	0.3778 (0.0538)	0.9938
0.2472 (0.1002)	0.0026	0.0675 (0.0127)	−0.2630 (0.1410)	0.0533
0.3307 (0.1016)	0.9745	0.0549 (0.0032)	0.4406 (0.0360)	0.9787
0.0892 (0.1243)	0.9302	0.0832	(0.2653) (0.0921)	0.9289
T = 0.0090			T = 0.0116	

ANNEX *(continued)*

Sector number	*Commodity group*	*Model I*			*Model II*		
		b	*C*	R^2	*b*	*C*	R^2
			All India Rural: Group III				
1	Cereals	0.2922 (0.0138)	5.3016 (0.3097)	0.9510	0.2340 (0.0247)	6.4452 (0.1954)	0.9459
2	Milk and milk products	0.1368 (0.0144)	−0.2778 (0.2062)	0.9147	0.1471 (0.0076)	−0.0082 (0.0571)	0.9192
3	Edible oil	0.0309 (0.0120)	0.2393 (0.1201)	0.9660	0.0307 (0.0018)	0.3198 (0.0131)	0.9716
4	Meat, fish and eggs	0.0402 (0.0121)	0.0431 (0.1163)	0.9204	0.0394 (0.0023)	0.1450 (0.0168)	0.9242
5	Sugar and gur	0.0440 (0.0104)	0.0536 (0.0933)	0.9366	0.0466 (0.0026)	0.1390 (0.0135)	0.9393
6	Other food items	0.1439 (0.0134)	0.7053 (0.1756)	0.8310	0.1751 (0.0146)	0.8181 (0.1099)	0.8164
7	Clothing	0.0812 (0.0127)	0.1706 (0.1770)	0.5922	0.0966 (0.0077)	0.2856 (0.0916)	0.6725
8	Fuel and light	0.0546 (0.0129)	0.6433 (0.1560)	0.9517	0.0452 (0.0032)	0.8816 (0.0319)	0.9510
9	Other non-food items	0.1760 (0.0136)	−0.0160 (0.2504)	0.9371	0.1853	0.4236 (0.1537)	0.7267
			T = 0.0180			T = 0.0180	

1	Cereals	0.1791 (0.0116)
2	Milk and milk products	0.1560 (0.0118)
3	Edible oil	0.0230 (0.0113)
4	Meat, fish and eggs	0.0296 (0.0117)
5	Sugar and gur	0.0453 (0.0114)
6	Other food items	0.1098 (0.0116)
7	Clothing	0.1370 (0.0113)
8	Fuel and light	0.0343 (0.0114)
9	Other non-food items	0.2859 (0.0115)

All India Rural: Group IV

9.4852 (0.2607)	0.9304	0.1880 (0.0241)	8.3591 (0.4009)	0.9298
1.7259 (0.2451)	0.9275	0.1551 (0.0099)	0.8119 (0.1384)	0.9239
0.7174 (0.0880)	0.9557	0.0239 (0.0018)	0.5650 (0.0220)	0.9570
0.5780 (0.0916)	0.8731	0.0323 (0.0028)	0.3763 (0.0344)	0.8811
0.6368 (0.0949)	0.9082	0.0399 (0.0031)	0.4592 (0.0296)	0.9368
2.7443 (0.1645)	0.9696	0.1119 (0.0062)	2.1223 (0.0993)	0.9737
1.9618 (0.2472)	0.8164	0.1357 (0.0103)	0.1013 (0.1610)	0.8083
1.5380 (0.1078)	0.8528	0.0322 (0.0038)	0.1349 (0.0560)	0.8819
3.1264 (0.4716)	0.8602	0.2810	1.2488 (0.4074)	0.8561
T = 0.0082			T = 0.0082	

ANNEX *(continued)*

Sector number	Commodity group	Model I			Model II		
		b	C	R^2	b	C	R^2
			All India Rural: Group V				
1	Cereals	0.0802 (0.0084)	12.8666 (0.4785)	0.8573	0.0801 (0.0102)	11.0851 (0.5601)	0.8599
2	Milk and milk products	0.0726 (0.0084)	5.1797 (0.4627)	0.7493	0.0703 (0.0080)	3.5670 (0.4723)	0.7456
3	Edible oil	0.0279 (0.0083)	1.3852 (0.2542)	0.7946	0.0269 (0.0025)	0.7384 (0.1414)	0.7905
4	Meat, fish and eggs	0.0128 (0.0083)	1.1775 (0.2104)	0.5842	0.0153 (0.0020)	0.7403 (0.1055)	0.6026
5	Sugar and gur	0.0282 (0.0083)	1.6719 (0.2349)	0.8237	0.0260 (0.0028)	1.1228 (0.1300)	0.8294
6	Other food items	0.0722 (0.0083)	1.5253 (0.4139)	0.9210	0.0703 (0.0044)	3.7599 (0.3046)	0.9293
7	Clothing	0.1216 (0.0084)	5.9263 (0.8339)	0.7242	0.1202 (0.0118)	2.8124 (0.8728)	0.7339
8	Fuel and light	0.0213 (0.0085)	2.3541 (0.2659)	0.6803	0.0209 (0.0026)	1.8529 (0.1476)	0.6949
9	Other non-food items	0.5633 (0.0084)	11.9762 (0.3540)	0.9520	0.5700	−2.9908 (3.3455)	0.9485
			T = 0.0166			T = 0.0171	

1	Cereals	0.4870 (0.0092)
2	Milk and milk products	0.0439 (0.0088)
3	Edible oil	0.0451 (0.0082)
4	Meat, fish and eggs	0.0290 (0.0083)
5	Sugar and gur	0.0303 (0.0088)
6	Other food items	0.1753 (0.0089)
7	Clothing	0.0183 (0.0091)
8	Fuel and light	0.0741 (0.0092)
9	Other non-food items	0.0970 (0.0092)

All India Urban: Group I

0.5702	0.9724	0.4862	1.0101	0.9714
(0.3174)		(0.0177)	(0.2630)	
−0.0637	0.7007	0.0452	−0.0296	0.7026
(0.0524)		(0.0051)	(0.0279)	
−0.0067	0.9073	0.0456	0.0334	0.9158
(0.0494)		(0.0030)	(0.0216)	
0.0095	0.8078	0.0356	0.0070	0.7941
(0.0429)		(0.0035)	(0.0196)	
0.0170	0.7823	0.0336	0.0315	0.7760
(0.0531)		(0.0048)	(0.0239)	
0.1326	0.9357	0.1799	0.2688	0.9526
(0.1126)		(0.0109)	(0.0855)	
−0.2903	0.0286	0.0172	0.0013	0.0002
(0.0658)		(0.0095)	(0.0589)	
0.2148	0.9134	0.0700	0.3259	0.9088
(0.0888)		(0.0048)	(0.0556)	
0.2529	0.7737	0.0867	0.4261	0.7797
(0.1025)			(0.0896)	
T = 0.0263			T = 0.0258	

ANNEX *(continued)*

Sector number	Commodity group	Model I			Model II		
		b	*C*	R^2	*b*	*C*	R^2
			All India Urban: Group II				
1	Cereals	0.3446 (0.0116)	2.1788 (0.2148)	0.9880	0.3428 (0.0204)	2.4493 (0.2485)	0.9874
2	Milk and milk products	0.9934 (0.0115)	−0.2113 (0.1050)	0.8868	0.0947 (0.0072)	−0.1482 (0.0635)	0.8876
3	Edible oil	0.0437 (0.0091)	0.1403 (0.0712)	0.9688	0.0493 (0.0040)	0.1362 (0.0263)	0.9694
4	Meat, fish and eggs	0.0345 (0.0094)	0.0354 (0.0692)	0.9076	0.0415 (0.0034)	0.0154 (0.0238)	0.9092
5	Sugar and gur	0.0317 (0.0110)	0.1181 (0.0926)	0.9084	0.0250 (0.0052)	0.1941 (0.0345)	0.9210
6	Other food items	0.1690 (0.0110)	0.5544 (0.1181)	0.9823	0.1741 (0.0126)	0.6480 (0.0928)	0.9813
7	Clothing	0.0192 (0.0105)	0.0583 (0.1048)	0.0108	0.0364 (0.0121)	−0.0778 (0.1176)	0.0001
8	Fuel and light	0.0752 (0.0116)	0.3424 (0.1255)	0.9781	0.0640 (0.0041)	0.5098 (0.0544)	0.9785
9	Other non-food items	0.18879 (0.0118)	0.0832 (0.1750)	0.9473	0.1721	(0.2402) 0.1482	0.9471
			T = 0.0115			T = 0.0121	

1	Cereals	0.1722 (0.0088)
2	Milk and milk products	0.1474 (0.0090)
3	Edible oil	0.0486 (0.0080)
4	Meat, fish and eggs	0.0513 (0.0083)
5	Sugar and gur	0.0452 (0.0088)
6	Other food items	0.1824 (0.0088)
7	Clothing	0.0358 (0.0083)
8	Fuel and light	0.0581 (0.0088)
9	Other non-food items	0.2592 (0.0089)

All India Urban: Group III

4.6711	0.9813	0.1798	4.3387	0.9815
(0.1282)		(0.0138)	(0.1736)	
0.0689	0.9362	0.1419	−0.1129	0.9362
(0.1241)		(0.0071)	(0.0859)	
0.4038	0.9805	0.0457	0.3523	0.9817
(0.0713)		(0.0031)	(0.0271)	
0.1487	0.9411	0.0457	0.1239	0.9448
(0.0702)		(0.0025)	(0.0275)	
0.2415	0.9403	0.0379	0.2409	0.9522
(0.0807)		(0.0032)	(0.0275)	
1.4274	0.9785	0.1727	1.2359	0.9817
(0.1324)		(0.0088)	(0.0951)	
0.3963	0.0464	0.0676	−0.0683	0.1220
(0.0876)		(0.0103)	(0.1444)	
0.9141	0.9748	0.0576	0.8084	0.9765
(0.0997)		(0.0029)	(0.0500)	
0.0842	0.9629	0.2511	0.4240	0.9629
(0.2156)			(0.1975)	
T = 0.0098			T = 0.0115	

ANNEX *(continued)*

Sector number	Commodity group	Model I			Model II		
		b	*C*	R^2	*b*	*C*	R^2
			All India Urban: Group IV				
1	Cereals	0.0371 (0.0095)	6.6035 (0.1151)	0.9584	0.0488 (0.0133)	6.2814 (0.2068)	0.9600
2	Milk and milk products	0.1506 (0.0097)	1.1009 (0.2123)	0.9608	0.1512 (0.0072)	0.4521 (0.1862)	0.9608
3	Edible oil	0.0435 (0.0092)	0.7991 (0.1179)	0.9748	0.0402 (0.0030)	0.6654 (0.0505)	0.9765
4	Meat, fish and eggs	0.0524 (0.0095)	0.4175 (0.1225)	0.9061	0.0406 (0.0032)	0.3991 (0.0604)	0.9118
5	Sugar and gur	0.0278 (0.0094)	0.6564 (0.1207)	0.9403	0.0270 (0.0025)	0.5469 (0.0441)	0.9432
6	Other food items	0.2289 (0.0097)	2.4216 (0.2811)	0.9694	0.2260 (0.0122)	1.5243 (0.2756)	0.9720
7	Clothing	0.0647 (0.0086)	1.0932 (0.1487)	0.5360	0.0855 (0.0084)	0.3337 (0.2056)	0.5892
8	Fuel and light	0.0424 (0.0092)	1.5716 (0.1418)	0.9614	0.0480 (0.0027)	1.2476 (0.0726)	0.9671
9	Other non-food items	0.3527 (0.0095)	2.6596 (0.4622)	0.9679	0.3327	1.2888 (0.4690)	0.9677
			T = 0.0076			T = 0.0081	

All India Urban: Group V

1	Cereals	0.0124 (0.0065)	7.6253 (0.1420)	0.8655	0.0186 (0.0048)	6.6439 (0.2751)	0.8692
2	Milk and milk products	0.0924 (0.0065)	6.3125 (0.4460)	0.9158	0.0921 (0.0038)	1.2018 (0.3760)	0.9305
3	Edible oil	0.0126 (0.0065)	1.9846 (0.1531)	0.9434	0.0123 (0.0009)	1.2884 (0.0636)	0.9469
4	Meat, fish and eggs	0.0247 (0.0065)	1.9692 (0.1774)	0.8219	0.0252 (0.0015)	0.5903 (0.1184)	0.8716
5	Sugar and gur	0.0110 (0.0065)	1.5293 (0.1514)	0.8870	0.0110 (0.0009)	0.8835 (0.0604)	0.8853
6	Other food items	0.1443 (0.0065)	10.6081 (0.6607)	0.8808	0.1288 (0.0128)	3.6358 (0.8979)	0.8979
7	Clothing	0.0994 (0.0065)	5.5231 (0.5542)	0.8997	0.0993 (0.0044)	−0.8298 (0.5540)	0.8959
8	Fuel and light	0.0268 (0.0065)	3.3279 (0.2173)	0.9074	0.0281 (0.0012)	1.4927 (0.1368)	0.9378
9	Other non-food items	0.5766 (0.0065)	23.3548 (0.2986)	0.9813	0.5847	−13.2699 (2.9875)	0.9759
			T = 0.0112			T = 0.0126	

Note: The T's are Thiel's average information inaccuracy parameters.

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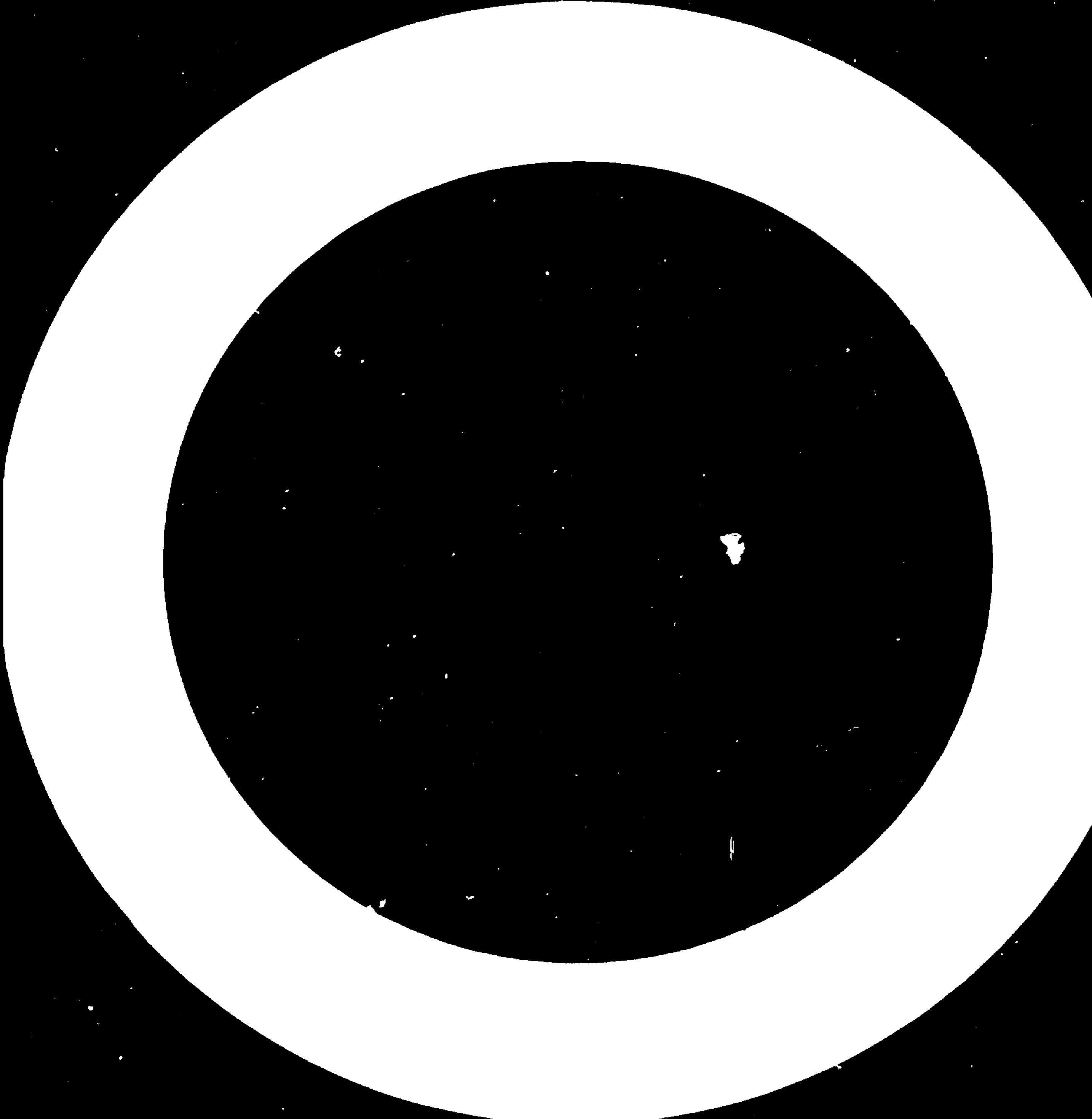
Part three

Summary



Where are we now? A short account of the development of input-output studies and their present trends

Richard Stone*

In what follows the present stage of I/O analysis will be set in its historical context. After that the growing points for the future will be identified. Although economists and statisticians engaged in I/O analysis will continue to be largely concerned with problems that have preoccupied them in the past, the cumulative effects of the new developments are likely to bring about considerable changes in the subject as it is known today.

The background

I/O analysis began with Wassily Leontief's paper of 1936 [1] and his famous book which followed it five years later [2]. It is possible to find precursors to his work. François Quesnay, whose *Tableau Économique* dating from the mid-eighteenth century has been set out in I/O form by Phillips [3] and Barna [4], is often referred to. The construction of equations relating input and output was suggested by Walras [5] and Dmitrief [6]; and a "chessboard" table for the Union of Soviet Socialist Republics was published by the USSR Central Statistical Board [7]. These precursors did not quite achieve I/O analysis. In his paper on the use of mathematics in economics, Nemchinov [8] lists what he considers as Leontief's initial contributions to the subject, including the all-important formulation of the first model connecting inputs and outputs, which made it possible to calculate indirect as well as direct inputs and thus to carry out the many, now familiar, analyses which depend on being able to do this.

Thus, it was some forty years ago that Leontief provided an I/O model and a data base in the form of small tables for the United States in 1919 and 1929. The new ideas were accepted and were rapidly developed in a number of areas.

The construction of I/O tables

In the second edition of Leontief's book [2], published in 1951, a table was added for the United States in 1939; and in the same year the documentation for the first official table for that country, constructed by the Bureau of Labor Statistics and relating to 1947, began to appear [9]. Other early examples were: in

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Denmark, Department of Statistics [14, 15]; in Italy, Chenery, Clark and Cao Pinna [18]; in the Netherlands, Central Bureau of Statistics [16, 17]; and in the United Kingdom, Barna [10, 11], Central Statistical Office [12] and Stewart [13]. More and more countries followed suit and, at a later date, lists of tables and their main characteristics were set out by the United Nations Statistical Office [19, 20].

I/O taxonomy

Work on the construction of tables naturally gave rise to discussions on questions of classification, definition and treatment. After publishing its standardized system of national accounts, the Economics and Statistics Division of the Organization for European Economic Co-operation (OEEC) [21], under Milton Gilbert, arranged meetings on specialized aspects of a complete system. In the case of I/O it seemed that the time was not ripe for agreement on taxonomy. There was not yet much experience in many countries and what agreement there was had not been the subject of wide discussion. Much later, the United Nations tried to deal with the main issues [19, 20].

I/O and the national accounts

In the beginning I/O was often regarded as something quite separate from the national accounts and, in some countries, the group constructing I/O tables was distinct from the group constructing the national accounts. Initially this may not have been a bad idea since it ensured that each subject was developed on its merits before an attempt was made to integrate them. But with the development of disaggregated models covering many aspects of the economy the separation became a nuisance because the different bodies of data tended to develop their own taxonomies and more work had to be done to make them compatible. Stone and Utting contributed a paper on I/O in a system of national accounts [22], to the first conference on I/O techniques (it was not called that at that time) which was held at Driebergen in the Netherlands in 1950. It was a cosy affair. The promoter was Jan Tinbergen and there were about twenty-seven participants. A comparison with the numbers attending this conference, the seventh in the series, gives some idea of how the subject has grown in three decades.

The setting of I/O in the framework of the national accounts played an important part in the report by Stone [23], published by the OEEC and arising from the discussions of the fifties. The problems were worked through in detail in terms of experience of the United Kingdom and data by the group engaged on the Cambridge Growth Project; and the social accounting matrix (SAM), which formed the main data base for the group's model of the economy of the United Kingdom was published in Cambridge Department of Applied Economics [24]. This experience provided some ideas for the revision of the United Nations system of national accounts, published 1968 [25].

Prices and quantities

It was known from the outset that along with the quantity model, which enables total outputs to be derived from a knowledge of final demands there is also a price model, which enables the total cost per unit, or price, of each product to be derived from a knowledge of the primary input cost per unit. This makes it possible to study price repercussions and the transmission of inflationary pressures. Interesting early work in these areas can be found in Goodwin [26, 27] and in Rasmussen [28]. From a different point of view the model system provides a basis for the construction of consistent systems of index numbers of prices and quantities as described by Stone [29] and by the United Nations Statistical Office [25]. This approach to index numbers, which involves the measurement of inputs as well as outputs, leads naturally to the method of double deflation. It might be supposed that this method would always be superior to single deflation, in which a single indicator is attached to a net output weight. But, as is shown by Hill [30], this presupposes that indicators are not subject to error; if they are, it may be preferable to use an indicator of output or input by itself.

Dynamics

Leontief formulated a dynamic version of the model [31] by adding a term in the change in the output vector premultiplied by a matrix of capital coefficients. This led to two developments: theoretical work on the mathematical properties of the dynamic model, and empirical work on the construction of matrices of capital coefficients. Dynamic analysis in an I/O methodology was set out in Dorfman, Samuelson and Solow [32] and various aspects arising in connection with the Cambridge Growth Project were discussed in the Cambridge publication [33] and in Stone and Brown [34]. The dynamic inverse appeared in Leontief [35].

The stability of coefficients, updating and projection

I/O coefficients change and the reasons for this and the problems to which it gives rise have been studied from the beginning. The matter is discussed in Leontief and others [31] and, among many other studies, there are Arrow and Hoffenberg [36], Tilanus [37] and Carter [38]. Sevaldson published a series of studies of the stability of coefficients based on Norwegian experience. In Barker [39, 40] there are some results of updating and projecting intermediate demand based on British experience. In Stone [41] there is a short account of various methods that have been used.

Regional studies

From an early stage I/O has been used as a method of regional analysis. As far as the author knows, the original model was proposed by Isard in 1951 [42]. In it the United States was divided into three regions each with twenty industries. Since it was a fixed-coefficient model, an industry in any region which required a given input would obtain it in fixed proportions from each region in which it was made. A different kind of model, making use of the distinction between locally and nationally balanced commodities, was described in Leontief and others [31]. As far as nationally balanced commodities are concerned, this model shows each region's vectors of imports and exports but not the source of the imports or the destination of the exports. In Moses [43] a model complementary to the preceding one is set out which provides for each commodity a trading pattern connecting the regions pairwise.

Shortly after these theoretical developments, local and regional I/O tables began to appear. Examples are the work of Amsterdam Municipal Bureau of Statistics for Amsterdam [44], Artle for Stockholm [45], Bauchet for the region of Lorraine [46], Chenery, Clark and Cao Pinna for Italy [18], Derwa for the region of Liège [47, 48], Kansai Economic Federation for the Kinki region and the rest of Japan [49], and Moore and Petersen for the State of Utah [50]. This kind of work has continued and by now a great many regional tables have been constructed.

From a wider point of view, countries can be regarded as the regions of a larger area and their industrial structure can be compared in terms of standardized I/O tables. A method for doing this was proposed in Chenery and Watanabe [51]. The matter was discussed at the fifth I/O conference in 1971 but the voluminous material provided by Ambica Ghosh (Economic and Social Commission for Asia and the Pacific (ESCAP) member countries), Vera Nyitrai (Council for Mutual Economic Assistance (CMEA) member countries), Vittorio Paretti (European Economic Community (EEC) member countries) and Jiri Skolka and others (Economic Commission for Europe (ECE) member countries) was not published in the proceedings. Some information for the ESCAP region is, however, contained in Ghosh [52].

The standardization of tables paves the way for the construction of supranational tables. This was apparently first done for the OEEC countries in 1953 by Kirschen and associates [53].

It would not be difficult to think of other topics that have engaged the attention of I/O analysts more or less regularly over the past 30 years, though the 7 topics mentioned cover the main themes. Yet, there are always new themes arising from development of the subject. First, there is the development of the I/O model regarded as a disaggregated model mainly concerned with product flows and the primary inputs needed to sustain them. Second, there is the extension of this model to handle pollution, whether or not it is treated, and to take account of such matters as the distribution of income and questions of money and finance which were not in the past treated on a disaggregated basis in general models of the economy. Third, there is the application of I/O accounting techniques to the organization of demographic and social data and to the building of socio-demographic models. And, finally, there are the applications of computer programming to data processing and to model construction and solution. Let us now take a look at each of these.

The development of the input-output model

In the early stages the I/O model consisted of a matrix of intermediate product flows bordered to the right by one or more vectors of final demand and below by one or more vectors of primary inputs and other production costs such as provisions for depreciation and indirect taxes. Input coefficients were calculated, as a matter of simple arithmetic, by dividing the elements in the intermediate product vectors by the corresponding output total. Apart from a certain number of arithmetical and accounting identities, these coefficients constituted the relationships of the model. They could be arranged in a matrix, usually denoted by A, from which the Leontief inverse, or matrix multiplier, $(I - A)^{-1}$ could be calculated, though with a good deal more time and trouble than is involved nowadays. In the quantity version of the model this inverse transforms final demands into total outputs; in the price version, its transpose transforms primary input and similar costs per unit of output into total costs per unit of output, or prices. Many other interesting calculations can be made with this inverse; the main ones are illustrated in Stone [23].

Forty-five years ago Tinbergen started to build models of the economy. The first related to the Netherlands and various versions of the model can be found in Tinbergen [54, 55, 56]. Shortly afterwards, in 1939, came his well-known studies for the League of Nations [57, 58]. These early models, and the many that followed them in the same spirit, covered many aspects of the economy but in terms of a comparatively small number of macro-variables. They were dynamic and intended for purposes of policy analysis and prediction. They were the forerunners of the macro-economic short-term forecasting models of today.

By contrast, I/O models have developed into disaggregated medium-term planning models. This transition involves a number of steps of which the following are among the more important.

Endogenizing the exogenous variables

Instead of taking final demands as given, we could introduce explanations, at least of some of them, into the model. Consumers' expenditures are an obvious candidate for this treatment and the linear expenditure system provides a convenient formulation since, for a fixed set of prices, expenditure on each commodity is a linear function of total expenditure. The first paper of this author on the subject [59], was presented to the second conference. It is now known from Deaton [60] that this simple system has a serious defect, which it shares with all systems based on additive preferences, namely that it imposes severe restrictions on total expenditure elasticities in relation to own-price elasticities. One thus has the choice of putting up with the defects of the linear expenditure system because of its convenience, whilst adopting other demand equations despite the fact that they may not fit neatly into a general model or searching for a revised formulation which will combine both advantages. The almost ideal demand system (AIDS) proposed by Deaton and Muellbauer [61] is a move in this direction.

Corresponding efforts can be made with respect to the other components of final demand. See, for example, the paper on foreign trade by Barker [62] which was presented at the fifth conference. An exception, perhaps, is government current expenditure on goods and services which seems very difficult to model and can best be based on statements of government intentions or some form of trend projection.

Interactions and feedback

Once the components of final demand cease to be fixed exogenously they are free to vary in response to what is happening in the model; just as outputs depend on final demands, so final demands depend on incomes, which in turn depend largely on outputs but also on taxes and other transfers. As a consequence it is necessary to build consumption functions into the model which will close the output-income-expenditure loop.

Generalizing I/O production functions

The early recognition of the fact that input coefficients change and the efforts that were made to update and project them were mentioned earlier. The methods used represent one way of doing something that has to be done but for the most part they are wholly empirical and do not throw much light on the causal factors at work. For instance, it seems fairly clear that input coefficients must be influenced by relative prices. This suggests that the simple relationships derivable from an I/O table should be generalized and an attempt made to test the generalization.

At the same time there is another issue to be considered. In the initial model, primary inputs per unit of output are either exogenous or residual. In an attempt to explain them one might set up production functions connecting value added and primary inputs. But one would then have, for no very good reason, different functions for primary and intermediate inputs. There is much to be said, therefore, for starting not from value added but from gross output and relating this to all inputs. This was done by Wigley using a vintage model of production [63].

This approach was carried a good deal further by Peterson [64] in a paper presented to the sixth conference. Following Shephard [65] he switched from production functions to cost functions and applied the generalized Leontief function suggested by Diewert [66] which under certain conditions reduces to the fixed-coefficient hypothesis of I/O analysis. For computational reasons Peterson began by aggregating inputs into five groups—new investment goods, fuels, materials, services and labour—and then adopted a two-stage procedure. At the first stage, producers decide on their aggregate inputs of each of the five groups, depending on the expected level of output and a set of group-price indices; and at the second stage, they decide on their demands for individual inputs, taking account of their decision at the first stage and the prices of these inputs.

The method can be modified and the stages applied in the opposite order. For instance, in a paper presented to this conference the first stage was to estimate the input-mix of 7 fuels into 15 industry groups and 2 final expenditure groups, public administration and households, on the basis of British data for the years 1955–1975. The results show the importance of technical change (or changes in tastes) and price substitution in determining this mix and the unreality of assuming that the fuels are demanded in fixed proportions. The second stage was to estimate for each of the using groups a demand equation for energy, that is the aggregation of the individual fuels. These equations fit well and show that in many cases relative prices, temperature and residual time-trends, as well as output levels, are important. By combining the two stages the input of each fuel into each using group can be estimated.

Factor prices and commodity prices

The I/O price model enables commodity prices to be calculated as an accumulation of input costs per unit of output. In the simplest case, certain prices, say import prices, are assumed to rise, and the repercussions on domestic commodity prices are worked out on the assumption that domestic factor prices do not rise. Clearly this is not very likely to happen and the opposite extreme assumption, namely that domestic factor prices rise so as to maintain the purchasing power of domestic factors, can be worked out by treating the household sector as endogenous. When this is done, domestic factor prices will rise in line with the rise in import prices and the international transmission of inflation will be immediate. But this, too, is unlikely to happen and the question is to find out by how much factor prices are likely to be affected in different parts of the economy.

This is a difficult question and the model builder may wish to keep open the option of treating factor prices as exogenous. At the same time there is a very large amount of literature on price and wage equations. This literature should eventually provide a basis for completing this aspect of the model.

Dynamics

The early disaggregated planning models concentrated on a future target year without going into the question of the path from the present to that year. These models may have contained time lags in a number of their equations, and so dynamic elements were present, but these were not sufficient to calculate a path except, perhaps, under rather restrictive assumptions about the development of the economy and the normality of the target year. For instance, in a static model it seems acceptable to determine stockbuilding by means of fixed coefficients whereas in fact stockbuilding is likely to prove a volatile element exerting a dynamic influence, if only a transient one, on the economy.

With the introduction of dynamics, disaggregated models come to share an important feature with macro-economic models of the Tinbergen type, which have been dynamic all along though their time horizon is usually not much more than 8–24 months.

Testing the model as a whole

If one tries to explain an econometric model to natural scientists their first reaction is usually to inquire what has been done to test the model's performance. It can usually be said that bits of the model have been tested; that is, the consumption, production or foreign trade functions give a good fit over the period of observation and even over later years if they are supplied with the subsequently observed values of the relevant determining variables. But if the model is limited so that, implicitly or explicitly, it always refers to normal years or if important variations are fixed by assumption, it is difficult to carry out and interpret tests of the model as a whole. A complete dynamic model allows us to get over this difficulty since it enables one to make predictions, always given the expected values of the exogenous variables, rather than highly qualified projections. The model can be started off at the outset of the observation period and run through it into the future. One is then able to see, for example, how the consumption functions work when they are presented with disposable income and other endogenous determining variables as worked out by the model rather than with the observed values of these variables, as in the partial tests.

Policy simulation, control and optimization

One of the purposes of building econometric models is to contribute to policy making. This can be attempted in various ways.

The first is policy simulation, an essentially exploratory technique involving neither control nor optimization, which consists of running the model on different assumptions and, in particular, assumptions relating to instruments considered relevant to the policy issues under discussion. The method is useful but has the disadvantage that, unless the issues and the acceptable means of tackling them are narrowly defined, the number of simulations to be performed may be very large.

The second method, which provides the model builder with a clearer picture of what is to be aimed at, is the targets and instruments approach introduced by Tinbergen [67, 68]. This consists of setting up a number of targets, such as the level of the balance of payments, the number of unemployed and so on, and then working out how to set policy instruments so as to hit these targets. An account of how this can be done and a number of illustrations of doing it drawn from the Cambridge Growth Project's static model of the British economy can be found in Livesey [69]. The application of the method to a dynamic model is a larger problem because it is necessary to think in terms of the time paths of targets and instruments. Such an application has not yet been attempted but it does not seem to involve difficulties of principle.

The main problem with this method is that while it is possible to work out trade-offs between instruments it does not provide a basis for working out trade-offs between targets.

The third method, which does allow these calculations to be made, is programming. But this may be thought to require too much information since one must be able to set up a utility function to guide the choice of the degree to which different targets should be approximated. Further, it must be possible to give the problem a particular form if it is to be solved by known computational procedures. Thus, the standard quadratic programming problem requires the variables to be non-negative, the constraints to be linear and the utility function to be the sum of a linear and a quadratic form which must be negative definite if the global optimum is to be unique. In the present context an application of this method would seem both difficult and uncertain. Difficult because it is hard to see how the utility function could be constructed. Uncertain because of the restrictive conditions required to reach a solution.

Extensions of the input-output model

In the preceding section the intention was to trace the changes that have been taking place in the original I/O model. This section deals with what can fairly be called extensions in the sense that they were not regarded as part of I/O analysis in the early days.

Accounting for pollution

A method of introducing pollutants and the corresponding treatment services into an I/O table was proposed by Leontief [70]. It consists, essentially, of setting up a number of additional columns containing the cost structures of the treatment services and an equal number of rows containing the emissions of pollutants which could be handled by the corresponding service. Emissions appear among the cost elements of the various industries because they provide a measure of the output of a treatment service needed to remove them. This arrangement can be applied to final users as well as to productive activities.

This neat solution leaves for further discussion the question of how much of each pollutant should be treated. If total emissions appear in the cost structures it is implied that they should be treated in full, but the community might prefer only to treat them partially and, as a consequence, have more resources to devote to the production of "regular" goods and services. Furthermore, the community is likely to be more interested in the state of the air, land or water after treatment than in the amount of treatment carried out. These issues were discussed in Stone [71] and Meade [72].

From a practical point of view there is the further point that a certain amount of pollution has always been treated either by the polluter himself or by some public service. Interesting accounts of the conceptual and statistical issues that arise in environmental measurement and of the amount of national expenditure on pollution abatement and control, which have been prepared by the United States Department of Commerce since 1972, are given in Cremeans [73]. Clearly the full implementation of Leontief's original proposal would be an extremely difficult undertaking.

Papers on pollution have featured in our last two conferences, including an application of Leontief's scheme to the problem of air pollution by Leontief and Ford [74] at the fifth conference; and contributions by Cumberland and Stram [75], Hartog and Houweling [76] and Thoss [77] at the sixth conference. Further results from Thoss are given in Thoss and Wiik [78].

The distribution of income

I/O started off as a means of analysing the productive system and disaggregation was confined to branches of production and their products. For this purpose there is no need to disaggregate the income and outlay accounts of sectors; and this was not done in standard systems of national accounts either. Even the revised SNA [25] contains only a small number of industries and no division of the household sector. At the time, the United Nations Statistical Office was working on a complementary system of distribution statistics for households; provisional guidelines were published in 1977 [79].

Given the necessary information it is possible to set up an I/O system for the income and outlay accounts for sectors which receive income, make transfers among themselves, spend for consumption and save. This is demonstrated in Pyatt, Roe and associates [80] and is further illustrated in the papers presented at the conference on social accounting methods in development planning held in Cambridge in the spring of 1978 under the sponsorship of the World Bank Research Program.

Another development in this field, but one which is more akin to the socio-demographic models treated below, is to treat the generation of income as a Markov process. This was proposed by Champernowne in the 1930s in a thesis which was not published at the time but eventually appeared in revised form in 1973 [81]. For other publications in this field by the same author see [82, 83]. Many applications of this method have been made, for instance by Vandome [84], Esberger and Malmquist [85], and Shorrocks [86].

Wealth and the flow of funds

As was pointed out by Stone [87], capital account and balance sheet data can be set up in an I/O format and used as a basis for simple linear models of wealth and the flow of funds. This type of question was discussed at the fifth conference by Isayev [88] and Roe [89]. The second of these authors drew some conclusions from an application of the simple model which he had worked out on the basis of British data in [90]. As was to be expected, this model provides some insight into structural relationships but is unsatisfactory as a forecasting device because of changing coefficients. No doubt something could be done, as it is in applications to the productive system, but it is doubtful if in this instance it would work very well since financial coefficients are likely to be extremely volatile, so that a more sophisticated model is required.

International trade and world models

Although the subject of international trade and world models has perhaps always been recognized as a part of I/O analysis, world trade models until about 10 years ago were all quite small. In Taplin [91] it was recommended that a multiregional, multicommodity model be built in terms of 10 or 12 regions and 6 categories of goods. A step in this direction is described in Thorbecke and Field [92]. Their model is a short-term, demand-oriented model based on annual data over the period 1953–1967 and appears to perform fairly well over the observation period.

In the meantime a world income and trade model, MEGISTOS, was constructed, which is described in detail in [93]. Its purpose was to provide a coherent system of projections for the main economic aggregates at the world level in 1975. The countries of the world were grouped into 12 regional zones and linked together in terms of trade, aid and borrowing. Another type of model, Project LINK, is discussed in [94] and [95]. The first of these volumes describes the conceptual framework within which the integration of individual country models is taking place and presents some of the underlying empirical work. The second sets out the equations used in the models of the 13 collaborating countries, as they were in late 1973, together with those of the regional models for developing areas prepared by UNCTAD.

The subject was represented at the sixth conference in papers by Tokoyama and others [96], Petri [97] and Panchamukhi [98]. Another recent study is given in Nyhus [99].

The latest contribution in this field, described in the study by Leontief and others, is *The Future of the World Economy* [100]. This is a highly disaggregated model with 15 regions and 45 branches of economic activity. It is static and does not contain explicit feedback mechanisms or optimization. It embodies exogenous population estimates and handles international trade in terms of a single international trading pool to which all exports are delivered and from which all imports are drawn. It was constructed in the first instance for the study of development in relation to environmental questions, and provision is made for calculating the cost of pollution abatement. It is, however, a general-purpose economic model which can be used to analyse the evolution of the world economy from many points of view.

New worlds to conquer

About 15 years ago work began on the flow of pupils and students through the British educational system and it was shown that an I/O scheme could be very useful in organizing information of this kind [101]. Later a paper on what could be called demographic I/O was contributed to the fourth conference [102]. Shortly afterwards, at the suggestion of Philip Redfern, who had treated similar problems [103], the format was altered although the scheme remained essentially dynamic. In the later version the numbers in the opening and closing stock-vectors of the population are connected by flows from the beginning of the year to the end of it. Just as in the economic quantity model the matrix multiplier transforms final demands into total outputs, so in the demographic model it transforms the new entrants, that is the year's births and immigrations, into the closing population stock-vector. As in the economic case, there is also a price model which combines with the quantity model to generate the equivalent of the identity national income equals national expenditure.

As can be seen from [104] and [105] the possibilities of applying this model in social demography are almost unlimited. In such applications the place of I/O coefficients is taken by transition proportions; and if these can be interpreted as probabilities, then the process studied is an example of an absorbing Markov chain. An analysis of the Norwegian educational system from this point of view is given by Thonstad [106].

Among the many writers who have contributed to this kind of analysis are Andorka and Illés [107], Coleman [108], whose paper was presented at the fifth conference, Cooper and Schinnar [109], Fox [110], Rees and Wilson [111], Schinnar [112, 113] and Shishido and others [114].

Automating model building and analysis

Over the last generation the amount of data available for economic and social model building has greatly increased and the models have tended to become larger and larger. As can be seen from the first paper given at this conference by Barker, Peterson and Winters, the present version of the Cambridge multisectoral dynamic model, MDM3, contains 2,759 equations, of which 759 are stochastic, 7,484 variables and 12,884 coefficients. It is solved year by year by the Gauss–Seidel iterative method, each year requiring 10 or 25 iterations to obtain adequate convergence and taking 4 to 6 seconds of computing time.

The increase in the size of models has had two consequences. First, the amount of data processing, estimation, solving and testing has grown enormously; and, second, large models are inconvenient for many experimental purposes in which the amount of detail they contain is irrelevant. Fortunately, at the same time, the speed and power of computers has also been growing and new algorithms and programming methods have been devised. These developments have made a number of procedures possible which greatly ease the task of the model builder.

The adjustment of social accounting matrices

Social accounting matrices are compiled from a variety of sources and this is also true of parts of them such as I/O tables. Inevitably the data used are in some degree incomplete, inconsistent and of varying reliability. When the pieces are assembled a number of discrepancies are revealed and there are also some gaps: not so very long ago a number of countries derived their estimates of consumers' expenditure as a residual. The discrepancies could be removed by applying the classic technique of the adjustment of conditioned observations by the method of least squares, provided one was willing to make a subjective estimate of the variance matrix of the errors. This application was suggested in Stone, Champernowne and Meade [115] and on a number of subsequent occasions small examples to illustrate the method were worked out. For various reasons it did not catch on. The most important was the amount of computing involved. This difficulty is overcome by Byron [116] in which the problem is reformulated in terms of a quadratic loss function and use is made of the conjugate gradient algorithm. The method is applied to a matrix of order 46 and it is feasible to adjust a monster matrix, say of order 1,000.

A generator for input-output tables and models

Since 1962, research has proceeded at the IFO Institute in Munich on the possibilities of automating the construction of tables and the estimation of the parameters of the corresponding models; and in 1975 these ideas began to be applied to the construction of interregional tables, as reported in the paper by Gehrig and others of the University of Frankfurt to this conference. It is recognized that tables are based on statistics taken from sources of varying reliability and on estimates and guesses made with varying degrees of expert knowledge. Detailed instructions must therefore be given to the computer not only on the definitions and classifications to be used on the type of tables to be constructed but also on the means of combining the basic data. This brings us back to the problem considered above.

These developments seem likely to prove of great value because most of the large models that are built these days are intended to continue. It is only if they do continue that they, and econometric model building in general, are likely to improve. In maintaining such models there is the continuing need to introduce data for the latest year, to revise data from the past, to rebase index numbers and so on. Even with good worksheets and data banks this is a laborious process and one which cannot easily be undertaken at any moment as a new batch of data becomes available. The same is true of re-estimating parameters. By encouraging model builders to reduce everything to rules, the problems created by inevitable changes in staff would be diminished.

A program for model solution

In the second paper presented to this conference by Barker, Peterson and Winters, the experience gained in solving the Cambridge Growth Project's series of models has been used to write a general program which can be used to solve any model of the general type of MDM. This has been useful in forcing one to think through every aspect of the solution of large econometric models of the economy. It is hoped that it will be useful to others who would prefer to devote their energies to the basic statistics and underlying economics of their model rather than to devising an efficient method of solving it.

The condensed form

As econometric models grow in size, it becomes more and more difficult to see what is going on in them and they become less and less convenient for experimental purposes. It would be interesting to know therefore if the model could be boiled down to a small number of equations connecting its macro-variables without destroying its essential features and losing much more than detail.

This model of the model was termed the "condensed form" in [117] and was applied to the static model. In the first paper presented to this conference by Barker, Peterson and Winters it is applied to MDM3, the latest version of our dynamic model. As can be seen, the results are satisfactory, doubtless because MDM3 has a comparatively simple structure although it is very large. It is thought that this development will be useful for pedagogic and experimental purposes.

Conclusions

I/O analysis has immense vitality shown by its ability to transform itself from simple beginnings into more and more complicated forms; its capacity to throw light on branches of the social sciences other than the one in which it originated, and its contribution to new ways of using computers which its very success has stimulated.

There is still a good deal of life in the traditional topics though it is hoped that before too long questions of how to construct I/O tables and of I/O taxonomy are settled. The development of the I/O model seems to be leading in directions in which its I/O core is becoming less and less discernible. This is as it should be, because it shows the possibility of improving the very simple relationships which were used initially. But it must be remembered that it is precisely simple relationships like these which got disaggregated econometric model-building off the ground and that without them the first step forward might have been long delayed. The extension of I/O ideas to other aspects of economics shows the power of those ideas. Probably they too will eventually give way to more sophisticated formulations. Applications to social demography may be considered more directly connected with Markov chains than with I/O; however, there have been sessions on these applications at the last three conferences and the subject is represented at this one. Moreover, the recognition that for each quantity equation there is a corresponding cost equation seems a distinctive contribution from I/O. As regards developments involving computing, these are only to be expected in subjects which involve data processing and parameter estimation on a large scale and the solution of large systems of non-linear equations.

It seems clear enough that I/O has developed enormously in its 40-odd years of life. In making this survey it is regretted that a most unbalanced account of the literature in terms of sources and languages has been given. One tends to cite the contributions with which one is most familiar; it is hoped that other authors will correct this one-sidedness.

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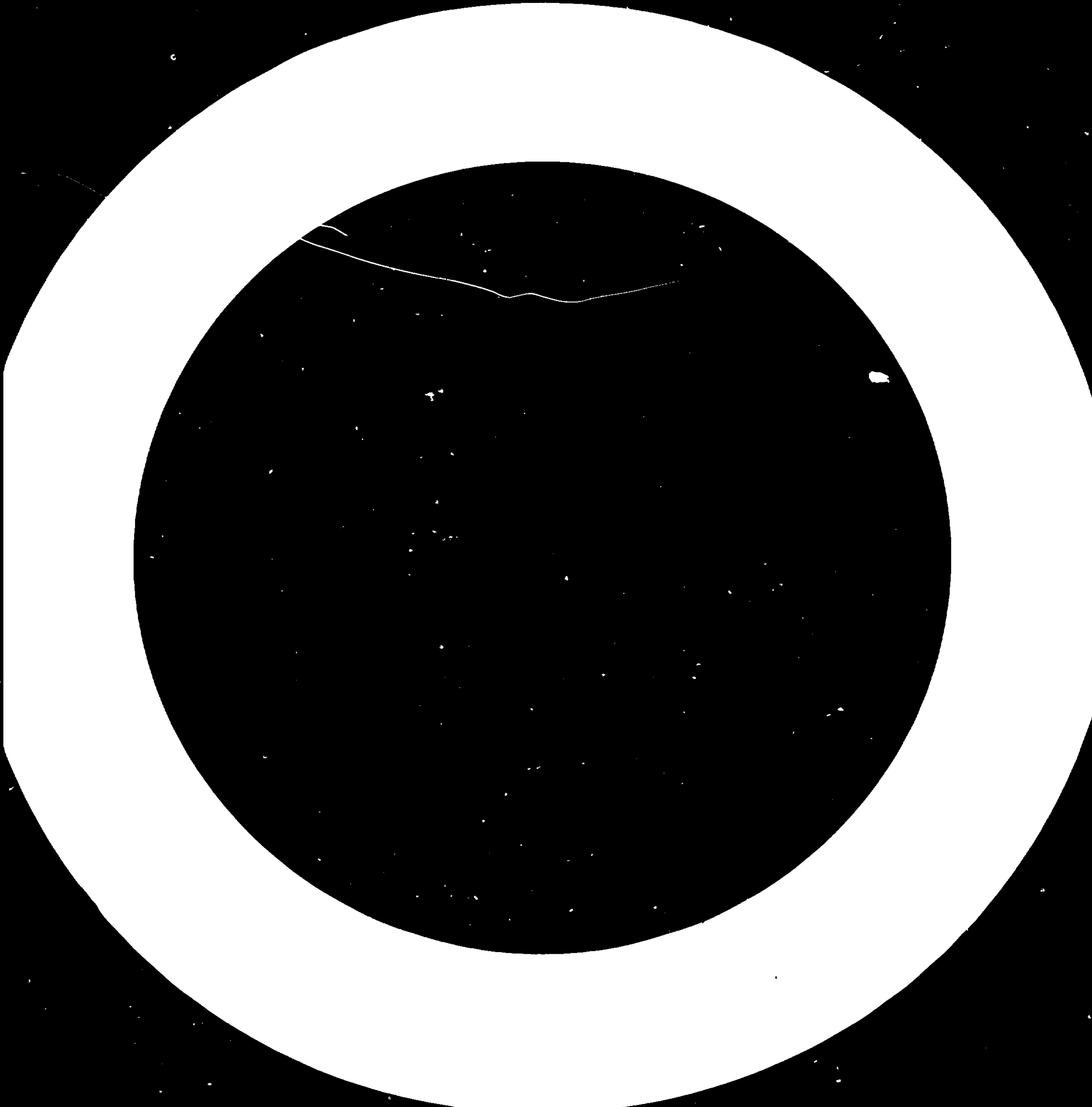
Appendices



Appendix I

WORKSHOPS

Title	*Discussion leader*
Multi-regional models and economic growth	J. Waelbroeck
Adjustment and the use of RAS	A. Parikh
Demographic and distributional factors	S. Bhattacharya
Changes in input-output coefficients	E. Wolff
Optimization methods in input-output models	J. Tsukui
Dynamic exploratory models	M. Augustinovics
Modelling prices and price sensitivity	J. Schumann
Interdependence between financial and technical structures	Y. Torii
Compilation of input-output tables by official statistic offices	K. R. Blackburn
Changes in coefficients: Input-output models	A. Parikh
Planning	V. Kossov
International comparisons	F. Fink
Energy and resource problems	J. Segura

Appendix II

SESSIONS OF THE CONFERENCE

Session	*Topic*	*Chairman*
1	Global and Multinational Models	W. Leontief
2	Patterns of Economic Growth in Developing Countries	A. Ghosh
3	Income Distribution and Social Welfare	A. P. Carter
4	Europe in 1985 and After—Joint Meeting with the European Scientific Association of Applied Economics	E. Fontela/A. Bródy
5	Input-Output in Large Econometric Models	J. Tsukui
6	Planning and Optimization	E. Baranov
7	Estimation, Adjustments and Comparisons	J. Skolka
8	Where Are We Now?	R. Stone

List of papers presented at Session 1

Title	*Authors*
The Inforum-IIASA International System of Input-Output Models	Clopper Almon*
A Variable Sector World Input-Output Model	Anthony Bottomley, John Dunworth, Colin Carpenter, Donald Nudds, Michael Lloyd, Martin Curtis, Alexander Diediw, Refat Al-Kamizy, Mark Berka, Richard Darwin, William Mould, Philip Williams
A Six Sector World Model	Anthony Bottomley, M. J. Lloyd
The University of Bradford Input-Output Data Base	Anthony Bottomley, M. J. Lloyd
The United Nations World Model: Experimental Calculations on the Basis of Alternative Procedures	Antonio Maria Costa
Some Lines on Development of the United Nations Global Input-Output Model	A. Granberg, A. Rubinshtein
The Global Model Behind the World Development Report of 1978 of the World Bank	A. Gupta, A. Schwartz, R. Padula
World Bank Global Modelling Research	Syama P. Gupta, Jean Waelbroeck
Projet d'articulation de moise et d'un modèle dynamique multinational	Gerard Lafay
The Population Effect on Leontief-Strout Gravity Coefficient and Derivation of Quantitative Implications of Inter-Regional Co-operation among Developing Countries	P. N. Mathur*
Canadian Implications of the United Nations World Input-Output Model	Harry H. Postner*
An Econometric Model of International Trade	Jean-Jacques Snella
Converting Regional and Multiregional Trade Coefficients into Endogenous Variables	George I. Treyz, Benjamin Stevens, Karen R. Polenske

*The asterisks refer to papers personally presented by their authors at Sessions 1–8.

List of papers presented at Session 2

Title	*Authors*
IDIOM — An International Dynamic Input-Output Model	Terry Barker, William Peterson, L. Alan Winters
Input-Output Tables for Israel	S. Bar-Eliezer
A Methodology of Cross-Sectional Comparisons of Production Structures	J. M. Blin, E. A. Stohr
Planning Tools in Tunisia Models and Input-Output Tables	Mamou Hamza
Long-Term Projections of Economic Growth in ESCAP Member Countries	Y. Kaya, A. Onishi, H. Smit, S. Abe*
Sources of Industrial Growth and Structural Change: A Comparative Analysis of Eight Countries	Yuji Kubo, Sherman Robinson
Financial Interdependence and Economic Growth: An Input-Output Approach	A. A. Kubursi, M. G. Vargas
Structural Interdependence and Economic Development — An Interpretation of International Time-Series Data	S. K. Mukhopadhyay
A Social Accounting Matrix for Egypt	Osman Mohamed Osman
Technological Change and the Pattern of Economic Development	I. Ozaki, M. Shimizu*
Meaning and Measurement of Real Growth	Utz-Peter Reich
Public Consumption Turnpike Model and the Long-Term National Planning of Japan	Hideto Sato, Tetsuki Hirose, Hiroshi Niida, Kazuhiko Tokoyama, Jinkichi Tsuki
Impact of Redistribution of Financial Resources on Employment and Industrial Structure in Italy	Gustav Schachter, Solomon Kenea
Economic Development and Changes in Linkage Structure: An Input-Output Analysis of Korea and Japan	Y. Torii, K. Fukasaku*
A Non-Linear Input-Output Model in Physical Units and its Application in China	Chen Xikang, Sun Shizheng

List of papers presented at Session 3

Title	*Authors*
The Utilization of Input-Output Analysis for Planning Basic Structures Affecting the Satisfaction of Needs in the German Democratic Republic	J. Behr, M. Kraft, U. Ludwig

Income Determination in the Inforum Model of the U.S. Economy	David Belzer
Adjustment Properties of Input-Output Models, Markov Interregional Trade Models, and Population Systems	Ivar E. Bradley, Stephen F. Seninger
The Forecasting Balance of Population and Labour Resources Drift	B. D. Breyev, O. V. Staroverov
Fixed Capital without Joint Outputs: Disequilibrium Growth in a Vintage Multi-Sector Economy	T. A. Broadbent
Généralisation d'une analyse sectorielle de la valeur par les tableaux I/O	Jacqueline Fau
The Introduction of a Non-Homogeneous Labour Force in an Input-Output Framework	Luigi Filippini, Carlo Filippini
Income Distribution and Production Structure in Input-Output Framework	A. Ghosh, A. K. Sengupta*
Demographic Changes and Economic Consequences: Demo-Economic Multiplier for Austria	M. Lubtacik, I. Schmoranz
Income Distribution and Trade Policy in a Multisector Planning Model	Jame de Melo,* Sherman Robinson
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Title	*Authors*
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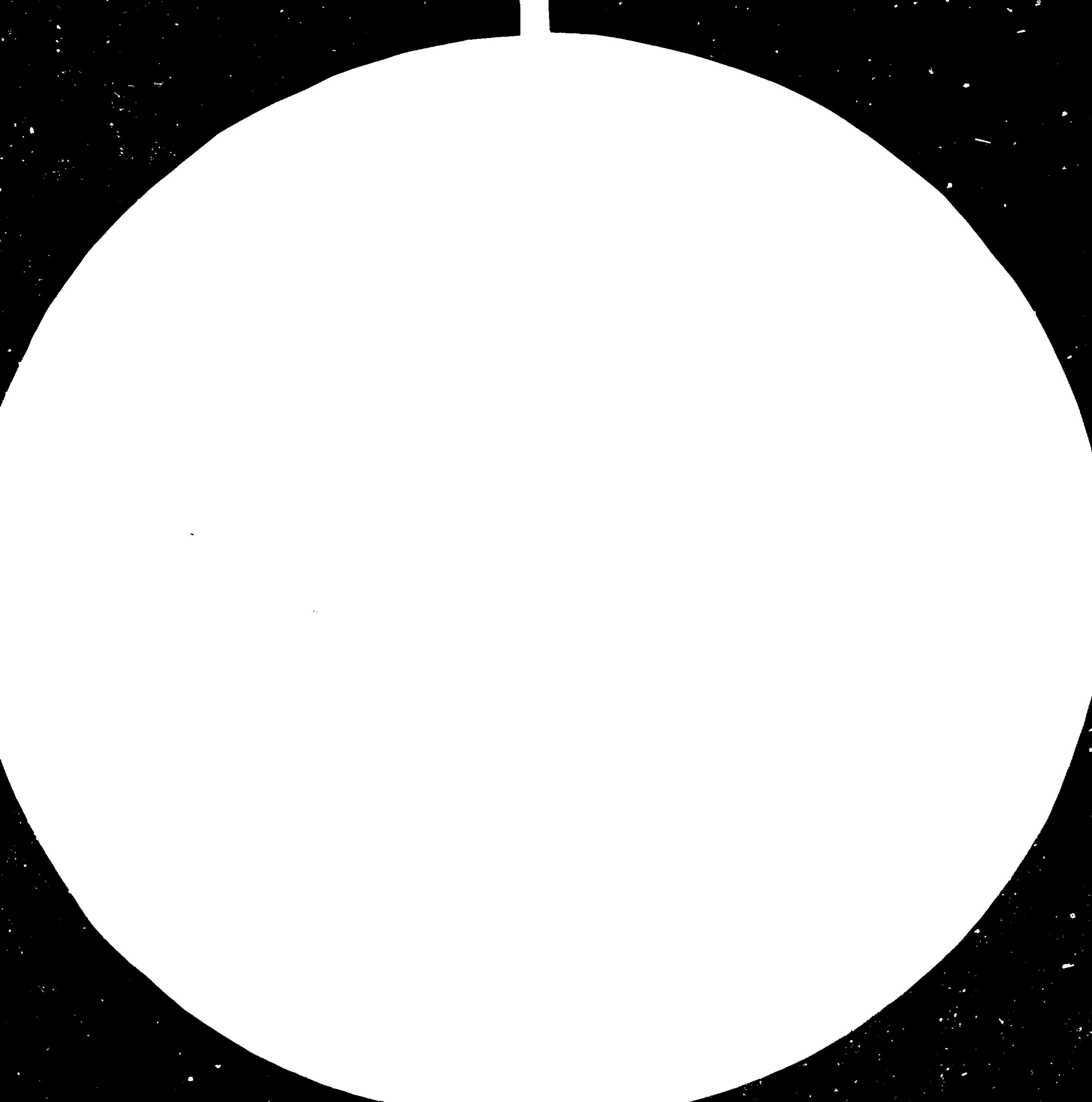

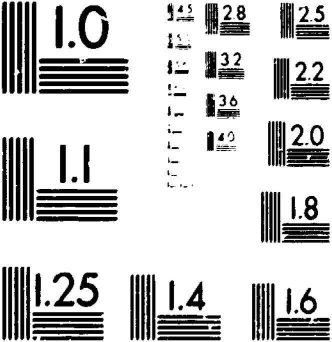

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