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Using Statistics for Process Control and Improvement

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An Introduction to Basic Concepts and Techniques

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Using Statistics for Process Control and Improvement

An Introduction to
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PREFACE

Competitiveness, competitive advantage, competitive success: These are key words in nearly every discussion about industry in a global environment. Among the determinants of competitive success of industries are conventional factors like labour costs, capital investment, workers' skills and technological capability. However, there is also one fairly recent addition to the list of sources of competitiveness which is usually circumscribed by terms like quality assurance, total quality management or continuous improvement. This has to do with the way in which inputs to production are deployed. A firm's prosperity, as well as a whole industrial subsector's ability to compete in the international market, will increasingly depend also on the adoption of techniques for continuous improvement.

Normally continuous improvement involves a large number of minute adaptations or innovations in day-to-day operations. While potentially such modifications are an important source of competitive strength, a precondition for realizing this potential is a systematic approach to improvement. In order to fulfil this requirement, statistical analysis is needed. Statistical methods enable users to identify those areas that need improvement, to separate the essential from the not-so-important, and to ensure that adaptations do not have conflicting effects. In short, the application of certain statistical tools guarantees objectivity and ensures that progress is genuine and lasting. And this is true for large and small firms in all sectors of economic activity, both in developing and developed countries.

The present text sets out to provide a first introduction to and basic guidance in the use of statistics for process study and improvement. It aims to achieve this goal by following three guidelines of presentation. First, from the broad range of statistical tools in this area, a representative set is selected for discussion. Second, these tools are discussed in considerable detail, comprising an outline of the objectives of their use, an operational description of underlying calculations and accompanying graphical techniques, and ample illustration by way of examples of industrial processes. Third, in presenting statistical methods, an attempt is made to outline—wherever it seems appropriate—the broader context of management issues in which these methods have to be embedded.

As well as serving as a basic introduction to the above subject for a general audience, the present monograph can be used in at least two specific ways. One is as part of the material used in an introductory course on statistical methods of process improvement. Another is for self-study by managers or production engineers. For the latter use, the main target audience targeted would be readers with managerial or technical functions in an organization that is planning to introduce methods of quality management or continuous improvement. In particular, personnel of small or medium-sized firms with limited access to training in improvement methods may benefit from this elementary text on objectives, principles and methods.

The plan of the book is as follows: A brief general discussion of the role that variation and analysis thereof play in efforts at continuous improvement of processes provides the starting point in Chapter 1. Chapter 2 further prepares the ground by outlining basic facts and principles of the collection and summarizing of data to be used for process study. Chapter 3 introduces control charts and their practical application for the case of counts data, while Chapter 4 describes the analysis of other data on product attributes. Chapter 5 introduces the major concepts underlying control charts for variables data, discusses the construction and use of those charts and outlines the analysis of process capability. Chapter 6 gives a concise overview of the analysis of components of variance in the context of process improvement, emphasizing the important role of subgrouping to ensure usefulness of the results of statistical process analysis. Finally, Chapter 7 deals in some detail with the analysis of measurement processes and their impact on process analysis. The brief text is completed by a set of practice problems and a short bibliography. The latter is intended to point the reader to material not only on the statistical aspects of the subject but also on the broader managerial issues and the general background of total quality management.

Acknowledgements

The United Nations Industrial Development Organization (UNIDO) has been active in the related fields of quality control, statistical process control and standardization for a number of years. The contribution to this book by UNIDO staff with experience in the aforementioned areas is gratefully acknowledged. Special thanks are due to Dr. Richard Sanders and Dr. Mary Leitnacker of the Management Development Center at the University of Tennessee who served as consultants, providing a manuscript on statistical process control. Much of this material was used, when Helmut Forstner of UNIDO's Industrial Statistics Branch drafted the present publication. Finally, thanks are due to Brigitte Leutner for graphical work and Suzanne Abdou, Elizabeth Cruz and Anna Padickakudy for preparing the manuscript for print.

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CHAPTER 1

CONTINUOUS IMPROVEMENT AND THE STUDY OF VARIATION

The present text is intended to provide basic guidance in the statistical analysis of systems and processes of production. It has to be seen in connection with the use of indicators to evaluate a firm's operations where the goal usually is that of improving performance in various respects. For achieving this goal it is crucial to take into account in the analysis the variation in performance measures. In particular, the sources of such variation have to be identified and correctly interpreted. Such interpretation together with an understanding of the effects of variation on system performance are prerequisites for successful systems management.

1.1 The interpretation of variation

Monitoring performance by use of indicators is a well known and widely accepted business practice. Depending on circumstances, such indicators might include throughput (per direct labour hour), the cost of materials purchased (per unit of production), the value of goods in inventory, and many other data. Numbers of this kind can reveal trends in overall business activity, performance, efficiency and help to assess differences in performance among production sites. More generally, monthly values of indicators are a means for monitoring business and provide a basis for improving performance.

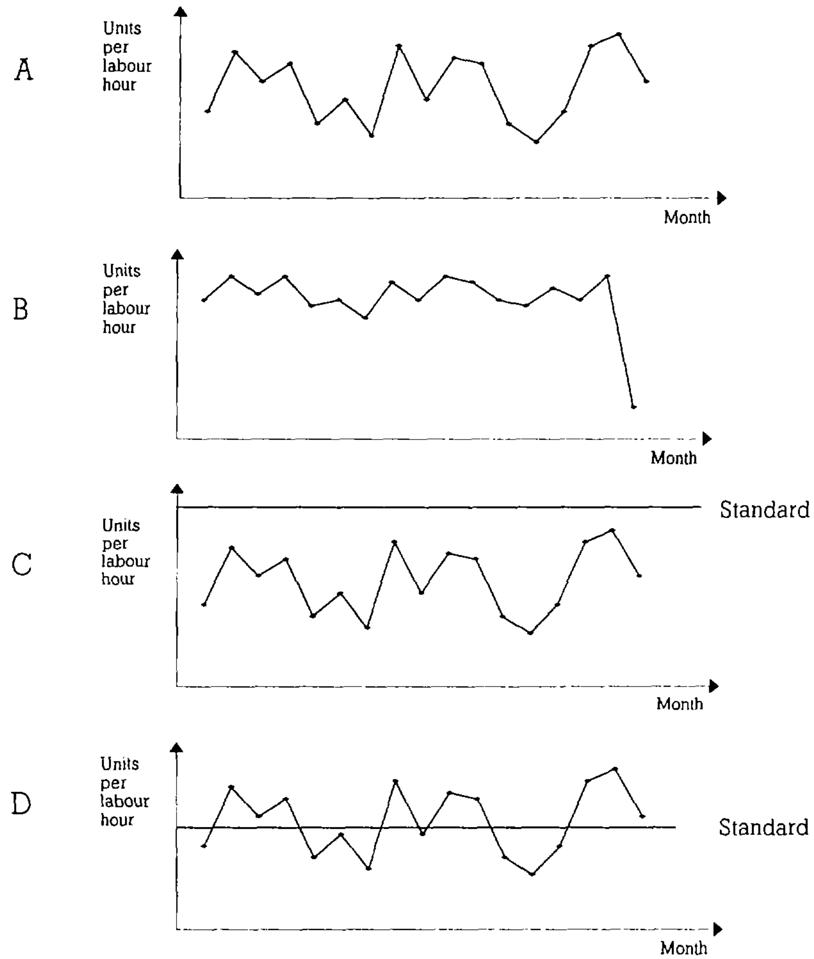
Any user of performance indicators knows about their **variation**. Managers, e.g., understand that the recorded numbers are the result of numerous activities and decisions, many of which they can hardly influence. At the same time they realize that there are strategies to effect changes in the values of at least some of the indicators. These strategies presuppose an understanding of the variation in observed indicator values as well as of its sources. At issue are the depth of this understanding and also the resultant strategic options for process improvement.

In an evaluation of performance results by use of monthly indicators, two types of reference points are normally used: (1) an indicator value representing a forecast, goal, standard or some other type of expectation or prediction; (2) a value observed in a previous time period. This practice raises some fundamental questions about deviations from the two types of reference points which will be addressed below. Quite independently of the answers to these questions the evaluation can provide some guidance towards how to impact crucial outcomes. Of course, any action with this objective largely depends on the intentions of management, and guidance for it is not likely to be found in the numbers themselves. Nevertheless, treating a given result as one member of a series of results and taking into account the variation exhibited by that series, can yield valuable insights.

In the analysis it is of great importance to distinguish between **common** (or chance) and **special** (or assignable) **causes of variation**. Common causes affect each one of the observed results and their impact is experienced continually. **Figure 1.1.A** illustrates the working of common-cause sources of variation. At first glance the most recent value in the series seems to indicate a deterioration in performance. However, it still lies within the boundaries of variation observed in the historical series. Hence it does not represent an exceptional case, but can be interpreted as the effect of (a) common cause(s). Accordingly, reacting to this particular outcome without understanding the nature of its cause(s) may not have the intended effect.

By contrast, special causes of variation are those which affect only certain results. Special-cause sources of variation (in addition to common-cause sources) might have an effect on measured outcomes as depicted in **Figure 1.1.B**. Given both the level and the degree of variation of that series, the most recent one of the values plotted there has to be considered exceptional. Thus, it must be suspected that the observed deviation is due to a special cause.

Figure 1.1 Four time patterns of process outcomes



In theory, the distinction between common and special causes appears simple and straightforward; in practice, however, it often requires thorough investigation. The concept of special cause is meant to help identifying sources of erratic changes in outcomes and subsequently eliminating them. This is not to say, however, that a system subject only to the impact of common causes of variation is ideal. Quite independently of the source of variation, it is the manager's job to evaluate a system relative to what is required and make judgements on the basis of well-defined criteria. As was pointed out previously, numerical indicators of performance are usually reported each month along with a **standard** indicating what is required of the system. **Figures 1.1.C** and **1.1.D** are plots of the time series depicted also in Figure 1.1.A . The line drawn on the plot represents the standard of acceptable performance. This standard changes the view of variation compared to Figure 1.1.A.

The use of standards to judge performance is prevalent in many industries. Typically, a manager who is required to report performance measures will at the same time be required to explain deviations from a given standard. However, sensible explanations can often not be found so easily due to the nature of standards.

Performance standards may have been set for a number of different reasons: In some instances they are meant to describe the results of best practices, while in others they serve as a goal. The major purpose of a given standard must be taken into account when it is used for evaluation. The same is true for the capability of the system that is to be examined. If, for example, the setting of a (high) standard does not properly take into account capabilities of the given system, but sees the standard only as a desirable goal, deviations from it cannot provide useful information about system performance.

There are still other problems associated with the use of deviations from a standard as a management signal. For example, if a deviation is seen as a one-time occurrence, common causes get often treated as special causes without justification. To avoid such pitfalls deviations have to be examined in the context of the given system and by use of statistical concepts. The system could have yielded any one out of a range of possible values for the variable under study. Consequently, what is conventionally seen as a deviation should be interpreted in light of probable events within the studied system. A major argument for this approach derives from the 'natural' concept of work as a process taking place in the context of a system.

Interpreting a deviation from a standard as a one-time event obscures the fact that a given deviation is only one realization out of an array of possible values that the system could generate. Thus, the approach makes no use of any knowledge about the range of deviating values observed in the past. Consequently, the probabilistic nature, the historic record and the capability of the system are ignored.

Without adopting a system-wide view, managers would not be in a position to distinguish between special and common causes of variation. Without such distinction, however, there would be little incentive to analyze and improve the system; indeed, the possibility of such analysis and improvement may not even be perceived. Likewise, if statistical concepts like probability or predictability were not invoked, the issue of system variability could not be examined properly.

1.2 Evaluating the effects of variation

As a rule, variation of output is not deliberately created within a system. It rather results from the interaction of procedures, personnel, material, and equipment used in producing a product or a service. More specifically, variation is a consequence of the ways of 'doing business' in an organizational system. Neither are the sources of variation always known, nor — if they are known — are they always understood. The situation is similar when the effects of variation within a production system are considered. Knowledge of these effects — in terms of cost, capability, or performance — is essential to decide whether a reduction of variation could be beneficial. The following examples illustrate some of the possible effects of variation.

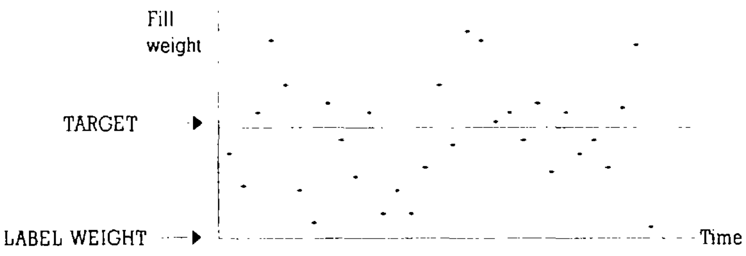
Fill weight for a granular product shipped in containers with a given label weight provides one of the examples. There is a target value for container content which is based upon a lower specification and the assumption that there is variability in the process. If the filling process could be managed in such a way as to produce stable variation in fill weights, managers could confidently determine the target taking into account the known magnitude of variation. However, without such knowledge target selection must be governed by lack of 'confidence' in the ability to perform predictably. Since a minimum value must be attained under all circumstances, erratic, unpredictable variation generally increases the target value in order to meet the lower specification. **Figure 1.2** provides an illustration of this point.

The advantages of predictability and of a decrease of variation in fill weight are easy to understand. Stable variation can guide the setting of an appropriate target value as well as the estimation of costs associated with a given amount of variation. Decreased variation in fill weight provides management with the possibility of lowering the target, and hence reducing costs and increasing efficiency.

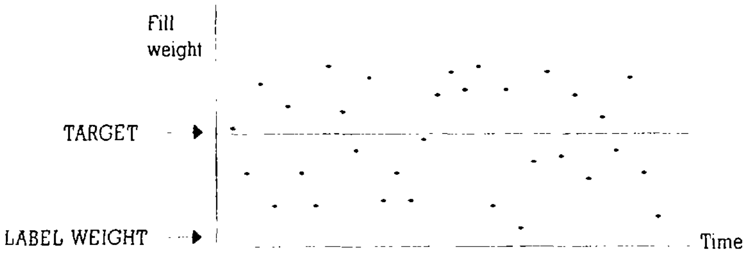
In addition to illustrating the advantages of predictable or decreased variation, the preceding discussion suggests to process managers specific action like measuring and analyzing variation in actual fill content. Furthermore, operating practices have to be studied, as well as those characteristics of material, equipment and environment investigated that affect variation and its stability over time.

In the present example a large variation in net content implies financial losses due to overfilling as a means of maintaining a specified minimum.

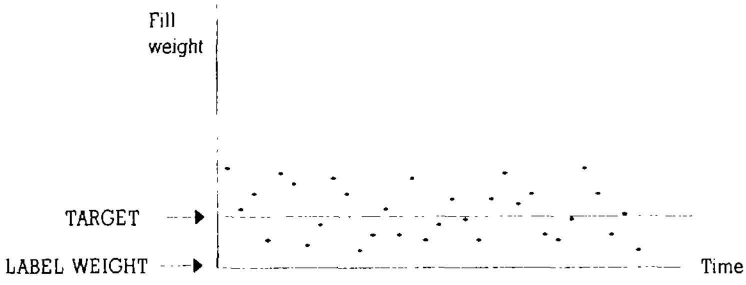
Figure 1.2. Three possible descriptions of fill weight measurements



Erratic, unpredictable variation (high target set)



Stable (predictable) with large variation



Stable with small variation

In other cases like that of gelatine capsules in use in the pharmaceutical industry the relationship between costs and benefits is not so easily understood. Product characteristics sometimes open a different perspective on what variability might mean in the context of management practices. An important property of the above capsules is wall thickness.

Usually a target value is specified for wall thickness with a view to providing for the material in the capsule the necessary protection from the environment and ensuring compatibility of the capsule with customers' filling equipment. For this reason, average wall thickness is determined externally. High process variability implies that consistently capsules are produced that have a wall thickness outside specifications. The consequences of this shortcoming are easy to see: First, capsules must be sorted in order to remove those of an incorrect size -- a process which uses up resources. Second, by implication a loss of output occurs. The costs associated with this loss are not easy to identify by current cost-accounting practices. As a consequence, costs are not correctly assessed and benefits that may arise from improvements of the given process are unlikely to be realized.

Financial gains of improvements internal to the process of capsule production may appear to be modest. However, more significant benefits may be identified external to the process. When shipments of capsules have characteristics lying within a specified range, a customer can have greater assurance for being able to process the capsules without machine stops and therefore with higher equipment utilization and improved cycle times implying attendant economies. Thus, improvement in the material properties of capsules can provide a competitive advantage. For the producer of capsules a competitive advantage may be derived from increased process knowledge stemming from efforts to reduce variation. This knowledge in turn allows the manufacturer of capsules to better adjust to new specifications of wall thickness, or other customer needs.

It is important to note here that the benefits of decreased variation in wall thickness do not result in an immediate return to the manufacturer of capsules. Managers must understand the wide implications of variability of this process parameter, which are not evident from process knowledge only. It must also become clear what customers might value, in order to realize potential gains. Experience indicates that financial audits are usually focused upon a narrow, internal evaluation of costs and benefits, thus directing attention away from the wide range of advantages to be derived from decreased variability. Traditional financial models do not allow to identify all the financial gains which may accrue from managing variation.

Because of the complexity of any analysis of the effects of variation, it is imperative for managers to work through in detail what excessive and erratic variation might imply. Against this backdrop, determination of the costs associated with variation is one of the essential tasks with which management is charged. Hence, a good understanding of the effects of variation is vital to an objective analysis of the benefits of reducing it.

1.3 Changing practices

Understanding both the sources and the effects of variation in output is an essential part of process control and improvement. In order for this understanding to lead to improvement, motivation of various types has to be fostered. In particular, attaining stability of variation is to be perceived as one of the responsibilities of management. Decreased variation results from system change and incremental improvements in which managers take the leading role. Therefore, it is crucial that managers understand the working of a particular system in order to correctly assess the variation of results and any benefits of its reduction or elimination.

As an illustration of the foregoing remarks the objective of increasing the throughput rate of production shall be discussed in some detail. From the outset the rationale for this objective will not be questioned here. Rather, the discussion will be about a strategy for achieving the set goal where the selection from among various options and the reasons for choosing one of them shall be examined.

Typically, expectations for increased throughput are expressed forcibly at plant level. However, management and staff may not have considered realistically how to achieve the required gains. Some tactics for attaining an increase may be stated explicitly, such as working to eliminate a known bottleneck or requiring that the throughput rate of each unit in the facility be increased. However, this may not be consistent with the system sources of variation that impact current throughput levels. Also an overemphasis on throughput itself may promote practices that are in conflict with other expectations and needs of the business.

For example, under pressure to increase throughput a department may release poor quality material. By making this choice, the throughput rate may increase while the yield stays the same. As a further consequence, the opportunity to achieve improved throughput by increasing the ability to consistently produce high quality products at each stage of operation is foregone because of the concentration on schedule rather than on other system parameters.

There are other potential losses like the failure to broaden, as well as deepen process knowledge of management and staff. In particular, the management group is likely to fail to acquire improved practices for use in future tasks and sustained experience in reducing defects may be lacking. In summary there is no assurance that the approach on the whole will yield appreciable benefits.

More generally, there cannot be much confidence that working on the input side exclusively will result in improved throughput capability. Under pressure to attain or surpass a set schedule, equipment may be run without adequate maintenance entailing probable adverse impacts on plant capability. Shifts, treated as if they were independent production units, are usually goaded toward quota achievement. Likely effects are either delaying appropriate maintenance or foregoing the opportunity to choose the best time for maintenance. In addition, if each shift is pressured to achieve a certain production quota, the effect is often to set one shift working against the others. In reaction to such practice, production knowledge is hoarded and work is moved to other shifts or times by a variety of means. Each shift concentrates on achieving its own quota, often emptying the line of all work in progress in order to achieve the objective. The consequence is a larger start-up job for the next shift. Thus the idea of running the operation in a smooth, consistent fashion over consecutive shifts is not honoured, nor are the benefits of improved throughput realized.

A significantly different practice of working to improve throughput levels would invoke the understanding of sources of variation in order to suggest changes to improve the existing system. Implicit in this approach is the concept that there exists a multitude of sources which affect each other as well as they impact throughput rate. Consequently, the idea of searching for one, or the most prevalent, cause of deviations from a standard is to be replaced by that of understanding system behaviour and the variation in those components of the system that affect throughput levels. In examining the whole productive system, managers might begin to look at specific activities in a different manner and find ways for improvement previously unexamined.

Finally, numerous set-up changes and within-run modifications to accommodate raw material variation often reveal the fact that purchasing has not been included in the production subsystem. Here at least two issues surface. First, there is the obvious adverse impact on productivity and efficiency of frequent set-ups and modifications. Second, there is evidence of a 'system break' between the purchasing and the production subsystems. Examining the interface between purchasing and production offers large returns for the work of management.

Another simple example which can serve to illustrate the contrast between practices that recognize variation and its implications and practices that do not is tool quality. Inconsistent tool quality leads to erratic and frequent tool changes with a consequent effect on throughput in terms of quantity and quality. However, it also indicates that management has probably not paid sufficient attention to the issues of tool requirements, an understanding of why tools might differ over time, and cooperation with vendors as a means to prevent problems with tool quality from resurfacing. The sources of variation evidenced by frequent tool changes may in a first round be characterized by the physical properties of the tools themselves. At a more fundamental level, however, at least some of the variation may result from management practices that fail to develop an adequate approach to problems involving tool requirements and the relationship with vendors.

In general, within an organization attention to sources of variation can be focused in a variety of ways. One possible focus would be on the operational or functional level. Although this approach is valuable and desirable, it is at the same time incomplete in that it leaves unattended many opportunities for improvement. For example, work on improving throughput in a particular function might have been motivated by noting the different capabilities of several machines which are performing the same operation. Improvements in the process would be effected by identifying this source of variation and then making changes to ensure adequate performance of all machines. Precisely because the operational focus is deemed to be useful, work on variation may be limited to this level of the organization. In fact, a previously espoused role of management has been to empower personnel involvement in this work to investigate processes with a view to implementing such changes.

However, the operational focus alone is inadequate to address many of the major sources of variation. Ensuring that all machines operate in a consistent manner may provide a valuable improvement. Yet it is perhaps more important to deal with management behaviour and actions that allowed the machines to operate inconsistently in the first place. Here, a possible fault might have been that no system was in place to bring new machines on line in accordance with other machines. In this case purchasing would need to become involved. Another possibility is that the effects of differences in machine performance were not monitored; here management would have to go for a change of standard operating procedures. Still another possible source is that of differences in maintenance practices. Again, it is management's role to understand these issues and address them.

Finally, the manner in which management addresses the above issues across functions is also important. In much the same way as empowering people to examine and make changes is not sufficient to manage variation, merely facilitating improvement work across functions is also too limited a role for management. While it is not justified to conclude that the 'management-as-facilitator' approach has not provided valuable improvements, the passive role of a facilitator is clearly not adequate to address the improvement issues confronting management. The content of the work of management ought to be enriched by knowledge not only of the existence of variation, but also of methods of its quantification, and of an assessment of its effects. Such broadening of the view of managerial tasks in the present context is only possible on the basis of a systems approach.

1.4 A systems perspective of managing variation

The above examples should have provided a first impression of what variation might mean to a manager. Other cases where variation plays an important role are presented in the following paragraphs. All the examples discussed here appear to plead for a systems view mainly because it renders a comprehensive picture of the relationships under study. In addition to outlining the systems perspective, these examples also provide a full account of variability issues, involving the magnitude, the sources and also the predictability of variation.

As was shown previously, the view that results can be usefully managed by examining deviations from a standard is quite limited. In particular, it often presupposes that there is only one cause for an observed deviation. Actions taken on such assumptions are likely to have unintended consequences. By identifying and acting on only one cause, other possibly hidden causes do not get addressed. This in turn may have the consequence of moving the observed variation from one part of the system to other parts, where it appears perhaps in a different form or at a different time. This may still be an appropriate strategy, but it could also have unintended negative effects on other system parameters. These effects may in turn impair performance in terms of cost, quality or delivery. Another potential negative effect of the strategy is that no new knowledge is gained which could support system change or improvement in future action.

To illustrate the previous arguments with an example, a situation is considered where shipping dates are not being met. Here deviations from committed dates are observed, attention is drawn to these deviations, and a programme to improve shipping results is initiated. Performance in relation to shipping dates will most likely improve, at least until attention turns to other concerns. The reasons behind achieving this improvement and effects on other parts of the business may be complex in nature. Thus, for example, the achievement of shipments leaving on schedule may be bought at the cost of material of questionable quality. Alternatively, among the reasons for an improvement in shipping performance could be that overtime has been incurred, additional crews have been employed or more capacity has been made available at the cost of a delay or even cancellation of maintenance.

The actions listed above as potential sources of an improvement in shipping performance may be appropriate, but need not. In any case, they are likely to be unintended. To illustrate this point further, it might be hypothesized that shipping dates are manipulated through an ongoing series of negotiations with customers. Such renegotiated dates could then be met and performance marks improved. However, in that case real customer needs regarding timing, quality, and quantity would not get addressed and on account of that the business might be discredited in the future.

By contrast, if management took a systems point of view there would be strong indications that the issue of improvement needs to be considered far beyond any efforts to better meet shipping date commitments. Systems issues underlying the difficulties in meeting shipment schedules would have to be addressed. Such issues may be found in rather prosaic matters such as quality or assembly problems, which in turn may be related to design, equipment capabilities or maintenance, or material purchases. Likewise the inability to meet scheduled shipping dates may be due to the process for assigning a particular order mix to the plant, or possibly to the lack of a focus as to which products to produce first for which customer. If the potential effects on other results such as labour cost, future equipment performance, quality, and future sales were not considered and attempts were made to directly affect performance in relation to shipping schedules, the outcome could be damaging to current and future business performance.

Another example for the usefulness of a systems approach is provided by a manufacturing firm that incurs a large fraction of direct costs in the acquisition of raw materials, components, or parts. Since management is certainly aware of these costs, attention quite naturally becomes focused on the costs of incoming materials and parts. Frequently, pressure to reduce these costs results in searches for a low bid price, a desirable and intended result. However, gaining a reduction in purchase price may result in variations in other costs, many of which may not be directly linked to the purchasing function and thus may go unnoticed. For example, the quality of incoming material may suffer, leading to internal sorting and reworking with the result of an increase in production costs. In addition, inferior materials can easily decrease productivity because of ensuing difficulties in working, forming or assembling purchased parts.

There are a number of other ways in which the acquisition of raw materials, components, or parts can contribute to system deficiencies. There is no doubt about justification for the general managerial principle of attempting to control and decrease costs. However, it is important to identify the type of costs to be decreased as well as the criteria and the means for their reduction from the organization's overall perspective. If the guidelines for managing costs are entirely functional in form and content, then functional optimization may move the variation and attendant waste from one part of the business to another, possibly with little or no overall benefit.

1.5 Conclusion

In order to be able to improve performance in a comprehensive manner, a manager must possess profound knowledge of system capability as it is expressed in measurable parameters. He/She must also be able to judge whether this capability is appropriate. Averages of the parameters concerned and parameter variability must inform decisions regarding the management of the processes or systems of the organization. Evidence of system instability indicates that either unmanaged sources of variation are impacting system performance or that management activity has an inconsistent result on performance. This in turn suggests that managerial action is required to address this issue. By contrast, evidence of system stability sends a different type of signal to the manager and calls for a different approach.

The following chapters will introduce statistical tools for process and systems management. The methods presented here are intended to appraise process or system variation, assess the impact of sources of variation on process or system outcomes, and guide the management of that variation. Competence of the managers and engineers of an organization in using these tools is certain to improve the capability to deliver products and services that are valued by users or customers.

CHAPTER 2

BASICS OF DATA COLLECTION

Managing process variation is one of the means to improve business systems. In this connection various types of **evaluation** are essential to the improvement effort. Such evaluation usually produces information that is vital to the systematic investigation of a given process. In order to obtain such information efficiently, managers or engineers have to use a number of well-defined approaches and techniques for identifying, collecting and organizing data.

In the context of production, evaluation is carried out for a variety of reasons. For example, products are often evaluated in order to determine what to do with them next: At a given stage in the production process a decision may have to be taken about whether a certain (intermediate) product is fit for further use or not. Alternatively, products data, along with process data, are evaluated to judge the performance of processes. For the latter reason, for example, counts of the number of pieces of scrap are made and reported regularly. Similarly, the number of flaws in an assembly, certain properties of a product, or the amount of impurity in a volume of raw material are assessed and recorded. Thus, in many instances data are developed on the performance of equipment and processing lines in a fairly regular fashion. These data often include also cost information for a whole business unit as well as measures of the degree of customer satisfaction.

Evaluation is usually based on some form of **measurement** of a broad number of variables. Generally, any bundle of activities aimed at qualitative or quantitative evaluation is called a measurement process. The following discussion begins with an outline of the major motives behind measurement and describes the basic components of a typical measurement process. In this connection four broad types of data are introduced together with some simple methods of summarizing them. In addition, guidelines for the design of data collection forms are provided. The chapter concludes with a brief introduction to the concept of control chart.

2.1 Purposes of measurement and record keeping

Evaluation of a product or service output may be carried out for various purposes at any of several points in the process of producing this output. Five of the most common of these purposes are:

- to assess the disposition of a product or service
- to develop or update a history of product or service results
- to measure the variation of results and assess the stability of this variation
- to change process or input factors
- to develop an understanding of the impact on the process of causal factors.

2.1.1 Disposition

The first purpose refers to the decision about what is to be done with a product at a particular point in the process. Here the three basic options are to deliver the product to the next stage, to rework it, or to scrap it. Inspection for disposition may in the short term be dictated by economic considerations. By contrast, an analysis with longer-term objectives would lead to process improvement that makes redundant further inspection for disposition. Nevertheless, upstream improvement need not be motivated only by the benefits deriving from the by-passing of inspection. Other (internal) benefits include a reduction of the complexity of operations, reduced requirements to keep records as well as smaller fluctuations in workloads for reworking or repairing.

2.1.2 History

Measurement of product or service characteristics adds to a history of product or service results. A product or service profile may be maintained to meet regulatory or other reporting requirements or for warranty or reliability reference. If measurement is consistent, such a history documents the stability of variation in process results and allows to evaluate the capability of meeting future requirements. Although establishing and maintaining a product or service history may be entirely legitimate, such histories often are the result of a one-time request for information that is never used thereafter. Thus, poorly designed measurement procedures involving large capital investments in equipment may be introduced and prove difficult to eliminate afterwards. Only a thorough review of the intent of and the need for institutionalized measurement can rid the system of such redundancies.

2.1.3 Stability of variation

Product or service evaluation is also performed to assess the degree of stability of variation in process results providing an indicator of the effectiveness with which the process is engineered, managed and operated. This effectiveness in turn builds on the current knowledge base of process management and the consistent use of that knowledge.

2.1.4 Process and input factors: statistical process control

Actions taken to change process or input factors may have a temporary effect on process results. Measurement as part of a manual or automated 'control' system is usually taken as the basis for impacting a process and its outputs. The term '**statistical process control**' (SPC) refers to monitoring process variables or output characteristics and adjusting the monitored process accordingly. Hence, SPC is a form of control system which is intended to maintain steady and predictable variation in the output stream. Continuous adjustment with the goal of rendering the process stable is an integral part of such services.

The measurement of product or service characteristics may be part of a wider engineering or managerial feedback control system. Such a system is usually intended to monitor output results and thereby gain information for input or process adjustment. Here the term 'control' is used not in the statistical sense, but rather means certain operating procedures for process intervention. Such procedures may use statistical criteria to determine the need for intervention and adjustment and to gauge the extent of the latter.

When a process is unstable, i.e., subject to systematic change, monitoring and intervention in order to attain a steady state is an important objective. However, two critical points are often missed in the design and operation of the pertinent feedback system. First, such a system in many instances does not provide for the reduction of variation below routine levels. In this case the best result achievable is that of maintaining the level of variation typical for the process. Second, many managerial feedback control systems as well as some engineering control systems do not recognize or appropriately interpret the 'noise' in the results of a stable process. Consequently, there is considerable risk of intervening in the process for the wrong reasons with the possible consequence of increasing its variation.

An understanding of the concept of stable variation provides the necessary basis for a comprehensive improvement in results. Furthermore, learning about causal factors behind variation and subsequently acting on those factors is likely to benefit both measurement procedures and the strategies of intervention and adjustment. For these reasons, engineering and managerial control systems should recurrently be improved upon.

Intermediate evaluation is often performed on processes of physical production or the delivery of services. Such evaluation may suggest correction or adjustment at various process steps. Upstream product and service evaluation usually leads to final outcomes that are more likely to meet customer requirements. For example, in surgical procedures a particular outcome from a physiological evaluation, performed prior to or during surgery, may require that a surgical team modify their procedures to protect the patient's welfare or to achieve the surgical objective. In much the same way, intermediate evaluation in manufacturing processes is often part of a manual or automated feed-forward control strategy which alters downstream processing conditions. Such a strategy may be justified by economies that downstream adjustment carries in comparison with changes in upstream conditions. However, no control strategy should be adopted without in-depth consideration of the economic issues involved.

2.1.5 Causal factors: the use of cause-and-effect diagrams

The purpose of evaluation is often that of understanding how causal factors and their interactions affect product or service results. In this case, evaluation results help to gain insight into the mechanisms that generate a product or service. The objective here is to stabilize and then improve processes in order to create improved value for customers. The latter goal is multi-dimensional in nature including quality, cost, timeliness and performance in use of the product or service. When a first attempt is made at stabilization, the factors behind observed variation may not be known or at least not be well understood. In order to improve results, the effects of potential causal factors must be carefully analyzed. The ideal situation would be that of investigating all factors that could possibly affect outcomes.

An effective tool to help employees organize their ideas and work methods is the **cause-and-effect diagram** (also called **fishbone diagram**). In a first step employees usually agree to study a certain process characteristic. A diagram, relating this process characteristic or problem to the main contributing factors as well as more detailed factors, is then constructed. Careful study of such a cause-and-effect diagram should help the manager in answering questions such as: What is known about the impact of certain factors on the problem at hand? Which of these factors should be controlled? What types of interactions between causes affect the process and the characteristic under study?

There is no single set of rules or guidelines for constructing a cause-and-effect diagram. In any event is it essential, however, to single out potential causes of the problem at hand. These causes generally fall into one of the following categories:

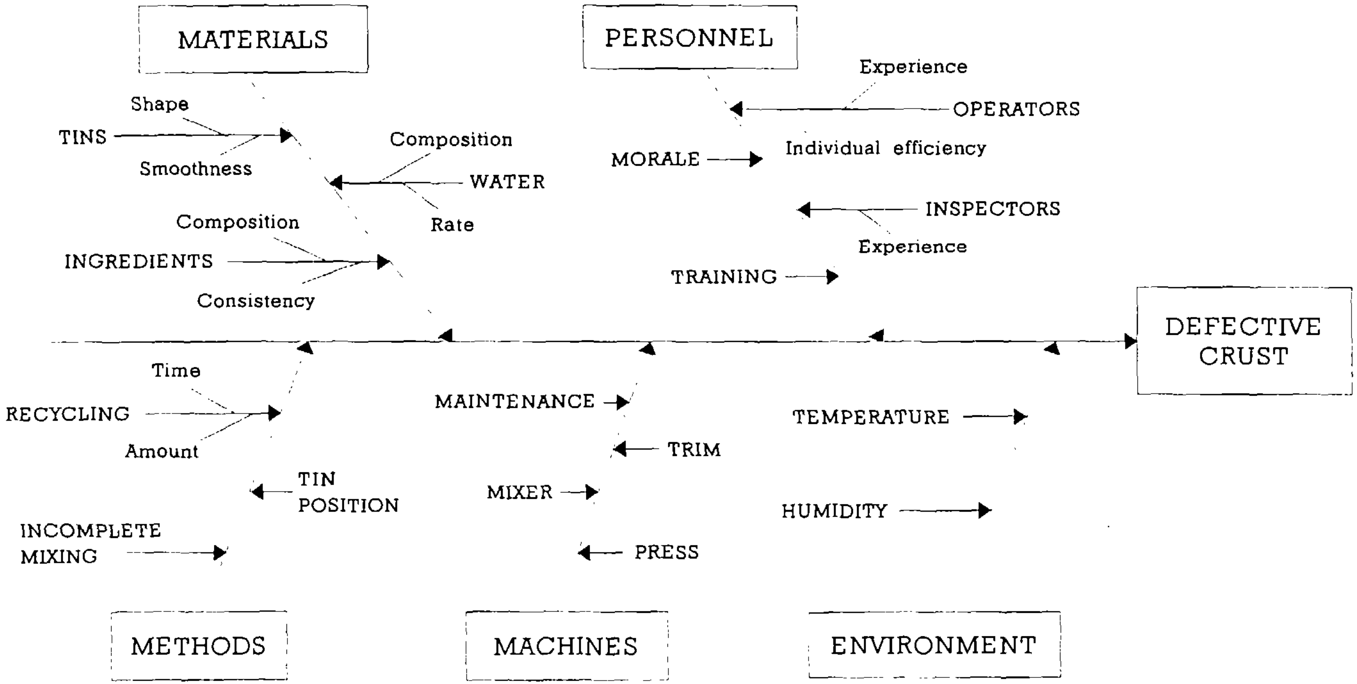
- Personnel
- Materials
- Equipment
- Production methods
- Measurement
- Production environment

In a cause-and-effect diagram like that of Figure 2.1, the result of concern is placed in a box on the right-hand side of the figure and referred to as the effect. Major types of causal factors which may contribute to the dispersion in results form the main branches in the diagram. In the given example these major categories are materials, personnel, methods, machines and environment. Specific factors within these major categories are then placed as stems on the corresponding branches.

The plant studied in the example of Figure 2.1 produces individual servings of frozen meat pies (pot pies) in metal trays. Standard practice is to inspect every bottom crust produced and to remove any crust that has breaks in the dough or does not completely cover the bottom of the metal tray. A large number of crusts (18 per cent) has to be scrapped after they have been placed in the metal trays for filling.

The management group charged with introducing improvements in the process finds it hard to admit that the large proportion of nonconforming crusts represents a 'quality' problem. Some argue that there is no problem, since every crust is inspected and defective ones are removed from the production line. Nor do they believe there is any waste because

Figure 2.1 Cause-and-Effect Diagram



the discarded dough is reused at a later stage in the process. Other members of the group disagree with this view, arguing that the machine and operator time used to produce items which cannot be sold is an unnecessary expense. They also believe that the taste and texture of the crust deteriorates if the dough is being reworked, making the standard procedure an inappropriate solution. As a consequence, the management group decides that more information is needed and a cause-and-effect diagram should be developed. This diagram is reproduced in Figure 2.1 which – in addition to the major factors – shows details on hypothesized causal relationships.

As a general rule for the construction of cause-and-effect diagrams, it is important to solicit input and ideas from as many persons as possible. Another important consideration is that the cause-and-effect diagram is only a tool for identifying potential causes. It would be too weak a basis for making process changes without careful analysis of underlying causal relationships.

2.1.6 More on measurement

Up to this point little has been said about actual measurement of input characteristics, process factors or operating conditions. For obvious reasons measurement of input characteristics is of high significance in the framework of feed-forward control systems. In some industries, the evaluation of process factors and operating conditions is a matter of meeting regulatory requirements. The pharmaceutical industry provides an example where specifications of process conditions, such as temperature and processing times, must be met in order to satisfy regulatory requirements. At the same time measurement of process factors and operating conditions can provide inputs to feedback control systems. For example, a control system for a machining process uses measurement results on machine temperature, machine forces and tool characteristics as inputs to a control algorithm. In order to develop an understanding of how causal factors and their interaction affect output characteristics, plans for measurement of inputs and process factors must address the issue of how the corresponding measurement results are associated with output measures.

Since measurement and the subsequent summary and analysis of data usually have multiple objectives, it is important that those who initiate and carry out measurement understand the major purposes of data collection. Thus, data collected to maintain a historical record may not be useful for assessing process stability or implement feedback control, because in such information the source of data and the sequence of production are usually not recorded. This makes it difficult to obtain information on process changes and on potential causes of variation. For the development and improvement of process knowledge, by contrast, the timely collection of specific data along with careful documentation of process changes is vital. More specifically, such data must provide information on process changes and must support the investigation of hypothesized cause-and-effect relationships.

Measurement of process inputs, factors, methods, and environmental conditions, carried out with the objective of process investigation, serves more than just the purpose of a simple description of what currently exists or of what has happened in the past. The motives behind such measurement are to understand the effects on process output of changes in material, equipment, methods or environmental conditions, to judge the correctness of the current level or average performance, and to assess the potential benefits of reducing variation. Overall, the prime objective is to make changes to the process with the effect of improving future outcomes.

2.2 Components of a measurement process and factors affecting measurement results

Measurement processes can be as straightforward as making a judgement and recording the result. They can, however, also be complex involving prescribed multi-step protocols for chemical, mechanical or electronic analysis. Regardless of the sophistication of the equipment and procedures applied in a particular measurement process, in all such processes a few common components can be identified:

(i) **persons** who carry out the measurement,

(ii) **materials**, comprising the items measured as well as materials used in measurement,

(iii) **equipment** utilized in the measurement process,

(iv) **methods** of measurement.

With a view to variation in the results of measurement, the **environment** in which measurement takes place becomes also important, since fluctuations in environmental factors may be reflected in measurement fluctuations. Examples of such factors are the selection of personnel, training practices, pressure to meet schedules, equipment procurement policies and a host of managerial practices that create the environment in which measurement is carried out.

2.2.1 Personnel and methods

Methods of measurement relate to the way in which materials, equipment and the items to be measured are prepared, the way in which equipment is used, the way in which observations are carried out — e.g., visual judgement relative to a specified standard, reading a dial etc.— and the way in which an observation is translated into a recorded result. For each one of these issues there should be a clear description of the standard practice to be followed. A useful way of providing such a description is a flow diagram outlining each activity and decision in their proper sequence. Inconsistency of measurement methods is a very likely outcome, if the personnel involved is not sufficiently trained. As a rule, every plan for training should provide for continuity of training activities in order to ensure maintenance of the standards of the above procedures.

When plans are drawn up for the collection of data for process study, an important consideration is that a new 'technology' may have to be introduced in the process environment. For example, the use of certain techniques demands skills relating to the arithmetic of data summary and charting. In this context it is often assumed that production staff would already possess these skills. This assumption, however, proves to be erroneous in many cases. Accordingly, any plan for measurement, data summary and analysis usually has to include as separate components an assessment of relevant skills of the staff concerned and provision of training to the extent required.

2.2.2 Materials

Measuring a certain characteristic of a product often implies that the product reach a certain state (temperature, age, cleanliness etc.) before measurement can be carried out. For this purpose an established procedure of preparing the product may have to be followed, as, e.g. cleaning the item or removing lubricants from its surface, cutting it into sections, heating, etc. Failure to follow such a procedure can introduce variation into measurement results and render them less reliable than expected. Likewise, materials of measurement, such as gauge masters, chemical additives, filters, and so on, usually require proper handling and storage to avoid changes in those materials that may in turn change measurement results. Established procedures for handling such materials should be rigorously followed in order to prevent any undesirable impact on measurement results.

2.2.3 Measurement equipment

Gauges and other types of measurement devices are subject to deterioration due to age and use. Such devices should therefore be subjected to the same kind of preventive maintenance as applied to equipment used in producing products or services. This is particularly valid for high-precision measurement devices which require carefully controlled temperature, humidity, cleanliness, freedom from vibration etc.

The 'reading' of measurement values is another important issue. Measurement equipment should be designed in such a way that measurement errors due to the difficulty of 'reading' signals are kept to a minimum. More specifically, the resolution of the measurement readout should not be greater than what the measurement equipment can detect. At the same time it should be fine enough to allow for variation in product characteristics to be detected. If, for example, this variation is on the order of a thousandths of an inch, then the combined resolution of the measurement equipment and its readout (dial, digital display etc.) should be on the order of ten-thousandths of an inch.

2.2.4 Environment

As was mentioned previously, there are a number of environmental factors that may affect measurement results. Among them are effects on the ability of personnel to observe and evaluate. Straightforward examples are the adequacy of lighting or freedom from distraction. Another important 'environmental' factor is the consistent use of definitions in assessments of whether a product is acceptable or not. If, for example, the goal of meeting production schedules overrides all other considerations, there may be strong pressure to change immediately the ways of evaluating products. In this case the impact of managerial policies and practices on the social environment of measurement becomes evident. Consequently, such policies have a powerful, if only indirect, influence on all kinds of evaluation.

The preceding list of factors that could influence the quality of measurement results is notoriously incomplete and the same holds for their description. Additions to the list will be made in later chapters and likewise will methods for investigating the effects of such factors on measurement results be discussed in some detail later on.

2.3 Operational Definitions

Measurement results are the information link between process outcomes and action directed towards improvement. Communication between individuals, groups or functions includes summaries and analyses of measured results. The quality and consistency of this communication depend crucially on a common understanding of what is measured, how it is measured, and how measurement results are to be used.

When a product is delivered from a supplier to a customer (internal or external to the enterprise) both parties involved in the transaction need to learn about the current properties of the items. This is necessary in order to be able to determine any need for adjustment or the conditions of payment. The required information usually refers to product characteristics that must meet certain specifications. Such specifications would have little meaning and could become a source of confusion and conflict, if operational definitions of the relevant characteristics were not available. Any operational definition must meet the following three minimum conditions:

(i) It should be stated clearly what is considered to be an acceptable process outcome, and this statement should be communicable.

(ii) There should be an agreed method of evaluating a given characteristic, and both the equipment and the material to be used in measurement should be specified.

(iii) Finally, a procedure needs to be established to reach clear and replicable decisions on whether or not requirements have been met.

As a rule, operational definitions of key characteristics help to avoid disagreement and uncertainty about product delivery. **Figure 2.2** provides an example of a definition relating to a particular specification for sheets of paper. It may be useful to emphasize here that a definition is neither right nor wrong, but simply represents a unequivocally stated agreement between producer and user of the item at hand.

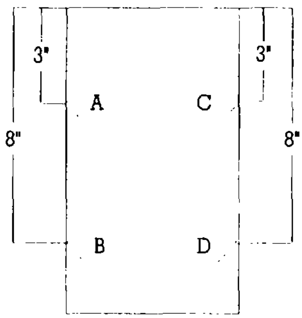
2.4 Using flow diagrams and data collection sheets

The usefulness of measurement results depends crucially on a strategy for data collection. In many organizations, a large amount of historical data is collected and stored as part of a routine. However, such data often do not meet the purpose of assessing the variation in process outcomes and

Figure 2.2 A definition for a specified characteristic

Criterion for determining whether a sheet of paper meets a specification on width of 8.5 ± 0.001 inches

Choose one long edge of the sheet. Orient it to the left.
Measure down the left edge from the upper left corner with a (type specified) measuring device a distance of 3 inches. Mark the point A.
Measure down the left edge a distance of 8 inches. Mark the point B.
Repeat the marking process for the right edge to obtain points C (3 inches down) and D (8 inches down).

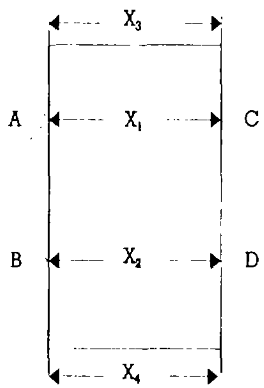


Measure the distance from A to C. Call the result X_1.

Measure the distance from B to D. Call the result X_2.

Measure the length of the top edge of the sheet. Call the result X_3.

Measure the length of the bottom edge of the sheet. Call the result X_4.



Criterion:

If all of the measurements (X_1, X_2, X_3, X_4) are numbers between 8.499 and 8.501 inclusive, then the sheet meets the width specification.
If any of the measurements is smaller than 8.499 or greater than 8.501, then the sheet does not meet the width specification.

identifying its sources. If the measurement process is to support the above objectives, they must guide the data collection strategy.

One prerequisite of any rational planning for data collection is a good understanding of the production process in its entirety. Above all it provides the basis for determining critical points at which measurement needs to be taken. Thus, prior to identifying measurement points and collecting data, the process should be described in an accurate and detailed fashion with particular emphasis on causal relationships.

To identify key points in the process **flow diagrams** or **flow charts** prove very helpful by providing a schematic picture of operations. Furthermore, flow charts often allow to detect redundant and costly process steps so that immediate changes can be made to reduce process complexity without loss. The flow chart of **Figure 2.3** can serve as an example where - in the shaded areas - non-value added stages in the process are shown.

The effective use of measurement results also depends on their being recorded correctly as well as on auxiliary information. Therefore, **data collection forms** should be designed in such a way as to make correct recording easy, efficient and as little prone to error as possible. In particular, a data collection form should allow not only for space to record the data themselves but also the time of data collection, identification of the collector, and other auxiliary information. In the present context required auxiliary information depends on which factors may causally affect measurement results. For example, information on the source of the items measured, such as lot numbers for incoming supplies, machine identification for parts, time or product etc., might be recorded. Observed (changes in) conditions that are considered important for the interpretation of recorded data – such as ambient temperature, maintenance action, machine adjustments, machine stops and starts – may need to be recorded as well. In general, it is useful for the form to provide for flexibility in the selection of recorded information.

If a running record of results is to be examined for statistical control, it is often helpful to use graphical methods as well to represent the information in question. **Figure 2.4** shows a data collection sheet for maintaining inspection records of counts of defects and recording their types . From this sheet a running record of counts can be plotted.

2.5 Types of data and their analysis

The results of repeated applications of a measurement process, collected and summarized, form a set of data. Appropriate analysis of such data

Figure 2.3 Flow diagram of manufacturing process of horn pad assemblies

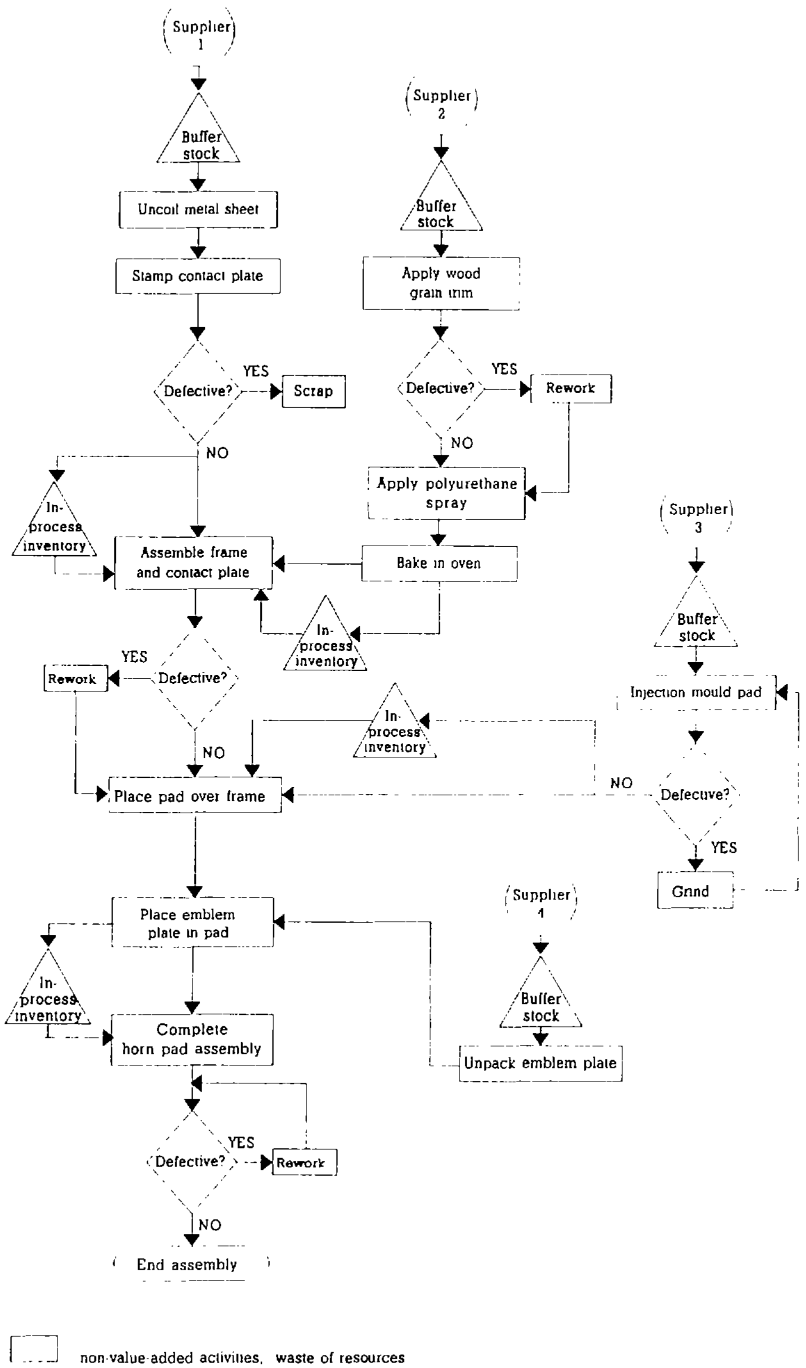


Figure 2.4 *Data collection sheet - horn pad assemblies*

MACHINE	SHIFT	DATE 03-06-94	03-07-94	03-08-94	03-09-94	03-10-94	COMMENTS
1	Day	A D I	E A	AA F D			
1	Night	B F	A D C	D E			
2	Day	AE G	A I	E B			
2	Night	E A G	A D E	E A A D D			
3	Day	D D I	E AB	D			
3	Night	A B	E F	D D B			

Types of Defects:

A: defective urethane spray	D: surface defects	G: blow pressure wrong
B: improper assembly	E: excessive flash	H: loose nuts
C: missing parts	F: bad trim plate	I: other

requires first of all an assessment of their nature. In traditional quality control, two basic data types are usually distinguished, namely **attributes data** and **variables data**. Whenever a measurement process requires a count of items or a count of occurrences of a specified qualitative characteristic, the resulting data are of the attributes type. They comprise the subgroups of categorical data, counts of events or items and rank data. By contrast, variables data are generated by measurement of a quantitative characteristic, as for example, the weight of a particular item.

To summarize the above distinctions, the types of data underlying an analysis of measurement results can be grouped as follows:

(i) categorical data

(ii) counts of events in space or in time

(iii) ranks or ratings

(iv) variables data

Each data group will be discussed briefly below.

2.5.1 Categorical data

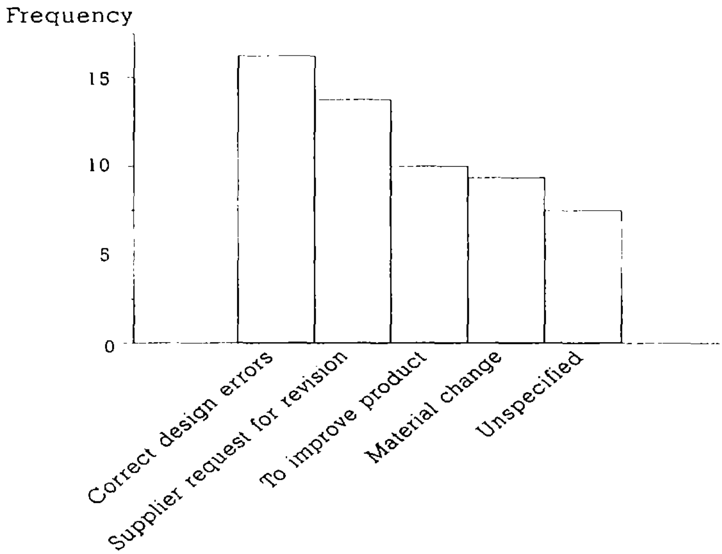
These data arise from classifying each member of a set of individuals, items or events into one of two or more categories. The resulting data are usually the counts of members in each category. Examples of categorical data are:

- Members of a collection of parts are classified as scrap, repair or good. Recorded data could indicate, e.g., that among a total of 100 parts, 3 parts are scrap, 8 parts are slated for repair and 89 are good.
- Respondents to a survey questionnaire are classified by their age. The resulting data consist of the number of respondents in each age category.
- Notices on engineering change are classified by the reason for change. Recorded data indicate that out of 50 notices, 9 changes were made with the objective of product improvement, 12 as a result of supplier requests for revision of standards, 14 to correct design flaws or errors, 8 to institute material changes and the remaining 7 for unspecified reasons.
- An overnight package delivery service defines a delivery as being "on time" if it is made before 10.30 a.m. on the specified day. A week's deliveries in a given region are then classified as "on time" or "not on time" with the latter category including lost packages. The resulting data consist of the number of on-time deliveries out of the total number due for a given week.
- Processing line interruptions are classified by the type of interruption. The resulting data consist of counts of interruptions by type that occurred in a month. Typically this kind of information is used to set priorities in attempts at reducing interruptions.
- Sheets of material are examined for surface defects of various kinds. The resulting data consist of counts of defects of various kinds found in a given number of sheets.

Categorical data consisting of counts of items in two or more categories are sometimes summarized using bar graphs. **Figure 2.5** shows a bar graph of the data on engineering change notices described above.

The statistical model used for the study of variation in data consisting of counts of individuals or items in one of two categories is the **Bernoulli** or **binomial probability model**. Generalization of this model to more than two categories produces the **multinominal model**. In the study of

Figure 2.5 *Bar graph of number of engineering change notices by reason for change*



variation in a sequence of counts of items in two or more categories, so-called *np* or *p* charts are usually employed.

2.5.2 Counts of events in space or in time: the use of Pareto diagrams

Such counts are made under a broad variety of circumstances. Some of them, like the number of processing line interruptions in the previous example, arise as a result of observing occurrences of a certain event in a given period of time. Further examples are counts of the number of customer complaints per month, the number of accidents per labour hour, the number of fires per month, the number of equipment failures per thousand parts produced, the number of requests for service or the number of service interruptions per unit time.

Similar treatment is required for counts of the existence of flaws in two or three dimensions or in a given volume of liquid or gaseous material. The above example of surface defects involves counting of flaws of a certain kind in a two-dimensional area. Similarly, the number of dirt particles in the paint on the hood of an automobile or the number of scratches on the journals of a crankshaft are counts of events in a given area. An example of a volumetric count of events is the number of voids and fractures in nuclear reactor vessel welds. Other examples are certain types of air quality measurement which are intended to reflect the concentration of various kinds of particles per unit volume.

The basic statistical model for the study of variation in data consisting of counts of events in space or in time is the **Poisson probability model**. When studying the variation in a sequence of such counts, so-called *u charts* are usually employed.

Counts of events in three or more different categories, such as defects by type, accidents by type or impurities by type, are often summarized by use of a **Pareto diagram**. A Pareto diagram is a bar graph used to compare frequencies of events. It is constructed by first selecting the categories for the summary, summarizing the counts accordingly and rank ordering them. Rare and unrelated categories may be grouped together as a residual category (with the label "other"). In the resultant bar graph the horizontal scale indicates categories, the vertical scale category frequencies and the height of each bar is proportional to the frequency of counts in the associated category.

Inspection of the assembly of horn pads provided the data for **Table 2.1** below. In addition to keeping records of the number of defective items, records on the types of defects were maintained. The table gives summary counts of the number of defects of different types among 1,000 horn pad assemblies with the counts arranged in descending order of magnitude.

The Pareto diagram for the different types of defects in horn pad assembly helps to determine those stages in the production process for which improvement efforts would be most promising. Examination of the data summarized in Table 2.1 and in **Figure 2.6** suggested to rearrange them with a view to this objective. On the one hand, surface defects and excessive flash are 'produced' in the pad-moulding process. On the other hand, improper assembly, missing parts and loose nuts are directly connected to the assembly process. Rearranging the counts of defects according to their points of origin in the production process produced the Pareto diagram shown in **Figure 2.7**. Further examination of that diagram suggests that the pad-moulding process produces the greatest number of defects and is therefore a natural point of departure for improvement work.

A warning should be sounded, however, about potential pitfalls in the above method of setting priorities. One of them is failing to recognize

Table 2.1 *Counts of the number of defects by type obtained in the final inspection of horn pad assemblies*

Type of Defect	Number	Percent of Total
Defective urethane spray	65	25.0
Surface defects	64	24.6
Excessive flash	63	24.2
Improper assembly	24	9.2
Bad trim plate	12	4.6
Other	11	4.2
Blow pressure wrong	9	3.5
Missing parts	8	3.1
Loose nuts	4	1.5
TOTAL	260	

variation over time in the number of problematic occurrences generated by a stable process. As a norm such a process creates varying numbers of the different types of defects. Due to normal variation in the outcomes of the process the ranking of these types of defects is likely not to be stable over time. As a consequence such rankings cannot provide reliable guidance towards setting priorities for improvement efforts. Instead different methods have to be used to distinguish between problems that are chronic and others that occur only sporadically. On the basis of such knowledge, priorities for improvement work can be set that enjoy a higher degree of confidence in its effectiveness.

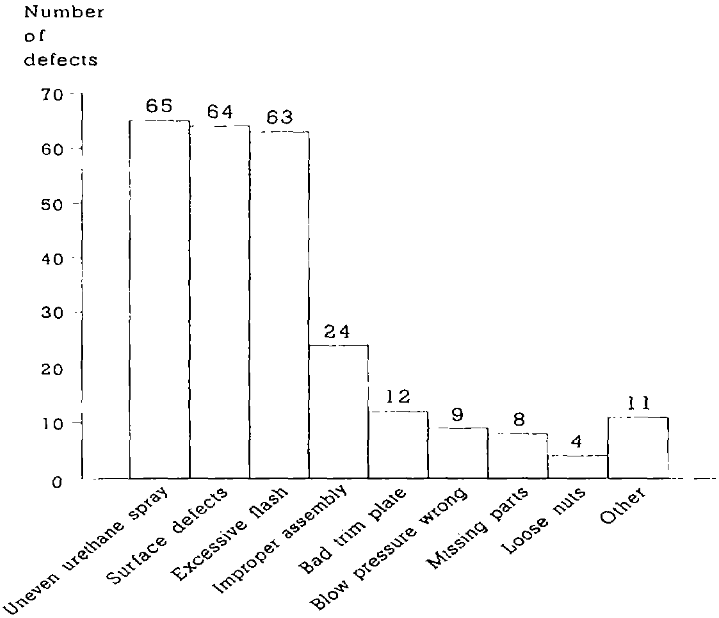
A second potential pitfall arises in connection with the problem-solving procedure itself. Examining only defective items can be misleading and contribute to faulty analysis and ineffective, if not incorrect, action on processes. If problem-free as well as problem items are produced within the same time span, some of the problem-free items should also be examined for the reason that otherwise the causes of the problem at hand might be misjudged. Since a process generates both good and bad items, factors that account for a sufficiently large variation of the quality of items should be identified.

The following example – built around an assembly designed to spray a liquid into a chamber – provides an illustration of the above points. Leakage from the assembly is of course considered to be a critical defect. In inspections of production output several assemblies were found to suffer from this fault. In a next step only leaking assemblies were examined to identify possible causes of leakage with the result that such assemblies were found to contain a particular component that was contaminated by foreign matter. Since already in the past contamination of components had been associated with leakage, this product characteristic was taken as the cause of leakage. As a result, the supplier of the component in question started preparing to include a costly decontamination step in the production process. However, examination of some non-leaking assemblies (manufactured during the same time period) produced the surprising result that non-leaking assemblies showed both the same type and the same degree of contamination as leaking assemblies. The fact that contamination existed in both leaking and non-leaking assemblies made it necessary to search for other possible causes of leakage. In this connection the variation in some of the dimensional characteristics of the assembly was investigated. Only on the basis of this investigation, excessive variation in component dimension could be revealed as the major factor behind leakage. In turn successful attempts at reducing that variation solved the leakage problem and rendered unnecessary the addition of a decontamination step to component production. This example illustrates faulty problem-solving logic as well as failure to recognize the complex nature of the process under study. Both the variation in the number of problem results generated by a stable process and major traits of the problem-solving process itself should be taken into account in attempts at understanding cause-and-effect relationships.

2.5.3 Data consisting of ranks

Such data are created by ordering a set of n individuals or items according to some criterion — like best to worst or largest to smallest — and assigning the numbers 1 to n to the members of this ordered set. Such data are called **ordinal** data. Similar data arise from assigning a **rating** (e.g., excellent, good, mediocre) to each member of a given set of individuals or items. The results of customer evaluation of products or services often consist of responses of this type.

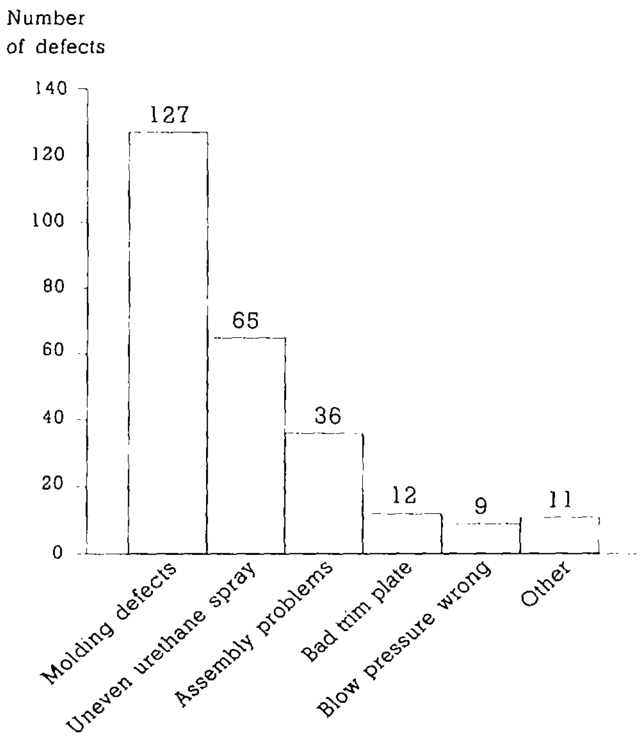
Figure 2.6 *Pareto diagram for counts of horn pad assembly defects by type*



2.5.4 Variables data: dot plots, histograms and the normal distribution

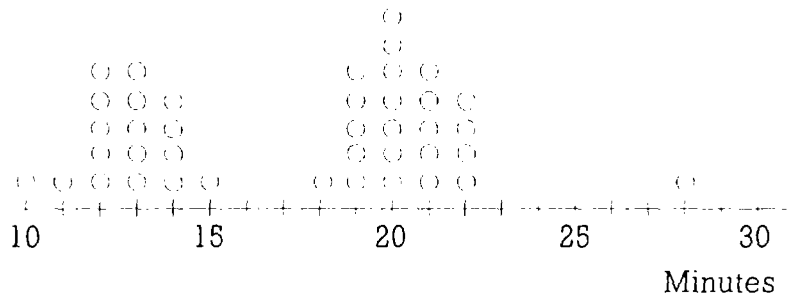
Variables data arise from measurement of a quantitative characteristic of an individual, item or group. For example, measurement of a person's age, height and weight results in three values of the variables type. Dimensions, forces, velocity, rates of flow, temperature, and other quantities that are expressed by numerical values on a continuous scale are also data of the variables type. And the same holds for time data, such as the time required to accomplish a particular service or task. Measurement of the time that elapsed until the first occurrence of a failure or breakdown or of the time between such occurrences is commonly used in the analysis of product reliability. Under certain circumstances, count data of the two kinds mentioned earlier can be treated as variables data, but the conditions for adopting this approach need to be examined carefully.

Figure 2.7 *Pareto diagram for horn pad assembly defects organized by process step*



There is a variety of methods of summarizing a collection of variables data. **Dot plots** may be used to picture the pattern of variation of a small number of measurement values over the potential scale of measurement. A dot plot is constructed by drawing a horizontal scale for the range of possible values and plotting a dot for each measurement above the appropriate point on the scale. **Figure 2.8** shows an example of a dot plot based on data on the time required to transport shipments of parts from one plant to another. This plot suggests that two different processes exist which result in significantly different values of transport time. It also shows an outlying data point.

Figure 2.8 *Data plot of transport times for shipments of parts*



Histograms can be used to roughly indicate the shape of the distribution of a larger set of measurement values. **Figure 2.9** is an example of a histogram derived from 85 values of the measurement of available chlorine (AVCL) in production samples of a household cleaning product.

A histogram can be constructed in the following steps:

1) Find the smallest and largest measurement values in the collection of data.

2) Divide that portion of the scale of measurement that covers the smallest and largest measured values into 5 to 20 equal contiguous intervals that do not overlap, so that each measurement value belongs to only one interval. (With more data points, more intervals can be used).

3) Count the number of measurement values falling in each interval.

4) Draw a horizontal scale representing the scale of measurement and mark off the intervals on that scale.

5) Draw a vertical scale representing frequency.

6) Draw vertical bars over the intervals with heights that correspond to the number of measurement values lying in each interval.

Figure 2.9 *Histogram constructed from 85 measurements of available chlorine in samples of a consumer cleaning product.*

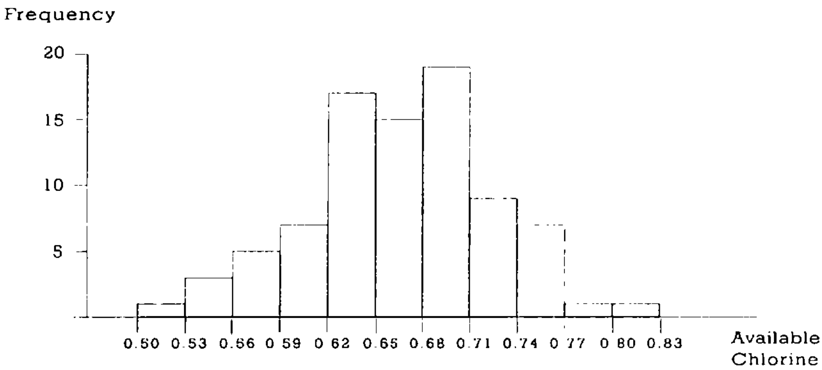


Table 2.2 contains the 85 measurement values the frequency distribution of which is summarized in Figure 2.9

Table 2.2 *85 Measurements of available chlorine in samples of a household cleaning product*

.72	.75	.67	.70	.67	.69
.69	.57				
.64	.70	.68	.71	.73	.69
.75	.74				
.68	.72	.66	.68	.61	.64
.53	.51				
.62	.75	.77	.80	.70	.69
.69	.66				
.69	.63	.59	.64	.63	.58
.53	.62				
.65	.68	.71	.65	.69	.70
.66	.60				
.64	.64	.61	.67	.63	.60
.63	.62				
.67	.67	.64	.62	.65	.54
.58	.72				
.69	.76	.75	.66	.67	.73
.69	.66				
.68	.62	.58	.59	.64	.57
.62	.67				
.59	.71	.72	.75	.70	

An important point to be kept in mind when examining a histogram is that of arbitrariness of the choice of both the number and the width of intervals. This point is significant for the simple reason that the shape of the underlying distribution of values indicated by the histogram is dependent upon that choice. From this it follows that any interpretation of a histogram has to invoke a high degree of caution.

Both dot plots and histograms are summaries of a collection of measurement values depicting not only the location of observed values in relation to a measurement scale but also approximating the distribution of the data. If measurement values refer to characteristics of ongoing processes, any summary of the data that ignores the time sequence of results may obscure one of the most important aspects. It might be useful to consider, as an illustration, the time sequence plots shown in **Figures 2.10** and **2.11**. In each case, the series of measurement values has been summarized by use of a histogram on the right side of the figure where the scale for the histogram is given by the vertical scale of each figure. Although the summary histograms in Figures 2.10 and 2.11 are identical, the sequence plots shown there reveal the existence of two very different processes. The series of results plotted in Figure 2.10 shows no systematic pattern, whereas the series shown in Figure 2.11 exhibits a downward trend. The latter figure suggests that a systematic change occurred in the process raising the question about what might be the cause(s) of that change. If merely the histogram had been used, important information concerning systematic change in the underlying process could not have been obtained. Similarly, the dot plot of transport time shown in Figure 2.8 may obscure useful information available in the original time series.

The above methods are descriptive and produce records of the past. Historical results can usually not form the sole basis for prediction of future results; knowledge of at least the major causal factors of variation is essential to good predictions.

The model most commonly used to study variation in data of the variables type is that of the **normal distribution**. More generally, this model provides the basis for statistical analysis of so-called control charts for variables data. One of its essential characteristics is symmetry of the distribution of measurement values as sketched in **Figure 2.12**.

Figure 2.10 *Time-ordered plot of measurements taken on successive results of a process (random)*

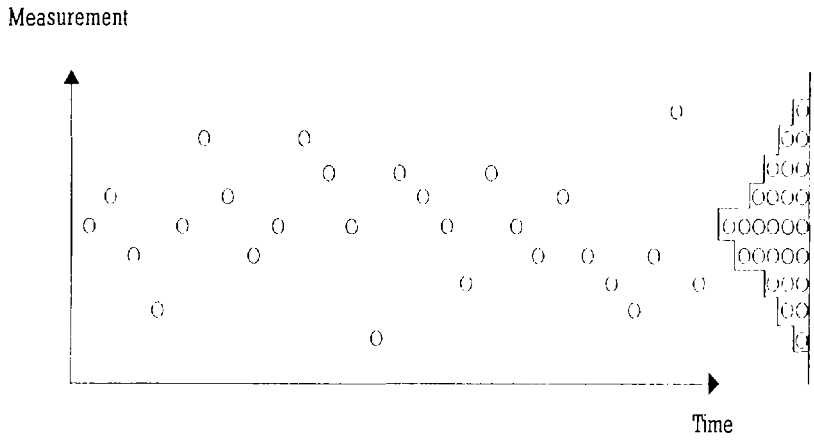


Figure 2.11 *Time-ordered plot of measurements taken on successive results of a process (trend)*

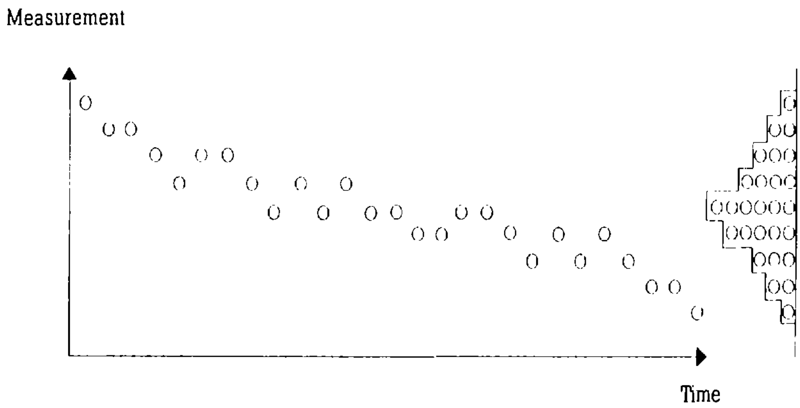


Figure 2.12 *A normal curve*

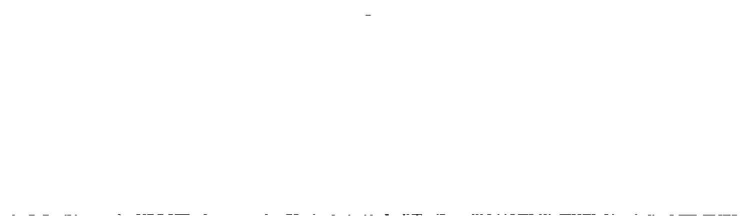


Figure 2.13 *A skewed distribution*

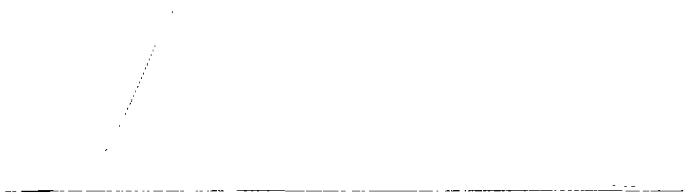


Figure 2.12 as well as **Figure 2.13** represent the relative frequency distribution of a large number of measurement values. Such a distribution should be seen as the limiting case of a sequence of relative frequency histograms with increasing numbers of measurement values. When the number of class intervals grows very large and their width approaches zero, the normal distribution results.

By contrast, other kinds of variables data, such as waiting time, the time elapsing until failure occurs, or certain types of chemical measurement have non-symmetrical distributions. Figure 2.13 depicts such a skewed distribution model. In extreme cases of lack of symmetry the use of non-standard statistical methods may be required. In particular, the time between occurrences of events may be more appropriately analyzed by use of methods other than those based upon the normal model.

It is vital for an understanding of the methods discussed in the present text, that statistical control and the application of the normal distribution model be not substituted for each other. Statistical control, based on the concept of variation, deals with fluctuations over time in the results produced by a process. In particular, it addresses the question of whether or not these fluctuations are around a constant average and also of a steady magnitude. The accumulated results of a process producing statistically controlled variation may indicate a distribution that bears no resemblance to a normal curve. Examples that come to mind in this connection are values of a fraction defective or counts of occurrences of events produced by a stable process. Such data may exhibit a distribution that is heavily skewed.

Practitioners often believe that a histogram representing an accumulation of process data over a particular time span can be used to "test" for statistical control. Likewise, it is often held, that non-normality implies the absence of statistical control or that, conversely, an approximately normal shape of the probability distribution indicates the presence of statistical control. A glance at Figure 2.11 makes it clear that when a sequence of results is summarized by a histogram, the shape produced can appear to be approximately normal, even though the sequence of results is clearly non-random. Furthermore, the absence of extreme outlier results from a histogram is not sufficient evidence for the state of statistical control of process results. Only control charts can provide a reliable answer to the question of whether or not process results are statistically controlled.

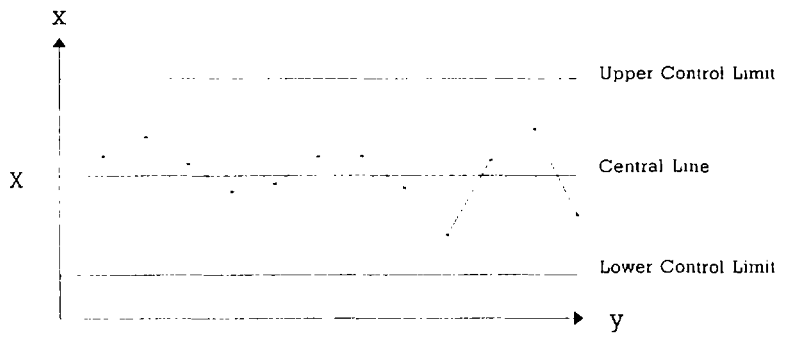
2.6 Introducing control charts

Once the methods of data collection have been determined and preliminary investigations carried out with the help of graphical techniques described previously, analysts are ready to monitor patterns of variation. The major tool in this task is that of **control charts.** These charts are fairly easy to construct and serve several purposes: They help managers to identify different patterns of variation, interpret their meaning, conclude on the causes underlying variation and gauge the success of efforts to improve the studied process.

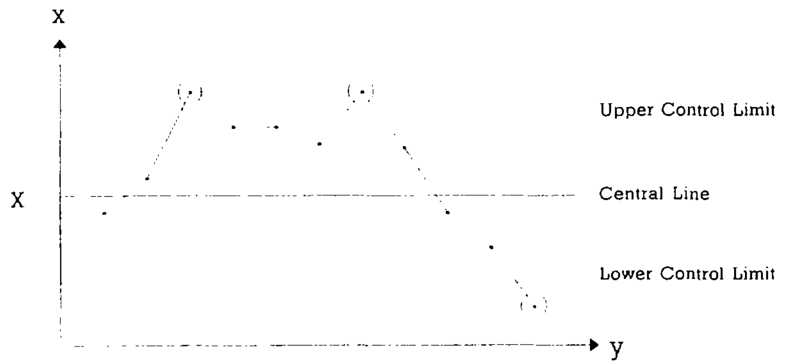
Depending on the type of underlying data and the information sought, different versions of control charts are constructed. All, however, share common features which are shown in **Figure 2.14**. They include a **centre line** representing an estimate of the average value for all observations taken into account and **control limits** defining the boundaries for acceptable variation. A widely adopted principle of obtaining **upper and lower** control limits is that of adding to and subtracting from the average value **three-times** an estimate of the **standard deviation** of the plotted statistic. Throughout the present text this approach (for which a rationale is provided in Annex A) will be followed.

Figure 2.14 *Schematic control charts*

a) A Manufacturing Process that is in Control



b) A Manufacturing Process that is out of Control



The manufacturing process represented by the control chart can then be characterized in the following way: The process is considered as being **'in control'** if all data points on the chart fall **inside control limits** and exhibit **no other sign of non-random behaviour**, as, for example, a trend or other distinctive pattern.

Should one or more observations lie outside the range between the two control limits, this can be taken as an indication of a special cause of variation and the process is termed as being 'out of control'. If control limits are reasonably symmetrical around the centre line, additional criteria can be used to judge whether variation within control limits is random. The most common among these criteria are those relating to runs, trends and periodicity as signals of non-random variation. Accordingly, the following rule-of-thumb indications of non-randomness are often used as a guide to detect out-of-control conditions:

1) a 'run' of at least seven successive points falling on one side of the centre line (rule of seven, run of seven);

2) at least seven successive increases or decreases of data points on the chart indicating a trend;

3) data points showing periodic up-and-down trends for the same interval on the chart.

Figure 2.14 provides an illustration of the use of control limits. Later on in the text reference will also be made to the rules of thumb stated above. The following chapters will discuss in considerable detail the construction and use of the major types of control charts. This will be done on the basis of the general guidelines presented in this section and with ample illustration of techniques by means of real-life examples. In addition, the discussion will frequently refer to the broader context in which control charts and their applications have to be seen.

CHAPTER 3

ANALYZING ATTRIBUTES DATA

The effective use of charts requires competence in the application of the appropriate statistical methods. However, a number of issues other than purely statistical ones must be addressed concurrently with the use of charts. Among them are the intent for developing control charts, the manner in which process knowledge is being recorded, the use which should be made of this knowledge for designing samples and subgroups of data, and the tasks for which management uses statistical analyses of a process.

This chapter will not attempt to discuss all of these issues in detail, but will consider the more prosaic aspects of constructing and interpreting a particular type of chart for the case of attributes data. Against this backdrop conditions for the effective use of such charts will be discussed. Finally, several case studies which illustrate the appropriate use of the techniques presented here will be provided.

3.1 Attributes data that require the use of *p* or *np* charts

Information on the performance of a process is often obtained by counting the number of items which possess a certain attribute. Such attributes data were labelled categorical in Chapter 2. Each one of the items examined falls in one of two categories, according to whether it possesses the specified characteristic or not. In quality-improvement work these data arise from a count of the members of a collection or subgroup of items which are judged to be **nonconforming** in relation to a given set of criteria.

Illustrative examples are:

1) Counts of the number of damaged cans in lots of a given size. Each can is judged as being 'damaged' or not.

2) Counts of the number of defective shafts in samples of a given size. Here 'defective' is defined as the diameter failing to meet a stated specification. Each shaft either meets or fails to meet this specification.

3) Daily counts of the number of those motors out of each day's production that fail to pass a certain voltage test. Each motor is judged as passing or failing the voltage test.

4) Weekly counts of the number of invoices which contain at least one error. Each invoice contains either one or more errors or is correct.

There are at least three objectives of evaluation that lead to the collection of attributes data. The first one has to do with the need to obtain general information on quality. To give an example, a merchandiser may record the number of dented cans in a lot of canned food purchased from a producer in order to assess the quality of the lot. A second objective is to draw up a product or service profile which reflects certain characteristics of the product or service. The data in **Table 3.1** are an example describing the performance of automobile engines that have been subjected to extensive testing. From each shift's production at the considered plant ten engines were selected for extensive testing. In the tests, 47 different characteristics were checked on each engine. If an engine failed on any one of these characteristics, it was recorded as rejected. The numbers in the table are the counts of the numbers of engines rejected in the production of each one of 21 successive weeks. When, as in the present case, attributes data are collected over time, the stability of variation in the results can be assessed. In the case of stability the future performance of the process can be predicted with regard to the characteristic studied.

Finally, generating data on nonconforming items as such may be the objective. Attributes data are then used to gain insight into the mechanisms of the process or the dynamics of the system which generated the product under study. In this instance, current knowledge of the process and system is used to guide data collection. An analysis of attributes data then serves the purpose of updating current (system) knowledge as well as indicating possibilities for process (system) improvement.

Table 3.1 *Number of rejected engines*

Subgroup	n	np	Subgroup	n	np
1	100	6	12	100	5
2	100	8	13	100	7
3	100	2	14	100	8
4	100	3	15	100	11
5	100	5	16	100	3
6	100	10	17	100	5
7	100	4	18	100	9
8	100	7	19	100	5
9	100	2	20	100	7
10	100	12	21	100	2
11	100	10			

3.2 The construction of *p* charts

In an analysis of attributes data of the present type, *p* and *np* charts are used. This is illustrated at the case of the data of Table 3.1. The entire data set is first subdivided into 21 subgroups representing the 21 weeks over which data were collected. For each of these subgroups containing 100 engines the counts of nonconforming engines were recorded. The following notation will be used to describe the analysis in detail:

k denotes the number of subgroups
n denotes the number of items in a subgroup
p denotes the fraction of non-conforming items

3.2.1 Plotting points on a *p* chart

The construction of a *p* chart begins by plotting the nonconforming fraction *p* for each subgroup. This fraction is calculated for each subgroup by dividing the number of nonconforming items by the total number of items in the subgroup. An examination of Table 3.1 shows that, for example, out of the 100 engines tested during the first week six were found to be nonconforming resulting in a nonconforming fraction of .06 for the first subgroup. The *p* values for all 21 subgroups are calculated in an analogous manner and then plotted on a chart with the subgroup number along the horizontal axis and the range of the nonconforming fraction along the vertical axis. **Figure 3.1** was constructed in this way. It is important to note that the data were recorded and also plotted in the same order in time in which they were collected. The significance of plotting the data in this fashion will emerge from the analysis.

3.2.2 Calculating the centre line for a *p* chart

The centre line of a control chart represents the average value of the data which the chart represents. This average value of nonconforming items is denoted *p* (bar) and given by:

$$\bar{p} = \frac{\text{total number of nonconforming items}}{\text{total number of items inspected}}$$

In the example of Table 3.1 out of the 2,100 engines inspected 131 engines were rejected, so that the equation for the average reads:

$$\bar{p} = \frac{131}{2100} = 0.0624$$

This value of the average has been used to position the centre line in the *p* chart of figure 3.1.

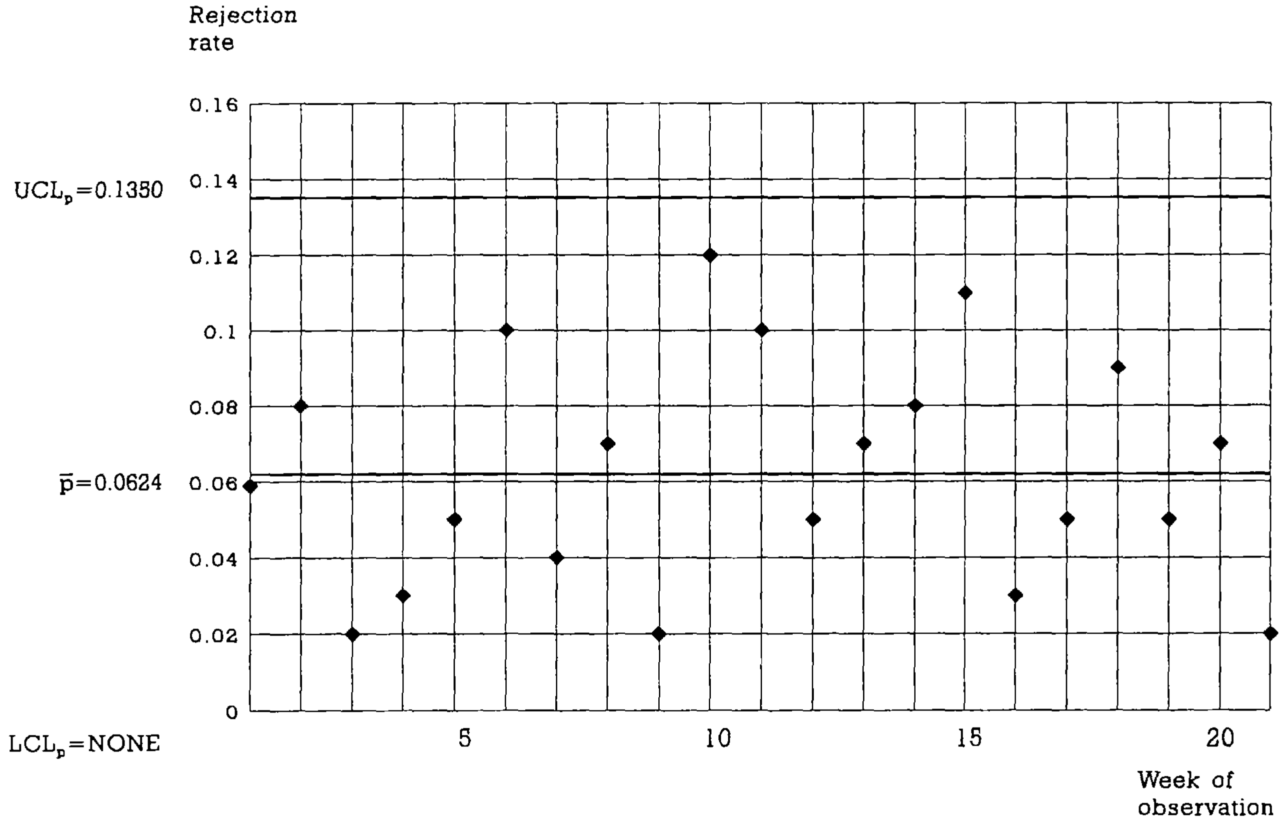
3.2.3 Calculating the control limits for a p chart

The upper and lower control limits define the amount of variation which might be expected to occur in p values if the process is stable. For the p chart these limits are:

$$LCL_p = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$UCL_p = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

Figure 3.1 Plot of proportion of rejected engines



where UCL (LCL) stands for upper (lower) control limit. For the data on the engine tests

$$UCL_p = 0.0624 + 3\sqrt{\frac{0.0624(1-0.0624)}{100}} = 0.1350$$

and

$$LCL_p = 0.0624 - 3\sqrt{\frac{0.0624(1-0.0624)}{100}}$$

The latter value would result in a negative number. Since a negative fraction of nonconforming items is impossible, the convention of saying that there is no lower control limit will be used in this text. Thus, for the engine testing data we have:

$$LCL_p = \text{none}$$

Figure 3.1 also displays the values of upper and lower control limits.

Summary of the construction of *p* charts

1) Each item in a subgroup comprised of n items is classified as either conforming or nonconforming. For each subgroup the number of nonconforming items is recorded.

2) For each of the k subgroups *p* (the nonconforming fraction) is calculated.

$$p = \frac{\text{number of nonconforming items in the subgroup}}{n}$$

3) The average of nonconforming fraction is calculated and used to draw the centre line on the chart:

$$\bar{p} = \frac{\text{total number of nonconforming items}}{\text{total number of items inspected}}$$

4) Upper and lower control limits are calculated and the corresponding lines drawn into the graph:

$$UCL_P = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$LCL_P = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

3.3 Statistical control

The p chart constructed from the data on engine tests provides specific information about the process producing these engines. What is to be noted about this chart is first of all the variation in the values of p. The smallest p value plotted is 0.03, whereas the largest such value is 0.12. An obvious question relates to what can be learned about the process from the observed variation in p values. For example, the fact that the largest p value occurred during the tenth week suggests to investigate what was different about the tenth week of operation. Likewise, the fact that the lowest p value was observed both for the fourth and the sixteenth week poses the question of whether better work was done during these weeks than during others. Answers to such questions can be found using the concept of a **stable process** or a **process in statistical control.** In this connection, it has to be specified how much variation is to be expected in the outcomes of a stable process, and what this implies for the interpretation of empirical data.

A stable process may be thought of as a process which produces a constant level of variation over time. Loosely speaking, stability can be described as the variation in outcomes being of predictable size. Such is the case with a process for which the sources of variation are '**common causes**', i.e., causes which are common to all outcomes. By contrast, an unstable process is under the impact of '**special causes**' — usually in addition to common causes — of variation which act only on some outcomes but not on all.

The two types of causes of variation can be illustrated with the help of the engine test data previously discussed. Common causes are those that are present throughout the time during which data were gathered. These causes may have to do with the capability used for engine manufacture, the maintenance practices for this equipment, the original design of the engine. For example, the fact that over time the metal from which the engine block is cast varies in its degree of hardness means that machining operations require varying amounts of time to complete. Sometimes adequate machining may not be done because of the difficulty in accommodating the machining equipment to the varying degree of hardness. Consequently, some engines are improperly machined, possibly resulting in oil leakage or inadequate combustion characteristics. This type of problem might occur at any time throughout the manufacture of the engine and would thus be common to all outcomes.

An example of a special cause of variation in the fraction of nonconforming engines is the type of cutting tools or grinding wheels being used in the machining operation. If for example over time these tools were purchased from different suppliers, the type used in one week may not be the same as that used in another week. Differences in results stemming from inadequate tools purchased from one supplier would then affect some but not all outcomes of the process over the given time period. When an inadequate tool is used, a marked increase in nonconforming engines may be expected. Thus additional variation due to the use of inadequate tools would appear to have a special cause.

Earlier in the discussion the question was raised whether the p value of 0.12 in week ten was large enough to indicate a difference between the way engines were produced in this particular week and other weeks. This question can now be rephrased as: "Does the value of p=0.12 differ sufficiently from other p values recorded to indicate a special cause in week ten?"

An answer to the above question or to questions of a similar type is of considerable importance to the study of the process at hand. More specifically, if stability could be established, the process could be predicted to behave in the future in a fashion similar to that of the past. Conversely, if a process is not stable and the special causes of instability cannot be identified, future outcomes cannot be predicted.

The significance of predictions can be made clear from the engine example outlined previously. Predictions of the proportion of conforming engines are the basis for reliable production cost estimates, efficient management of schedules for engine production, and the provision of supply of engines to assembling within clearly specified bounds.

The benefits of any improvement due to enhanced knowledge of the process may be gauged, for example, from a brief description of the effects of poor scheduling. Schedules must usually be met irrespectively of the possibility to run the process in a stable manner. If there exists a large discrepancy between a schedule and the potential to meet that schedule, ways have to be found to cope with the ensuing difficulties. These ways may include overtime to meet the schedule, hold-ups at the next stage of production, and negotiated allotments with customers. If remedial measures become too painful, it may be decided to increase the estimated scheduling time of a job. In other words, lack of confidence in the ability to meet a given schedule may result in overestimates of the time scheduled as a means of avoiding the consequences of unmet schedules.

In addition to its significance for business planning, the study of stability of a process is an important element of any quality-improvement program. Improvement of a process requires not only comprehensive engineering knowledge, but also analytical information about outcomes. In the current example of engine testing only little use was made of any technical knowledge about process mechanisms. Instead, fairly standard data were collected without considering particular technical requirements. Notwithstanding the lack of technical detail, a stability analysis of the recorded outcomes provides important inputs to improvement efforts.

If for a process instability is diagnosed, the next step is to identify the underlying special cause(s). In the case of a special cause accounting for increased levels of nonconforming items, the question arises about what should be done to eliminate this cause. In the alternative case of reduction of the levels of nonconforming items, ways should be found to incorporate the special factor in the standard operating procedure. If in the context of the engine test example the value of p=0.12 differed so much from other p values that a special cause appeared to be operating, there would be a need to identify that special cause and then try to eliminate it. If in addition the value of p=0.03 were so much smaller than every other p value that a special cause seemed to be acting on that particular outcome, this would necessitate to identify the associated special cause and attempt to make it part of the standard operation of the process.

Alternatively, the conclusion that a process is stable and can therefore be subject only to common causes of variation suggests that only changes that refer to one or more such causes can be expected to result in improvement. Thus in order to improve a stable process information is needed regarding the impact of common causes on process outcomes. This is why the data on engines presented in the previous example would most likely be insufficient to support process improvement. The collected information provides little insight into how various factors impact the level of variation in the production process under study.

The distinction between common and special causes is important for judging the appropriateness of actions to improve a process. Without knowing, for example, whether or not a *p* value of 0.12 points to a special cause, it would be very difficult to propose appropriate remedial action. If the process were in control, it would be doubtful whether any action could effect improvement. At worst, such action could, due to overadjustment, produce more variation in outcomes than previously.

The completed *p* chart in Figure 3.1 shows that none of the points plotted falls outside control limits. Since the chart shows no other signs of non-random behaviour either, the conclusion must be that there is no evidence of instability. In other words, the process appears to be in statistical control.

3.4 The example of pot-pie production revisited

In order to bring out more clearly some of the points mentioned in previous sections, the example of 'pot-pie' production (introduced in Chapter 2) shall be taken up again.

As was mentioned in a first brief outline of the example, management decided to compile available information in order to summarize process knowledge. **Figure 3.2** presents the flow chart of pot-pie production while **Figure 3.3** provides a more detailed description of that portion of the production process which produces 'bottom crusts'. In addition, a cause-and-effect diagram - displayed in Figure 2.1 - reflects information that was provided by several of the operators of the pot-pie line.

The cause-and-effect diagram led to a lengthy discussion about which of the causes listed there were effective in producing nonconforming pie crusts. At this juncture it was realized that much of the difference in opinion about the reasons for nonconforming crusts had to do with incomplete information about the current capability of the pot-pie line. For instance, there was only scant knowledge of whether more nonconforming crusts were observed on some days in comparison with others. If there was a difference between days regarding the number of nonconforming crusts, the group would have to investigate more closely the

Figure 3.2 Flow chart of pot pie production

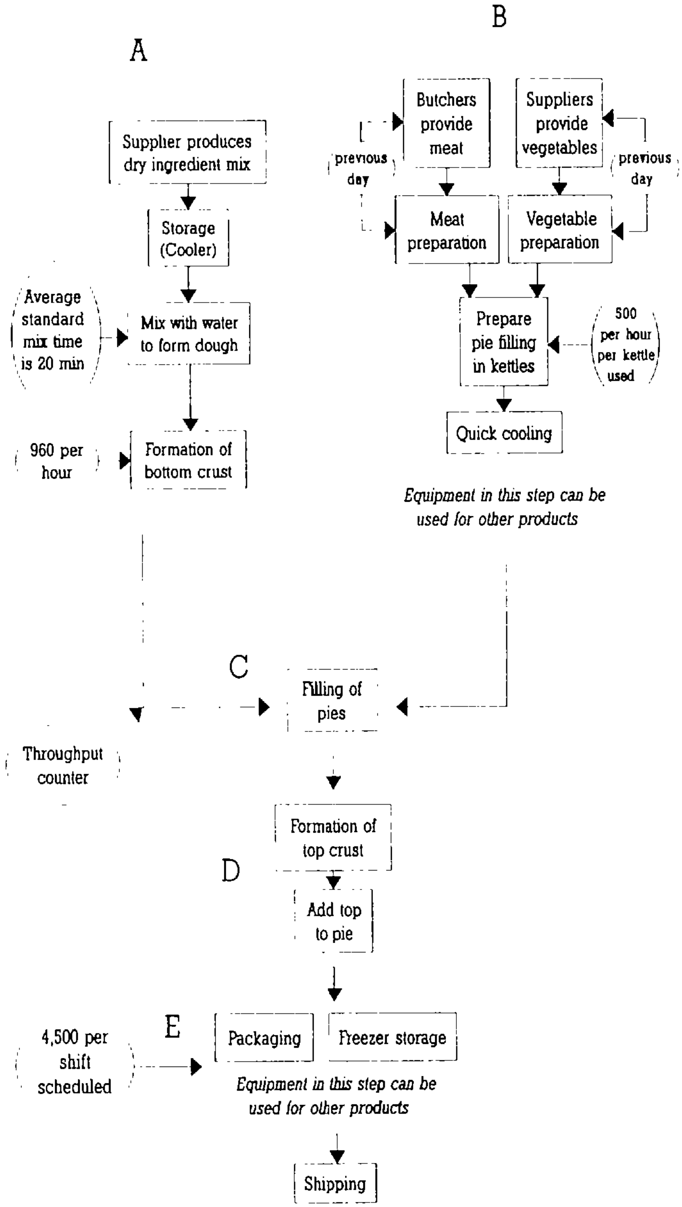
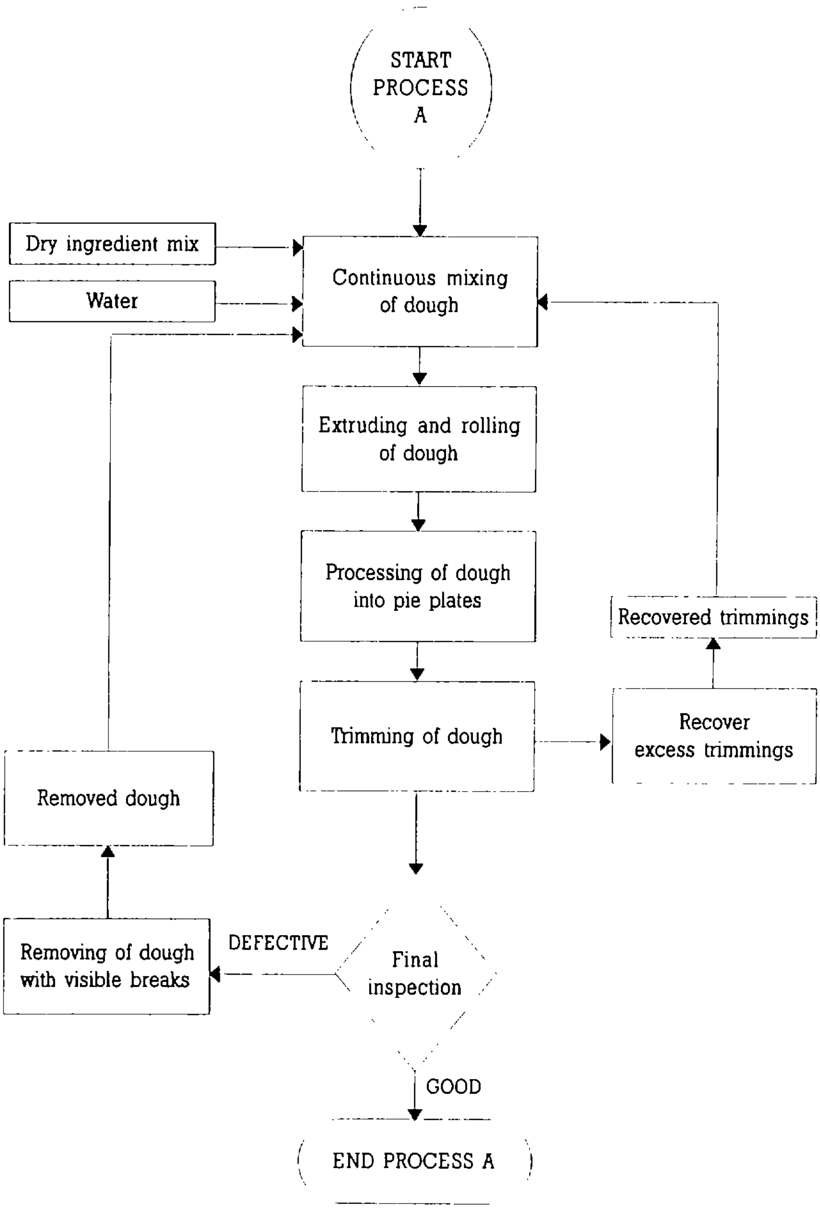


Figure 3.3 Flow chart of the production of bottom crusts



causes that were effective on some days but not on others. Further, the group possessed no knowledge about whether the number of nonconforming crusts was larger at a certain time compared to the rest of the day. Knowledge of process performance during a day would help identify causal factors behind the production of nonconforming crusts. Furthermore, thorough knowledge of how the process was currently operated would provide a reference point from which to judge future improvements.

In order to better understand the given process, the management group decided to collect also information on the number of crusts discarded throughout a single day's operation. The flow chart in Figure 3.3 describes the operation of the process and also illustrates at which points data collection was performed. The production of pie crusts begins with the dough being extruded from the vat in which it is mixed into a thick sheet. From this point the dough is pulled into the forming machine, which rolls it into a thin sheet. This is then cut into wide strips by the forming machine with the strips covering 24 metal trays. Subsequently, the dough is pressed into pie plates and excess dough is trimmed. The forming of a group of 24 crusts is referred to as a cycle, and the operator who runs the forming machine controls the time at which the cycles occur. The same operator examines the pie crusts after the forming operation to see that there are no visible breaks in the dough and that all metal trays are completely covered by dough. It was the large number of crusts discarded at this point of the operation which caused concern. Therefore, it was decided to collect information on nonconforming crusts at precisely this point of the process.

3.5 The construction of *np* charts

As indicated above, the management group trying to improve on the pot-pie production process decided to collect information throughout the day and over several days on the number of nonconforming crusts being produced. In order to achieve this goal the operator who was inspecting the crusts was instructed to record the number of nonconforming crusts produced in four consecutive cycles once each hour. This collection of information was carried out for one working week, i.e., five working days. The resultant data are shown in **Table 3.2.**

For an evaluation of the information contained in these data a p chart could be utilized. An alternative is to plot the *number* of nonconforming crusts rather than their fraction. The result is an np chart which contains the same information as a p chart, however in a different form. In the case

Table 3.2 Number of nonconforming crusts in four consecutive cycles of 24 crusts

Date	Time	in subgroup	nonconforming
11-08	8:00 a	96	15
	9:00 a	96	16
	10:00 a	96	17
	11:00 a	96	20
	12:00 a	96	12
	1:00 p	96	19
	2:00 p	96	13
11-09	8:00 a	96	18
	9:00 a	96	27
	10:00 a	96	16
	11:00 a	96	21
	12:00 a	96	22
	1:00 p	96	20
	2:00 p	96	19
11-10	8:00 a	96	9
	9:00 a	96	14
	10:00 a	96	18
	11:00 a	96	23
	12:00 a	96	19
	1:00 p	96	12
	2:00 p	96	13
11-11	8:00 a	96	15
	9:00 a	96	16
	10:00 a	96	18
	11:00 a	96	18
	12:00 a	96	16
	1:00 p	96	12
	2:00 p	96	16
11-12	8:00 a	96	11
	9:00 a	96	17
	10:00 a	96	11
	11:00 a	96	18
	12:00 a	96	15
	1:00 p	96	14
	2:00 p	96	19
			579

of each subgroup containing the same number of items examined the choice between a *p* chart and an *np* chart is merely a matter of convenience.

3.5.1 Plotting the points on an *np* chart

Table 3.2 contains information on the time of inspection, the number in a subgroup, and the number of nonconforming crusts in a subgroup. As mentioned previously, on an *np* chart the **number** of nonconforming items is plotted.

In **Figure 3.4** these values are plotted in time order so that information on the time pattern of performance is retained. As in the case of a *p* chart the subgroup numbers appear along the horizontal axis while along the vertical axis the range of the number of nonconforming items is indicated. Analysis of the information contained in such a plot proceeds in a manner analogous to that applied in the case of a *p* chart. Accordingly, a centre line is drawn into the chart and so are upper and lower control limits. Like in the case of the *p* chart these control limits serve to determine the level of variation which is compatible with the data originating from a stable process.

3.5.2 Determining centre line and control limits for an *np* chart

On an *np* chart the value used to draw the centre line is the average number of nonconforming items in a subgroup. This average is denoted *np* (bar) and given by

$$n\bar{p} = \frac{\text{total number of nonconforming items}}{\text{number of subgroups}}$$

For the data of Table 3.2 (with a total of 579 nonconforming crusts in the 35 subgroups) the value

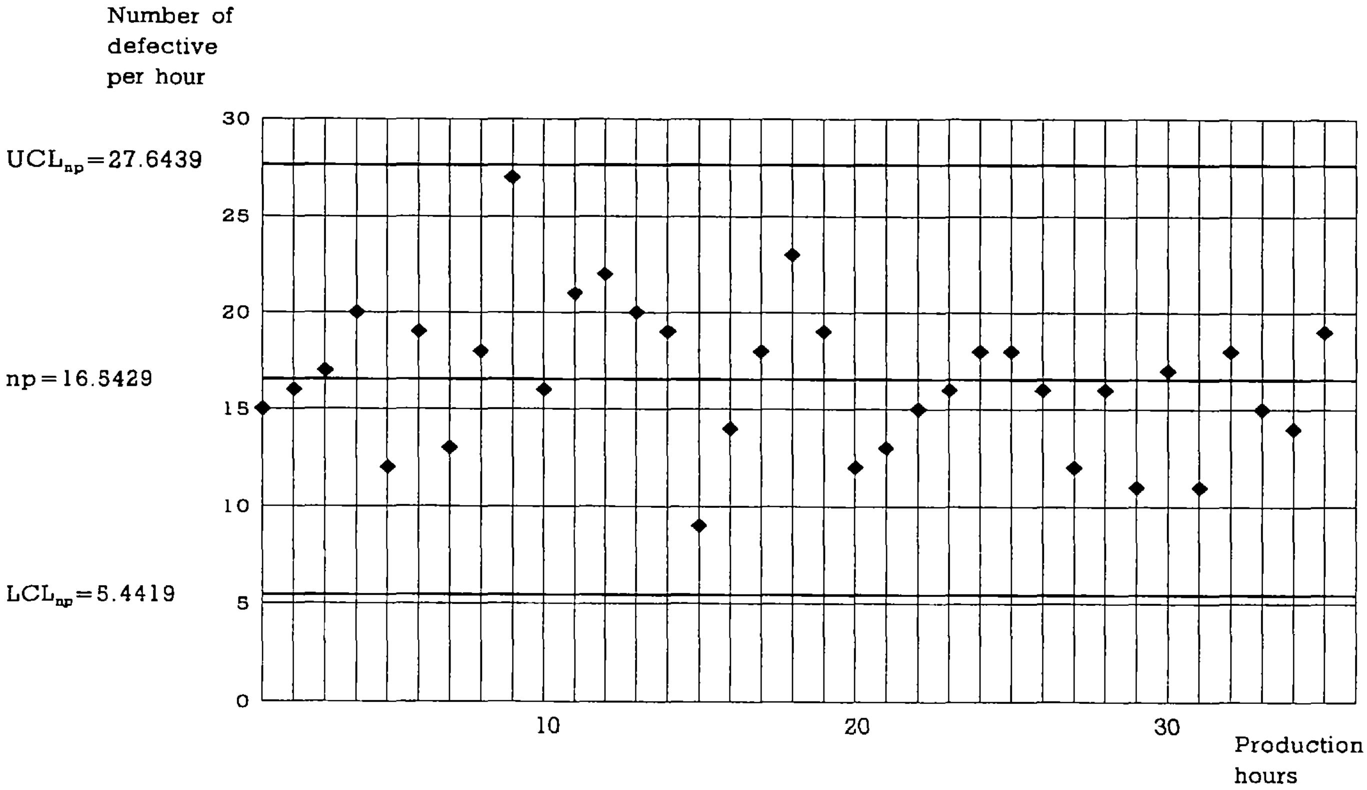
$$n\bar{p} = \frac{579}{35} = 16.5429$$

is obtained and used to position the centre line in Figure 3.4.

The calculation of upper and lower control limits for an *np* chart requires the calculation of the value of *p*(bar) by the formula given in Section 3.2.2. Alternatively, *p*(bar) could be calculated by dividing *np*(bar) by *n*, the subgroup size.

$$\bar{p} = \frac{16.5429}{96} = 0.1723$$

Figure 3.4 Illustrative np chart: monitoring the occurrence of defective pie crusts



The upper (UCL) and lower (LCL) control limits are then calculated as follows:

$$UCL_{np} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$LCC_{np} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$$

For the data in Table 3.2, these limits are:

$$UCL_{np} = 16.5429 + 3\sqrt{16.5429(1-0.1723)} = 27.644$$

$$LCL_{np} = 16.5429 - 3\sqrt{16.5429(1-0.1723)} = 5.4419$$

The above values have been used to draw the upper and lower control limits in Figure 3.4. The completed chart shows no indication of a special cause operating. From the available data, it appears that hour to hour, throughout the week studied, the proportion of nonconforming pie crusts is stable with about 17% nonconforming crusts being produced on average.

At first glance the summary of the results of the *np* chart seems to provide the same information as the initial statement about the pot-pie line, namely, that it was producing about 18% nonconforming items. However, there is additional evidence arising from the *np* chart. This evidence is about the direction that improvement work might best take. The data behind the points on the chart were collected on an hour to hour basis over a period of one week. The indication is that the process is stable when judged in this fashion. Consequently, any investigation of sources of variation that would result in differences between days or hours is not recommended at this juncture, since there is no evidence of a special cause acting at some hours and not at others. This implies that there is no need to consider differences between hours or days in the performance of the process. However, those causes on the chart that might affect the number of nonconforming crusts in the course of an hour are promising candidates for further examination. And the cause-and-effect diagram can be used to aid in identifying causes of the production of nonconforming crusts that might be acting inconsistently within an hour.

3.6 Selecting the subgroup

In the study of pie crusts each subgroup on the *np* chart consisted of four groups of 24 crusts. These four groups came from four consecutive cycles of the machine which formed 24 pie crusts at each operation. Data for the 35 subgroups shown in Figure 3.4 were collected at various points in time throughout one day. Obviously the manner in which subgroups are selected determines which special causes of variation can be detected.

One of the possible causes identified on the cause-and-effect diagram for nonconforming pie crusts had to do with the position of each individual tin at the moment when dough is placed over tins. Tins occupying an end position at that moment are more likely to be defective, since the dough may be too thin at this location or may not have been properly rolled out so that it does not cover the pie tins completely, or else the dough may at the ends dry more quickly than in middle positions. If rejection of a pie tin is associated with its position at the time of filling, it would be expected that different proportions of tins are rejected at different positions. In this connection two questions arise:

1. Are the numbers of nonconforming crusts formed at different locations stable over time?
2. How do the numbers of crusts rejected at different locations compare?

The previous subgrouping strategy was not geared to providing answers to these questions as each subgroup contained both "middle" and "end" crusts. In order to determine whether the end positions do indeed produce a higher proportion of defective crusts, a different subgrouping strategy has to be devised. In the context of this strategy the subgroups are selected in such a way that some contain only end crusts and some only middle crusts.

Collecting data in this fashion requires a much larger investment of time, since each crust has to be identified by its location in addition to noting whether it is nonconforming. The resulting data are recorded on a form like the one shown in **Figure 3.5**. The circles on the form correspond to the 24 locations of the pie tins when they are being covered with dough. The places labelled 1 through 4 and 21 through 24 are end positions. Crusts placed in these eight positions in the first four consecutive runs formed the first subgroup. The number of nonconforming crusts in this group with sample size $n = 4 \times 8 = 32$ is given in **Table 3.3**. The second subgroup consists of the remaining 64 crusts placed in middle positions and pertaining to the same four consecutive runs from which the first subgroup was drawn.

Figure 3.5 Form for recording pie crusts by location

Inspection time	1	3	5	7	9	11	13	15	17	19	21	23

Inspector												
--------	2	4	6	8	10	12	14	16	18	20	22	24
Inspection time	1	3	5	7	9	11	13	15	17	19	21	23

Inspector												
--------	2	4	6	8	10	12	14	16	18	20	22	24
Inspection time	1	3	5	7	9	11	13	15	17	19	21	23

Inspector												
--------	2	4	6	8	10	12	14	16	18	20	22	24
Inspection time	1	3	5	7	9	11	13	15	17	19	21	23

Inspector												
--------	2	4	6	8	10	12	14	16	18	20	22	24

The rest of the information in Table 3.3 was recorded in an analogous fashion. Odd numbered subgroups were formed from end crusts and have a subgroup size of 32. Even numbered subgroups were formed from the middle crusts and have a subgroup size of 64. All 40 subgroups were collected over one shift of operation. Because they do not all have the same value of n (subgroup size), a control chart for varying n had to be constructed for these data.

Table 3.3 *Number of nonconforming end crusts and middle crusts in four consecutive cycles*

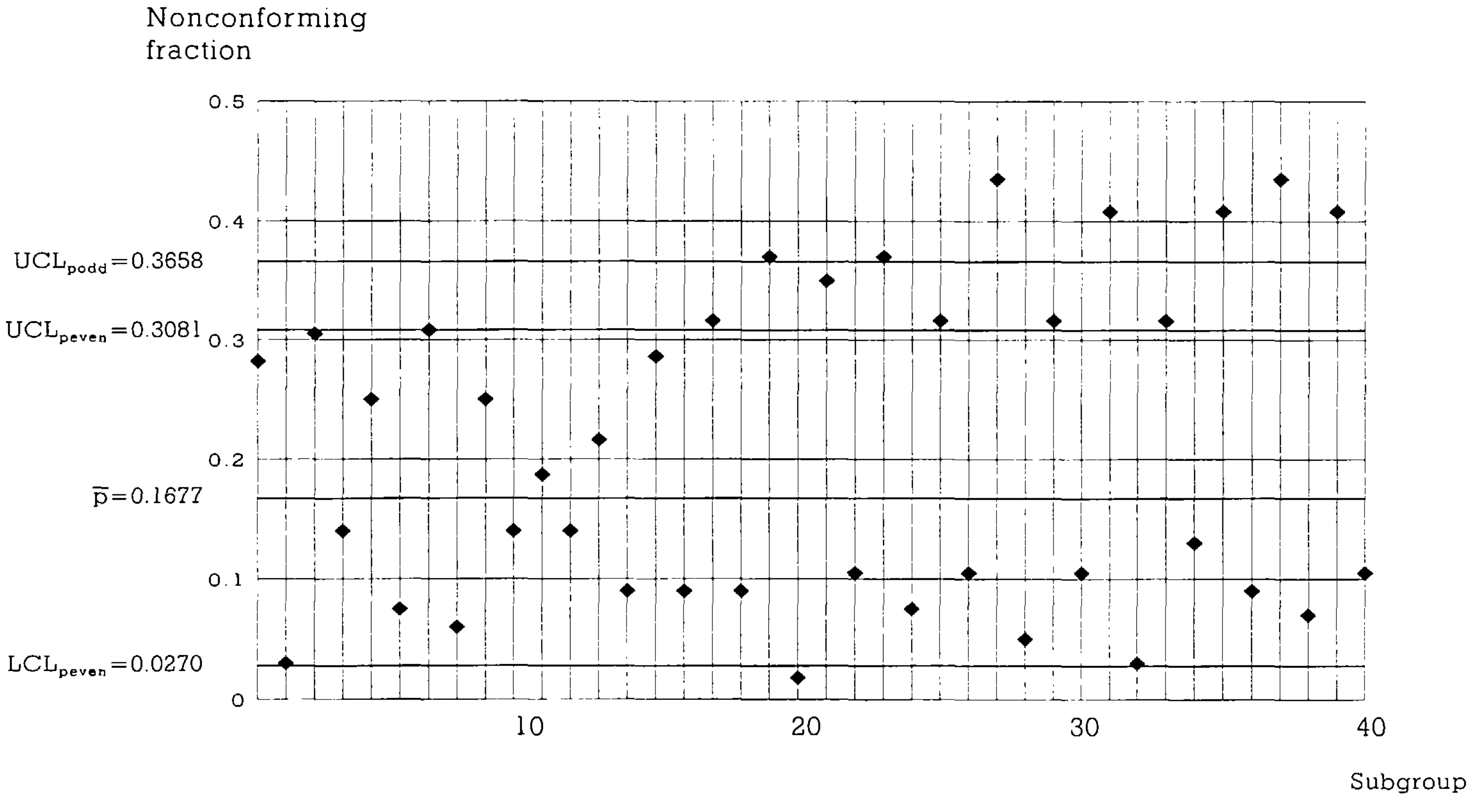
Subgroup	n	np	p	Subgroup	n	np	p
1	32	9	.281	2	64	2	.031
3	32	10	.313	4	64	9	.141
5	32	8	.250	6	64	5	.078
7	32	9	.281	8	64	4	.063
9	32	8	.250	10	64	9	.141
11	32	6	.188	12	64	9	.141
13	32	7	.219	14	64	6	.094
15	32	9	.281	16	64	6	.094
17	32	10	.313	18	64	6	.094
19	32	12	.375	20	64	1	.016
21	32	11	.344	22	64	7	.109
23	32	12	.375	24	64	5	.078
25	32	10	.313	26	64	7	.109
27	32	14	.438	28	64	3	.047
29	32	10	.313	30	64	7	.109
31	32	13	.406	32	64	2	.031
33	32	10	.313	34	64	8	.125
35	32	13	.406	36	64	6	.094
37	32	14	.438	38	64	5	.078
39	32	13	.406	40	64	7	.109
		208				114	

3.7 The construction of p charts when n varies

Whenever a set of data on nonconforming items is grouped in subgroups of varying size, the appropriate chart for analyzing these data is a p chart. Plotting the points and drawing the centre line on the chart is done in the way described in Section 3.2. For the data in Table 3.3 the chart in **Figure 3.6** shows the values p of the nonconforming fraction plotted by subgroup together with the centre line. The value for positioning the latter is given by

$$\overline{\mathrm{p}} = \frac{208+114}{(32\times 20+64\times 20)} = 0.168$$

Figure 3.6 P chart for the data of table 3.3



The formulae for upper and lower control limits for p charts given in Section 3.2.3 were a function of subgroup size n. One way of interpreting these formulae is that in a stable process the amount of expected variation in the nonconforming fraction also depends on the number of items in a subgroup.

When the size of subgroups varies as in the case of the data in Table 3.3, different expressions have to be used. Here the control limits used for the p values plotted depend on the number of items used in the calculation of p values. Thus for points based on a sample of 32 crusts, the upper and lower control limits are given by:

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$UCL = 0.168 + 3\sqrt{\frac{0.168(1-0.168)}{32}} = 0.366$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$LCL = 0.168 - 3\sqrt{\frac{0.168(1-0.168)}{32}}$$

$$LCL = \text{none}$$

For points which were calculated as the fraction of nonconforming items out of 64 crusts the upper and lower control limits are:

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$\text{UCL} = 0.168 + 3\sqrt{\frac{0.168(1-0.168)}{64}} = 0.308$$

$$\text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$\text{LCL} = 0.168 - 3\sqrt{\frac{0.168(1-0.168)}{64}} = 0.027$$

Limits determined on the basis of these formulae have been inserted in the *p* chart of Figure 3.6. Odd-numbered subgroups had a group size of 32 implying an upper control limit of 0.366 and no lower control limit according to the above formulae. The remaining points correspond to a subgroup size of 64. Therefore the control limits for these points are 0.308 and 0.027, respectively. The chart in Figure 3.6 shows several points above the upper control limit and one *p* value below the lower control limit. Since all points above the upper control limit belong to subgroups of end crusts, it appears that such crusts tend to have larger nonconforming rates than middle crusts.

The strategy for analyzing the aforementioned difference between crusts can be summarized as follows: A possible source of variation in the fraction of nonconforming crusts was identified as having to do with tin position. In order to prove this conjecture, subgrouping distinguished between end crusts and middle crusts. The fact that points associated with end crusts are out of statistical control suggests that a special cause is in all likelihood connected with end crusts.

3.8 Non-random behaviour: the example of pot-pie production continued

The present example can serve to illustrate further systematic investigation of non-random behaviour in a production process. According to the results reported in the previous section, improvement work on the pot-pie line has to be based on an understanding of why more nonconforming crusts are produced at the ends rather than in the middle. How this work is to be conducted depends on whether or not the number of nonconforming crusts - when considered separately by type - is stable over time. If, for example, that part of the process associated with the production of end crusts were statistically out of control, the underlying special causes would have to be identified. On the other hand, if stability were diagnosed, improvement work would have to take into account common causes of variation in the quality of end crusts.

In the present example, separate p charts of the nonconforming fractions among end crusts and middle crusts, respectively, were constructed. The chart for end crusts revealed non-random behaviour in the form of a trend of the fraction of nonconforming end crusts to increase over time. This observation called for efforts to identify the cause(s) behind this apparent trend. Of course, nothing on the chart itself indicated what led to the perceived systematic pattern. The chances to identify its causes in part depend on the observation skills of managers, technical personnel, and operators as well as on their ability to correlate observations with process performance at large.

In the search for special causes of variation it is important to draw on various types of information about the process. Marking on the chart events such as changes in material, changes of operating personnel, line shut downs, etc. can be of great help. However, consideration - prior to data collection - of the broad range of factors which might impact process output is the best strategy. Although a complete list of potential causes of variation in outcomes is unlikely to be compiled before the collection of data begins, any preliminary analysis of causes and effects will be of much help. In this task the pool of knowledge of personnel employed in the studied process is an invaluable source of information.

Regarding the present example, a number of possible causes of the gradual increase in the number of nonconforming end crusts had been listed previously. Among them two seem to deserve closer scrutiny:

(i) Changing environmental conditions

At the beginning of the day, the moisture level in the dough is set by adjusting the rates at which dry and liquid ingredients flow into the mixing vat. Moisture levels are thought to be critical as dough that is too dry will tear easily. However, as temperature and humidity in the plant change throughout the day, the set rates may not suffice to maintain appropriate levels of moisture. Data need to be collected and analyzed in order to substantiate this hypothesis.

(ii) Recycling of excess and scrap dough

Cutting dough to fit the tins in which it is moulded generates a certain amount of scrap dough. It is current practice to return this dough to the mixture from which other pie crusts are then rolled. Additionally, dough from nonconforming pie crusts is also returned to the mixture. The supposition is that this practice may cause deterioration of the dough throughout the day resulting in increased numbers of nonconforming crusts.

Although these two factors were identified as possible reasons for the observed trend in nonconforming pie crusts, it was not clear whether either one of them was actually effective. One method to find an answer to this question is to eliminate the influence of one particular hypothesized factor and assess the consequences. For example, the effect of recycling dough could be studied by discontinuing this practice. If as a consequence of this measure no trend were observed, the factor may safely be excluded from the list of candidate causes. Likewise, ambient temperature and humidity can be kept constant in an attempt to assess the impact of environmental conditions. In still another approach the cessation of recycling under a variety of temperature and humidity conditions could be studied. Finally, investigations of potential causes of the observed trend would have to take into account factors other than the ones described previously.

In the reported example, the team studying the process - and, in particular, the trend in nonconforming end crusts - gathered data from the line over several days before as well as after recycling was stopped. While environmental conditions were not controlled, changes in temperature and humidity were recorded throughout the day. These changes were also recorded on several successive days on which recycling had been discontinued: Temperature and humidity were seen to change in a fashion similar to that of earlier days. Finally, data on end crusts were collected on one of the days on which recycling had been discontinued. A p chart representing these data showed no evidence of non-random behaviour of any kind. This supports the hypothesis that recycling of dough caused the increase in the number of nonconforming end crusts.

The above result immediately raises the important issue of confirming the outcome of a given study. In general, confirmation of results is of great significance for improvement work giving rise to a particular kind of study. Such confirmation studies have to be performed repeatedly in order to establish if improvements can be maintained under a variety of conditions. Too often an effect which is noted at one point in time - such as the reduction in nonconforming crusts when recycling is halted - may not recur at a later time. Sometimes it happens that focussing on a particular area results in improvements in that area, but that these improvements may not be sustained after attention has been directed elsewhere. Thus improvements may incorrectly be attributed to some change made to the process rather than to increased attention. Also causes other than those investigated may bring about the observed trend in the number of nonconforming end crusts. Here confirmation studies could decide whether a given improvement occurred for the suspected reason or not.

In the present example of the pie-crust line, confirmation studies might simply consist in applying the same tests to an enlarged set of data. Information collected over a longer time span which should preferably include a variety of operating conditions would usually provide the data basis. In addition, the same analysis could be performed on data collected on other shifts and other lines as a means of validating previous results.

3.9 Five ideas supporting the effective use of control charts

This section presents some general ideas on how the use of charts can be most effectively integrated in a program for continuous improvement. These ideas should be read as suggestions rather than rules. While there will certainly be situations in which not all of them can be followed, it appears to be important that the practitioner know about the options they offer.

- **A process flow diagram is a prerequisite of data collection.**

Before a set of data is collected for process study, the process needs to be understood in terms of how it affects the processes and systems in which it is embedded and is affected by them. This is particularly important for those managers who set the priorities of process study. It may be possible to spell out alternatives for process improvement only after close scrutiny of the entire system set-up, where a process flow diagram is an indispensable tool of investigation.

In view of the above it is clear that a diagram simply giving the order in which operations in the process should occur is insufficient. Instead a flow chart is called for to describe the process in sufficient detail, with the additional requirement that this chart needs to be revised with every significant change in the operations that it describes.

- **An operational definition of the specifications, characteristics or attributes under study is also needed.**

Benefits usually accrue from involving managers, at appropriate levels, in selecting and defining the characteristics that are to be measured. Some of these benefits have to do with the strengthening of the team working on the process due to a deepened understanding by team-members of why a given operational definition and its consistent application are important. In addition, the validity of measurement and the reliability of outcomes are enhanced for obvious reasons. Furthermore, operational definitions need to be checked on a regular basis. In this context procedures should be established for the revision of definitions and specifications in the case of product or process changes.

- **It should be clear how the various types of information gained from a *p* chart will be used.**

Management and supervisory directions about how to identify, understand, and eventually eliminate special causes need to be specified. Without such directions there is a high probability that fundamental causes of variation do not get addressed. 'Band-aid' problem fixing, based upon restricted resources and, perhaps, limited understanding of the technological aspects of special causes is then a very likely result. Management has to provide plans for the systematic study of the process. For example, a list of potential special causes could be established and the associated factors analyzed. Furthermore, individuals that are to be involved in related work should be identified by management with due consideration of the issues of knowledge, skills, responsibility and authority. An additional requirement is to take into account in this exercise departmental and functional representation.

Other elements in a plan for process study should account for cause-and-effect relationships. Since the ultimate goal is to improve the process, bringing it into stable operation can only be the first step. Subsequently, subgroup formation and sample frequency need to be considered with a view to understanding cause-and-effect relationships. In this endeavour a *p* chart can prove very helpful.

- **A *p* chart will be most effective for process study, if it relates to only one specification, standard, or non-conformity.**

It is considered as a fundamental mistake in resource allocation to report on various specifications or characteristics on one single chart. In terms of time, effort and other resources, data collection and basic 'charting' is by far the least expensive, whereas problem identification, determination of causes, and testing and implementation of solutions are considerably more resource intensive. Notwithstanding these cost relationships multiple characteristics are often included in a single chart with adverse effects on its usefulness. Thus it may happen that a trend in the nonconforming fraction pertaining to one characteristic is obscured or offset by contrary movements in the fraction of another characteristic. Similar problems can arise in attempts to identify special causes of variation in a certain characteristic. Numerous common-cause systems and various types of special causes may be active but the chart will provide little information about any one of these. With multiple characteristics on one chart, the time spent on inspection and data collection efforts will most likely be wasted since the form of data presentation does not aid problem identification.

There are surely exceptions to the above policy. For example, before making suggestions about process or product improvement typical problems must have been identified together with assessing the relative frequencies with which they occur. A p chart that includes all types of nonconforming attributes may be useful for reporting on the current state of a process. A Pareto diagram may then be used to support the initial investigation and put in relief some of the larger problems. Of course judgement will have to be exercised when setting priorities on the basis of this work. The severity of various problems will have to be appraised, their impact considered from the customer's viewpoint, and their possible long run nature evaluated.

- **Where technically and economically feasible, the results of inspection for the purposes of process and product improvement should be recorded in the form of a variable.**

Sometimes work on quality and productivity involves measurement, comparison of the result to specifications, and then recording 'conforming' or 'nonconforming' as the response. If only this attribute is recorded valuable information is discarded. Although the p chart does provide information about failing to meet specifications, the information is too vague for work on process improvement. There is no clue as to whether the problem lies with excessive variation, incorrect averaging or both. A full evaluation of process performance requires information on the sources of variation, and these data are not available when only conformance to specifications is recorded.

CHAPTER 4

MORE ON THE ANALYSIS OF ATTRIBUTES DATA

There are situations in which process data consist of counts other than the type discussed in the previous chapter. Data are often collected, for example, on the number of individual flaws or defects found in a single item or a collection of items. Examples are the number of surface flaws on a sheet or several sheets of material, or the number of impurities found in a volume of a continuously manufactured or batch-produced product. Such counts are different from those analyzed by a p or np chart because they arise from counting occurrences of some specified type rather than classifying each item into one of two categories. Likewise, in a business environment counts of occurrences of events in time are often made. Examples are counts of the number of service or transmission interruptions, production line stops, accidents, calls for service or customer complaints. Such counts often exhibit variation over time due to changing conditions. This variation needs to be studied to better understand process and system performance.

4.1 Attributes data that require the use of c or u charts

Counts of occurrences of events in time or of nonconformities on a unit of material are analyzed by use of c or u charts. The methods of analysis depend on the way in which such events occur when there are only random or common-cause sources of variation. The four criteria listed below summarize the conditions under which the use of a c or u chart is suggested:

Criterion 1: The counts are independent of each other.

Criterion 2: The number of possible occurrences is large.

Criterion 3: The chance of an occurrence at any specified time or place is small.

Criterion 4: The expected number of occurrences is proportional to the amount of time or material included in an inspection unit.

The study of counts of flaws on sheets of manufactured material provides an illustration of the application of these four criteria. For example, in the manufacture of aluminium cans it is critical that there be no holes in the aluminium used. Assume that at one plant past experience has shown that the aluminium sheets received from the supplier have no large holes, but that occasionally pinholes are detected. The material is therefore exhaustively inspected prior to its use for can manufacture and the number of pinholes in a sheet of aluminium is recorded.

If the occurence of pinholes is subject only to common-cause sources of variation, holes are scattered in a random fashion over a sheet of aluminium. If by contrast some special cause were active, this would supposedly have the effect of concentrating pinholes in one area of the sheet. Furthermore, if pinholes tended to appear in clusters, detecting a hole in one position would increase the probability of occurrence of others in the vicinity. Criterion 1 for the use of a c or u chart refers to the above distinction. When only common causes of variation are at work, occurrence of a pinhole at a particular location is independent of that of others in the vicinity and holes are randomly scattered across the sheet.

If the size of a whole sheet of aluminium is compared with that of a pinhole, clearly Criterion 2 is seen to hold. In fact, if a sheet of aluminium were subdivided into many tiny pieces, each one of these pieces might possibly contain a pinhole. However, there would be a great number of pieces without a pinhole, quite in accordance with Criterion 3.

If the sheet of aluminium were divided in half, the expected number of pinholes on one-half would be the same as that on the other half. Or conversely, on two sheets of aluminium about twice as many pinholes would be expected as on one sheet. This is the idea behind Criterion 4: The expected number of pinholes is proportional to the amount of aluminium inspected. Consistency between sheets of the expected number of pinholes reflects the assumption of stability of the process over time. Departure from this behaviour — evidenced, for example, by the occurrence of a considerably larger number of holes on one sheet compared to other sheets — would indicate the presence of a special cause. It would then be useful to identify such causes in the form of different conditions, materials or other factors.

Another example for the use of a c or u chart is in the analysis of counts of the number of production line stops in a shift. Here the unit of inspection is a unit of time, in particular, one shift. If the production line is shut down at planned intervals for a tool change, use of a c or u chart to summarize the information on counts would not be appropriate. By contrast, the c or u chart would be considered for studying a situation where randomly occurring events are causing some difficulty which may result in stopping the production line. An example is in the production of paper where a number of different causes, acting together or separately, result in breaks in the paper. The times at which breaks might occur cannot be predicted, they are distributed randomly.

The four criteria for using c and u charts have to be applied in a slightly modified fashion when the inspection unit is a unit of time. Criterion 1 states that the counts of occurrences should be independent. In the present situation, this implies that if one production stop occurs, this would not influence the probability of production stops in the near future. Rather, the next production stop would occur at random some time in the future, with the same chance of occurrence as if the previous one had not occurred.

When considering the use of a c or u chart, the following question might be raised regarding the above assumption: "Is it possible that under certain conditions production stops tend to occur in clusters?" An answer in the affirmative would not rule out application of a c or u chart. Limits established on the basis of a c or u chart would then indicate the number of occurrences that could be expected if the process performed according to the above criteria. On the other hand, unusual performance reflected on the chart in points lying outside these limits would indicate non-random behaviour of the process.

For the case of production stops, applicability of Criteria 2 and 3 could be considered in much the same way as for the pinholes example. If the unit of inspection for line stops were a shift, this could be divided into many small increments of time, for example, seconds. A stop might occur in any one of these small time increments. Thus the number of occurrences is possibly very large. However, the chance of seeing an occurrence at a large number of these time increments is very small; most of the time increments will not contain a line stop.

Criterion 4 states that the expected number of occurrences should be proportional to the length of the time period over which an inspection is carried out. Usually careful examination of the ways of data collection is required in order to see whether this criterion is met reasonably well. For example, after a stop the production line might be down for as long as an hour. If this were the case, an eight-hour shift might not be a good choice for the unit of time over which production line stops are counted. Instead, a more appropriate unit of time for analysis might be one that corresponded to actual running time of the equipment.

4.2 The construction of *c* charts

Both *c* and *u* charts are used to examine the stability of a process over time when the information collected consists of counts of occurrences of a given event. The ***c* chart** is used to study the counts of occurrences when the time period or the amount of material subjected to inspection remains the same over the course of the study. The data in **Table 4.1** represent a situation in which the amount of material inspected remains constant in the course of the study. The data in this table originated from the above example of aluminium can production and reflect the counts of pinholes in each of 20 subgroups. Each subgroup was formed by inspecting 10 rolls of aluminium in a shipment received from a supplier. Each of the rolls of aluminium had the same number of feet per roll, so that for each subgroup the amount of material inspected remained the same. The following notation will be used in the present discussion of c charts:

k denotes the number of subgroups

c denotes the number of nonconformities in each subgroup

As in the case of constructing a *p* chart, the construction of a *c* chart begins with plotting the data. With a *c* chart, the points plotted represent the counts of nonconformities. **Figure 4.1** shows the plot of the number of pinholes in each of the 20 subgroups. The horizontal axis corresponds to the subgroup number while the vertical axis corresponds to the count of pinholes. The time order represented by the horizontal axis corresponds to the listing given in Table 4.1 of the subgroups. The subgroups match

Table 4.1 Number of pinholes in aluminium rolls

Subgroup	Number of pinholes	Subgroup	Number of pinholes
1	22	11	15
2	29	12	10
3	25	13	33
4	17	14	23
5	20	15	27
6	16	16	17
7	34	17	33
8	11	18	19
9	31	19	22
10	29	20	27
	Total		460

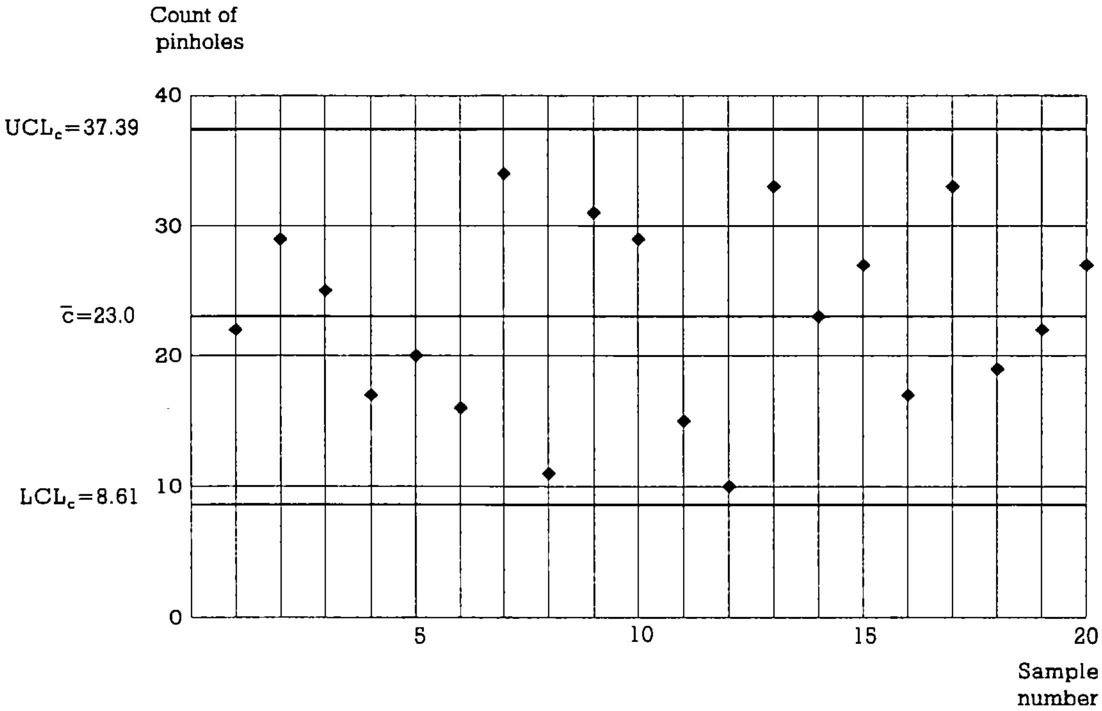
with the time at which the shipments were received: subgroup 1 corresponds to the inspection of a shipment which arrived at the plant prior to the material inspected for subgroup 2, etc. It should be noted, though, that the ordering in time of shipments is not necessarily reproduced in that of manufacture.

The centre line on a *c* chart represents the average number of nonconformities per inspection unit. Since the number of inspection units on a *c* chart is the same as the number of subgroups, the average $\overline{c}$ is simply the total number of nonconformities divided by the number of subgroups.

For the data in Table 4.1, there was a total of 460 pinholes in 20 subgroups, so that:

$$\overline{c} = \frac{460}{20} = 23$$

Figure 4.1 Plot of number of pinholes in 10 rolls for each of 20 subgroups



This value has been used to insert the centre line in the *c* chart in Figure 4.1.

The upper and lower control limits for the *c* chart define the amount of variation that might be expected in the recorded nonconformities if only common-cause sources of variation are present. Stated differently, the control limits describe the amount of variation that the recorded c's would be expected to exhibit, if criteria 1 and 4 governed the process in the previous section. The formulae used to calculate the upper and lower control limits are, respectively:

$$UCL_C = \bar{c} + 3\sqrt{\bar{c}}$$

$$LCL_c = \bar{c} - 3\sqrt{\bar{c}}$$

For the data on pinholes the upper control limit is:

$$UCL_C = 23 + 3\sqrt{23} = 37.4$$

The lower control limit is given by:

$$LCL_C = 23 - 3\sqrt{23} = 8.6$$

Figure 4.1 displays the completed control chart. Because there are no points on or outside the control limits or any signs of non-random behaviour in the plotted points, the completed chart shows no indication of a special cause operating. It would appear that the occurrence of pinholes in the aluminium is randomly spread out over time. One subgroup of 10 rolls does not have a significantly higher or lower number of pinholes to indicate that the process may have been behaving differently when those rolls were made.

Construction of c charts

1. Count the number of nonconformities on each inspection unit (in each subgroup) . The values are plotted on the c chart.
2. Calculate the centre line on the chart as:

$$\bar{c} = \frac{\text{total number of nonconformities}}{\text{total number of units inspected}}$$

3. Calculate the control limits as:

$$UCL_C = \bar{c} + 3\sqrt{\bar{c}}$$

$$LCL_c = \bar{c} - 3\sqrt{\bar{c}}$$

4.3 Comparing Processes

The previous example of counting pinholes in rolls of aluminium referred to a company which buys aluminium rolls in order to produce aluminium cans. The number of pinholes is of concern to the company since portions of aluminium which contain holes need to be removed before the aluminium is cut into pieces to form cans. Since the c chart showed the process to be consistent, the value of 23 provides an estimate of the average number of pinholes that would be expected to be found in 10 rolls of aluminium. Managers of the plant for which these data were collected felt that an average of 23 per 10 rolls (or 2.3 per roll) was too high to be accepted. The supplier of aluminium was contacted in order to communicate the findings of the previous study and solicit the vendor's help for addressing the above issue.

The supplier of aluminium was surprised by the value for the average number of pinholes reported by the manufacturer of cans. Inspection at the aluminium producing plant revealed a considerably lower number of holes per roll. Yet, inspection techniques used at the two sites were different in at least two respects. Different equipment was used by the two groups to count pinholes. Also the number of pinholes at the supplier's plant were counted prior to rolling the aluminium while at the user's plant counting took place after unrolling. The observed differences suggested that pinholes might be formed during the rolling or unrolling of aluminium.

In order to follow up this clue, the two groups proposed the following joint study. The supplier of aluminium agreed to count the number of pinholes in 10 rolls of aluminium in each shipment. These 10 rolls would be tagged. As they were unwound by the can manufacturer, the number of pinholes would again be counted. **Table 4.2** presents - for 10 shipments of aluminium - the counts of nonconformities observed by both the aluminium supplier and the manufacturer of cans.

There are several ways of examining the data presented in Table 4.2. The question raised here is about whether consistent counts were being generated by the two different groups producing the data. Combining the counts shown in Table 4.2 and calculating centre line and control limits yields the values 19.3, 32.5 and 6.1 for $\bar{c}$, UCL_C and LCL_C, respectively.

Table 4.2 Study of pinhole counts performed by aluminium supplier and can manufacturer

Shipment	Number of pinholes found by supplier	Number of pinholes found by manufacturer
1	22	29
2	18	19
3	17	23
4	14	33
5	18	10
6	11	27
7	17	34
8	12	15
9	9	17
10	14	27
Total	152	234

From the data it appears that the counts obtained by the supplier are lower than those obtained by the manufacturer.

To identify the reason for the above discrepancy necessitates further investigation for which only the broad directions are given here. It is possible that the equipment used by the two groups is responsible for widely differing counts. However, there is also the possibility that the rolling of the aluminium by the supplier or certain phases of the shipping process or the unrolling generate additional pinholes counted by the can manufacturer. In the context of the investigation it is useful to scrutinize the data generated by the supplier in order to reveal whether the process supplying the aluminium is stable. In general, the supplier must be interested in reducing the number of pinholes found by the can manufacturer. Therefore any information provided by the manufacturer should be valuable to the supplier too.

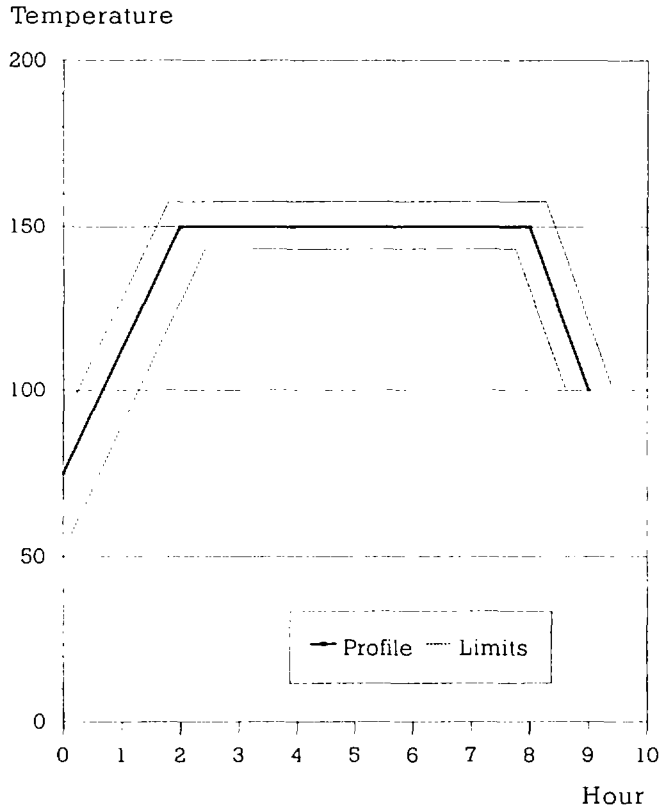
4.4 The construction of *u* charts

Like *c* charts also *u* charts are used to examine process performance when the data are counts of occurrences. The distinguishing feature of *u* charts is that the amount of material or the unit of time defining a subgroup may vary among such groups. The following example is intended to illustrate the construction and use of a *u* chart.

A chemical firm produces many of its products in batches. At one site of the company attention is drawn to the question of how well the temperature profile for these batch processes is managed and whether improvements in this management promise improvement in yield and throughput of the processes. Soon it is realized by the group working on the problem that the understanding of this characteristic of batch production is deficient in many respects. Therefore initial efforts are directed at developing pertinent operational definitions and gaining knowledge about how well temperature profiles are maintained.

The graph in **Figure 4.2** illustrates what is meant by a temperature profile. The horizontal axis is the processing time in a tank and refers to one stage of a batch process. Temperature is graphed on the vertical axis. From this graph it can be seen that in this stage of the batch process the temperature in the tank is raised from 75 degrees to 150 degrees in the first two hours, maintained at 150 degrees for the following six hours, and then reduced to 100 degrees over the final hour. This graph is referred to as temperature profile. It should be noted that those parts of the profile that reflect the rise in temperature during the initial hours and the drop during the final hour

Figure 4.2 Temperature profile with limits



are more explicit than the description given in the batch protocol. The protocol requires a gradual warming and drop in temperature, where however the rate of change is not specified. In particular, the protocol does not require temperature change to be linear. However, in order to establish how well the protocol is being followed, an explicit description of the way in which temperature should change as well as a definition of what constitutes a significant deviation from the protocol are required. Initially, a judgement was solicited from chemists and technical staff as to which deviation of temperature from the profile would be labelled as 'temperature excursion'. Figure 4.2 also shows limits drawn around the profile in order to define how close to this profile temperature should be held. In an operational way a 'temperature excursion' is now defined as the temperature exceeding one of these limits.

Table 4.3 Number of excursions from temperature profile

Subgroup	Batch type	Hours	Number of excursions	u
1	A	10	3	.300
2	B	8	0	.000
3	C	12	4	.333
4	A	10	7	.700
5	B	8	1	.125
6	A	10	2	.200
7	C	12	7	.583
8	D	8	1	.125
9	B	8	4	.500
10	C	12	1	.083
11	D	8	1	.125
12	A	10	9	.900
13	C	12	2	.167
14	A	10	6	.600
15	B	8	2	.250
16	D	8	3	.375
17	C	12	0	.000
18	D	8	2	.250
19	A	10	8	.800
20	C	12	3	.250
	Total	196	66	

An initial study of temperature excursions was conducted by use of data that had been collected over a three week period. These data are summarized in **Table 4.3**. The second column in this table is labelled batch type. Since the processing equipment under study is used for producing different types of material, records have been kept of the type of batch being produced. Different types of material have, of course, different temperature profiles which need to be maintained. The profiles in turn show varying levels of temperature and are also of different duration. The third column in Table 4.3 . represents the duration in hours of the temperature profile. Finally, the number of temperature excursions throughout the recorded hours is given in the fourth column.

These counts of temperature excursions provide an example of counting within varying time units of inspection. Batch type C covers twelve hours of processing time, whereas type B covers only eight hours. Thus, there is a longer time period during which temperature excursions could occur with batch type C. The fifth column in Table 4.3 is labelled 'u.' The values in this column are the average number of excursions per hour. The counts of occurrences per inspection unit— in the present case one hour— are plotted on the u chart as shown in **Figure 4.3**. Here instead of points the letters corresponding to the type of batch have been plotted.

The centre line for the *u* chart is plotted at the level

$$\bar{u} = \frac{\text{total number of nonconformities}}{\text{total number of units inspected}} = \frac{66}{196} = .337$$

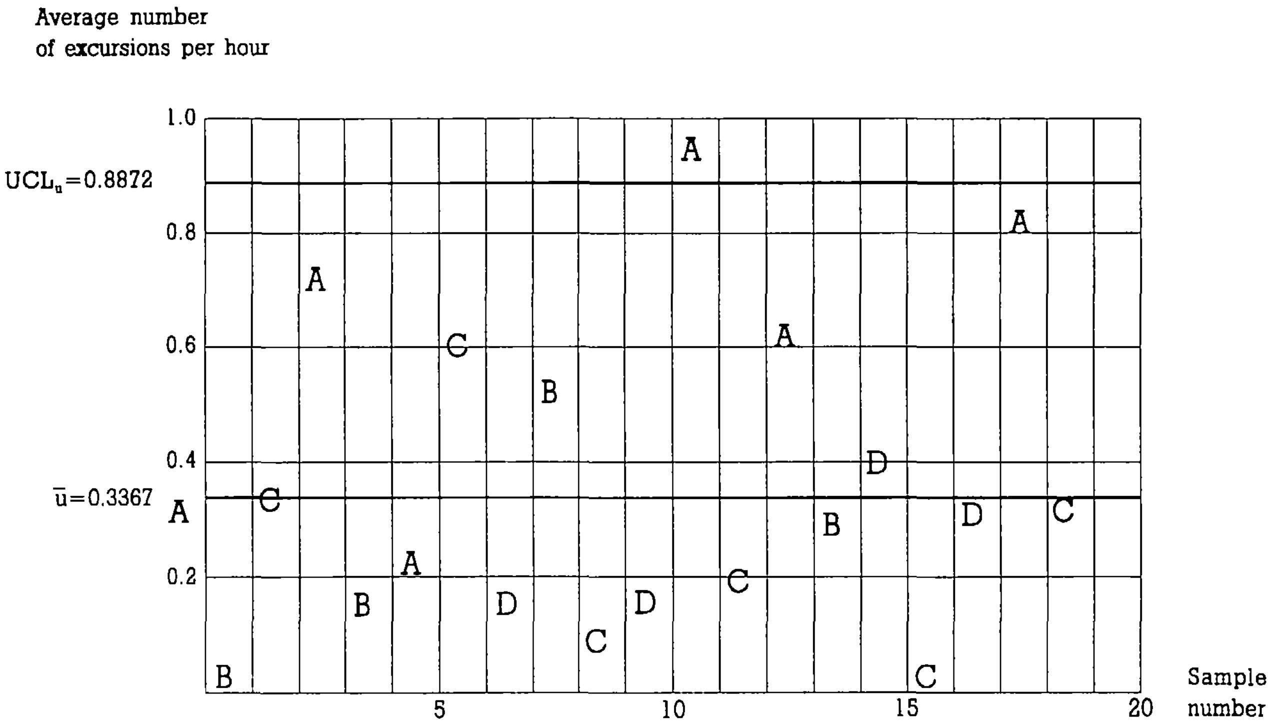
The upper and lower control limits of a u chart are a function of n, the number of inspection units in a subgroup. In Table 4.3, the third column, labelled 'hours,' contains the values for n. The values of the upper and lower control limits for the first subgroup corresponding to batch A are given by:

$$\text{UCL} = \bar{u} + 3\frac{\sqrt{\bar{u}}}{\sqrt{n}} = 0.337 + 3\frac{\sqrt{0.337}}{\sqrt{10}} = 0.887$$

$$\text{LCL} = \bar{u} - 3\frac{\sqrt{\bar{u}}}{\sqrt{n}} = 0.337 - 3\frac{\sqrt{0.337}}{\sqrt{10}} = \text{none}$$

Control limits for other subgroups are given in Table 4.4, while Figure 4.3 reveals the twelfth point plotted to lie above the upper control limit for batch A. This indicates that the ability to manage temperature profiles is not uniform across batches and time. It is impossible to decide on the basis of the available information whether the difference noted at the twelfth point is due to a particular event which occurred at that time point or rather

Figure 4.3 Plot of temperature excursions (u chart)



to the fact that temperature profiles for batches of type A are more difficult to manage than for other batch types. Nor is it possible to solve this problem without more specific information about the process, about the equipment used to maintain temperature profiles, and about how the equipment performs its role of increasing , decreasing or maintaining temperature at a given level. The initial objective of the study of temperature profiles was to find a connection between the ability to manage the temperature profile, on the one hand, and yield and throughput of the batch process, on the other hand. This objective could not be achieved because of the observed inconsistencies in the management of temperature profiles which were not fully understood.

To provide additional insight into current operations, the analysis of the excursion data needs to be modified.

Table 4.4 *Control limits for u chart on number of excursions from temperature profile*

Batch type	Hours (n)	UCL	LCL
A	10	.8872	—
B	8	.9522	—
C	12	.8392	—
D	8	.9522	—

Table 4.5 *Control limits for u chart on number of excursions from temperature profile using distinct centre line values*

Batch type	Hours (n)	$\bar{u}$	UCL	LCL
A	10	.583	1.307	none
B	8	.219	.715	none
C	12	.283	.744	none
D	8	.219	.715	none

Construction of u charts

1. For each subgroup, calculate u, the number of nonconformities per inspection unit.

$$u = \frac{c}{n}$$

where c is the number of nonconformities in the subgroup and n the number of inspection units.

2. Plot the u 's on the control chart.

3. The centre line of the control chart is calculated by:

$$\bar{u} = \frac{\text{total number of nonconformities}}{\text{total number of units inspected}}$$

4. The upper and lower control limits for each plotted u will depend on the number of inspection units used to calculate the value of u. The formulas for the upper and lower control limits are given by:

Upper control limit: $$UCL_u = \bar{u} + 3\frac{\sqrt{\bar{u}}}{\sqrt{n}}$$

Lower control limit: $$LCL_u = \bar{u} - 3\frac{\sqrt{\bar{u}}}{\sqrt{n}}$$

In **Figure 4.4** data are plotted by batch type instead of time order. Moreover, centre lines and limits for this *u* chart are calculated separately for each type. For example, batch type A was run for a total of 60 hours and showed 35 temperature excursions over this period. Therefore the centre line drawn for batch type A is at the value 0.583. The upper and lower control limits for batch A are based on these values.

Figure 4.4 *U chart with separate calculations by batch type*

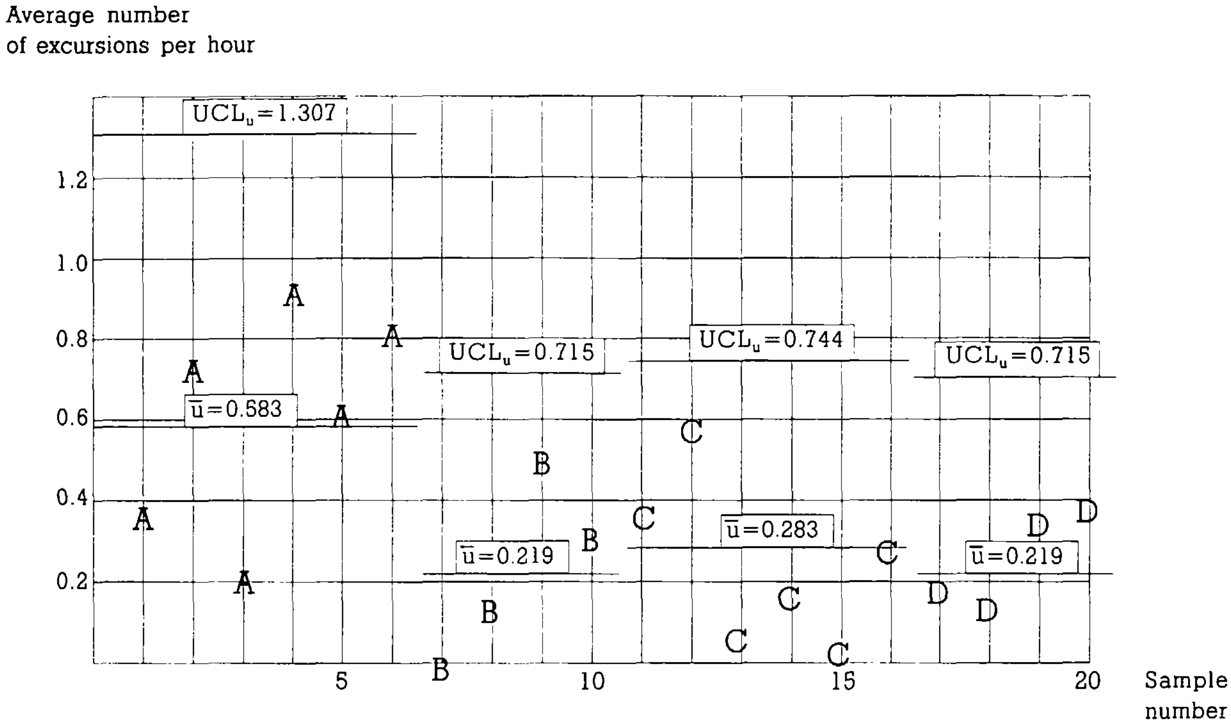


Table 4.5 shows all centre lines and control limits for the four batch types while Figure 4.4 actually contains four separate control charts which, for reasons of convenience, are plotted on the same graph. In the present example, however, there are insufficient data on any of the batches to allow for reliable inferences from the four separate charts. Nevertheless, the plot in Figure 4.4 indicates that the four batch types are consistent within themselves as regards the number of temperature excursions experienced. Further data on each batch type are needed to assess consistency of the excursion rates for different batch types.

The conclusions reached so far are that the numbers of excursions are inconsistent across time and that this inconsistency may be influenced by batch type. Personnel at the chemical plant used this information to plan further studies of the effect of temperature excursions on batch processing. An immediate goal of further analysis was to determine the nature of the differences in excursion rates. Although all of the batch types showed different temperature profiles, these profiles were similar in that an initial step of increasing temperature was followed by maintaining the elevated level and finally decreasing temperature. On the basis of this common pattern temperature excursions were now recorded not only by batch type but also by the stage of temperature control. In other words, for each temperature profile, the number of temperature excursions was recorded during the heating, the maintaining and the cooling stages. It was thought that this type of information might provide useful insights into the observed inconsistencies. A better understanding of the reasons for inconsistent performance by batch type was believed to help plant personnel to reduce the number of temperature excursions.

CHAPTER 5

ANALYZING VARIABLES DATA

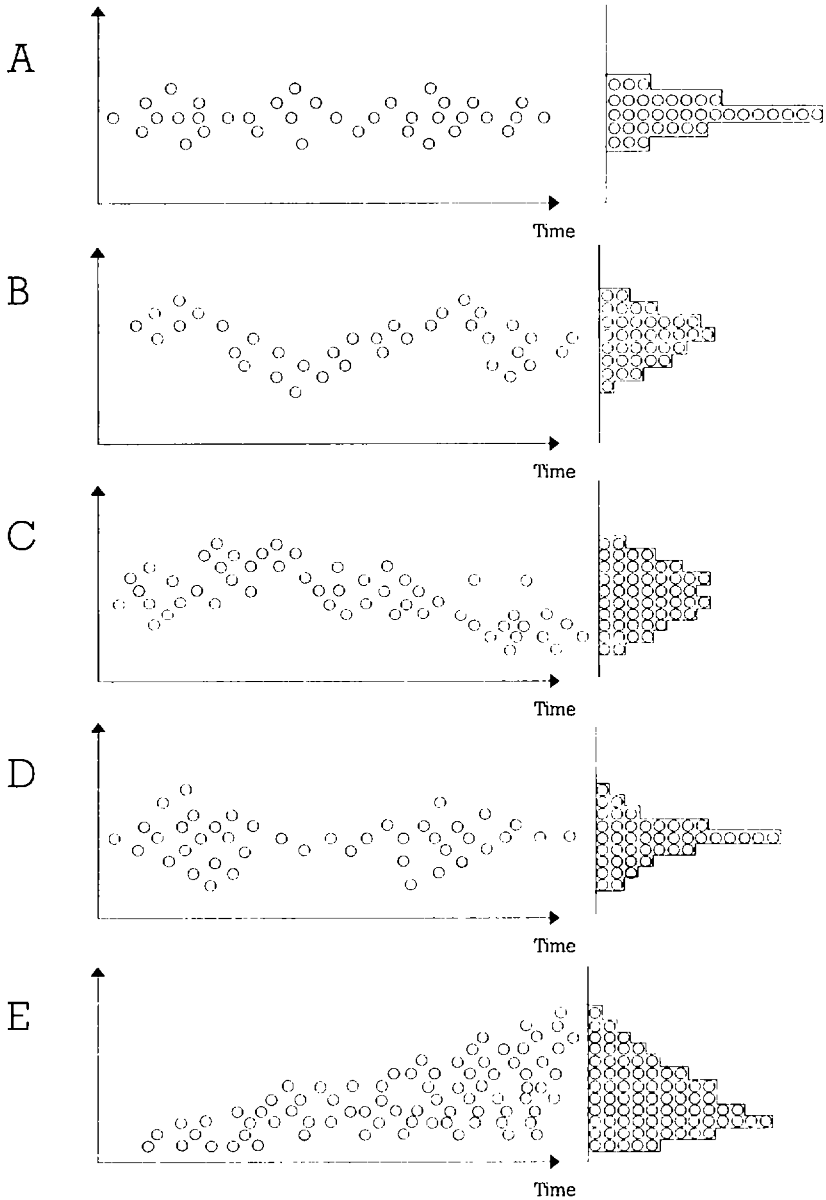
The measurement of characteristics of processes, goods and services is the basis of qualitative or quantitative evaluation. The former type of evaluation draws on counts of nonconforming items, errors, defects, omissions etc. as was discussed in preceding chapters. By contrast, the latter type usually produces variables data which indicate the degree to which a process or service possesses a given characteristic. For example, variables data can result from the measurement of physical properties like density, pressure, temperature, resistance, force, hardness and others. Information of this nature and the methods to analyze it are treated in the present chapter.

5.1 Different types of variation in process data

The data analysis outlined here is expected to contribute above all to identifying and evaluating sources of variation in measured results. These sources usually affect the magnitude of **short-term variation**, the **average**, and any **change** of both these attributes. Consequently, data analysis should be directed toward measuring process variation and process level or location as well as evaluating the predictability of process outcomes.

An introductory discussion of the above issues can be based on a series of simple examples. In **Figure 5.1** data plots from five processes, named A, B, C, D and E are shown. In each case, the vertical scale corresponds to measurement of a process input, a process parameter or a process output with the measured values referring to properties like viscosity, length, density, flow rate, temperature or thickness. The horizontal scale

Figure 5.1 Process examples



corresponds to time. Measurement is carried out over an extended period and the results are plotted in time order. Although the given examples of time patterns of measurement results are certainly not exhaustive, they illustrate some of the issues central to data analysis. These issues have to do with variation, averages, and predictability.

First, the data plot of measurement results for process A is examined where it is assumed that the results relate to some process output. Like all the other processes investigated here, process A does not produce exactly the same result for each point on the time scale, but exhibits variation. The magnitude and the nature of such variation are of prime interest also here.

For process A, the range between the smallest and the largest measurement values observed during any short time span appears to be about the same throughout the whole period over which data have been collected. Because of this uniformity in variation, the **short-term variation** of the process is said to be stable or predictable.

Long-term variation which relates to changes in 'process level' or average values of measurement is another phenomenon to be studied. More precisely, such variation concerns the level around which individual values tend to fluctuate. In the example presently studied process A appears to operate at a nearly constant average value throughout the period of data collection. Because of this apparent constancy in average value also the process level is said to be predictable.

In the present context of analyzing variables data, a process is considered as being in statistical control, if both its short-term variation and its level are virtually constant. The discussion below will show how near constancy of variation and level can be assessed.

Comparison of process A with the other processes studied here yields additional insights. For process B short-term variation is predictable or stable and the same holds for process C. Moreover, for processes B and C, respectively, point-to-point fluctuation or short-term variation is of the same order of magnitude as that of process A. However, results for processes A, B and C are not equivalent in regard of process level. Process B, in particular, does not operate at a constant average over the time period represented by the entire horizontal scale, and the same applies to measurement results of process C. More specifically, process B shows a smooth decline in its average displaying apparently cyclical behaviour over longer stretches of time. In contrast to such smoothness in change, process C exhibits abrupt shifts in its average. The performance of both processes is different from that of process A: While the latter stays at a constant average, B and C do not have predictable averages.

Process D operates at a constant average over time. However, it differs from A, B, and C in that it displays short-term variation of a changing magnitude. Finally, process E shows a trend in its average level together with increasing variation of individual measurement values.

To summarize, the above processes can be compared with respect to their being 'in statistical control' or not . Process A is said to be in control with respect to variation and average, since over a short interval fluctuations in individual values are of the same magnitude throughout the period of observation and the average value remains constant over that period. Processes B, C, D, and E which do not fulfil both of these conditions simultaneously are said to be unpredictable or 'out of control'. Data from B and C, respectively, display consistent short-term fluctuation of individual values on the one hand, but a changing average on the other hand. By contrast, process D maintains a constant average throughout, but shows inconsistent fluctuation around this average. Finally, process E is unstable with respect to both variation of individual values and the average. In summary, Figure 5.1 provides sufficient information for determining the state of control of the processes described there.

It is obvious that both changes over time in the average as well as deviations of individual values around average values contribute to **total variation** in process output. All the values plotted for process A have been used to construct the histogram which appears to the right of the time plot in Figure 5.1. Similar histograms are shown for processes B, C, D and E in the same figure. From there it can be seen that the values from processes B, C, and D show a larger total variation than those in the distribution derived from process A, although the fluctuations around the average during any short period of time appear to be of about the same magnitude for these four processes.

The obvious next step in the data analysis outlined here aims at learning more about changes in short-term variation and process level. Again, investigations have to be directed at potential sources of variation as well as the reasons for observed changes in short-term variation or in the average level of measured outcomes of a process.

5.2 Identifying variation and learning about its sources

In the above example processes A through E exhibit different types of variation over time, suggesting that different causes are behind different patterns of variation in outcome. Data plots like those shown in the previous section can aid the analysis by showing the way in which process output changes over time. However, time is only a proxy variable which stands for numerous changes in processing circumstances that potentially affect variation or the average of process output.

For example, component parts or raw material almost certainly change over time and so do their characteristics. Such changes, in turn, may give rise to increased fluctuation around an average or alternatively cause this average to shift. Another example of change over time is that in management, engineering or operating personnel with the likely consequence of changes in the ways of operating machines, assembling components, or mixing materials. Furthermore, environmental variables such as temperature and humidity are also likely to change with obvious consequences for the variation in output characteristics. Finally, the efficiency of operating process and laboratory equipment will not remain constant over time and thus be crucial to output variation.

Process analysis will usually proceed in several steps. At the outset information of the type displayed in Figure 5.1 will often not be available so that the first step in the procedure is the search for data on process performance. These data will serve to judge, whether or not the process is in control. If the process under study is found to be out of control, the immediate task is to get it under control. Once this has been achieved, control has to be maintained in order to render the variation of crucial outcomes predictable.

Subsequently comparisons can be drawn between current output and required results. In other words, once control has been achieved and is being maintained, the focus shifts towards process improvement. In an attempt at defining such improvement it could be said that it results in reduced variation and, if required, in a preferred average of the measured outcome. From this it becomes clear that process evaluation and improvement work are iterative activities that are expected to help achieving the desired consistent results.

5.3 Control charts for ranges and averages

Table 5.1 presents a set of data on the basis of which some standard statistical techniques of process analysis can be demonstrated. Each number in the table represents measurement of a single characteristic on a collection of items. The data are organized in subgroups of four readings each. This implies that the four values were collected under the same conditions, or at the same point in time, or from the same source. In general, there should always be some systematic basis for subgrouping the data.

There are several characteristics which need to be derived for this data set. Some of them will be required for each subgroup, while others refer to all subgroups together. For each subgroup, the subgroup range will be computed as a measure of how much the four values differ among themselves. In addition the average value will be compiled for each subgroup as an indication of its location. All of the subgroup ranges will be used to provide information about the stability of short-term variation and its magnitude. The subgroup averages, by contrast, provide information on process centring and predictability.

5.3.1. Measure of location: the arithmetic average

In the following discussion 'n' represents the number of measurement values in a subgroup where in the present example n=4. The symbol 'k' represents the number of subgroups with k=20 in the example of Table 5.1.

A subgroup value is represented by the symbol X_i, where i indicates the position in a sequence of observations. The average value is given by:

$$\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$

where the symbol Σ (sigma) designates the sum of the values X_i taken from i = 1 to i = n. Averages for all subgroups are shown in Table 5.1 together with the original data.

The arithmetic mean or average is the most frequently used measure of location. It can be interpreted as the balance point, or centre of mass, for a collection of measurement values. The average value need not itself appear as a number in the subgroup. In subgroup 1, for example, no observation has the value of 35.27 which represents the subgroup average. The average value also need not divide the subgroup into two equal portions of values. Taking the example of subgroup 3, it can be seen that this group contains three values larger than the average and one that is smaller. In some data sets comprising extreme measurement values, the

Table 5.1 A practice data set

	Observations					
Subgroup	X_1	X_2	X_3	X_4	R	$\overline{X}$
1	36.13	32.85	34.05	38.04	5.19	35.2675
2	38.68	34.95	32.36	33.68	6.32	34.9175
3	34.34	35.69	35.06	29.72	5.97	33.7025
4	33.37	31.73	33.45	35.58	3.85	33.5325
5	32.42	35.58	34.35	35.79	3.37	34.5350
6	30.62	34.10	34.75	36.91	6.29	34.0950
7	31.76	33.29	37.41	31.50	5.91	33.4900
8	34.94	33.79	33.68	36.90	3.22	34.8275
9	34.66	32.02	36.34	33.50	4.32	34.1300
10	39.37	34.82	33.47	31.30	8.07	34.7400
11	33.06	38.97	35.88	36.07	5.91	35.9950
12	37.42	36.39	34.68	33.52	3.90	35.5025
13	37.18	34.43	36.34	33.88	3.30	35.4575
14	32.19	34.90	36.34	33.41	4.15	34.2100
15	33.36	34.36	33.38	33.68	1.00	33.6950
16	33.22	31.18	32.95	32.51	2.04	32.4650
17	34.22	33.01	36.63	35.10	3.62	34.7400
18	32.68	33.03	38.15	35.47	5.47	34.8325
19	31.49	35.84	31.00	35.47	4.84	33.4500
20	34.08	28.97	34.76	35.53	6.56	33.3350

average value need not be a faithful representation of other values in the data set.

5.3.2. Measure of subgroup variation: the range

Numerous methods have been devised to measure the variation in a set of numbers. For the purposes addressed in this text only one of them, the subgroup range, will be used to characterize variation within a subgroup. The subgroup range, indicated by the symbol 'R' is defined as the difference between the largest (maximum) and the smallest (minimum) value. The range for subgroup 1, for example, is:

$$R = 38.04 - 32.85 = 5.19$$

The range is the simplest and most direct method for measuring variation in a subgroup of size n with its value depending only on the two extreme observations. The values of the range for all the subgroups of the above example are reported in Table 5.1.

5.4 The construction of range and average charts

The average range of the entire data set can be calculated from the ranges for all subgroups as:

$$\overline{R} = \frac{\sum_{j=1}^{k} R_j}{k} = \frac{93.30}{20} = 4.665 \cong 4.67$$

Lower and upper control limits for the range are found by applying the following formulae:

$$LCL_R = D_3\overline{R} \text{ and } UCL_R = D_4\overline{R}$$

where D_3 and D_4 are constants whose values depend on the number of observations within a subgroup. The values of D_3 and D_4 for different numbers of observations are given in **Table 5.2**. For the example of Table 5.1 (with n=4) D_4 assumes the value of 2.282 and hence the upper control limit for the range is:

$$UCL_R = 2.282 \text{ x } 4.665 = 10.646 \cong 10.65$$

Table 5.2 Factors for use with average and range charts

Number of observations in subgroups	Factor for $\bar{X}$ chart	Factors for range charts LCL	UCL	Factor for estimating σ
n	A_2	D_3	D_4	d_2
2	1.880		3.267	1.128
3	1.023		2.574	1.693
4	0.729		2.282	2.059
5	0.577		2.114	2.326
6	0.483		2.004	2.534
7	0.419	0.076	1.924	2.704
8	0.373	0.136	1.864	2.847
9	0.337	0.184	1.816	2.970
10	0.308	0.223	1.777	3.078
11	0.285	0.256	1.744	3.173
12	0.266	0.284	1.716	3.258
13	0.249	0.308	1.692	3.336
14	0.235	0.329	1.671	3.407
15	0.223	0.348	1.652	3.472
16	0.212	0.364	1.636	3.532
17	0.203	0.379	1.621	3.588
18	0.194	0.392	1.608	3.640
19	0.187	0.404	1.596	3.689
20	0.180	0.414	1.586	3.735
21	0.173	0.425	1.575	3.778
22	0.167	0.434	1.566	3.819
23	0.162	0.443	1.557	3.858
24	0.157	0.452	1.548	3.895
25	0.153	0.459	1.541	3.931

For subgroups of size six or less no lower limit for ranges can be calculated, since the value of D_3 is not defined. Consequently, there exists no lower control limit for the range in the present example. (Annex A provides more information on constants like those above.)

When it comes to use the above results on the range, it has to be recalled that control charts are meant to provide information on changes in the process. As was indicated in Section 2.6, such changes are usually identified

(a) by the magnitude of variation in the points (i.e., if all points lie within control limits, the process is judged to have remained stable)

or

(b) by any non-random pattern in the points which is taken to indicate the presence of special or unusual events. In the present context a non-random pattern can be defined as

i) a run of seven or more consecutive points lying on the same side of the average line for the chart,

or

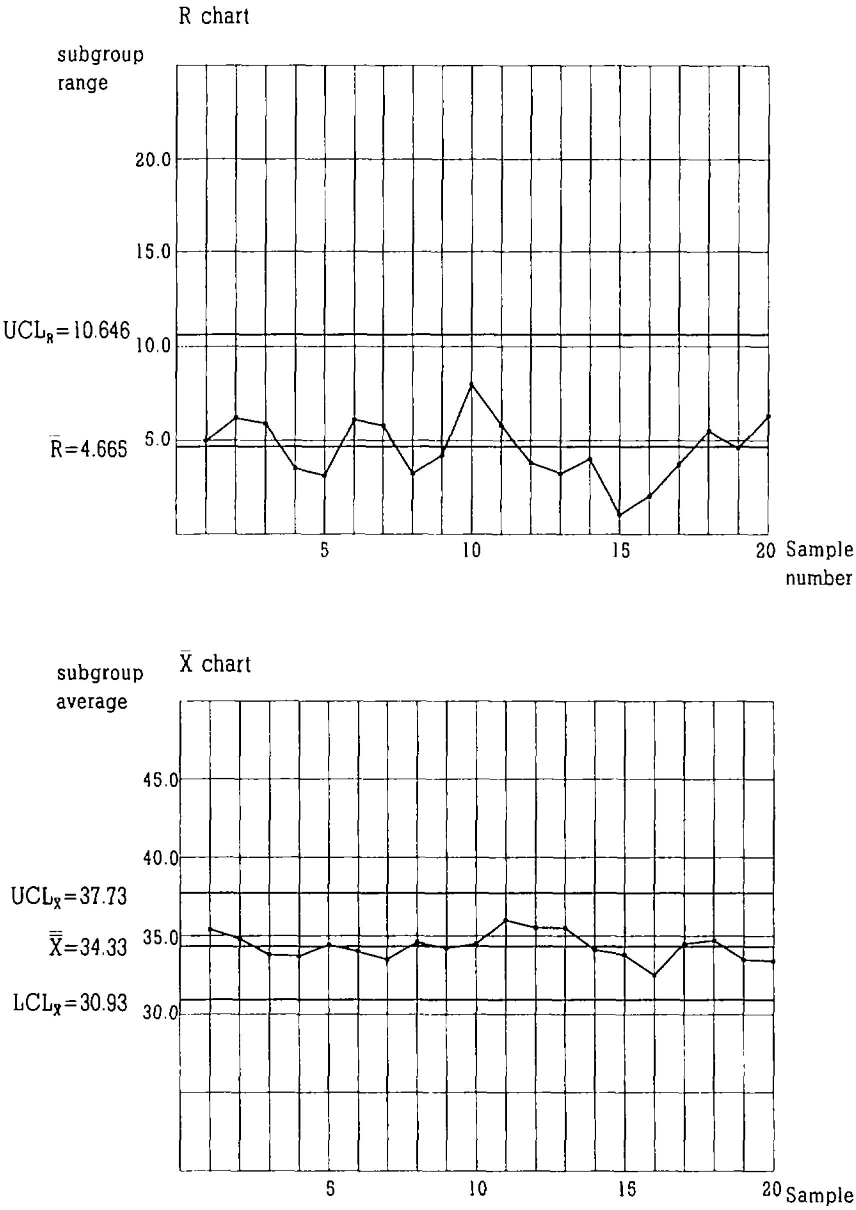
ii) a trend of seven consecutive increases or decreases.

According to the above conventions there is no indication of the presence of special causes in the range chart shown in **Figure 5.2**: All values of R lie below the upper control limit. There is also no pattern in the data points which would suggest the presence of systematic influences. The conclusion therefore is that the variation in the values of R is due to common causes.

Evaluation of the average chart also shown in Figure 5.2 depends to some extent on the results obtained for the range chart. If there was evidence of the presence of special causes of variation, it would not be appropriate to place control limits on the average chart, although average values could be plotted to identify systematic patterns. Nevertheless, it is only after the range has been stabilized, that a useful analysis of the process average can be conducted.

The present example allows for an examination of the limits for the variation that the subgroup average values may display. As in the case of other control charts the average chart too requires drawing of a centre line and of control limits. The position of the centre line is determined by the average of the subgroup averages. Formulae for this average of averages

Figure 5.2 Range and average charts



as well as for upper and lower control limits are given below, along with the numerical results for the present example.

$$\overline{\overline{X}} = \frac{\sum_{i=1}^{k} \overline{X}_i}{k} = \frac{686.92}{20} = 34.35$$

$$UCL_{\overline{X}} = \overline{\overline{X}} + A_2\overline{R} = 34.35 + 0.729 \times 4.67 = 37.75$$

$$LCL_{\overline{X}} = \overline{\overline{X}} - A_2\overline{R} = 34.35 - 0.729 \times 4.67 = 30.95$$

In the evaluation of the average chart the same rules apply as in the case of the range chart. In the present example all subgroup averages lie within control limits so that there is no evidence of special causes of deviation. If the pattern of data points is examined in addition, no evidence is found for any special causes of changes in average values.

It appears to be useful to examine the formulae for upper and lower control limits for the average chart from a process perspective. As the formulae indicate, the limits on average charts depend on the average value of the range. This points to the effect on limits of common-cause variation within subgroups. And the process average is judged to be in control or not according to common-cause variation within subgroups. This variation in turn is subject to two major influences: the sampling strategy for subgroups and the measurement process from which data result. Thus, if either the measurement process or the sampling strategy were to change, the basis for judging the stability of the process average is likely to change too.

In the example presently analyzed the process is stable and predictable in terms of both variation and average. It has to be noted, however, that stability or 'control' is a purely operational concept which says nothing about the utility of process outcomes. A process being in control means simply that the effects of changes in material, equipment or methods are consistent over time. Yet a controlled process is not necessarily a satisfactory process.

5.5 Summaries of process performance

Once a process has been judged as stable with respect to both variation and average, estimates of process parameters can be obtained. In addition, a histogram can be constructed on the basis of individual measurement values which provide additional insight into process performance. Average, standard deviation and distribution of process outcomes not only characterize the way in which a process is operated but also provide descriptors by which actual outcomes can be compared to what is required. The information contained in these three descriptors will be illustrated by a further examination of the previous data set.

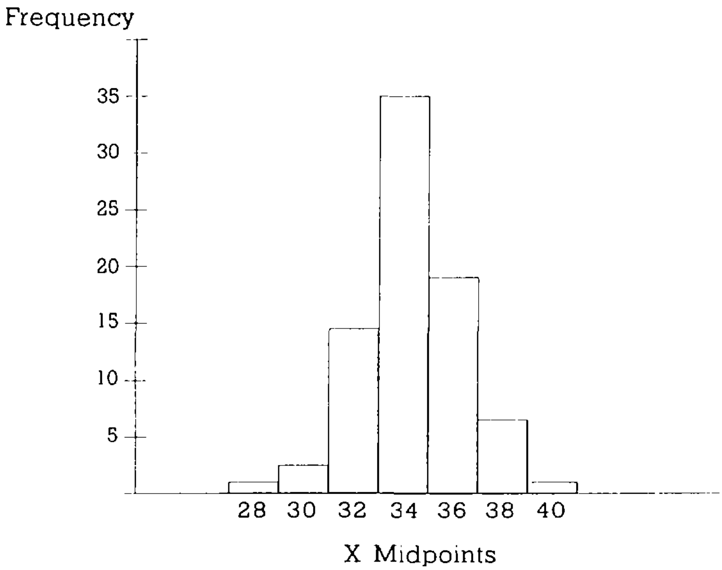
5.5.1 The interpretation of histograms

The 80 individual measurement values of the data set presented in Table 5.1 were used to construct the histogram of **Figure 5.3** which illustrates major characteristics of the process. The first one of these characteristics is the shape of the distribution of measurement values. Two terms that are used to describe the shape of the histogram in Figure 5.3 are 'mound-shaped' or 'bell-shaped.' These terms refer to the fact that most of the observations appear to cluster around a centre value and that observations seem to be grouped about that centre in a symmetric manner. Such a distribution of measurement values is fairly close to the model of the normal distribution. Thus, the data set of Table 5.1 might be termed as being 'normally distributed.'

The four histograms in **Figure 5.4** illustrate some other shapes of distributions which might occur in practice. Histogram A appears to have a large number of values either right above the lower specification limit or right below the upper specification limit. A possible reason for the observed distribution might be that material which was near the upper or the lower limit was reworked to insure it fell within specifications. Another possible explanation is that values that were close to the upper or lower specification limits were remeasured until a reading was obtained which fell within specifications.

Histogram B is chopped off at the upper and lower specification limits. As with histogram A an explanation for this shape can be expected from a closer examination of the process generating the results. One interpretation of the results is that the material being measured has been sorted prior to the point at which measurement was carried out. Material which fell above or below the specification limits, respectively, was then removed. If this were correct, the process would be expected to have more variation than can be tolerated. Histogram C shows a pattern similar to that of B, but with a more dramatic drop at the lower specification limit.

Figure 5.3 Histogram of values from Table 5.1

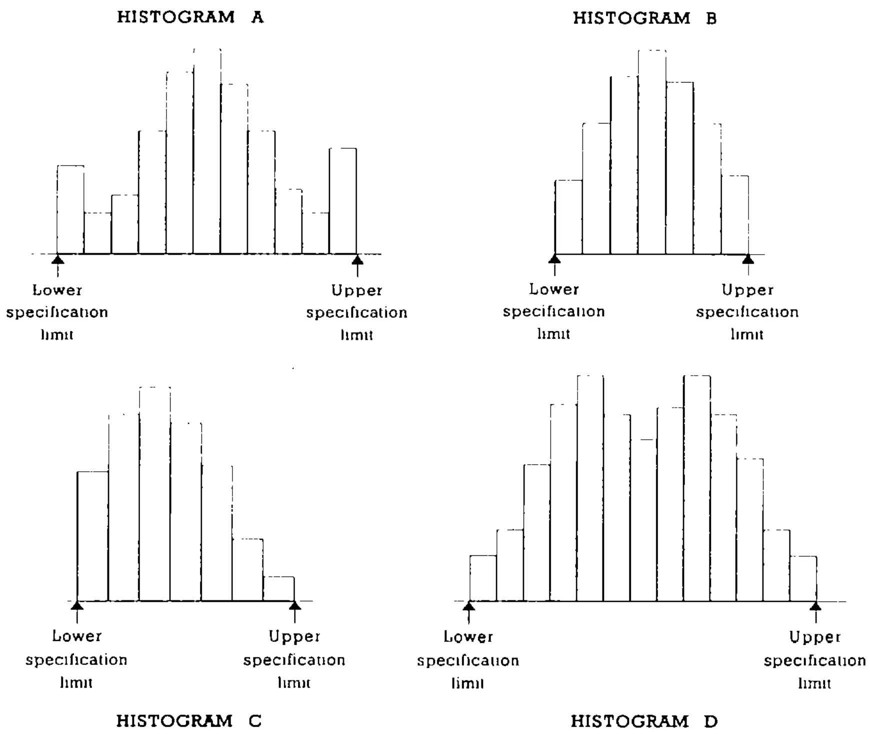


Again, the suspicion is that there is more variation in the process than can be tolerated. Furthermore, one might suspect that the material measured prior to sorting would have a normal distribution with the measured values clustering around a central value. Histogram C then suggests that this centre does not fall in the middle between specification limits.

Finally, histogram D shows two distinct peaks. A number of reasons for such a distribution could be given, but there is need for further investigation. One hypothesis is that there are two separate processes generating the observed data. For example, if the measured output came from two machines, an obvious possibility would be that each machine produced material centred at different values.

One of the uses of a histogram is to provide insight into process operation; another one is the description of measurable output produced by the process. The latter use has as one of its aims predictions about process output, which in turn are dependent on process stability. The histograms of Figure 5.4 provide an illustration of this point in the following way. The histograms for processes B and C look similar in that both can be described as mound-shaped. However, because both histograms represent processes which are inconsistent over time, neither of the two

Figure 5.4 Histogram examples



histograms can be relied on to describe process outcomes. The histograms only capture the shape of process outcomes over the period investigated. Inconsistencies in the processes mean that for different time periods the histograms will likely show different descriptions of process outcomes.

5.5.2. Estimating process variation

The histogram of Figure 5.3 indicates variation in measurement values as another important property of the process. Closer scrutiny of the histogram shows that its 80 values are spread over a range from about 28 to 40. The **standard deviation** is a numerical measure which provides information on process variation. Since the process which generated the data in Table 5.1 was seen to be stable, the standard deviation derived from these numbers also indicates what the standard deviation for other process values would be. It is denoted by s and given as

$$s = \sqrt{\frac{\sum_{i=1}^{n} (X_i - \overline{X})^2}{n-1}}$$

where in the present example $\overline{X}$ is the average of the 80 observations and n equals 80.

Examining the formula for the standard deviation helps to understand in which way it captures information about process variation. The value for s is based on squared deviations from the average. The farther away from the average a data value lies, the larger will be its contribution to the value of the standard deviation.

In the above example s is based on only 80 measurement values. One could imagine that a very large number of measurement values were available and the standard deviation was calculated on the basis of these numbers. (This task is clearly a conceptual one and is discussed here only to give meaning to the term **process standard deviation**.) The number which would result from this calculation is referred to by the Greek letter σ. It is taken to describe the variation over the whole process, and is consequently referred to as the process standard deviation. The calculation of s on the basis of only 80 measurement values can be considered as one way to estimate σ. Alternatively, an estimate of the process standard deviation can be derived from the average range which was calculated previously as $\overline{R}$ = 4.665. The formula for estimating σ in this way and the resulting value in the present example are:

$$\hat{\sigma} = \frac{\overline{R}}{d_2} = \frac{4.665}{2.059} = 2.27$$

where the value for d_2 is taken from Table 5.2. The symbol σ indicates the standard deviation while the symbol '^' indicates that the reported numerical value is an estimate of the process standard deviation. Because the range chart did not contain any indication of abnormal performance and the average chart demonstrated a stable process average, $\hat{\sigma}$ can be taken as a reliable estimate of the process standard deviation.

The standard deviation of the process is a useful descriptor of process variation. It provides a measure by which the variation observed can be compared to what is required. Such a comparison is often made by using the standard deviation to estimate the range of measurement values which may result from the process. For processes which follow a normal distribution, almost all measurement values should fall within a range of six standard deviations. For the data in Table 5.1 the standard deviation of the process was estimated to be 2.27. Thus, an estimate of the range over which measurement values can be expected to be distributed is $6 \times 2.27 = 13.62$. The histogram in Figure 5.3 shows that the measurement values fall in a range from 28 to 40. This spread of 40 - 28 = 12 is comparable to the value estimated by six standard deviations.

5.5.3. Estimating the process average

The histograms of Figures 5.3 and 5.4 were described as having distributions of measurement values which clustered around a central value. The process average yields information about this central value. When signs of abnormal process performance are absent from both the range and the average charts, the average of subgroup averages (which is the level of the centre line in the average chart) can be taken as an estimate of the central value of the process. In the case of the data in Table 5.1 the value for the process average is 34.35.

5.5.4 Process capability

In its most general form, process capability refers to delivery by the process of what is required. In a narrower sense process capability refers to whether the measured outcomes of a process exhibit sufficiently small variation to fall within some set specification limits. If, for example, specifications are such that the measured dimension of the data from Table 5.1 should fall between 30 and 40, then the **engineering tolerance** (ET) for the measurement is:

$$ET = 40 - 30 = 10$$

By contrast, the **natural tolerance** (NT) for the process refers to the range of measurement values over which the process will produce material.

Typically, the natural tolerance for the process is estimated by:

$$NT = 6\sigma$$

Thus, the natural tolerance for the process described by the data in Table 5.1 is 6 x 2.27 = 13.62. A comparison of NT with ET shows whether the process spread is small enough so that specifications could be met if the process were properly centred. In the present example NT is larger than ET so that the process has to be termed 'not capable'.

Various **capability indices** have been defined to quantify the ability of a process to meet specifications. Two of the more common indices are C_p and C_{pk}. C_p is defined by the ratio:

$$C_p = \frac{ET}{NT}$$

If the value of C_p is greater than or equal to one, the process is said to be capable. It is frequently stated that the preferred range of values for this index is 1.33 or more.

A drawback of the use of C_p to report on process capability is that the index does not take into account the process average. A process could have a large C_p ratio and yet be producing a lot of product outside specifications if it is not targeted. The C_{pk} index on the other hand represents an attempt to account not only for the impact of variation but also for that of the average on the capability of the process. C_{pk} is defined to be the smaller one of the numbers C_{pU} and C_{pL}, which are calculated according to the formulae:

$$C_{pU} = \frac{USL - \overline{\overline{X}}}{3\sigma} \qquad C_{pL} = \frac{\overline{\overline{X}} - LSL}{3\sigma}$$

Here USL refers to the upper specification limit and LSL to the lower specification limit. The data of Table 5.1 yield the following estimates of process characteristics:

$$\overline{\overline{X}} = 34.35$$

$$\hat{\sigma} = 2.27$$

$$NT = 13.62$$

The upper and lower specification limits for the process are:

$$USL = 40$$
$$LSL = 30$$

And from the above information, C_p is calculated as:

$$C_p = \frac{10}{13.62} = 0.73$$

Since the value of C_p is less than one, the process is judged as being not capable. The number 0.73 could be given the interpretation that 73 percent of NT is 'used' by ET.

In the present example C_{pU} and C_{pL} attain the values:

$$C_{pU} = \frac{40 - 34.35}{3 \times 2.27} = 0.83 \qquad C_{pL} = \frac{34.35 - 30}{3 \times 2.27} = 0.64$$

Thus, the smaller of these two numbers yields 0.64 as the value for C_{pk}. The fact that C_{pk} has a value even smaller than that for C_p can be explained by comparing the process average to the centre of the specification limits, or the nominal value. The nominal value in this instance is 35. The process is not centred on this nominal value, but is targeted somewhat below 35 at a process average of 34.33.

5.5.5 The use of capability indices: assumptions and limitations

The previous section illustrated the calculation of two capability indices assuming that the process under study was operating in statistical control. In general, one should be sceptical about reported values of C_p and C_{pk}, as these indices may be reported without any prior testing of the stability of the process. Often C_p or C_{pk} are determined by a one-time application of a control chart or, even worse, by collecting a number of consecutive readings from a process. If this were the case, then the derived values for $\overline{X}$ and σ could not be relied on to summarize process performance.

The calculation of either of the two capability indices also assumes that the distribution of measurement values closely resembles a normal distribution. To verify this assumption a histogram of individual results needs to be constructed in order to identify the shape of the distribution of measurement values. Furthermore it would be highly questionable to state as a requirement that C_p or C_{pk} be reported on all measured outcomes of a process. Many measured values cannot reasonably be expected to be part of a normal distribution.

Capability indices provide summary measures of how current process performance compares to certain stated specifications. However, their use as an aid to directing improvements efforts is limited. For example, when C_p or C_{pk} are reported as summary results of a process, the focus of study is often on the results of the process, rather than on the characteristics which need to be studied to improve the results. The sources of variation in outcomes need to be studied in addition.

Unfortunately, it is common practice in many organizations to state goals for C_p or C_{pk} for many, if not all, processes. In light of the above remarks on underlying assumptions and limitations of the use of the indices, such practices very often do not provide useful directions to improve process behaviour.

5.6 On the effective use of range and average charts

The major purposes for the use of the above two types of control charts may be briefly described as:

1) The charts serve to maintain process control.

2) The charts are used to judge the effect of deliberate process changes. The plotted information provides data for comparing process outcomes before and after a process change. It also provides evidence as to the magnitude, direction and stability of the effects of process change.

3) After a process change has been implemented and verified as to its effect, the charts provide ongoing confirmation that the change is maintained. In this respect, the charts help to maintain a gain achieved by a previous process change. Over a period of time this purpose merges into that of maintaining control.

4) The charts represent data in a way which allows operators, engineers, and managers to discover and evaluate the effect of various sources of variation.

Obviously, a major purpose of the discussed analysis is to maintain control over a process as defined previously. In particular applications, baseline data are collected, out-of-control conditions identified, and efforts made to eliminate these causes. In practice, there is often confusion regarding the control purpose. Engineers or operators may see a process go out of control, make an offsetting change in one or more process parameters and mistakenly consider 'control' as having been achieved. This is a wide-spread mistake made in both process control and improvement work. Such practices in fact build a culture that works against process control and improvement. Although it may be necessary to sometimes make offsetting changes, it is nevertheless necessary that the root causes of out-of-control conditions be identified, verified and eventually eliminated.

As improvement work proceeds, the process will be made predictable, i.e., it will be brought into a state of control. Outcomes will be monitored by means of range and average charts indicating progress in relation to variation and average. As the process becomes stable or predictable, information about standard deviation, average, and the distributional shape will gain credibility.

Predictability, or control, is a permanent concern and must constantly be addressed for the simple reason that processes do not naturally stay in control. Data that are collected and plotted continuously support good control: the plotted points indicate where the process is or where it might be headed. They also reveal factors which have to be eliminated in order to reestablish process control.

Successful implementation of these practices requires considerable discipline and effort. A prerequisite of success in this respect is for managers to communicate the intention to run the respective processes in control. Process engineers, line supervisors and operators must know how to measure, sample and evaluate process results and must be aware of the fact that these duties are expected of them. Extensive process knowledge must be built up, recurrently verified and deployed. For example, past and current behaviour of key input variables and work practices must be known in detail and so must the effects of variation in these variables on process outcomes.

It is often assumed that charts on outcome variables can serve as part of an engineering 'feedback' mechanism. Whether or not this view is correct depends very much on the process. In this connection it is often thought that the purpose of charting is to represent current conditions and could therefore serve the aforementioned engineering purpose. But the use of results in this way is often disappointing. The major reason is that the sampling and measurement methods underlying the data for these charts are usually not geared to serve an engineering feedback control system. For example, the statistical limits previously derived are not related to process dynamics which determine the design of a successful engineering feedback mechanism.

For purposes of control, the dominant issue is whether the essential causal factors are known and managed correctly and consistently. Understanding the management of common causes of variation provides the basis for process improvement. It is with this intention that the charts are best used for 'control.' Process management without reference to common causes provides for only reactive behaviour and cannot guarantee control. As was stated previously, the discussed charts provide indicators not only of control but also of the effectiveness of process change. Once a process is in control, process change may be recommended to affect common-cause sources of variation. Here the intention may be to reduce short-term variation, move the average to a more favourable value or prevent long-term shifts in averages. In any event the specific nature of the intended effect has to be defined. Process changes are normally rooted in machine changes, revised tolerances for machine parameters, changes in material, its characteristics or specifications, or in revised work methods or protocols.

Charts on range and average will characterize the stable process. Baseline data are then used for judging the effect of process changes. A first set of range and average charts is based on data that represent the process as it is prior to a specific process change. After change has been made, data are collected and plotted again with the measurement and sampling strategy remaining unchanged. The 'new' data are usually plotted directly onto the first set of range and average charts to facilitate comparison. In such an exercise several outcomes are possible:

a) The change has had no effect on the process. Evidence for this conclusion stems from the observation that charts are very similar. In other words, the new data are consistent with the old ones and the charts do not reveal signs of sustained process change.

b) The change has had the anticipated effect on either short-term variation or on the average. A decrease in short-term (within-subgroup) variation is revealed by a decline in the average range. Likewise, a shift in the average value to a new, preferred level is revealed by the respective average charts.

c) The change has had an undesirable effect on the process. It may be that the change has thrown the process 'out of control'. In this case, it may be that an improved process would result if the new process could be brought into control. The change may have resulted in process deterioration in other respects: for example, the short-term variation may have become larger or the average shifted to a value not anticipated or wanted.

5.7 The special case of subgroup size 1

There are numerous processes for which it is not feasible to design subgroups of two or more measurement values. Typical examples are processes where measurement is expensive or requires long periods of time to obtain. Other examples are found in processes where measurement is possible only hourly, daily, weekly or even less frequently. In these situations — which may obtain in manufacturing, service or administrative processes — it may not be possible to form homogeneous subgroups of measurement values. In manufacturing, subgrouping may not be possible when large homogeneous batches of material are produced or when records on process parameters such as temperature, pressure or amperage are to be studied. Administrative examples include the study of average overtime hours per full-time employee, daily utilization rates of equipment or other resources, accounting data on shipments and orders, monthly numbers of items produced per direct labour hour, and monthly deviations of actual sales from forecasts. In each of these examples, only one observation is available to represent a given set of circumstances. When there is only one measurement value available for any one set of conditions, the notion of subgroup range becomes meaningless. If short-term variation is to be utilized to judge long-term process movements, a modification of the method outlined previously is needed. Short-term variation is now calculated as the average of the absolute values of the deviations between two consecutive values. In turn, this average deviation is used to calculate limits for the chart of individual values. The following example illustrates this procedure.

5.7.1 The construction of moving-range and individuals charts

The data for the following example are taken from a batch process producing a sterilized, concentrated baby formula. While there are several important product characteristics, only one of them, the Brookfield viscosity, is reported here. Specifications on this property are 900$\pm$100 and data from 20 consecutive batches are shown in **Table 5.3**.

Table 5.3 Viscosity readings

Batch	Viscosity	Moving range
1	805	-
2	976	171
3	901	75
4	929	28
5	927	2
6	942	15
7	904	38
8	804	100
9	874	70
10	944	70
11	850	94
12	941	91
13	992	51
14	795	197
15	952	157
16	832	120
17	809	23
18	878	69
19	936	58
20	888	49

The sampling scheme for the data of this table requires that one specimen (sample) be taken from a randomly selected position in the completed batch. Thus, each reported number represents a finished batch. These individual numbers, one per batch, constitute the values that will be plotted on the **'individuals'** chart for which an average or centre line and control limits have to be determined. The average of individual values is computed in the usual way. The control limits for the chart for individual values are computed from information on the chart for moving ranges discussed in the following paragraph.

The moving range (MR) is defined as the absolute difference between consecutive values. Thus, the first moving range recorded in Table 5.3 is the difference between the first two values, i.e.:

$$MR = 976 - 805 = 171$$

Once moving ranges have been calculated for the baseline data set, these values are plotted on a so-called **moving range chart**. (It should be noted that there will be one moving-range value less than there are individual values in the data set). The average value for the moving range in the present example is given by:

$$\overline{MR} = \sum MR / n = \frac{1477}{19} = 77.7$$

The upper control limit for the chart for moving ranges is then computed in the same way as for subgroups of size two, yielding an upper control limit of

$$UCL_{MR} = D_4\overline{MR} = 3.267 \times 77.7 = 254.0$$

If all values on the moving range chart lie below the upper control limit, this is again an indication that short-term variation is stable over time. Since the average moving range reflects the size of short-term variation, this average can be used to judge the stability of individual readings too. Here the quantity

$$\frac{\overline{MR}}{d_2}$$

describes the standard deviation expected for individual readings, if the short-term variation captures all the variation affecting such readings. The limits on the individuals chart reflect this short-term component of variation. If additional sources of variation become effective over the long term, this additional variation should emerge as an out-of-control signal on the chart of individual values. The control limits for the individuals chart are computed as:

$$UCL_x = \overline{X} + 3(\overline{MR} / d_2)$$
$$LCL_x = \overline{X} - 3(\overline{MR} / d_2)$$

where the control chart constant d_2 is taken from Table 5.2. Like in the case of the moving range chart, also the d_2-value is taken for subgroup size two. In the present example , the centre line, the upper and the lower control limits for individual readings on viscosity are given by:

$$\overline{X} = \frac{17879}{20} = 894.0$$

$$UCL_x = 894.0 + 3 \times \frac{77.7}{1.128} = 1100.7$$

$$LCL_x = 894.0 - 3 \times \frac{77.7}{1.128} = 687.2$$

The stability of variation in individual viscosity readings can be judged by use of the above values.

5.7.2 Effective use of moving range and individuals charts

A prerequisite for the appropriate use of any statistical chart in process management is a good understanding of the sampling strategy. For the present process this strategy was to select one sample from each completed batch so that one value per batch was recorded. Hence each individual observation represents one batch.

The sources of within-batch variation and also of batch-to-batch variation are process based. It is likely that variation originates from two general sets of sources, namely those which tend to make one batch differ from another and those leading to non-homogeneous batches. An example of the first type of source can be found in the amounts and densities of raw material used to manufacture a batch; these might vary from batch to batch in ways that are not monitored and will tend to result in differences between batches. There are other practices and methods which are equipment- and personnel-related and could be added to the list of sources of variation which make one batch different from another. Once a particular batch is created, there are also process sources that result in batches that are not homogeneous but display within-batch variability. The random sample selected from a batch and measurement conducted on that specimen ensure that variation in the individual values potentially reflects all these causes. And the moving range displays this variation.

The average value of the moving range indicates the average difference among consecutive batches. The upper control limit for the moving range chart provides information on the magnitude by which consecutive batches could differ from each other. Points above the upper control limit for the moving-range chart reveal abnormally large shifts or changes from one batch to the immediately following batch. It is recommended not to apply the usual runs or patterns tests to the moving-range chart as a way of testing for stability of short-term variation. Moving ranges are calculated from consecutive observations and so each one of them shares an observation with another moving range. This characteristic prevents values from being independent, while independence is a mathematical prerequisite for the appropriate use of the runs or patterns tests. Nevertheless, a possible dependence between values of the moving range does not significantly affect estimates of process variation based upon the average for the moving range.

CHAPTER 6

SUBGROUPING AND COMPONENTS OF VARIANCE

When improvement of systems and processes is attempted, the value of statistical analysis depends on the rationale and purpose of the analysis, on its concordance with the objectives of managers and other actors involved and on the timeliness and relevance of the underlying data. In this context the sampling or subgrouping strategy of data collection has a significant role to play. The collection and statistical analysis of data will contribute to process improvement only if it helps to identify causes of unsatisfactory process performance. In general this is possible only if this intention already guides the collection of data. As a consequence, the sampling or subgrouping strategy is to be seen as a major determinant of success or failure of the statistical analysis.

An example of a poor sampling strategy would be one that suggested to collect data at only one point in time and thereby prevented the study of factors that are not at work continually. Likewise, failing to give due consideration to sources of variation included in a cause-and-effect diagram would not be appropriate. Typically, the routine collection of data at inspection stations suffers from such shortcomings. As data are mainly collected for the purpose of judging process output, they will often not provide useful information on the causal factors behind variation in process outcomes.

In process study the major objective is to acquire knowledge that is sufficient for operating a process in a consistent fashion. This goal demands that sources of variation are identified and their effects understood. The next objective to be pursued is to improve the process according to certain criteria. This may take the form of reducing output variation or improving the capability to target the process. The present chapter deals with the role that subgrouping and analysis techniques can play in this context.

6.1 Components of total variation

Whether process variation can be reduced, largely depends on an understanding of the factors that contribute to it. Such understanding above all concerns the nature and the significance of the sources of total variation. To give an example: In the production of a porous membrane, a characteristic of critical interest is the variation in the pore size of the membrane. While in all likelihood the material used to produce the membrane is the cause for much of the observed variation, its nature too needs to be understood if variation is to be reduced. Usually, the material used to produce membranes is received in large containers. Is the variation in pore size largely due to differences between containers, or is its source a factor that operates within each container? To answer this question information has to be obtained on the relative contributions of within-container and between-container sources of variation.

Another example is the total variation which is observed in the fill weights of containers coming off a filling operation. The filling operation uses three filling machines each of which has four heads. In assessing the magnitude of variation in fill weights, it may be discovered that variation is too large. Success in decreasing the variation will be predicated upon knowledge of the relative contributions of differences in filling heads and of differences in filling machines to the total variation observed.

A third example is provided by the production of material in a batch process. Here the variation in particle size of the material produced is of critical importance. Is there a large variation in particle size observed within each batch? Or is it the case that within a batch particle size is fairly uniform, whereas successive batches differ much among each other with the result of a large variation across batches? Collecting data to answer these questions will require a sampling strategy that helps to assess the relative contributions of within-batch and between-batch sources of variation to total variation.

The following section discusses a technique for collecting and analyzing data that helps to understand the contributions of components of variation to total variation. Although the example used to illustrate the study of components of variation comes from a batch process, the application of the technique is not confined to batch processes.

6.2 Study of a batch process

The assumption underlying the following example is that a team has been charged with the study of the variation of a given batch process. This team has constructed a process flow chart and identified the major quality characteristics of the product. One of the factors suspected to affect final product quality is viscosity of the batch at an intermediate stage of processing. The range of measurement values of viscosity has been specified to extend between 83 and 85. At the time when work on the process began, little was known about how well viscosity was being maintained. Therefore, the team decided to take one sample from each batch in order to assess both variation and average of the process. Initial data for the 20 batches studied are given in **Table 6.1**.

An analysis of range and average (not reproduced here) supports the hypothesis that viscosity is consistent. However, an estimate of the standard deviation of viscosity reveals an unacceptably high level of variation:

$$\hat{\sigma}_t = \frac{\overline{MR}}{d_2} = \frac{0.61}{1.128} = 0.54$$

The natural tolerance, NT, of the process is found to be:

$$NT = 6 \times 0.54 = 3.24$$

Since the engineering tolerance, ET, is only 2, the variation in the process is much wider than the stated requirements for viscosity. Because of the large variation in viscosity readings the team decided to try to identify its sources.

6.2.1 Sources of total variation in a batch process

Efforts to reduce the variation in viscosity readings need to be based on a thorough understanding of the sources affecting total variation. In this context two basic scenarios of the batch process can be considered. A first scenario is one in which there is little difference in viscosity values measured for three different batches: Each one of the batches has about the same range and centres at about the same average. Therefore, total variation in the combined output from the three batches can be explained in terms of the large variation within batches. For a second scenario a

Tables 6.1 Viscosity measurements for a batch process

Batch	X	MR
1	84.6	-
2	84.6	0
3	84.1	0.5
4	83.7	0.4
5	84.0	0.3
6	84.1	0.1
7	84.1	0
8	83.1	1.0
9	83.0	0.1
10	84.3	1.3
11	83.9	0.4
12	83.7	0.2
13	83.9	0.2
14	84.4	0.5
15	82.6	1.8
16	84.2	1.6
17	83.7	0.5
18	84.8	1.1
19	83.8	1.0
20	84.3	0.5

smaller level of within-batch variation for each one of three batches can be assumed. At the same time the assumption is made that there are larger differences between the three batches. While the variation in combined output in scenario two is similar to that for scenario one, the explanation differs: The source of large total variation in combined output of scenario two has to be sought in the difference between batches.

In general, it is important to understand how the sources of variation, both within and between batches, contribute to total variation. Different causal structures are suspected to underlie within-batch variation as opposed to between-batch variation. For example, mixing practices of the batch process might be the cause of variation observed within a batch, whereas they would have little effect on differences observed for the level of viscosity from one batch to the next. By contrast, a large variation in the amount of a viscous ingredient added in the initial stage of production might result in large differences in viscosity from one batch to the next without impacting within-batch variation. Therefore, if for a given process scenario one can be taken to represent observations correctly, attention should be directed towards sources of the large within-batch variation. An analogous conclusion holds for scenario two.

Any attempt to analyze the effects of within-batch and between-batch sources of variation requires data collection to follow a suitable subgrouping strategy. In order to understand within-batch variation, multiple readings of viscosity from each one of several successive batches will be needed. On the other hand, assessing differences from one batch to another requires readings from many different batches.

6.2.2 Assessing stability and magnitude of within-batch variation

The team in the present example decided to collect five samples from randomly selected locations in each of 20 successive batches. On the basis of these data, stability and magnitude of the within-batch component of variation is to be assessed. The averages and ranges of the five measured values from each batch are recorded in **Table 6.2**. Before analyzing these data, potential sources of variation should be considered. Since each of the five values in a subgroup came from the same batch of material, the magnitude of the range reflects those sources of variation active within a batch. Of course, the amount of variation within a batch may not be consistent between two batches. Usually a range chart is drawn to determine whether the within-batch component of variation is stable across the 20 batches included in the study.

Table 6.2 Averages and ranges of five viscosity readings selected from each of 20 batches

Batch	Average	Range
1	84.04	0.4
2	83.96	0.4
3	83.52	0.7
4	84.70	0.8
5	83.40	0.8
6	84.22	1.2
7	84.36	0.7
8	83.58	1.3
9	84.00	0.4
10	84.58	0.9
11	84.30	0.8
12	83.18	0.5
13	83.94	0.8
14	84.82	1.1
15	83.82	0.8
16	84.14	0.5
17	83.64	0.9
18	83.68	1.0
19	83.90	0.5
20	83.24	1.0

In the present example such a chart (not shown here) provides evidence for consistency of within-batch variation. The magnitude of this variation is estimated in terms of the standard deviation given by:

$$\hat{\sigma}_{\text{within}} = \hat{\sigma}_{\text{w}} = \frac{\overline{R}}{d_2} = \frac{0.775}{2.326} = 0.333$$

For obvious reasons, the within-batch standard deviation does not capture all of the variation in the process.

6.2.3 Assessing stability and magnitude of between-batch variation

The data presented in Table 6.2 can also be used to identify between-batch sources of variation. This can be achieved by analyzing batch averages. Since each average is an estimate of the mean level of a batch, the differences between batch levels will be reflected in the variation observed in the averages. Thus, an analysis of batch averages will shed light on the between-batch component of variation.

As a way of evaluating the stability of the between-batch component of variation, moving ranges of batch averages have been calculated. **Table 6.3** contains these moving ranges together with the batch averages that already appeared in Table 6.2. The moving ranges reflect the short-term variation which is observed between batches. Of course, short-term batch-to-batch variation is not the only type of variation captured by the moving ranges. They also reflect within-batch components of variation.

Figure 6.1 shows the completed moving-range chart. The calculations underlying this chart are as follows:

$$\overline{MR} = 0.606$$

$$UCL_{MR} = D_4\overline{MR} = 3.267 \times 0.606 = 1.98$$

$$LCL_{MR} = \text{none}$$

Table 6.3 Moving range for averages of viscosity readings

Batch	Average	Moving range
1	84.04	-
2	83.96	0.08
3	83.52	0.44
4	84.70	1.18
5	83.40	1.30
6	84.22	0.82
7	84.36	0.14
8	83.58	0.78
9	84.00	0.42
10	84.58	0.58
11	84.30	0.28
12	83.18	1.12
13	83.94	0.76
14	84.82	0.88
15	83.82	1.00
16	84.14	0.32
17	83.64	0.50
18	83.68	0.04
19	83.90	0.22
20	83.24	0.66

where the constants for samples of size n = 2 are used. Since the moving-range chart is in control, short-term between-batch variation is proved to be stable. Using the average moving range, control limits for the chart of batch averages can be calculated. The standard deviation for batch averages is given by:

$$\frac{\overline{MR}}{d_2} = \frac{0.606}{1.128} = 0.537$$

and the centre line for the average chart is located at the average level

$$\overline{\overline{X}} = 83.95$$

The control limits for the average chart are found by adding and subtracting, respectively, from the above value three standard deviations of the batch averages:

$$UCL_{\overline{X}} = 83.95 + 1.61 = 85.56$$

$$LCL_{\overline{X}} = 83.95 - 1.61 = 82.34$$

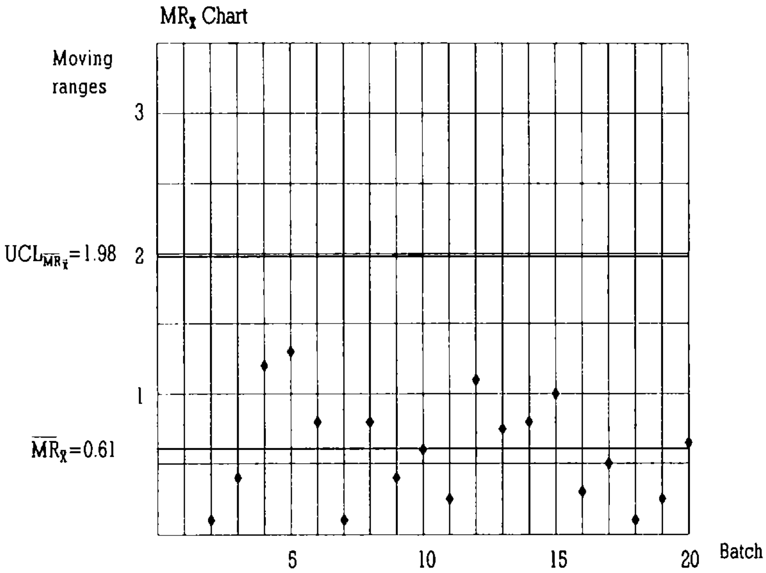
Since the completed average chart is in control, the long-term batch-to-batch component of variation too can be considered as being stable over time.

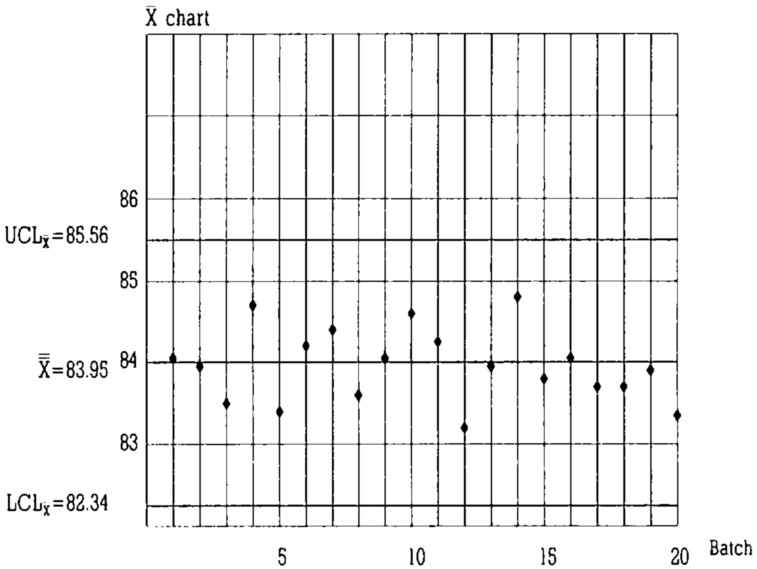
In summary, neither the short-term nor the long-term batch-to-batch component of variation appears to be subject to variation deriving from special causes.

For comparison with the within-batch component of variation, an estimate of the between-batch component is derived. This estimate, σ_b, is based on the standard deviation calculated for batch averages. The formula for the standard deviation of the between-batch component of variation is given by:

$$\hat{\sigma}_b = \sqrt{\left(\frac{\overline{MR}}{d_2}\right)^2 - \frac{\sigma_w^2}{n}}$$

Figure 6.1 Moving- range chart (range based on batch averages)





where n is the number of observations used to calculate each batch average. Accordingly, the between-batch standard deviation is found to be:

$$\hat{\sigma}_b = \sqrt{(0.537)^2 - \frac{(0.333)^2}{5}} = 0.516$$

6.2.4 The contributions of within-batch and between-batch variation to total variation

Based on the estimates of within-batch and between-batch variation, an estimate of the total variation of the batch process can be obtained. This estimate reflects the fact that the total variance of the batch process is found by summing the variances of the within-batch and between-batch components. Thus, the standard deviation for the batch process is given by:

$$\hat{\sigma}_t = \sqrt{\hat{\sigma}_b^2 + \hat{\sigma}_w^2} = \sqrt{(0.516)^2 + (0.333)^2} = 0.614$$

This standard deviation is another estimate of the value that the process standard deviation approximated. The process standard deviation (estimated on the basis of a moving-range chart) was obtained by taking one sample from each of 20 successive batches. By contrast, the process variation calculated above was found by first estimating the two components of total variation and subsequently combining these components to derive an estimate of total process variation.

Another way of capturing the relationship between total and component variation is to examine the relationship in terms of the total, the between-batch, and the within-batch variances. This relationship can be described as:

$$\hat{\sigma}_t^2 = \hat{\sigma}_b^2 + \hat{\sigma}_w^2$$

Estimates of the values of the above variances are given as:

$$\hat{\sigma}_t^2 = 0.377145$$

$$\hat{\sigma}_b^2 = 0.266256$$

$$\hat{\sigma}_w^2 = 0.110889$$

The description of total, between-batch, and within-batch variation in terms of the corresponding variances is useful in that it points out the relative contributions of the components to total variation. In the present example, the 'within' contribution accounts for about 30 percent of total variation due to:

$$\frac{\hat{\sigma}_w^2}{\hat{\sigma}_t^2} \times 100\% = \frac{0.110889}{0.377145} \times 100\% = 29.4\%$$

leaving about 70 percent of total variation to be accounted for by differences between batches. This implies that reducing the variation in viscosity readings between batches promises a greater reduction in the overall variation of the batch process than working on within-batch sources of variation.

6.3 Summary of the analysis of components of variance

The analysis of the components of variance for the viscosity readings from a batch process required several stages of plots and calculations. To facilitate recalling the sequence of operations in this analysis, a detailed discussion of eight steps is presented below.

Step one identifies the type of data required for analyzing the between and within components of variation contributing to total variation. In studying a batch process, the required data are repeated readings of n values from each one of k batches. The n readings from each batch form one of k subgroups. The selection of the k batches studied crucially affects information about between-batch variation. As a rule, at least 20 batches should be selected for further study. However, more important than the total number of batches is the way in which selection takes place. Here the guideline is to choose the k subgroups in such a way as to allow identification of the sources of between-variation. For example, it can be supposed that a change in the source of raw material will affect between-batch variation: Therefore, if the k subgroups did not include batches from different raw material sources, the data could not capture any effect of raw material variation on between-batch variation.

Although the present example describes the study of batch processes, the same data collection strategy can be used for the purpose of studying within and between components of variation for other types of processes as well. The process of making porous membranes (discussed in Section 6.1) provides another example. In this process it was of interest to understand those portions of variation in pore size which can be attributed to container-to-container variation or to within-container variation. Data collected to analyze these two sources of variation would have to consist of subgroups of n membranes made from the same container, and these repeated readings on pore size would need to be made for k different containers.

Once the data are available, step two consists in constructing a range chart for the ranges of each of the k subgroups. The constant used to find the correct value of D_4 is n, the number of readings in each subgroup. The range chart is used to evaluate stability of the within-batch, or within-subgroup, component of variation. If the range chart is out of control, the reason for the within-subgroup variation being inconsistent needs to be identified. A cause-and-effect diagram which captures the factors affecting within-subgroup variation will be helpful. This diagram can be explored to provide some indication of those sources which may be acting intermittently to create unstable variation within batches.

If the range chart is not in control, it is not possible to investigate the between-batch component of variation. In other words, in such a case it is inappropriate to construct a moving-range chart of the batch averages for mainly two reasons. One of them is theoretical, namely, that statistical theory does not provide for an evaluation of the moving-range chart of batch averages when within-subgroup variation is unstable. The other reason is pragmatic. The fact that the range chart is out of control already indicates the direction for future improvement work: An immediate goal of such work must be to discover and eliminate the sources of instability of within-subgroup variation.

Step three in the analysis is about estimating the within-subgroup or within-batch standard deviation. (Of course, it would not be appropriate to estimate this component of variation if the range chart was out of control.) The value of D_2 used in the formula of this step corresponds to n, the number of readings in each of the subgroups.

Step four consists in constructing a moving-range chart from the subgroup or batch averages. It should be noted here that the value of D_4 used to calculate the upper control limit corresponds to the one for samples of size two. This value is used because there are two averages involved in the calculation of each moving range. It also needs to be recalled that the moving-range chart is constructed in order to evaluate the stability of short-term batch-to-batch variation. If the task was to evaluate the variation in pore size of a porous membrane, then the moving-range chart would provide a way for evaluating the stability of short-term between-container variation. In both cases, if the moving-range chart is out of control, work should first be directed towards identifying the reasons for the observed instability. For example, if the moving ranges of the average viscosity readings had shown evidence of inconsistency, attention would have to be directed towards identifying why at some points in time one or more batches are very different from others, whereas it would be inappropriate to construct the average chart.

Step five in the analysis consists in the construction of the average chart. This chart provides information on the stability of long-term between-subgroup, or between-batch, variation. For example, if there were trends or cycles in the batch averages, the average chart should provide evidence of this.

If the average chart is in control, steps six and seven can be completed. In these steps the between-subgroup components of variation as well as total variation are estimated. Finally, step eight is about calculating those percentages of total process variation that are due to within-subgroup sources and between-subgroup sources of variation. These percentages are helpful to set priorities in the work for improving the process. If within-subgroup sources of variation are the major contributor to total variation, then attention needs to be directed at the corresponding causes. In the example of viscosity readings on a batch process, investigating within-batch sources of variation might involve an examination of the mixing procedures of batches, since poor mixing might be one of the reasons for large variation in viscosity within a batch.

CHAPTER 7

MEASUREMENT PROCESSES

Any measurement of acceptable quality usually relies on a measurement process. This process - like a production process - has to be examined in order to assess whether it delivers the expected output. Any output of a measurement process will undoubtedly exhibit variation. As in the case of a production process, an understanding of the nature of this variation — both regarding its magnitude and its stability over time — as well as of the conditions of measurement variation is critical.

The variation and average of the outcomes of a measurement process have profound effects on the information that measured values provide about a production process. If a measurement process, for example, exhibits unstable variation, then the interpretation of measurement values will be impeded. If such instability goes unnoticed, the unstable variation in outcomes may erroneously be attributed to an instability in the production process on which measurement is taken. Likewise, if a measurement process has stable, yet large variation, then the potential for understanding production process variation will be reduced. If a measurement process does not deliver values at a consistent level, it becomes impossible to evaluate the average of the underlying production process. For reasons such as these, the study of the process generating measurement values is indispensable for the analysis of a production process. The example provided in the next section illustrates the application to a measurement process of techniques described in previous chapters.

7.1 Evaluating a measurement process

A chemical manufacturing firm relies on laboratory measurement to check the purity levels of a herbicide, called product M. In order to ensure the quality of the measurement process, the laboratory carrying out measurement routinely calculates the purity level of a sample (called a standard) which contains a known amount (96 units) of product M. Each time the laboratory runs an assay to determine the amount of M in one or more production samples, the amount of M in the standard is also measured. Since product M is a stable compound, the actual amount of M measured in the standard sample should not change. **Table 7.1** contains the measured amount of M in the standard for the 30 most recent assays. The variation in these numbers is not, of course, due to changes in the purity of material. Instead, the observed variation is characteristic of the measurement process.

It is useful to consider - before examining control charts of the data in Table 7.1 - how sources of variation affecting the measurement process are reflected in the above data. Among potential causes of variation are the following:

(a) In the analytical laboratory four different chromatographs are in use, any one of which might be employed in a given assay for product M.

(b) Even if the same device were used for each assay, it might read differently from one assay to the next.

(c) Any one of eight different laboratory technicians may perform the assay.

(d) The assay requires a fairly involved sample preparation. Even if only one technician performed the assay, there would be slight differences in the sample preparation which would produce variation in results.

(e) The reagents used to carry out the assay may change over time.

(f) At infrequent intervals the equipment itself changes, as the chromatographs have columns which need to be repacked.

Table 7.1 Amount of "product M" in standard sample

Date	Assay value	Moving Range
07/26	95.9	---
07/27	95.7	0.2
07/29	96.7	1.0
08/04	95.8	0.9
08/05	96.9	1.1
08/07	95.5	1.4
08/10	96.8	1.3
08/10	96.0	0.8
08/11	95.6	0.4
08/12	96.4	0.8
08/14	96.0	0.4
08/14	95.2	0.8
08/16	96.2	10
08/16	96.0	0.2
08/16	95.2	0.8
08/18	95.5	0.3
08/19	96.2	0.7
08/20	96.1	0.1
08/20	96.7	0.6
08/21	96.2	0.5
08/22	96.2	0.0
08/22	96.5	0.3
08/24	95.3	1.2
08/26	96.0	0.7
08/27	95.5	0.5
08/28	95.5	0.0
08/29	96.0	0.5
09/02	97.2	1.2
09/04	95.3	1.9
09/05	96.4	1.1

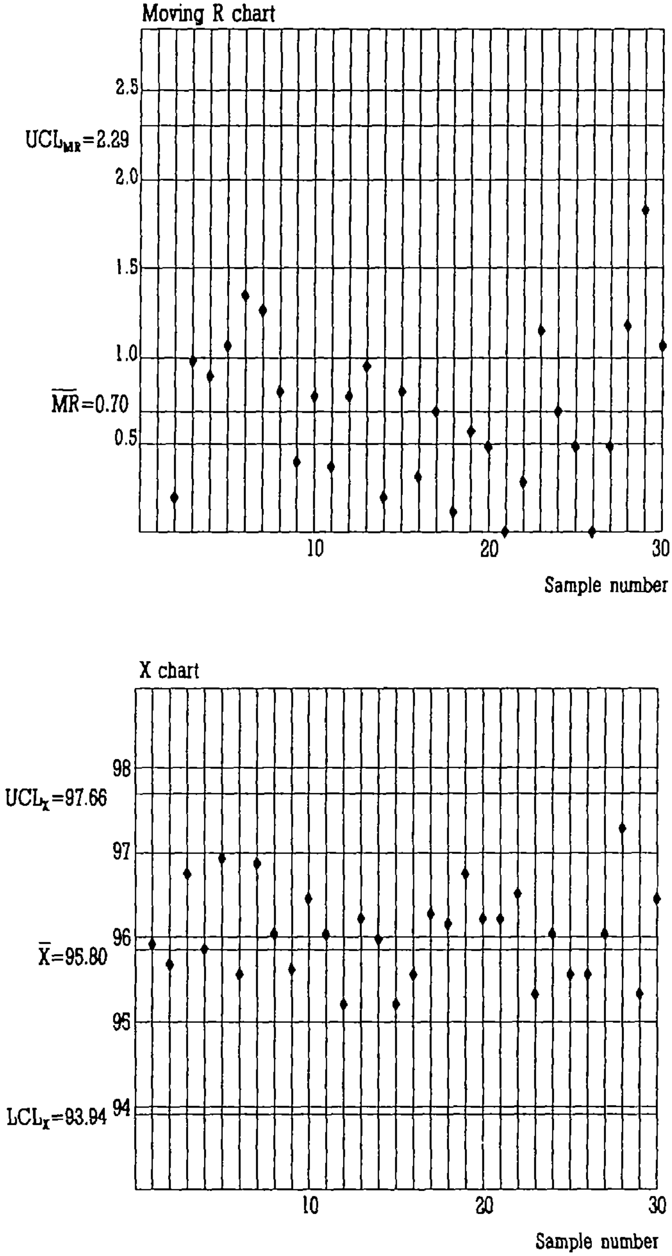
Together with recorded values moving ranges for the standard sample are also reported in Table 7.1. Since an assay may have been performed by any of several technicians on any of several devices, differences among individuals or devices would have an impact on the magnitude of the ranges. Furthermore, changes due to differences in sample preparation or set-up would also affect the size of the moving ranges.

Since — as **Figure 7.1** shows —the moving-range chart is in control, there is no evidence for assay-to-assay variation to be inconsistent across the range of data examined. Taking into account those factors behind variation that influence the ranges, it might be concluded that set-up variation, operator variation and device-to-device variation is fairly consistent over time.

Since the moving-range chart is in control, limits can be constructed for the values on the X-chart. However, before examining the completed X-chart, the sources of variation in X-values need to be scrutinized. If the reagents used to conduct the assay deteriorated over time, this would probably result in a trend in the values for the standard. Likewise, if the column were repacked during the course of the study, the average value might be affected. The completed X-chart in Figure 7.1 does not indicate any such trend or shift in the average reading over time. Thus, the measurement process appears to be reading to a stable average of 95.8, a number very close to 96, the amount actually contained in the standard.

Since both the moving-range chart and the X-chart are in control, the standard deviation from the average of the moving ranges can be estimated. This estimate takes the value of 0.62 which was used to calculate the control limits for the X-chart. This standard deviation indicates variation in the measurement process for which it is useful to identify the major sources. Since the samples are run by an analyst on any one of several machines and the values are separated by at least one day, the standard deviation could be thought of as describing measurement variation of the laboratory. More precisely, the standard deviation is seen to reflect that contribution to variation in measured values that is due to the measurement process. An alternative procedure would be measurement of the standard sample on the same device throughout. If this were the case, the standard deviation would be expected to be smaller. However, if product M is routinely measured on any device, such an estimate would not accurately reflect the amount of variation contributed by the measurement process.

Figure 7.1 Control charts for purity of product



Another practice for estimating measurement variation is to run 30 or more standard samples during the same assay. In other words, the sample preparation for the 30 samples would be made at the same time and the amount of product M in the samples would be reported. There would be little justification for placing these numbers on control charts, as they are not spread out over time. Nevertheless, it is possible to calculate the sample standard deviation. This standard deviation provides an estimate of within-assay variation for one assay only. Nothing could be learned about the stability of within-assay variation over time, nor would the effect of different operators or devices be captured by the sample standard deviation. This practice of estimating within-assay variation might be useful for understanding sources of variation in the measurement process, but it clearly cannot be used as a description of measurement variation.

The 30 measurement values in Table 7.1 represent important evidence on the performance of the measurement process over a little less than two months' time. Good laboratory practice would call for analyzing the standard sample frequently, if not with every assay, since the variation in measurement values may not remain stable. New operators or changes in equipment, reagents or procedures could cause changes in the variation. Therefore, the practice of continuing to monitor measurement variation is crucial for establishing reliable measurement.

The above analysis has only described the current state of the measurement process in the given example. Whether the variation in this process is small enough will depend on what is required in terms of the quality of output. With a view to these requirements the following section outlines techniques for assessing the adequacy of a measurement process.

7.2 Characterizing a measurement process

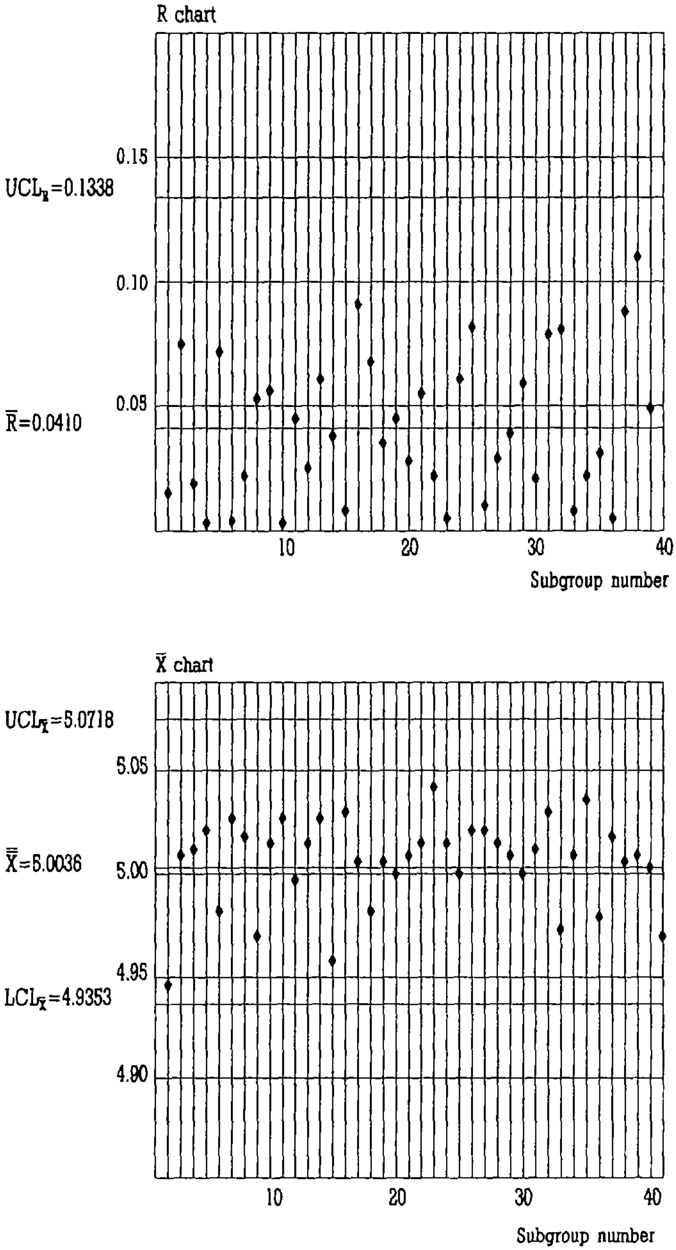
A natural starting-point for characterizing a measurement process is – as in other cases – an examination of the sources of measurement variation. In establishing a list of possible sources of such variation, the conditions under which measurement will actually be made have to be considered. In the example of the previous section, measurement in the laboratory was carried out by several operators using several devices. Reagents used in the assay would change over time; in addition, the device itself was subject to changes. A characterization of the measurement process in hand therefore required taking into account these sources.

To give another example, measurement of the weight of a bar of hand soap can be considered. For the checking of the scale used in measurement, a standard weight of 5 ounces is purchased. Twice in each shift, the standard is weighed on the scale which is also used for weighing the bars of soap. The measurement values recorded over the past 40 shifts are presented in **Table 7.2**. These data have also been used to construct the charts in **Figure 7.2** The fact that the range chart is in control, indicates that the variation in the measured weights within a shift remains consistent across the duration of the study. Since the average chart is also in control, the average weight read on the scale appears to be consistent over time.

Table 7.2 Measured weights of 5-ounce standard

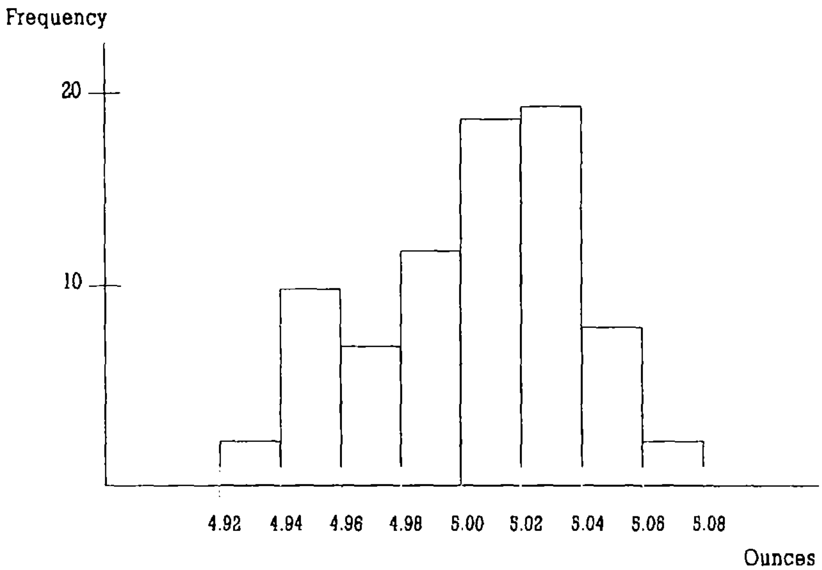
Subgroup	X1	X2	R	$\bar{X}$
1	4.948	4.943	.005	4.9455
2	4.971	5.046	.075	5.0085
3	5.019	5.001	.018	5.0100
4	5.018	5.018	0	5.0180
5	4.942	5.016	.074	4.9790
6	5.029	5.025	.004	5.0270
7	5.025	5.002	.023	5.0135
8	4.995	4.942	.053	4.9685
9	5.041	4.985	.056	5.0130
10	5.023	5.026	.003	5.0245
11	5.016	4.970	.046	4.9930
12	4.999	5.024	.025	5.0115
13	4.994	5.055	.061	5.0245
14	4.974	4.937	.037	4.9555
15	5.022	5.031	.009	5.0265
16	5.051	4.959	.092	5.0050
17	5.014	4.946	.068	4.9800
18	5.019	4.987	.032	5.0030
19	4.975	5.022	.047	4.9985
20	5.021	4.993	.028	5.0070
21	4.985	5.039	.054	5.0120
22	5.029	5.050	.021	5.0395
23	5.017	5.012	.005	5.0145
24	5.044	5.020	.024	5.0320
25	5.059	4.975	.084	5.0170
26	5.014	5.024	.010	5.0190
27	5.028	5.000	.028	5.0140
28	5.028	4.990	.038	5.0090
29	5.026	4.967	.059	4.9965
30	5.021	5.001	.020	5.0110
31	5.066	4.987	.079	5.0265
32	5.009	4.927	.082	4.9680
33	5.009	5.001	.008	5.0050
34	5.028	4.966	.062	4.9970
35	4.959	4.990	.031	4.9745
36	5.019	5.013	.006	5.0160
37	5.048	4.959	.089	5.0035
38	5.061	4.951	.110	5.0060
39	5.021	4.982	.039	5.0015
40	4.951	4.984	.033	4.9675

Figure 7.2 Control charts for weights of standard



Since the weighing of the standard weight portrays a stable measurement process, a histogram can be constructed on the basis of the 80 measurement values of Table 7.2. This histogram is shown in **Figure 7.3**. It provides a visual representation of the variation that results from weighing the same piece of material (a five-ounce standard) repeatedly. Of particular interest in this context is the centre or average of the distribution of values as well as its spread or range. In the characterization of a measurement process the concept of **accuracy** is used to indicate how closely on average the measurement process delivers the value of a known standard. By contrast, the concept of **precision** refers to the size of the variation that is characteristic of a measurement process.

Figure 7.3 Measurements of standard weights



7.2.1 Precision of a measurement process

Figure 7.4 contains histograms from four different measurement processes. For each of these processes, a five-ounce standard was weighed repeatedly over different time periods and under a variety of conditions. The measurement processes were found to have stable variation and average. Histograms A and B show that the underlying measurement processes have less variation than those which generated the values for histograms C and D. Hence, measurement processes A and B are said to be more precise than processes C and D. In general the precision of a measurement process refers to the amount of variation which repeated measurement of the same unit of a product or material produces.

In the measurement underlying the histograms of Figure 7.4, only a single standard weighing five ounces was used. Consequently, the statements about precision of the measurement processes under study should be qualified. Precision in the present case has only been determined for the weight of five ounces. It is perfectly possible that measurement process A exhibits significantly larger variation when the object measured weighs seven ounces instead of five. Any information derived from Figure 7.4 about the precision of measurement processes refers only to a weight of five ounces. Thus, if the weight of soap varies over a wide range, the above results on precision of the measurement process become inapplicable. In the case of varying weights of output, precision of the measurement process would have to be evaluated at various levels of weights. Since measurement precision may change with changing product values, weights used for standards should be selected with care. A reasonable choice would be to use a standard close to the target value of the production process.

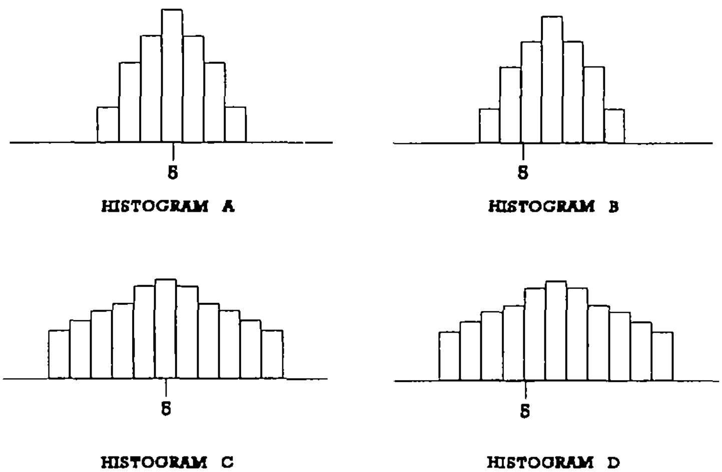
The data of Table 7.2 resulted in a range chart which is in control with an average range of 0.04095. Thus, the precision of the process can be estimated by use of an estimate for the standard deviation. For the measurement process in the present example, the precision is estimated by:

$$\hat{\sigma}_e = \frac{\overline{R}}{d_2} = \frac{.04095}{1.128} = .03630$$

7.2.2. Accuracy of a measurement process

The four histograms of Figure 7.4 also illustrate the concept of accuracy. Histograms A and C both are centred at five ounces, the actual weight of the standard measured. Since the average delivered by these two measurement processes is almost the same as the quantity measured, measurement processes A and C are said to be accurate. More generally, accuracy of a measurement process refers to its capability to deliver, on average, the recognized value of a standard. By contrast to processes A and C, the measurement processes underlying histograms B and D are not accurate, but biased. This does not indicate however, that these measurement processes cannot be used for evaluating product characteristics. The reason is that if the bias is understood, the measured values can be adjusted . Just as in the case of precision of a measurement process, care should also be taken when extrapolating accuracy from the case of measuring one value to that of measuring a whole range of values.

Figure 7.4 Measurement processes



Here again, the accuracy of a measurement process over a range of possible production values can only be determined by actually investigating the measurement process over the indicated range of values.

The centre-line in the average chart of the data in Table 7.2 provides an estimate of the average value that the measurement process produces when measuring a five-ounce standard. Since this average is 5.00355, the measurement process is considered to be accurate.

7.3 The effect of measurement variation on process study

If the amount of measurement variation is large, both the analysis and the improvement of a given process become difficult. An examination of the relationship between measurement variation and the variation measured on a process illustrates these difficulties. The respective variances are related in the following way:

$$\sigma_m^2 = \sigma_p^2 + \sigma_e^2$$

where

σ_m^2 is the variance of values measured on a production process

σ_p^2 is the variance of the output of the production process

and

σ_e^2 is the variance of the measurement process.

Thus, the above equation states that the variance of values measured on output from a production process is equal to the sum of the actual variance of the output and the variance of the measurement process.

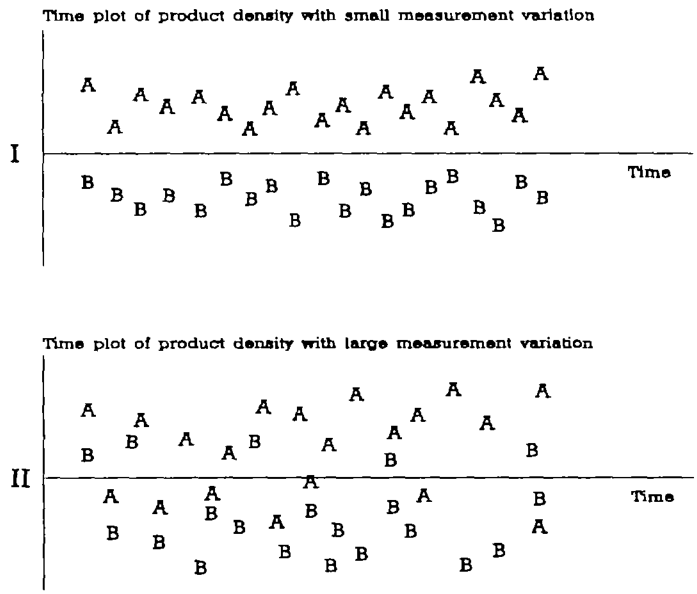
Figure 7.5 provides a graphical illustration of how large measurement variation can hinder process study. Plot I of Figure 7.5 is a time plot of the density of a plastic part produced at a plant. For the case of the first plot, measurement variation, σ_e^2, is assessed as being small.

In the production of the considered part, two sources of raw material are used. The letter A is used to plot the density of a part produced from raw material A and B for a part produced from raw material B. The difference between raw materials is readily apparent from plot I.

Although both processes appear to be stable, the average densities pertaining to the two materials clearly differ. Hence, the differences between the two raw materials are a source of variation in density of the part produced at the plant.

Plot II of Figure 7.5 is similar to the previous plot where, however, the process underlying measurement of densities is known to have large variation. From plot II it can be seen that the difference in average densities of parts produced from the two different types of raw material is masked by measurement variation: the difference in average density witnessed by plot I is therefore not readily detected in plot II.

Figure 7.5 Measurement variation



Since the magnitude of measurement variation impacts the usefulness of information about a process, criteria are needed to judge the adequacy of a measurement process in view of its variation. Two methods can be suggested in this connection. The first one is to calculate the percentage of variation in measured product output that is due to measurement variation. The second one involves the use of control charts to describe the suitability of a measurement process in relation to the need to discriminate between different values of product output.

7.3.1 Measurement variation as a percentage of measured output variation

In Section 7.1 a method for studying the process underlying the measurement of the purity of a product was described. An examination of part of the data led to the conclusion that the measurement process was stable over time and an estimate of 0.62 was obtained for the measurement standard deviation. The same measurement process supplies data on production regarding the purity levels of every batch produced. These data are routinely plotted and examined. An analysis of the moving-range and average charts constructed from these data shows that the process is in statistical control with an estimated process average of 94.4 and an estimate for the process standard deviation of 1.6. This standard deviation captures both the variation in purity levels as well as measurement variation. Using the notation introduced earlier, the above information on process and measurement variation can be summarized as follows:

$$\sigma_e = 0.62 \qquad \sigma_e^2 = 0.3844$$

$$\sigma_m = 1.60 \qquad \sigma_m^2 = 2.56$$

The variance of the measured purity, σ_m^2, is the sum of the variance of the actual product plus the measurement variance, σ_e^2. In addition, the following relationship can be considered

$$\frac{\sigma_e^2}{\sigma_m^2} \times 100\% = \frac{0.3844}{2.56} \times 100\% = 15.0\%$$

This relationship indicates that 15 per cent of the variation observed in measured purity levels can be attributed to measurement variation. An immediate question is about whether this percentage is high or low. Although there is no straightforward answer to this question, a value for this percentage of 10 or less is generally accepted as an indication of the measurement process being adequate for the study of process variation. Since for the process under study measurement variation accounts for 15 per cent of the total variation observed, the adequacy of the measurement process is to be questioned and the sources of measurement variation need to be studied.

Since measurement variation includes instrument-to-instrument variation, it might be useful to set aside one instrument for continually performing this assay and work to further reduce the variation contributed by that one instrument. Nevertheless, work directed at the reduction of measurement variation might prove unsuccessful. In this case, laboratory personnel may want to reconfigure the process for taking measurement by routinely performing multiple assays on both the standards used as well as on every product sample. The reported levels of purity would then be the average of the multiple readings. If n multiple readings are obtained for each sample, the measurement variance should be reduced by a factor of $\sqrt{n}$. It will not, of course, be sufficient to assume that the measurement variance is decreased by this amount. A re-evaluation of the measurement process which includes collecting multiple readings on every specimen of material needs to be performed.

7.3.2 Using control charts to evaluate the discriminative power of measurement processes

Large measurement variation reduces the potential to evaluate the characteristics of a product and, hence, the process producing this product. One method of evaluating the effects of measurement variation on measured product variation is to determine the percentage of observed product variation that is due to measurement variation. Another method is to use control charts as a means of assessing the potential of the measurement process to discriminate between product outcomes. In an example discussed below, this method was used by a customer and a vendor who disagreed on the measurement of an important quality characteristic. The vendor reported an average and a standard deviation for every outgoing lot of material. When the lot arrived at the customer's plant site, some parts of the incoming material were selected and another assessment of the average and standard deviation of the lot was made. Disagreement arose out of the fact that the difference in the results reported by the vendor and by the customer was at times dramatic. While the actual sources of this difference were not obvious, sampling differences or differences in measurement methods were considered as likely candidates.

In a first step towards reconciling their differences, managers from the two plant sites agreed on conducting a measurement study of the characteristic in question. The study was performed on 20 parts selected at random from a manufactured lot. These 20 parts were measured twice at the vendors' facility on two consecutive days. After the two measurement values had been obtained on the 20 parts, the parts were shipped to the customer's facility and there passed through the inspection procedure on each of two consecutive days. The results of the outlined measurement study are reported in **Table 7.3**. This table presents the range of the measurement values of one given part for both the vendor's and the customer's measurement as well as the average measurement value obtained by the vendor and by the customer. Before conducting a statistical analysis of the data, it will be instructive to consider the sources of variation suggested by the ranges and averages.

The ranges reported for the measurement values obtained by the vendor are an indication of the precision of the vendor's measurement process. Each one of these ranges describes the difference observed from one day to the next when the same part is measured. Thus, by constructing a range chart from these ranges, the stability of measurement variation across parts can be evaluated. If parts of different size resulted in different measurement variation, then this should be taken as a hint of a special cause of variation. In other words, the range chart allows for checking the stability of the measurement process across parts, but not over time. In applying these results to the vendor's measurement process, stability of its variation over time cannot be judged. In order to make such a judgement the vendor has to monitor his measurement process over time and across a variety of conditions. Similar information on the customer's measurement variation is supplied in Table 7.3 and Figure 7.6 presents the two range charts constructed from these data.

Both range charts in Figure 7.6 are in control. In other words, both the vendor's and customer's measurement processes appear to deliver consistent measurement variation across the 20 parts used in the study. However, even though both processes are consistent, differences among the measurement processes become immediately apparent from the two charts.

Table 7.3 Comparison of two measurement processes

	Vendor's process				Customer's process			
Part	Measurement		Range	Average	Measurement		Range	Average
1	28.5	32.9	4.4	30.70	34.7	36.1	1.4	35.40
2	32.5	28.1	4.4	30.30	32.3	30.7	1.6	31.50
3	28.0	22.8	5.2	25.40	29.9	30.8	0.9	30.35
4	41.4	50.5	9.1	45.95	43.9	42.7	1.2	43.30
5	54.6	54.7	0.1	54.65	47.5	50.5	3.0	49.00
6	37.5	46.8	9.3	42.15	45.7	43.5	2.2	44.60
7	36.8	38.3	1.5	37.55	44.8	44.1	0.7	44.45
8	32.4	40.3	7.9	36.35	36.7	36.5	0.2	36.60
9	41.0	40.4	0.6	40.70	41.1	41.3	0.2	41.20
10	37.0	40.4	3.4	38.70	33.9	36.3	2.4	35.10
11	53.5	45.4	8.1	49.45	40.7	42.0	1.3	41.35
12	35.3	34.5	0.8	34.90	36.3	36.3	0.0	36.30
13	34.2	38.7	4.5	36.45	39.7	38.7	1.0	39.20
14	49.9	46.2	3.7	48.05	45.0	46.5	1.5	45.75
15	46.4	39.7	6.7	43.05	48.9	45.9	3.0	47.40
16	37.0	37.3	0.3	37.15	36.3	37.1	0.8	36.70
17	46.9	45.9	1.0	46.40	47.0	46.5	0.5	46.75
18	39.8	25.9	13.9	32.85	31.3	30.0	1.3	30.65
19	46.3	42.3	4.0	44.30	46.4	43.4	3.0	44.90
20	35.5	41.3	5.8	38.40	41.2	40.0	1.2	40.60

The average range for the vendor's measurement process is considerably larger than that of the customer's. An examination of the respective average charts for the vendor and the customer provides a way of evaluating the effect which the additional measurement variation from the vendor's measurement process has on the potential to evaluate product characteristics.

Another point of interest is that of sources of variation affecting the average charts. Each of the averages calculated by the vendor results from two instances of measurement made on one of the 20 parts. Although some of the differences among the 20 averages are due to variation from the measurement process, it would be expected that the averages exhibit genuine variation among the 20 parts. In fact, if the process applied to measure the parts had negligible variation, then all the variability in the averages would have to be ascribed to part-to-part differences.

In constructing the average charts of **Figure** 7.7, the average range in the respective range charts has been used to calculate upper and lower control limits. Thus, the only source of variation taken into account for these limits is measurement variation. This implies that any variation in addition to that of measurement should result in an out-of-control signal on the average chart. Since average values are subject to part-to part variation, as well as to measurement variation, average charts would be expected to be out of control.

An examination of the two average charts reveals that both of them are out of control. The conclusion reached on this basis is that both the vendor's and the customer's measurement processes are capable of discriminating between parts. Since only measurement variation was used to construct the control limits on the average charts, differences between parts were observed on the average chart; in fact, the part-to-part differences were picked up as a special cause on the average charts. Again, it is useful to note the differences between the two average charts. The average chart representative of the vendor's study does not have as many points outside the control limits as does that of the customer's study; nor are the points as far outside the control limits on the vendor's chart. From this qualitative difference it can be concluded that, because of the smaller variation experienced by the customer's measurement process, this process has a greater potential to discriminate among parts than has the vendor's measurement process.

Figure 7.6 Range charts for two measurement processes

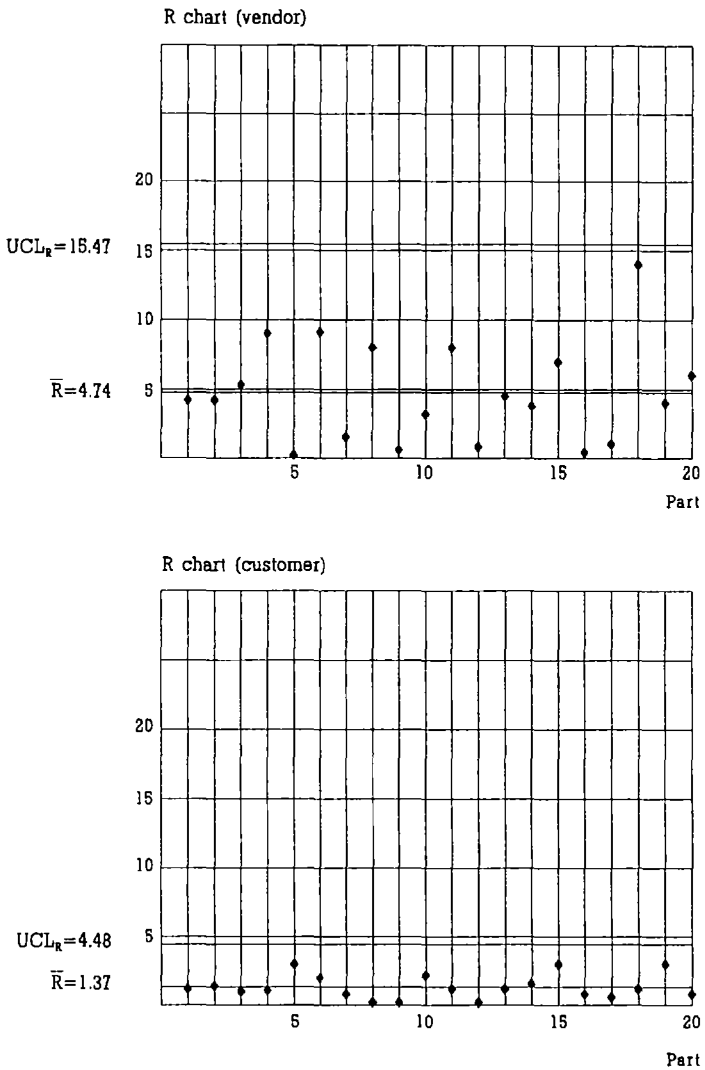
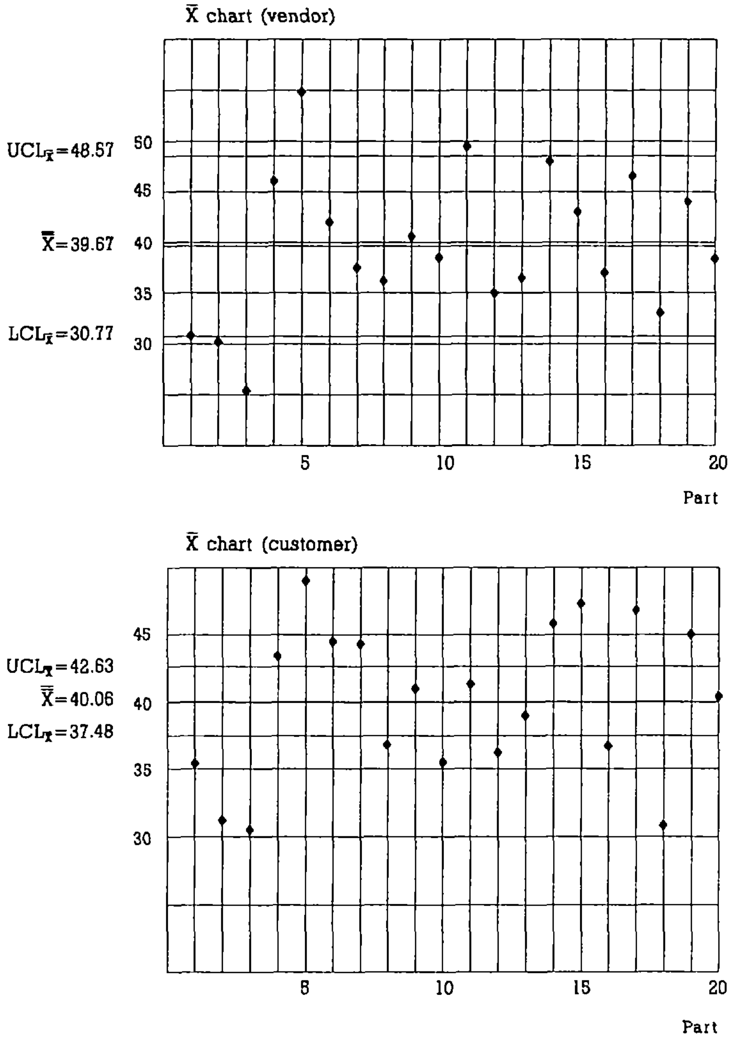


Figure 7.7 Charts for part averages



7.4 Concluding remarks on the study of measurement processes

Investigation of the process should begin with a description of how the process operates and also of potential sources of variation in measured outcomes. Again, process flow charts and cause-and-effect diagrams provide useful tools for performing this task. Once information about the sources of variation pertinent to a measurement process has been obtained, a data collection plan can be devised for gathering evidence on the operation of the measurement process. For example, in the study of the measurement of purity levels in Section 7.1, the data collected about the measurement process attempted to capture the effect that different operators, different measuring devices, different reagents, and also equipment changes might have on the measurement of purity in a herbicide. Data collection should provide the basis for evaluating the stability of the variation in measurement values over time and - if the measurement process is stable - to assess the precision, accuracy and discriminative power of this process.

The studies of measurement processes in the present chapter have focused on the design of measurement studies. It has to be noted, however, that such studies need to be performed not only once, but evaluation of a given measurement process has to become a continuous activity. As materials, personnel, techniques etc. change over time, the measurement process under study will tend to change too. Therefore, this process should also be continually monitored.

ANNEXES

A. STATISTICAL ANNEX

The following sections outline some statistical concepts and arguments underlying the methods described in the main text. (Detailed and comprehensive discussions of the statistical background to process control can be found in most of the statistical texts listed in the bibliography.)

1. Populations, samples and statistics

In statistical analysis the totality of items under consideration is called a **population.** In the examples of the present text a population is a whole lot or a process in its entirety. Since it is usually impossible or uneconomical to gather information about a total population, a **sample** of one or more items is taken for the purpose to obtain information on the population. In other words, a sample is used for estimating characteristics of the entire population and should therefore be taken in such a way as to reflect these characteristics faithfully. The commonly used approach is that of random sampling, where there is equal probability for any member of the population to be chosen.

Data used in process analysis are obtained by measuring the characteristics of interest for each member of a sample. From these data inference can be drawn about the characteristics of the whole sample. This is usually achieved by use of a so-called **statistic,** i.e., a value calculated from the sample or more precisely, a function of sample observations. Statistical theory provides the means to infer the value of a (constant) population parameter from the sample-dependent value of a statistic.

2. Distributions

The notion of distribution is central to statistical inference also in the area of process analysis. In the main text it had been shown how **frequency distributions** can be constructed on the basis of sample observations. Statistical theory provides the tool of **theoretical or probability distribution** for more convenient analysis by way of mathematical equations. A central task in the analysis of a sample is to describe in an approximate way the distribution of actual observations in terms of a theoretical distribution. In the process analysis described presently three types of distributions provide the mathematical basis for most assessments and predictions: the binomial distribution, the Poisson distribution and the normal distribution.

Binomial distribution

This distribution deals with a series of events each one of which has only two possible outcomes. In process control the typical application is that of defectives in population and sample: Based on the number of defectives found in a sample, the binomial distribution allows to infer the number of defectives in the total population.

Poisson distribution

This distribution deals with the number of times a random event occurs within a given length area, volume or period of time. In contrast to the binomial distribution, non-occurrence of the above event is not considered here. The Poisson distribution applies in cases where the possibility of a defect is continuous, but where only a few defects actually occur.

Normal distribution

This distribution deals with continuous variables, not with discrete events. It is concerned with the way in which measurements cluster around a central value of maximum frequency. The greater the deviation of a measurement from the central value, the less likely is the occurrence of this measurement. The result is a bell-shaped distribution (as shown in Fig. 2.12) which can be described by its mean value (central tendency) and its standard deviation (dispersion about the mean).

3. More on the statistics of control chart analysis

The above distributions provide the basis for statistical inference on the behaviour of processes. In a first step sample observations (measurements) are used to estimate the parameters of the underlying probability distribution under the assumption of process stability. A second step consists in confronting the empirical distribution of observations with the derived theoretical distribution. This provides for a test of the hypothesis of process stability: If the empirical distribution of observed values is compatible with the probability laws of the derived theoretical distribution, the assumption of a stable process is maintained. Otherwise it is rejected and a change in process parameters suspected. In the language of control chart analysis, underlying distributions are identified under the assumption that the studied process is subject only to common or chance causes of variation in process outcomes. Significant deviations of single observations or a group of observations from the characteristics of the assumed distribution are taken as an indication of special or assignable causes of variation.

The rules usually followed in the assessment of control charts can best be made plausible with reference to the normal distribution. The bell-shaped curve of this distribution (as depicted in Fig. 2.12 on p. 40) can be described in probability terms in the following way:

(i) The distribution is symmetrical, 50 percent of measurements fall on either side of the central value.

(ii) On either side of the central value, only 16 percent fall outside 1 standard deviation,

(iii) 2 percent outside 2 standard deviations and

(iv) 0.1 percent outside 3 standard deviations from the central value.

From (iv) can be derived the 3-sigma rule applied throughout the present text with regard to control limits. If upper and lower control limits are set at a distance of 3 standard deviations from the central value, there is a chance of only 1 in 1000 for a measurement value to be found outside control limits by chance. In 999 out of thousand cases this must be the result of a change in process parameters, i.e., the loss of process stability.

For the calculation of the values of control limits first the distribution of the statistic used has to be determined. On this basis, 3-sigma limits can be calculated where constants that are characteristic of the identified distribution can be used. Values of these constants are usually tabulated (as shown in Table 5.2) and can be found in most texts on statistical process control.

The discussion of control charts in the main text concentrated on 3-sigma limits as the major analytic criterion. In addition to these control limits a few rules of thumb for detecting non-randomness in charts were stated in Section 2.6. On the basis of the previous outline of statistical arguments some of these rules can be given a precise meaning and a few more rules can be established.

(1) A process is regarded as out of control when a 'run' or series of successive observations all fall on one side of the 'centre line' of the chart. If this centre line is taken to be the median (the middle point in the ordered set of observation points on the chart), the occurrence of runs can be given an exact probabilistic interpretation. Thus, for example, the probability to observe among 20 points a run of at least seven points on one side of the median line is 5 percent. From this derives the fairly strict 'rule of seven' (cited in Section 2.6) which under all circumstances takes a run of seven consecutive points as an indication of abnormality. Similar rules state that a process should be considered out of control when: (i) at least 10 of 11 consecutive points fall on one side of the centre line; (ii) at least 12 of 14 consecutive points are on one side of the centre line; (iii) at least 16 out of 20 consecutive points meet this criterion.

(2) Patterns that exhibit a clear trend are other signals for the process being out of control. Should a series of consecutive observations form a monotonically increasing or decreasing portion of the total curve of points, an abnormality in the process has to be assumed. For reasons similar to those stated above a length of seven for such a trend is considered critical.

(3) Periodicity is said to occur when repeatedly an up-and-down cycle is observed over similar intervals on the chart. Unlike for the cases of runs or trends, there is no simple rule for assessing periodicity in a control chart. The best strategy here is to track the movement of data points over time. If a periodic pattern is observed on a regular basis, this can be taken as evidence of non-random behaviour.

(4) Further hints to the existence of special causes of variation are found in the clustering of observations around control limits or the centre line. The term used for such patterns is 'hugging' (of control limits or centre line) and for the analysis of such cases additional boundaries have to be introduced into the chart.

(5) Similar to the use of control limits, that of so-called warning limits is often suggested on the basis of property (iii) of the normal distribution stated above: If observations fall outside the 2-sigma limits, this could be taken as a warning of possible problems. More precisely, in a control chart it is about equally unlikely to find two out of three successive points outside the 2-sigma lines (or four out of five successive points outside the 1-sigma lines) as to have one point outside the 3-sigma limits.

4. Beyond control charts

The main text has focused on the important tool of control chart for an introductory demonstration of the use of statistics for process control and improvement. In this context the most frequently used types of control charts were introduced and discussed extensively. These charts are essentially those that Dr. Walter A. Shewhart proposed as tools to serve three major goals of the study and control of repetitive processes: (i) to define the goal(s) or standard(s) that should be attained; (ii) to serve as an instrument to attain those goals; (iii) to provide a tool for judging whether the goal(s) has(have) been reached. Thus, control charts can be used in specification, production and inspection of industrial processes with the added advantage of integrating these three phases.

Although the discussion of the standard 'Shewhart' set of control charts touched upon most of the important aspects of the tool, it was not exhaustive in that it left aside a number of charts which are commonly employed nowadays and cover special areas of use. In particular, there are many special types of control chart for variables data like the cumulative sum (CUSUM) chart, the exponentially weighted moving average (EWMA) chart, the modified control chart, and the acceptance control chart. Discussions of these charts are found in any of the more comprehensive texts on statistical process control.

Besides the broad field of control chart analysis other statistical methods are often used in industrial research. Among theses methods are tests of various hypotheses about proportions, means and variances, tests of normality, and the analysis of variance. Another set of methods has to do with examining the relationship between two or more variables where correlation and simple or multiple regression methods have to be used. The case of two variables is important, for example, in the context of examining partial relationships as described in the framework of a cause-and-effect (Ishikawa) diagram. Here the simple tool of a scatter diagram (scatter plot) which pictures the studied relationship in two dimensions can provide the starting point for a formal analysis by way of correlation or regression methods. Again standard texts on statistical quality and process control can be consulted on the broader range of methods applied in industry statistics.

Finally, there are a number of subject areas closely related to the application of the methods discussed in the present text or in more advanced presentations. One of them is the wide field of sampling techniques which have to do with the observation of a portion of a population with the objective of obtaining information about the population itself. Another related field is that of the design of experiments which are intended to analyze the effects of given factors on observed outcomes.

B. PRACTICE PROBLEMS

Each of the following practice problems is assigned to that chapter in the text that covers the method(s) required to solve it.

Problem 1 (Chapter 3)

At a food packaging plant, a recent increase in the number of customer claims concerning defective cans has prompted your investigation into the situation. An initial concern was whether or not the inspection process is stable and predictable. You have been informed that operational definitions have been developed and an inspection training program has been implemented in the recent past. Assuming, therefore, that the inspection process is consistent, your attention turned to the packaging process. Knowing that the sealed metal containers pass through a final inspection, where they are checked for proper can height, label application, vacuum, and other surface characteristics, you have requested data on the proportion of defective cans found in final inspection. You are provided with the following:

Fractions defective in 22 samples of 1000 cans each
(Total number of defective units: 67)

.003 .004 .008 .006 .003 .006 .004 .006 .002
.005 .004 .003 .004 .001 .002 .001 .002 .000
.001 .001 .000 .001

a. Construct an appropriate control chart for the data.

b. Comment on the method by which these data were obtained.

c. In order to determine the current status of the process and to analyze the possible causes for the increase in defective cans, what will be your course of action?

Problem 2 (Chapter 3)

The shift 2 supervisor has expressed his belief that the number of defective cans are higher in shift 1 than in his own shift. In order to check this claim, you request that, on each of 26 successive shifts, 700 cans are to be inspected, and the number of defective cans in each sample recorded. You are later presented with the following data:

Sample	*Shift*	*Number defective*	*Sample*	*Shift*	*Number defective*
1	1	4	14	2	2
2	2	0	15	1	1
3	1	2	16	2	2
4	2	1	17	1	3
5	1	3	18	2	3
6	2	1	19	1	2
7	1	2	20	2	4
8	2	0	21	1	2
9	1	2	22	2	3
10	2	0	23	1	3
11	1	2	24	2	1
12	2	1	25	1	2
13	1	7	26	2	2

a. Draw a control chart.

b. Based on your analysis of this chart, what would your conclusions be concerning the stability of the process?

c. Construct separate sets of control charts for the two different shifts.

d. As a manager, how would you respond to the shift 2 supervisor's claim?

e. Are there any reasons that make you sceptical about the data and, consequently, about the information provided in these control charts?

Problem 3 (Chapter 3)

In your company, individual work centre performance is measured on the basis of whether or not processes are kept in control. After your area has manufactured crankshafts, they are sent to the line where they are used in engine assemblies. Due to several complaints you have received from the line concerning defective crankshafts, you decide to investigate your process for control through use of a control chart. The data are obtained by 100% visual inspection of 24 lots of varying sizes. Out of a total of 5,544 crankshafts inspected, 367 of the units are found to be nonconforming. Data for two particular points on the chart are given below:

Lot	*Lot size*	*Number nonconforming*
22	200	23
23	275	5

a. Calculate the values which would be plotted on the control chart and the control limits for these two lots.

b. Discuss the impacts that methods of performance appraisal can have on the use of data to improve systems.

Problem 4 (Chapter 4)

In an electronics manufacturing facility, radios are checked for nonconformities after the assembly process is complete. For some time, the process has been in control at an average of six nonconformities per radio.

a. If the process were to be monitored by inspecting three radios from time to time and recording the total number of nonconformities per group, what type of control chart should be used?

b. What should the centre line and control limits be for the chart?

Problem 5 (Chapter 4)

In a potato chip plant, after the chips are drained, they enter into one of three lines where seasonings are applied. Line 1 applies seasoning to flat chips, while the other two lines season the "angled" chips. After the chips are seasoned, they are sampled by one of three inspectors on that shift. The inspectors are responsible for sampling chips out of each seasoned batch and determining whether the batches are consistent with respect to chip presentation and amount of seasoning. If the inspectors find that a series of several batches are inconsistent, they can stop the line for adjustment. The line can also stop because of machine breakdowns, change of seasonings, shortage of chips at the start of the seasoning lines, etc. You are on the work team which is investigating the stops in the production line.

In order to help determine where the specific causes of the line stops may be, the flow diagram of the process has been developed. In addition to the flow diagram, you have suggested that a cause-and-effect diagram would also help the team to identify possible causes of line stops. Before constructing the cause-and-effect diagram, you questioned several people on the lines. The following are comments that were made:

"As I see it, the line stops can't be prevented until someone changes the way the system is run. There are just too many different people who can stop the line."

"The inspectors are the problem. There's one inspector who calls for a line stop for every little thing."

"If we didn't always have to wait for the chips to be delivered to the line, the situation would not be so bad."

"I can tell you where the problem is: It's with maintenance. These machines are just patched so we can keep running. If they would fix them right, it would make a big difference in the long run."

a. The following data report the number of line stops per day (24 hours) for each of the three lines. Construct a control chart to determine whether there exists a difference in the number of line stops between the three lines. (Put individual sets of control limits on the chart for the different lines.)

Day	*Line 1*	*Line 2*	*Line 3*
1	8	10	14
2	11	9	13
3	7	14	13
4	9	11	10
5	8	16	12
6	11	7	8
7	5	13	13
8	8	16	15

Day	*Line 1*	*Line 2*	*Line 3*
9	7	15	14
10	9	13	13
11	14	13	9
12	6	11	14
13	9	10	10
14	3	8	12

b. Your team has asked you to make recommendations on what the next step in the improvement effort should be. What will your recommendations be?

Problem 6 (Chapter 4)

A company manufacturing oilcloth determines all cost estimates and prices on the basis of 100 square yards of oilcloth. The following data are obtained from inspection:

Lot number	*Square yards inspected*	*Total number of defects*
1	200	5
2	250	7
3	100	3
4	90	2
5	120	4
6	80	1

Construct the appropriate preliminary control chart(s) for these data.

Problem 7 (Chapter 5)

A chemical product is produced in large batches. An important characteristic of the product is the level of active chlorine in the material. It is suspected that there exist two major sources of variation in the process which affect the level of active chlorine in the product. One possible source of variation is that the mix of ingredients is not the same from batch to batch. Another potential source is that the batches may not be thoroughly mixed. In order to study the active chlorine levels, three samples from each of 30 successive batches are taken. The levels of active chlorine in the three samples are measured. The range and the average of the three samples are calculated and plotted on control charts for ranges and means. The sum of the sample means is 1.648 and the sum of the sample ranges is 0.151.

a. Draw the range and average charts.

b. Compute the control limits for the range chart and, if appropriate, the average chart.

c. Which of the two major sources of variation is reflected in the range values?

d. Which source of variation would tend to cause the range of variation in the subgroup averages to be larger than that predicted by the control limits on the average chart?

e. From visual inspection of the charts, which of the two sources of variation appears to be causing most of the product variation?

Problem 8 (Chapter 5)

In the production of steel bars, square billet stock is heated and rolled. After the bars are cut to length and straightened, they are shipped to one of two converters where they are sized. Your plant requires the following size specifications to be met by the two converters: 8.75 ± 0.025. After the steel bars are sized at the converters, they are returned to your plant, where they are stored in lots. The lots generally consist of a mixture of the steel bars sized at the two converters. Prior to shipping to your customers, a final inspection is performed where a sample of bars is inspected for defects and size data is collected on the sample. Because of recent claims by customers that the steel bars are not meeting the size specifications, 10 bars are randomly selected from each of 20 lots and measured.

Measurement											
1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*X-bar*	*Range*
8.760	8.749	8.743	8.793	8.772	8.779	8.743	8.741	8.779	8.775	8.7634	0.052
8.780	8.781	8.743	8.757	8.760	8.778	8.760	8.732	8.767	8.745	8.7600	0.049
8.757	8.779	8.753	8.768	8.722	8.786	8.770	8.775	8.779	8.774	8.7663	0.064
8.779	8.769	8.730	8.771.	8.774	8.774	8.758	8.752	8.746	8.779	8.7632	0.049
8.772	8.749	8.753	8.755.	8.784	8.775	8.789	8.778	8.784	8.751	8.7690	0.040
8.764	8.772	8.755	8.760	8.787	8.766	8.785	8.783	8.752	8.737	8.7661	0.050
8.738	8.774	8.783	8.781	8.762	8.746	8.777	8.780	8.771	8.776	8.7688	0.045
8.785	8.736	8.782	8.787	8.760	8.756	8.737	8.762	8.745	8.754	8.7604	0.051
8.782	8.785	8.775	8.761	8.781	8.777	8.782	8.751	8.750	8.744	8.7688	0.041
8.725	8.788	8.776	8.778	8.779	8.740	8.760	8.749	8.765	8.776	8.7636	0.063
8.784	8.779	8.782	8.728	8.758	8.741	8.750	8.756	8.757	8.780	8.7615	0.056
8.778	8.779	8.783	8.775	8.747	8.746	8.788	8.782	8.779	8.776	8.7733	0.042
8.781	8.745	8.748	8.748	8.742	8.761	8.779	8.765	8.780	8.776	8.7625	0.039
8.794	8.743	8.768	8.757	8.747	8.789	8.772	8.774	8.799	8.775	8.7718	0.056
8.759	8.745	8.739	8.755	8.747	8.790	8.774	8.788	8.766	8.785	8.7648	0.051
8.759	8.758	8.773	8.750	8.740	8.783	8.787	8.768	8.773	8.783	8.7674	0.047
8.774	8.751	8.737	8.732	8.749	8.740	8.754	8.773	8.777	8.775	8.7562	0.045
8.770	8.770	8.755	8.763	8.773	8.767	8.754	8.780	8.778	8.741	8.7651	0.039
8.750	8.759	8.739	8.752	8.759	8.791	8.783	8.773	8.755	8.739	8.7600	0.052
8.752	8.757	8.784	8.781	8.772	8.785	8.768	8.751	8.755	8.758	8.7663	0.034

a. Construct the appropriate control charts to analyze the stability of the process. Is the variability of the sizing process in control? Is the process average in control?

b. If appropriate, compute an estimate of the process standard deviation. Determine if the process is capable of meeting the specifications.

Problem 9 (Chapter 6)

Oven dried moisture tests are conducted by taking a 12 x 12 sample from each finished roll of paper. Specifications on moisture are 6%±1%. Data from the last 20 rolls are reported below.

a. Using two-item moving range and individuals charts, determine if the process is in control over this time period.

b. Is the process capable of meeting specifications?

Roll number	*X*	*MR*
1	6.4	–
2	4.8	1.6
3	6.7	1.9
4	4.9	1.8
5	6.4	1.5
6	4.2	2.2
7	5.8	1.6
8	6.4	0.6
9	5.9	0.5
10	5.5	0.4
11	6.3	0.8
12	6.4	0.1
13	7.1	0.7
14	5.9	1.2
15	6.1	0.2
16	4.9	1.2
17	5.4	0.5
18	7.1	1.7
19	6.9	0.2
20	5.9	1.0

Problem 10 (Chapter 7)

Bottles of shampoo are filled on an automatic line using a five-head filler. You have been assigned to the team responsible for analyzing the process, determining process capabilities, and recommending ways in which to improve the process. In order to begin a study of the fill weights, you have decided to check the calibration of the scale to determine whether or not the measurement process is stable. Three members of your team have suggested the following different subgrouping methods for this purpose.

1. A subgroup is to consist of the weights of five consecutive bottles from one head.

2. One bottle should be selected from each head. The subgroup consists of four weights of the same bottle from one head.

3. A subgroup should consist of one bottle selected from each of the five heads, and each bottle should be weighed once.

a. What are the sources of within-subgroup variation in the subgrouping methods (1), (2) and (3), respectively?

The team has decided to use the second subgrouping method, whereby a bottle is chosen from one head, and four repeat measurements of the weight are made.

Sample number	*Head*	*Measurement* *1*	*2*	*3*	*4*	*X-bar*	*Range*
1	1	479.17	478.93	479.54	479.12	479.19	0.61
2	2	481.13	481.75	480.97	481.21	481.27	0.78
3	3	475.71	476.24	475.95	475.48	475.85	0.76
4	4	485.37	485.17	484.70	484.89	485.03	0.67

Sample number	Head	Measurement 1	2	3	4	X-bar	Range
5	5	476.80	477.12	477.07	477.10	477.02	0.32
6	1	479.01	479.23	479.12	479.10	479.12	0.22
7	2	481.16	480.83	481.10	481.30	481.10	0.47
8	3	475.82	476.14	476.17	475.93	476.02	0.35
9	4	485.08	485.29	485.52	484.92	485.20	0.60
10	5	477.11	476.68	477.32	477.54	477.16	0.86

b. Construct a range chart from these data.

c. What sources of variation are captured in the range chart?

d. If appropriate, estimate the within-subgroup variation.

e. Construct the average chart based on the above range chart. Discuss the issue of using the value of R-bar to put the limits on the average chart for this subgrouping strategy.

f. Estimate the percentage of total variation that is due to within-subgroup variation.

g. What does the above information tell you about variation in your measurement process?

C. SOLUTIONS TO PRACTICE PROBLEMS

Problem 1

a. p-bar = 0.003045
UCLp = 0.008275
LCLp = none
All points are inside control limits.

b. There is no information available on how the data were collected. Are the data time ordered? Without this information runs tests are inappropriate. There is also no information available on the method of subgrouping. The most easily obtainable data do not always provide sufficient information about the process.

c. Course of further action:

Draw the process flow chart in order to define the process.

Analyze the potential causes of defective cans by a cause-and-effect diagram.

Develop a data collection strategy to provide the desired information.

Problem 2

a. np-bar = 2.12
UCLnp = 6.47
LCLnp = none

b. The process is not stable. There is one observation above the upper control limit.

c. The number of measurements in the sample is 700.

Shift 1:

p-bar = 0.0038
np-bar = 2.6923
UCLnp = 7.6053
LCLnp = none

Shift 2:

p-bar = 0.0022
np-bar = 1.5385
UCLnp = 5.2554
LCLnp = none

d. The process of shift 1 is stable and predictable. The same holds for the process of shift 2. As a consequence, the average proportions of defective cans between the two shifts can be compared. It appears that shift 1 has a lower fraction of defective cans produced.

e. More questions concerning the data:

Who took the measurements?

Is the inspection process consistent between the two shifts?

Are there operational definitions?

Problem 3

a. p-bar = 0.0662

Lot 22:

n = 200
p = 0.1150
UCLp = 0.1189
LCLp = 0.0135
All points are inside control limits.

Lot 23:

n = 275
p = 0.0182
UCLp = 0.1112
LCLp = 0.0212
There are points outside control limits.

b. Methods of performance appraisal can have the following impact on the use of data for system improvement:

1. Possible misrepresentation of data.
2. Data may be used to support individual performance instead of process performance.

Problem 4

a. In this analysis a c chart should be used.

b. The centre line is at the level 18.

UCLc = 30.7279
LCLc = 5.2721

Problem 5

a. A set of c charts based on the inspection unit of 1 day need to be constructed.

Line 1:

c-bar = 8.2143
UCLc = 16.8125
LCLc = none

Line 2:

c-bar = 11.8571
UCLc = 22.8173
LCLc = 1.5269

Line 3:

c-bar = 12.1429
UCLc = 22.5969
LCLc = 1.6889

There are no points outside control limits.

b. The focus should be on reducing the number of stops on lines 2 and 3. Since there are no special causes acting on the process, improvement efforts must be directed at the system.

Problem 6

Using 100 square yards as an inspection unit the following numbers are obtained:

Lot number	*n*	*c*	*u*	*UCLu*	*LCLu*
1	2.00	5	2.50	6.052	–
2	2.50	7	2.80	5.690	–
3	1.00	3	3.00	7.474	–
4	0.90	2	2.22	7.737	–
5	1.20	4	3.33	7.051	–
6	0.80	1	1.25	8.047	–

u-bar = 2.619
UCLu = 7.474
LCLu = none

Problem 7

a,b. Range chart: The centre line is at the level 0.00503

UCLR = 0.0129
LCLR = none

Average chart: The centre line is at the level 0.05493

UCL = 0.0601
LCL = 0.0498

c. Within-batch variation is reflected in the range values.

d. Between-batch variation - the variation due to the ingredients being different between batches - causes the range of variation in subgroup averages to be larger than that predicted by the control limits on the average chart.

e. The variation between batches appears to be causing most of the variation in the process.

Problem 8

a. Range chart: The centre line is at the level 0.048

UCLR = 0.085
LCLR = 0.011

No points are outside control limits.

Average chart: The centre line is at the level 8.765

UCL = 8.780
LCL = 8.750

No points are outside control limits.

b. An estimate of the process standard deveiation is 0.0156

NT = 0.0936
ET = 0.0500
The process is not capable.

Problem 9

a. Calculations for the moving-range chart:

MR-bar = 1.037
UCLMR = 3.387
LCLMR = none
The moving-range chart is in control.

Calculations for the individuals chart:

X-bar = 5.95
UCLX = 8.71
LCLX = 3.19
The individuals chart is in control.

b. An estimate of the standard deviation of X is 0.919.

NT = 5.516
ET = 2
The process is not capable.

Problem 10

a. Sources of within-group variation for

method (1):

within-head variation - lack of homogeneity
bottle-to-bottle variation within the same head
measurement variation

method (2):

measurement variation

method (3):

head-to-head variation
measurement variation
bottle-to-bottle variation

b. Range chart: The centre line is at the level 0.564.
UCLR = 1.287
LCLR = none

c. The range chart captures within-subgroup variation and measurement variation.

d. Since the range chart is in control, it is appropriate to estimate within-subgroup variation. An estimate is 0.274.

e. Average chart:

The centre line is at the level 479.695
UCL = 480.106
LCL = 479.284

When the value of R-bar is used to put control limits on the average chart, only within-subgroup variation is taken into consideration. There are two other sources of variation (head-to-head and bottle-to-bottle) that should be considered as part of the system, but are not captured in the within-subgroup variation.

f. Calculations for measuring between-subgroup variation:

Moving-range chart:

MR-bar(X-bar) = 5.6754
UCL = 18.5417
LCL = none

Average chart:

X-double-bar = 479.695
UCL = 494.7891
LCL = 464.0085

An estimate of the between-batch standard deviation is 5.0295. An estimate of total variation is 25.3711. Therefore the percentage of total variation due to within-subgroup variation is 0.30%.

g. Variation in the measurement process is small compared to total process variation.

D. BIBLIOGRAPHY

I. A Few Suggestions for Further Reading

1. Measurement and Statistical Methods

Burr, I. W. (1976), *Statistical Quality Control Methods,* Marcel Dekker Inc., New York.

Duncan, A. J. (1986), *Quality Control and Industrial Statistics,* Irwin Press, Homewood.

Grant, E. L. and R. S. Leavenworth (1980), *Statistical Quality Control,* McGraw-Hill, New York.

Ishikawa, Kaoru (1982), *Guide to Quality Control,* Asian Productivity Organization, Tokyo.

Kume, Hitoshi (1985), *Statistical Methods for Quality Improvement,* AOTS, Tokyo.

Small, B. B., ed. (1984), *Statistical Quality Control Handbook,* AT&T Technologies, Indianapolis.

Shewhart, W. A. (1980), *Economic Control of Quality of Manufactured Products.* Van Nostrand, New York; republished by ASQC Quality Press, Milwaukee.

Wadsworth, H. K. Stephens and A. Godfrey (1986), *Modern Methods for Quality Control and Improvement,* John Wiley & Sons, New York.

2. Managing Continuous Improvement

Asian Productivity Organization (1972), *Japan Quality Control Circles,* Asian Productivity Organization, Tokyo.

Bhote, Keki R. (1987), *Supplier Management,* American Management Association, New York.

Biech, E. (1994), *TQM for Training,* McGraw-Hill, New York.

Brocka, B. and S. Brocka (1992) , *Quality Management: Implementing the Best Ideas of the Masters.* Irwin Press, Homewood.

Capezio, P. and D. Morehouse (1993), *Taking the Mystery Out of TQM: A Practical Guide to Total Quality Mangement,* Career Press, Hawthorne.

Deming, W. Edwards (1982), *Quality, Productivity and Competitive Position,* MIT Center for Advanced Engineering Study, Cambridge.

Deming, W. Edwards (1986), *Out of the Crisis,* MIT Center for Advanced Engineering Study, Cambridge.

Ernst and Young Quality and Improvement Consulting Group (1992), *Total Quality: A Manager's Guide for the 1990s,* Kogan Page Ltd., London.

Hall, Robert (1987), *Attaining Manufacturing Excellence,* Dow-Jones-Irwin, Homewood.

Kinlaw, D. (1992), *Continuous Improvement and Measurement for Total Quality: A Team-based Approach,* Irwin Press, Homewood.

Mizuno, Shigeru (1988), *Company-Wide Total Quality Control,* Asian Productivity Organization, Tokyo.

Porter, Michael E. (1985), *Competitive Advantage: Creating and Sustaining Superior Performance,* Free Press, New York.

Schmidt, W. and J. Finnegan (1993), *TQManager: A Practical Guide for Managing in a Total Quality Organization,* Jossey-Bass, San Francisco.

Schonberger, Richard J. (1986), *World Class Manufacturing: The Lessons of Simplicity Applied,* Free Press, New York.

Stahl, M. and G. Bounds (eds) (1991), *Competing Globally Through Customer Value,* Quorum Books, New York.

Walker, J. P. (1988), *A Disciplined Approach to Continuous Improvement,* Packard Electric.

II. Selected General References

Abegglen, J. and G. Stalk (1985), *Kaisha: The Japanese Corporation,* Basic Books, New York.

Cole, W. and J. Mogab (1995), *The Economics of Total Quality Management,* Blackwell Publishers, Cambridge.

Drummond, H. (1992), *The Quality Movement: What Total Quality Management Is Really All About,* Nichols Publishing Company, London.

Hayes, R. H., S. C. Wheelwright and K. B. Clark (1988), *Dynamic Manufacturing,* The Free Press, New York.

Juran, J. M. (1974), *Quality Control Handbook,* McGraw-Hill, New York.

Mann, N. R. (1989), *The Keys to Excellence: The Story of the Deming Philosophy*, Prestwick Books, Los Angeles.

Porter, M. (1990), *The Competitive Advantage of Nations*, The Free Press, New York.

Schonberger, R. J. (1982), *Japanese Manufacturing Techniques*, The Free Press, New York.

Suzaki, K. (1987), *The New Manufacturing Challenge*, The Free Press, New York.

Wilson, S. R., R. H. Ballance and J. Pogany (1995), *Beyond Quality: An Agenda for Improving Manufacturing Capabilities in Developing Countries*. Edward Elgar Publishing, Aldershot.

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