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Research and Development of Stirling Cycle Engines

FINAL REPORT

Subject: evaluation of manufactured Stirling  
cycle engines, performance calculation  
and test methodology.

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Engine Technology. MSTU

Russian Federation, Moscow, December 1993

## 1. SELECTION OF THE ENGINE'S STRUCTURAL DIAGRAM

As a converting mechanism in Stirling engines one uses most often: an axial crank mechanism, a rombic drive, a taper washer and a crank-back-balance mechanism.

Accumulated experience in designing and manufacturing Stirling engine allows to recommend the use of displace type structural diagram for low-power engines (about 1-10 kW). This design is of coaxial round heat exchangers making possible to have uniform temperature fields around heat exchangers elements what eliminates the occurrence of hazardoid temperature stresses. However in order to eliminate a one-way force action of gas pressure on a working piston and therefore on crankshaft bearings, there must be another chamber under the piston in this design with the pressure near to the mean cycle pressure (or sometimes the minimal cycle pressure). Such chamber is called a buffer chamber.

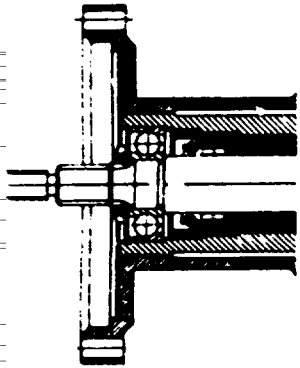
The presence of a special buffer space required the application of an a addition rod seal and furthermore it considerably increases the engine's overall dimensions for height.

The use of the engine's crankcase as a buffer space is an alternative approach. The engine becomes more compact and is characterized by smaller friction losses in seals as only one seal is arranged between the crankcase and the external space and that one being for sealing of the crankshaft outlet end during rotation instead of reciprocating piston rods as in the previous case.

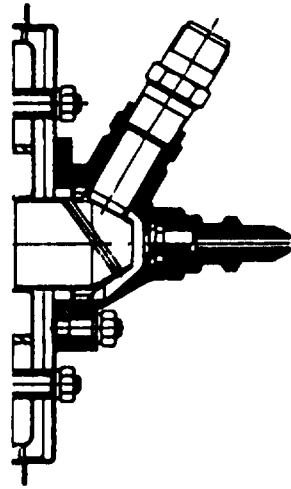
An undoubted advantage of the buffer space arranged in the crankcase is the possibility of using a dry crankcase, i.e. the application of roller bearing elements in mechanism friction assemblies sealed and lubricated for the whole operation period or being equipped with channels (piston cups) hard coats are utilized to replace greasing.

Stirling engine of the displacer type forming the basis of the present development uses a cam driving mechanism. See fig. 1

## *Stirling engine*



GoldStar do not agreed to open this part  
to third party, because GoldStar is on  
the application of patents at present.



*Fig. 1*

The reduce the engine's total cost one most utilize low- and medium- allow materials and lower the working temperature of elements up to a level (650-700)<sup>o</sup> C. Besides, resistance to scaling of low-allow steels can be increased at the expense of applying thin-layer ceramic protective coating.

The lower is the maximal pressure level in the internal circuit, the lower are stresses occuring in most critical parts of the heating circuit. In this case the engine litre power goes down and obviously a trade- off shoud be search for.

The point of the working medium selection is rather complex. If there is a problem on designing a Stirling engine with high specific indices for mass and overall dimensions then selection of a working medium is conclusive - hidrogen or helium. For low-cost engines air or nitrogen are most appropriate.

Such effective parameters as engine's power and economical operation are completely difined by structural perfection of a heat input system. The most complex item here is the necessity of correct combination of structural dimensions and the shape of the combustion chamber with the engine's heater.

## 2. CALCULATION OF THE THERMODYNAMIC CYCLE AND INTERNAL CIRCUIT HEAT EXCHANGERS

The indicated calculations are carried out to determine geometric dimensions of working chambers (hot and cold chambers), heat exchangers (heater, regenerator, cooler) in the internal circuit as well as the engine driving mechanism providing the given power and economical operation for the chosen cyclicity, sort of gas and levels of maximal temperature and pressure. Calculating has been of program "3tir2" and "Tep12".

The results of the conducted study of the thermodynamic cycle and heat exchangers in the internal circuit are given in tables 1, 2, 3, 4 and on diagrams in Figs.2-10.

Based on those investigations design dimensions and shape of the cylinder - piston unit as well as of the engine heat exchangers were chosen.

All the design documentation on the engine has been developed and passed to the customer in accordance with the SE 6000 project.

## INITIAL DATA

Table 1

Hot chamber temperature	923.0	K
Cold chamber temperature	363.0	K
Required power	1.0	kW
Rotation frequency	3000.0	min
Maximal pressure	1.2	MPa
Ratio of maximal volumes	1.0	
Factor of rod influence	0.96	
Relative clearance volume	2.20	
Heater relative volume	0.40	
Regenerator relative volume	0.40	
Cooler relative volume	0.20	
Degree of regeneration	0.97	
Specific heat at $T_e$	5.0	kJ/kg/K
Specific heat at $T_c$	4.0	kJ/kg/K
Universal gas constant	4.157	kJ/kg/K
Isothermal deviation in compression	0.95	
Isothermal deviation in expansion	0.93	
Heat energy efficiency	0.010	
Cooler energy efficiency	0.020	
Regenerator energy efficiency	0.010	
Available heat fraction	0.18	
Mechanical efficiency	70.0	%
Heat of combustion	44600.0	kJ/kg
Thermodynamic unit number	1.0	
Sc-De ratio	0.20	
Rc-Re ratio	1.0	
Charging temperature	300.0	K
Conductive transfer fraction	0.03	
Calculation step	20.0	deg

CALCULATED PARAMETERS

Table 2

Temperature ratio	0.393	
Angle beta	70.0	deg
Phase angle phi	126.67	deg
Ratio of maximal volumes	1.125	
Phase angle psi	70.0	deg
Phase angle theta	107.15	deg
Compression ratio	1.571	
Pressure ratio	0.333	
Regenerator hot end temperature	604.77	K
Regenerator cold end temperature	596.09	K
Regenerator mean temperature	600.43	K
Heater temperature factor	0.393	
Cooler temperature factor	1.0	
Regenerator temperature factor	0.605	
S	1.318	
Relative heat of underregeneration	0.028	
Regenerator relative load	0.907	
Qc-Vh ratio	0.455	
Mean indicated pressure	0.179	MPa
Indicated efficiency	31.39	%
Mean effective pressure	0.125	MPa
Effective efficiency	21.97	%
Effective fuel rate	0.37	kg/kW/h
Working volume	159.64	sm**3
Hot cylinder diameter	9.21	sm
Cold cylinder diameter	9.21	sm
Re	1.197	sm
Re	1.197	sm
VI	159.54	sm**3



Table 3

Heater maximal heat power	5850.9	W
Cooler maximal heat power	2294.3	W

Table 4

WORK PER CYCLE:	
WORK OF EXPAND	68.920
POSITIVE WORK OF EXPAND	172.457
NEGATIVE WORK OF EXPAND	-103.537
WORK OF COMPRESS	-22.911
POSITIVE WORK OF COMPRESS	132.698
NEGATIVE WORK OF COMPRESS	-155.609
WORK PER CYCLE	16.008
POSITIVE WORK PER CYCLE	153.445
NEGATIVE WORK PER CYCLE	-107.437

Fig. 2 Curves of volumes

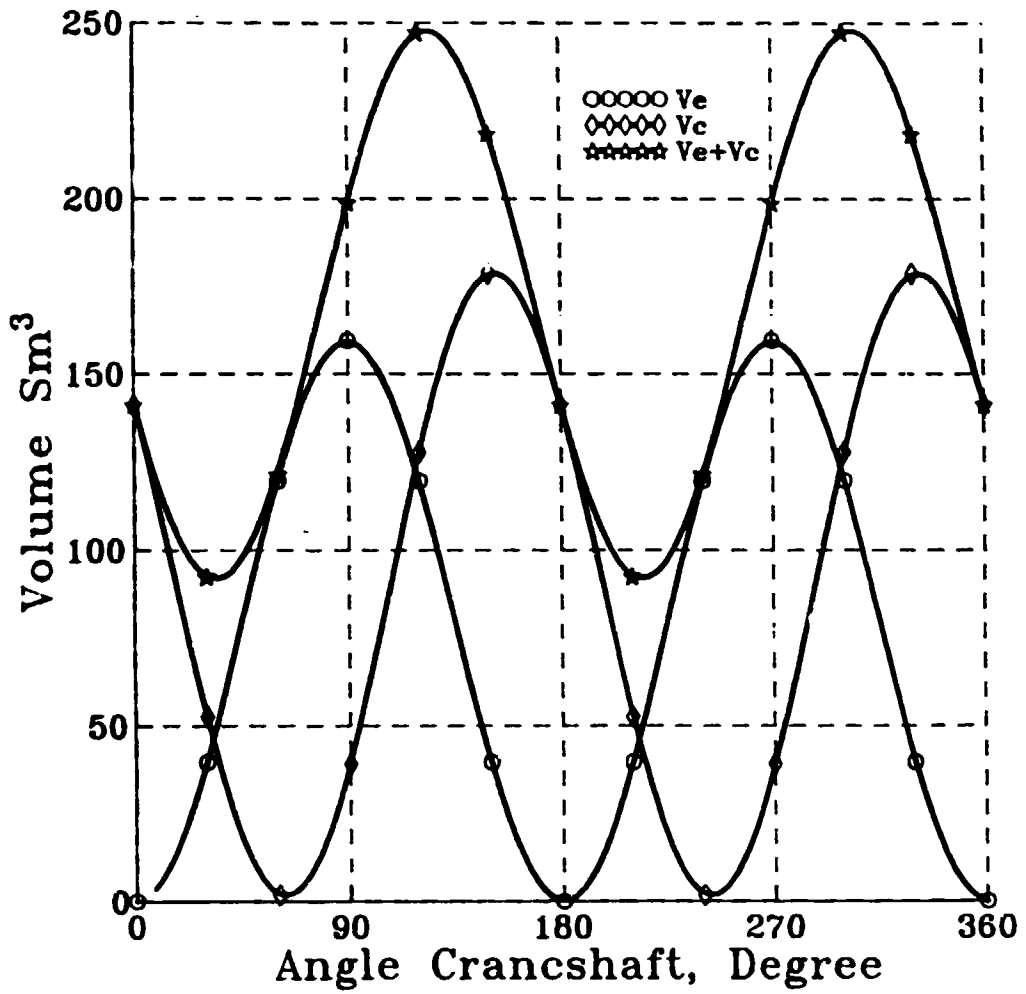


Fig 3 P-V diagram for the total volume

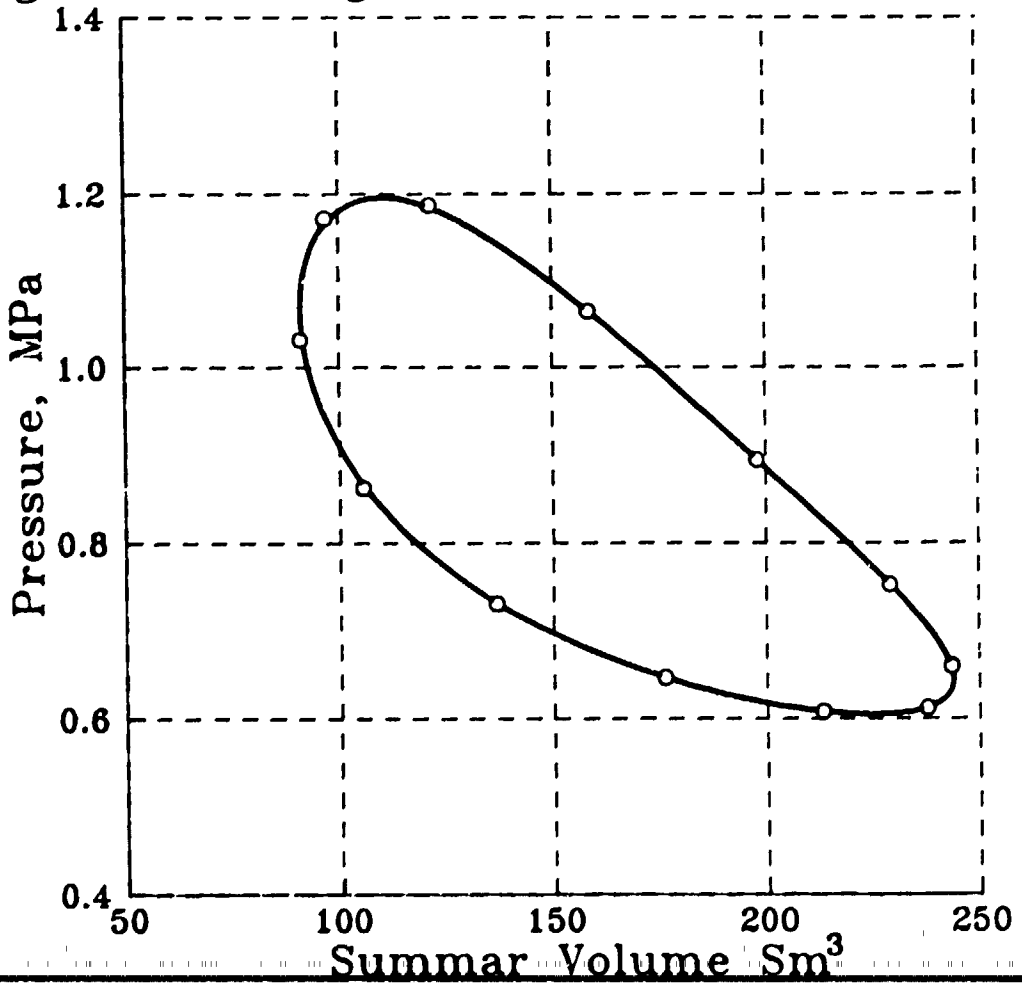


Fig. 4 Working mass flow rates

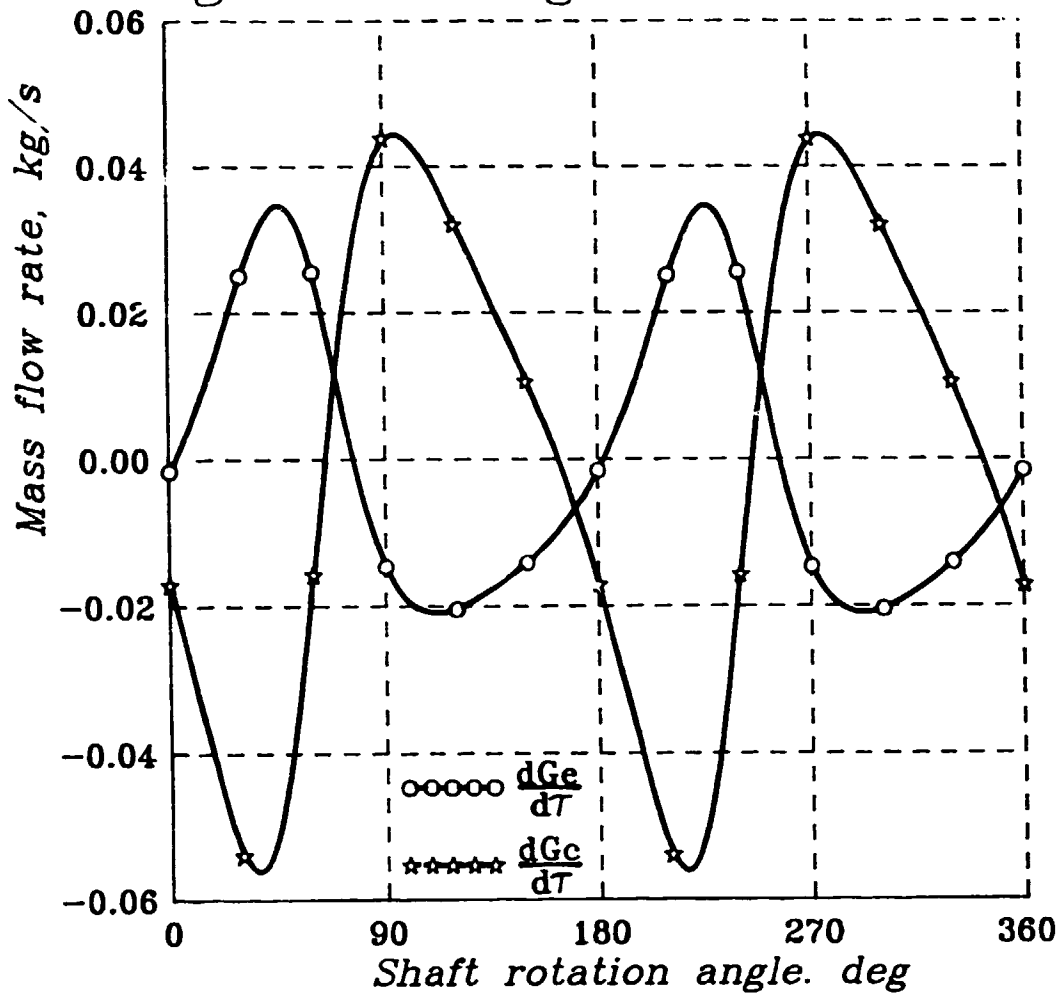


Fig. 5 Working medium velocity

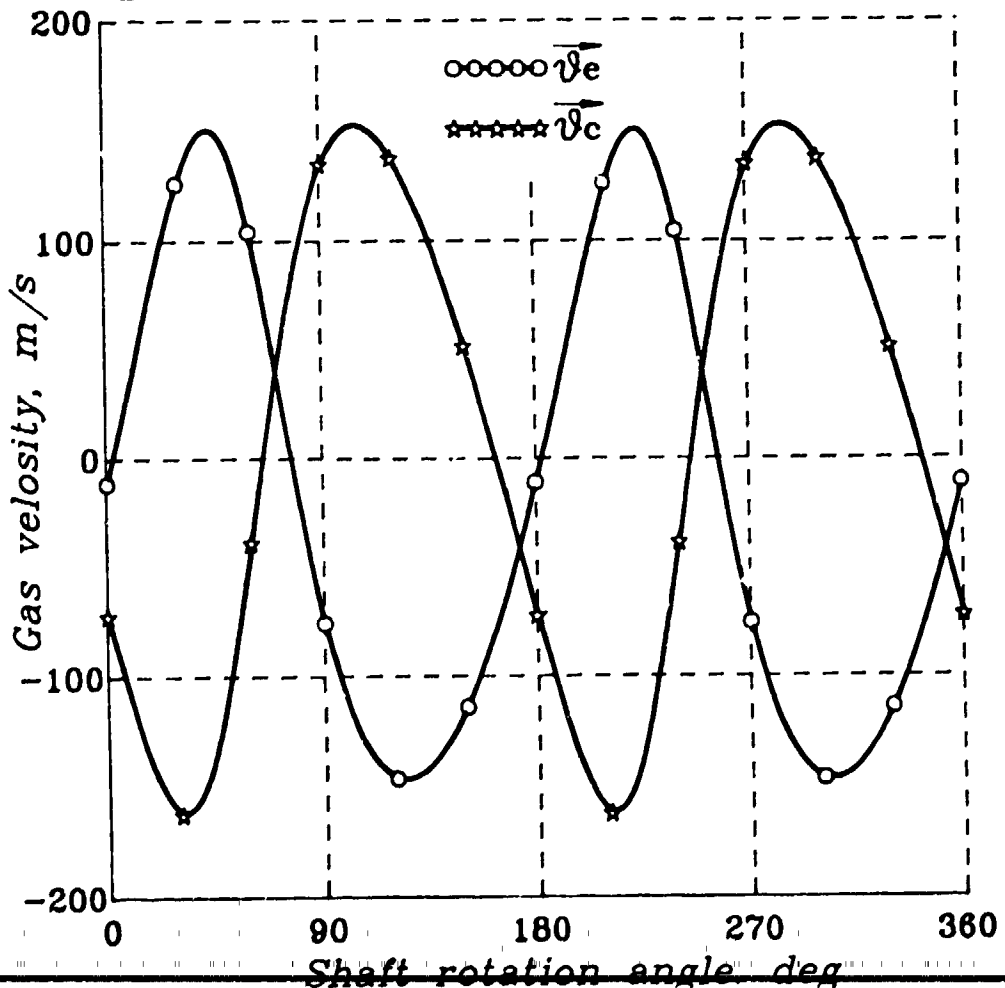


Fig. 6 Reynolds number

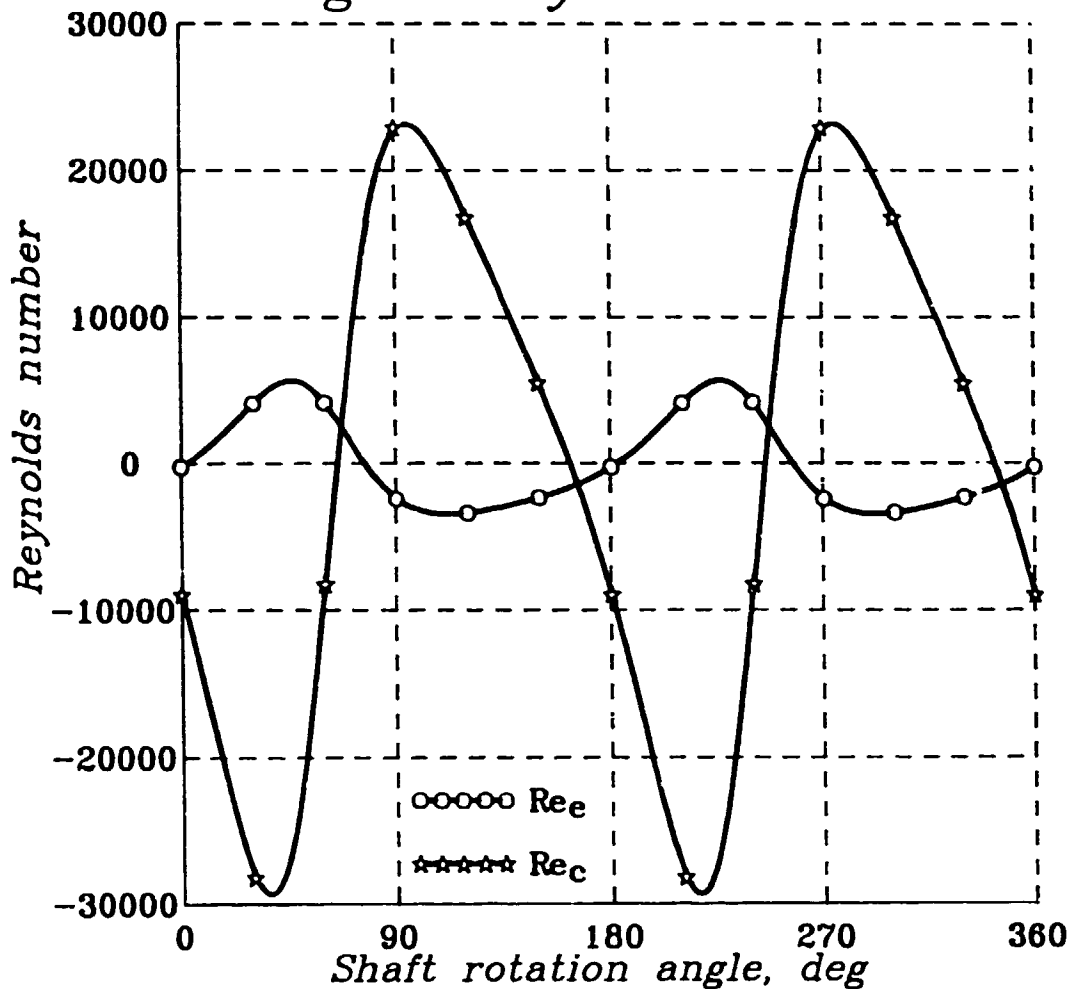


Fig. 7 Heat transfer coefficient

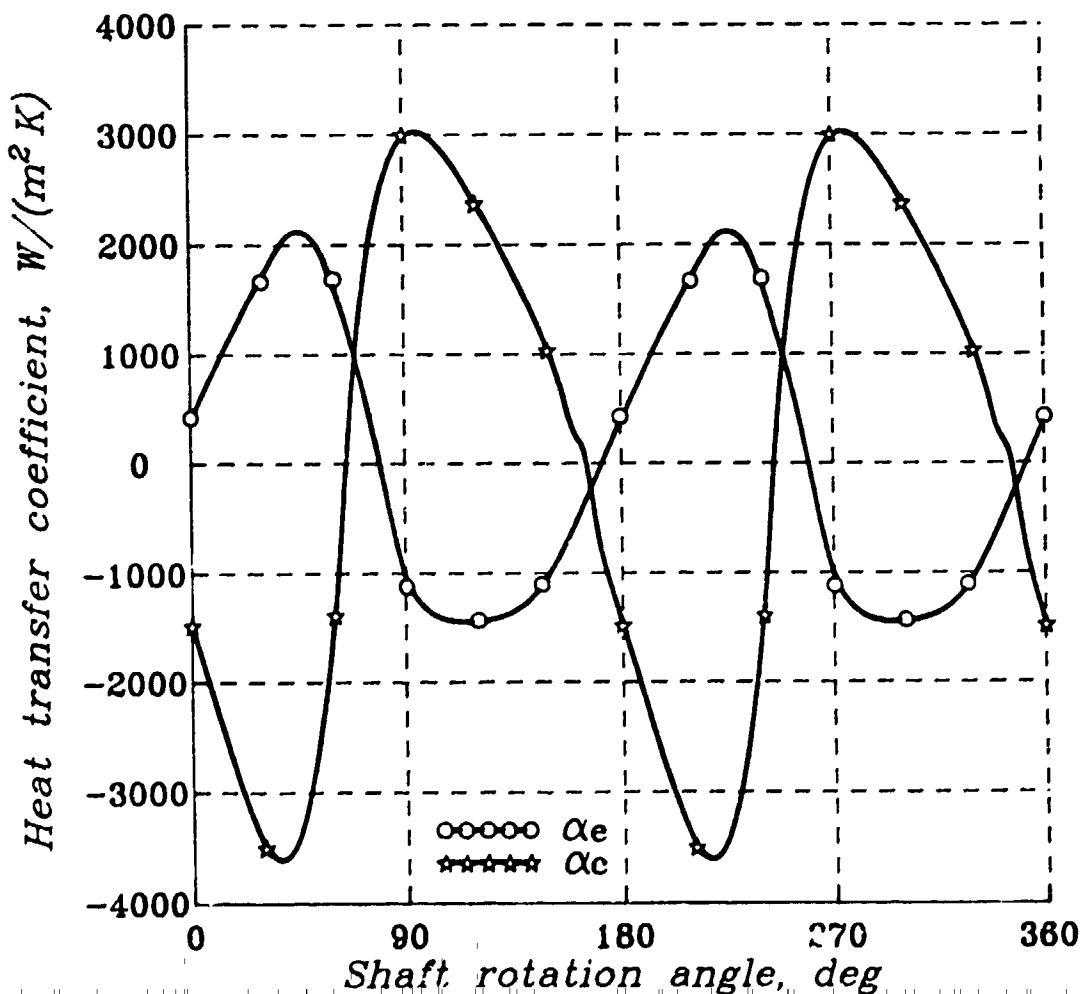


Fig. 8 Amount of heat

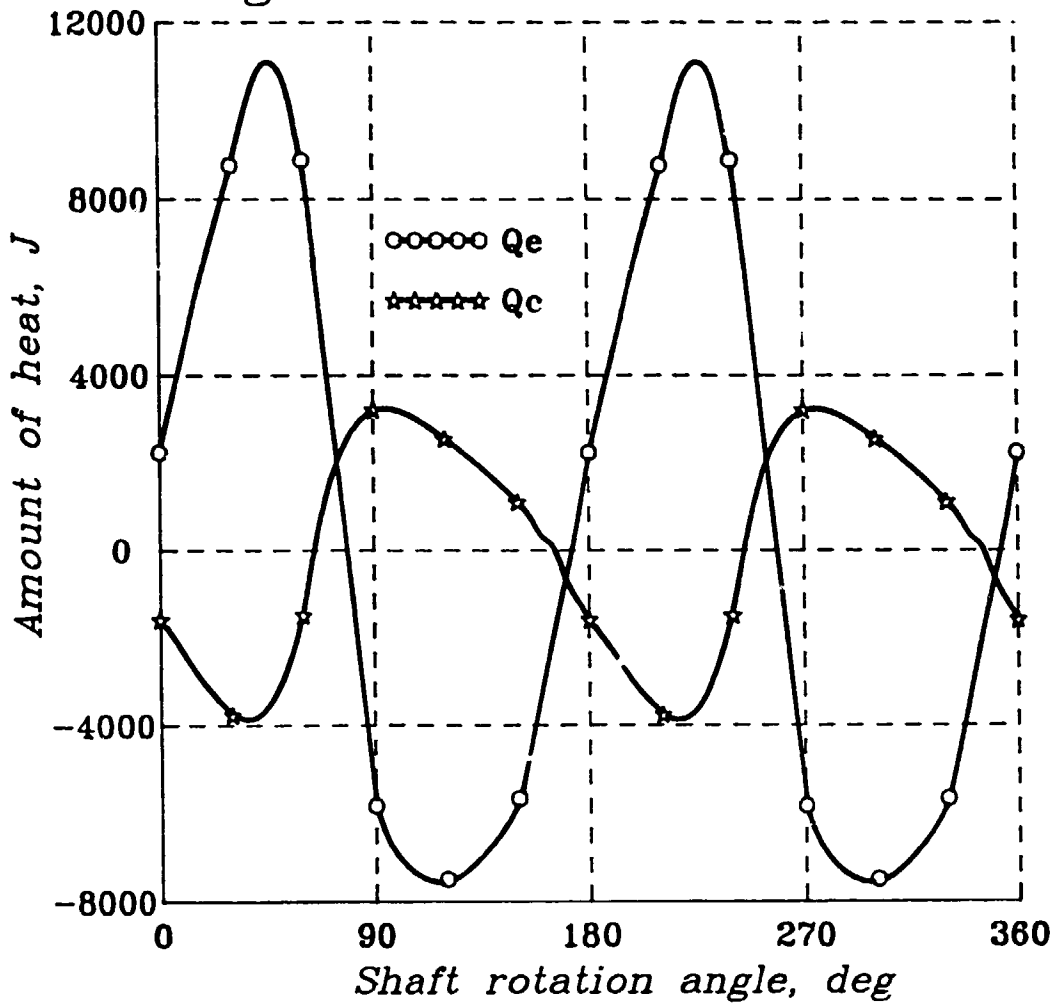


Fig. 9 Pressure loses

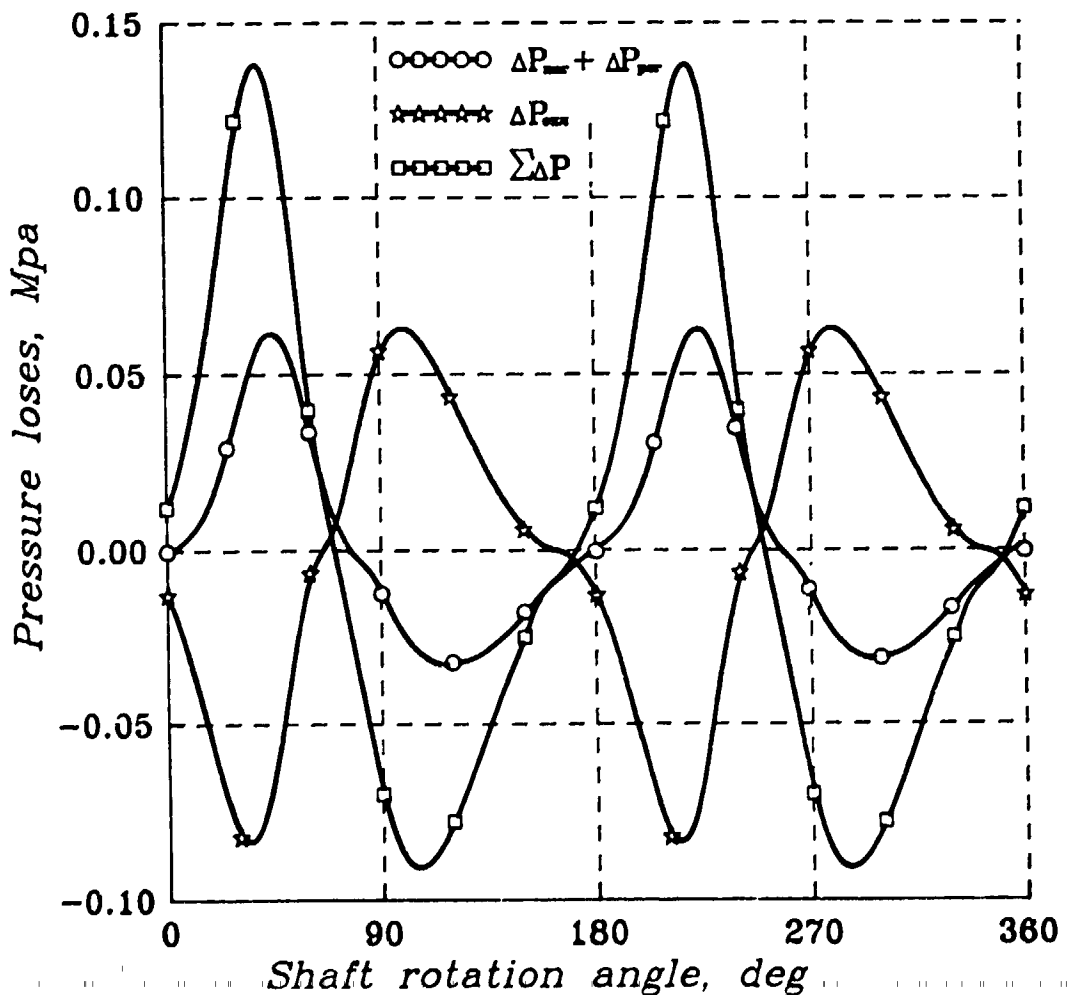
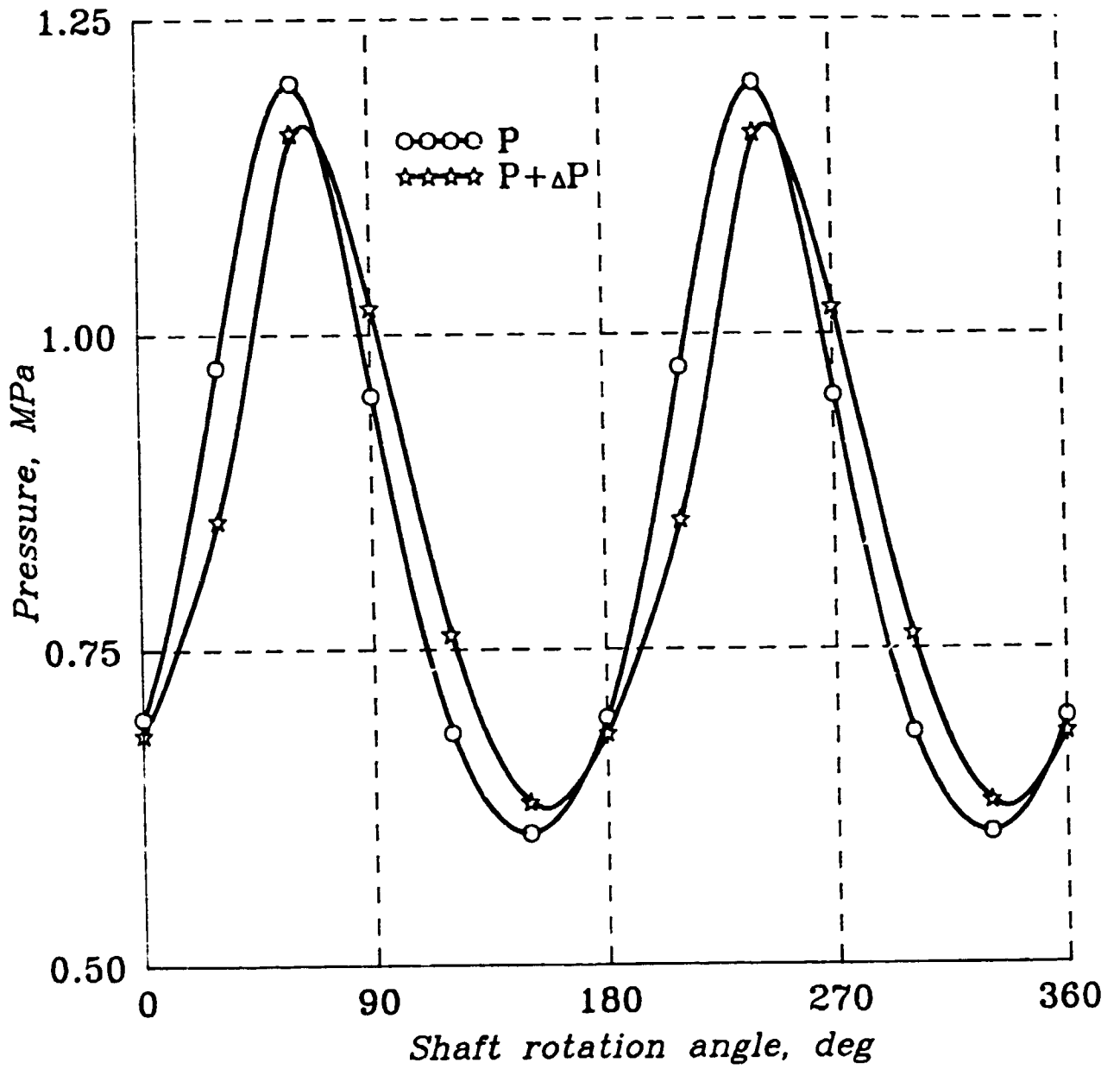


Fig. 10 Resultant pressure curve



### 3. DRIVING MECHANISM

#### 3.1. Selection of driving mechanism

Cam mechanism which hasn't been applied earlier is selected for the engine to be designed. This mechanism comprises a cylindrical rotor with a closed sinusoidal recess of rectangular section made on its lateral surface. Two pairs of rollers whose axes are rigidly connected with the working piston and piston-displacer are moving in the recess.

In comparison with conventional mechanisms utilized in Stirling engines the given mechanism has a number of obvious advantages:

- Owing to the fact that the rotor rotation axis of the cam and the consumer shaft axis coincide with the cylinder axis, it's possible to build up a compact plant occupying a minimal area;
- Relatively small number of parts makes the design simple and rather cheap in manufacture;
- The mechanism operation doesn't impose stringent requirements for accuracy of the sinusoidal recess fabrication;
- After carrying out additional investigations on determining optimal law of pistons motion to a great extent approximating the ideal one this can be easily realized in the proposed mechanism by making a recess differing in shape from a sinusoid;
- This mechanism construction allows with reasonable facility to realize one of the most economic methods of power control in the Stirling engine by varying a phase angle between the working piston and the piston-displacer at the cost of moving the casing recess receiving the piston-displacer reactive moment;

- In the cam driving mechanism its rotor can as a flywheel for passing dead centres and this also favours the reduction of mass and overall dimensions of the plant as a whole.

Calculating of kynematics and dynamics of cam drive mechanism has been in program "Stt" (see Fig. 11-16).



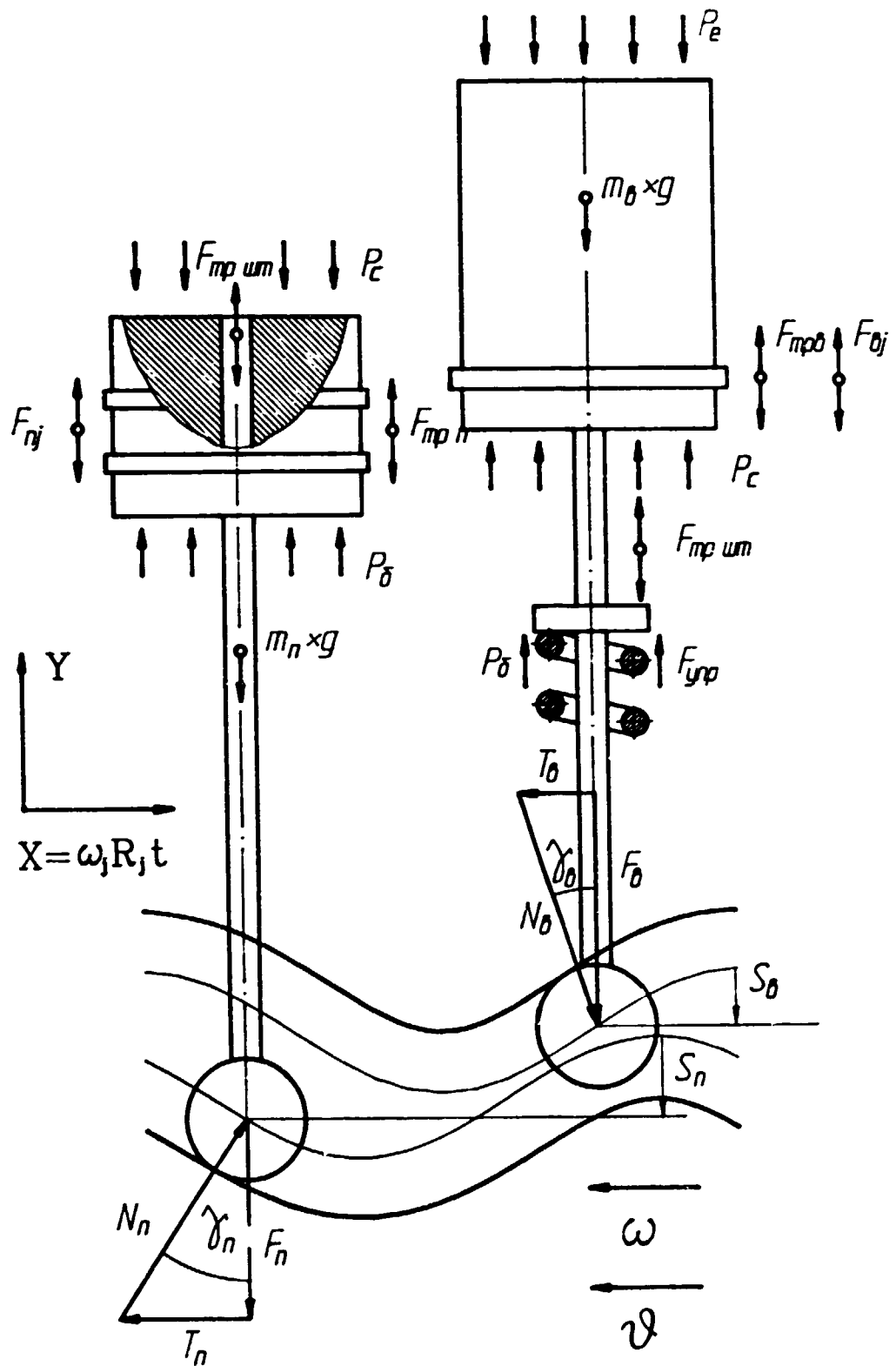


Fig. 11 Force diagram of drive loading

Fig. 12 Piston accelerations

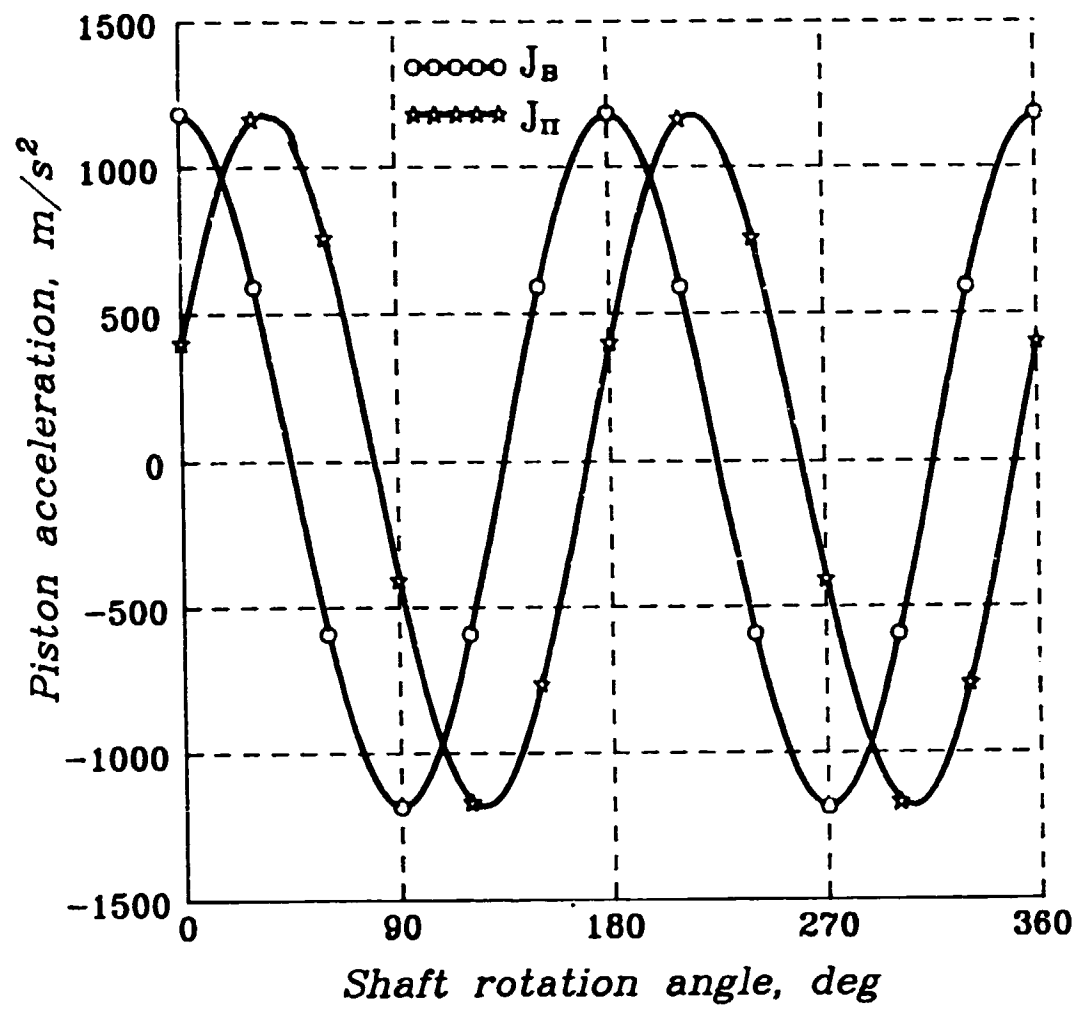


Fig. 13 Inertial force

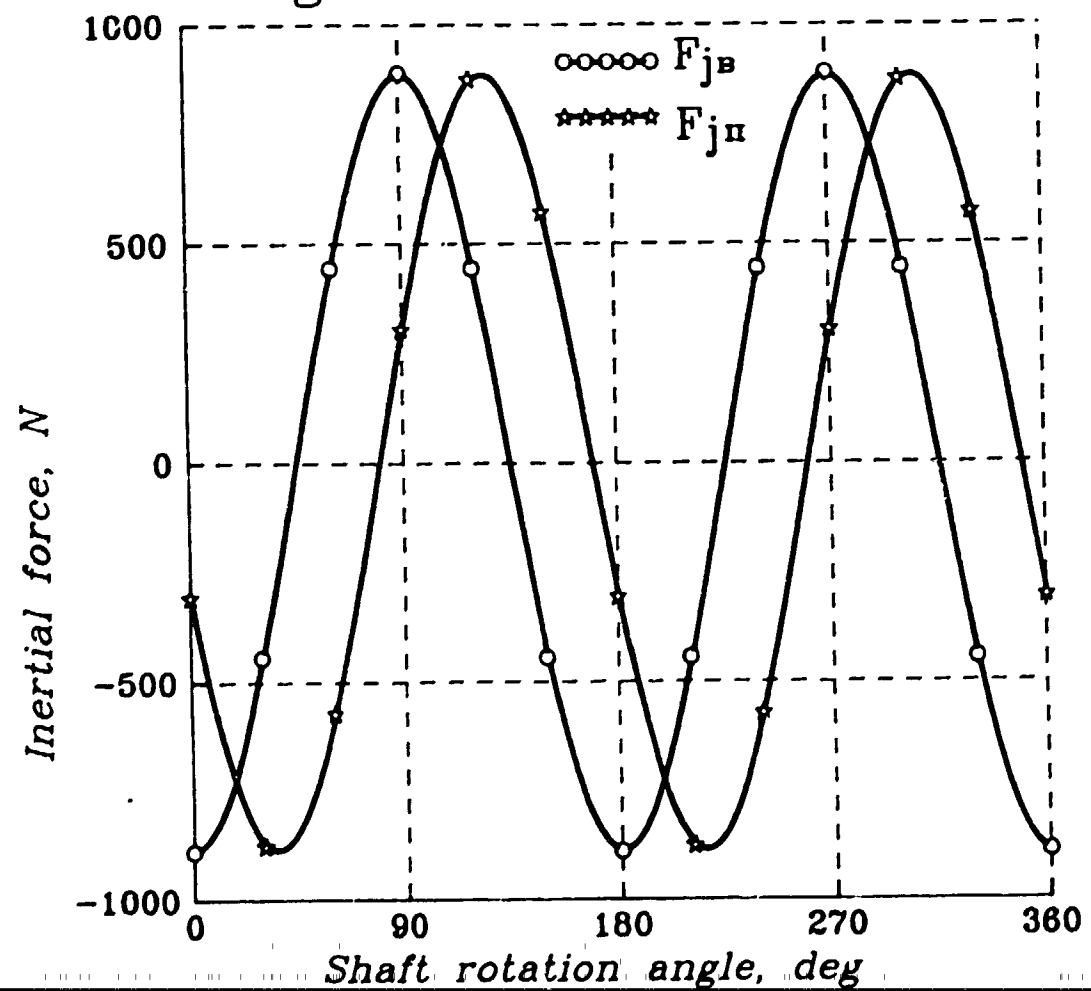


Fig. 14 Total forces

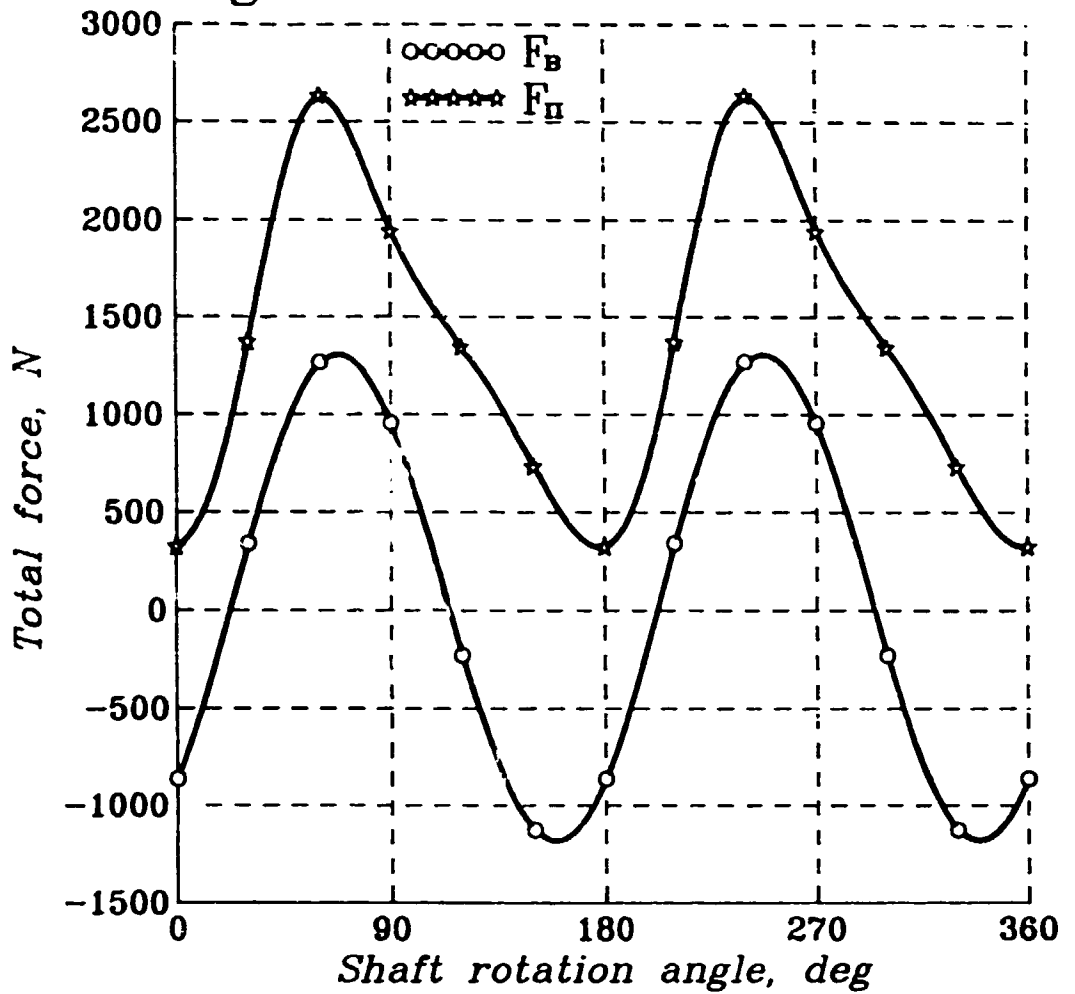


Fig. 15 Tangential forces

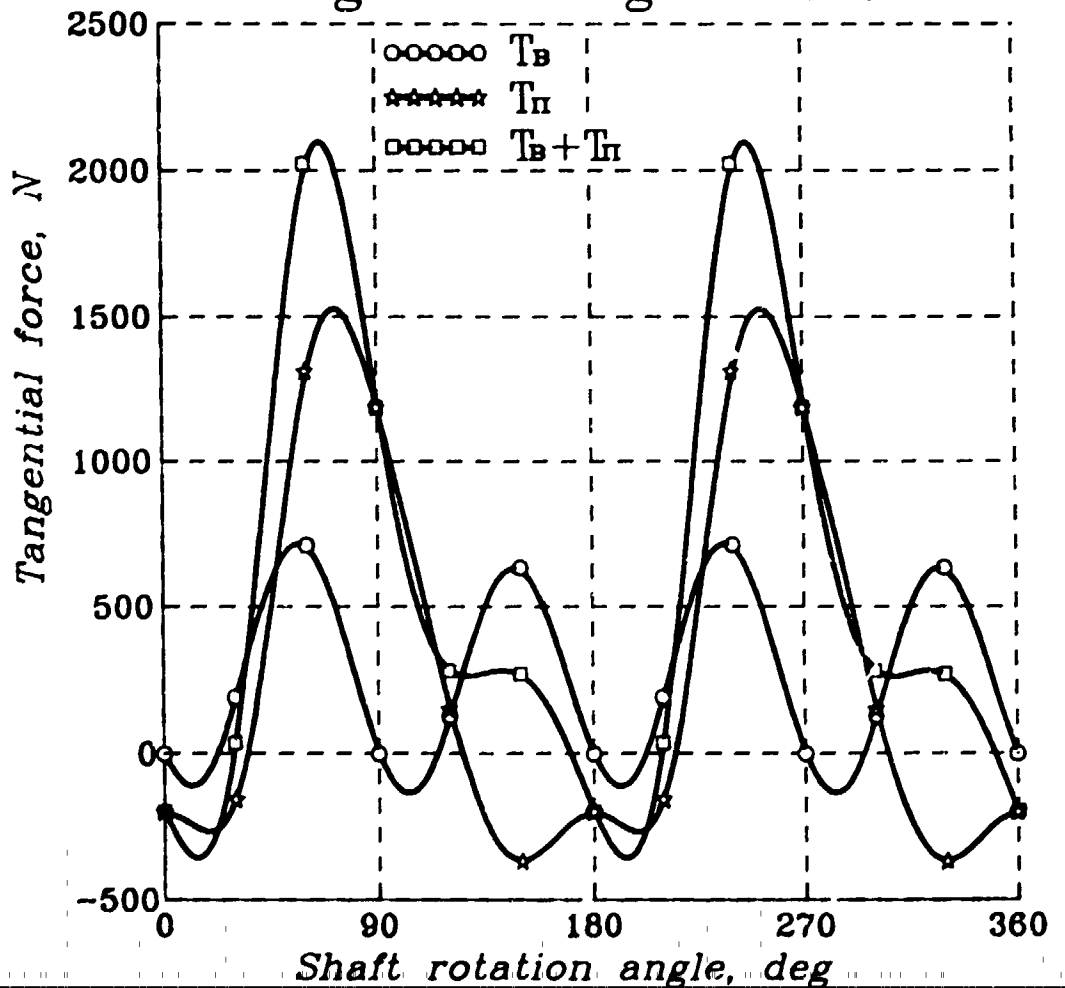


Fig. 16 Normal forces

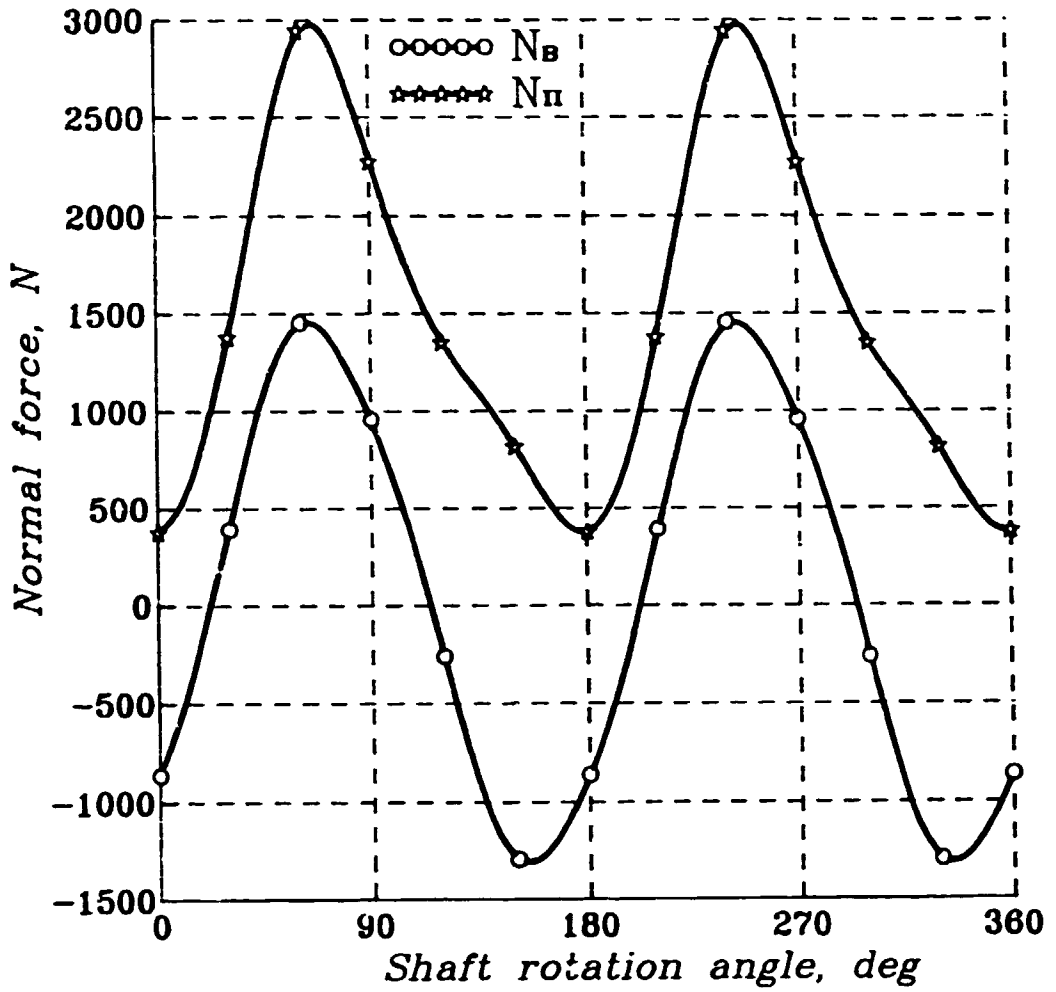
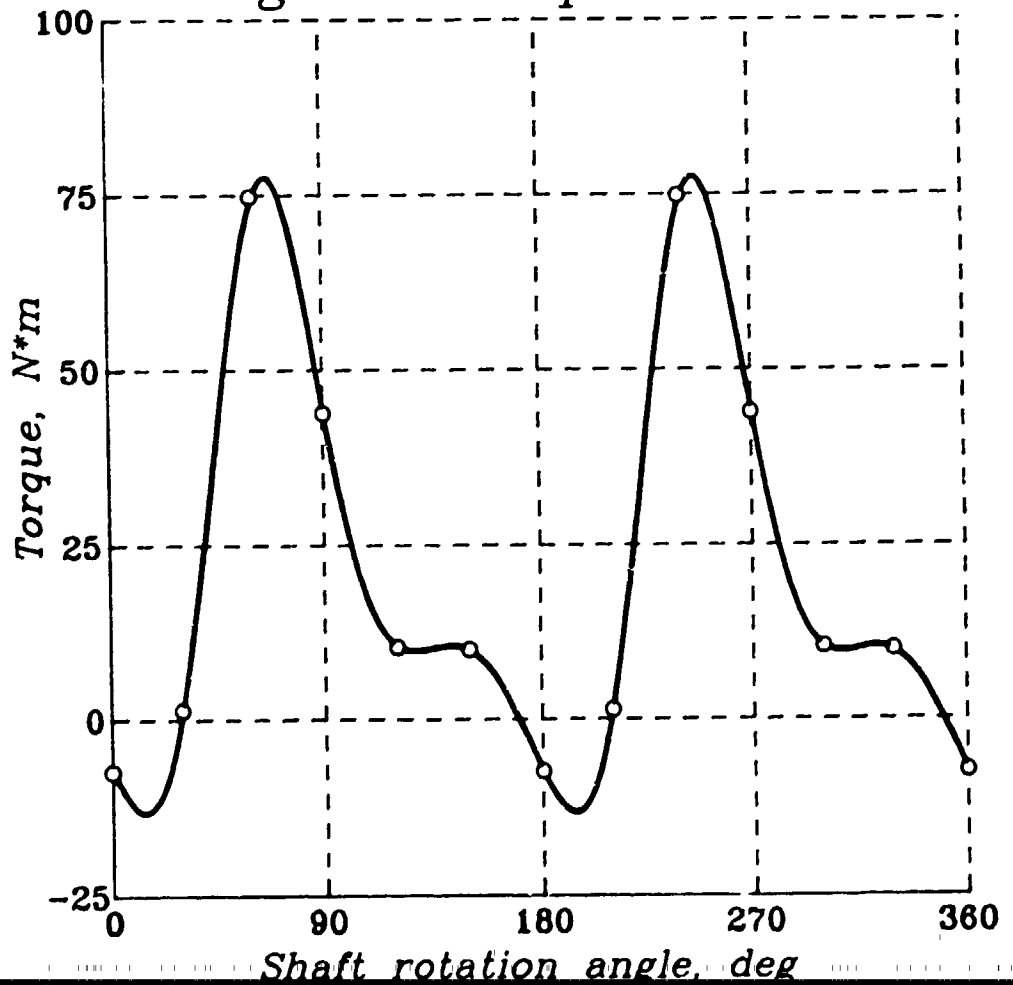


Fig. 17 Torque



#### 4. HEAT TRANSFER PROCESSES

In the stationary mode of the combustion chamber operation heat fluxes to flame tube walls from the inside are equal to heat fluxes from the tube to the outside (due to small thickness of the flame tube wall longitudinal conductive heat flux can be neglected). In line with this an equilibrium temperature is being established at any point of the flame tube wall. Physical model of the described heat exchange process is given in Fig.17.

Under stationary conditions heat flux from the inside of the combustion chamber to any elementary segment of area  $\Delta F_{w1}$  of the flame tube wall should be balanced out by heat flux to the outside of the same element, i.e.:

$$(R_1 + C_1 + K)\Delta F_{w1} = (R_2 + C_2)\Delta F_{w2} = K_{1-2}\Delta F_{w1} \quad (1)$$

For thin walls of the flame tube the longitudinal conductive heat flux  $K$  is negligible in comparison with other heat fluxes, therefore in this case we set  $K=0$ . Besides for the same case it may be considered that  $\Delta F_{w1} \approx \Delta F_{w2}$ . Then Eq.(1) takes a simpler form:

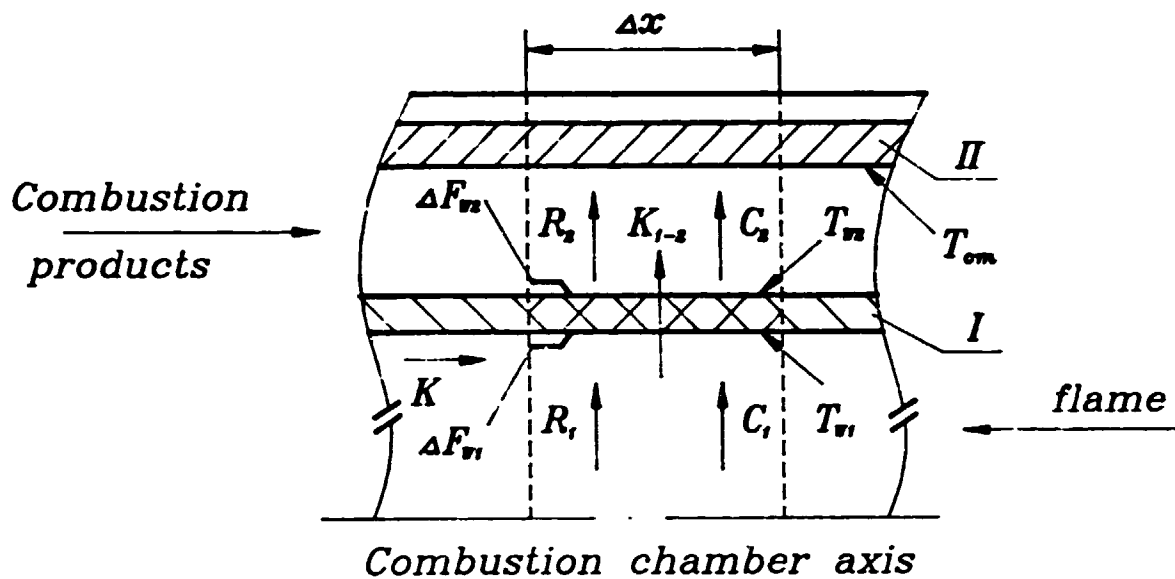
$$R_1 + C_1 = R_2 + C_2 = K_{1-2} \quad (2)$$

where  $K_{1-2}$  - specific conductive heat flux across the flame tube wall being defined by temperature gradient  $t_w$  across the wall thickness:

$$K_{1-2} = \frac{K_w}{t_w} (T_{w1} - T_{w2}) \quad (3)$$

- $K_w$  - thermal conductivity of the wall;
- $T_{w1}$  - flame tube inner wall temperature;
- $T_{w2}$  - flame tube outer wall temperature.

Fig. 18 Physical model of heat transfer



## RADIATION OF GASES

Resultant specific radiant flux is determined by the formula:

$$R_1 = \sigma(\epsilon_r T_r^4 - \alpha_r T_{w1}^4) \quad , \quad ( 1 )$$

where:  $\sigma = 5,67 \times 10^{-8} \text{ W}/(\text{m}^2 \text{K}^4)$  - Stefan-Boltzmann constant;

$\epsilon_r$  and  $\alpha_r$  - gas radiating power and absorbing capacity, respectively.

An approximating dependence has been found for  $\alpha_r$  over a wide range of temperature changes:

$$\alpha_r / \epsilon_r = (T_r / T_{w1})^{1,5} \quad . \quad ( 5 )$$

Average temperature of combustion products  $T_r$  is calculated by means of adding gas temperature increment due to fuel combustion to the combustion chamber inlet temperature  $T_{bx}$ :

$$T_r = T_{bx} + \Delta T_c \quad . \quad ( 6 )$$

For small values of  $\epsilon_r$  Eq.(6) reduces to a more convenient and accurate formula:

$$\epsilon_r = 290P (\times l_b)^{0,5} T_r^{-1,5} \quad , \quad ( 7 )$$

where:  $P$  - pressure, kPa;

$T_r$  - temperature, K;

$l_b$  - beam path length, m.

where  $l_b$  with a satisfactory accuracy for practical problems can be calculated from the formula:

$$l_b = 3,4 V/A \quad , \quad ( 8 )$$

where  $V$  and  $A$  - volume and surface of gas containing chamber, respectively.

## RADIATION OF WALLS

Radiant heat flux  $R_2$  from the flame tube wall to be heater wall can be calculated on the following assumption: both walls are grey, they have even emissivity factors  $\epsilon_w$  and  $\epsilon_{CT}$  and temperatures  $T_{w2}$  and  $T_{CT}$ .

$$R_2 F_{w2} = \frac{\sigma(T_{w2}^4 - T_{CT}^4)}{(1-\epsilon_w)/\epsilon_w F_w + 1/F_w A_{wCT} + (1-\epsilon_{CT})/\epsilon_{CT} F_{CT}}, \quad (9)$$

where:  $F_w$  - surface area of a flame tube element;  
 $F_{CT}$  - surface area of a heater shell element;  
 $A_{wCT}$  - angular coefficient between the flame tube and the heater shell.

Therefore it's possible to consider  $T_{CT}$  to be an allowable temperature for a wall material of the heater shell and to set the angular coefficient equal to 1.

In this case expression (9) is brought to the form:

$$R_2 = \frac{\epsilon_w \epsilon_{CT}}{\epsilon_{CT} + \epsilon_w (1-\epsilon_{CT})} \frac{\sigma(T_{w2}^4 - T_{CT}^4)}{F_{CT}}. \quad (10)$$

Accurate values of emissivity factors for different materials can be found in corresponding handbooks. The estimation of  $R_2$  for tubular type combustion chambers and for a housing of stainless steel can be broadly carried out by the formula:

$$R_2 = 0,6\sigma (T_{w2}^4 - T_{CT}^4). \quad (11)$$



## CONVECTIVE HEAT TRANSFER TO THE FLAME TUBE

The calculation of internal convective heat transfer from gas to the flame tube wall is the least exact.

$$C_1 = 0,02 \frac{k_r}{D_{\text{HT}}^{0,2}} \left( \frac{\dot{m}_r}{S_{\text{HT}} \mu_r} \right)^{0,4} (T_r - T_{w1}) \quad (12)$$

where:  $D_{\text{HT}} = 4S_{\text{HT}} / \Pi_{\text{HT}}$  - flame tube hydraulic diameter;  
 $S_{\text{HT}}$  and  $\Pi_{\text{HT}}$  - flame tube cross section area and perimeter, respectively;  
 $k_r$  - gas thermal conductivity.

## CONVECTIVE HEAT TRANSFER FROM THE FLAME TUBE TO THE HEATER SHELL WALL

Nusselt number for turbulent flow in an annular passage when  $q_1 = q_2$  is calculated by the formula:

$$Nu_1'' = Nu_2'' \approx 0,95Nu_0'' \quad (13)$$

Where:

$q_1, q_2$  - specific heat fluxes on inner and outer tubes;

Local Nusselt number for turbulent flow in plain circular tubes is calculated by the formula:

$$Nu_0'' = \frac{\xi/8 \operatorname{Re} \operatorname{Pr}}{K + 4,5\sqrt{\xi} (\operatorname{Pr}^{2/3} - 1)} C_t C_x, \quad (14)$$

where:  $K = 1 + 900/\operatorname{Re}$ ;  $\xi = (1,82 \lg \operatorname{Re} - 1,64)^{-2}$ ;

$C_t$  - correction factor for nonisothermal character of the flow; for gases in heating:

$$C_t = (T_{CT} / \bar{T}_x)^m, \quad m = -(0,3 \lg(T_{CT} / \bar{T}_x) + 0,36);$$

in cooling  $C_t = 1,0$ .

$T_{CT}, \bar{T}_x$  - surface temperature of the tube wall and average mass flow temperature (heat content average);

$C_x$  - correction factor for a thermal stabilization initial section of hydrodynamically steady liquid flow;

$$C_x = 1 + 0,48(d_r/x)^{0,25} \left[ 1 + \frac{3600}{\operatorname{Re} \sqrt{x/d_r}} \right] \exp(-0,17x/d_r), \quad (15)$$

$x$  - distance from the tube inlet.

Formulae (14) and (15) are applicable over the range:  
 $\operatorname{Re} = 4 \times 10^3 - 5 \times 10^4$ ;  $\operatorname{Pr} = 0,7 - 1,0$ ;  $x/d_r \geq 0,6$ .

For the range of quantity changes  $\operatorname{Pr} \approx 0,7$ ;  $\operatorname{Re} = 3 \times 10^3 - 5 \times 10^4$  and  $x/d_r > 2$  a simpler relationship for determining  $C_x$  is appropriate:

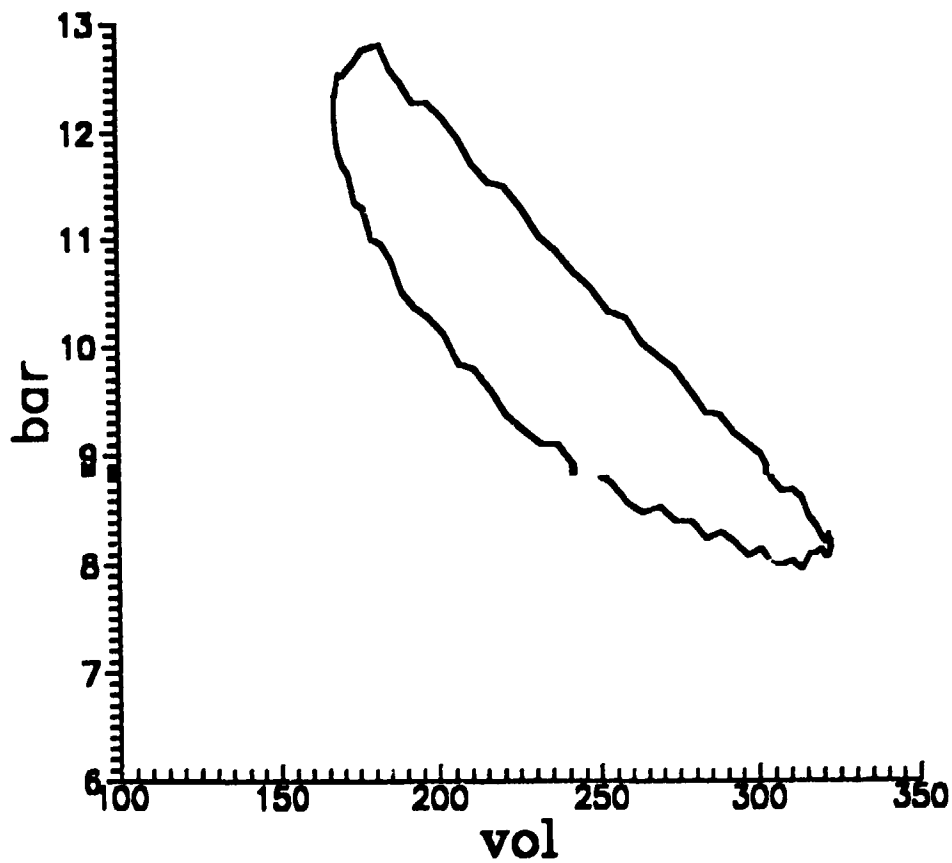
$$C_x = 1 + (0,8 + 5,6 \times 10^4 \text{Re}^{-3,2}) d_r/x \pm 5\% \quad . \quad ( 16 )$$

Convective heat flux from the flame tube to the heater shell wall is determined from the formula

$$C_2 = \bar{\alpha} (T_{w2} - \bar{T}) \quad , \quad ( 17 )$$

where:  $T_{w2}$  - temperature of the flame tube outside surface;  
 $\bar{T}$  - average mass (heat content average) flow temperature at the annular passage inlet;  
 $\bar{\alpha}$  - average heat transfer coefficient per length  $x$ ,  $\text{W}/(\text{m}^2 \text{ } ^\circ\text{C})$ .

*Fig. 19 Experimental P-V diagram  
of Stirling Engine*



## FINAL

Manufacturing according calculating and projective construction documentation Stirling engine (project SE6000) show his work (see Fig.13 work P-V diagram engine). For having more high data in power and efficiency we need on far work from sepa-rating elements and mechanisms engine.