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United Nations
Centre for Industrial Development — Original: English

Inter-regional Symposium on Industrial Project Evaluation — [illegible].17 Discussion paper

Prague, Czechoslovakia
11 - 29 October, 1965

07614

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65-41306

I. Introduction

The field of industrial project evaluation is a relatively new branch of economic analysis, and as such is still in its formative stages. Numerous gaps still exist in the available literature, and in many cases alternative approaches to problems have been suggested which entail differences of concept that are as yet unresolved. These facts have determined the design of this survey. An attempt has been made to take a constructive and forward-looking approach, focusing on gaps, weaknesses, and unresolved issues in the field and attempting to contribute to an improvement of existing procedures wherever possible.

Because the great bulk of the literature available in the U.S.A. concerned project evaluation in predominantly private enterprise or mixed economies, the study has been confined to such cases, making no attempt to consider the case of completely centrally-planned economies. But it is recognized in what follows that social benefits and costs do not always coincide with private pecuniary benefits and costs. Indeed it may be said that one of the principal concerns of cost-benefit analysis is to appraise costs and benefits from a social point of view in cases where these diverge from the pecuniary costs and benefits perceived by the individuals in the marketplace.

Section II focuses on the controversial problem of the relevant rate of discount fo us in cost-benefit analysis. First the advantages and disadvantages of the internal rate of return are discussed, and it is concluded that, though useful as a summary indicator of a project's profitability, the internal rate of return should not be used as the basic criterion for project evaluation. Then market rates of interest on bonds, the "social rate of time preference" and the marginal productivity of capital in the private sector are considered. It is concluded that the optimal rate for use in discounting costs and benefits, in a market economy, is the marginal productivity of capital in the private sector of the economy, defining this marginal productivity in such a way as to include all social benefits and costs in the calculations. Finally, the question of the variation of the discount rate through time is considered. It is concluded that the appropriate discounting of flows of benefits and costs should normally be done at rates which may vary from year to year, the principle being that flows occurring in year ten should be discounted back to year nine at the marginal productivity of capital expected to prevail in year nine, that these flows, in turn, should be discounted back to year eight at the marginal productivity of capital expected to prevail in that year, etc. This principle of a variable discount rate is necessary in order to reach proper decisions on project timing and scale, and is particularly important in reaching valid decisions in years in which investible funds are either particularly abundant or particularly scarce relative to existing investment opportunities.

Section III focuses on the measurement of benefits and costs, and particularly on how to project the path of expected benefits and costs through time. Initially, the basic principles underlying demand projections are reviewed, and subsequently the principles underlying the projection of prices, wages, and other costs are considered. The main conclusion of Section III is that it is necessary to project the prospective costs and benefits of a project year by year through the entire expected life of the project, incorporating expected changes in prices and costs directly into the analysis. Projects can then be evaluated and compared on the basis of the excess of discounted benefits over discounted costs, thus projected. Particular importance is attached to the fact that in a developing economy, wages must be expected to rise relative to product prices in general, so that the excess of the price of a project's output (which is the first-approximation measure of its benefits per unit of output) over costs may frequently be expected to decline as real wages rise through time. Attention is also paid to the problem of projecting the path through time of the exchange rate and of cost components other than wages. Finally the problem of measuring the indirect costs and benefits of a project is briefly surveyed.

Section IV discusses the use of accounting prices in project evaluation. It finds that divergences between social costs and market prices can be significant in many cases, and thus endorses in principle the use of accounting prices. The main effort of this section is, however, to discuss the appropriate ways of estimating accounting prices. In the case of labor, the need for having distinct accounting prices for labor of different skills and types, and in different regions, is emphasized. It is suggested that a minimum estimate of the accounting price for urban labor of a given type may often be obtained from the wage rate received by labor of that type employed within the urban complex in activities in which wages are not influenced either by minimum wage legislation or by union agreements. It is explicitly concluded that the marginal productivity of labor in agriculture is not a relevant measure of the accounting price of urban labor. The method of setting the accounting price of foreign exchange is then outlined, the principle involved being the estimation of the market value of the goods that would likely be imported as a consequence of the availability of additional foreign exchange. The possibility of using accounting prices for materials inputs is then examined, the conclusion here being that, although accounting prices may in some cases be justified for such inputs, equivalent results are achieved by generally valuing all materials inputs at their market prices, and considering separately, as indirect benefits of the project, any surplus of benefits over costs generated in the material-producing industry as a direct consequence of the project in question. Finally, the question of accounting prices for the output of a project is considered, the focus being particularly on cases in which this output is subject to indirect taxation. The conclusion is reached that, except in unusual instances in which the indirect tax was itself placed on the product in order to countervail an existing external diseconomy associated with the product's production or consumption, the social benefit associated with the output of a project is to be measured by its price including tax.

A brief addendum to Section IV considers the possibility of obtaining appropriate accounting prices through the use of linear programming models for the entire economy. Here it is concluded that in order to make a linear programming model for the whole economy feasible, the characteristics of the economy must be so drastically oversimplified as to make the resulting accounting prices highly unreliable.

In Section V, problems of timing are considered. First, the influence of high discount rates on projects of different productive lives and gestation periods is reviewed, then the question of when to construct a given project is considered, and finally the question of how to deal with risk is faced. The key conclusions are: (a) The timing of the construction of a project is a problem of considerable importance. Construction should not be undertaken at the moment when the present value of benefits exceeds the present value of costs, but should be delayed to the point where the excess of the present value of benefits over the present value of costs is a maximum. For a particular class of cases, it is shown that this rule entails the delay of a project until such time as the benefits of its first year of operation exceed the interest charge on the capital invested in the project. (b) If benefits and costs are appropriately projected, so as to take account of possible reductions in the value of benefits of a project stemming from future improvements in productive technique, there is no need to add a risk factor to the discount rate used in cost-benefit analysis.

In Section VI, interrelations among projects are considered.

The importance of analyzing separately the contribution of all separable components of a project is emphasized. Finally, the principles for deciding which of a set of interrelated projects should be undertaken are briefly set out.

II. Present Value Criteria versus the Internal Rate of Return

A. Advantages and Defects of the Internal Rate of Return

The internal rate of return on a project (P) is obtained by the solution of the following equation

$$\sum_{t=0}^{N} \frac{B_t - C_t}{(1+\rho)^t} = 0 ,$$

where B_t represents the benefits anticipated to accrue in year t of a project's life and C_t represents the costs anticipated to be incurred in year t. N is the length of life of the project. Costs are defined to include capital outlays, labor, materials, energy and transport costs, and maintenance and repair expenditures. Costs do not include depreciation charges or actual or imputed interest charges, as the internal rate of return itself reflects the implicit "net interest yield" of the project, and in this sense allows for the depreciation of the project's cost. Thus, if a project has a capital cost of 100 in year 0, and yields a benefit of 120 in year 1, with an operating cost of 20, the net effect of the operation of the project would be -100 in year 0 and +100 in year 1. The capital invested would be just barely recovered one year later. Such a project would have an internal rate of return of zero, indicating that no more than capital recovery can be expected from it. On the other hand, if the project were to have a benefit of 130 in year 1, with an operating cost of 20 in that year, its internal rate of return would be 10 per cent, indicating that the capital invested in the project will produce a yield of 10 per cent after allowing for capital recovery. Finally, if the benefit in year 1 were merely 110, together with an operating cost of 20, the value of $B_1 - C_1$ would be 90, and the internal rate of return would be -10 per cent, indicating that the project is incapable of yielding sufficient benefits to cover the cost of the invested capital.

The great advantage of the internal rate of return lies in the fact that it can be calculated on the basis of project data alone. In particular, its calculation does not require data on the opportunity cost of capital, which, as will be seen below, is critical to the present value technique and can often be exceedingly difficult to estimate. Thus, when a project evaluator has several different projects to be surveyed, he may independently calculate the internal rate of return on each, and use the resulting figures as one basis of comparison among the projects.

The disadvantages of the internal rate of return are severe, however— so severe as to warrant the greatest caution in its use. In the first place, there are some projects for which it is not possible to determine the internal rate of return uniquely. Figure 1a shows the time-profile of net benefits $(B_t - C_t)$ for a typical project. In it an initial period of investment, during

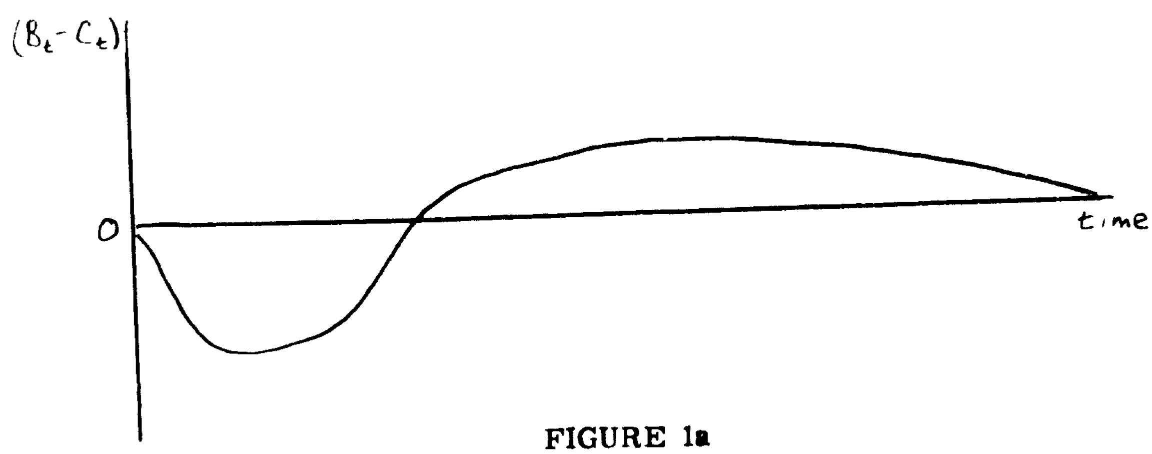

FIGURE 1a

which the value of $B_t - C_t$ is negative, is followed by a period in which the net benefit of the project is always positive. For all cases of this type there is a unique solution for the internal rate of return. However, if the time-profile of net benefits crosses zero more than once, there will be multiple solutions for the internal rate of return. Examples of such projects are cases in which major items of equipment must be replaced relatively frequently, giving rise to negative net benefits, say, every five years when these replacements are accomplished (see Figure 1b); or cases in which the termination of a project entails substantial net costs (e.g., of restoring rented facilities to their former state) (see Figure 1c).

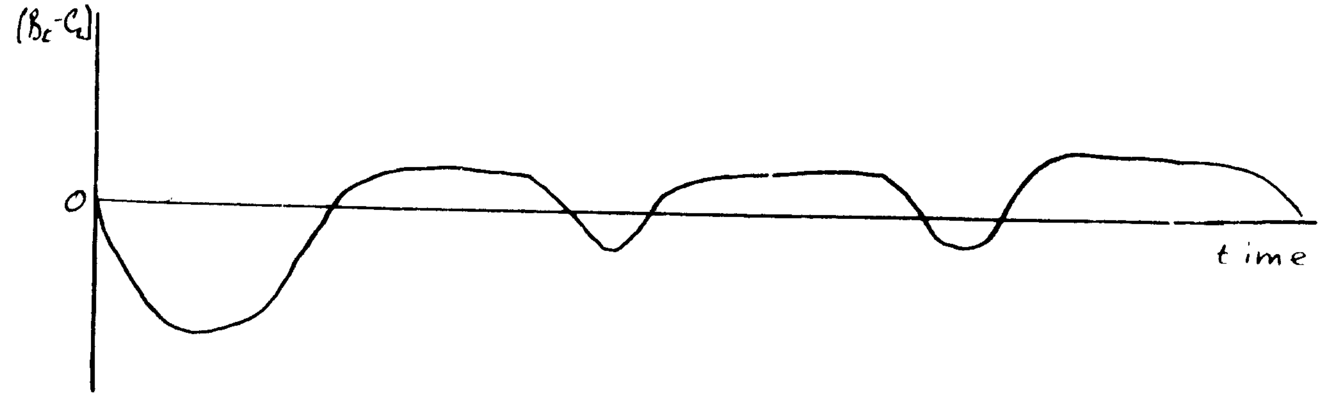

FIGURE 1b

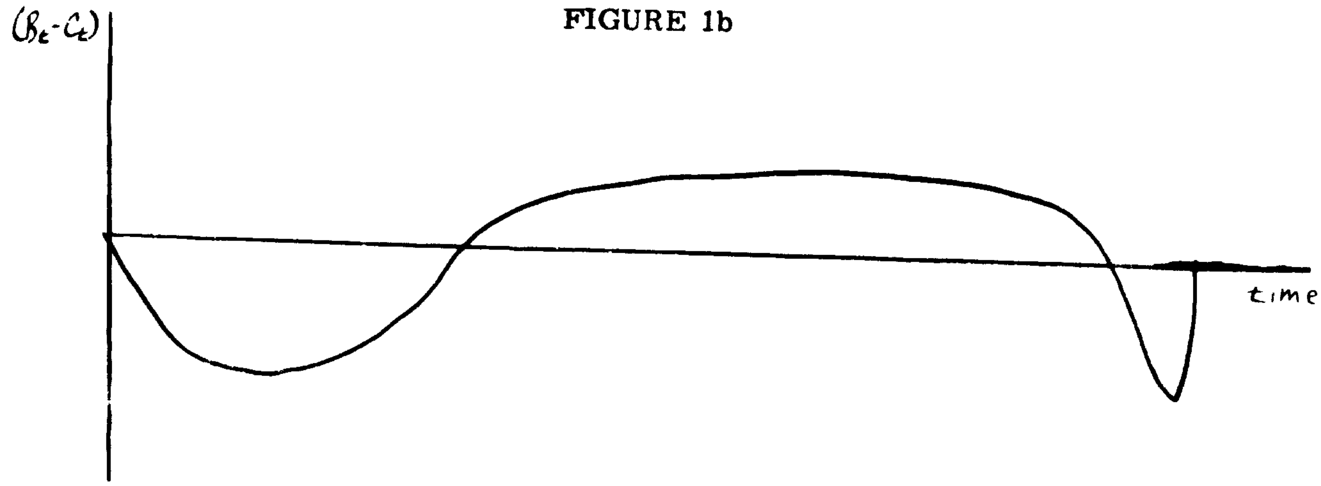

FIGURE 1c

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All cases of the types illustrated in Figures 1b and 1c will yield multiple solutions for the internal rate of return; these multiple solutions are a mathematical necessity and present a problem of choice from which there is no escape. Consider the simple case of an investment of 900 in year zero, a net benefit of 1900 in year 1, and a net cost of 1000 in year 2. Obviously, one solution for the internal rate of return is zero, for at a zero discount rate the present value of benefits is just equal to the present value of costs. But another solution is a 11.11 per cent rate, for setting $\left(\frac{1}{1+\rho}\right) = .9$, and $\left(\frac{1}{1+\rho}\right)^2 = .81$ we obtain

$$-900 + \frac{1900}{1+\rho} - \frac{1000}{(1+\rho)^2} = -900 + 1710 - 810 = 0.$$

Even where the internal rate of return can be unambiguously calculated for each project under consideration, its use as an investment criterion encounters other difficulties when some of the projects in question are alternatives to each other. Consider first a case in which all projects are strictly independent. In such a case the internal rate of return criterion will work well. By arranging the projects in descending order of internal rates of return, one can select first that project with the highest internal rate, then that with the next highest, etc., proceeding in this way until the available investible funds are exhausted. Suppose that the last project qualifying under this procedure has an internal rate of return of 8 per cent. Then 8 per cent will represent the opportunity cost of investible capital, and it becomes appropriate to evaluate the costs and benefits of all projects using this rate. Given that all the projects accepted under the internal rate of return criterion have internal rates higher than 8 per cent, and assuming these internal rates to be unique (i.e., assuming that the projects have net benefit profiles of the kind shown in Figure 1a), the present value of all accepted projects, evaluated at 8 per cent will be positive, and the present value of all rejected projects, again evaluated at the 8 per cent rate, will be negative. In this case the internal rate of return criterion leads to no contradictions.

Now, however, consider a case in which two projects are alternatives. Let project A have an internal rate of return of 20 per cent and project B have one of 12 per cent. The internal rate of return criterion would lead one always to choose project A, yet it can be shown that B might very well be preferable. If the available investible funds are exhausted, as in the previous example, at an 8 per cent rate of return, we take 8 per cent to be the opportunity cost of investible capital, and calculate the net present values of all projects using this rate. It can very well occur that the project B, in spite of its lower internal rate of return, has a higher present value than project A. For example, suppose that project B has a net benefit of $240,000 per year in perpetuity on a capital investment of $2,000,000, while project A has a net benefit of $64,000 per year in perpetuity on a capital investment of $320,000. The present value of Project B's benefits, evaluated at 8 per cent, is $3,000,000 while the project cost is $2,000,000, yielding a net excess of benefits over costs of $1,000,000. On the other hand, the present value of Project A's benefits is $800,000, from which, deducting the project cost of $320,000, one obtains a net excess of benefits over costs of $480,000. In spite of Project A's higher ratio of benefits to costs. Project B is preferable, because if one invests $320,000 in A rather than $2,000,000 in B, the best alternative use for the $1,680,000 thus saved is a "marginal" investment with an internal rate of return of only 8 per cent, on which the excess of benefits over costs, evaluated at the opportunity cost rate of 8 per cent, would be nil.

Discussions of the internal rate of return as a criterion can be found in Friedrich and Vera Lutz The Theory of Investment of the Firm (Princeton Princeton University Press, 1951), pp. 16-48, in Roland N. McKean Efficiency in Government through Systems Analysis (New York John Wiley and Sons, 1958), pp. 89-92, and in J. Hirshleifer, "On the Theory of Optimal Investment Decision," Journal of Political Economy (August, 1958). All of the above writers recognize the disadvantages indicated above.

B. Choice of Discount Rates for Use in Connection with a Present Value Criterion

1. The Marginal Productivity of Capital in the Private Sector.

It was shown above that the use of capital in a given project was justified if the benefits of the project exceeded its costs, evaluated at a discount rate reflecting the opportunity cost of capital. One highly recommended measure of the opportunity cost of capital is the expected marginal productivity of typical capital investments in the private sector of the economy. If a public sector project is to be financed by borrowing from the private sector, it is to be presumed that the funds so mobilized could, in the absence of this project, have been used to finance private sector investments; hence in this case there is a direct sense in which private-sector investment can be considered as the relevant alternative to the project. When, on the other hand, the funds to be used are part of the savings of the public sector, the connection between a public-sector project and its private-sector alternatives need not be so clear cut. If the funds available to the public sector investment authorities are sufficiently ample, it may work out that, in order to use all the available funds within a given set of projects being considered, the public sector authorities make investments having a yield of 5 per cent, even though capital in the private-sector has an expected rate of marginal productivity equal to 10 per cent. Given that the yields in both cases are worked up on the basis of social benefits and social costs, the acceptance of public sector projects with rates of return lower than those to be anticipated from additional private sector investments must be considered uneconomic. It would be preferable in this case to accept only those public sector projects exhibiting a social yield of 10 per cent, or more, and to invest any remaining public sector funds in financing additional private sector investments with an expected yield of 10 per cent or more. Thus in this case the optimal use of the funds available to the public sector leads to a result in which the private sector investments are the relevant alternatives to marginal investments in the public sector.

One case in which the marginal productivity of investments in the private sector might not be the appropriate criterion for public-sector decision-making is where the investible funds of the public sector are so severely limited that they can be exhausted on public sector projects, all of which have a higher expected yield than a typical private-sector investment. In such a case, if the limitation on public-sector funds is a [illegible] constraint, the relevant opportunity cost for public sector investments would be that rate of discount which, when used as the basis of a present value criterion, would result in the acceptance of a group of projects whose cost was just barely sufficient to exhaust the available funds. For example, in a case of severe budgetary stringency, it might turn out that using a 16 per cent rate of discount, the projects yielding a positive excess of benefits over costs would not fully exhaust the available funds, but that using a 15 per cent rate of discount sufficient additional projects would pass the present value test so as to just exhaust the given budget. In this case the opportunity cost of capital for a public sector project would be 15 per cent, in spite of the fact that private sector investments have an expected marginal yield of only 10 per cent. However, this result occurs only when the budgetary restriction on public sector projects is binding. Otherwise, in a case such as that just described, the optimal result can be achieved by the public sector authorities accepting all projects having benefits greater than their costs, evaluated at a discount rate of 10 per cent, and borrowing the required additional funds from the private sector.

Thus, the opportunity cost of capital is best measured by the marginal productivity of capital in the private sector in virtually all cases, the only serious exception being the case of a binding budgetary constraint on the investible funds of the public sector, in which case the private-sector marginal productivity of capital still remains as a lower limit to the discount rate relevant for public-sector investment decisions. We turn therefore to the problem of estimating the marginal productivity of private-sector capital. Consider any line of activity in the private sector, the line of activity being defined as including all operations producing a given product by similar production methods, for sale in the same market. An increase in the amount of capital invested in such a line of activity will augment the supply of the product in question, and may affect its price. If it does affect the price of the product, it will alter the marginal productivity of the capital previously invested in the line of activity in question, but it will similarly affect the rate of return perceived by the owners of this previously invested capital. Thus, where the newly-invested capital is of the same type as that already existing, the private rate of return to capital in the line of activity in question may be taken as a rough first approximation to the marginal productivity of capital in that line. Some problems must, however, be noted immediately. If a technological advance has occurred, it may be true that new investment—using the new technique—will have a marginal productivity—and a rate of return—equal to, say, 20 per cent, but the introduction of this technique may reduce the price of the product to the point where the return on capital invested in the old technique is but 5 per cent. The rate of return on all capital invested in the industry will be a weighted average of the 20 per cent rate on the new technique and 5 per cent on the old. And indeed it will be true, if no other complications enter into the calculation, that the marginal productivity of capital is 20 per cent for that invested in the new technique and 5 per cent for that invested in the old technique. The overall marginal productivity of capital in the activity in question will also, in this case, be a weighted average of 20 and 5 per cent, and will be measured (again barring additional complications) by the rate of return on the total capital invested in that activity. The problem here is that any new investment that occurs will use the new technique, so that the marginal productivity of capital that is relevant for current and future decisions is that 20 per cent rate obtainable from the new technique. The use of the observed rate of return in the entire activity (on both old and new techniques) therefore underestimates the rate relevant for the evaluation of current and future projects. This error could be avoided by considering the two techniques as separate lines of activity and using, in the calculation of the marginal productivity of capital to be used in project evaluation, only the 20 per cent rate arising from investment in the new technique. The difficulty with this approach stems from the way in which the available data typically appear, i.e., from the financial accounts of enterprises. In these accounts there is no way of distinguishing how much capital is invested in a new technique and how much in an old one, and likewise there is no way of allocating the income earned by an enterprise into a part attributable to a new and a part attributable to an old technique. Thus the data automatically yield rates of return which are, in our example, weighted averages of the 20 per cent and the 5 per cent rate, and it must be recognized that these estimates are biased downward when significant technological advances have recently occurred.

A second source of bias in estimating the marginal productivity of capital on the basis of observed rates of return is the presence, in some lines of activity, of monopoly elements. The effect of monopoly is to restrict production of the monopolized product and to raise its price. As a consequence, the value of the marginal product of all factors of production is raised above their respective prices. If prices are raised 10 per cent above costs, the consequence would be an element of monopoly profits consisting of 10 per cent of the wages paid, 10 per cent of the cost of materials used, and 10 per cent of the true cost of capital. The difficulty presented for measuring the marginal product of capital by the observed rate of return is that the profits appearing on the accounts of a company include the full amount of monopoly profits plus the true cost of capital; whereas for a proper measurement of the marginal productivity, they should include, in this example, only 110 per cent of the true cost of capital. Thus the measured rate of return tends to overstate the true marginal productivity of capital when monopoly elements are present.

The construction of series on the rate of return to capital in the private sector is dealt with in some detail in George J. Stigler, Capital and Rates of Return in Manufacturing Industries (Princeton: Princeton University Press, 1963), Appendix A, and in John W. Kendrick, Productivity Trends in the United States (Princeton: Princeton University Press, 1961), the latter dealing principally with the problem of measuring the stock of capital. The literature of the subject is as yet very weak on the problems of measuring the social as distinct from the private yield on private-sector capital.

There are a number of possible sources of divergence between the social and private benefits of private investment; but of these, by far the most important consists of taxes. Corporation income taxes typically account for between 25 and 50 per cent of the income generated by capital in the corporate sector, the social yield of capital (including the corporation income taxes) can thus easily be 12 per cent, even though the private yield is only 6 per cent. It would accordingly be erroneous to proceed on the assumption that the private yield on capital reflected its full opportunity cost. Of two investments with the same private yield, one of which generates corporation income tax payments equal to its private yield, and the other of which generates no tax payments at all, the former is clearly socially preferable, as it either enables the public sector to have more command over real goods and services, or, alternatively it permits the public sector to reduce some other tax and thus permits the private sector to buy more real goods and services. The indicated procedure is therefore to include corporation tax payments generated in any industry as part of the social return to capital in that industry. And if the social rate of return to capital is being estimated for the private sector as a whole, the entire yield of the corporation income tax should be added to the income perceived by private enterprises in order to convert the latter to a social concept of "income generated by capital."

Where indirect taxes exist on a final product, they lead to a situation in which the value of the marginal product of each factor of production involved in that good's production exceeds the income earned by that factor by the percentage rate of indirect tax. In this case the income from capital (gross of corporation income tax) should be augmented by a fraction of the receipts from the indirect tax, the fraction being capital's share in the value added in the industry in question.

Other sources of divergence between the private and the social rate of return on capital can arise out of divergences of the market prices of factors of production from their opportunity costs. These will be discussed in Section IV, below, in some detail, and will not be dealt with further at this place.

For an attempt at estimating the social rate of return to capital in an underdeveloped country, in which explicit account is taken of the effects of taxes and of certain possible divergences between market prices and opportunity costs, see A. C. Harberger, "Investment in Man versus Investment in Machines: the Case of India," in C. A. Anderson and M. J. Bowman eds., Education and Economic Development (Chicago: Aldine Publishing Company, 1965).

2. Market Interest Rates.

The conventional way of converting costs and benefits to present values is by the use of some market rate of interest. Market rates of interest generally substantially underestimate the opportunity cost of capital, because they fail to reflect the taxes that are paid on account of the profits of private-sector projects, and because they neglect other external benefits generated by private-sector investments, particularly where there are divergences between market prices and opportunity costs of factors of production or goods.

Two examples of the conventional view are the following:

> "It is recommended that estimates of benefits and costs [illegible] at various times should be made comparable by adjustment to a uniform time basis through the use of projected long-range interest rates. Pending the development of such rates, the average rate of return, i.e., yield, on long-term Federal bonds over a sufficiently long period of time to average out the influence of cyclical fluctuations is considered appropriate for uniform application by all agencies on the condition that adequate allowance has been made for uncertainties and risks." U.S. Inter-Agency Committee on Water Resources, Proposed Practices for Economic Analysis of River Basin Projects (Washington: Government Printing Office, 1958), p. 24.

> "Interest rates are a measure of the value attached to time differences and, hence, provide a means of converting estimates to a common time period. In calculating the costs of developing a project, interest should be charged on the project for its entire economic life and reduced to an annual basis in order to compose annual costs and benefits. The rate of interest to charge a project depends upon the rate you must pay for financing the project. Generally government financed projects can be financed at a lower rate than private industry. The government rate of borrowing is relatively risk free because the security is the general taxing power and because the overall degree of security for the loan is relatively certain. In view of these considerations, it is recommended that the expected average long-term government bond rate be used as the basis for calculating public investment costs and that higher rates be used for private investment costs." H. W. Singer, "Development Projects as Part of National Development Programmes," in United Nations, Formulation and Economic Appraisal of Development Projects (1951), pp. 123-24.

The approach reflected in both of the above quotations fails to appreciate the difference between the market interest rate on bonds and the opportunity cost of capital. Tinbergen, in advocating the use of accounting prices, has a much clearer appreciation of this distinction. He says that

> "[accounting prices of factors of production] represent the value of the marginal product to be obtained with their aid. . . . The interest rate to be applied should express the real scarcity of capital, to be derived from the marginal yield of projects as well as from the marginal rates to be paid for foreign loans" (J. Tinbergen, The Design of Development [Baltimore: The Johns Hopkins Press, 1958], pp. 40, 42.)

Tinbergen suggests the use of a 10 per cent interest rate, which is far above the rates applying to government bonds in most countries, and which undoubtedly lies closer to the opportunity cost of capital than the government bond rate.

Likewise, the Industrial Development Division of ECOSOC recognizes the unsuitability of bond rates:

"More specifically, . . . accounting interest rates may be set at least double the rates on government securities or on international loans and possibly at as high as 20 per cent" from "Evaluation of Projects in Predominantly Private Enterprise Economies," Industrialization and Productivity Bulletin No. 5 (United Nations New York, 1962), p. 30.

3. Other methods for setting discount rates.

Some of the theoretical literature rejects both the rate of interest on bonds and the private-sector marginal productivity of capital in favor of what is called the "social rate of time preference" or the "social rate of discount." This concept attempts to represent the relative valuation which society puts on a marginal amount of consumption in different time periods. For example, if "society" considered $1.10 of extra consumption next year to be subjectively equivalent to $1.00 of extra consumption this year, the social rate of time preference between the two years would be 10 per cent.

The main ground on which this part of the literature rejects market rates of return is the belief that the market, which reflects the resultant of individual, atomistic savings and investment decisions, does not give any weight to the preferences of future generations and hence tends to save "too little," with the result that the market rate of return on investment is "too high."

As Eckstein puts it,

"Social policy, as derived from the political process, may prefer rejection of present intertemporal preferences in favor of a redistribution of income toward future generations. Much of the conservation philosophy can be interpreted in these ter . Resource development is a field particularly suited to this kind of redistribution because there are genuine opportunities for making investments, part of the benefit of which will accrue in the far future. And perhaps equally important is the fact that it is in the resource area that the idea of making provision for the future of the country has caught the imagination of the public. It is not logically inconsistent for the same person to be willing to borrow at high interest rates to increase his present consumption while voting to spend tax money to build a project from which future generations will benefit, for in the case of a vote to tax, he can be sure that the other individuals in the society will be compelled to act similarly. . . . Our notion of efficiency is relative to the distribution of income; should we seek to redistribute income to future generations, the interest rate loses its meaning as an efficient price." Otto Eckstein, Water Resource Development: The Economics of Project Evaluation (Cambridge. Harvard University Press, 1958), pp. 99-100.

A more detailed discussion of this view can be found in O. Eckstein, "Investment Criteria for Economic Development and the Theory of Intertemporal Welfare Economics," Quarterly Journal of Economics (February, 1957). A somewhat similar position is expressed in Stephen A. Marglin, "The Social Rate of Discount and the Optimal Rate of Investment," Quarterly Journal of Economics (February, 1963). and by A. K. Sen in "On Optimizing the Rate of Saving," Economic Journal (September, 1961).

The difficulty that emerges from the Eckstein-Marglin-Sen position is that when the social rate of time preference is low, its use in evaluating benefits and costs is likely to lead to the acceptance of a great many projects—in all likelihood more than can be financed.

Eckstein says:

"I propose the following compromise, which is designed to preserve the long-term perspective of the federal program, yet would assure that only projects are undertaken in which capital yields as great a value as it would in its alternative employments: let the government use a relatively low interest rate for the design and evaluation of projects, but let projects be considered justified only if the benefit-cost ratio is well in excess of 1.0" (op. cit., p. 101). (See also J. V. Krutilla and O. Eckstein, Multiple Purpose River Development [Baltimore: Johns Hopkins Press, 1958].)

Marglin, in a more elaborately developed discussion than Eckstein's, develops formulas for measuring the opportunity cost of public investment when the social rate of discount lies below the marginal productivity of capital in the private sector. His formulas depend on the manner in which the public sector funds are raised—he considers the "cost" of \$1 of public funds raised at the expense of current consumption to be \$1, while the cost of \$1 of funds raised at the expense of investment is considered in his basic model to be $\$\rho/r$, when ρ is the marginal productivity of capital in the private sector and r is the social rate of discount. This assumes that \$1 of private investment would have a perpetual yield of $\$\rho$ per year, which, discounted back to the present at the social rate of discount, would have a present value of $\$\rho/r$. If the fraction θ of public funds are raised at the expense of investment, and the fraction $(1 - \theta)$ at the expense of consumption, the present value of the foregone alternatives of a dollar of public funds will be $\$[(\theta\rho/r) + (1-\theta)]$. Marglin then proceeds to recommend that the present value of the benefits stemming from a dollar of public investment should be at least equal to $\$[\theta\rho/r + (1-\theta)]$. See Stephen A. Marglin, "The Opportunity Costs of Public Investment," Quarterly Journal of Economics (May, 1963). A somewhat similar approach is followed by Peter O. Steiner in "Choosing among Alternative Public Investments in the Water Resource Field," American Economic Review (December, 1959).

The solutions reached by Eckstein, Marglin, and Steiner are all subject to a single, decisive criticism: they may lead to results in which the rate of return to investments in the public sector lies below that which could be obtained by placing the same funds at the disposal of the private sector, or by investing directly in private-sector type activities. Future generations lose, rather than gain, if funds are used for a 5 per cent public-sector investment rather than for a 10 per cent private sector investment. The public sector can, and in many countries does, provide both equity and debt financing for the private sector, and can thus assure itself that its financing of private sector activities does not entail the granting of a subsidy to the private sector but rather simply enables the public sector to obtain the same rate of return that prevails on private sector investments. Once the public sector is prepared to accept this degree of flexibility in its use of investible funds, the criterion for project evaluation reduces once again to the marginal productivity of capital in the private sector of the economy, discussed in II. B. 1. above.

The fact that the social rate of discount may lie below the marginal productivity of capital says only that the rate of investment should be expanded; it does not say that, for a given rate of investment, capital should have different marginal rates of productivity in the public and private sector. The end result of an optimal investment policy, with the social rate of discount taken as given, would therefore be a situation in which the marginal productivity of capital in both the private and the public sectors was equal to the social rate of discount. During the transition from a position in which the marginal productivity of capital in the private sector lies above the ultimate social rate of discount to a position where these are equal, the optimum path would entail so allocating the investible resources of the economy as to maintain continuing equality of the marginal rates of productivity of capital in the public and private sectors, with these rates declining together from their initial (high) level to their ultimate (lower) level as a consequence of a stepped-up rate of investment.[1]

4. Changes in the relevant discount rate through time.

The case cited in the preceding paragraph gives only one of many possible ways in which the relevant discount rate may vary through time. Another possibility—more optimistic from the standpoint of economic development—is that through adoption of superior techniques, through better management and organization, and through an improved mix of social overhead investments, the marginal productivity of capital might rise rather than fall through time. This corresponds, in technical economic language, to upward shifts in the production function through time, which more than outweigh the downward pressure on the marginal productivity of capital stemming from the effects of increased capital-intensity of production.

Actually, for those courtries for which it has been possible to estimate the marginal productivity of capital over substantial periods of time,

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[1]This view is supported in a recent paper by Arrow, who says that, so long as public investment can be financed by bonds or taxation, "the rates of return in the two sectors (public and private) should be equated at every instant of time, but the government through its bond and tax policies should aim at driving the common rate towards the natural rate of interest. The optimal policy may well involve negative bond financing, i.e., government lending or retirement of the national debt" (Kenneth J. Arrow, "Discounting and Public

there appears to have been no very significant upward or downward trend in this magnitude. Stigler, for example, finds the private rate of return to capital in U.S. manufacturing to have fallen in the 1930's to less than half the level of the late 1920's, then to have risen in the late 1940's to about 1 1/2 times the level of the late 1920's, and finally to have fallen by the late 1950's to approximately the same level as the late 1920's (Stigler, op. cit., p, 203). This experience is suggestive of the possibilities that may emerge in other contexts. In the 1930's, the conditions of the U. S. economy were such that an abnormally low rate of return on capital prevailed; in the late 1940's, on the other hand, the need to restore the capacity for production of non-military goods created a situation where an extraordinarily high yield on investment could be obtained. In neither of these instances could one reasonably expect that the then-prevailing rate of marginal productivity would be maintained indefinitely into the future. Similarly, it may occur that an underdeveloped country may face a situation in which investible funds are abnormally scarce relative to investment opportunities (e.g., when large debt service payments are due and available investment opportunities are particularly good) or one in which investible funds are abnormally abundant relative to opportunities (e.g., when the country receives a particularly large amount of foreign aid, or when its main export product experiences a

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Investment Criteria," paper presented at the 1965 Western Resources Conference, Seminar on Water Resources Research, 6 July 1965).

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temporary large increase in price, without investment opportunities expanding correspondingly).

In circumstances like these, the country should attach a "price" to the use of investible funds which is higher than the expected future price if funds are relatively scarce, and lower if funds are relatively abundant. This can be done by attaching to each year a discount rate that corresponds to the expected marginal productivity of capital in that year. Thus, if we have a project with an expected life of three years, we would discount benefits and costs expected to accrue one year hence at the rate r_1 to bring them back to the present. Likewise we would discount benefits and costs accruing two years hence by the rate r_2 to bring them back to one year from now, and then by the rate r_1 to bring them back to the present. Thus, the acceptance or rejection of a three year project would turn on whether the sum.

$$(B_o - C_o) + \frac{(B_1 - C_1)}{(1 + r_1)} + \frac{(B_2 - C_2)}{(1+r_1)(1+r_2)} + \frac{(B_3 - C_3)}{(1+r_1)(1+r_2)(1+r_3)}$$

was greater or less than zero. The general form of this criterion, for a project of N years duration, is

$$(B_o - C_o) + \sum_{i=1}^{N} \frac{(B_i - C_i)}{\prod_{t=1}^{i} (1+r_t)}$$

It is unfortunate that the great bulk of the literature on cost-benefit analysis has been based on the simplifying assumption of a constant discount rate, because this assumption fails to give guidance as to how to overcome periods of unusual stringency in the supply of capital funds or how best to take advantage of a temporarily large availability of such funds. One notable exception is the work of Pierre Massé, in which changing discount rates are discussed explicitly, and which of the analyses is carried out in such terms. See Pierre Massé Optimal Investment Decisions (Englewood Cliffs: Prentice-Hall, 1962), pp. 10-20. For an earlier discussion of the same problem, see Irving Fisher, The Theory of Interest (New York: Macmillan Co., 1930).

III. The Measurement of Benefits and Costs.

A. Projections of Demand for the Affected Product.

Projections of demand for the affected product are an important element in estimating the economic feasibility of a project and in determining its appropriate scale. The techniques of projection appropriate to any given case can only be determined by a careful study of the case itself, but certain general statements can be made.

a) The potential market for the product must be ascertained (e.g., local, regional, national, international).

b) Factors influencing the intensity of demand for the product in this market must be isolated and projected.

c) On the basis of (b) the overall level of demand for the product must be projected.

d) The prospects of expansion of existing alternative sources of supply must be examined and corresponding projections made.

e) The prospects of new sources of supply appearing in the future must be evaluated, and, if they are likely to appear, supply from these sources must be projected.

For any market, a key factor influencing demand is the level of income, and the projection of this magnitude is therefore of key importance. Unfortunately, there is no touchstone to estimating the rate of growth of income. In particular the rate of growth of income is not directly tied to the rate of capital accumulation in the community, but is the resultant of many factors, of which capital accumulation is only one. [See, for example, R. M. Solow, "Technical Change and the Aggregate Production Function," Review of Economics and Statistics (August 1957), and E. F. Denison, The Sources of Economic Growth in the United States and the Alternatives Before Us (New York: Committee for Economic Development, 1962).] This fact introduces considerable uncertainty into all income projections, and suggests that basing such projections mainly or exclusively on capital-output ratios is unwise. The most appropriate procedure appears to be to assess the relative contribution of certain key factors (capital formation, labor force increase, improvement in labor force quality, and technical advance) to past economic growth, to assess their probable future strength, and to estimate the likely rate of income growth on this basis.

Once having projected the rates of growth of income, population, etc., the problem of estimating demand for a particular product depends on the nature of the product. For most consumer goods, income and relative price appear to be the key determinants of demand; so that demand, expressed as a function of price, can be projected once the course of income is known. However, for products which are materials or intermediate goods, the best procedure is to estimate the demand for each type of end-use separately, and to project the demand for the material according to the projected growth of each of its corresponding end-uses. (See U.S. President's Materials Policy Commission, Resources for Freedom [Washington: Government Printing Office, 1952], Vol. II, chapter 22, "Projection of 1975 Materials Demand," United Nations, ECLA, Analyses and Projections of Economic Development, I [New York: United Nations, 1955], pp. 32-33, and United Nations, ECLA, Manual on Economic Development Projects [New York: United Nations, 1958], p. 24.) Care must be taken, however, to allow for possible future changes in the quantity of the material used per unit of each end-use product. Capital goods demand should be projected on the basis of the amounts expected to be required for replacement, plus the additional amounts needed to produce projected increases in the final product of the activity in which the capital goods are used. (For an example, see "Projection of Demand for Industrial Equipment," United Nations, Industrialization and Productivity Bulletin No. 7 [1964].) Once again, it is important that prospective development of improved and competing types of equipment be taken into account.

B. Projections of Product Prices.

Since the market price of the output of a project is the principal element in estimating the benefits to be obtained, it is important that a project analysis should include projections of the probable path of this price through time. Project analyses need not be concerned with possible movements in the general level of all prices (i.e., general price inflation or deflation), as a parallel movement of all prices and costs would not alter the real cost benefit relationship. However movements of relative prices can have a determining influence on the worthwhileness of a project.

The best general procedure for projecting the prices and costs relevant for a project's analysis is to project their movements relative to the general price level. Concerning the price of the output of a project, one must therefore attempt to judge whether the price will move more or less than the general price level, and if so, by how much. Having projected in index form the relationship P_i / P_g, where P_i is the price of the project's output, and P_g is the general price level, for each year of the expected life of the project, this index is then applied to the initial year's product price, P_{io}, in order to express future year's prices in monetary units of the initial year's purchasing power. Thus the projected price series would be of the form $P_{io}(P_{it}/P_{io})(P_{go}/P_{gt})$.

The factor $(P_{it}/P_{io})(P_{go}/P_{gt})$, will average out to unity over the whole economy when the appropriate weighted average is taken, for

$$\sum_i \frac{Q_{io}P_{io}}{(\sum_i Q_{io}P_{io})} \cdot \frac{P_{it}}{P_{io}} \cdot \frac{P_{go}}{P_{gt}} = \left(\frac{\sum_i Q_{io}P_{it}}{\sum_i Q_{io}P_{io}}\right) \cdot \frac{P_{go}}{P_{gt}}$$

$$= \frac{P_{gt}}{P_{go}} \cdot \frac{P_{go}}{P_{gt}} = 1.$$

Thus for a typical commodity, the projection of a constant product price is likely to be justified. However, relative prices do exhibit substantial variations over time, and it is important to attempt to identify situations in which a particular price is likely to rise or fall relative to the general index of prices. In general, for industrial products, the course of prices will be the resultant of changes in input costs on the one hand and improvements in technology (including economies of scale) on the other. Since the wage component of input prices is likely to rise over time, the question largely centers on whether future technological advances will be sufficient to offset this force. In many industries, some indication of the likely force of future technological advances can be obtained from the processes that today are being studied for possible future application, and projections can be made on that basis. In some cases, the present market for the product may be found to be abnormal, in the sense of a current shortage of output causing an unusually high price or a current glut of supply causing an unusually low price. It is particularly important that such situations be identified, as in these cases it is highly unlikely that the assumption that the price will remain at its present level will be warranted.

Although most discussions of cost-benefit analysis pay lip-service to the principle of taking expected price changes into account, they generally do not go beyond this. Probably the most extensive treatment of the problem—itself not very extensive but at least attempting to face the major issue—is to be found in the ECLA Manual, pp. 26-28.

C. Projections of Cost Components.

1. Wages.

One of the gravest deficiencies in the existing literature on project evaluation is its failure to allow, explicitly and systematically, for the expectation that wage rates will rise regularly in the future, relative to product prices. In an economy experiencing successful economic development, it can be anticipated that real wages will rise at a rate of 2 per cent per year or more. Thus, whereas the price of the average product will change in accordance with movements in the general price level, wages will increase at a significantly greater rate. The rise of wages at a greater rate than that of prices is possible because of the continued improvement of productive techniques. But in a given project, the technique of production is often determined by the design of the project itself. In this case, labor requirements will be determined by the layout of the plant, the types of machinery installed, etc. Future rises in wages will not in this case be accompanied by reductions in labor requirements, hence project costs will increase to reflect the rise of real wages.

A proper evaluation procedure should surely take into account expected rises in real wages. In those cases where future labor-saving innovations are anticipated, which will be applicable to the project in question, these may be taken into account, including in the project analysis the expected cost of introducing the innovations as well as the reduction in labor requirements that is expected to follow.

2. The Exchange Rate.

The exchange rate is an exceedingly important price for project evaluation, and an adequate projection of its expected future course through time is therefore necessary. As with other types of prices, it is movements of the exchange rate relative to the general price level which are of interest. Three key questions should be borne in mind in developing exchange rate projections.

a) Does the present exchange rate reflect the normal forces of demand and supply, or are certain abnormal forces present which produce an exchange rate that is unlikely to be maintained in the future? Abnormal forces might reflect unusually high or low prices for key export (or import) commodities, unusually large capital movements and/or receipts of foreign aid, etc.

b) What are the likely trends in the basic demand for imports and the supply of exports? Here one must take into account not only the effects of secular income growth, but also the effects of the changing composition of production. Thus, projected expansions of export production, or of that of import substitutes, would influence the probable future course of the exchange rate.

c) What are the likely changes in government policy with regard to import restriction? Here one can expect that the liberalization of trade controls will produce a higher price of foreign currency, and their tightening a lower price, than would be the case with unchanged policies.

3. Other Costs.

The prices of inputs that are manufactured products can generally be projected by the same method as was suggested above for projecting the price of the output of a given project; i.e., as a resultant of expected changes in input costs and expected improvements in the technique of production. This procedure is based on the generally valid assumption that the prices of manufactured goods are largely cost-determined.

Minerals and agricultural products, however, are not typically as elastic in supply as manufactured goods. Hence their projection requires an analysis of the likely movements in both supply and demand. Moreover, because of the characteristically low price-elasticity of demand for these products, it can readily occur that the price observed currently is far different from the price to be expected in the longer term future, after the level of production can be adjusted to accommodate the demand situation.

4. "Annualized" Benefits and Costs.

The many possibilities listed above of prices and costs changing over time, as well as the likelihood (discussed in Section II) that the relevant discount rate will itself change over the life of a project, indicate the necessity of carrying out project evaluation by projecting expected benefits and costs on a year by year basis, and then discounting them back to the present by the appropriate discount factors. The often-recommended procedure of attempting to put all benefits and costs on an annualized basis (see ECLA Manual, pp. 198ff.) entails the possibility of dangerous oversimplification. As it leads one to presume that all the relevant components of benefits and costs will be (comparatively) constant over time, the "annualization" approach tends to distract attention from the whole set of problems considered in this section.

D. Indirect Benefits and Costs.

In addition to its direct benefits and costs, a project may induce a series of indirect effects, which in principle should be taken into account in its evaluation. These indirect effects are the results of changes that take place in the rest of the economy as a consequence of the project in question having been undertaken. Obviously any project is likely to have some perceptible effect on the demand and supply of goods produced by other industries, the main effects of this type being in the industries which supply the materials used by the project, and the industries which supply goods which are either complementary to or competitive with the project's output. If, as a consequence of a project, changes occur in the output of an industry for which at the margin, social benefits equal social costs, no adjustment need be made. But if changes occur in the output of industries for which benefits exceed or fall short of costs, at the margin, an adjustment is in order. The appropriate adjustment is the difference between marginal social benefit and marginal social cost, per unit of output in the industry in question, times the change in the output of that industry which is induced by the project under consideration.

The task of measuring indirect benefits thus can be reduced to (a) ascertaining those industries or activities in the economy for which marginal social benefit (MSB) is likely to differ from marginal social cost (MSC), (b) estimating the magnitude of the difference, for each such industry, per unit change in its output, and (c) estimating the likely change (ΔQ) in the output of such industries as a consequence of the project being evaluated. Having done this, the estimation of indirect benefits can be calculated by the formula $\sum_i (MSB_i - MSC_i)\Delta Q_i$, where the subscript i varies over all industries for which $MSB_i \neq MSC_i$. (See U.S. Inter-Agency Committee on Water Resources, Proposed Practices for Economic Analysis of River Basin Projects, p. 8.)

IV. Use of Social or Accounting Prices in Industrial Project Evaluation

The early work on cost-benefit analysis did not advise the use of social or accounting prices. An example is the following.

"Ideally, measurement standards in project evaluation should reflect the interests of society as a whole; as such, these standards should be concerned with 'real' costs and benefits. However, it is not practicable to establish and apply 'real' costs and values. Estimates would be in theoretical terms rather than in terms of a monetary unit. All things considered, the most satisfactory approach would result from using prices estimated as they are expected to be at the time when costs are incurred and benefits received. . . . This procedure is recommended as the best available method. It permits a useful working relationship with repayment determination. It takes account of future prices and price relationships based on the best judgment at hand" (H. W. Singer, "Development Projects as Part of National Development Programmes," in United Nations, Formulation and Economic Appraisal of Development Projects [1950], Book I, pp. 121-22).

This view is in marked contrast with the tone of the more recent literature.

"The market price would represent the true value of goods and services, if the law of supply and demand operated freely, under perfect competitive conditions, with full employment of all resources and complete mobility of all factors. If because of any interference, obstacles, or regulations, these conditions do not exist, then the price system will be distorted; it will not correspond to that ideal system of equilibrium nor represent the value of the factors from the point of view of the community as a whole. It is therefore considered necessary to correct market prices, in order to obtain what has been termed the 'social cost' of the factors." United Nations, Economic Commission for Latin America, Manual on Economic Development Projects (New York, United Nations, 1958), p. 203.

"As in the choice among sectors, the basic criterion that is recommended for comparing projects is the social return on the capital invested in each alternative use. . . . Labor, imported materials, and export and import substitutes are valued at accounting prices. The remaining inputs are valued at market prices except for a few important elements, such as electric power and transport, for which the market price may seriously understate the amount of resources used in their production. In these cases accounting prices should be calculated also." United Nations, Economic Commission for Asia and the Far East, Formulating Industrial Development Programmes (Bangkok: United Nations, 1961), p. 39.

"Under the circumstances, a selection of projects based on market prices will result in a misallocation of resources, in the sense that there will be a heavy strain on the resources that are under-priced while part of the resources that are over-priced will be left idle, so that the aggregate yield of the selected projects will fall short of the maximum yield that could have been obtained from the available resources. It is thus necessary to introduce into the evaluation procedure a device intended to restrain the use of under-priced factors and stimulate the use of the over-priced ones. [illegible] can be accomplished, . . . [by [illegible]] the evaluation on 'shadow' or 'accounting' prices instead of the market prices. The accounting prices are intended to reflect as accurately as possible the intrin[illegible] values of the factors involved" ("Evaluation of Projects in Predominantly Private Enterprise Economies" in United Nations, ECOSOC, Industrialization and Productivity Bulletin, No. 5 [New York], 1962), p. 29.

There can be no doubt that the recent trend toward consideration of accounting prices represents in principle a substantial advance over [illegible] alternative position, since it attempts to take into account the effects of divergences between market prices and social costs while the alternative approach does not. However, the problem still remains of obtaining adequate estimates of the appropriate accounting prices to use, and it must be admitted that this aspect of the problem has not been thoroughly explored in the literature. We turn therefore to the examination of this question for the main types of prices.

A. Accounting Prices for Labor.

The "shadow wage," or accounting price of labor, is an elusive magnitude to estimate, particularly because of the great variety of skills and types of labor, and because of regional immobility of that factor. It can therefore readily occur that the opportunity cost of agricultural labor might be quite low, but the opportunity cost of employing the same labor in industrial projects in the cities might be considerably higher. It is necessary, when considering the accounting price of labor, to be specific both as to region and as to skill, and to recognize that it is generally not possible to obtain even the most unskilled labor in urban areas at wages rates similar to those paid such labor in rural places. Thus the accounting price of urban labor should not be considered to be the actual wage received by similar labor in rural employments, but should rather be based (a) on the wage that is required in order to voluntarily attract this type of labor from rural to urban employment, plus (b) an adjustment factor to reflect the higher costs of providing social overhead facilities for urban as against rural workers and their families. It is not correct, as suggested in the ECLA Manual on Economic Development Projects (p. 205), to consider the agricultural wage as the opportunity cost or accounting price of labor diverted to urban employment.

Similarly, the existence of unemployment should not be taken to mean that the accounting price of labor is zero, unless the unemployment is so widespread as to include substantial fractions of the labor force of every type and skill. In general it is [illegible] that the more highly skilled grades of labor will have accounting prices at or very near to their market prices, as these grades of labor are typically in relatively scarce supply even in periods when the unemployment rate for the total labor force is relatively high. Even for the lower skill grades, the phenomenon of unemployment cannot be taken as direct evidence that the accounting price of labor is substantially below its market price. The unemployment rate must be viewed as the outcome of a number of forces: plant shutdowns, normal labor force turnover, migration to the city, etc. Suppose that as a consequence of these forces, six per cent of the urban labor force is at any moment unemployed, and that a new project is established which will occupy 1000 workers. This new project will also have plant shutdowns, seasonal variation in its demand for labor, normal turnover, etc., and it can very well be that over the year this new plant will engender for these reasons an average unemployment equal to six or more per cent of its own labor force. In this case it might be concluded that the opportunity cost of labor for the new plant was given by the market wage rate, in the sense that at that wage rate it would be drawing 1000 workers from the market, who would have been employed 94 per cent of the time and unemployed 6 per cent of the time, and it will itself employ them 94 per cent of the time and leave them unemployed for the remainder.

[illegible] is not contended that the [illegible] should be used as a guide in attempting to arrive at accounting prices for labor; it is merely [illegible] as an example of a case in which the existence of reasonably significant unemployment might plausibly be interpreted as being consistent with an accounting wage equal to the market wage.

Actually, the estimation of accounting wage rates for labor classified by different skills, types, and regions is an extremely complex and important area of research which deserves much deeper study than it has had. Such research should take into account not only the forces of seasonality, normal turnover, and shutdowns mentioned above, but also should investigate the forces which are operating to keep the market price of labor above its opportunity cost. These latter forces include wage rates set either legally or by union agreement, but often there are large segments of the labor force which are unprotected by either of these means. It is generally to be presumed that in these segments of the labor force the wage rate reflects opportunity cost; and such wage rates can often be taken as minimum estimates of the accounting prices of labor of similar skills and types in the industries and activities in the same region in which labor is protected by minimum wage rates and/or union agreements.

Attempts to specify the nature of the discrepancies between market and accounting price for labor are necessary for another reason as well—the projection of how these discrepancies are expected to change in the future. It is to be anticipated that, in a developing economy, gross differences between market and accounting wages will tend to be eliminated over time, but the process and speed by which this occurs depends upon the source of the initial discrepancy. In any event, it is reasonable that a cost-benefit analysis should allow for at least the gradual reduction over time of such discrepancies—thus confronting us once more with the importance of carrying out a project analysis through

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a year-by-year projection of benefits and costs rather than attempting to summarize these solely through annualized estimated largely based on the current situation.

B. The Accounting Price of Foreign Exchange.

Whereas labor is characterized by great heterogeneity and substantial immobility, foreign exchange, at least in a world of convertible currencies, is a basically homogeneous commodity that can readily be shifted from one use to another. Thus, where in principle numerous accounting prices will be required for labor, just one will typically be required for convertible foreign exchange. Nonetheless, serious difficulties arise in estimating this accounting price, owing to the many distinct uses to which foreign exchange can be put. This can easily be seen by considering a tariff structure in which some items are not taxed at all, while others are taxed at, say, 20 per cent, and still others at, say, 50 per cent. If the exchange rate is 5 rupees to the dollar, a dollar spent on imports of category 1 will bring in goods having an internal value of just 5 rupees, while a dollar spent on category 2 will bring in goods having an internal value of 6 rupees, and the same dollar spent on category 3 will bring in goods having an internal value of 7 1/2 rupees. The value produced by the dollar thus varies with its use.

The key to estimating the accounting price of foreign exchange is to estimate the likely pattern in which incremental dollars would be distributed over the various categories of goods. If it was anticipated that extra dollars would be spent 50 per cent on category 1, 30 per cent on category 2, and 20 per cent on category 3, then the internal value of a marginal dollar would be (.5)(5) + (.3)(6) + (.2)(7.5), or 5.8 rupees.

This procedure for estimating the accounting price of foreign exchange can also be applied to goods which are subject to licensing or other restrictions rather than tariffs, but here one must estimate independently on the basis of available market evidence what is the internal value of a dollar's worth of each type of goods so restricted.

The basic difficulty with the suggested procedure is estimating the pattern in which incremental foreign exchange will be distributed among imports, but this can be at least roughly estimated on the basis of past marginal distributions of foreign exchange, e.g., by ascertaining [illegible] import statistics how the increase in foreign exchange availabilities from, say, 1960 to 1965 was in fact used. More accurate estimates could be obtained by serious econometric study of the demand for different categories of imports. In some cases, the exchange licensing authorities might themselves have a policy indicating how they would allocate any additional sums becoming available.

The procedure outlined above assumes that the incremental foreign exchange will be used to augment the total supply, i.e., that it will not force down the price which exporters receive for foreign currency. If it does this, then the above procedure would be applied to estimate the value of the net increment to the supply of foreign currency, and the rate of exchange applicable to exports would reflect the value attaching to the use of incremental foreign currency to displaced exports.

This procedure is closely attuned to the economic reality, as such it is far preferable to the procedure recommended in the ECLA Manual (p. 204) of arriving at the accounting price of foreign currency on the basis of a purchasing power parity formula. The great difficulty with the purchasing power parity approach is that it is valid only when the causal factors at work between the two situations being compared were completely monetary, i.e., differential rates of inflation in the two countries whose currencies are being compared. But the function of the exchange rate in cost benefit analysis is basically as a guide to resource allocation. Rather than looking backward to a base year and being concerned with monetary changes having taken place in the past, cost-benefit analysis looks at the present and the future, and attempts to evaluate alternative projects in "real" terms. There can be no doubt that a direct effort at estimating the value of the economy today of the goods an extra dollar is likely to buy forms a better basis of judgment of the value of foreign exchange than a mechanical extrapolation from some past year. By the same token the analysis of the current value of foreign exchange, in the manner indicated above, provides the most reasonable starting point for the projection of the time path of this variable in the future.

C. Accounting Prices for Inputs of Materials.

The problem of arriving at accounting prices for materials is in some respects similar to that for foreign exchange. Suppose that the market price of a material is $5 and its social cost of production is $4. A project under consideration will use some of this material, and the question arises of setting the appropriate accounting price. The problem that faces us can be summarized by considering two extreme possibilities. On the one hand, the output of the material may remain constant, and the supply for the project under consideration may be diverted from other uses. In this case the appropriate accounting price is the market price which can be taken to represent the marginal value of the material in its other uses. On the other hand, the project's demand for the material might be met by increasing its supply: in this case it appears that the appropriate accounting price is $4, the true economic cost of producing each added unit. This apparently plausible conclusion is, however, not always correct. For suppose that the materials-producing industry were to augment its output by the same amount, in the absence of the project being considered. This increased output could, presumably, be sold at prices in the neighborhood of $5—say, between $4.90 and $5.00—on the open market. Some reduction in price would presumably have to occur in order to induce additional sales, but unless the demand of the project in question were very great indeed relative to the initial level of production of the material, or unless the overall demand for the material were very inelastic, the required reduction in price would not be very great. Thus even if the production of the material does expand in response to the project's additional demand, the opportunity cost of the project's use of the material can be approximated by the market price of the material rather than by its social cost of production.

The use of the market price of materials as their social or accounting price has another advantage in avoiding the double-counting of benefits among projects. Suppose that project A is a construction project, in which substantial amounts of cement will be used. Suppose, further, that project B is a project to expand capacity in the cement industry. If cement is valued at its market price in evaluating project B, and is also valued at its market price in evaluating project A, we can be sure that there will be no double counting of benefits. But if cement is valued at $5 in evaluating project B and at $4 in evaluating project A, the difference of $1 per unit will be counted as benefits for both projects—clearly a dubious procedure. In order for $4 to be a valid accounting price for the cement used in project A, project B must meet two stringent conditions. (a) present value of benefits equals present value of costs, when the cement is priced at $4, and (b) the cement produced by project B must have a value no greater than $4 in alternative uses (other than project A).

Having thus indicated the grounds for preferring the use of market prices for materials inputs, it is imperative to qualify this preference by noting that when the output of a material will in fact expand as a consequence of a given material-using project, and where that material does not have an alternative use in which its value lies above the cost of producing the material, and where the market price is nonetheless above the cost of producing the material, an accounting price equal to the cost of production of the material is appropriate for use in evaluating the material-using project. Examples of cases meeting these conditions can indeed be found. Perhaps the clearest case is one in which (a) the material [illegible] an infinite elasticity of supply at a price equal to its unit cost of production, and (b) a tax exists which makes the market price higher than unit production cost. In this case any expansion in the industry has social benefits greater than social costs by the amount of the additional tax collections. Moreover, even though with a cost of $4 and a market price of $5 (=$4 plus $1 tax), added production of the material could be sold if offered at a price of $4.95, it will not be so sold because this would entail a loss to the producers. In fact, the expansion of output of the material is strictly contingent on the emergence of additional demand at a price of $5, and, so long as the tax remains at $1 and the net-of-tax supply price remains at $4, each increment of demand at the price of $5 will in fact generate the additional supply necessary to meet it. And, assuming the supply price truly reflects the social costs involved, the net-of-tax price can in such cases be used as the accounting price of the material. Even in such a case, however, it might be preferable to use the market price of the material in the basic calculations of the direct costs and benefits of the material-using project, and to count the extra tax payment generated by the project on account of the expansion of material supply as an indirect benefit of the project. The two procedures amount to the same thing, and counting as indirect benefits the excess of benefits over costs generated in other activities as a consequence of a given project permits one to adopt the standard rule that accounting prices of materials should always be their market prices.

D. Accounting Prices for the Output of a Project.

Where products are freely sold at the market price, the social benefit attaching to such products should be measured by their market prices. Where, however, goods are subject to rationing or licensing, accounting prices different from market prices are indicated. In this case the accounting prices should attempt to reflect the intrinsic value of an increment of output to those who purchase it.

Where products are subject to indirect taxation, the market price inclusive of tax should be used as the measure of benefits. This is seen clearly by the U.S. Inter-Agency Committee on Water Resources:

> "To the extent that taxes are reflected in the market prices of goods and services, such taxes . . . will have been considered in estimating the value of the goods and services produced by . . . development projects. No deductions for taxes in market prices should be made since this would reduce the value of benefits below the actual appraisal of the market as indicated by consumers' preferences of willingness to pay" (op. cit., p. 30).

The ECLA Manual, on the other hand, recommends elimination of taxes and subsidies on the ground that "greater or lesser customs duties or sales taxes cause variations in selling prices, unrelated to the effort involved. . . . Thus variations in the amount of sales tax, or the list of goods to which it is applicable, can vary the apparent productivity of projects employing such goods or services, distorting their relative position in the priority scale, although there have been in fact no changes in productivity. Similar observations can be made for subsidies, inasmuch as they are 'negative taxes.' " (op. cit., p. 203).

The position taken in the ECLA Manual is difficult to interpret; as it does not distinguish clearly between taxes upon materials inputs and taxes upon the output of a project. In the example given, the reference appears to be to materials inputs. If correction for taxes and subsidies on materials inputs is all that is meant, then no exception can be taken to the statement. Taxes on materials, as indicated above, mean that benefits exceed costs in the materials-producing industry, and a project can in this case legitimately consider the additional taxes generated on account of its increased use of materials to be an indirect benefit of the project.

On the other hand, if the statement is taken to refer to taxes on the output of a project as well as on materials, one must take exception to it, the value to purchasers of the product being the price that they pay for it, which clearly includes the tax.

The only exception to the general rule that taxes paid on the product of an activity are to be included in the benefits of that activity is the case in which the taxes are designed to correct a previously-existing disequilibrium between social benefits and market price. Thus, if an activity produces a product with a price of $1, but the consumption or production of that product engenders external diseconomies of $.10, the market equilibrium will be one in which, at the margin, consumers of the product receive a benefit of $1, but others suffer an added cost of $.10 for each unit consumed. In this case a tax of $.10 would be indicated as a corrective measure. The price including tax would be $1.10, the consumers of the product would have a benefit, at the margin, of $1.10, but other consumers would lose $.10 per unit, so that the total social benefit would be $1 per unit, in this case being the market price less the tax. Since in actuality virtually no taxes are levied for the purpose of overcoming the external diseconomies associated with the consumption or production of a product, the general rule should be to measure benefits by market prices including taxes, and to deduct from such benefits any identifiable external diseconomies. In short, since no presumption can be established that the existing taxes are an appropriate measure of external diseconomies, or that existing subsidies are an appropriate measure of external economies, market prices gross of taxes should be taken as the proximate measure of benefits, and in the project analysis itself the attempt should be made to correct for external economies or diseconomies associated with a project, either in the production of the project's output or in its consumption. It is to be anticipated that cases of significant external effects of this type will be rare, and not closely related to the amounts of tax or subsidy on the product in question.

E. Accounting Prices Obtained from Linear Programming Models.

It has sometimes been suggested that accounting prices be obtained on the basis of a linear programming model. (See H. B. Chenery and Paul G. Clark, Interindustry Economics [New York. John Wiley and Sons, 1959], chapter 11.) This approach has proved highly valuable in the programming of activities within a firm, and its successful extension to problems of greater scope is a distinct possibility. However, it is unlikely that this technique will be able to yield relevant accounting prices for a national economy as a whole. In principle, this would require an accurate description of all actual and potential productive processes within the economy, and an accurate inventory of its resources. Moreover, it should also entail a study of the transferability of resources from one category to another (i.e., how many factory operatives could work effectively as carpenters? how many could be trained to do so at a given cost? etc.). These requirements of basic data go far beyond the foreseeable possibilities. As a consequence, the application of linear programming techniques in practice requires that the problem be drastically oversimplified— by assuming that one or two or three processes can describe the activities available to an industry, by aggregating industries into a few broad groups, by considering all labor to be homogeneous, etc. The resulting "shadow prices" that emerge from the analysis can, unfortunately, be very sensitive to the way in which the simplification is done, and as a consequence one cannot place much faith in the results emerging from any particular simplification. Since drastic simplification is an unavoidable necessity for using linear programming models for an entire economy, there is no way of avoiding serious uncertainty as to the validity of the resulting "shadow prices."

V. Problems of Timing

A. Choices among Projects of Different Productive Lives and Different Gestation Periods

As the analysis of Section II showed, the problem of placing projects with different time profiles on a comparable footing reduces to the problem of obtaining an appropriate set of discount rates, reflecting for each point in time the opportunity cost of capital at that time. Once this set of discount rates (r_1, r_2, ..., r_N, for years 1, 2, ..., N in the future) is obtained, the relevant criterion for project choice is to maximize the net present value of the entire investment operation, considering investments to be made this year as well as investments to be made in future years.

The particular relevance of the pattern of discount rates to choices among projects lies in the fact that high discount rates weigh heavily against projects with long gestation periods and long productive lives. Thus a project with a one-year gestation period and a total cost of $1000, would have to yield $200 per year in perpetuity in order to be justified at a 20 per cent discount rate, starting a year from now. But a project whose construction costs were spread out over five years, in equal quotas of $200 per year, would have to yield $318 per year in perpetuity, starting five years from now, in order to pass the 20-per cent test. The required absolute benefit increases rapidly with the length of the gestation period—if the same $1000 of investment were spread evenly over a 10-year gestation period, the required yield would be $559 per year in perpetuity, in order to make the investment worthwhile at a 20 per cent rate of discount.

By the same token, the length of duration of the benefit stream takes on less importance at high discount rates than at low ones. The present value of $100 per year, in perpetuity, at a 20 per cent rate, is $500; but the present value of $100 per year for just the next ten years is $419, or nearly as much as that of the perpetual stream.

These considerations come to be of crucial importance when the relevant discount rates are high, which is likely to be the case for most underdeveloped countries. Not only is it true that the private rate of return tends to be high (probably 10 per cent or more) in these countries, but this rate of return has to be adjusted upward to reflect both taxes attributable to capital and differences between the market prices and opportunity costs of associated factors of production, in order to arrive at an estimate of the social rate of return. In particular, any substantial excess of the market price of labor over its opportunity cost is likely to raise the social rate of return to capital significantly above the market rate. This point is clearly seen by Tinbergen, when he says, "Very probably the equilibrium level of wage rates will be considerably less than market wages. On the other hand, equilibrium interest rates probably are much higher than market rates" (Design of Development, p. 39).

Considerations of gestation periods and productive life are important in the choice of the scale of a project as well as in choosing among different projects. Obviously, the scale of a project will affect the pattern of both costs and benefits through time. The optimal scale for a project at any given point in time is that scale for which the present value of benefits minus costs is a maximum. If scale is a continuous variable, then optimum scale is reached when the increment to the present value of benefits stemming from a small expansion of scale is just equal to the increment in present value of costs associated with that same expansion. On these points see Friedrich and Vera Lutz, op. cit., pp. 22-32, and Pierre Massé, op. cit., pp. 42-81.

B. Criteria for Deciding when to Postpone a Given Project

The existing literature on cost-benefit analysis typically is not at all explicit on the question of when to initiate a project. Failure to consider this choice can lead to serious mistakes, however. Suppose, for example, that a project could be constructed this year for a capital cost of $1000, and would then produce a stream of expected net benefits having a present value of $1050, evaluated at the relevant set of discount rates. It appears that this project is worth doing. Yet suppose that the same project, constructed next year, would have an expected capital cost of $1050, and an expected present value of net benefits of $1150. The net present value of the project would be $100, evaluated as of next year, or $\$100/(1+r_1)$, evaluated as of this year. Obviously, it pays to postpone construction of the project, so long as r_1, the rate of discount applicable for comparisons between this year and next, is less than 100 per cent.

The solution to the pure timing problem, of when to do a particular project, is simply an application of the general present value rule. Let N_i be the net present value, evaluated as of today, of the project in question if it is to be constructed in the year i. The optimum construction time is then that year i^* for which N_i is at its maximum. N_i can vary with i because the capital costs of the project will depend on the date of its construction, and/or because the net benefit accruing in any future year will vary, depending on the date of construction (i.e., depending on the age of the project), and/or because of the fact that in postponing a project for a year we lose the first year's net benefits and gain an extra year's net benefits at the end of the project's life. All of these elements are incorporated in the calculation of N_i for various starting times, and in the procedure of choice which choses i^* to maximize N_i.

A particularly simple special case of the timing choice occurs when net benefits accruing in any year depend only on the year (in the sense of calendar time) and not on the age of the project, and in which the anticipated capital cost of constructing the project does not change through time, and in which the project has an infinite life. In this case, provided that the net benefit stream is an increasing function of time in the neighborhood of the optimal construction date, the optimal construction date is that point in time i^* in which the first year net benefits of the project are just equal to its cost of construction times the interest rate r_{i^*+1}. The reasoning behind this is simple. Regardless of whether the project is constructed in the year 0 or in year 1, it will be in operation from year 2 onwards. Therefore all net benefits from year 2 onwards will be present in either case, and the decision whether or not to postpone the construction of the project from 0 to 1 cannot depend on them. The postponement decision turns simply on the question whether the net benefits to be obtained in the year 1, which will be enjoyed if the project is already constructed by then, are sufficient to compensate for the cost of constructing the project one year earlier (i.e., in year 0 rather than year 1). The cost that is entailed in constructing earlier is simply the interest rate reflecting the opportunity cost of capital between 0 and 1, which we have denominated r_1. Thus the fact that net benefits will increase in the future does not justify the construction of a project now. The time to construct the project is when the immediately forthcoming benefits are sufficient to justify the immediate use of the capital funds in question.

In a slightly less simple case, if construction costs are expected to increase between this year and next, the requirement for construction this year is that the net costs of postponement (which now consist of the net benefit of year 1 plus the increase in construction costs between year 0 and year 1) be less than r_1 times the capital cost of constructing the project in year 0. Thus a project whose capital costs are expected to increase with postponement will qualify for earlier construction, while one whose capital costs are expected to decrease with postponement will require further delay of construction than was indicated in the previous example, which assumed capital costs not to vary with the date of construction. These modifications can be of some importance, for in some industries expected improvements in technology can lead to reduction over time in the capital cost of a project, while in other lines expected rises in labor and materials costs can work in the opposite direction.

An excellent discussion of the timing problem, including a consideration of the case of projects of finite life, which reveals only minor differences from that just outlined for the case of infinite life projects, is to be fou'.d in Stephen A. Marglin, Approaches to Dynamic Investment Planning (Amsterdam: North Holland Publishing Co., 1963), esp. pp. 9-34.

C. The Relation of Investment Decisions and Timing to Uncertainty and Risk

The conventional approach to making allowance for risk is well reflected in the following quotation:

> "It is recommended that net returns exclude all predictable risks, either by deducting them from benefits or adding them to project costs, usually on a present worth or annual equivalent basis. Allowance for uncertainties or unpredictable risks in benefit accrual should be made indirectly by use of conservative estimates of net benefits, requirement of safety margins in planning, or including a risk component in the discount rate." U.S. Inter-Agency Committee on Water Resources, Proposed Practices for Economic Analysis of River Basin Projects (Washington, 1958), p. 23.

The difficulty with this statement, and indeed with most discussions of the subject, is that it is not explicit on how to cope with uncertainties or "unpredictable risks." Virtually all writers agree that predictable risks of fire, hazard, etc., should be dealt with on an insurance basis. But when it comes to other types of risk or uncertainty, a wide divergence of opinion emerges. Eckstein argues that a premium in the interest rate is "the most useful adjustment for risk in project evaluation" (Otto Eckstein, Water-Resource Development, p. 90,) and Hirshleifer comes to a similar conclusion in "Risk, The Discount Rate, and Investment Decisions," American Economic Review [May, 1961]). But Arrow (op. cit.) argues that the government should not display risk aversion, i.e., should not incorporate a risk premium in the discount rate it uses, and Marglin (op. cit., pp. 31, 71-72) maintains that where the net-benefit stream is rising over time, the criteria arrived at in Section V.B, above, give appropriate guides to investment decisions and their timing, without adjustment for uncertainty.

The issue in question appears to be in large part (though not entirely) semantic. Eckstein asserts that future changes in technology will, if they occur at all, be improvements, reducing the net benefit to be obtained from an investment made today (which would in this case become obsolescent). Clearly, if the probability of such changes has not already been taken into account in the estimation of future net benefits, it must be considered at some point, and one way to do this is to give relatively less weight to future benefits by raising the discount rate applicable to them. Likewise, if future technological changes have not been adequately foreseen, taking them into account may alter the shape of Marglin's rising net benefit stream, and turn it into one which first rises and then falls, or one which falls uniformly with time. In this case the fact that next year's net benefits covered the interest cost on this year's investment in a project would not be a sufficient basis for justifying the project's construction; one would have to check further to see whether the present value of the (adjusted) net benefit stream was in fact greater than or equal to the capital cost.

In principle, Arrow and Marglin appear to be closer to the truth than those who would place an explicit risk premium on future net benefits, but this assumes that all estimates of future benefits and costs have been adjusted to incorporate our best guesses as to expected changes in these magnitudes. It may be concluded that because an adjustment of the discount rate for a risk premium on future benefits is, by its nature, likely to be applied quite generally, implicitly assuming that "unadjusted" calculations of future benefits should be adjusted in the same way regardless of the type of investment, line of activity, etc. Since expected changes in product prices and factor costs, and expected improvements in technology as such are likely to be very different for different types and lines of investment, the adjustment for these changes should be carried out by as detailed as possible an extrapolation of individual cost and benefit items on each project separately, rather than being dealt with by a global risk premium attached to future discount rates.

This procedure suggested here implicitly assumes that the government does not have risk aversion as such—which appears to be a fair assumption since the wide variety of governmental investments ensures a substantial amount of risk-reduction through diversification. Moreover, the private sector investments which yield the marginal productivity of capital that should be used as the discount rate in public investment decisions are themselves extremely widely-diversified, and when taken in the aggregate as distinct from individually, appear to entail very little risk. Thus with both the public sector package of investments and its private-sector alternative being widely diversified and therefore of relatively low risk, the assumption that the public authorities are neutral to risk appears quite reasonable.

VI. Interrelations Among Projects.

A. Separability of Components of a Project.

Like the choice of scale of a project, the problem of dealing with separable components is readily handled using the present value rule. As the U.S. Federal Inter-Agency Committee on Water Resources puts it:

> "Net benefits are maximized if the scale of development is extended to the point where the benefits added by the last increment of scale or scope are equal to the cost of adding that increment. The increments to be considered in this way are the smallest increments on which there is a practical choice as to inclusion or omission from the project. The same principle applies when selecting a number of projects to form a program or system of projects to meet a given objective. To be justified for inclusion in a plan, each project in a group, each purpose of a project, and each separable segment of a project should add as much or more benefits as it adds costs" (op. cit., p. 14).

This principle is indeed the correct one to apply so long as all projects having a positive excess of benefits over costs can be financed, a proviso that we have assumed to be met, given the possibility of government borrowing. However, it is important to recognise that the principle applies to large as well as small components of a project. A case in point occurred in the evaluation of the benefits of the publicly-owned beet-sugar refining industry in Chile. Here large benefits were attributed to the indirect effect of the extension services given to farmers upon their general efficiency of operation. On the presumption that similar extension services could be given even if no sugar-beets were cultivated, the benefits in question should be attributed to the extension operation and not to the overall sugar-beet project. Thus, one could view the extension operation as a separable component, and evaluate it separately from the rest of the project. When this was done, the main project turned out to be of dubious validity, even though the extension component was quite clearly worthwhile. (See Ernesto R. Fontaine, "Un Analisis de los Costos y Beneficios Sociales de la Industria Azucarera Nacional, S.A.," in Estudios de Economía (Santiago: Universidad Católica de Chile, 1961), pp. 31-32.

The careful examination of possibilities of separating components from a project is as important an aspect of appropriate design and evaluation procedures as the study of possibilities of adding components. It is, moreover, an aspect of cost-benefit analysis which has not received sufficient attention to date.

B. Criteria for the Evaluation of Groups of Projects.

The evaluation of groups of projects is quite similar in nature to the problem of dealing with separable components. There is no need to consider groupings of projects when their benefits and costs are independent, but when the benefits or cost associated with one project will be different, depending upon whether or not another project is undertaken, the analysis of the projects so related should be done jointly. The appropriate method is as follows.

Let PVB (A) stand for the present value of the benefits of project A if it is undertaken alone, and PVC (A) stand for the present value of its costs (including both capital and operating costs) if undertaken alone. Correspondingly, PVB (B) represents the present value of the benefits of project B, undertaken alone, and PVB (AB) represents the present value of benefits of A and B taken together. Similar notation will be used for costs. Two projects are independent on the benefit side when PVB (AB) = PVB (A) + PVB (B); they are independent on the cost side when PVC (AB) = PVC (A) + PVC (B). The projects are:

a) complementary on the benefit side when $PVB(AB) > PVB(A) + PVB(B)$

b) substitutes on the benefit side when $PVB(AB) < PVB(A) + PVB(B)$

c) complementary on the cost side when $PVC(AB) < PVC(A) + PVC(B)$

d) substitutes on the cost side when $PVC(AB) > PVC(A) + PVC(B)$

Let N = PVB - PVC be the net present value of any project or group of projects. The principle of choice is to maximize the total net present value. Thus if there are three projects which are interrelated either on the demand or cost side or both, there will be seven possible options. One can undertake A, B, or C alone, or A and B together, A and C together, or B and C together, or, finally, one can undertake all three projects together. The criterion for choice in this case reduces to finding which of the following seven magnitudes if the largest: N(A), N(B), N(C), N(AB), N(AC), N(BC), N(ABC), and investing in that project or combination of projects. This criterion for choice among groups of projects can be extended to any number of interrelated projects. It automatically takes account of the effects of any given project on the benefits and/or costs of other projects in the group. Moreover, it can also handle the problem of timing, simply by including as separate projects in the list the possibilities of constructing a given project at different times. Thus if we had 2 projects, A and B, and were considering the benefits of constructing either or both of them, with options of timing in years 1, 2, and 3, there would be fifteen possible options whose net present values would have to be compared. A_1, A_2, A_3, B_1, B_2, B_3, A_1B_1, A_1B_2, A_1B_3, A_2B_1, A_2B_2, A_2B_3, A_3B_1, A_3B_2, A_3B_3, and the problem would reduce to finding which of these options had the greatest net present value, when the benefits and costs of all of them are discounted back to the same point in time.

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