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CAPITAL BUDGETING AND PRICING TECHNIQUES

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I

Introduction: Objectives and Policy Instruments

Selecting the "best" of alternative projects or facilities to be constructed and identifying appropriate pricing policies for the selected facility is a perplexing and pervasive problem. To be determined are prices which lead to the "correct" amount of capital investment in physical capacity, and to its efficient utilization, while recognizing the opportunity costs of resources used.

The general capital budgeting and pricing technique described in this paper differs in several significant aspects from more conventional capital budgeting procedures. Above all, pricing policies are recognized as planning or administrative instruments to be used in achieving certain specified objectives established by a particular physical project or facility. We further assert that pricing and capital budgeting are closely interrelated aspects of the same planning problem and that capital budgeting is not rationally executed in abstraction or in isolation from the consideration of pricing problems.

The more difficult pricing decisions confronted by modern managements and public planning officials usually involve the question of how to "recapture" capital, research, development, administrative, and similar costs neither easily nor directly related to variations in output. In large measure, these less directly related or traceable costs derive from commitments of an investment character, so that the problems of evaluating these investments, and their "recapture" (either actual or potential) through price assessments become closely intertwined. Even though capital charges or prices may not be assessed directly to consumers in all cases (as a matter of established public policy, for example) the necessity to evaluate the market possibilities remains, so long as efficient allocation of investment resources is desirab

Attention will not and should not be focused, moreover, only on narrow economic issues affecting the particular facility under consideration. Pertinent broader issues can and do influence capital budgeting and pricing decisions, particularly in the public sector. Accordingly, other broader and more qualitative variables are accommodated explicitly in the planning process here outlined, at specified "decision points." Thus, important qualitative aspects interact upon the planning decision, and upon the strictly economic optimizing process, in a clearly specified manner and in contrast with the more informal procedures of much conventional practice.

A dichotomy is sometimes defined in the set of procedural methods available to the public or private planner between a comprehensive, systems-wide or general equilibrium approach on the one hand and a project-by-project methodology on the other. The approach outlined here belongs more with the latter category. The efficacy of either type of approach depends, of course, upon a host of social, political, and economic conditions surrounding its application. A more decentralized or disaggregated administrative and planning framework, for example, would tend to favor a project-by-project approach. A highly centralized political and economic framework, on the other hand, would make a comprehensive, economy-wide evaluation more likely and feasible.

Such problems as data generation and acquisition also affect the selection of the proper planning approach. In this respect, project planning as contrasted with a full systems analysis may be mobilized with smaller, less extensive data input requirements. Thus the "first step" for implementation of our present technique should be not quite the hurdle that gearing up for a comprehensive systems model would be. An essential distinguishing feature between the two planning approaches, one which should loom large in the selection of either, is that the broad, comprehensive model by its nature internalizes within the planning process many effects of a particular capital investment which are considered external in the project planning methodology. Whether the possible gain in planning efficiency from such internalization of these effects over many projects is worth the added cost of a systems model implementation, at any particular point in time, is again dependent upon the existent planning and administrative procedures, and the extent to which consistent treatment of different sectors of the economy is both sought and feasible.

The capital budgeting procedures outlined here also necessitate greater emphasis upon explicit estimation and definition of demand and supply curves. Estimation problems as such are not a central concern here, however. Rather, it is assumed that the requisite supply and demand functions are given or known. In addition, estimates are required of the present cost and the discount rate (or rates) relevant to the facility under consideration. With these data a determination is then made as to whether the facility is economically rational under the given parameters, and, if it is, the pricing schemes which are optimal. These steps, moreover, are undertaken with reference to specified objectives for the project or facility in question.

Project comparisons are based upon the present net value technique. Future gain and cost streams are discounted to the present and then the net present value is found by deducting the present cost from the discounted gain (or revenue or profit). The selection of the best alternative project then involves simply finding the one with the largest net present value under a particular pricing regime.

Of the alternative pricing regimes that might be considered those aimed at maximizing revenue could have a particular appeal for both private and public sector planners, especially if projects in the public sector are to be discounted at an approximation of the market or private sector's rate of interest. The justification would be that if public investments are to be evaluated at the same rate of interest as private, then similar pricing strategies should be adopted or allowed when computing benefits. Of course, such a procedure implicitly assumes that no substantial differences exist in the degrees of market or monopoly power in the public and private sectors. Even if some "social rate" of discount is used for the public sector, the criterion of maximum net present value might remain valid in determining the best ordering of alternative capital investment projects within the public sector. In such circumstances, while the private sector planner likely would continue to follow profit maximization in determining his real pricing strategy, actual pricing procedures (due to different objectives) might be different in the public sector from those used for making capital budgeting decisions.

With regard to objectives, the feasible set could be extremely large, depending upon the detail of specification. Two general extremum conditions, however, are conventional in the economic literature. They are: 1) maximize the net present revenue (profit) generated from use of the facility (at the specified discount rate), which normally implies high opportunity costs of capital in alternative uses; or 2) maximize the consumer use of the facility subject only to the constraints of maintaining at least a non-negative net present value and every user paying a price no lower than the short-run marginal costs of what he consumes. (That is, use is to be encouraged, but the facility must recover its capital and maintenance costs). Between these two polar extremes lie innumerable possible combinations.[1] The second objective, maximum use subject to self-recovery of facility costs, has strong

[1] See Section III below, page 22, for a brief discussion of a third type of possible objective for public sector projects.

social and intuitive appeal and probably lies not too far from rational economic and public policy practice in most cases. It is, nevertheless, an arbitrary limitation.

By way of illustration, some broader issues possibly could force a public policy which required a certain amount of net revenue or, conversely, even a deficit operation of a facility. The present model will identify the pricing schemes relevant to such an objective, or almost any other for that matter. At the same time, it will signal to the administrator or planner either the differential facility use, or else the amount of monetary deficit created by pursuit of other objectives. Again, the procedure is entirely flexible and amenable to any combination of the different objectives of profit or use maximization. Thus the model can be of use both to private managements, whose objectives and specifications would tend toward the maximum profit pole, or to public sector planners, where the maximum use pole may more often be of interest.

The feasible set of alternative pricing schedules --the main controllable variables of the budgeting and pricing problem-- can be divided into two subsets corresponding to the two basic polar objectives, that is, to achieve maximum revenue or maximum use. The type of objective stated, in turn, affects the kind of pricing policy which should be used. Thus, the maximum use objective is closely interrelated with the phenomenon of demand peaking, as exhibited by almost every conceivable good or service (depending on the time length of cycle considered). For example, demand for Christmas trees and for bathing suits peaks once annually (in a specified climatic zone). Similarl daily peaks are observed in the demand for street capacity in urban areas, or for electric power, or for water supply. By dividing the time intervals considered in the model (one year) into two periods, peak and off-peak,[1] the best price to charge peak and slack period consumers in order to achieve maximum use of the facility can be identified (subject to the constraint that net present value is not less than zero). By contrast, if the objective is maximum revenue or profit, the price or prices which generate maximum net present revenue are charged, regardless of use during peak or off-peak periods.

In achieving specified goals, three general types of price differentiation can be identified. First, there is a <u>cyclical</u> price discrimination or differentiation[2] which is applicable to situations where very sharp seasonal, daily, or other variations occur in the rate at which the service or product is consumed. Such discrimination can be a means of ameliorating the high costs and other problems associated with

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[1] More than two periods can be accommodated if the added complexity in computation seems warranted.

[2] Jack Hirshleifer, in "Peak Loads and Efficient Pricing: Comment," <u>The Quarterly Journal of Economics</u>, Vol. LXXII, No. 3 (August, 1958), pp. 451-62, claimed a fundamental distinction between "discrimination" and "differentiation," if social marginal costs are defined in the opportunity sense. Given such a definition, no discrimination exists in the case of different charges for peak and off-peak consumers because the relevant social marginal costs of service also differ. Such a definition stirs up murky definitional problems of marginal costs, however, and we prefer to use the term "discrimination" in its usual looser sense.

very intensive peak use or demands or as a means of "smoothing" demands and at the same time encouraging maximum total use.

Second, there is inter-consumer price discrimination which involves different price levels for different categories of consumers, established within any single time interval. The several levels will be assumed to remain constant for these different consumer groups over and between time intervals and throughout the planning period or life of the facility. Administrative difficulties can be encountered with this type of pricing policy, but they need not necessarily be insuperable, especially if the demand for the facility's output is sufficiently inelastic, at least within certain delineable categories.

The final type of price differentiation to be considered is inter-temporal discrimination. This refers to a situation in which one price is charged all consumers in all peak or slack periods, but the price level rises or declines over time. The length of time any particular price is charged may change within the total planning period, in order to retrieve a non-negative amount of present revenue. A price level declining over time would appear to be the most administratively desirable, but either alternative is feasible within the model's algorithms.

The particular pricing scheme chosen depends not only upon the stated objectives, moreover, but also upon the economic and political ambiance affecting the given facility. Desirable attributes of any pricing policy include simplicity both in administration and in comprehension by the consumer, social and political feasibility, and efficacy in achieving the desired objectives. Underlying all of the pricing schemes considered, therefore, is the assumption that in order to enhance administrative feasibility and consumer acceptance, the number of separate price levels set for the facility should be kept as limited as possible, while allowing the desired objective to be achieved. Thus, if a single price to all users over the life of a facility will accomplish a desired purpose, and if it meets with the specified financial and economic constraints, this price is considered optimal. In short, only after testing single price possibilities and finding them inadequate is consideration given in the model to price discrimination or differentiation schemes.

II

Basic Demand and Supply Function Characteristics

As stated, the present model requires more attention to explicit demand and supply curve estimation than the processes based on arbitrary cost-allocations common to conventional capital budgeting methods. The difference is mainly one of emphasis. In reality, the demand curve lies, explicitly or implicitly, at the core of most procedures for estimating the benefits to be derived from individual investment projects. Only in broader "systems" or "national income" analyses of certain large public works is this reliance on the demand function as a basis for benefit evaluation likely to be seriously mitigated. For private enterprises, management may call the demand estimation exercise a "market" forecast or analysis, but the objective is still the same as determining a demand function: namely estimating how much people are "willing to pay" for specified quantities of the product or service to be produced. Public officials contemplating civil investments follow much the same procedures. For example, when road-builders try to estimate the cost reductions a new highway will confer on users, they really are trying to estimate how much the users might be willing to pay for the facility. Therefore, their efforts represent indirect attempts to estimate the demand function. The same is true of attempts to evaluate the "direct benefits" (for example, flood control, irrigation, navigation) of dam or reclamation projects. Of course, public officials may also count indirect benefits in their evaluations. This practice, however, has been increasingly questioned by economists on grounds that it represents an improper accounting of simple transfers from one group to another in society (rather than creation of new wealth) or it promulgates an "unwarranted" bias in favor of public as against private investments. Indeed, since private investors normally cannot recoup the secondary or indirect benefits of their undertakings, a strong case exists for stressing potential "recoupability" for evaluating the benefits of public investments, at least so long as some sort of parity is sought in capital accumulation and evaluations by the public and private sectors. Again, any such stress on public-private parity, implies that estimates of demand are the central consideration in evaluating public projects.

It is well to digress briefly, therefore, on just what information supply and demand functions convey and, explicitly, whether the usual demand curves of economic analysis measure benefits in a meaningful way. Indeed, a rather extensive literature and controversy exists on this point.

One difficult problem that arises when using demand curves to estimate benefits is that demand curves necessarily are expressed in monetary sums while, in a strict sense, what should really be measured is the satisfaction or utility people derive from money. Of course, if every unit of money were like every other unit in terms of the extra satisfactions it would buy, this would not be a problem. It is, however, difficult to imagine real circumstances in which every unit of expenditure indicated by a demand curve would be of constant benefit value --or, more technically, of constant marginal utility.

This proposition can be illustrated by considering the basic character of consumer decisions. A rational consumer might be expected to rank all the possible ways he could spend his money income according to the satisfaction they yielded and would spend his money by proceeding down this hierarchial ordering until all his funds were exhausted.[1] Assuming that every product can be consumed in eactly the desired amounts (i.e. that there is perfect product or service divisibility), the rational consumer would spend on every product until the marginal satisfaction from the last unit of money spent on each product is equal to the marginal satisfaction derived from the last unit spent on every other product --otherwise he could make himself better off by transferring funds from a product yielding low satisfaction to one supplying large satisfaction. Furthermore, if money income were increased, the consumer would proceed down the list further than before and the last dollar expended probably would yield less utility than before. Contrarily, with a reduction in income, the last unit of money spent would supply more satisfaction than before.

It is worth noting the possible consequences if a price

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[1]Savings should be treated as one form of expenditures.

increase or decrease occurs on one of the products consumed. Say the price increases. If the consumer does not want to reduce by much the quantity of the product consumed, the result might be that his total money outlay on the product increases. However, to increase outlay on this product, the consumer must decrease expenditures on other products -- assuming that money income remains the same. A decrease in expenditure on the other products means, though, that the last dollar spent on each of these products probably yields more utility and the marginal utility of money therefore has increased to the consumer because of the price change.

On the other hand, the consumer might curtail his consumption so that he spent less in toto on the product after the price increase. This, in turn, would free funds for making increased expenditures on other products, with the marginal utility of the last dollar spent thereby being reduced.

When most consumers are in the former situation --that is, when more is spent on a product after a price increase-- demand for that product is said to be inelastic. The demand elasticity is defined to be less than unity when demand is inelastic and greater than unity when it is elastic. A unitary demand elasticity is the case for which total expenditure on a product remains unchanged in the face of a price change.

The demand elasticity, as a rule, will be different at different points on most demand functions. The usual assumption is that the demand elasticity is greater than unity at high prices, and less than unity at low prices. The concept of "demand elasticity" is very useful in analyzing the meaning of the suggestion that benefits should be measured in dollar units of constant marginal utility. In essence, this proposition implies that a demand function relating quantity to dollars of constant marginal utility should be used to measure benefits rather than a normal demand function. It is well, therefore, to consider the relationship between a constant marginal utility demand function and regular demand function.

In the elastic portion of the conventional demand curve any increase in price decreases total expenditures on the product or service under analysis and frees money for expenditure on other products. This forces the marginal utility of the last dollar spent downward. To restore the marginal utility of the last dollar to its original position would require taking money away from the consumer. This in turn would depress consumption of the product being analyzed below its original level and make the depressive effect of the price increase even greater than it would otherwise have been. In the inelastic portion of the curve the situation would just be reversed. In short, only when demand is unit elastic will there be no induced income effects on demand which would require compensatory action to restore the marginal utility of money to its original position.

Thus, for measuring benefits in dollars of constant marginal utility, the usual demand curve will provide an improper estimate unless demand happens to be just unit elastic. The degrees of the overestimate depends on the extent to which the two demand functions diverge; this, in turn, is largely a function of how important an item of consumption the product under analysis happens to be. For goods that absorb a big proportion of income, the divergence will be large; for goods that account for only a small percentage of total consumer expenditures, the bias should not be great.

Because of these difficulties, it has sometimes been suggested that the proper approach to benefit valuation is not to attempt measuring benefits in dollars of constant marginal utility but rather to determine what would be the maximum number of dollars, regardless of utility value, that people would pay rather than do without a product or service. The reasoning behind this approach is that if this sum is larger than the total costs of providing the good or service, production is economically justified.

Again, the area under the demand curve usually is accepted as a reasonably valid first approximation to the amount to be estimated. Remembering that a demand curve indicates the price that must be charged to bring a certain number of customers into the market, this sum would be identical to that realized by a monopolist practicing perfect price discrimination. Such a monopolist would arrange his customers according to the maximum each was willing to pay rather than do without his product and would extract from each customer this maximum.

A regular demand curve, however, actually would overestimate the maximum amount that customers would pay because extracting every penny available to be spent on a product changes the basic assumptions under which demand curves are usually constructed. Specifically, perfect price discrimination means that more income will be spent on the product under analysis at every level of output (after the initial) than would otherwise be the case. Thus, under a system of perfect price discrimination, less money income might be expected to be available at every level of consumption than would be available without price discrimination. Assuming that the product under analysis is not an inferior good,[1] less of the product being subjected to price discrimination (and less of other non-inferior products as well) would be demanded for a given price than would otherwise be the case because of the reduction in income.

The preceding is, however, strictly a partial analysis. It overlooks the fact that one man's purchase is another's sale. Consequently, if in a system with price discrimination those who gained income had the same marginal propensity to consume and exactly the same marginal product preferences, on balance, as those who lost income, the effect would be to restore demand to the initial state. It is doubtful, of course, that these assumptions would be met in reality. But neither it is clear what the net effect would be of permitting

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[1]An inferior good is defined to be one whose consumption rises when income declines because it is substituted for other goods of higher price. Hamburger (in place of steaks and roasts) and rye flour (in place of wheat flour) are usually thought to be illustrative examples of inferior goods.

price discrimination. Depending on preference and consumption patterns, the ultimate effect on the sales of a particular product or service at a given price could be an increase, a decrease, or no change. If an increase occurred, the area under demand curves would tend to underestimate benefits; if a decrease was the result, the tendency would be to overestimate benefits.

Still another objection might be leveled against either the price-discriminating-monopolist or constant-marginal-utility of money concepts of benefit measurement. As compared with the usual competitive criterion, both essentially establish a second or double standard for determining whether or not production of a good is economically justified. The competitive test for determining whether a good should be produced is that at some level of output, the price that consumers are willing to pay is greater than the supply price at that output. By contrast, the full areas under the constant-marginal-utility of money or price-discriminating-monopolist demand curves normally will result in higher estimates of benefits. These criteria therefore could suggest production of goods that would be excluded on the single-price competitive standard. But to obtain this production, either legalized monopoly and price discrimination or government subsidization would have to be instituted for those goods which, though justified by a "full-area" criterion, do not have demand curves that ever lie above their supply functions.

It seems very doubtful that acceptance could ever be obtained for the rather major change in the economic institutions of most countries that complete adoption of any "full-area" criterion would imply. There are few signs at least, that most western societies would want to undertake a large scale subsidization program or abandon competitive pricing in favor of monopolistic price discrimination on any extensive scale. At a minimum, any systematic shift to a "full-area" benefit calculation in deciding capital outlays and investments (broadly construed) in a decentralized private enterprise economy would pose substantial administrative problems.

Of course, under certain circumstances a double standard might be defensible. For example, a "full-area" count of benefit might be sanctioned for voluntary non-profit and public service activities. In a sense, even western economies already use such a mixed system since many educational and cultural activities (e.g., symphony orchestras and art museums) as well as certain public utilities are financed in this way. Still, adoption of a "full-area" concept of benefits is clearly a policy decision and not a strictly economic one.

There is still another, simply pragmatic difficulty with any "full-area" measure of benefits. This is the need to know the shape of demand functions much beyond the normal range of available data or experience. As a rule, the investigator trying to estimate a demand function has only a limited number of price and quantity observations, perhaps as illustrated in Figure 1, where each dot represents one observation. The usual procedure in estimating a demand function is to fit a line or curve, depending on the circumstance, to these data according to some criterion of best fit (as a rule, least squares). Where there are data, this procedure should yield reasonably good results. But it is obvious that not much of substance is known about the shape of the demand function out beyond the limited range of the available price and quantity data. For "full-area" measures of benefits, this is a serious handicap.

No one of the previously stated objections may be overriding when taken by itself. But the cumulative effect of these many criticisms could be of substantial magnitude. Furthermore, potentially difficult policy decisions often must be made at certain stages in the evaluation of a proposed facility or project. From the technical standpoint this suggests that the best capital budgeting procedure would be one that minimizes the number of applicable criticisms and difficult policy decisions. The purpose should be to arrive at a correct budgeting or investment decision with a minimum of assumptions and required information gathering. One obvious way to do this is to proceed sequentially. Projects initially should be tested for feasibility with a minimum of assumptions and avoiding a maximum of difficulties. If proven infeasible under these conditions, additional assumptions and decisions should be introduced in approximate order of "defensibility", testing for feasibility at each stage. The process should continue until conditional feasibility or total infeasibility was clearly established. It is this sequential

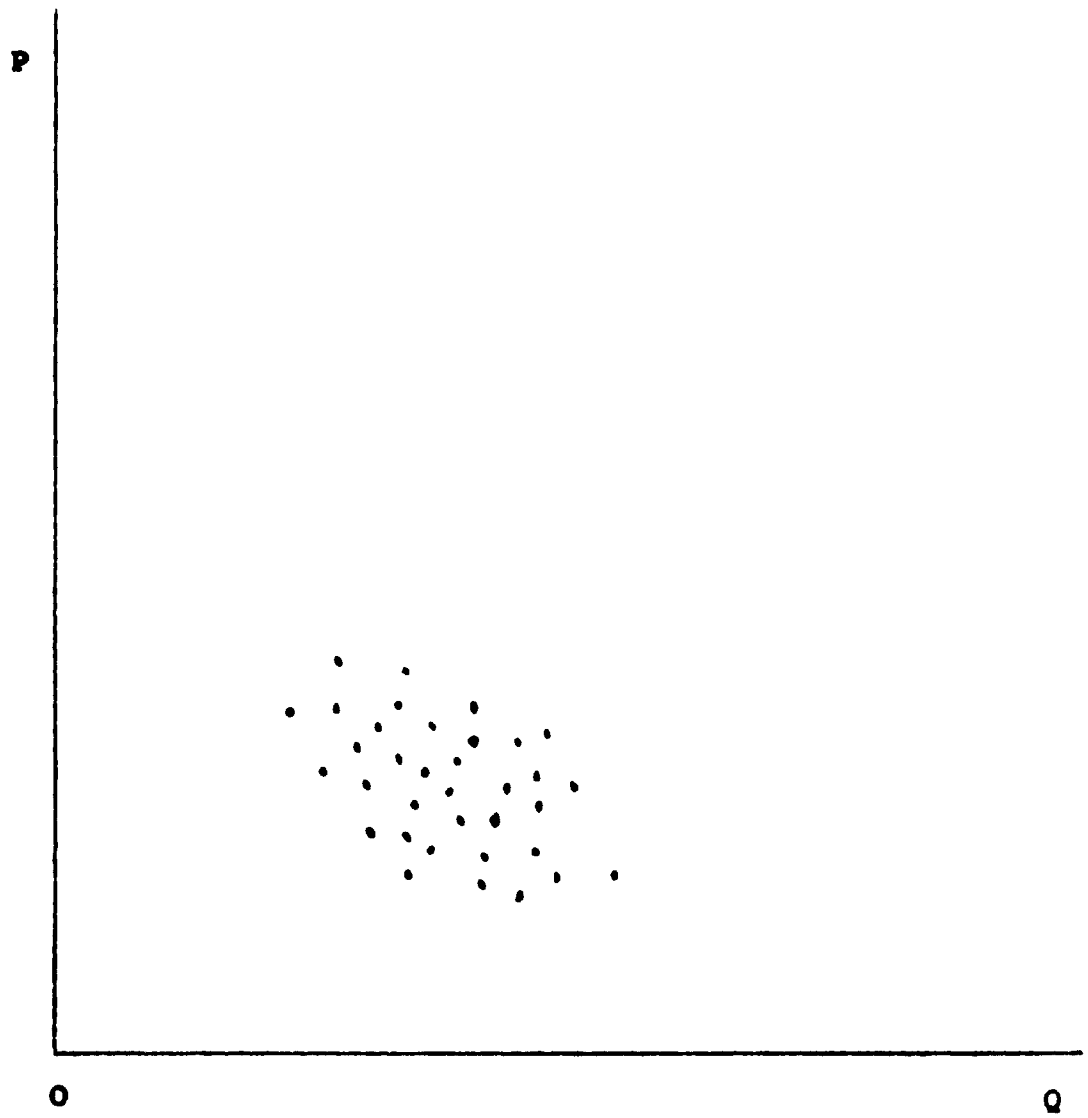

FIGURE 1: Observed demand data points.

process that we refer to when seeking explicit identification of qualitative decision points.

Description of the Computational Procedure

In addition to the characteristics of closer integration of pricing and capital budgeting, explicit identification of qualitative decision points, and the conscious use of pricing schemes as planning instruments, it is desirable to construct a procedure which keeps the needed series of computations relatively simple and straightforward. Although our model was programmed for an electronic digital computer[1], our assumptions of strict linearity of demand and supply functions enable an ordinary desk calculator to suffice if an electronic computer is unavailable.[2]

The linearity constraint on the demand and especially on the supply functions can be relaxed without too great an increase in computational labor, but departure from linearity for what we call the "composite net outlay" curves creates considerable arithmetical difficulty. These problems are strictly computational, however, not conceptual. Indeed, conceptually the procedure is almost completely general; the

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[1] The IBM 7094 at the Harvard University Computing Center.

[2] An estimate of the man hours required for the necessary computations is difficult to state in abstraction from a specific case. The work involved, however, is no more time consuming than most simple statistical regressions or engineering calculations.

only requirements are that present capital costs of the facility and its demand and supply functions over the relevant planning period be estimated, and an appropriate rate of discount (or range of rates) be specified. In short, even if the linearity constraints are relaxed, the conceptual procedure remains essentially as described.

As noted, exogeneous information needed as inputs to the model include estimates of the demand and supply functions for each time interval of the total planning period, including peak and off-peak period demand estimates.[1] The supply functions, furthermore, pertain only to all factors other than the fixed or capital facility under investigation. Depending upon circumstances, the functions could shift but retain constant slope over time, or the slopes could change over time also. The demand functions are assumed to be monotonically decreasing and can be expressed in symbols as:

$$D = f(P,\ T,\ K,\ Y,\ N,\ \ldots)\ ,$$

where P = price of product or service,

T = time,

K = cost of physical facility capacity,

Y = consumer income, and

N = growth in population.

Similarly, the supply functions for all factors other than

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[1] We specify a time interval arbitrarily as one year. The length of the planning period, normally about the same as the economic life of the system, can easily be changed to reflect different estimates of economic life.

the fixed facility are:

$$S = g(P, T, K, Y, N, \ldots)$$

for any one time interval. These supply functions, again, pertain to the offering of all labor, administrative and other skills needed to "complete" the facility's productivity when joined with the fixed capital outlay or project under investigation. It would be expected, moreover, that every different investment alternative normally would have a different supply function. In particular, differences in the productive capacity of different alternatives would be expressed by differences in the slope or shape of the supply functions.

The effective demand for a facility or its "net outlay curve" (as we shall term it) can be obtained for every time interval by substracting the supply from the demand functions in each interval. These net curves represent the derived demand for the facility itself after all cost other than charges for facility use have been subtracted, and are thus net effective demands for the facility's capacity.[1] Assumption of linear supply and demand curves greatly facilitates the derivation of net outlay curves, as shown in Figure 2. As Figure 3 illustrates, however, assumption of a curved or even a kinked supply curve does little to increase mathematical complexity if only the two axes' intercepts are used to specify the several net outlay curves. As shown in Figure 3, the

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[1] These curves are akin to the derived demand curves of A. Marshall. See his Principles of Economics (8th edition), [illegible]

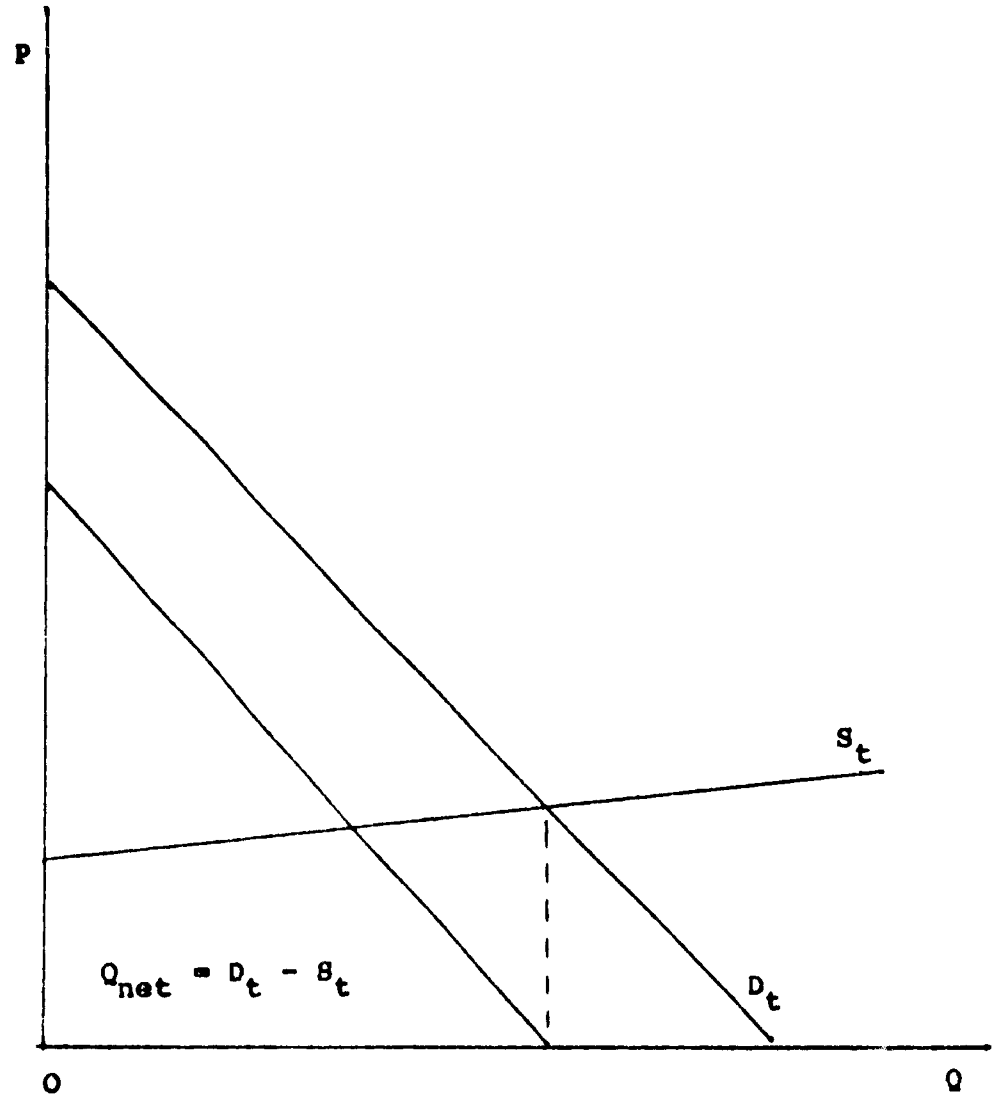
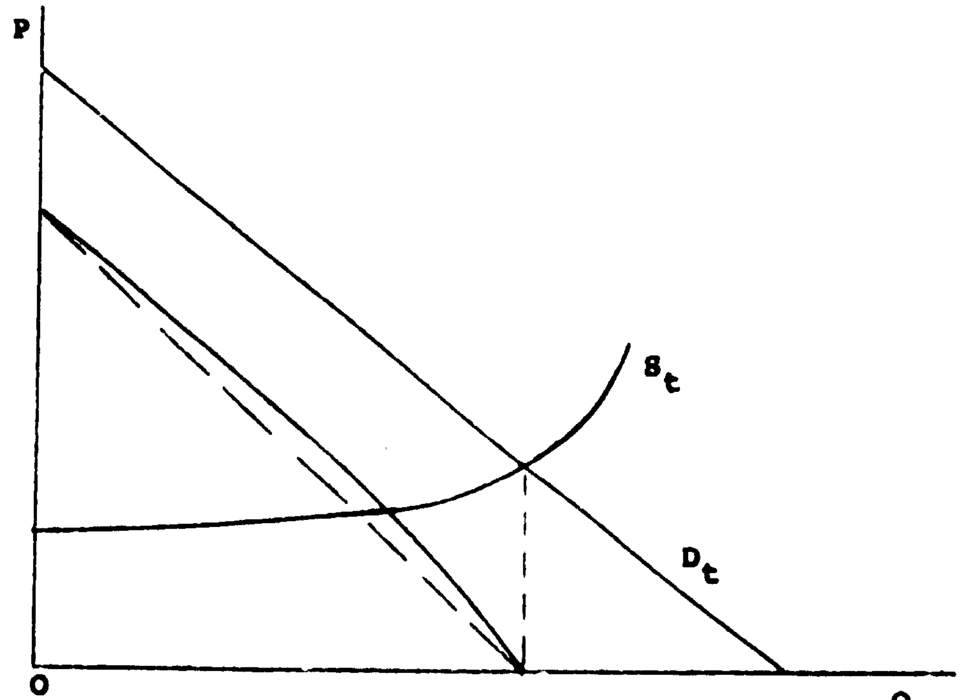
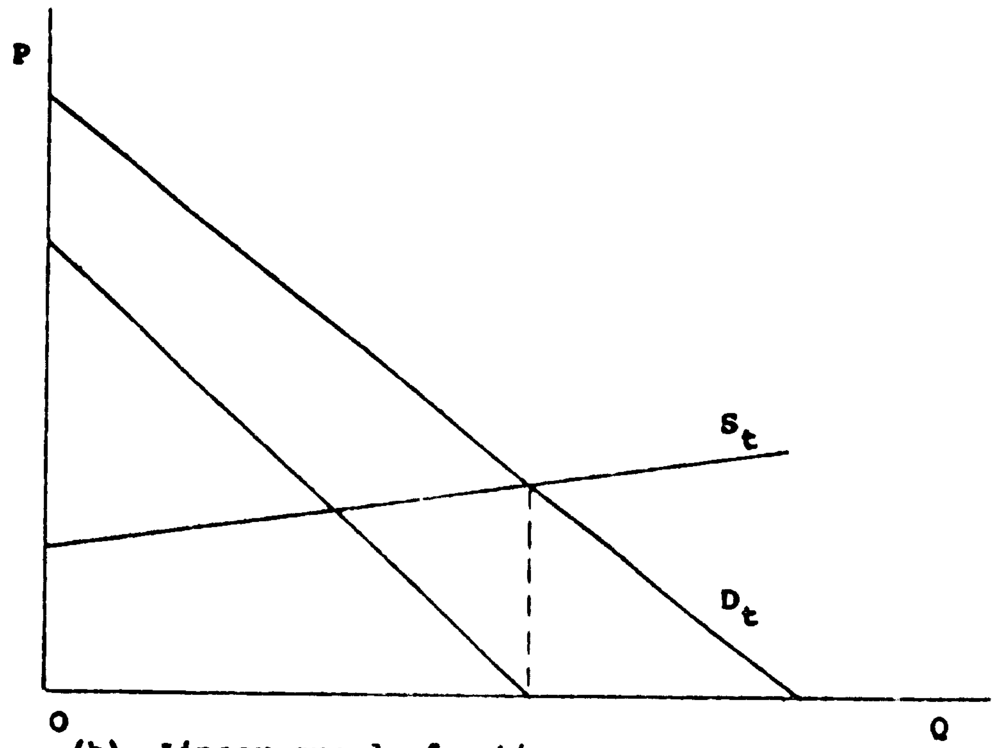


FIGURE 2: Net outlay curve (Q_{net}) derived from linear demand and supply curves in time interval t.



(a) Curvilinear supply function.



(b) Linear supply function.

FIGURE 3: Derivation of linear net outlay curves, with assumptions of linear

resulting linear approximation is generally conservative over the length of the net outlay function.

These net outlay curves provide a basis for evaluating different pricing policies and thereby the potential benefits of the capital investment. Moreover, two types of net outlay curves could be determined for each time interval: one for peak and one for slack period demands. The computations described subsequently apply identically to each type, with aggregate values being the sum of values found using each type of net outlay curve. As Steiner has pointed out,[1] such net outlay curves can be very useful in analyzing peak and off-peak pricing problems for productive facilities, particularly where a so-called "shifting peak" may exist. To determine the maximum amount of capacity justified under peak and off-peak demand conditions, the net outlay curves must be added vertically, as shown in Figure 4. Steiner assumes that β in Figure 4 is the constant cost of providing a unit of capacity, independent of the amount of capacity required. Thus, P_1 and P_2 are the peak and off-peak prices respectively

[1] P. O. Steiner, in "Peak Loads and Efficient Pricing," The Quarterly Journal of Economics, Vol. LXXI, No. 4 (November, 1957), pp. 588ff. For an earlier treatment of peak load pricing which arrived at essentially the same conclusions see Marcel Boiteaux, "Peak Load Pricing," in James R. Nelson (ed.) Marginal Cost Pricing in Practice (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1964), pp.59-89.

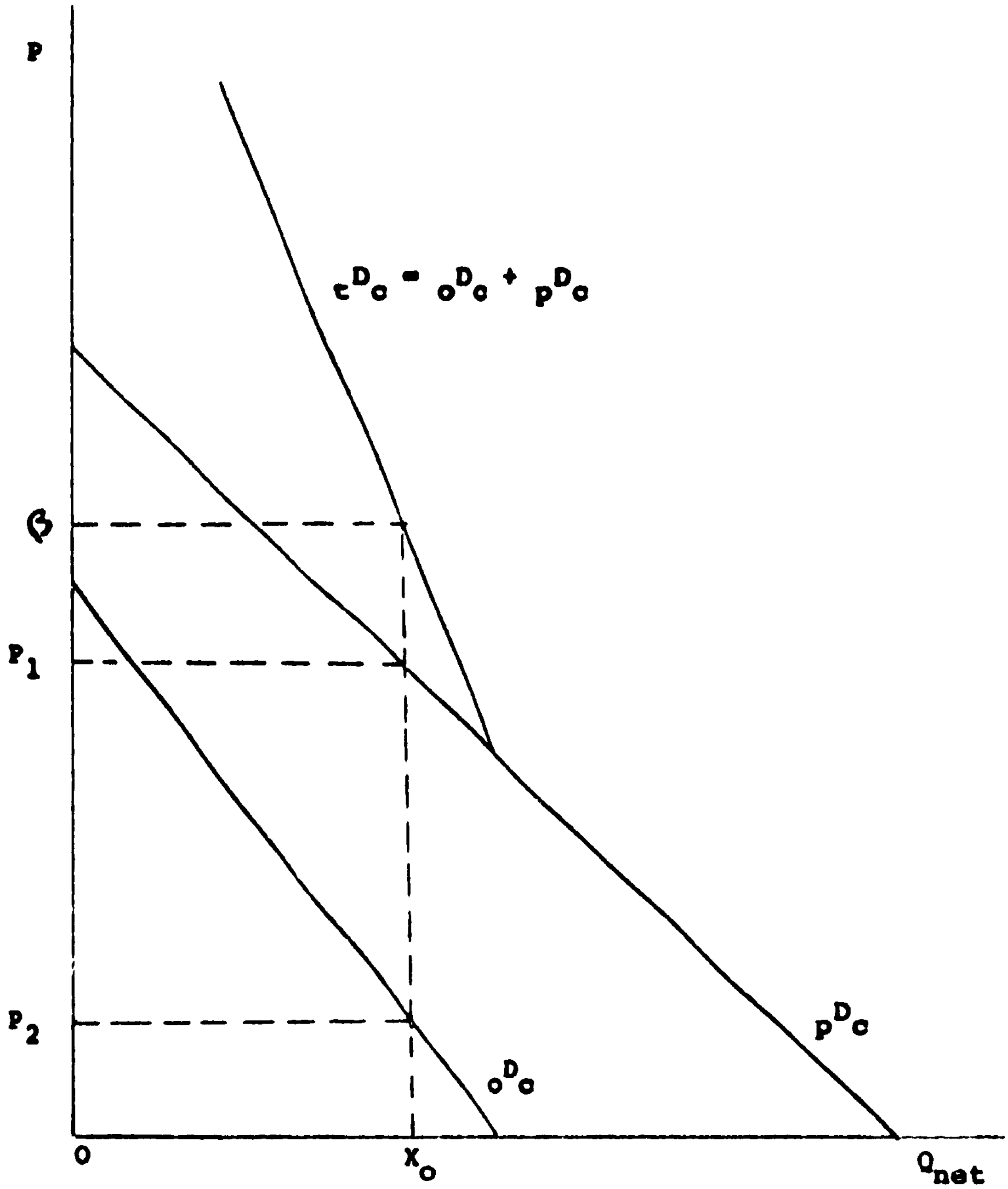


FIGURE 4: Vertical addition of peak and slack period net outlay curves, showing a "shifting peak" case. (Adapted from Steiner (*op.cit.*), Figure 1, p.598).

and X_0 represents the units of capacity required.[1]

Determining revenues available to finance an investment under different pricing policies also can be done expeditiously by simply adding the peak and slack period net outlay curves of each time interval (appropriately discounted), but in this case summing horizontally rather than vertically. That is, the net outlay curves for each time interval should be discounted to the present using the appropriate present value factor[2] according to the relevant time interval and the discount rate selected. Since the discounted net outlay curves of each type are to be aggregated horizontally along the quantity axis, they are expressed for convenience mathematically with relation to that axis. That is, the intercept A in the linear equation

$$Q_{net} = A - BP$$

is the quantity (Q) axis intercept and B is the slope with reference to the vertical price (P) axis. Composite peak and slack period discounted net outlay curves for the total planning period are shown in Figure 5, and are piecewise linear functions. Strictly linear proxies for the piecewise linear

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[1]Steiner (op.cit.), p. 587. The assumption of a constant seems rather unrealistic. Capital cost should vary according to the cost parameters of the particular facility under investigation, and its physical design and size. Therefore, capacity cost varies as a function of output in the present model.

[2]The computer program is designed to allow a range of discount rates to be tested. Thus, the sensitivity of the planning decision to the interest rates can readily be determined.

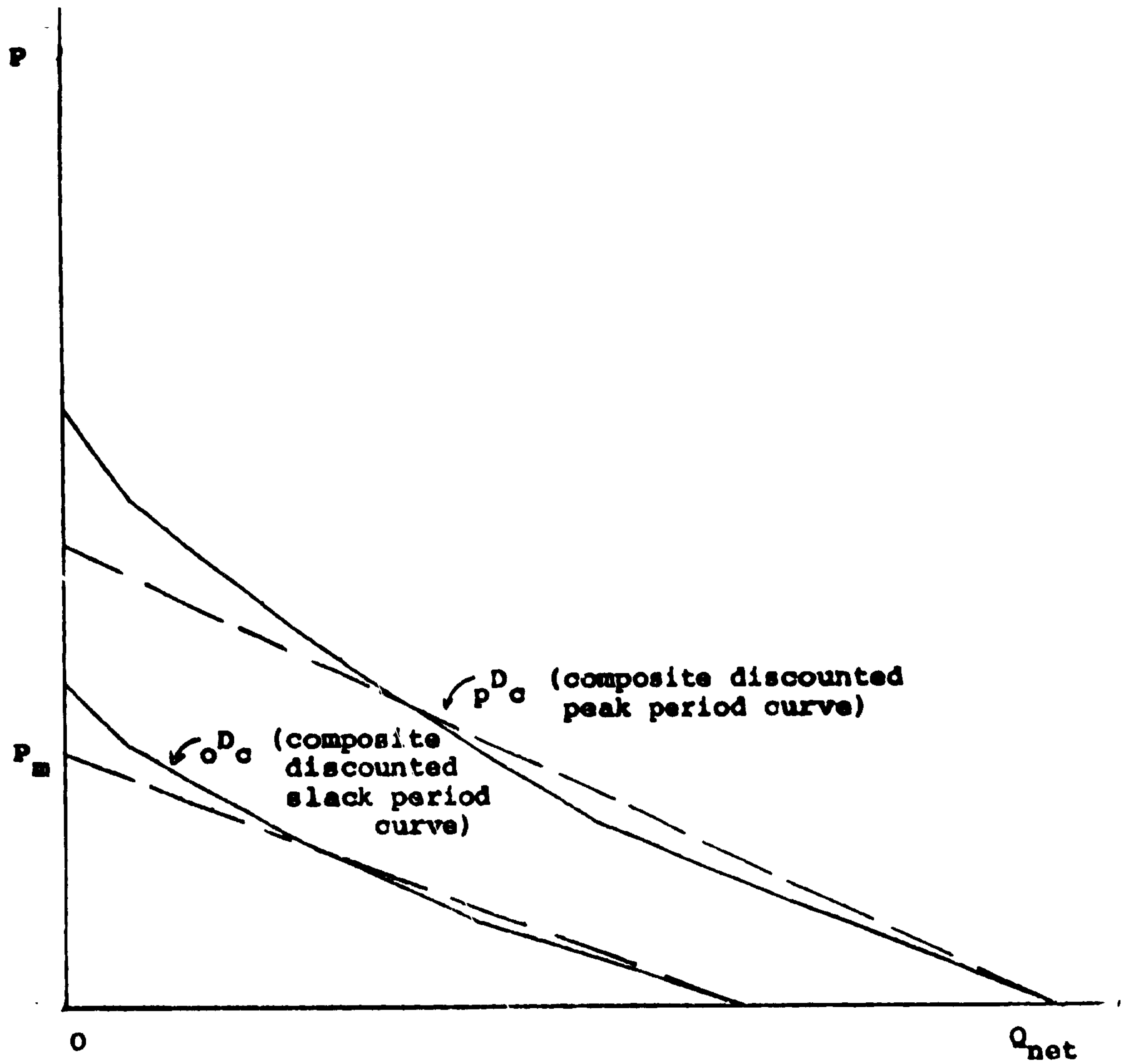

FIGURE 5: Composite net outlay curves with linear approximations.

composite curves can be found by taking the price axis intercepts to be the arithmatic mean of all the individual net outlay curve price intercepts weighted by their relevant present value factors. Once the weighted mean price intercept is found (P_m in Figure 5) the slope of the proxy curve in each case follows easily and the functions are completely described. The area under each linear approximation is equal to that under the relevant piecewise linear curve.

The question arises, obviously, of how much accuracy is sacrificed by using such linear approximations. Only if the price axis intercepts of the individual net outlay curves vary over a wide range, or else if there are very few time intervals being considered --say less than five-- will the approximations affect the price levels subsequently determined, and then only prices in the higher portion of the pricing schedule. If the axis intercept is small relative to the quantity axis intercept, loss of accuracy with respect to price levels determined is negligible.

Having obtained our linear composite discounted net outlay curves for peak and off-peak consumers, experiments can be conducted with different pricing schemes and pertinent qualitative decision points can be identified. First, a contingency check should be made to see if the alternative projects proposed are economically feasible even with total discounted consumers' surplus (the entire are under the composite net outlay curves) counted as benefits. If present gross benefits less present costs prove to be negative after this check, it is time to stop and reconsider the need for the facility or at least its estimated design and construction costs. In some instances, of course, there may be prevailing, non-economic arguments favoring construction of the facility. Here they should be specified and considered, and their importance weighed at least qualitatively against the economic deficit resulting from construction of the facility.

For the alternative projects passing the first contingency check, selection of the optimal project could proceed, if so desired, by identifying that facility which generates the maximum revenue. The prices in the peak and slack periods which will return maximum revenue for each facility are those at the unit elastic points on the discounted composite net outlay curves for peak and off-peak periods respectively, as shown in Figure 6.

If the best of the available alternatives selected in this fashion provides a positive net present revenue, then for the private sector (assuming maximum profit from the facility as the criterion), the optimal pricing schedule is simultaneously determined. In the public sector case, where maximizing use of the facility may be the objective, along with self-recovery of facility costs, the pricing scheme illustrated in Figure 7 would be optimal. By recursive testing, the rectangle OABC, whose area represents the cost stream of the facility discounted to the present, is found to correspond to the lowest price consistent with the facility being self-sufficient; it is fitted under the vertically combined peak and off-peak net outlay curves. The net peak and off-peak prices, then, as shown in Figure 7, are P_p and P_o, for maximum use. If the "present cost rectangle" should fit below the kink in the vertically added composite curves (for example, ODEF), a single peak period only price, at F in Figure 7, for all peak period consumers and a zero facility price for off-peak users will generate maximum usage.

Of course, the objective of pricing policy, in a situation characterized by sharply different peak and off-peak demands might not always be oriented to either profit-maximization or use-maximization (subject to the constraints that users pay the marginal costs of what they consume and for all or most of the capacity). For example, the objective could be a single yet fully compensatory price for all users. The grounds might be simply administrative expediency or some simple "equity" concept that everyone should pay part of the cost. Alternatively, price elasticities of demand[1] might be so low as to make price discrimination simply not worth the effort or extra administrative cost

For example, the situation might be as illustrated in Figure 8. The demand curves represent the effective demands for capacity: D_o for the off-peak, D_p for the peak, and D_s for the horizontal sum of the two demand curves. P_c is the common or single price that will yield sufficient revenue to pay for the capacity if levied against both peak and off-peak users. P_p is the price that would yield the same requisite

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[1]See Section II above, for a discussion of demand elasticities.

amount if charged to peak hour users only. As drawn in this example, off-peak users would be charged nothing for capacity since no shift in the peak would occur even at this minimum charge. The effect of levying a common price in both periods as against price discriminating, therefore, is to reduce usage in the off-peak from Q_o' to Q_o and to increase it from Q_p' to Q_p in the peak and at the same time redistribute the burden of the capacity costs away from peak and onto slack period users. In this particular illustrative case, total use is decreased by a common price since the price elasticity of demand in the off-peak is shown to be greater than during the peak. The reverse, however, could be true in real applications.[1] Again, it is entirely possible that under some circumstances any reduction in usage occasioned by a single price might be viewed as a small "cost" to be paid for the administrative simplicity of a single price system.

Of course, even the best alternative project might fail to generate a non-negative net present revenue under any type of non-discriminatory pricing scheme, including differentiation between peak and slack period users. Two additional types of price discrimination schemes might then be tested, inter-consumer and inter-temporal, to see if by systematic price discrimination of either type, a positive or at least zero net present revenue can be achieved.

In the inter-consumer case, separate price levels can be

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[1] Indeed, such a reversal may well be the case of travel demand for urban expressways in the United States.

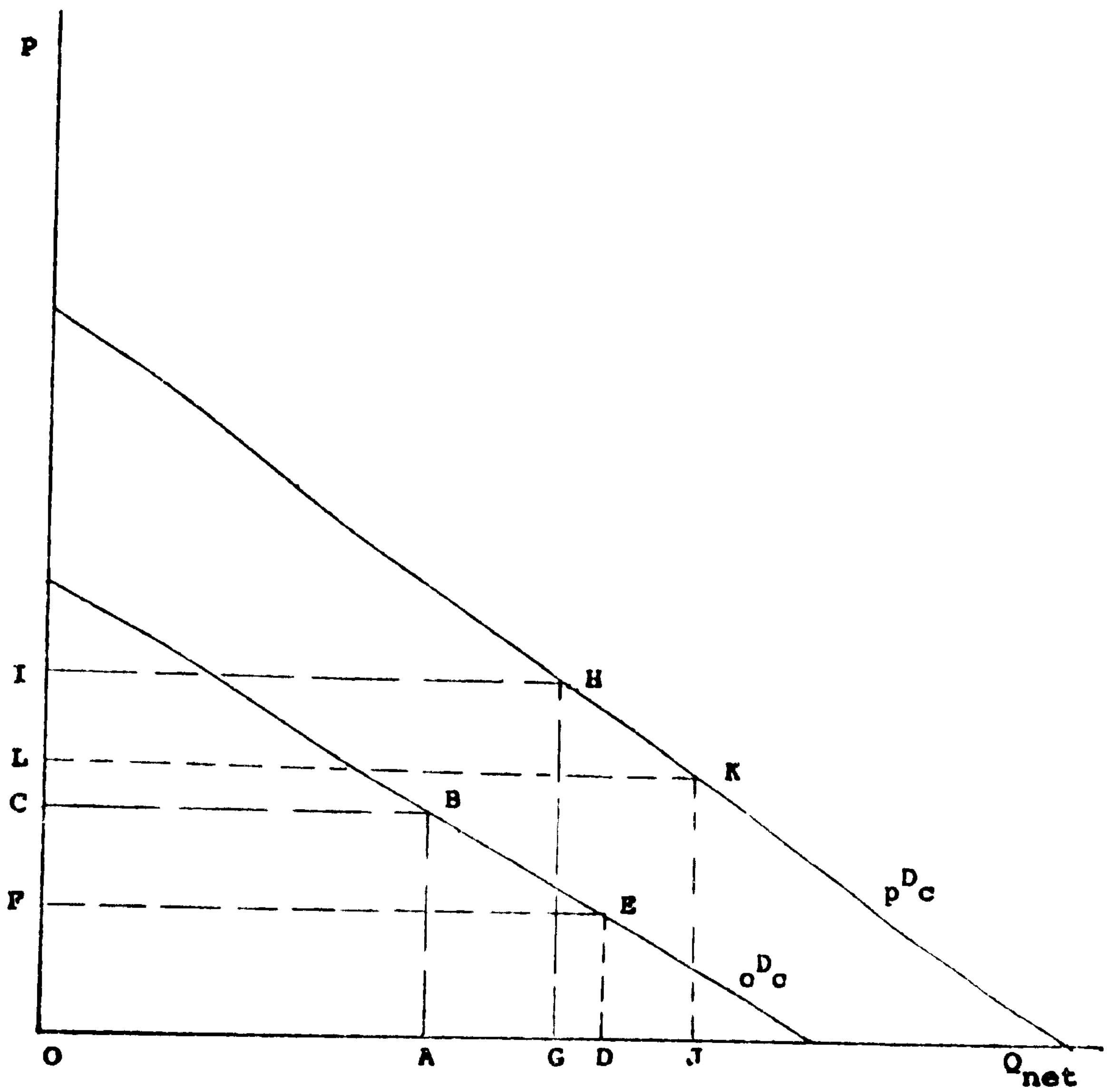


FIGURE 6: Maximum revenue and use price levels.

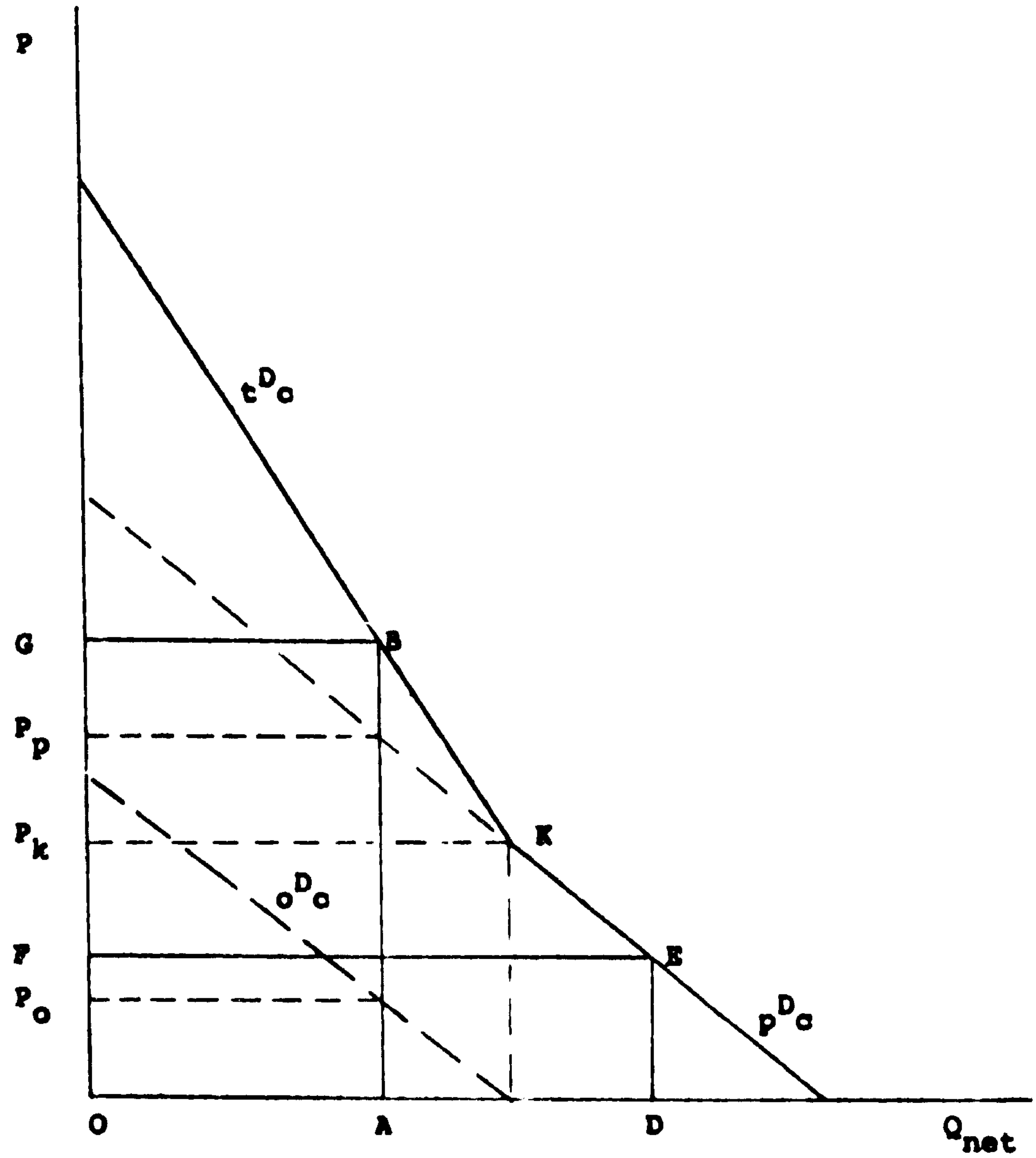


FIGURE 7: Optimal peak and off-peak prices for maximum use with net present value equal to zero.

[redacted]

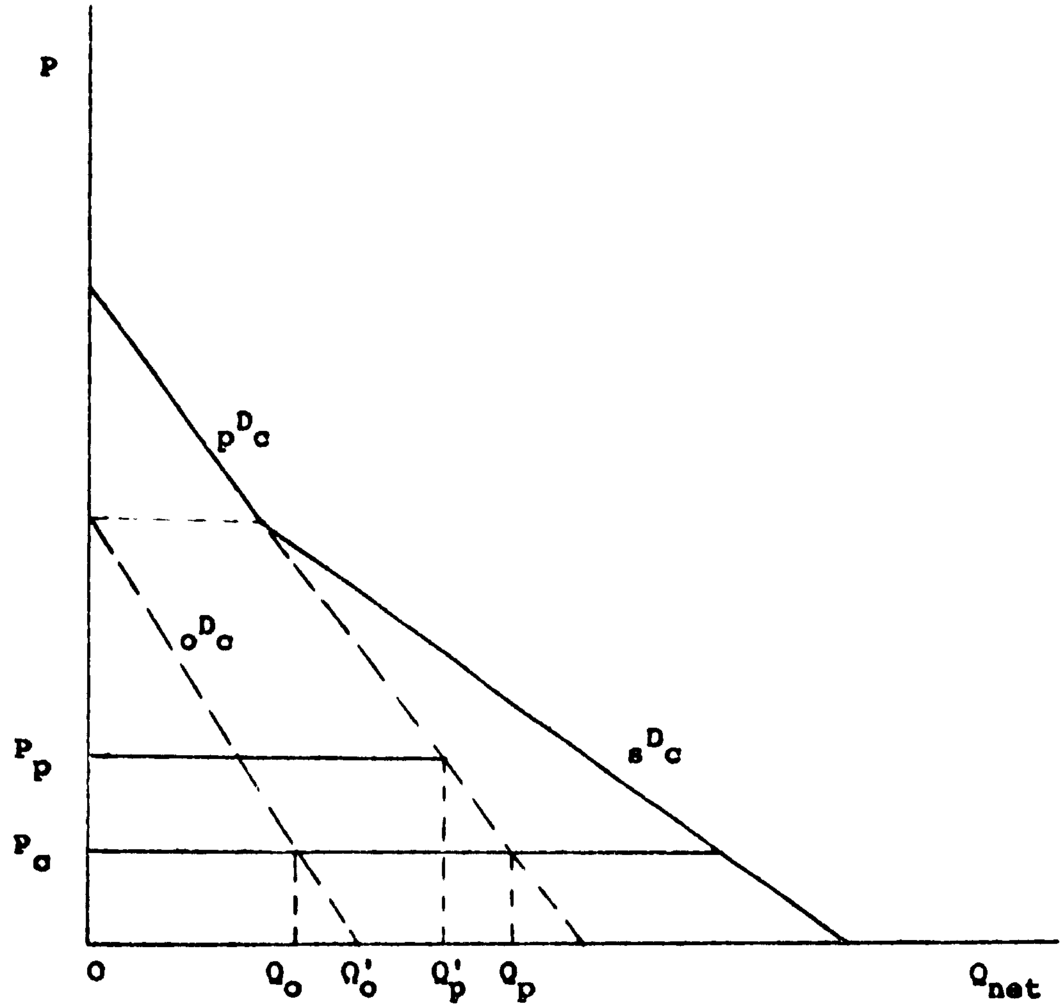


FIGURE 8: Effects of a single and fully compensatory price charged to all consumers, in a peak and slack period demand situation.

found by straightforward trigonometric manipulation of the area under the discounted composite peak and off-peak net outlay curves. As soon as enough revenue is obtained to fulfill the specified objective and constraints, the number of discriminatory levels of peak and off-peak prices is fixed. In keeping with our plausible assumptions that administrative simplicity and consumer acceptance are desirable, the testing within the model follows a recursive sequence of permutations of price levels and moves from most simple and probably administratively feasible to least. Table I indicates the sequence of inter-consumer price testing, with a maximum number of six price levels: three for peak and three for slack period consumers. This arbitrary upper limit of six levels could be increased, but at a concommitant relative loss of computational and administrative simplicity and probable consumer acceptance.

Determination of optimal or acceptable inter-temporal pricing schemes relative to some objective is somewhat more involved computationally than for the inter-consumer case. Suppose the objective is to determine the optimal inter-temporal pricing scheme with a maximum of two price levels (again, an arbitrary upper limit easily changed) over the total planning period. The problem is composed of two closely interrelated parts. We must ascertain 1) the lengths of the two subperiods of the total planning period, and 2) the price levels within each subperiods for both peak and off-peak users. Additional assumption, mentioned earlier, which incidentally helps keep the number of possible permutations in

[redacted]

TABLE I

MODEL SEQUENCE OF INTER-CONSUMER PRICE LEVELS

PERIOD	TESTS (NUMBER OF SEPARATE PRICE LEVELS)							
	1	2	3	4	5	6	7	8
Peak	1	1	2	1	2	3	2	3
Off-peak	0	1	1	2	2	2	3	3

the model within reasonable bounds, is that prices must decline (or rise) over time. For example, in a two level inter-temporal pricing scheme, the second price charged might be constrained to some fraction of the initial price. Such a constraint seems quite realistic, particularly when considering the introduction of new products or services. The opposite assumption is also permissable within the model, but it may often seem less plausible. If the second type of constraint is assumed, then the model helps to identify the minimum increment of increase in price, of all the feasible increments, which meets the objective.

As an illustrative example of the inter-temporal algorithm, consider a planning period composed of, say, twenty time intervals. The extremum conditions for the two-level inter-temporal scheme are: 1) charge the initial price in the first time interval and the secondary price in the remaining nineteen, or 2) charge the initial price over the first nineteen periods and the secondary price in the last time interval only. For any subperiod of more than one time interval, the best single price for that subperiod is found in a manner analogous to finding the single best price for the total period described above. It is more convenient within the algorithm, however, particularly with computers, to obtain optimal prices in a somewhat different manner from that described previously. It is more convenient to find first the single best price in a subperiod, defined to be the arithmetic mean of the unit elastic prices of each of the component time intervals of the subperiod weighted by their associated output quantities suitably discounted to the present. The prices for peak and slack users found by this method is equivalent to that of the method described above and illustrated in Figure 8, but the mathematical format is more amenable to making subsequent iterations as necessary. The weighting factor for any time interval unit elastic price is

$$W_t = \frac{Qt}{(1+r)^t}$$

and the single optimal price within a subperiod of the total planning period composed of, say, intervals one to five, is then

$${}_1P^*{}_5 = \frac{P_t W_t}{W_t}$$

where P_t is the unit elastic price for time interval t.

The computer algorithm proceeds iteratively to determine the optimal price levels for each possible combination of subperiods within the total planning period until the net present revenue generated is non-negative and the constraint of decreasing or increasing price levels is met. The number of inter-temporal price levels can be increased beyond two at a substantial increase in computer time required for the calculation. A sample listing of the requisite values obtained by hand methods for a five-interval subperiod and for peak period demand only, is given in Table II. Appendix A contains a tabular guide for hand calculation of a single optimal peak period price using hypothetical data. Appendix B is a flow diagram of the computer program for the model.

TABLE II

INTER-TEMPORAL PRICING SCHEME CALCULATIONS FOR THE BEST SINGLE PEAK PERIOD PRICE, WITH AN INTEREST RATE OF TWO PERCENT AND A FIVE YEAR PLANNING PERIOD (t = 1, . . . , 5)

t	P_t	Q_t	W_t	W_tP_t
1	5.00	4.54	4.46	22.28
2	6.00	5.54	5.33	31.98
3	7.00	6.36	6.00	41.98
4	8.00	7.27	6.72	53.75
5	9.00	8.18	7.41	66.70
			Σ = 29.92	Σ = 216.69

$$ {}_1P_5^* = \frac{\sum_1^5 W_tP_t}{\sum_1^5 W_t} = \frac{216.69}{29.92}, \quad {}_1P_5^* = 7.24 \text{ units} $$

NOTES:

1. t is time interval in years.
2. P_t is unit elastic price level in time interval t.
3. Q_t is associated output quantity at price P_t, obtained from Q_{net} curve in time interval t.
4. $W_t = Q_t / (1 + r)^t$.
5. See Appendix A for input data.

Summary

This paper presents some techniques which could be used in confronting complex capital budgeting and pricing problems. The suggested procedures differ in several significant aspects from more conventional capital budgeting procedures. For example, it is asserted that capital budgeting should not be accomplished in isolation from pricing considerations, and a procedure is suggested wherein pricing problems and capital budgeting are more closely integrated. Additionally, pricing policies are recognized as instruments for planning which can be used directly to help achieve some stated objective for the capital facility. Another attribute of the model is that broader issues and variables in the social and political context of the proposed capital facility, variables perhaps not susceptible to facile quantification, can nevertheless be considered explicitly at specified decision points within the planning process.

A central argument of this paper has been that sensible capital budgeting requires knowledge of the effective demand or "net outlay" for the capacity created by any prospective capital investment. This, in turn, means that the supply prices of all other factors required to produce the final product or service in question must be known as well as the demand function for that final output. Furthermore, knowledge

of the effective demand for new capacity can only be converted into an estimate of prospective revenues or benefits if some stipulation is made of the objectives sought by those persons or organizations controlling pricing policy together with the constraints, administrative or economic, under which these policies are executed.

To accurately define these objectives and constraints requires information or decisions on many matters: the extent to which consistency is sought in the decision processes used by different classes of policy makers, both public and private; whether profit or use maximization is considered the socially most suitable goal; the possibility and justifiability of subsidization; the value attached to simplicity in administrative mechanisms; and so forth. Many or most of these issues are not likely to be easily settled, of course. All can involve some subjective and political judgements of considerable complexity. A contribution of the present procedure is that some of the complexity is reduced, and subtle interrelationships are clarified, at least in the quantifiable portions of the planning analyses.

APPENDIX A

SAMPLE CALCULATIONS TO IDENTIFY THE BEST SINGLE PRICE FOR MAXIMUM REVENUE, WITH AN INTEREST RATE OF TWO PERCENT AND A PLANNING PERIOD OF FIVE YEARS ASSUMED. (PEAK PERIOD CURVES ONLY)

INPUT DATA ARRAYS

TIME INTERVAL (YEARS)	D_t^a	D_t^b	S_t^a	S_t^b
1	20*	1.0	10*	.1
2	22	1.0	10	.1
3	24	1.0	10	.1
4	26	1.0	10	.1
5	28	1.0	10	.1

* In demand and supply units

NOTES:

1. Assume constant demand and supply slopes; right-shifting (increasing) peak period demand over time.
2. Present value factor (PRVF) is $1 / (1 + r)^t$.
3. D_t^a is price axis intercept of peak period demand curve at time t.
4. S_t^a is price axis intercept of supply curve at time t.
5. D_t^b is slope of peak period demand curve in time interval t.
6. S_t^b is slope of supply curve in time interval t.

[redacted]

TABULAR CALCULATIONS FOR BEST SINGLE PEAK PERIOD PRICE

t	PRVF	PI_t	A_t	B_t	$DISA_t$	CS_t	DCS_t
1	.98039	10.00	9.09	.909	8.91	45.45	44.56
2	.96117	12.00	10.91	.909	10.48	65.45	62.91
3	.94232	14.00	12.73	.909	11.99	89.09	83.95
4	.92385	16.00	14.55	.909	13.44	116.36	107.50
5	.90573	18.00	16.35	.909	14.82	147.27	133.39
					59.65	463.63	432.32

NOTES:

1. PI_t is the price axis intercept of the peak period net outlay curve in time interval t: $PI_t = D_t^a - S_t^a$.
2. A_t is the quantity axis intercept of the peak period net outlay curve in time interval t: $A_t = PI_t / D_t^b + S_t^b$.
3. B_t is the slope of the peak period net outlay curve with reference to the price axis in time interval t: $B_t = A_t / PI_t$.
4. $DISA_t$ is the A_t intercept discounted to the present.
5. CS_t is the consumers' surplus in time interval t.
6. DCS_t is CS_t discounted to the present.
7. DAC is the quantity axis (Q) intercept of the composite net outlay curve discounted to the present:

   $$DAC = \sum_1^5 DISA_t \text{ ; } DAC = 59.65 \text{ units.}$$

8. PIC is the composite peak period net outlay curve price axis intercept.
9. BC is the slope of the linear composite peak period net outlay curve:

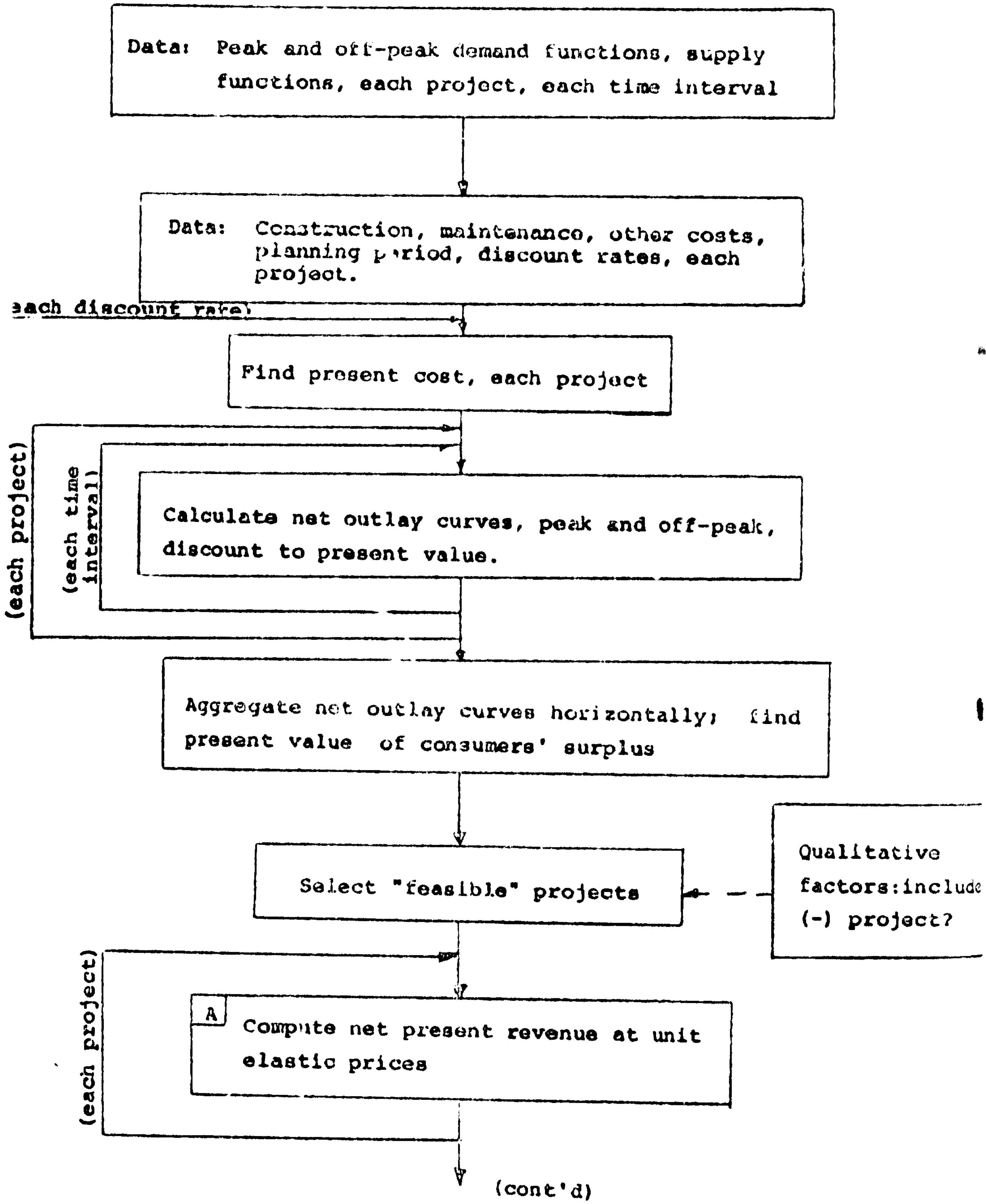
   $$BC = DAC / PIC$$

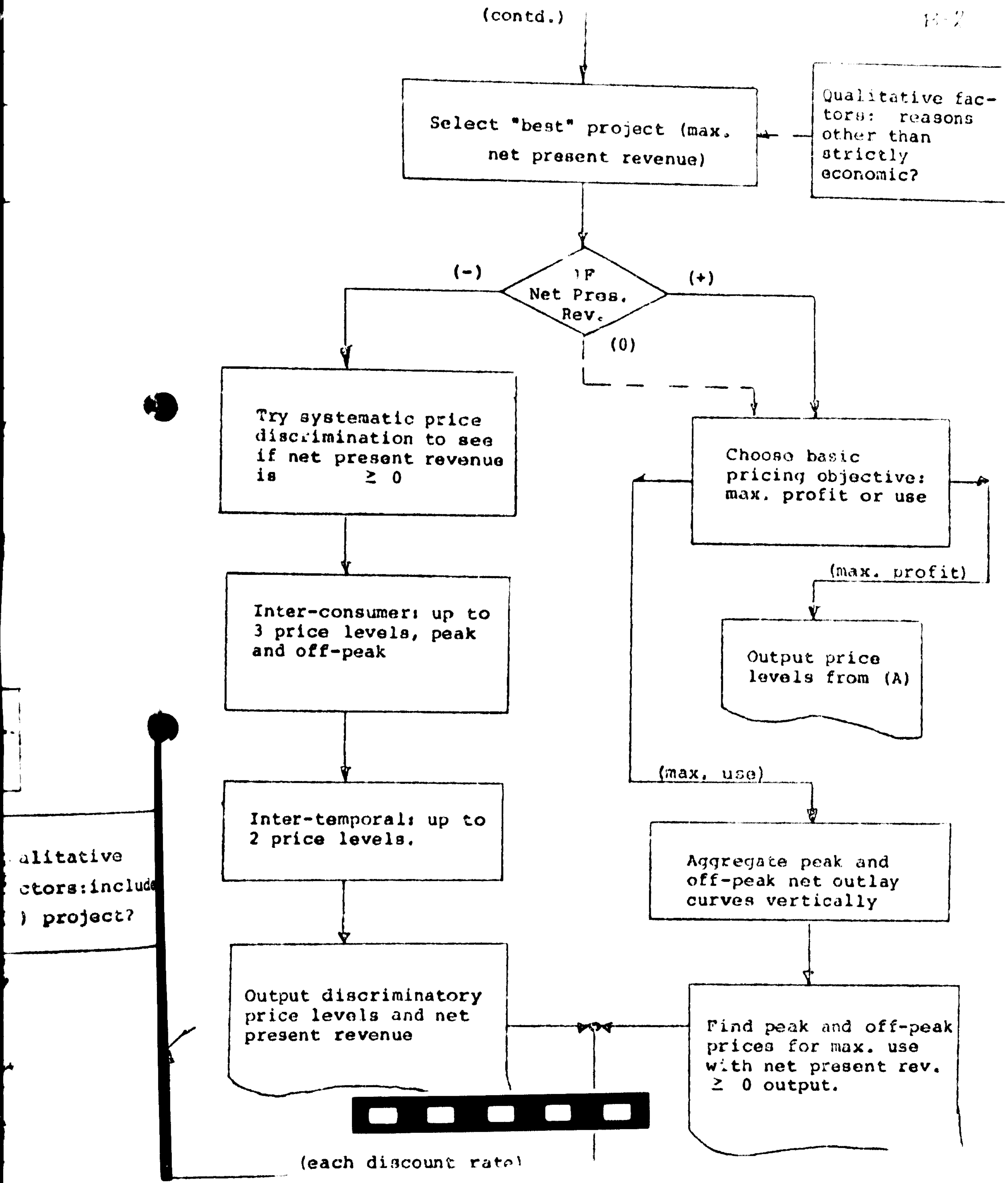
10. P^* is the single best peak period price for maximum revenue:

    $$PIC = 2 \times DCS / DAC$$
    $$P^* = PIC / 2 \qquad P^* = 7.25 \text{ units.}$$

APPENDIX B

DIAGRAM OF COMPUTATIONAL PROCEDURE




(contd.)
Select "best" project (max. net present revenue)
Qualitative factors: reasons other than strictly economic?
IF Net Pres. Rev.
(-)
(+)
(0)
Try systematic price discrimination to see if net present revenue is ≥ 0
Inter-consumer: up to 3 price levels, peak and off-peak
Inter-temporal: up to 2 price levels.
Output discriminatory price levels and net present revenue
Choose basic pricing objective: max. profit or use
(max. profit)
Output price levels from (A)
(max. use)
Aggregate peak and off-peak net outlay curves vertically
Find peak and off-peak prices for max. use with net present rev. ≥ 0 output.
(each discount rate)
alitative
ctors:includ
) project?

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