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NEW METHODS FOR COMPUTING ELASTICITIES [1]

by

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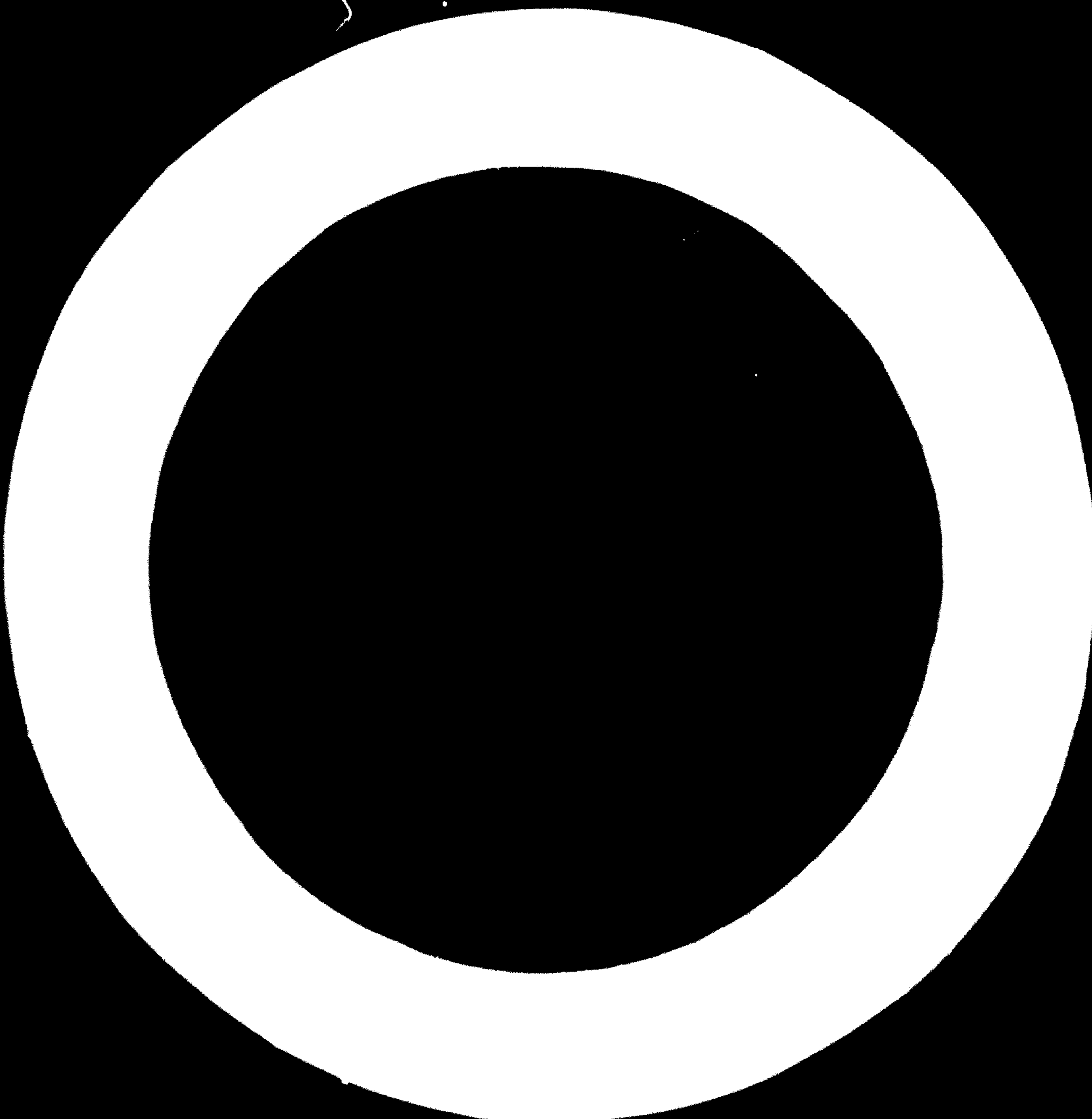

INTRODUCTION

Econometric data frequently come in the form of time series. In order to transform a nonstationary stochastic process (e.g. a series which contains a trend) it seems reasonable, especially in the case of short series, to utilize finite differences (Tintner 1940). This might be considered a generalisation of the usual practice in econometric investigations to utilize first differences. The choice of the appropriate difference is a difficult multiple choice problem. The complicated true distribution of the ratio of the variance of two consecutive difference series can be simplified by using the results of a circular population as convenient approximations (Rao and Tintner 1962). There tables are computed for small samples. Numerical approximations can also be used in order to deal with the multiple choice problem (Rao and Tintner 1963).

Having found the appropriate order of the differences involved we may use the method of transformations in order to estimate statistically the very form of the econometric relations within the class of linear relations and power transformations. The zero power corresponds to the natural logarithm. (Box and Cox 1964, Tintner and Kadekodi 1973).

Let $x_0, x_1, \dots x_p$ be economic variables, which have already been transformed into differences. We want to estimate the relationship:

$$(1) \quad x_0^{\lambda_0} = a_0 + a_1 x_1^{\lambda_1} + \dots + a_p x_p^{\lambda_p} + e$$

With just two variables x_0 and x_1 we have for $\lambda_0 = 0$, $\lambda_1 = 1$ the log, for $\lambda_0 = 1$, $\lambda_1 = 0$ the semilog, $\lambda_0 = 0$, $\lambda_1 = -1$ the loginverse relations.

We want to estimate the constants $a_0, a_1, \ldots a_p$ as well as the powers $\hat{\lambda}_0, \lambda_1, \ldots \lambda_p$. Assuming a normal distribution for the deviations e, this can be accomplished empirically by application of the method of maximum likelihood. The special case $\lambda_0 = \lambda_1 = \ldots = \hat{\lambda}_p$ should also be considered. The elasticities are then:

$$(2) \quad \eta x_0 / \eta x_1 = (\partial x_0 / \partial x_1)(x_1 / x_0) = \frac{a_1 \lambda_1 x_1^{\hat{\lambda}_1}}{x_0^{\hat{\lambda}_0} \lambda_0}$$

Fiducial and confidence limits for the estimate can be computed by using traditional regression theory.

A slight generalization of these methods is possible, if a dependent variable appears with two powers. Consider e.g.

$$(3) \quad x_0^{\lambda_0} = a_0 + b_1 x_1^{\lambda_1} + b_2 x_1^{\mu_1} + e$$

Here we have for $\lambda_0 = 0$, $\lambda_1 = -1$, $\mu_1 = 0$ the log/relation.

Again, assuming a normal distribution for e, we can use the method of maximum likelihood to estimate empirically the constants $b_0, b_1, b_2, \lambda_0, \lambda_1, \mu_1$. The elasticity is in this case:

(4) [illegible]

[illegible] standard errors, [illegible] and confidence limits by the methods of traditional regression theory.

CONTENTS

1. Importance of elasticities for economic policy.

Consider a deterministic relation between two economic variables:

(1) $x = f(y)$

Economic policy measures may change the variable y. Then we derive:

(2) $dx/dy = df(y)/dy = f'(y)$

the derivative of x with respect to y. This may be interpreted in the following way: If y changes by a small quantity Δy, the resulting change in x will be approximately Δx:

(3) $\Delta x = f'(y)(\Delta y)$

This result depends evidently upon the units, in which x and y are measured. If the relation (1) is computed for different countries, we wish for a measure of change which is independent of the units in which quantities, prices, etc. are measured in a given country, and which are typically different for different countries. The formula for the elasticity is already to be found in Cournot (1838). It has been popularized among economists by Marshall (1948), generalized by Allen (1949).

The formula for the elasticity is in our simple case:

(4) $$Ex/Ey = \lim_{\Delta y \to 0} (\Delta x/x) / (\Delta y/y) = (dx/dy)(y/x) =$$

$$= f'(y)(y/x)$$

The interpretation is now as follows:

Assume that y increases by a small proportion ($\Delta y/y$). Then we might expect that the proportional increase in x ($\Delta x/x$) will be approximately Ex/Ey.

To give an example. Assume that y increases by 1%. Then the anticipated increase in x will be approximately (Ex/Ey) %.

Models which use elasticities for economic policy purposes have been constructed by the author and his collaborators (von Hohenbalken and Tintner 1962, Tintner and Pollan, Tintner and Davila 1965). Here the exogenous variables are considered the instruments of economic policy (Tinbergen 1956, Theil 1964, Fox, Sengupta, Thorbecke 1966). If ceteris paribus one of the exogenous variables (instruments) increases by 1%, the calculated elasticities give an answer to the following question: By how many % will the endogenous variables increase?

In our simple model we consider x an endogenous variable and y an exogenous variable or instrument.

2. A model for computing elasticities.

Consider now the quite general transformations:

(5) $x^{(1)} = g(x)$

(6) $y^{(1)} = h(y)$

Writing our original model now in the form:

$x^{(1)} = f(y^{(1)})$

we derive for the computation of the elasticity the following formula.

(7) $\frac{Ex}{Ey} = \frac{f'(y^{(1)})h'(y)y}{g'(x)x}$

An especially interesting case is the power transformation.

(8) $x^{(1)} = x^{\lambda}$

(9) $y^{(1)} = y^{\mu}$

which has been considered by the statisticians (Box and Cox 1964). In fact, the idea is much older, a similar approach appears e.g. already in the book on probability by Lord Keynes (Keynes 1948).

If we modify the transformation slightly:

(10) $x^{(2)} = \frac{x^{\lambda} - 1}{\lambda}$

(11) $y^{(2)} = \frac{y^{\mu} - 1}{\mu}$

it can be seen, that the zero power ($\lambda = 0$, $\mu = 0$) corresponds to the natural logarithm:

(12) $\lim_{\lambda \to 0} (x^{\lambda} - 1)/\lambda = \log_e x$

(13) $\lim_{\mu \to 0} (y^{\mu} - 1)/\mu = \log_e y$

Using the transformations $x^{(1)}, y^{(1)}$ we derive for the elasticities:

(14) $\frac{Ex}{Ey} = \frac{f'(y^{(1)})\, \mu\, y^{\mu}}{\lambda\, x^{\lambda}}$

Using the transformations $x^{(2)}, y^{(2)}$ we have for the elasticity:

(15) $\frac{Ex}{Ey} = \frac{f'(y^{(2)})\, y^{\mu}}{x^{\lambda}}$

Consider now the special and computationally convenient case of a linear relation between the transformed variable:

(16) $x^{(k)} = f(y^{(k)}) = a + b\, y^{(k)}$, $k = 1,2$.

where a and b are constants which should be estimated by statistical methods. We have:

(17) $f'(y^{(\lambda)}) = b$

The elasticities are then for the two transformations considered:

$k=1$

(18) $Ex/Ey = b\mu y / \lambda x^{\lambda}$

$k=2$

(19) $Ex/Ey = by / x^{\lambda}$

It should perhaps be remarked, that the linear form comprises functions which have been discussed in the literature:

Table 1

	$\lambda = 0$	$\lambda = 1$
$\mu = -1$	loginverse	inverse
$\mu = 0$	log log	semilog
$\mu = 1$	log	linear

Consider now a slight generalization, i.e. a linear model where the variable y appears with two different exponents μ and ρ:

(20) $x^{\lambda} = A + B y^{\mu} + C y^{\rho}$

where again A, B and C are constants which have to be estimated by statistical methods. Here we find again a model which has been considered in the literature:

Table 2

$\lambda = 0$	$\mu = -1$	$\rho = 0$
	loginverse log	

The elasticity is in this case:

$$(21)\quad \frac{Ex}{Ey} = \frac{[illegible]}{\lambda x^{\lambda}}$$

Another possible model is:

$$(22)\quad (x^{\lambda} - 1)/\lambda = A^{*} + B^{*}((y^{\mu} - 1)/\mu) + C^{*}((y^{\rho} - 1)/\rho)$$

In this case the elasticity is given by:

$$(23)\quad \frac{Ex}{Ey} = \frac{B^{*}y^{\mu} + C^{*}y^{\rho}}{x^{\lambda}}$$

3. <u>Difference transformations</u>.

If the data for the computation of the elasticity come in the form of time series, it is necessary to transform these series which in general contain a trend, i.e. are not stationary, into stationary time series. For this purpose the method of Variate Difference recommends itself. If the trends of the time series are smooth functions of time, they can certainly in the small be represented by polynomials. But a polynomial of order p has the property, that its p-th series of finite differences is constant, and hence differences of order p+1, p+2,.. are zero. (Anderson 1929, Box and Jenkins 1970, Grenader and Rosenblatt 1957, Hannan 1970, Kendall and Stuart 1966, Malinvaud1970, Strecker 1970, Tintner 1940, 1952, 1960, Wilks 1962, Yaglom 1955).

Consider a time series $z_1, z_2 \ldots z_N$ observed at equidistant intervals. Since it must be smooth, seasonal fluctuations have to be eliminated. We define the variances of differences:

(24) $$V_0 = \sum_{t=1}^{N} (x_t - \bar{x})^2 / (N-1)$$

(25) $$\bar{x} = \sum_{t=1}^{N} x_t / N$$

(26) $$V_k = \sum_{t=1}^{N-k} (\Delta^k x_t)^2 / {}_{2k}C_k (N-k), \qquad k = 1, 2 \ldots$$

where the differences are defined recursively:

(27) $$\Delta x_t = x_t - x_{t-1}$$

(28) $$\Delta^k x_t = \Delta(\Delta^{k-1} x_t)$$

And the binomial is:

$${}_{2k}C_k = (2k)! / (k!)^2$$

Unfortunately, the distribution of V_k is very complicated. In order to simplify the mathematical derivations, we assume a circular population.

Assume that the population is:

(29) $$\epsilon_1, \epsilon_2, \ldots, \epsilon_N, \epsilon_1, \epsilon_2, \ldots, \epsilon_N, \epsilon_1, \epsilon_2, \ldots$$

and that this represents a normal distribution, where the individual items are independent and have mean value zero and constant variance σ^2:

(30) $$E\epsilon_i = 0$$

(31) $$E\epsilon_i^2 = \sigma^2$$

(32) $$E\epsilon_i \epsilon_j = 0 \qquad i \neq j \qquad i, j = 1, 2, \ldots, N$$

(Tintner 19.. [illegible] of a circular population was [illegible] by R.L. Anderson (1942). See also T.W. Anderson (1971).

We define now the circular variances of variate differences:

(33) $V_k^* = \sum_{t=1}^{N} (\Delta^k x_t^*)^2 / N \binom{2k}{k}$ $k=1,2,\ldots$

where $x_{N+s} = x_s$, $s=1,2,\ldots$

The exact distribution of the circular variances of variate differences can now be determined for small samples. (Tintner 1955). Also, it is possible to compute for small samples the exact distribution of V_1^*/V_0^*, V_2^*/V_1^*, V_3^*/V_2^* which may be used as convenient approximations for the non- circular ratios V_{k+1}/V_k.

The justification is that for large samples the difference between V_k and V_k^* becomes negligible. (Tintner 1960 p.291).

There is however still a complicated multiple choice problem. The choice is not really between the approximation (in the small) by a polynomial of order p-1 and a polynomial of order p, but between polynomials of order 0, 1,2,..... Following the lead given by T.W. Anderson (1962, see also T.W. Anderson 1971) we have also approximated numerically the significance points for the following situations:

[illegible]

[illegible] were

significant, [illegible] procedure allows the solution of the multiple [illegible] problem, if the highest order of the polynomial is four. But in practice, the ratios V_k^* / V_{k-1}^* of circular variances of variate differences should be replaced by the noncircular variances V_k / V_{k-1}, for which they are convenient approximations.

4. Power transformation.

Consider now the new variables:

(34) $x_t^{(p)} = \Delta^p x_t$

(35) $y_t^{(q)} = \Delta^q y_t$

where the orders of differences p and q have been estimated by the methods of section 3.

Our model is now:

$$x_t^{(p)\lambda} = a + b y_t^{(q)\mu} + u_t$$

$$t = 1,2,\ldots,N' = N - \max(p,q)$$

We assume that the errors or deviations u_t are (approximately) normally distributed, have mean value zero, constant variance σ^2 and show no autocorrelation. We transform for computational reasons our variables by dividing them by their geometric means:

(36) $x^{(3)*} = \left(\prod_{t=1}^{N'} x_t^{(3)}\right)^{1/N'}$

(37) $y^{(3)*} = \left(\prod_{t=1}^{N'} y_t^{(3)}\right)^{1/N'}$

(38) $x^{(4)} = x_t^{(3)}/x^{(3)*}$

(39) $y^{(4)} = y_t^{(4)}/y^{(4)*}$

$t = 1, 2 \ldots N'$

and we have in terms of the transformed variables:

(40) $x_t^{(4)} = a^* + b^* y_t^{(4)} + z_t$

where z_t is again an approximately normally distributed random variable, with mean value zero, constant variance σ^2 and no autocorrelation. We have evidently:

(41) $a^* = a\, x^{(3)*}$

(42) $b^* = b\, x^{(3)*}/y^{(3)*}$

The constants a and b are estimated from the normal equations:

(43) $\hat{a}^* N' + \hat{b}^* \sum_{t=1}^{N'} y_t^{(4)\mu} = \sum_{t=1}^{N'} x_t^{(4)\lambda}$

(44) $\hat{a}^* \sum_{t=1}^{N'} y_t^{(4)\mu} + \hat{b}^* \sum_{t=1}^{N'} \{y_t^{(4)\mu}\}^2 = \sum_{t=1}^{N'} x_t^{(4)\lambda} y_t^{(4)\mu}$

for given λ, μ.

[illegible]

[illegible]

(45) [illegible]

The log likelihood function [illegible] if we disregard an irrelevant con[illegible]

(46) $L_{max}(\lambda,\mu) = -(N'/2)\log_e S^2_{\lambda\mu} + N'\log$ [illegible]

and this function must be maximized numerically with respect to λ and μ.

The small sample distribution of $L_{max}(\lambda,\mu)$ is unfortunately not known. For large samples we may use the proposition that this quantity is distributed like χ^2 with one degree of freedom.

Hence the solution of the equation:

(47) $L'_{max}(\lambda,\mu) - L_{max}(\hat{\lambda},\hat{\mu}) = 1.92$

where $\hat{\lambda}$, $\hat{\mu}$ are the maximum likelihood estimates of λ and μ, may be used to determine 95% fiducial or confidence limits for λ and μ.

Because of (43), the elasticity computed at the geometric means of $x^{(3)}$, $y^{(3)}$ is simply:

(48) $Ex/Ey = b^* \mu / \lambda$

Consider now another transformation:

Our model is now:

(49) $$\frac{x_t^{(3)\lambda} - 1}{\lambda} = a^{**} + b^{**}\left\{\frac{y_t^{(3)\mu} - 1}{\mu}\right\} + u_t$$

where we assume that the errors or deviations u_t are [illegible] variables we have:

$$(50)\quad \frac{y^{(\lambda)} - 1}{\lambda} = a + b\,[\ldots]\,\frac{x^{(\lambda)} - 1}{\lambda}\ [\ldots]$$

where we have:

$$(51)\quad a = \frac{y^{(\lambda)}\lambda - 1}{\lambda} - a\, x^{(\lambda)}\lambda\ [\ldots]\ \frac{[\ldots]}{y^{(\lambda)}}\left(\frac{x^{(\lambda)} - 1}{\lambda}\right)$$

$$b = \frac{[\ldots]\, y^{(\lambda)}\lambda}{y^{(\lambda)}\lambda}$$

Approximately (apart from an irrelevant constant) log likelihood function is given by:

$$(53)\quad L_{max}(\lambda, \mu) = -(N/2)\log_e s^2\ [\ldots]\ N \log_e \lambda\ [\ldots]$$

where s^2 is an estimate of [illegible]^2, the sum of squares of deviations in relation (43). The function $L_{max}(\lambda, \mu)$ has to be maximized by numerical methods. For large samples, it is distributed like χ^2 with 1 degree of freedom. Hence by determining

$$(54)\quad \left| L_{max}(\hat{\lambda}, \hat{\mu}) - L_{max}(\lambda, \mu) \right| = 1.92$$

we can find a 95% confidence or fiducial region for λ, μ.

(55) [illegible]

[illegible]

(56) [illegible]

[illegible]

[illegible]

[illegible])

(57) [illegible]

which [illegible] record [illegible]

[illegible]

where S^2 [illegible] sum of the [illegible] from

relation (56).

[illegible]

(58) [illegible]

[illegible]

(59) [illegible]

The [illegible] likelihood function

is now (apart from an irrelevant constant))

(60) [illegible] $= -(N/2) \log_e S^2$ [illegible]

where S^2 is the empirical sum of squares of deviations from relation [illegible] has been determined by the classical method of least squares.

[illegible]

[illegible]

(61) [illegible]

5. Tests for autocorrelation.

Let now v_t be the error of deviation from the fitted relationship ([illegible]). We test the autocorrelation by using the circular test of R.L. Anderson (1942). But we consider his tabulated values (Tintner 1960, p. 322 as convenient approximations to the empirical non-circular autocorrelation-coefficient r_1:

$$(62) \quad r_1 = \frac{\sum_{t=1}^{N-1} v_t v_{t+1} - \left(\sum_{t=1}^{N-1} v_t\right)\left(\sum_{t=2}^{N-1} v_t\right)/(N-1)}{\sqrt{\left(\sum_{t=1}^{N-1} v_t^2 - \left(\sum_{t=1}^{N-1} v_t\right)^2/(N-1)\right)\left(\sum_{t=2}^{N} v_t^2 - \left(\sum_{t=2}^{N} v_t\right)^2/(N-1)\right)}}$$

If the empirical first order autocorrelation coefficient is statistically significant, we utilize it as an approximation for a Markov process of our residuals. In this case the standard error of the regression coefficient b has to be modified.

Following the modification proposed by Wold (Wold and Jureen 1953), we compute the first autocorrelation coefficient of the variable $y^{(4)}$. Let us denote this by R_1. Compute

$$(63) \qquad \rho = r_1 R_1$$

Then the standard error has to be multiplied with a factor:

$$(64) \quad f = \sqrt{(1+\rho)/(1-\rho)}$$

[illegible]

[illegible]

[illegible]

degree of [illegible] procedure [illegible] to

compute [illegible] confidence limits for the

elasticities, see formula (40).

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