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UNITED NATIONS INDUSTRIAL DEVELOPMENT ORGANIZATION
Vienna International Centre, P.O. Box 300, 1400 Vienna, Austria
Tel: (+43-1) 26026-0 · www.unido.org · unido@unido.org

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Economic-Mathematical models and methods of locating industrial enterprises

V.F.MEDVEDEV

Director of the Scientific Research Institute of Economics and Economic-Mathematical Methods of Planning, BSSR, Gosplan, Minsk.

V.A.KAGAN

Head of the Department of Mathematical Modelling of Location and Forecasting of Development of the Economy, BSSR, SRIEMP, Minsk.

Submitted by the Byelorussian Soviet Socialist Republic.

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The report has been prepared on the basis of experience in working out and applying methods and models of location of industrial enterprises by the State Planning Committee of the Council of Ministers of the Byelorussian SSR and its scientific research Institute of Economics and Economic Mathematical Planning Methods.

INTRODUCTION

1.0. Problems of finding ways of accelerating industrial development have always been timely and have aroused great interest both in their theoretical and in their practical aspects. Among these problems the most prominent are questions associated with the development and location of complete branches of industry and also of separate enterprises within the limits of a closed region. A great deal of attention has been devoted to the consideration and analysis of these problems at a number of international symposia and seminars within the framework of the United Nations Organisation [1,2,3,4].

2.0. Because of the complexity of industrialisation in the developing countries there is a great demand for the formulation and choice of objectives and instruments of economic politics. The circumstance should also be borne in mind that in different countries there exist unequal conditions for economic development, (different norms of accumulation and consumption, various degrees of influence of foreign trade on the development of separate branches, etc.) Experience in many countries, especially in countries which in the course of one generation have overcome economic backwardness and created basic branches of modern industry, provides important and useful data for the practical solution of problems of planning the development and location of production.

3.0. The Byelorussian SSR, with almost half a century of planning history, has accumulated a great deal of experience in working out short-term, middle-term and long-term micro- and macroeconomic models, schemes and plans for the development of the national economy. At the centre of attention of planning and economic organs of the Republic there has always stood the problem of determining the most effective ways of developing the economy, the overcoming of the heritage of economic backwardness bequeathed by the pre-revolutionary past, the attainment of the highest level of production and steady increase in the standard of living of the people. The solution of these problems has also required the working out of corresponding methods of planning which would ensure rational branch, interbranch and territorial (regional) proportions and tempos in the development of the national economy.

4.0. Accumulated experience in the field of the national economy, the production of high-speed computing machines (in particular the computer "Minsk 22"), advances in applied mathematics and economic science have brought us to a new stage in the development of planning methods for the national economy - to the working out and use in planning of a system of economic mathematical models and methods for choosing the most effective version of development both for the economy of the Republic as a whole and for its separate elements and branches, regions, towns and separate enterprises.

5.0. In the present investigation we consider economic mathematical models and methods of location of industrial enterprises worked out and applied in the practice of planning and industrial designing in the Byelorussian SSR, which are a composite part of the system of planning and modelling of the economic development of the Republic. They give quantitative expression to economic laws of location of social production. Methods and location models applied in the BSSR may be of definite interest for other countries and serve as a means of facilitating the finding of the optimal solutions of problems arising in the planning and designing of programs for the development and location of industrial enterprises.

I. ECONOMIC INTERPRETATION OF LOCATION MODELS OF INDUSTRIAL ENTERPRISES.

I.1. Long-term planning or programming of the development and location of production depends in considerable degree on the means of economic politics applied in different countries. Directions, volumes and effectivity of investments which are the instruments of economic politics acquire basic significance. Long-term planning of development and location of production is made necessary by requirements for dealing with conflicts in objectives between branch and regional aspects of direction and the liquidation of the arising disproportion.

2.1. The theory and practice of long-term planning, which is on the whole a continuous process, requires an apparatus which can account elastically for conflicts and dynamics, significance and uncertainty, and other points connected with the practical implementation of projects for the development and location of industry. At the basis of the theory of optimal planning and functioning of the socialist economy lies a direction method according to which, from a number of possible versions of a plan, the best must be chosen according to the criterion of the optimum which has been decided on. At the present time it is not necessary to compare several versions of a plan, based on calculations with pencil and paper, and then to "select" the best. The most effective method of comparing a number of versions of the plan is the automised variation of their technical and economic parameters, computed on an electronic computing machine which itself, by means of a special mathematical apparatus, chooses the best from a number of plans.

3.1. The theory of optimal planning of the socialist economy and its functioning includes as its main element the idea of construction of economic mathematical models and the working out of corresponding mathematical methods which can then be practically calculated on computing machines. The model is a mathematical expression containing variables whose behaviour is analagous to that of a real system. The construction of a mathematical model makes it possible to penetrate deeply into the economic process analysed. By constructing a model we obtain a set of mathematical relationships describing all feasible plans, i.e., those plans which can be carried out while observing all the limitations on the process under consideration. It is known that the more complex the structure of an economic process the more complicated the mathematical model.

4.1. The goal of working out an effective plan of location of industrial enterprises and choosing their capacity at an optimum has always existed. The practical solution of this problem is complicated by the fact that in drawing up a future plan it is impossible to limit oneself to the priority of organisational and technical solutions associated with the structure of the enterprise itself and the branch as a whole. In the plan it is necessary to link the organisational and technical complex closely with regional factors, for the purpose of ensuring such an optimisation of location of industrial enterprises by economic regions as would lead practically to the appearance of territorial economic complexes. From this it follows that a fairly layered macroeconomic system can be modelled.

5.1. The structure of most complex systems is hierarchial with successive solutions of each of the steps of the given complex. The hierarchial system of structural economic mathematical models is the following:

- the global model in the system of planning and functioning of the national economy is the dynamic model of optimal development of the economy, on the basis of which interbranch proportions are determined. For the practical implementation and normal functioning of the system it is also necessary to have a low-order model, whose objective would be to describe territorial and production and technical structures of economic regions and separate branches of the economy.

6.1. The basic model and at the same time the one coordinating the system is the model of interbranch balance of production and consumption of the output of the national economy as a whole, closely related to the models of optimal development of local branches. The direct connection between the given models is made by means of indexes of commodity output of branches of industry; the feedback, through coefficients of direct expenditures which change periodically depending on concrete results of plan solutions obtained in making local branch models. In turn, data of the model are coordinated through production and consumption coefficients, and through these same indexes, with a model of territorial structure of non-production consumption. In addition, local branch models of production develop-

development and location are interrelated with models of production and technical structure of industrial enterprises of a given branch.

7.1. Structural economic mathematical models of production location and development are worked out and used in Byelorussia at the present time both at the branch level of the national economy and at the level of separate industrial enterprises. Moving successively down the hierarchial system of vertically coordinated structural models is the practical way of optimising the whole planning system, in which economic mathematical models of future planning of location of enterprises play a considerable part.

8.1. The use of economic mathematical models and methods for the optimisation of a system of long-term planning of the development of the national economy requires successive elaboration in the following stages:

a) choice of optimum criterion;

b) formalisation of concrete conditions in which a system functions, their reflection in the model;

c) working out or using available numerical methods and algorithms for the solution of the problem;

d) preparation of initial information for solving the problem on the electronic computer in conformity with an approved method and program;

e) solution of the problem on the computer and subsequent economic mathematical analysis of results of the solution;

f) decisions by economic organs.

9.1. Final results of the action of an economic system are characterised by definite quantitative parameters pertaining to each of the versions obtained. The choice of criteria according to which the versions are objectively assessed leads to the concept of the optimum criterion expressing the objective of the solution. The optimum criterion of effectivity of a system is that at which in concrete production conditions the maximum value of the objective function is ensured. In selecting the optimum criterion one of the main conditions is the correspondence of local branch criteria to the global optimum criterion of the functioning of the economic system as a whole. The optimum criterion in long-term planning problems makes it possible to give an exact quantitative interpretation of the effectivity of versions of development and location of production in an investigated system.

10.1. The general optimum criterion in problems of planning the development and location of production is the maximum economic effectivity (8). As particular criteria depending on concrete conditions may be taken:

- total costs
- profit (price minus prime cost)
- investment profitability[1]

The first of these criteria is the sum of transport costs for the delivery of raw products and output, and converted production costs (in which extraordinary capital expenditures are commensurated with operating expenditures through the normative conversion factor) . Total expenditures (3) are de-

[1]Used conventionally. See formula p. 9.

termined according to the formula:

$$\text{Expenditures} = C + \mathcal{E} \cdot K + T_c + T_n$$

where C-is the annual operating cost excluding expenditures on the delivery of raw products and basic materials;

K-is the extraordinary capital investment for construction of new and reconstruction of existing enterprises, taking into account construction periods and absorption of design capacities;

$\mathcal{E}$-is the normative conversion factor of capital investments;

T_c-is the expenditure on delivery of raw products and basic materials to the points of location of enterprises;

T_n-is the cost of delivery of the output to the consumer.

II.1. The objective function in the formulation of the maximum profit problem has the form:

$$P = D_n - (C + T_c + T_n)$$

where D_n-is the income from the sale of the output.

In this formulation of the problem an important feature is the possibility of comparing alternatives of development and production location both according to expenditures and according to results (volume, structure, dynamics of output) (8).

12.1. One of the particular criteria whose application is also of sufficient interest in problems of optimal planning of the development and location of production is investment productivity. This is the relationship between profits and capital investments in the basic and working funds. In this the lowest and most feasible can only be the index at which total profit is ensured sufficient to cover deposits in the production funds and the creation of funds for material incentives and development. The level of profitability is determined by the formula:

$$R = \frac{P - (T_c - T_n)}{K_\phi},$$

where P - is the profit corresponding to definite capacities of industrial enterprises;

K_ϕ-is the capital investment in basic and working funds.

13.1. Formalisation of concrete conditions in which a system of planning development and location functions assumes preliminary description and separation of external from internal relations of the system and its separate elements. The group of elements considered must be logically limited, separating out the most essential, for example, nomenclature of output, versions of enterprise reconstruction and possible dynamics of their productive capacities, preliminary determination of points of possible location, conditions of transport of output, etc. A synthesized description of conditions and relationships is formally effected in the mathematical model.

14.1. The most important condition to be taken into account in planning models of production development and location is the condition of integrality in calculating technical economic indexes entering into the optimal criterion. They include indexes associated with the period of construction and reconstruction of enterprises, and also the time for absorbing their design capacities. It is known that the future economic effect which may be obtained only after a lengthy period is not equal to the direct present effect. Often to construct large enterprises with low operating costs requires a longer period for absorption of design capacities. For construction periods longer than one year and unchanged operating costs (C), as the design volume of capital investments we take their total value converted to the last year of construction:

$$K_{np} = \sum_{t=1}^{t_c} K_t (1+\varepsilon)^{t_c - t}$$

where t_c- is the construction period

K_t-is the capital expenditure for the corresponding construction year.

15.1. Calculation of regional factors in the territorial location of industrial enterprises is the most important condition of economic mathematical modelling. To these factors belong:

- labour force and living quarters
- energy and water supply
- construction site and spur tracks
- natural resources (land, water, forests etc.)

These conditions are allowed for in the model by means of establishing for each possible point of construction of a new or reconstruction of an existing enterprise its definite function of converted costs for production or profit, including expenditures on the delivery of raw products, depending on the capacity of the enterprise.

16.1. Setting up long-term planning problems of location of industrial enterprises and their formalisation in mathematical models can be subdivided in different ways, depending on different criteria:

a) according to the number of commodities considered in the problem, into one-commodity or multiple-commodity;

b) according to the method of assigning alternatives of production capacities, we distinguish discrete and continuous formulation. In discrete formulation, a certain number of alternatives of development and specialisation of units are given, in continuous, the feasible area of their existence.

c) according to level of importance of shipping costs, we distinguish transport-production and production problems;

d) according to the state of the system in time, for any given year - the static formulation of the problem, for a long term of T years - the dynamic formulation of the problem;

e) according to the method of subdividing systems: into single-stage and multi-stage problems.

17.1. The economic interpretation of the static one-commodity production-transport problem in discrete formulation reduces to the description of development and location of single-type plants. To them we may allocate, for example, enterprises for the processing of agricultural products (flax, beet, etc.), machine-building enterprises for the production of toothed wheels, hydraulic drives, heating apparatus, interbranch enterprises for the repair of machines of the same type, motors, assemblies, etc. In addition, to the one-commodity model we may allocate problems in which it is economically justifiable to reduce types of production similar to one another in construction and technological parameters to a single one through a conventional unit (conventional ton, set, machine, assembly, etc.).

18.1. Known magnitudes in such problems are:

a) volume of consumption of output per consumer;

b) points of location of existing enterprises, and points where new enterprises may be located (we shall call them production points).

c) the dependence of production costs on the capacity of enterprises at each production point.

d) Cost of shipping a unit of output from the production point to the consumer.

Unknowns in the problem are the volumes of output at the points of location of enterprises, and also volumes of output shipped from production points to the consumer.

Thus the problem may be economically formulated as follows:

The requirements for output in a territorial cross-section are worked out. The capacities of existing enterprises are determined and possible alternatives of their development are worked out. Such factors as labour resources, power supply, the availability of building sites, etc. are calculated and points are noted where new enterprises might possibly be located. To each of them is assigned a number of possible capacities with corresponding prime cost indexes of a unit of output and specific capital investments, and transport costs for shipping a unit of output from each production point to each consumer are calculated. A version of reconstruction and development of existing enterprises must be found and such points of location and volumes of production of new enterprises, and such a scheme for supplying output to the consumer worked out that the demands of the consumer for output will be met at minimum total cost,

II. MATHEMATICAL INTERPRETATION OF MODELS OF LOCATION OF INDUSTRIAL ENTERPRISES.

19.1. __Model 1.__ A mathematical model of the formulated problem is obtained by introducing the following symbols:

m - number of production points.

n - number of consumption points.

$\{a_{ih}\}$, $h = 1 \div r_i$ - a set of all possible capacities for the ith enterprise (enterprise located at the ith ($i = 1 \div m$) point;

$\{f_i (a_{ih})\}$, $h = 1 \div r_i$ - converted expenditures at the ith enterprise for full use of capacity.

b_j = volume of consumption of the output at the jth point;

C_{ij} = expenditure on the transport of one unit of the output from the ith point of production to the jth point of consumption;

X_{ij} = volume of transport of output from the ith point of production to the jth point of consumption (consequently, $\sum_{j=1}^{n} X_{ij}$-is the volume of production of the ith enterprise, which is sought).

Using these symbols the problem is formulated in the following way:

Find the number $X_{ij} \geqslant 0$, minimising the objective function

$$F(X) = \sum_{i=1}^{m} f_i \left(\sum_{j=1}^{n} x_{ij}\right) + \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} x_{ij}$$

(which is the sum of the converted and transport costs), observing the following conditions:

1. Volume of production at the ith ($i = 1 \div m$) enterprise is taken from the given set of capacities

$$\sum_{j=1}^{n} x_{ij} \in \{a_{ih}\}, \qquad h = 1 \div m$$

2. The requirements of the jth consumer are completely satisfied

$$\sum_{i=1}^{m} x_{ij} = b_j, \qquad j = 1 \div n$$

20.1. Model 2. The first of the two limitations is too strict and is in poor agreement with the actual situation. Actually, this limitation requires that the volume of production at the ith point exactly equal the design capacity of the ith enterprise. Practically, these requirements cannot be met. Usually the volume of production differs from the design capacity of an enterprise, and it is assumed that production may not fully come up to design capacity. In order to reflect this fact, instead of the indicated limitations we introduce the following:

$$\sum_{j=1}^{n} x_{ij} \in N_i, \qquad i = 1 \div m$$

where N_i is a set consisting of r_i segments.

$$\left. \begin{array}{c} \lambda \cdot a_{ih-1} + (1+\lambda) a_{ih} \\ 0 \le \lambda \le \dfrac{\varepsilon \cdot a_{ih}}{a_{ih} - a_{ih-1}} \end{array} \right\}, \qquad h = 1 \div r_i, \qquad a_{i0} = 0.$$

Here ε - is the maximum feasible percentage of unutilised capacity.

The converted cost function $f_i\left(\sum_{j=1}^{n} x_{ij}\right)$ - is given now not as a point set but as a set of segments N_i. In case of incomplete utilisation of design capacity it is natural to reflect this fact in the production cost function by introducing a certain penalty. Denoting this essentially new function of converted costs by f_i, as before, we now have to solve the system

$$\sum_{j=1}^{n} x_{ij} \in N_i , \qquad i = 1 \div m,$$

$$\sum_{j=1}^{n} x_{ij} = b_j , \qquad j = 1 \div n.$$

$$x_{ij} \geqslant 0 , \qquad i = 1 \div m, \quad j = 1 \div n$$

minimizing the objective function F(X).

21.2. Economic mathematical models of location of industrial enterprises become more complicated if we consider problems involving a group of commodities. In this case the specialisation of enterprises and their production structure must also be determined.

Let us consider the multiple-commodity problem. In this problem, from the number of given versions of development of existing and nev enterprises we choose those which ensure the consumer with the output of all types necessary to him at the lowest cost.

22.2. Model 3. Let us introduse the following symbols:

p_l - number of types of output ($l = 1 \div p$).

a_{ih}^{l} - volume of consumption of the lth commodity at the ith point according to the hth version of production.

b_j^{l} - volume of consumption of lth output at the jth point.

$\{f_i(a_{ih})\}, h = 1 \div \bar{h}_i$ - value of converted costs at the ith point accoding to the hth version of production.

c_{ij}^{l} - transport costs for transporting a unit of the lth output from the ith point of production to the jth point of consumption.

x_{ij}^{l} - is the volume of the lth commodity transported from the ith point of production to the jth point of consumption, which is sought.

The mathematical model of the formulated multiple-commodity problem is obtained from Model 1 if we assume:

$$a_{ih} = (a_{ih}, \dots, a^p_{ih})$$
$$b_j = (b'_j \dots b^p_j)$$
$$c_{ij} = (c'_{ij} \dots c^p_{ij})$$
$$x_{ij} = (x'_{ij} \dots x^p_{ij})$$

i.e., the symbols (a_{ih}, b_j, c_{ij}, x_{ij}) are represented as dimensional vectors.

23.2. Let us consider mathematical models of location of industrial enterprises, in which the optimum criterion is taken as the investment profitability.

<u>Model 4</u>.

We introduce the symbols:

m - Number of production points ($i = 1/m$)

n - number of consumption points ($j = 1/n$)

(a_{ih}), $h = 1/r_i$ - set of possible capacities for an enterprise at the ith point.

$\{f_i(a_{ih})\}$, $h = 1/r_i$ - profit at the ith enterprise for full utilisation of capacity.

$\{g(a_{ih})\}$, $h = 1/r_i$ - capital investments in basic and working funds at the ith enterprise.

b_j - volume of consumption of output at the jth point.

C_{ij} - Cost of transporting a unit of output from the ith point of production to the jth point of consumption.

X_{ij} - Volume of output transported from the ith to the jth point ($\sum_{j=1}^{n} x_{ij}$ — Volume of production at the jth point, which is sought).

The problem consists in finding the number X_{ij} maximising the objective function

$$L(X)=\frac{\sum_{i=1}^{m} f_i\left(\sum_{j=1}^{n} x_{ij}\right)-\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij}}{\sum_{i=1}^{m} g_i\left(\sum_{j=1}^{n} x_{ij}\right)}$$

(which is the total investment profitability observing the following conditions:

1. Volume of production of the ith enterprise is taken from the given set of capacities

$$\sum_{i=1}^{m} x_{ij} \in \{a_{ih}\}, \qquad h=1\div r_i.$$

2. The requirement of the jth consumer is fully met:

$$\sum_{i=1}^{m} x_{ij}=b_j, \qquad j=1\div n.$$

24.2. <u>Model 5</u>. To modify the model to take into account the volume of production as distinguished from capacity, instead of limitation (1) we introduce the following:

$$\sum_{j=1}^{n} x_{ij} \in N_i, \quad i=1\div m, \quad \left\{\begin{matrix} \lambda\cdot a_{ih-1}+(1-\lambda)a_{ih} \\ 0\le\lambda\le\dfrac{\varepsilon\cdot a_{ih}}{a_{ih}-a_{ih-1}} \end{matrix}\right\}, \quad h=1\div r_i, \quad a_{i0}=0.$$

The multiple-commodity economic mathematical model, according to the criterion of total investment profitability, is distinguished from model 4 in that the value p ($\ell=1\div p$) and the symbols (b_j, a_{ih}, C_{ij}, X_{ij}) are considered as p-dimensional vectors.

III. MATHEMATICAL METHODS OF SOLVING PROBLEMS OF LOCATION AND DEVELOPMENT OF INDUSTRIAL ENTERPRISES.

25.3. Formulated mathematical models are problems of multiple maxima and minima solved by the non-linear discrete programming method. This is due to the fact that the function of converted costs, profits, and also capital expenditures are of a non-linear nature, and the values of capacities of industrial enterprises are given discretely, since they depend on the productivity of the basic technological equipment. It is as a rule impossible to solve these problems by considering each of the possible alternatives, even on electronic computers, since the number of alternatives will become astronomical even for a small number of variables. Therefore, the solution of problems of such a type require the working out of effective algorithms, which would direct our search and enable us to find an optimal version. As an illustration of the methods of solving one-commodity problems of location according to the total cost criterion we shall consider the idea of one of the algorithms (5). The essence of the algorithm lies in the following. At each stage of operation of the algorithm 1) to a set of all possible alternatives of location there conforms a set of approximate costs (not exceeding actual) for the implementation of these alternatives; 2) from the set of all alternatives is chosen that in which these expenditures are at a minimum. If the approximate cost of implementation of the chosen alternative is equal to the actual one, then the alternative is optimal, and the algorithm has done its work. If not, then the approximate expenditures for the realisation of the alternative increases (approaches real expenditures) and the transition to the next stage takes place. The gap between approximate and actual costs is then closed by a fuller calculation of transport costs.

27.3. A more complex problem is the one of location according to the total investment profitability criterion. This is due to the fact that the objective function is the relationship between two functions, which must be maximised. One of the methods of solving this problem, worked out by scientific workers of the Byelorussian SSR, is as follows:

A set of all possible alternatives of location is broken down into sub-groups. Each sub-group is assessed and, as the sub-groups are broken down, these estimates are made more precise.

In the mth stage the algorithm selects from all constructed sub-groups that one which gives the maximum value of:

$$max \frac{\sum_{i=1}^{m} f_i \left(\sum_{j=1}^{n} x_{ij}\right) - \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}}{\sum_{i=1}^{m} g_i \left(\sum_{j=1}^{n} x_{ij}\right)}$$

The maximum of the given relationship is reached when the numerator has a maximum and the denominator a minimum value. The solution in the numerator, generally speaking, does not coincide with the solution of the denominator. The selected set is broken down into two by the following rule: the first of them is obtained from the selected one if, at the point at which the capacity is not yet determined, we substitute the value which enters into the solution of the problem.

$$max \sum_{i=1}^{m} f_i \left(\sum_{j=1}^{n} x_{ij}\right) - \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} .$$

In the numerator of the estimation relationship of the sub-group obtained, transport costs are specified, and the denominator $\sum_{i=1}^{m} g_i \left(\sum_{j=1}^{n} x_{ij}\right)$ does not diminish. Thus, the estimate of the subgroup obtained can either remain the same or decrease. As far as the sub-group is concerned, it is obtained from the selected one, if in it we prohibit substitution of the capacity fixed in the first sub-group. The numerator of the estimation relationship of the second sub-group does not increase and the denominator does not decrease, and consequently the estimate of the second sub-group can also remain as before, or decrease. After this the transition to the $\mu + 1$ stage takes place. The solution of the problem is found when the maximal estimate is obtained for a sub-group alternative the sum of whose substituted capacities is equal to $\sum_{j=1}^{s} b_j$. This version will also be the optimal.

IV. USE OF ECONOMIC MATHEMATICAL MODELS AND METHODS IN THE PRACTICE OF PLANNING LOCATION OF INDUSTRIAL ENTERPRISES IN THE BYELORUSSIAN SSR.

28.4. The mathematical models and methods considered have found practical application in the Byelorussian SSR in the solution of problems of location of enterprises of branches of industry, in particular

- in light industry (flax-processing plants, shoe enterprises);
- in machine-building and metal-working (assembly repair and machine repair plants, centralised foundries);
- in the electrotechnical industry (electrical repair plants);
- in agriculture (specialised enterprises for repairing agricultural machinery, slaughter houses);
- in the construction industry (enterprises for the production of agglоporite, plants for manufacturing reinforced concrete products, dwelling-house building combines);
- in the food industry (meat combines, dairies, etc.)

29.4. The solution of these problems on the electronic computer "Minsk 22" manufactured at the Minsk Electronic Computer Plant has shown, in addition to the high efficiency of the machine itself, a sufficiently rapid convergence of the algorithms worked out (see Table 1).

Table 1.

PARAMETERS (DIMENSIONS) OF CERTAIN PROBLEMS OF LOCATION OF INDUSTRIAL PLANTS AND TIME OF THEIR SOLUTION ON THE ELECTRONIC COMPUTER "MINSK 22".

No.	No. of Points of Possible Location	No. of Consumers	No. of possible capacities of Enterprises	Expenditures of Machine Time on Solution of Problem
1	2	3	4	5
1.	11	28	8	21 min.
2.	20	28	14	12 min.
3.	11	28	7	51 min.
4.	12	28	8	21 min.
5.	16	28	7	31 min.
6.	16	28	9	21 min.
7.	24	62	8	160 min.
8.	9	22	6	51 min.
9.	10	20	5	51 min.
10.	22	22	8-10	26.2 min.
11.	40	30	4	50 min.

30.1. Economic mathematical analysis of solutions of these practical problems have shown that by setting up optimal plans of development and location of industrial enterprises an alternative may be found in which total costs can be decreased by 10-20%.

31.1. When solving these algorithms on the electronic computer "Minsk 22" it is possible to work in two routines: in an exact and approximate routine (with a degree of precision given beforehand). As can be seen from Table 1, the parameters (dimensions) of problems solved in the Byelorussian SSR are comparatively small. In solving problems of larger dimensions on the electronic computer, the existing algorithms are not sufficiently effective in the sense that they converge slowly.

32.4. In practice, in planning the development and location of enterprises in the Byelorussian SSR there arises the question of the solution of large dimension problems (of the order of 120 x 120 and over). Such problems can be solved on the machine in a reasonable time on the basis of the algorithms set forth in [7]

33.4. An important feature of algorithms worked out in the Byelorussian SSR is the possibility of obtaining not only one optimal version according to a selected criterion, but a series of optimal versions according to a selected criterion in the order of decreasing objective function. Additional information of this type makes it possible to take into consideration external economic factors coordinating specialisation and location of enterprises of various branches of production in the industrial complex, and to select a version with the maximum degree of correspondence to the economic optimum.

V. CONCLUSIONS AND RECOMMENDATIONS.

34.5. One of the main objectives of the present investigation consists in elucidating a definite group of theoretical and practical problems associated with the final choice of one of many versions of location of industrial enterprises in the framework of an open regional unit. The formal apparatus used for the practical solution of the problems considered synthesizes into a complex economic mathematical models, numerical methods and algorithms, electronic computers, and, in the final count, supplies a system of decisions.

35.5. Economic mathematical modelling by comparison with traditional methods of long-term planning of location of industrial enterprises ensures that results are obtained rapidly, that they are realistically assessed, and that a considerable number of variables can be calculated. Results are obtained rapidly due to effective programs for the electronic computer, and also technological construction parameters (high speed, storage capacity, etc.) of the machine itself. A realistic assessment of versions of plans obtained proceeds from the features of the economic mathematical model, in which future requirements, the established set of capacities and costs (current and extraordinary) are associated with real dependencies. The possibility of calculating a considerable number of variables makes it possible to set up a group of equations and to formulate rules for coordinating them in complex solutions.

36.5. In working out economic mathematical models of long-term planning of location of industrial enterprises we must give priority to the multiple-step method of planning, based on the use of the macroeconomic system of modelling, and obtaining from these models the most important ("strategic") ones with subsequent solution on the electronic computer. Formally the procedure of ordered sub-systems reduces to the method of successive approximations, which is sufficiently well described in the literature, and on concrete models gives approximate data at the initial information preparation stage (in particular, the requirements for the year sought, volumes of production, etc.)

37.5. The complex of solutions (group of optimal versions) obtained on the electronic computer from economic mathematical models is supplementary standard material facilitating a final decision on the optimal version of location of industrial enterprises in the regional unit. In this sense especial importance is attached to the ability of the computer to supply an ordered group of optimal alternatives, making it possible to scrutinize them and compare them in subsequent economic mathematical and technical economic analysis for the purpose of coordinating local branch decisions with the regional optimum.

An example of the solution of a problem of location of specialised enterprises for the repair of grain combines for a definite territorial economic unit (in static formulation.)

Economic Mathematical Model

m - number of points of possible location of repair enterprises (i = 1, 2 ... 22)

n - number of exchange points (j = 1, 2, ... 22)

(a_{ih}), h = 100, 300, 500, 1000, 2000, 3000, 3500 - set of possible capacities for an enterprise located at the ith point.

$(f_i (a_{ih}))$, $h = 1 \% r_i$ - converted costs for the ith enterprise depending on its capacity (see Table 1).

b_j - requirement for repair at the jth exchange point (see Table 2).

Table 2

Grain combine	1	2	....	22
	135	175	. . . .	3500

C_{ij} - cost of transporting a unit of spare units and parts from the jth exchange point to the ith repair plant (and return). (See table 3).

Table 3

Km.	10	20	. . . .	700
Rub.	26.9	33.1	. . . .	356.7

X_{ij} - Volume of goods transported from the ith exchange point to the jth repair plant, which is sought.

In conventional symbols the problem is formulated in the following way:

- to find a non-negative value X_{ij} minimising the functional:

$$F(X)=\sum_{i=1}^{22} f_i\left(\sum_{j=1}^{22} x_{ij}\right)+\sum_{i=1}^{22}\sum_{j=1}^{22} c_{ij}\, x_{ij}$$

while observing the following conditions:

1. Capacity of a repair enterprise at the ith point is taken from the given set

$$\sum_{j=1}^{22} x_{ij} \in \{a_{ih}\}, \quad h = 0 \div 3500.$$

2. Requirements of the jth exchange point for repair is fully met

$$\sum_{i=1}^{22} x_{ij} = b_j, \quad j = 1 \div 22.$$

For the solution of the conversion problem we used the algorithm given in (7).

Results of the solution of the problem on the electronic computer "Minsk 22" showed that a specialised plant for the repair of combines with an annual production capacity of 3500 units per year at the 10th point would be the most effective. In this version the total expenses are 1.74 million rubles per year, including transport costs - 12%.

CONVERTED COSTS

Table 1. (in Rubles)

Capacity	Prime Cost	Coefficient of Effectivity	Specific Capital Expenditures	Converted Cost/ unit	Total Converted Costs
1	2	3	4	5= (2+3x4)	6= 1+5
000	000	000	000	000	000
100	0817	0.15	1495	1041.25	104125
300	0657	0.15	0945	798.75	239625
500	0592	0.15	0763	696.45	348225
1000	0515	0.15	0580	602.0	602000
2000	0449	0.15	0438	514.7	1029400
3000	0415	0.15	0373	470.95	1412850
3500	0401	0.15	0248	438.2	1533700

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