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# Development of Metalworking Industries in Developing Countries

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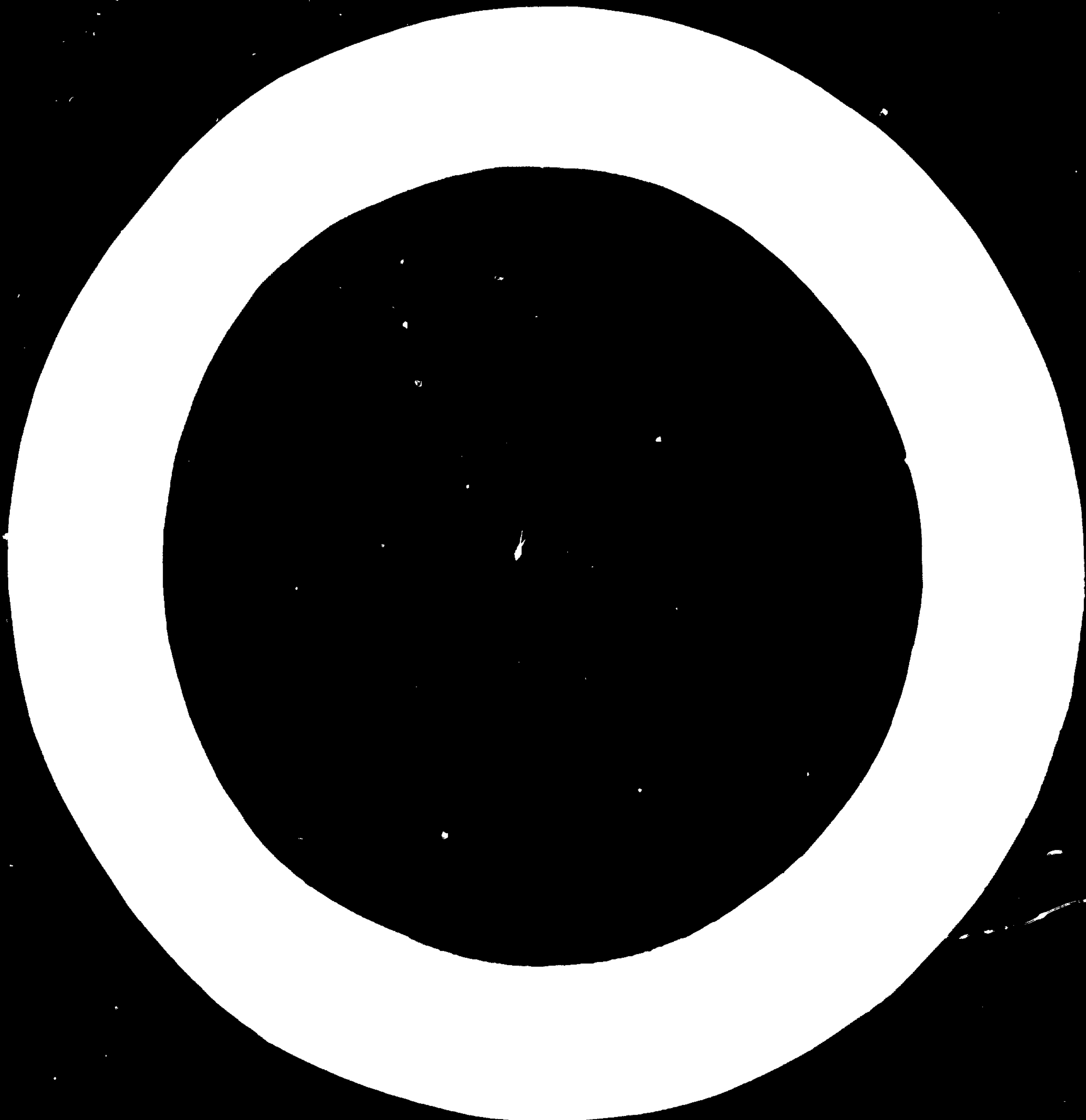

# DECISION RULES FOR EQUIPMENT INVESTMENTS IN METAL-PRODUCT INDUSTRIES WITH SPECIAL REFERENCE TO METAL-CHIPPING AND METAL-CUTTING MACHINES

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## I. Trends and figures

### *Productivity ranking (1)*

First, we will try to rank the metal-product industries by world regions, according to a productivity criterion. From the United Nations *Statistical Yearbook, 1963*, we obtain a percentage breakdown, by region, of the total value added in the metal-product industries in 1958. Also from the same source, we obtain the percentage breakdown of the total number of persons in the group in the same regions. By taking the ratio of the percentage distribution of value added over the percentage distribution of the number of persons engaged in this activity, a ranking of regional productivity is possible by using the ratio as a productivity indicator.

Productivity in the North American continent, according to our ranking procedure, is highest; in succession come the Soviet Union and Eastern Europe, and Oceania (primarily Australia and New Zealand). Europe (mainly Western Europe) is only in fourth place. Africa and the Middle East, Latin America and Asia (East and South-east), take fifth, sixth and seventh places in the ranking order (table 1).

### *Productivity ranking (2)*

A second productivity comparison will be made by relating index numbers of industrial production for certain industrial activities with the index number of industrial employment. We will compare the individual index numbers among industries per region. In addition, we will take the ratio of the index number of industrial production over the index number of industrial employment as an indicator for labour productivity in a specific year for a certain industrial activity per region. The ratio for 1958 is A, for 1962, B. The same regions as indicated in table 1 are participating in the comparisons. The indices and ratio A are presented in table 3 for ten industrial activities as defined in table 2.

If we compare among industries per region, using the industrial production indices from table 3, we notice that the index for the metal-product industries (ISIC 35–38) has the highest value, compared to all other industry groups, for the following regions: World, Soviet Union and Eastern Europe, North America, Latin America and Asia. For Europe, the index is the third highest and for the European Economic Community it is second.

Looking now at the industrial employment index in table 3, we notice that this industry has the highest value compared to all other industry groups for the USSR and Eastern Europe, North America, East and South-east Asia, Europe and the European Economic Community. For Latin America the index is second highest, and for the world only seventh highest (together with ISIC 23, 24, 29).

This comparison shows that the metal-product industries are of outstanding importance from a production as well as an employment point of view.

*Table 1*

PERCENTAGE DISTRIBUTION IN 1958 OF METAL-PRODUCT INDUSTRY (ISIC 35–38)

| *Regions* | *Value added in industry* | | *Number of persons in industry* | | *Ratio 1/2* | | *Productivity ranking* |
|---|---|---|---|---|---|---|---|
| **Africa and Middle East** | | 0.6 | | 1.3 | | 0.5 | 5 |
| **North America** | | 38.7 | | 17.9 | | 2.2 | 1 |
| **Latin America** | | 1.2 | | 3.1 | | 0.4 | 6 |
| **Asia (East and South-east)** | | 3.1 | | 12.2 | | 0.3 | 7 |
| Excluding Japan | 0.6 | | 6.7 | | 0.1 | | |
| **Europe** | | 28.7 | | 36.8 | | 0.8 | 4 |
| E.E.C. | 15.6 | | 20 | | 0.8 | | |
| E.F.T.A. | 11.5 | | 13.5 | | 0.9 | | |
| **USSR and Eastern Europe** | | 26.5 | | 27.3 | | 1 | 2 |
| **Oceania** | | 1.2 | | 1.4 | | 0.9 | 3 |
| *Total* | | 100 | | 100 | | | |

*Table 2*

DEFINITION OF INDUSTRY GROUPS

| *International Standard Industrial Classification (ISIC) number* | *Description of industry* |
|---|---|
| 1 1-3, 511, 512 | Mining, manufacturing, electricity and gas |
| 2 20-22 | Food, beverages and tobacco |
| 3 23, 24, 29 | Textiles, clothing and leather products |
| 4 25-26 | Wood products and furniture |
| 5 27 | Paper and paper products |
| 6 11, 13, 30-32 | Coal and crude petroleum, chemical, coal, petroleum and rubber products |
| 7 14-19, 33 | Non-metallic minerals and products |
| 8 12, 34 | Metal mining and basic metals |
| 9 35-38 | Metal products |
| 10 511, 512 | Electricity and gas |

If we now look at the ratio of index 1 over index 2, ratio A as a kind of indicator for labour productivity, we notice that, for the following regions, this ratio is higher for the metal-product industries than for any other industry group: World, USSR and Eastern Europe (together with ISIC 14-19, 33), East and South-east Asia.

For North America the ratio comes at the third place (behind ISIC groups 51-512 and 11, 13, 30-32). For Latin America the ratio also comes at the third place, for Europe at the seventh place and for the European Economic Community at the sixth place.

From this we may conclude that the growth of the index of industrial production for the metal-product industries has been less than the growth in the index of industrial employment. Of the seven regions distinguished, the industrial index was highest in five and the employment index was highest, also, for five (but different regions) of the seven, but the ratio of the two indices, however, is highest only in three of the seven regions. Table 4 summarizes the results of the ranking procedure.

This gives evidence of the following:

(*a*) The metal-product industry is a labour intensive industry;

(*b*) Mechanization and automation are more limited than in certain other industries;

(*c*) Choices in equipment with different levels of mechanization exist but are presumably not made in an optimal way in many cases.

An optimal choice is not necessarily a choice for the most mechanized equipment. Elsewhere, this author has

*Table 3*

PRODUCTIVITY INDICATOR A (1961 OR 1962; 1958 = 100)

| *ISIC index ration* | *1-3, 511-512* *1* | *20-22* *2* | *23, 24, 29* *3* | *25, 26* *4* | *27* *5* | *11, 13, 30-32* *6* | *14-19, 33* *7* | *12, 34* *8* | *35-38* *9* | *511, 512* *10* |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | *World (1961)* | | | | | |
| 1 | 126 | 115 | 117 | 126 | 122 | 125 | 128 | 126 | 132 | 130 |
| 2 | 109 | 107 | 106 | 109 | 110 | 102 | 110 | 110 | 106 | 105 |
| 1/2 = A | 1.16 | 1.07 | 1.10 | 1.16 | 1.11 | 1.23 | 1.16 | 1.15 | 1.25 | 1.24 |
| | | | | | *USSR and Eastern Europe (1962)* | | | | | |
| 1 | 150 | 131 | 127 | 151 | 131 | 138 | 159 | 144 | 175 | 160 |
| 2 | 114 | 105 | 110 | 115 | 111 | 109 | 114 | 115 | 126 | 113 |
| 1/2 = A | 1.31 | 1.25 | 1.15 | 1.31 | 1.18 | 1.27 | 1.39 | 1.25 | 1.39 | 1.42 |
| | | | | | *North America (1962)* | | | | | |
| 1 | 126 | 114 | 120 | 120 | 122 | 126 | 118 | 120 | 134 | 134 |
| 2 | 104 | 99 | 103 | 102 | 107 | 97 | 103 | 100 | 109 | 99 |
| 1/2 = A | 1.21 | 1.15 | 1.17 | 1.17 | 1.14 | 1.30 | 1.15 | 1.20 | 1.23 | 1.35 |
| | | | | | *Latin America (1961)* | | | | | |
| 1 | 119 | 111 | 116 | | 122 | 122 | 116 | 116 | 129 | 125 |
| 2 | 106 | 112 | 98 | | 115 | 104 | 103 | 123 | 112 | — |
| 1/2 = A | 1.12 | 0.99 | 1.18 | | 1.06 | 1.17 | 1.13 | 0.94 | 1.15 | — |
| | | | | | *East and South-east Asia (1961)* | | | | | |
| 1 | 164 | 121 | 131 | | 169 | 141 | 151 | 185 | 245 | 154 |
| 2 | 117 | 111 | 107 | 117 | 121 | 109 | 114 | 124 | 144 | — |
| 1/2 = A | 1.40 | 1.09 | 1.22 | | 1.40 | 1.29 | 1.32 | 1.49 | 1.70 | — |
| | | | | | *Europe (1962)* | | | | | |
| 1 | 127 | 119 | 115 | 126 | 129 | 133 | 130 | 122 | 131 | 135 |
| 2 | 108 | 107 | 103 | 107 | 110 | 97 | 107 | 106 | 115 | 105 |
| 1/2 = A | 1.17 | 1.11 | 1.12 | 1.18 | 1.17 | 1.37 | 1.21 | 1.12 | 1.14 | 1.28 |
| | | | | | *European Economic Community (1962)* | | | | | |
| 1 | 132 | 116 | 119 | 127 | 130 | 141 | 132 | 125 | 138 | 135 |
| 2 | 108 | 107 | 104 | 103 | 109 | 97 | 105 | 109 | 116 | 104 |
| 1/2 = A | 1.22 | 1.08 | 1.14 | 1.23 | 1.19 | 1.45 | 1.26 | 1.14 | 1.19 | 1.30 |

Index 1 — Industrial production. Index 2 — Industrial employment.

*Table 4*

RANKING ACCORDING TO THE HEIGHT OF INDICES AND RATIO A INTER-INDUSTRY PER REGION FOR METAL-PRODUCT INDUSTRIES

| *Region* | *Year* | *Ranking according to height of index of industrial* (1) *production* | (2) *employment* | *Ratio A* 1/2 |
|---|---|---|---|---|
| World .......... | 1961 | 1 | 7[a] | 1 |
| USSR and Eastern Europe | 1962 | 1 | 1 | 1[b] |
| North America .. | 1962 | 1 | 1 | 3 |
| Latin America ... | 1961 | 1 | 2 | 3 |
| East, South-east Asia .... | 1961 | 1 | 1 | 1 |
| Europe ......... | 1962 | 3 | 1 | 7 |
| European Economic Community ... | 1962 | 2 | 1 | 6[c] |

[a] Relative position shared with ISIC 23, 24, 29.
[b] Relative position shared with ISIC 14, 19, 33.
[c] Relative position shared with ISIC 27.

shown that the choice of equipment in the metal-product industries depends on: the wage rate and the interest rate; annual production (size of market), and the homogeneity of production (the size of production runs or lots).

In the second part of this study we will use an atomistic approach, in the sense that we will use highly disaggregated data to explore for which metal-chipping and metal-cutting operations there are choices or no choice in equipment with various degrees of mechanization, and how the optimal choice depends on the variation of such parameters as wage and interest rate and size of lots.

The aim of this first part of our study is only to show some trends and tendencies by comparison with highly aggregated figures.

### *Productivity ranking (3)*

The same procedure as carried out in table 3 for the years 1961 or 1962 was done for the year 1955, yielding ratio B.

Table 5 presents both cross-region ranking and inter-industry per region ranking order.

Noteworthy is the low ranking among industries of the metal-product industries in Europe. Cross-region, the metal-product industries in Europe rank low, especially

*Table 5*

RANKING ACCORDING TO INCREASE IN LABOUR PRODUCTIVITY INDICATOR FOR METAL-PRODUCT INDUSTRIES 1955–1961(62): ISIC 35–38

| *Region* | *Period* | *Ranking number* *Cross region* | *Inter-industry per region* |
|---|---|---|---|
| World ............ | 1955–1961 | 2 | 1 |
| USSR and Eastern Europe .. | 1955–1962 | 6 | 5[a] |
| North America .... | 1955–1962 | 1 | 2 |
| Latin America ..... | 1955–1961 | 7 | 6 |
| East, South-east Asia ........... | 1955–1961 | 3 | 5 |
| Europe ........... | 1955–1962 | 5 | 9 |
| European Economic Community ..... | 1955–1962 | 4 | 5 |

[a] Relative position shared with ISIC 511–512.

compared to North America. Some improvements in the efficiency of the group for Europe seem urgent on the basis of this comparison.

### *Annual production growth rates*

For a number of countries, the average annual rate of growth in the industrial production index of the metal-product industries is computed for the period 1953–1962. Table 6 shows the results. Some countries have amazingly high annual growth rates, such as Japan, 72.22 per cent, Taiwan, 42.59 per cent and Venezuela, 35.29 per cent, to mention only some countries with a growth rate higher than 30 per cent a year. Countries with a lower than 5 per cent annual growth rate during this period are: Argentina, Canada, the United States and the United Kingdom.

*Table 6*

AVERAGE ANNUAL RATE OF GROWTH, METAL PRODUCTS (PER CENT YEAR) (1958 = 100)

| *Countries* | *Index of industrial production* 1953 | 1962 | *Annual rate of growth* 1953–1962 |
|---|---|---|---|
| United States ....... | 111 | 135 | 2.40 |
| Canada ............ | 100 | 115 | 1.67 |
| Argentina ......... | 82 | 92 | 1.36 |
| Brazil (basic metals) . | 72 | 150 | 12.04 |
| Venezuela .......... | 34 | 142 | 35.29 |
| Hungary ........... | 80 | 172 | 12.78 |
| Czechoslovakia .... | 59 | 161 | 19.21 |
| Poland ............ | 52 | 195 | 30.56 |
| Soviet Union ....... | 49 | 175 | 25.71 |
| India .............. | 40 | 168 | 35.56 |
| Japan ............. | 40 | 300 | 72.22 |
| China (Taiwan) ..... | 36 | 174 | 42.59 |
| Belgium ........... | 79 | 126 | 6.61 |
| France ............ | 63 | 126 | 11.11 |
| Germany (F.R.) .... | 56 | 139 | 16.47 |
| Netherlands ........ | 72 | 145 | 11.27 |
| Italy .............. | 70 | 163 | 14.76 |
| Sweden ............ | 83 | 126 | 5.76 |
| United Kingdom ... | 83 | 114 | 4.15 |

Comparing the relative positions of the United States and Canada in table 6 with the relative position of North America in table 5, some evidence is found that efficiency increases have materialized, possibly by increased mechanization and automation.

### *Some further correlations*

In trying to interpret the height of the average annual growth rate of the index of the industrial production in the metal-product industry, the average annual growth rate in the index of *per capita* product for the same period was used as an indicator of over-all economic growth, assuming that countries with rapid economic growth would also experience rapid growth rates in the metal-product industries.

Table 7 shows the sample where the dependent variable $y$ stands for the average annual growth rate in the industrial index of the metal-product industry and $x$ stands for the average annual growth rate in the index of *per capita* product. The correlation coefficient is not high, 0.5944, with a standard error of 14.8246.

*Table 7*

**Average annual growth rate of *per capita* product index ($x$) and industrial product index ($y$) of metal-product industries**

| *Countries* | $x$ | $y$[a] |
|---|---|---|
| United States | 1.10 | 2.40 |
| Canada | 0.91 | 1.67 |
| Argentina | 1.81 | 1.36 |
| Brazil | 3.58 | 12.04 |
| Venezuela | 3.02 | 35.29 |
| Hungary | 5.82 | 12.78 |
| Czechoslovakia | 6.81 | 19.21 |
| Poland | 6.39 | 30.55 |
| Soviet Union | 9.74 | 25.71 |
| India | 1.17 | 35.56 |
| Japan | 11.54 | 72.22 |
| China (Taiwan) | 3.88 | 42.59 |
| Belgium | 2.78 | 6.61 |
| France | 4.37 | 11.11 |
| Germany (F.R.) | 6.96 | 16.47 |
| Netherlands | 3.70 | 11.27 |
| Italy | 6.53 | 14.76 |
| Sweden | 3.83 | 5.76 |
| United Kingdom | 2.17 | 4.15 |

[a] See table 6.

Finally, an effort was made to explain $y$, the average annual growth rate of the industrial index in the metal-products industries, using four independent variables: (*a*) the income *per capita*, (*b*) the size of the population; (*c*) the product of (*a*) and (*b*), and the average growth rate of the index in *per capita* product.

More independent variables were introduced, and the sample size then reduced to fifteen countries due to lack of information. The countries are: United States, Canada, Argentina, Brazil, Venezuela, India, Japan, China (Taiwan), Belgium, France, Germany (F.R.), the Netherlands, Italy, Sweden and the United Kingdom.

A multiple correlation coefficient of 0.8005 was obtained to which the variables of *per capita* income and the average annual rate of growth of the *per capita* product index contributed most. As the size of the market is not only the home market but also the foreign market, the introduction of an appropriate variable for the foreign market would, undoubtedly, have improved the results. Further work is intended to improve the sample and to find the best combination of independent variables.

## II. The analysis

In this section, we intend to make the analysis from which we will derive decision rules in the last part.

### *Aim*

The aim of the analysis is, as the title of this study suggests, the derivation of decision rules for equipment investments in metal-chipping and metal-cutting machines for a developing metalworking industry.

It is also suggested that there is something to decide, that a choice among alternative equipment types can be made. Everybody who has some experience with the metalworking industry knows that, for many metal-working operations (tasks) a choice among alternative machines does indeed exist. Even the highly aggregated data used in part I suggest the possibility of choices in metalworking equipment. If choices must be made, they should preferably be made in an optimal way and the present study thus should give answers to such questions as: which variables determine the optimum choice in metalworking equipment and how do these variables influence the optimum choice in metalworking equipment?

The practical questions follow directly: which type of analysis must be used to answer the first two questions most adequately and, subsequently, to whom are we going to direct ourselves, to private or to public decision makers? The last question is of importance because, in practical life, optimal choices for private or public authorities may mean different things. For this reason, we state as an additional aim of our analysis that it should yield results from which general rules can be derived, rules applicable to private as well as public decision makers. Hence, the final aim of our analysis can now be formulated: the derivation of decision rules on equipment investments in establishing or expanding a metal-product industries for private and or public decision makers.

### *Scope*

The scope of the analysis should be broad, in the sense that its results should cover:

(*a*) All countries in the world;
(*b*) All scales of production operation;
(*c*) All types of metal-products industries;
(*d*) All types of decision makers.

In order to achieve this, the analysis must take into account the production circumstances which simulate the real production characteristics for countries in various stages of economic development. To this end a model should be developed in which:

(*a*) Prices of primary inputs (stock and or flow) can be varied in a discrete way simulating the relative scarcity relationship of capital and labour (and possibly of foreign exchange) for most-highly, highly, semi- and under-developed countries.

(*b*) Lot sizes or production runs (defined as the number of identical parts produced with a single setup) can be varied, simulating small-, medium-, and large-scale production characteristics.

(*c*) A number of production tasks should be defined which vary according to shape, size, and precision, simulating in this way characteristics of all possible types of metal products.

The variations in wage and interest rates can be considered to reflect actual market prices or equilibrium prices, gross or net of inflation. Private decision makers will be inclined to work with market prices, gross of inflation; public decision makers should preferably make use of estimated equilibrium prices, net of inflation.

*Type*

We now have to decide on the type of analysis that will best meet our previously formulated aims and scope. Best fitted for our purposes is a sensitivity analysis: basically, we want to know how sensitive machine-tool optimality is when certain parameters are varied.

A machine is considered to be optimal if it can produce a given unit of output at lower total capital and labour costs than any alternative machine.

The sensitivity of which parameters on machine-tool optimality do we want to explore? First, of the wage- and interest-rate parameters (parameters are given exogeneously). Economically, the most-highly developed countries will be characterized by a relatively high wage rate and relatively low interest rate, under the assumption that in such countries capital is abundant relative to labour, while under-developed countries will be characterized with a reverse factor-price relationship—that is, a high interest rate and a relatively low wage rate under the assumption that labour is abundant relative to capital. The prices of labour and capital for the middle group, highly developed countries and semideveloped countries, will be set between the extremes of the most-highly and under-developed countries.

The next parameter that will be varied is the size of lots; the variation will be such that all the possible scales of production will be simulated.

The variation of interest rates for equipment capital and wage rates for labour and the variation in size of lots will be called sensitivity analysis A.

The introduction of costs of structures, a type of capital, and the introduction of a variation in equipment prices (e.g., transportation costs for equipment and the application of an equilibrium exchange rate instead of an over-valued official exchange rate causes equipment prices to be hiher in under-developed countries) will be explored in sensitivity analysis B. In this analysis, the efficiency rate of labour will be varied under the assumption that, in under-developed countries, this rate, as a result of such factors as less skill and less work discipline, will be lower than the efficiency rate in developed countries.

For the sensitivity analyses we will formulate total-cost functions, where total costs are a function of the equipment price, the interest rate, the labour time and the price of labour, the size of lots and the efficiency rate, from which unit-cost functions will be derived. Hence, as machine-tool optimality is defined as that machine with the lowest total capital and labour cost per unit, we will analyse how the variation of the above parameters influences the machine-tool optimality. As soon as we have established the sensitivity behaviour of machine-tool optimality we can then derive our general decision rules.

As we have now broadly indicated which type of analysis we will apply, something should be said about the level of aggregation on which the analysis will be carried out and which type of data will be applied. The analysis will be carried out at the most disaggregated level possible by applying engineering estimates of time data for metal-machining tasks which are defined for certain shapes, sizes, and precisions. For each task are listed alternative machines capable of carrying it out. Time data for each task on each alternative machine are used; time data consist of piece time (machine time and hand time) and setup time. Tasks are also used as the unit quantity of output; consequently a task unit is defined as an elementary machining operation with a particular shape, size and precision for a specific metal.

By assigning an investment cost to each machine and assuming that this study, as a first approximation, deals with only a one-machine—one-man relation, we know capital and labour requirements (piece times) per task per alternative machine.

By varying the prices of capital and labour in a discrete way, and by varying the lot sizes, we can analyse the price effect and the lot size effect on the optimal machine (sensitivity analysis A). The upper and lower sections of figure 1 illustrate respectively the lot size and the price effects on the total-cost function of one machine. The effect of a lot size increase on the total cost function of the individual machine, for fixed and given prices of capital and labour, is two-fold:

(*a*) The slope of the curve is affected as the labour requirements per unit of output decrease (the fixed setup time is divided by a larger number of units in the lot);

(*b*) The annual productive capacity of the machine is increased. In a fixed, given number of annual productive machine and man-hours (which are identical under the assumption of a one-man—one-machine relationship) more units can be produced as the production time per unit is decreased by producing larger batches. Note that within each lot size there are constant returns to scale, as can be easily observed in figure 1. Shifting from lot size 1 to lot sizes 2 and 3 causes increasing returns to scale. This latter effect is one of the subjects of investigation.

The unit costs will be measured at the full utilization level expressed in time units on a one shift basis of the machine. This assumption of full utilization is justified as we think in highly aggregated terms rather than at the level of the individual firm.

It could be argued that the various sizes of firms stand between the individual pieces of equipment and the aggregate output and that, at the firm level, the utilization is not necessarily 100 per cent; however, this author believes that, by varying the lot sizes between extreme boundaries, the variation in "productive utilization" of equipment covers all practical cases.

Figure 1 also shows what happens at the individual firm level, where the indivisibilities of machines are relevant. In an aggregate sense, the indivisibility aspect of the individual machines levels out. For this reason, we measure the lot size effect at the points of full utilization of the machines. Connecting the origin with the points of full utilization of a particular machine for a specified lot size, we obtain straight curves, as shown in the figure.

In figure 1B we illustrate the effect of price changes of capital and labour on the total-cost function of the individual machine for a given, fixed lot size. The annual productive capacity is now constant as the lot size is kept constant. Increasing the price of capital and decreasing the wage rate also has a twofold effect:

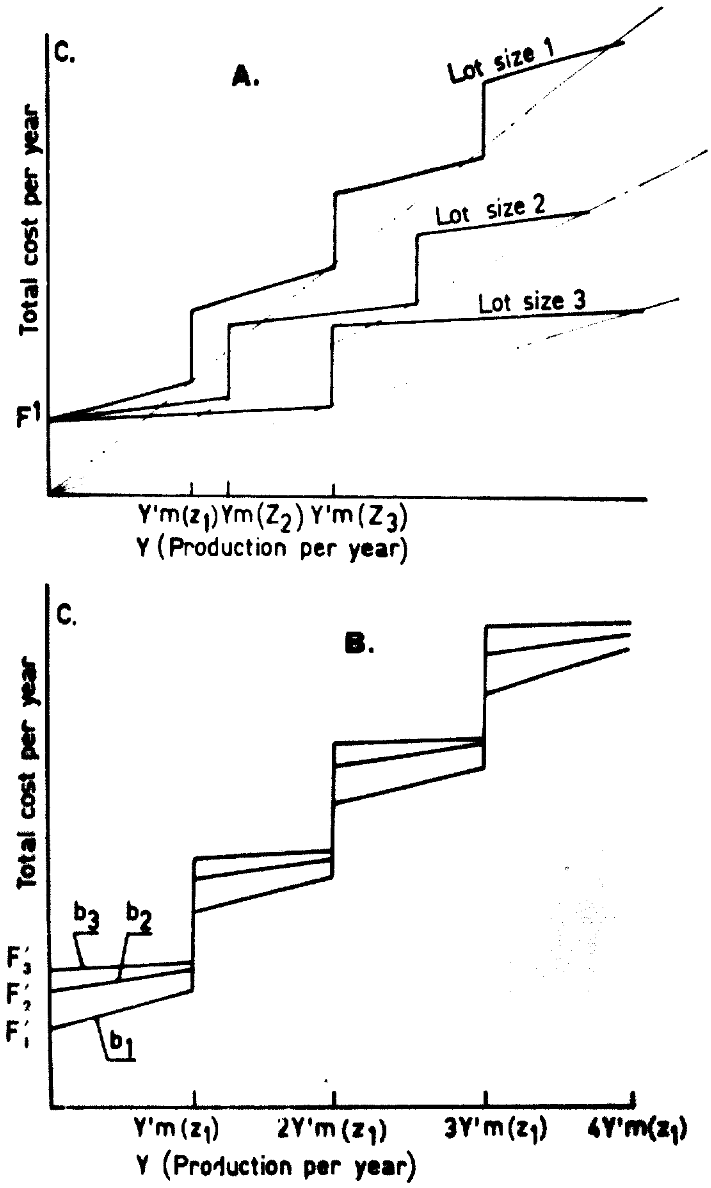


*Figure 1*

**EFFECT OF INCREASING RETURNS TO SCALE ON TOTAL COST FUNCTION FOR ONE MACHINE FOR A GIVEN AND FIXED PRICE OF CAPITAL AND LABOUR**

(*a*) The intercept, which measures the fixed cost, increases from $F_1$ to $F_2$ to $F_3$ in our example.

(*b*) The slope decreases as the price of labour is discretely lowered at the same time the price of capital is increased in a discrete way.

Unit cost will be measured at $y_m$ for each capital-labour price set (figure 1B), and at each lot size $z_d$ ($z_1$, $z_2$, $z_3$, etc., figure 1A).

For reasons of simplification, the figure shows only the total-cost function for one machine. In fact, for each individual task, there are as many cost functions as there are economically feasible machines to produce the task unit.

Analysis B is identical to analysis A except for the variations in the parameters mentioned earlier. By comparing the machine optimality in analysis B with the machine optimality obtained in analysis A, the sensitivity of machine optimality to the varied parameters as a group can be observed.

The analyses outlined above can best be characterized by the term sensitivity analyses, as they compare total capital and labour cost per task by keeping certain parameters constant and by varying certain others. From the changes in total unit cost, which lead to changes in optimality, the sensitivity of each parameter or group of parameters can be established.

### *Assumptions*

The assumptions on which this analysis are based are:

(*a*) Complementary or intermediate inputs for each machine per unit of product are the same and consequently omitted. Hence, the analysis counts only the capital (in analysis A only the equipment capital, in analysis B the equipment capital and the capital invested in structures) and labour cost per task unit per alternative machine.

(*b*) It is assumed that equipment is used 2,000 hours a year. Effective utilization, measured in physical output per year per machine, varies considerably as a function of lot size variation. It is assumed that the fluctuations in physical output per year reflect reasonably well the fluctuations in annual capacity utilization in actual production circumstances.

(*c*) The annual potential productive capacity of the machines remains constant over the years and is measured in physical units, in other words, gross benefits remain constant.

(*d*) The lifetimes of all the machines are equal and constant.

(*e*) The cost of capital includes interest and depreciation. The interest rate is assumed constant throughout the lifetime of the equipment. Interest and depreciation are maintained constant per year by applying a capital recovery factor (CRF) in computation.

(*f*) Labour is considered a variable input, but the price of labour remains constant over the lifetime of the equipment. Under these assumptions, benefits and costs are constant over the lifetime of the equipment and there is no need to introduce a discounting procedure in the calculation. Total costs can be computed on an annual basis and remain constant for a given price of capital and labour.

(*g*) The cost functions are linear step functions under the assumption that, for a given lot size, labour inputs are constant per unit of output.

(*h*) Prices of capital and labour are exogenously given. Each set of capital and labour prices is assumed to be representative for a specified geographical area with a certain degree of industrial maturity.

(*i*) For reasons of simplification, we assume that the same task unit is produced the year around, varying the size of lots. In reality, not the same but comparable task units are produced the year around; however, this assumption of a uniform task unit in production simplifies the analysis considerably and does not affect the conclusions.

($j$) Lot sizes are exogenously given under the assumption that they are dictated to the entrepreneurs by size and composition of demand.

### *The model*

We will now present some of the most essential equations of the model, which all refer to one task unit, $j$. The following symbols are introduced:

- $\hat{c}$ — total capital and labour costs for task $j$ on optimal machine
- $c^i$ — total capital and labour costs for task $j$ on machine $i$
- $k$ — equipment capital for task $j$ on machine $i$ for a given lot size $z_d$
- $j$ — task unit ($j = 1 \ldots 51$); subscripts $j$, however, will generally be omitted: the model refers to one task unit, $j$
- $l^i(z)$ — labour-output ratio for operator on machine $i$, task $j$, for a given lot size $z_d$
- $m$ — subscript for annual capacity output of a machine
- $n$ — number of shifts
- $p^i$ — piece time on machine $i$
- $r^*$ — price of capital (including depreciation) per year ($h = 1 \ldots 4$)
- $s^i$ — setup time on machine $i$
- $w^*$ — price of labour, per unit of time
- $z^*$ — lot size ($d = 1 \ldots 7$)
- $C^i$ — total capital and labour cost for annual production on machine $i$
- $E$ — highest integral number smaller than $U^i$ (if $U^i$ [illegible] 1)
- $F$ — efficiency factors
  - $F_a$ for illness, holidays, etc.
  - $F_B$ for allowances for rest and personal care
  - $F_v$ for general efficiency level inside and outside the factory, to the extent that it influences the productivity of the individual operator ($F_v$ is assumed to be 1 in analysis A)
- $H$ — potential maximum annual number of machine working hours on a one-shift basis
- $i$ — alternative machine, to produce a given task unit (superscript $i = 1 \ldots 5$)
- $H^i$ — efficient annual number of machine working hours
- $K^i$ — new price of machine $i$ capable of producing task $j$ in a given year and country, expressed in United States dollars
- $U_m^i$ — degree of utilization at capacity output
- $Y_m^i$ — annual capacity-output level of machine $i$ expressed in tasks $j$ for a specified shift pattern, lot size, and efficiency parameters ($Y_m^i$ indicates a side condition)

Starting point of the model is a function for total cost:

$$C^i(r,w) = K^i\left[E(U^i) + 1\right] r + \left[l^i(z)\right] Y_m^i w \qquad (1)$$

* The symbols $r$, $w$, and $z$ represent continuous variables; we consider only discrete values of these variables, indicated by subscripts $d$ and $h$; however, in the model, these subscripts will be omitted in order to simplify the notation.

where

$$l^i(z) = \frac{s^i}{z} + p^i \qquad (1.1)$$

and

$$H^i = F_a F_B F_v H \qquad (1.2)$$

and

$$Y_m^i(z) = \frac{nU_m^i H^i}{l^i(z)} \qquad (1.3)$$

then

$$\left.\begin{aligned} \frac{C^i(r,w)}{Y_m^i(z)} &= \left[\frac{K^i}{Y_m^i(z)}\right] r + \left[l^i(z)\right] w \\ c_m^i(r,w,z) &= \left[k_m^i(z)\right] r + \left[l^i(z)\right] w \end{aligned}\right\} \qquad (2)$$

where

$$\frac{C^i(r,w)}{Y_m^i(z)} = c_m^i(r,w,z) \qquad (2.1)$$

and

$$\frac{K^i}{Y_m^i(z)} = k_m^i(z) \qquad (2.2)$$

then

$$\hat{c}_m(z_d, r_h, w_h) = \min\left[k_m^i(z_d)\right] r_h + \left[l^i(z_d)\right] w_h. \qquad (3)$$

$$d = 1 \ldots 7 \qquad h = 1 \ldots 4.$$

Equation (1) gives the total unit cost equation with definitions of the basic relations in equations (1.1, 1.2, and 1.3).

Equations (2) give two versions of the unit-cost equation, with partial relations further explained in equations (2.1) and (2.2).

Finally, from equation (3), the solution of our problem comes as it states that the total capital and labour cost $\hat{c}$ for the optimal machine is a function of $z_d$, $r_h$, and $w_h$. By varying $d$ from 1 to 7 and keeping $h$ constant, changes in optimality of machines can be observed, which are due to lot size variation. By varying $h$ from 1 to 4 and keeping $d$ constant, changes in optimality that are due to price variation can be observed.

The foregoing refers to sensitivity analysis A. For analysis B, only minor changes in the basic model are needed. A new symbol is introduced $K_\lambda^i$, which indicates investments in space requirements needed for worksite around machine $i$; hence, the total investment for machine $i$ is $(K^i + K_\lambda^i)$. Equation (1.2) is revised as

$$H_i = 2F_a \, 2F_B \, F_v \, H \qquad (1.2.1)$$

Finally, instead of $K^i$, $2K^i$ is used in the equations for analysis B, as the machine investment is assumed as doubled.

### *Sources and significance of data used*

#### *The tasks*

So far, we have only occasionally referred to the data. The basic sample was collected in 1955. The sample is complete in so far as it includes all the conventional metal-chipping and metal-cutting machines. The more recently developed numerically or tape controlled metalworking machines are not included in the sample. With the conventional machines we mean all the metal-cutting and metal-chipping machine tools excluding electronically controlled machine tools.

As stated, the analysis is on the task level. How representative is such an analysis for the derivation of general conclusions? The point is that, in actual production circumstances, one works with parts. Very seldom does a part require only one machining task. Most commonly, multiple machining tasks have to be carried out on a part. If the time data by task, supplied by our basic sample, can just be added, yielding the same production time requirements as a part analysis could have given, there is no problem. In that case, one simply analyses which basic tasks have to be performed on whatever part one might be interested in and, after having determined the magnitude of the relevant parameters, the conclusions of our analysis are directly applicable.

After a careful investigation on this question, the conclusion is that for eleven tasks, except those involving the lathe family of machine tools, there is no significant difference between task and part analysis. For tasks involving lathing, there might be a difference because in lathes successive steps of mechanization can be distinguished most clearly. This means that, by carrying out a number of tasks in which a lathe is involved, one economizes on production times whenever, because of mechanization, tasks can be automatically changed, in various degrees, without interference by the operator.

For this reason, translating the task analysis into a part analysis will yield production times somewhat high for tasks involving turret lathes and automatic screw machines. However, after several trials with modified time data, our conclusion must be that the above indicated fact affects the outcome of our results in only a minor way.

*Task characteristics*

Tasks are characterized by shapes, size, and precision. Thirteen shapes, five sizes and three precision classes of the work piece were distinguished, as shown in table 8.

*Table 8*

CATEGORIES OF TASK CHARACTERISTICS

| No. | *Category I* Geometric shape | *Category II* Size of piece | *Category III* Precision |
|---|---|---|---|
| 1 | Flat surfaces, no contour | Very small | Semi-precision |
| 2 | Flat surfaces, external contour | Small | Precision |
| 3 | Flat surfaces, internal contour | Medium | High precision |
| 4 | Cylindrical surfaces, external | Large | |
| 5 | Cylindrical surfaces, interior | Very large | |
| 6 | Drilled holes | | |
| 7 | Cylindrical forms, external | | |
| 8 | Standard screw threads | | |
| 9 | Standard gear shapes | | |
| 10 | Complex shapes | | |
| 11 | Irregular periphery, flat surface | | |
| 12 | Multiple surfaces | | |
| 13 | Multiple holes, drilled | | |

*Prices*

Four discrete values of *h* (price set of production factors) are used which can be roughly identified as those prevailing in North America, Western Europe, the semi-industrialized countries, and the under-industrialized countries. A price set is defined as a wage rate and a capital rate that are used in conjunction. The prices are presented in table 9.

*Table 9*

CAPITAL AND LABOUR PRICES

| *Region* | *Interest rate per year* | *Lifetime of equipment (years)* | *Price set* *Capital recovery factor (CRF)* | *Wage rate in US $/hr* |
|---|---|---|---|---|
| North America .... | 5 | 10 | 0.12950 | 3.6 |
| Western Europe ... | 5 | 10 | 0.12950 | 2.0 |
| Semi-industrialized countries ... | 10 | 10 | 0.16275 | 0.45 |
| Under-industrialized countries ... | 15 | 10 | 0.19925 | 0.20 |

*Lot sizes*

For all tasks, seven lot sizes will be taken into account: 5, 10, 50, 100, 200, 300, and infinite. It is believed that the indicated lot size ranges cover all scales of possible production operations, that is, small-scale, medium-scale and large-scale production.

*Efficiency rate*

The basic data are corrected by certain allowances for fatigue and delay, varying with the type of machine, the degree of precision and the size of lots. In sensitivity analysis A they reflect normal annual production allowances common in the United States. In sensitivity analysis B, the allowances are doubled under the assumption that, because of differences in skill, differences in internal organization and differences in the organization of the economy at large, only half the efficiency can be obtained from those prevailing in the country with the highest industrial efficiency.

*Sample size*

Two sample sizes are distinguished. Sample size 1 includes all tasks for which (economical) feasible alternative machines are listed.

Sample size 2 includes tasks for very large pieces (size characteristics, category II, 5) and single observations. By single observations we mean tasks for which no alternative machines, no choice in equipment (or capital intensity or level of mechanization) exists. Sample size 1 includes fifty-one tasks; sample size 2, thirty-seven tasks of which four refer to size 5; and thirty-three to single observations. With the latter term we mean that for these tasks only one machine is listed as feasible and consequently no alternative can be chosen.

*Summary for parameter values*

In summarizing this section we present in table 10 the numerical values of the parameters that are varied in the analysis.

*Table 10*

NUMERICAL VALUES OF PARAMETERS

| *Price variation* $h$ | *Price set* | |
|---|---|---|
| | *Wage rate in US dollars* ($w_h$) | *Capital rate* ($r_h$) |
| **1** | 3.6 | 0.12950 |
| **2** | 2.0 | 0.12950 |
| **3** | 0.45 | 0.16275 |
| **4** | 0.20 | 0.19975 |
| *Lot size variation* $d$ | *Lot size* ($z_d$) *(task units)* | |
| **1** | 5 | |
| **2** | 10 | |
| **3** | 50 | |
| **4** | 100 | |
| **5** | 200 | |
| **6** | 300 | |

As stated, the parameters for sensitivity analyses A and B are the same; however, in analysis B, capital includes the amount invested in floor space; fatigue and personal and work delay allowances are doubled and equipment capital for all machines is uniformly doubled also.

*Problem formulation*

*Sensitivity analysis A*

*Problem*: Separate the influences of the following parameters on the optimum technique:

1. The variation of the prices of capital and labour (respectively $r_h$ and $w_h$, $h = 1 \ldots 4$).
2. The variation of the sizes of lots (batches or production runs) $z_d$, ($d = 1 \ldots 7$).

*Given*: The given parameters can be divided into two categories:

1. Parameters that are kept constant
   (*a*) Equipment prices for alternative machines $i$;
   (*b*) Potential maximum number of machine working hours per machine per year;
   (*c*) Time data for each task $j$, piece time ($p$ = machine labour time per task unit), and setup time ($s$ = make-ready time) for a number of identical task units on each alternative machine;
   (*d*) Shift pattern, degree of utilization, efficiency, etc.
2. Exogenously given, parameters that are varied in a discrete way e.g., parameters for lot size, wage rate, and capital rate.

*Procedure*:

1. Find the optimal capital intensity of the machine ($k$ = decision or independent variable) by determining the minimum total capital and labour costs ($\bar{c}$ = dependent variable) per task unit for each machine $i$.
   (*a*) Compute unit cost by equation (2) for each machine $i$ for each value of $r_h$ and $w_h$ for a given and constant $z_d$.
   (*b*) Determine $\bar{i}$ by equation (3) for each value of $r_h$ and $w_h$ for constant $z_d$.
   (*c*) Determine if $\bar{i}$ is changing, giving $r_h$ and $w_h$ different values as $h = 1, 2, 3, 4$, for given and constant $z_d$.
2. Determine whether or not and to what extent a variation in lot size affects the optimum technique.
   (*a*) Compute equation (2) for each machine $i$ for each value of $z_d$ for given and constant $r_h$ and $w_h$.
   (*b*) Determine $\bar{i}$ by equation (3) for each value of $z_d$ for given and constant $r_h$ and $w_h$.
   (*c*) Determine if $\bar{i}$ is changing, giving $z_d$ its values as $d = 1, 2, 3, 4, 5, 6, 7$ for each given and constant $r_h$ and $w_h$.

*Sensitivity analysis B*

Sensitivity analysis B is essentially the same as analysis A. Only parameters kept constant in analysis A, such as the equipment price and the allowances for labour, are varied in analysis B. In analysis B, the costs of structures for the worksite of the operator and the machine are included in the capital investment in addition to the price of the machine.

*Results*

By having discussed the model and the data we are now ready to feed the data into the model:

Labour-output ratios are computed with equation 1.1;
Annual capacity outputs per machine are computed with equation 1.3;
Capital-output ratios are computed with equation 2.2;
Total unit costs are computed with equation 2.
Equation 3 yields the result.

*Results of analysis A*

(*a*) Price effects of capital and labour occur whenever $\bar{i}$ shifts as a result of varying values of $r_h$ and $w_h$ for each given and constant value of $z_d$.

(*b*) Lot size effects occur whenever $\bar{i}$ shifts as a result of varying values of $z_d$ for given and constant values of $r_h$ and $w_h$.

Table 11 summarizes the degree of sensitivity to variation in the lot size parameters and indicates the variation in capital and labour prices for each task. The sensitivity to variation in lot size is defined as follows: for each task, for a given and fixed price set, the maximum number of changes in machine optimality due to lot size variation is equal to the number of economically feasible machines minus one, multiplied by four (the number of price sets).

*Table 11*

SENSITIVITY CLASSIFICATION TO VARIATION IN LOT SIZE PARAMETER

| *No. of alternative machines* | *Maximum no. of changes* | *Sensitivity classes (in number of changes per task in machine optimality)* | | | | |
|---|---|---|---|---|---|---|
| | | *N(none)* | *L(low)* | *M(medium)* | *H(high)* | *V(very high)* |
| **1** | 0 | 0 | | | | |
| **2** | 4 | 0 | 1 | 2 | 3 | 4 |
| **3** | 8 | 0 | 1, 2 | 3, 4 | 5, 6 | 7, 8 |
| **4** | 12 | 0 | 1 3 | 4 6 | 7 9 | 10 12 |
| **5** | 16 | 0 | 1 6 | 7 9 | 10 12 | 13 16 |

The sensitivity to variation in capital and labour prices is defined below. We distinguish the following optimality patterns, per task for each lot size, due to price variation. Each pattern of four X markings, as listed in table 13, has to be analysed independent of any preceding or following pattern.

Depending on the number of economically feasible machines, we classify the degree of sensitivity to price variation as in table 12.

*Table 12*

**SENSITIVITY CLASSIFICATION TO VARIATION IN CAPITAL-LABOUR PRICE PARAMETERS**

| *Number of alternative machines per task* | *Maximum number of changes in optimal machine* | *Sensitivity degree* N | L | M | H | V |
|---|---|---|---|---|---|---|
| 1 | | | | | | |
| 2 | 14 | | 2 | 3 5 | 6 10 | 11 |
| 3 | 21 | | 4 | 5 10 | 11 16 | 17 |
| 4 | 28 | | 6 | 7 12 | 13 20 | 21 |
| 5 | 28 | | 7 | 8 14 | 14 25 | 26 |

• The number of price variations in the analysis is 4, the number of lot size variations is 7. The number of changes in machine optimality, owing to price variation, depends on the number of alternative machines per task. The maximum number of changes is the number of price variations multiplied by the number of lot size variations.

*Table 13*

| *Optimality pattern* | *Number of variations in optimal machines for given and fixed $z_4$* |
|---|---|
| X X X X | 0 |
| X X X<br>X | 2 |
| X X<br>X X | 2 |
| X X<br>X<br>X | 3 |
| X<br>X X X | 2 |
| X<br>X X<br>X | 3 |
| X<br>X<br>X X | 3 |
| X<br>X<br>X<br>X | 4 |

In table 14, each task is classified according to its degree of sensitivity for lot size and capital and labour price variation. For sample size 2, there is no sensitivity to either lot size and price variation, except for task 52, which shows some sensitivity to lot size variation.

*Table 14*

**SENSITIVITY OF MACHINE OPTIMALITY TO LOT SIZE VARIATION AND CAPITAL AND LABOUR PRICE VARIATION**

Analysis A

| *Task no.* | *Lot size variation* N | L | M | H | V | *Price variation* N | L | M | H | V | *Number of economically feasible machines* |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | 4 | | | | | | 12 | | 3 |
| 2 | | | 2 | | | | | | 6 | | 2 |
| 3 | | | 2 | | | | | | 10 | | 2 |
| 4 | | 2 | | | | | | | 16 | | 3 |
| 5 | | | | | 4 | | 2 | | | | 2 |
| 6 | | | | 6 | | | | | 13 | | 3 |
| 7 | | | | | 4 | 0 | | | | | 2 |
| 8 | | | | | 4 | | | | 8 | | 2 |
| 9 | | | | 3 | | | | 4 | | | 2 |
| 10 | | | | | 4 | | 2 | | | | 2 |
| 11 | | | | | 4 | 0 | | | | | 2 |
| 12 | | | 12 | | | | | | 6 | | 2 |
| 13 | | | | 3 | | | | | 10 | | 2 |
| 14 | 0 | | | | | 0 | | | | | 1 |
| 15 | 0 | | | | | 0 | | | | | 1 |
| 16 | | | | 3 | | | | 4 | | | 2 |
| 17 | | | | | 4 | | 2 | | | | 2 |
| 18 | | | | 5 | | | | 10 | | | 3 |
| 19 | | | | | 4 | | 2 | | | | 2 |
| 20 | | | 2 | | | | | | 10 | | 2 |
| 21 | | | | | 4 | | 2 | | | | 2 |
| 22 | 0 | | | | | 0 | | | | | 1 |
| 23 | | | | | 4 | | 2 | | | | 2 |
| 24 | | | | 5 | | | | | 12 | | 3 |
| 25 | | | | | 4 | | | | 8 | | 2 |
| 26 | | | | 5 | | | | 9 | | | 3 |
| 27 | | | | | 8 | | | 9 | | | 3 |
| 28 | | | | 3 | | | | | 8 | | 2 |
| 29 | | 1 | | | | | 2 | | | | 2 |
| 30 | | | 2 | | | | | | 10 | | 2 |
| 31 | | | 2 | | | | | | 10 | | 2 |
| 32 | | | 4 | | | | | 10 | | | 3 |
| 33 | | | 2 | | | | | | | 12 | 2 |
| 34 | | 1 | | | | | | | | 13 | 2 |
| 35 | | | 3 | | | | | | 12 | | 3 |
| 36 | | | 3 | | | | | | 15 | | 3 |
| 37 | | 1 | | | | | | | | 14 | 2 |
| 38 | 0 | | | | | 0 | | | | | 1 |
| 39 | | 1 | | | | | | | | 12 | 2 |
| 40 | | 1 | | | | | | | | 14 | 2 |

## *Results of analysis B*

Machine price and efficiency rate effects do occur whenever $\bar{c}$ shifts as a result of doubling the machine price and reducing the efficiency rate of labour, keeping all other parameters and variations in parameters identical to analysis A.

The sensitivity of machine optimality to variation in the efficiency rate and variation in the equipment prices can be determined by comparing the optimality markings in analysis B with those of analysis A. Any change in machine optimality in analysis B compared to analysis A is the combined effect of the variation in the parameter values changed.

Table 15 indicates the sensitivity ranking for each task. For no task is a high or very high sensitivity observed. Considering sample size 1 from the fifty-one tasks, fourteen do not show sensitivity, twenty-six show low

*Table 15*

SENSITIVITY CLASSIFICATION TO VARIATION IN EFFICIENCY RATE AND MACHINE PRICE

| *Sensitivity* | *Number of changes per task in machine-tool optimality* |
|---|---|
| None (N) | |
| Low (L) | 1 or 2 |
| Medium (M) | 2 to 7 |
| High (H) | 7 to 14 |
| Very high (V) | 14 |

*Table 16*

SENSITIVITY OF MACHINE OPTIMALITY TO MACHINE PRICE AND EFFICIENCY RATE VARIATION

Analysis B

| *Task* | *Equipment prices and efficiency rate variation* | | | | | *Task* | *Equipment prices and efficiency rate variation* | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | *N* | *L* | *M* | *H* | *V* | | *N* | *L* | *M* | *H* | *V* |
| 1 | | 2 | | | | 26 | | 1 | | | |
| 2 | — | | | | | 27 | | | 3 | | |
| 3 | — | | | | | 28 | | | 3 | | |
| 4 | | 2 | | | | 29 | — | | | | |
| 5 | | 1 | | | | 30 | | 2 | | | |
| 6 | | | 6 | | | 31 | | 2 | | | |
| 7 | — | | | | | 32 | | | 4 | | |
| 8 | | 1 | | | | 33 | | | 3 | | |
| 9 | | 1 | | | | 34 | | 2 | | | |
| 10 | — | | | | | 35 | | 2 | | | |
| 11 | — | | | | | 36 | | | 4 | | |
| 12 | | 2 | | | | 37 | | 2 | | | |
| 13 | | 2 | | | | 38 | — | | | | |
| 14 | — | | | | | 39 | | 1 | | | |
| 15 | — | | | | | 40 | | 1 | | | |
| 16 | | 1 | | | | 41 | | 2 | | | |
| 17 | — | | | | | 42 | | 3 | | | |
| 18 | | 1 | | | | 43 | | 1 | | | |
| 19 | — | | | | | 44 | — | | | | |
| 20 | — | | | | | 45 | | | 3 | | |
| 21 | | 1 | | | | 46 | | 2 | | | |
| 22 | | 1 | | | | 47 | | | 3 | | |
| 23 | | 1 | | | | 48 | | 2 | | | |
| 24 | | | 5 | | | 49 | | | 5 | | |
| 25 | | | 3 | | | 50 | — | | | | |
| | | | | | | 51 | | 2 | | | |

sensitivity and eleven show medium sensitivity. The additional thirty-seven tasks of sample size 2 show no sensitivity to variation of the relevant parameters.

## III. DECISION RULES

From the results of the preceding, we can derive decision rules of a specific or a general character. We will first discuss the specific approach, then the general rules for decision on equipment purchases.

### *Specific procedure*

One has first to decide whether analysis A or B best fits the country under analysis. (It was made sure that the omission of structure costs for work sites in analysis A had only a minor effect on the optimality patterns.) For example, because of transportation cost and the extreme scarcity of foreign exchange, an equipment price twice the one prevailing in the United States might indicate an equipment investment better in many countries than the price of the internal United States market. Also, the efficiency of many workers in the newly industrializing countries cannot yet match that of the operators in mature industrialized countries. Although application of double work allowances in analysis B is somewhat exaggerated, in the context of this analysis it is better to overestimate rather than to underestimate the variations. The results of analysis B may then be considered as the least favourable limit, and analysis A as the most favourable limit of efficiency and equipment price variation. By presenting the upper and lower boundaries of machine-tool optimality patterns in this analysis, it is believed that worthwhile insight is supplied, the more so as this refers only to efficiency variation and equipment price variation, while the variations in capital-labour prices and lot sizes cover all possible production circumstances.

Generally speaking, analysis A refers to industrialized countries, analysis B to underindustrialized countries.

The next step for the decision makers is to determine, in detail, the task characteristics of their product mix. Hence, what kinds of shapes do we produce, which piece sizes, and what degree of precision do we need? If the annual product mix is roughly translated into tasks, as defined in this report, an estimate must be made about the average size of lots in which production will be carried out, say, in the next ten years. Also, the trend in the capital and labour prices has to be estimated. As equipment may last for ten to twenty years, we should make an optimal choice not on the production circumstances of today, but on some period that will be more representative in the future. This means, in general, that we have to count on a higher wage rate and a somewhat lower interest rate than the one prevailing today.

### *General procedure*

The more general approach for a decision procedure on machine optimality is to list which tasks are sensitive to which parameters and to make some generalizations from this. This information is useful as a basis for decisions on machine-tool optimality.

### *Generalizations from analysis A*

In table 14, the degree of sensitivity was indicated for lot size variation and for variation into the price of capital and labour for each task. From table 14 we derive table 17. In table 17 we distinguish ten categories of sensitivity for the effect of capital and labour price variation and for lot size variation on machine optimality.

Table 17 also can be aggregated into four major groups: A, B, C and D.

*Group A* consists of categories a, d and e. These are the subgroups which show a very high, or high, price sensitivity and respectively none, low and medium lot size sensitivity. Hence, the sensitivity to price is predominant in this group.

The practical implication is that, for tasks falling into this group, one should be alert to the level of capital intensity optimal in each country, as one may expect optimality to change for the four capital labour price areas, which we distinguished.

However, as the tasks involved are not very sensitive to

*Table 17*

SENSITIVITY CATEGORIES FOR PRICE AND LOT SIZE VARIATION ON MACHINE OPTIMALITY

| | *No. of tasks* | | *Definition of category sensitivity ranking* | | *Definition of task* | | | |
|---|---|---|---|---|---|---|---|---|
| *Category* | *Absolute* | *Per cent* | *Price* | *Lot size* | *Task no.* | *Shape* | *Size* | *Precision* |
| a | | 10 | Very high | (None) low | 34 | 7 | 3 | 1 |
| | 5 | 10 | | | 37 | 8 | 2 | 1 |
| | | | | | 39 | 9 | 2 | 2 |
| | | | | | 40 | 9 | 3 | 1 |
| | | | | | 49 | 12 | 3 | 2 |
| | | | | | 6 | 2 | 2 | 1 |
| | | | High | (Very) high | 8 | 2 | 2 | 3 |
| b | | | | | 13 | 3 | 2 | 1 |
| | 7 | 13 | | | 24 | 5 | 2 | 2 |
| | | | | | 25 | 5 | 2 | 3 |
| | | | | | 28 | 5 | 3 | 3 |
| | | | | | 45 | 12 | 1 | 1 |
| | | | | | 1 | 1 | 2 | 1 |
| | | | | | 2 | 1 | 2 | 2 |
| | | | (Very) high | Medium | 3 | 1 | 3 | 1 |
| c | 10 | 19 | | | 12 | 2 | 4 | 2 |
| | | | | | 20 | 4 | 3 | 1 |
| | | | | | 30 | 6 | 2 | 1 |
| | | | | | 31 | 6 | 2 | 2 |
| | | | | | 33 | 7 | 2 | 2 |
| | | | | | 35 | 7 | 3 | 2 |
| | | | | | 36 | 8 | 1 | 1 |
| | | | | | 4 | 1 | 3 | 2 |
| | | | | | 42 | 10 | 4 | 2 |
| d | 2 | 4 | High | Low | | | | |
| e | 2 | 4 | Medium | Medium | 32 | 7 | 2 | 1 |
| | | | Medium | Low | 41 | 9 | 3 | 2 |
| f | | | Low | Very high | 5 | 1 | 4 | 1 |
| | | | | | 10 | 2 | 3 | 2 |
| | 7 | 14 | | | 17 | 3 | 3 | 2 |
| | | | | | 19 | 3 | 2 | 2 |
| | | | | | 21 | 4 | 3 | 2 |
| | | | | | 23 | 5 | 2 | 1 |
| | | | | | 50 | 13 | 1 | 1 |
| | | | | | 9 | 2 | 3 | 1 |
| | | | | | 16 | 3 | 3 | 1 |
| g | | | Medium | (Very) high | 18 | 4 | 2 | 1 |
| | 9 | 18 | | | 26 | 5 | 3 | 1 |
| | | | | | 27 | 5 | 3 | 2 |
| | | | | | 46 | 12 | 1 | 2 |
| | | | | | 47 | 12 | 2 | 1 |
| | | | | | 48 | 12 | 2 | 2 |
| | | | | | 51 | 13 | 1 | 2 |
| | | | | | 29 | 6 | 1 | 1 |
| h | 2 | 4 | Low | Low | 43 | 11 | 2 | 2 |
| | | | | | 14 | 3 | 2 | 2 |
| i | | | None | None | 15 | 3 | 2 | 3 |
| | | | | | 22 | 4 | 3 | 3 |
| | 5 | 10 | | | 38 | 9 | 2 | 1 |
| | | | | | 44 | 11 | 3 | 2 |
| | | | | | 7 | 2 | 2 | 2 |
| j | 2 | 4 | None | Very high | 11 | 2 | 4 | 1 |

*Table 18*

SENSITIVITY CATEGORIES AND TASK CHARACTERISTICS

(Analysis A)

| Group | Category | No. of tasks | Shapes 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | Size 1 | 2 | 3 | 4 | Precision 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | a | 5 | | | | | | | 1 | 1 | 2 | | | 1 | | | 2 | 3 | | 3 | 2 | |
| | d | 2 | 1 | | | | | | | | | | | | | | | 1 | 1 | | 2 | |
| | e | 10 | 3 | 1 | | 1 | | 2 | 2 | 1 | | 1 | | | | 1 | 5 | 3 | 1 | 5 | 5 | |
| | | 17 | 4 | 1 | | 1 | | 2 | 3 | 2 | 2 | 1 | | 1 | | 1 | 7 | 7 | 2 | 8 | 9 | |
| B | b | 7 | | 2 | 1 | | | 3 | | | | | | 1 | | 1 | 5 | 1 | | 3 | 1 | 3 |
| | c | 2 | | | | | | | 1 | | 1 | | | | | | 1 | 1 | | 1 | 1 | |
| | | 9 | | 2 | 1 | | | 3 | 1 | | 1 | | | 1 | | 1 | 6 | 2 | | 4 | 2 | 3 |
| C | f | 7 | 1 | 1 | 1 | 2 | | 1 | | | | | | | 1 | 1 | 2 | 3 | 1 | 3 | 4 | |
| | j | 2 | | 2 | | | | | | | | | | | | | 1 | | 1 | 1 | 1 | |
| | g | 9 | | 1 | 1 | 1 | | 2 | | | | | | 3 | 1 | 2 | 3 | 4 | | 5 | 4 | |
| | | 18 | 1 | 4 | 2 | 3 | | 3 | | | | | | 3 | 2 | 3 | 6 | 7 | 2 | 9 | 9 | |
| D | h | 2 | | | | | | 1 | | | | | 1 | | | 1 | 1 | | | 1 | 1 | |
| | i | 5 | | | 2 | 1 | | | | | 1 | | 1 | | | | 3 | 2 | | 1 | 2 | 2 |
| | | 7 | | | 2 | 1 | | 1 | | | 1 | | 2 | | | 1 | 4 | 2 | | 2 | 3 | 2 |

lot size variation, there will be not much choice among optimal machines as to scale of operation such as between small, medium and large, within one price area.

In respect to generalization of task characteristics we can state that tasks, which involve shapes 7, 8, 9, 10 and 12 give very high and high price sensitivity and low lot size sensitivity. Shapes 1, 2, 4, 6, 7 and 8 give, also, very high price sensitivity, but now there is also a medium sensitivity to lot size. See table 18 for more details.

*Group B* consists of categories b and c. The tasks in these subgroups show a high or medium sensitivity to both lot size and price variation. The practical implication is that the optimality of the machines producing these tasks is, for areas with different levels of economic development, sensitive to the various price values of capital and labour and, within each area, sensitive to the scale of operation.

Primarily, shapes 2, 5 and 12 are involved with small piece sizes.

*Group C* consists of categories f, j and g. The tasks in these subgroups are predominantly sensitive to lot size variation.

Of the eighteen tasks involved in the three categories f, j and g, thirteen concern cases in which only two machines became optimal.

According to our sensitivity classes, when there are four changes because of lot size, they fall in the class of very high lot size sensitivity.

In all these cases a phenomenon occurs that we like to call an optimality break, which means that for all price sets a certain lot size variation causes a uniform shift in optimality. For other tasks, this optimality break occurs in two or three steps. We indicate below for which tasks this optimality break occurs respectively for 1, 2, 3 lot size steps.

The practical implication of this phenomenon is that for certain tasks, anywhere in the world above a certain critical lot size the same machine is optimal, independent

| Task no. | Task definition: Shape | Size | Prec. | Optimality break due to lot size variation: Lot size (2) | No. of changes | Lot size (3) | No. of changes | Lot size (4) | No. of changes |
|---|---|---|---|---|---|---|---|---|---|
| 5 | 1 | 4 | 1 | 10 | 1 | 50 | 3 | | — |
| 7 | 2 | 2 | 2 | | — | 50 | 4 | — | — |
| 9 | 2 | 3 | 1 | 10 | 2 | 50 | 2 | | — |
| 10 | 2 | 3 | 2 | — | — | 50 | 2 | 100 | 2 |
| 11 | 2 | 4 | 1 | — | — | 50 | 4 | | — |
| 16 | 3 | 3 | 1 | 10 | 1 | 50 | 2 | | — |
| 17 | 3 | 3 | 2 | 10 | 2 | 50 | 2 | | — |
| 19 | 4 | 2 | 2 | 10 | 2 | 50 | 2 | — | — |
| 21 | 4 | 3 | 2 | 10 | 2 | 50 | 2 | — | — |
| 23 | 5 | 2 | 1 | — | — | 50 | 3 | 100 | 1 |
| 50 | 1 | 1 | 1 | — | — | 50 | 3 | 100 | 1 |

of the price variation. For lower lot sizes than a critical range there is lot size and price sensitivity, hence for the smaller scale production processes. Although all these tasks are classified as highly sensitive to lot size variation, the variation itself is concentrated in a narrow range, mostly between lot sizes 10–100. From then on there is no, or very little, sensitivity to lot size, as well as to price variation.

*Group D* consists of categories h and i. The tasks in these groups show no or low sensitivity to both lot size and price variation. The practical implication is that metalworking tasks within these two sub-groups have to be carried out by the same level of mechanization anywhere in the world.

Generalizing further, we can say that the smaller the workpieces (task units), the lower the precision requirements; the more common the shapes of especially flat, cylindrical and multiple surfaces, the higher the price and lot size sensitivity will be.

The findings for sample size 2, the thirty-seven tasks with no price and lot size sensitivity confirms, in a sense, our generalization based on sample size 1.

In sample size 2, for example, eleven tasks concern very large pieces, nine tasks concern large pieces; eight tasks require high precision, fourteen tasks require precision, and eight tasks concern complex shapes and irregular periphery flat surfaces. Here we find, in general, larger sizes, higher precision requirements and less common shapes which tend to reduce the number of economically feasible machines and consequently the possibility of price and lot size sensitivity.

### *Generalizations from analysis B*

As to variation in the efficiency rate and the price of the machine, we notice from table 16 that no single task shows high or very high sensitivity.

From tables 19 and 20 we may conclude that primarily tasks (sensitivity ranking M) with shape 5 (cylindrical surfaces, interior), shape 7 (cylindrical forms, external) and shape 12 (multiple surfaces) show medium as well as low sensitivity.

Certain tasks (sensitivity ranking L, N) with flat surfaces (1, 2, 3), cylindrical surfaces (4, 5), drilled holes (6, 13), and standard gear shapes (9), show low sensitivity, certain others with the same task characteristics show no sensitivity.

In general, one can say that all tasks sensitive to price variation will be, in principle, also sensitive to a doubling of the equipment price.

Reducing the efficiency rate will favour the more mechanized equipment types relative to the less mechanized equipment types as, with the former, the speed of work is dictated more by the machine. This will, in principle, reinforce the lot size sensitivity. The combined effect of equipment price doubling, which favours the less capital-intensive alternative machines, and reducing of the efficiency rate, which favours the more capital-intensive alternative machines is, to a certain extent, compensatory.

### *Concluding remark*

The general decision rules are necessarily less precise. Most important is: what kind of tasks will be most frequently produced? In part III of this study we have indicated for which kind of tasks one may expect various degrees of sensitivity in machine optimality, for variation in the prices of capital and labour and in lot size (analysis A), and the sensitivity to variation in the price of the machine and the efficiency rate (analysis B). As soon as one, on the basis of this general information, may expect sensitivity, more accurate information can be obtained by following the decision rules outlined in the first section of part III.

*Table 19*

SENSITIVITY CATEGORIES FOR EQUIPMENT PRICE AND EFFICIENCY RATE VARIATION ON MACHINE OPTIMALITY

| *No. of tasks* | *Definition of category sensitivity ranking* | *Task no.* | *Definition of task* | | |
|---|---|---|---|---|---|
| | | | *Shape* | *Size* | *Precision* |
| | | 6 | 2 | 2 | 1 |
| | | 24 | 5 | 2 | 2 |
| | | 25 | 5 | 2 | 3 |
| 11 | Medium (M) | 27 | 5 | 3 | 2 |
| | | 28 | 5 | 3 | 3 |
| | | 32 | 7 | 2 | 1 |
| | | 33 | 7 | 2 | 2 |
| | | 36 | 8 | 1 | 1 |
| | | 45 | 12 | 1 | 1 |
| | | 47 | 12 | 2 | 1 |
| | | 49 | 12 | 3 | 2 |
| | | 1 | 1 | 2 | 1 |
| | | 4 | 1 | 3 | 2 |
| | | 5 | 1 | 4 | 1 |
| | | 8 | 2 | 2 | 3 |
| | | 9 | 2 | 3 | 1 |
| | | 12 | 2 | 4 | 2 |
| | | 13 | 3 | 2 | 1 |
| | | 16 | 3 | 3 | 1 |
| | | 18 | 4 | 2 | 1 |
| 21 | Low (L) | 21 | 4 | 3 | 2 |
| | | 22 | 4 | 3 | 3 |
| | | 23 | 5 | 2 | 1 |
| | | 26 | 5 | 3 | 1 |
| | | 30 | 6 | 2 | 1 |
| | | 31 | 6 | 2 | 2 |
| | | 34 | 7 | 3 | 1 |
| | | 35 | 7 | 3 | 2 |
| | | 37 | 8 | 2 | 1 |
| | | 39 | 9 | 2 | 2 |
| | | 40 | 9 | 3 | 1 |
| | | 41 | 9 | 3 | 2 |
| | | 42 | 10 | 4 | 2 |
| | | 43 | 11 | 2 | 2 |
| | | 46 | 12 | 1 | 2 |
| | | 48 | 12 | 2 | 2 |
| | | 51 | 13 | 1 | 2 |
| | | 2 | 1 | 2 | 2 |
| | | 3 | 1 | 3 | 1 |
| | | 7 | 2 | 2 | 2 |
| 14 | None (N) | 10 | 2 | 3 | 2 |
| | | 11 | 2 | 4 | 1 |
| | | 14 | 3 | 2 | 2 |
| | | 15 | 3 | 2 | 3 |
| | | 17 | 3 | 3 | 2 |
| | | 19 | 4 | 2 | 2 |
| | | 20 | 4 | 3 | 1 |
| | | 29 | 6 | 1 | 1 |
| | | 38 | 9 | 2 | 1 |
| | | 44 | 11 | 3 | 2 |
| | | 50 | 13 | 1 | 1 |

*Table 20*

SENSITIVITY CATEGORIES AND TASK CHARACTERISTICS

(Analysis B)

| | | *Task characteristics* | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | *Shape* | | | | | | | | | | | | | *Size* | | | | *Precision* | | |
| *Sens. cat.* | *No. of tasks* | *1* | *2* | *3* | *4* | *5* | *6* | *7* | *8* | *9* | *10* | *11* | *12* | *13* | *1* | *2* | *3* | *4* | *1* | *2* | *3* |
| **M** | 11 | — | 1 | — | — | 4 | — | 2 | 1 | — | — | — | 3 | — | 2 | 6 | 3 | | 5 | 4 | 2 |
| **L** | 26 | 3 | 3 | 2 | 3 | 2 | 2 | 2 | 1 | 3 | 1 | 1 | 2 | 1 | 2 | 11 | 10 | 3 | 12 | 12 | 2 |
| **N** | 14 | 2 | 3 | 3 | 2 | — | 1 | — | — | 1 | — | 1 | — | 1 | 2 | 6 | 5 | 1 | 6 | 7 | 1 |

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