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PROGRAMMING OF PRODUCTION AND EXPORTS FOR METALWORKING MODELS AND PROCEDURES[1]

by

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1/ The views and opinions expressed in this paper are those of the author and do not necessarily reflect the views of the secretariat of UNIDO.

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Introduction

1. In order to create a starting point for the discussion amid the endless difficulties and complexities characterizing the programming of the metalworking sector, three simple models will be presented which focus on some of the key characteristics of the suggested method of approach.

2. These models are formulated in linear programming format, as it has excellent synoptic qualities. As will emerge in the course of the discussion, some of the key problems are problems of nonconvexity which have their origin in economies of scale and indivisibilities. In the format of linear programming, these nonconvexities will be treated by specifying certain cost elements as fixed, i.e. the variables characterizing the corresponding expenditures can take on only the value of 0 and 1, or in the case of multiple facilities, 0, 1, 2, ... etc.: in other words, these are integer variables. Though integer programming is a much more difficult mathematical task then linear programming, for our current purposes integer programming creates no additional complexity since we are concerned with problem formulation which, in the format to be used, is not affected by integer variables. Once a problem is properly formulated, one can draw on a great deal of accumulated mathematical know-how to find a suitable solution to it. The lack of orientation characterizing the atmosphere in which planning decisions in the metalworking sector are currently undertaken is due mostly to the inability of finding simple problem formulations which offer the promise of satisfactory approximations. This is the task on which we shall concentrate.

3. We do not presume that the models to be presented can conclusively cope with all difficulties. They are, however, a basis from which a great many generalizations and modifications may be drawn as soon as these simple models are thoroughly worked through.

4. In Chapters 1 to 11 we shall confine ourselves to the presentation of models for the sector in isolation, leaving the connexions to the national economy as a whole implicit. In addition, these chapters will treat exports as exogenously given. This simplification, however, will be relaxed in Chapters 12 to 18 in which the sector connexions to the national economy are explored in detail.

5. The main concern of the latter part of this paper is the extension of the analysis to two situations of key importance which were previously abstracted from. The first is the embedding of sectoral programming within the operation of the

economy as a whole, with particular attention to economies of scale and indivisibilities. The second, closely linked to the former, is the explicit consideration of variable exports which change the seriality of individual production processes and the loading of productive capacities. The discussion of over-all policy implications closes the presentation.

1. The format

6. We shall use a slight modification of Tucker's combinatorial format (Tucker, 1963) to present the data of a given problem in a simple table. In such a table each row can be conceived of as a resource, each column as an activity. For example, in Model 1 the first seven rows correspond to "listed products" (New School of Social Research, 1967); each row is a balance of one specific listed product. Other rows may represent "resources" in a more generalized sense: any limit, restriction or constraint placed on the data creates an economic scarcity of one sort or another that will have a scarcity value like ordinary resources such as products or services. Examples of activities are production, imports, exports and so on.

7. The data (parameters) appearing in the models are placed inside the solid frame of each table. All data are constants. They represent either availabilities or requirements, according to whether they are positive or negative. Examples of availabilities (positive sign) are outputs and supplies. Examples of requirements (negative sign) are inputs and demands. Both availabilities and requirements are standardized to a unit level of the activity in whose column they appear. Thus, for example, the constant $(-m_1)$ appearing at the intersection of row 9 and column 15 in Model 1[1/] represents a requirement of m_1 units of foreign exchange (the resource whose balance appears in row 9) for the purpose of undertaking the activity of importing a unit amount of the first listed product (the activity of column 15). The number m_1 is measured in physical units of resource per physical unit of activity level, e.g. in foreign (not domestic) currency units per ton of product imported, where the units of foreign currency play the role of physical units (i.e. units not expressed in the common monetary valuation).

8. If a parameter is multiplied by the scale of the activity in whose column it appears, the total availability or requirement of the given resource connected with the activity in question is obtained. With reference to Table 1, which shows

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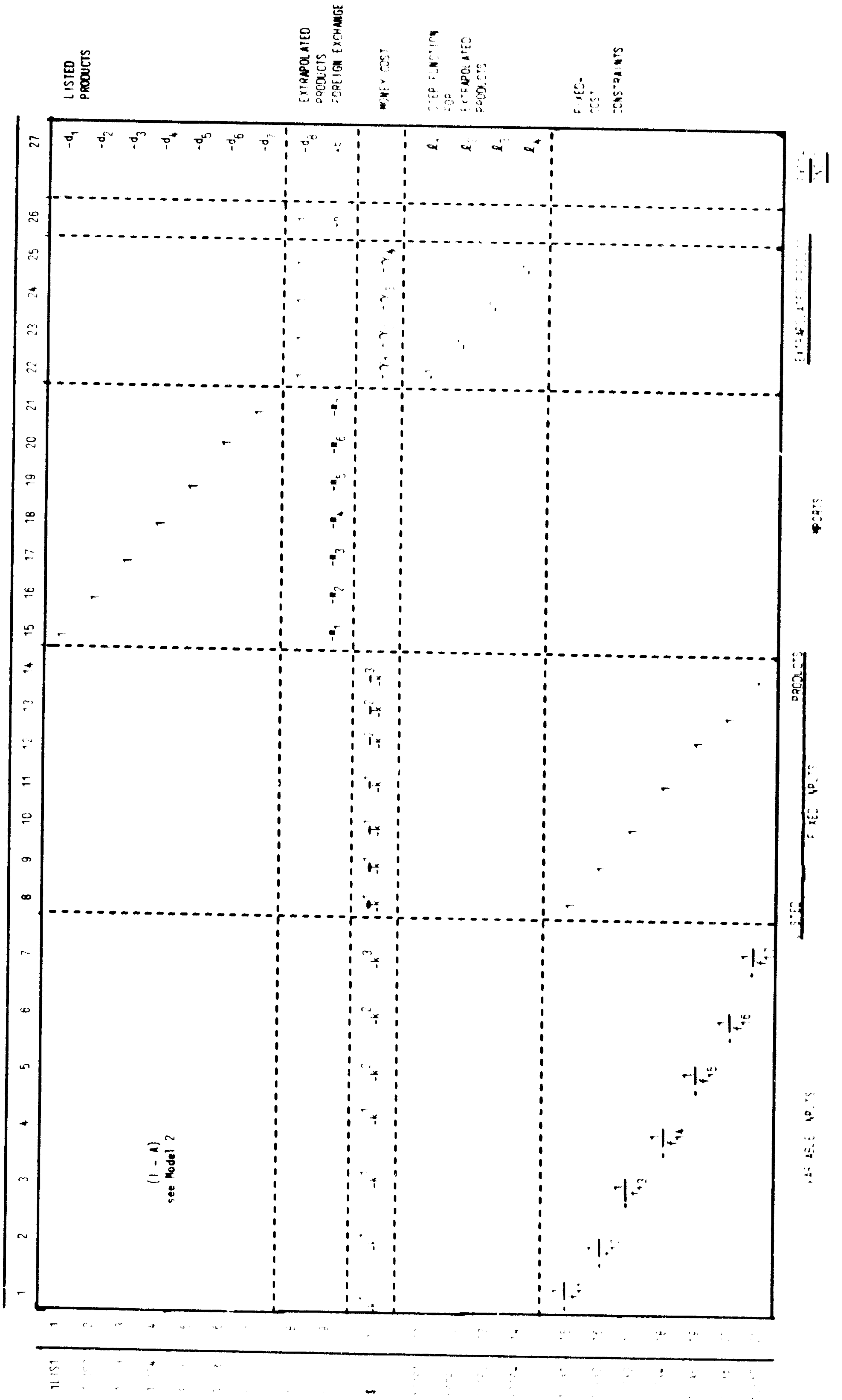
1/ Detailed explanations of notation for Models 1-3 are given in Annex I.

MODEL 1


LISTED PRODUCTS
EXTRAPOLATED PRODUCTS FOREIGN EXCHANGE
MONEY COST
STEP FUNCTION FOR EXTRAPOLATED PRODUCTS
FIXED-COST CONSTRAINTS
(I - A)
see Model 2
1LIS1
1LIS2
1LIS3
1LIS4
1LIS5
1LIS6
1LIS7
1LFX1
1LFX2
1LFX3
1LFX4
1LFX5
1LFX6
1LFX7
1LM1
1LM2
1LM3
1LM4
1LM5
1LM6
1LM7
1SIP1
1SIP2
1SIP3
1SIP4
1EXM
EXOG

schematically a linear economic system in the modified Tucker format used for Models 1 to 3, the scales of the activities are shown as $\underline{x}$ variables appearing at the foot of the column (activity) whose scale they represent. Thus the total availabilities or requirements of any resource $\underline{i}$ in connexion with activity $\underline{j}$ can be obtained by forming the product $(a_{ij})(x_j)$. If such products are formed for all a_{ij} in Table 1, then a row balance can be obtained for each row by algebraically adding the products in a given row. The algebraic sum represents the net availability, surplus (if positive), net requirement or deficit (if negative) of a resource in connexion with all activities. Since all x_j are treated as variables, the sum is also a variable, denoted by s_i. The s_i variables are shown in the left margin of Table 1; the equality sign following them refers to their definition as row balances. The symbol (*) appearing above the x_j variables denotes the operation of multiplication undertaken when forming row balances.

Table 1

A linear economic system in modified Tucker format

	$-\ell_1$	$-\ell_2$	$-\ell_3$	. .	$-\ell_n$		
	=	=	=		=		
s_1 =	a_{11}	a_{12}	a_{13}	. . .	a_{1n}	*	y_1
s_2 =	a_{21}	a_{22}	a_{23}	. . .	a_{2n}	*	y_2
s_3 =	a_{31}	a_{32}	a_{33}	. . .	a_{3n}	*	y_3
.	.	.	.		.		.
.	.	.	.		.		.
.	.	.	.		.		.
s_m =	a_{m1}	a_{m2}	a_{m3}	. . .	a_{mn}	*	y_m
	*	*	*		*		
	x_1	x_2	x_3	. . .	x_n		

9. In addition to row balances, it is also possible to form column balances. If a parameter a_{ij} is multiplied by y_i the price of the resource in whose row it appears (Table 1), the economic value of the availability or requirement of the resource is obtained, standardized to a unit level of the activity j. This value represents a revenue (if positive) or a cost (if negative) at unit activity level. If the products $(y_i)(a_{ij})$ are formed for all parameters a_{ij} in Table 1, then it is possible to get column balances by algebraically adding all products in a given column. These sums represent net revenues or profits (if positive) and net costs or losses (if negative). Since all y_i are treated as variables, the above sums are also variables; they are denoted by the symbols $(-\ell_i)$ which appear in the top margin of Table 1. The equality sign again refers to the definition of these variables by means of column balances.[2/]

10. The above form of a linear system is called "homogeneous". In this form all activity scales and resource prices are variable. Our task consists in finding values of these variables (a "programme") which will in some sense be optimal.

11. Optimality can be defined in two complementary ways by:

(a) Selecting a resource m whose surplus s_m will be maximized by varying the activity scales x_j, subject to the conditions that deficits (negative s_i) are avoided for all other resources and that no activity scale will be negative.

(b) Selecting an activity n whose profit $(-\ell_n)$ will be minimized by varying the resource prices y_i, subject to the conditions that profits (negative ℓ_n) are avoided for all other activities and that no resource price y_i will be negative.

12. Note that the conditions imposed in both cases boil down to the rule that no variable may be negative. This is common sense in regard to activity scales, since activities generally cannot be run in reverse[3/], for one cannot make pigs from sausages. Nor do negative prices make sense. The avoidance of deficits on any resource is again economic common sense, since we are aiming at a feasible and

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2/ It may be questioned why the symbols chosen to represent column balances are taken to be negative rather than positive as in the case of row balances. This is done conventionally in order to obtain the simplest scheme of algebraic manipulations for the linear system. Each ℓ_j represents a loss on an activity; if $(-\ell_j) > 0$, the activity is profitable.

3/ In some cases such conditions may be relaxed. For example, exports may be treated as negative imports provided that the export and import prices of a commodity are equal within a tolerable margin of error. In such cases the statement of optimality requires a slight revision.

practical resource allocation. The avoidance of profits, while at first sight paradoxical, corresponds to the maxim of neoclassical economics that, under perfect competition, profits are eliminated (with well-known favourable implications for the efficiency of resource allocation).

13. As for the maximum-minimum objectives, in the first definition the maximization of a resource surplus may mean either the maximization of net output or the minimization of net input. The resource in question can be a composite resource, if desired, for it may consist of a weighted average of several resources. In addition, it is necessary to introduce scarcity for the maximization to become meaningful. As long as all activities are treated as variable and thus can be indefinitely expanded, there is generally[4/] no limit on the expansion of the quantity to be maximized; some part of the system, however, has to be fixed. An activity is therefore selected whose scale is set to unity. It is convenient in sectoral planning problems to treat the exogenously given supplies and demands of the economic resources as fixed-scale activities. Whichever activity is thus fixed becomes the activity whose profit is minimized under the second definition of optimality. This offers a clue to the interpretation of profit minimization in the second definition: we are instructed to choose prices that will reduce the value of exogenous supplies and increase the value of exogenous demands, i.e. prices that will reduce the scarcity of limited supplies and enhance the benefit of prescribed demands.

14. In view of the problem before us, the following features of such linear programming models are particularly valuable:

(a) Linear programming models permit the representation of alternative activities. For example, the output of a given product may be obtained by domestic production or by imports, or there may be more or less labour-intensive activities for producing a product. Any alternative may appear with a zero scale in the optimal programme. The inclusion of inefficient alternatives in the model therefore does no harm.

(b) The models permit the representation of joint products, for a given activity may have more than one output (positive entries). This overcomes a limitation of Leontieff-type input-output formulations.

4/ At times it is impossible to find any programme with all x_i and s_i variables non-negative; then the question of optimization does not arise.

(c) Multiple restrictions may operate on the same activity or group of activities. In particular, the restrictions may be inequalities, such as an upper limit to the scale of an activity. If such a limit is written in the form $x_j \leq L$, it can be converted into an exact equality by adding the surplus variable s_i to its left-hand side:

$$s_i + x_j = L,$$

whence

$$s_i = (-x_j) + L,$$

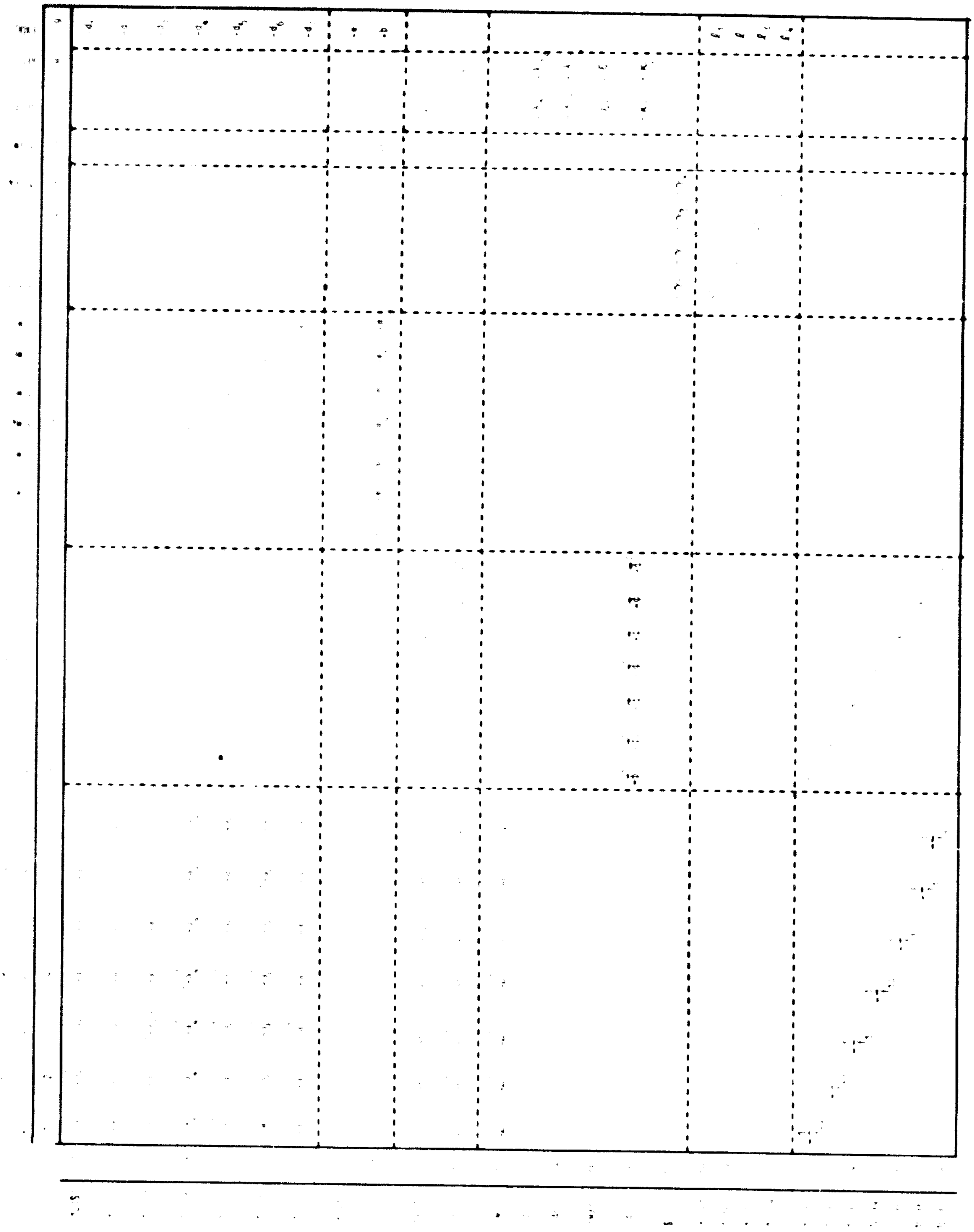
which puts the restriction into the conventional format applied to all resource balances, with L being an element of the exogenous vector. Any restriction may appear with a non-zero surplus in the optimal programme: such a restriction is ineffective and may thus be included in the formulation.

15. The models presented in this format, unless otherwise noted, are simple linear programming models which can be readily solved by a number of well-known methods, (for example, Dantzig, 1963). When some variables (in the models to be discussed always activity scales) are required to assume only integer values, the presentation of the model remains identical, but the mathematical and computational procedure for obtaining an optimal solution is rendered considerably more complex. In some cases, especially for small models, exact optimal solutions can be derived; in other cases only reasonable approximations may be drawn (Dantzig, 1963, Ch.26).

2. The simplest programming model

16. Models 1 and 2, both referring to a single branch, are two variations of the simplest programming model for the metalworking sector. As compared with Table 1, the <u>x</u>, <u>s</u>, <u>y</u>, and (-<u>l</u>) variables are not indicated in the margin, but rows and columns, in addition to being numbered sequentially, are given symbolic designations derived from the nature of the resource or activity they represent. While not explicitly shown, the former variables play exactly the same role as they do in Table 1; in particular, they intervene in the same way in the formation of row and column balances.

17. The products represented in Models 1 and 2 are listed products, numbered 1 through 7. The concept of listed products has been developed in a report by the New School for Social Research (1967). As indicated in this report the technical coefficients of a listed product are based on the technical coefficients of one or more typical products, the latter being studied in full engineering detail. The inputs per ton of product may be transferred without change from the typical product



to the listed product; this is the simplest procedure and has been adopted here for illustrative purposes. Alternately, the coefficients may be modified to some extent on the basis of simple parametric correlations of size, capacity, and so on. With each listed product we associate an activity (1LIS1, ..., 1LIS7) representing the domestic production of the product in question, and a row (also 1LIS1, ..., 1LIS7) representing the commodity balance. The coefficients in the first seven rows and columns, indicated in condensed (matrix) notation in Model 1, are written out in full in Model 2. In this 7 x 7 square block there is a diagonal of (+1) (unity) elements together with an (-a) element in every cell. The (+1) elements represent stated amounts of intermediate input of other listed products used in the production of the given listed product, for example, the requirements for an electric motor in the production of a pump. Generally, most of the a entries will be zero.

18. The formulation of such an input-output sub-matrix for listed products permits taking into account various stages of production. End-products, sub-assemblies and components can be designated as separate listed products, and the input requirements of each product can be given, including the requirements for lower-order intermediate commodities. It is also possible to include alternative ways of manufacturing a given listed product, although this is not shown in these models. Finally, while Models 1 and 2 refer to a single branch of the sector, in more comprehensive models (as in Model 3) input-output relationships connecting several branches of the sector will occur without posing any difficulty of formulation.

19. The superscripts are identical for the first four columns as they are for the next two. These superscripts refer to the serial number of the typical product from which the technical coefficients of the given listed product have been derived. In the present illustrative case, it has been assumed that four listed products are derived from the first typical product, two from the second and one from the third. The listed products derived from the same typical product differ formally among themselves only in regard to their seriality.

20. Typical products do not occur in the models as such; they are required only to derive the technical information for the listed products. Typical products, however, may and generally will appear as members of the product list within a branch; thus they will enter the model in the guise of listed products, without further distinction.

21. In Model 1 all other inputs of the production activities for listed products are condensed into a production-cost figure (-k) which has the same superscript as

the corresponding a coefficients; in Model 2 this production cost is broken up into resource-element capacity utilizations and direct material input. The only other coefficients of the production activities for listed products are the (-1/f) coefficients of the fixed-cost constraints, rows 1LFX1, ..., 1LFX7.

22. The fixed-cost constraints connect the first group of seven activities with the next group of seven activities in both Model 1 and Model 2. Activities eight through fourteen in both models are designated as 1LFX1, ..., 1LFX7 and refer to the activity of incurring fixed costs connected with setting up a production series. This postulates that before the manufacture of a product can be started, the costs of providing the required tooling, jigs and fixtures must be met. In addition it is necessary to set up the machinery with the aid of the former auxiliary devices before each individual production run. The amount of fixed costs is given as a single dollar figure (-k) in Model 1 but is broken up into a lump sum capital requirement and fixed capacity requirements of two different resource elements in Model 2 (more about this below). The fixed-cost activities are connected with the production activities in such a way that the entire fixed cost is incurred whenever a production activity is used. This converts the problem into a integer programming problem, as will be shown in detail below.

23. From an empirical point of view the properties of Models 1 and 2 make allowance for economies of scale arising out of the length of a production run in the manufacture of individual products, without allowing for economies of scale in regard to the size of productive facilities. All productive resources are still assumed to be infinitely subdivisible; the only consideration in their employment is the resource or money cost associated with their use.

24. The third block of seven activities in Models 1 and 2 relates to imports. Each of these activities has an entry of (+1), corresponding to the product which it makes available, and an entry of ($-m_i$) corresponding to the expenditure of foreign exchange per unit (ton) of product i, the world market price. Generally, it is assumed that imports are irreversible. At times it might be convenient to permit imports to behave as free variables, i.e. variables that may take on negative values, signifying exports. This introduces only a minor modification in the mathematical statement of the problem[5/].

5/ Whenever a variable x_i is to be treated as a free variable, the loss variable t_i in the upper margin (Table 1) must be zero, i.e. it is not permitted to be positive. In other words, not only the usual no-profit condition holds for this variable, but also a no-loss condition: the activity is required to break even exactly.

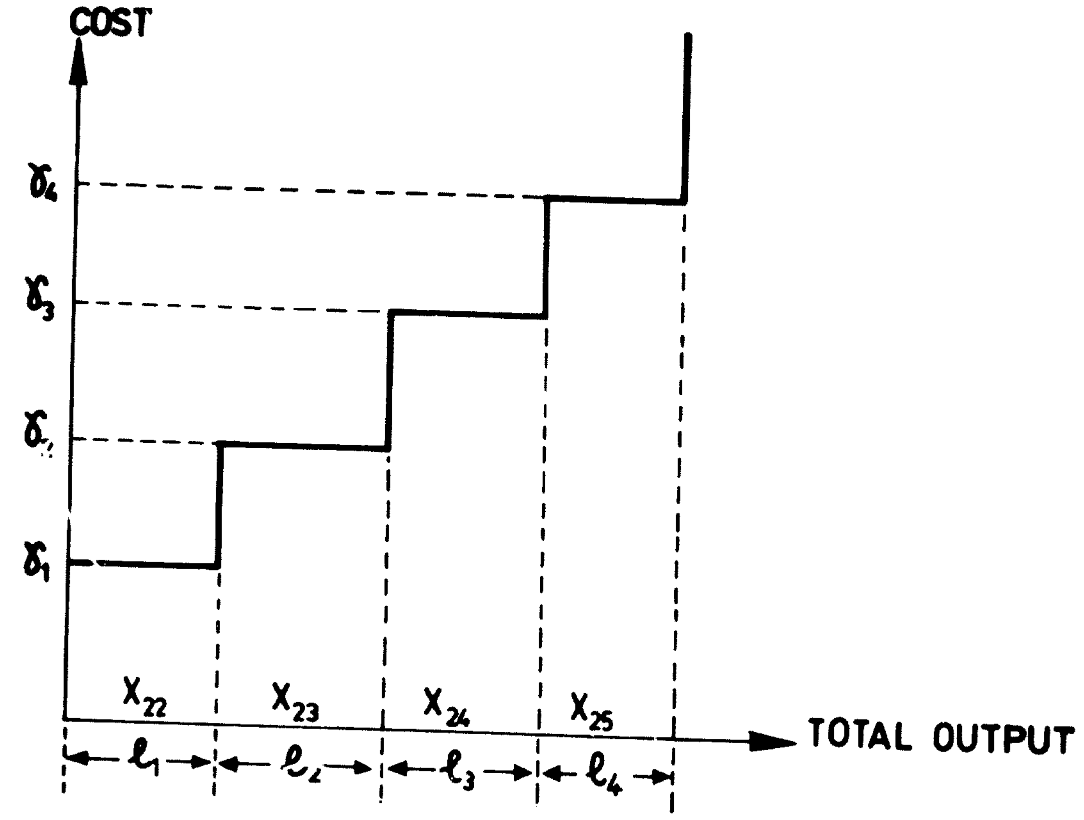
25. The next block of five activities refers to the extrapolated products (New School for Social Research, 1967) of the branch under study. In effect, products not listed individually within a branch are handled by means of a single cost function which attributes increasing domestic production costs to output as the output approaches the total demand for the branch. The device of extrapolation is meant to be used only for a minor part of the total demand within any branch; due to an inherent asymmetry in the distribution of demand for individual products, it is assumed that the major portion (perhaps 85-90 per cent) of total demand within the branch can be handled by the individual description of some 200-odd listed products. The cost trend of these listed products is to be extrapolated for the remaining products which can be several thousand in number. The rationale of this extrapolation will be developed below; for the moment it suffices to indicate that in Model 1 the increase in production costs is handled by a step function such as shown in Figure 1. With reference to Figure 1, the total output of the extrapolated product is represented as the sum of four step variables, $x_{22}, \ldots, x_{25}$ (these correspond to the columns with the same serial numbers in Models 1 and 2); each step has a given constant cost $\underline{\lambda}$ associated with it and is limited to a maximum length $\ell_1, \ldots, \ell_4$. In an optimizing model the lower-cost steps will always be used to the limit before switching to a higher-cost step; thus the steps will be used in the correct sequence, as specified by the shape of the step function, even without explicit sequencing instructions. The output of the extrapolated product may be measured either in tons or in foreign exchange units corresponding to the world market price, as will be discussed below. Activity 26 in Models 1 and 2 is an import activity for the extrapolated product. A comparison of Models 1 and 2 shows that Model 2 contains no detailed resource breakup for the extrapolated product (see Chapter 8).

26. In Model 1 there is only one further column: the column (No. 27) of exogenous supplies and demands. It includes the demands for the listed products, the demand for the extrapolated products, the exogenous supply (allocation) of foreign exchange and the limits associated with the step function of the extrapolated products. The exogenous column (No. 29) in Model 2 has the same structure.

27. Two additional activities are included in Model 2. These correspond to the input flows associated with maintaining given capacities of the two resource elements that appear in the Model. It is assumed that indirect materials input, capital input and two kinds of labour input are accounted for separately. The amount of detail can be increased at will without altering the structure of the model. The output of each of these activities is a (+1) entry associated with a unit of capacity held available.

Figure 1

Step function for extrapolated products



input and two kinds of labour input are accounted for separately. The amount of detail can be increased at will without altering the structure of the model. The output of each of these activities is a (+1) entry associated with a unit of capacity held available.

28. The objective function in Model 1 consists of row 10 and involves the minimization of total money cost.[6] This together with the usual conditions of non-negative variables, means that the cost of meeting all sectoral demands is to be minimized, while the supply of foreign exchange is not to be exceeded, and that all constraints pertaining to the step function and the fixed costs are to be observed. In Model 2 the objective function is defined as a weighted average of rows 12-17, where exogenously given prices of the resources in question (not shown in the Model) are to be used as weights. The price of money is set, by definition, to unity. The conditions of this optimization are the same as those in Model 1, except that two additional row-constraints (10 and 11) have to be satisfied and that the resource-element capacity requirements must be fully met.

3. The handling of fixed costs in the models

29. It has been pointed out that fixed costs are introduced into the models in the form of independent activities (such as 1LFX1, ..., 1LFX7)[7] which are connected with the corresponding production activities for listed products (1LIS1, ..., 1LIS7 to be referred to as variable-cost activities) in such a way that the entire fixed cost is incurred whenever the variable-cost activity in question is being used. This will now be formally clarified by reference to the above example, with the understanding that the principles presented here are applicable to the connexion between any fixed-cost and variable-cost activity. Later on fixed costs will also be introduced in connexion with the size of productive facilities and with the groups of skilled technical specialists which have to be established to support production.

30. The scale of a fixed-cost activity is a mathematical variable that can be interpreted as the number of times fixed cost is incurred; the fixed cost itself is given either in monetary terms (in Model 1, the $\bar{k}^i$ coefficients) or in terms of more detailed individual fixed resource inputs (in Model 2, the $\bar{c}^i_j$ and ^{i} coefficients). For example, the meaning of the relation $x_8 = 0.2$ (where x_8 is the scale of activity

6/ Formally, the surplus of the row, s_{10}, is being maximized.

7/ See Models 1 and 2.

8 in Model 1) is that the fraction 0.2 (20 per cent) of total fixed costs associated with the production of listed product (1) is being incurred.

31. Economically it makes no sense to represent a fixed cost as being incurred to the extent of 20 per cent since it is indivisible by its very nature: one cannot build half a factory or carry out only one fifth of a production programme. In other words, the scale of a fixed-cost activity should be represented by an integer variable which can only assume the values 0, 1, 2, ... etc. Where the fixed costs are incurred more than once, values larger than 1 have the economic meaning of multiple production facilities, production runs and so on.

32. The device used to compel fixed-cost incurrence in the models (to be referred to as the tie-in between the fixed-cost and variable-cost activities) consists of constraining the scale of the fixed-cost activity to be equal to or larger than some constant proportion (to be interpreted below) of the scale of the variable-cost activity. (This tie-in is provided, e.g. for listed product (1) by row 15 in Model 1, or row 22 in Model 2.) As long as the variable-cost activity is not used, e.g. the production scale x_1 of listed product (1) is zero, the scale of the fixed-cost activity x_8 can also remain zero. In this case no fixed cost has to be incurred. But as soon as the scale of the variable-cost activity x_1 rises above zero (no matter by how little) the tie-in with the fixed-cost activity x_8 forces the scale of the latter also to rise by at least a small amount above zero. Up to this point there is nothing to prevent the scale of the fixed-cost activity x_8 from assuming a fractional value; in fact if there were no further restrictions, the optimal solution would actually contain such fractional values. But now the integrality requirement for the fixed-cost-activity scale x_8 steps in and forces this scale to move upward to the nearest integer in the direction in which the tie-in constraints permit an inequality.[8/] Thus the full fixed cost is incurred at least once: when the scale of the fixed-cost activity x_8 (determined as a constant proportion of the variable-scale activity x_1) is between zero and one, prior to the application of the integrality requirement. If x_1 is larger than one, fixed cost will be incurred more than once.

8/ Inequalities are converted into equalities for representation in the models by adding a positive surplus s to the smaller side. The greater the difference between the two sides of the inequality, the larger the surplus s. Thus as the fixed-cost activity scale x_8 moves up to the next integer value, the corresponding surplus (s_{15} in Model 1 or s_{22} in Model 2) increases.

33. The connexion between the production and fixed-cost activities is further elucidated in Figure 2 which illustrates the tie-in between columns 1 and 8 in Model 1. With reference to this figure, the scale of the production activity (column 1 in Model 1) is x_1 and that of the fixed-cost activity (column 8) x_8. The variable x_8 measures the number of times the fixed cost is incurred. The fixed cost ($-\bar{k}^1$) is measured along the vertical axis denoting cost, the minus sign being omitted since all costs are inherently negative. The horizontal axis measures x_1, the scale of the production activity. The variable cost ($-k^1$) is the slope of the total-cost line AB. To interpret the nature of the fixed-cost tie-in parameter $1/f_{11}k$, it is assumed that a maximum production scale, f_{11}, exists associated with the expenditure of a single fixed cost. If there is an upper limit on yearly production, this can be identified with f_{11}.[9] In other instances, when the variable costs are tied to investment in a fixed productive facility, the capacity of this facility can be identified with the corresponding tie-in parameter, as will be shown in Model 3. The row balance in row 15 of Model 1 can be written out in full as follows:

$$s_{15} = (-1/f_{11})(x_1) + (1)\ (x_8).$$

When this row balance holds without surplus, $s_{15} = 0$, and

$$0 = (-x_1/f_{11}) + x_8;\ \text{thus}$$

$$x_8 = x_1/f_{11}.$$

34. In Figure 2, x_1 is drawn at about three-quarters of the way toward the maximum; therefore, $x_1/f_{11} = 3/4$. Accordingly, the scale of the fixed-cost activity will be (at least) this much, i.e. 3/4; the amount of fixed cost incurred will be 3/4 of $\bar{k}^1$, or point C, which is drawn to be at about 3/4 of the elevation of OA, the latter being equal to the full fixed cost $\bar{k}^1$. The fixed-cost constraint thus prescribes that the fraction of fixed costs incurred must be at least equal to the fraction of maximum production actually undertaken. At this point the integrality requirement for x_8 steps in to ensure that as soon as this fraction exceeds 0, it will rise all the way up to unity: fixed cost incurred will rise to OA.

35. In the example illustrated in Figure 2, x_1 was chosen smaller than f_{11}; accordingly x_8 was less than unity, prior to the application of the integrality requirement. In Figure 3, x_1 is assumed to be $(1.5).(f_{11})$, i.e. larger than the

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9/ If there is no economically meaningful upper limit of this kind, f_{11} is simply set to an upper bound on the practically occurring values of the variable-cost-activity scale. This ensures that fixed cost will be incurred no more than once.

Figure 2

Fixed costs

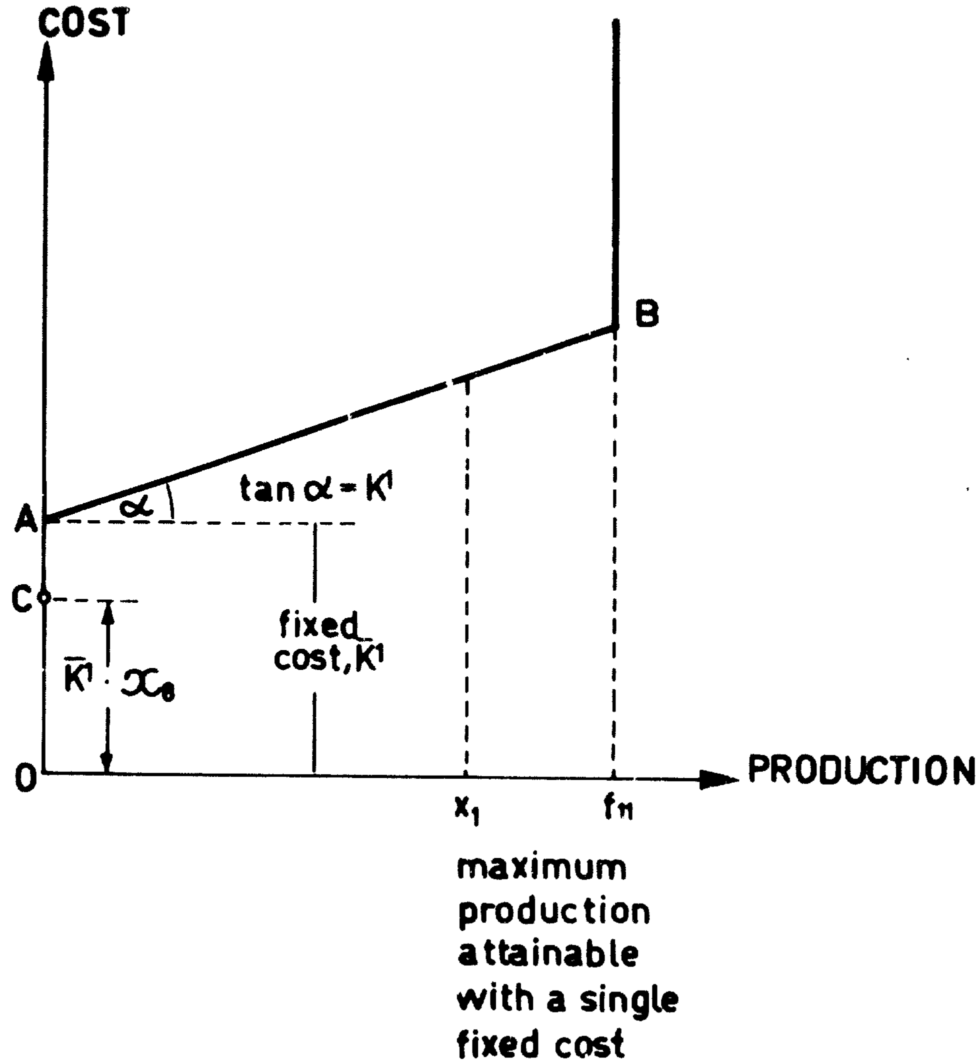
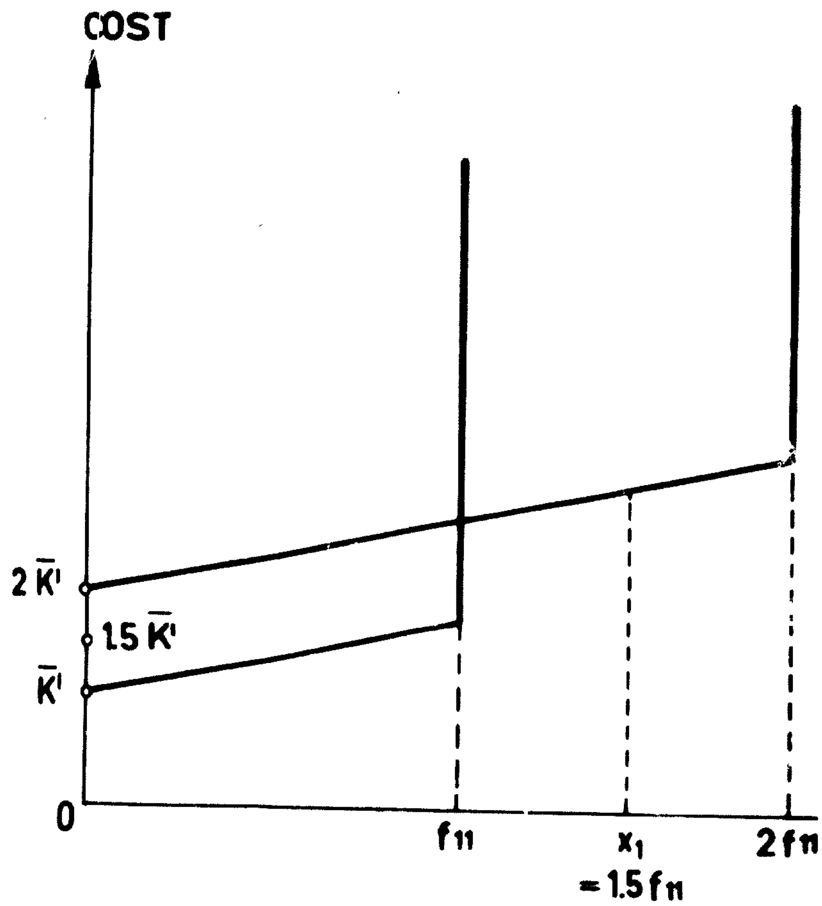


Figure 3

Multiple fixed costs



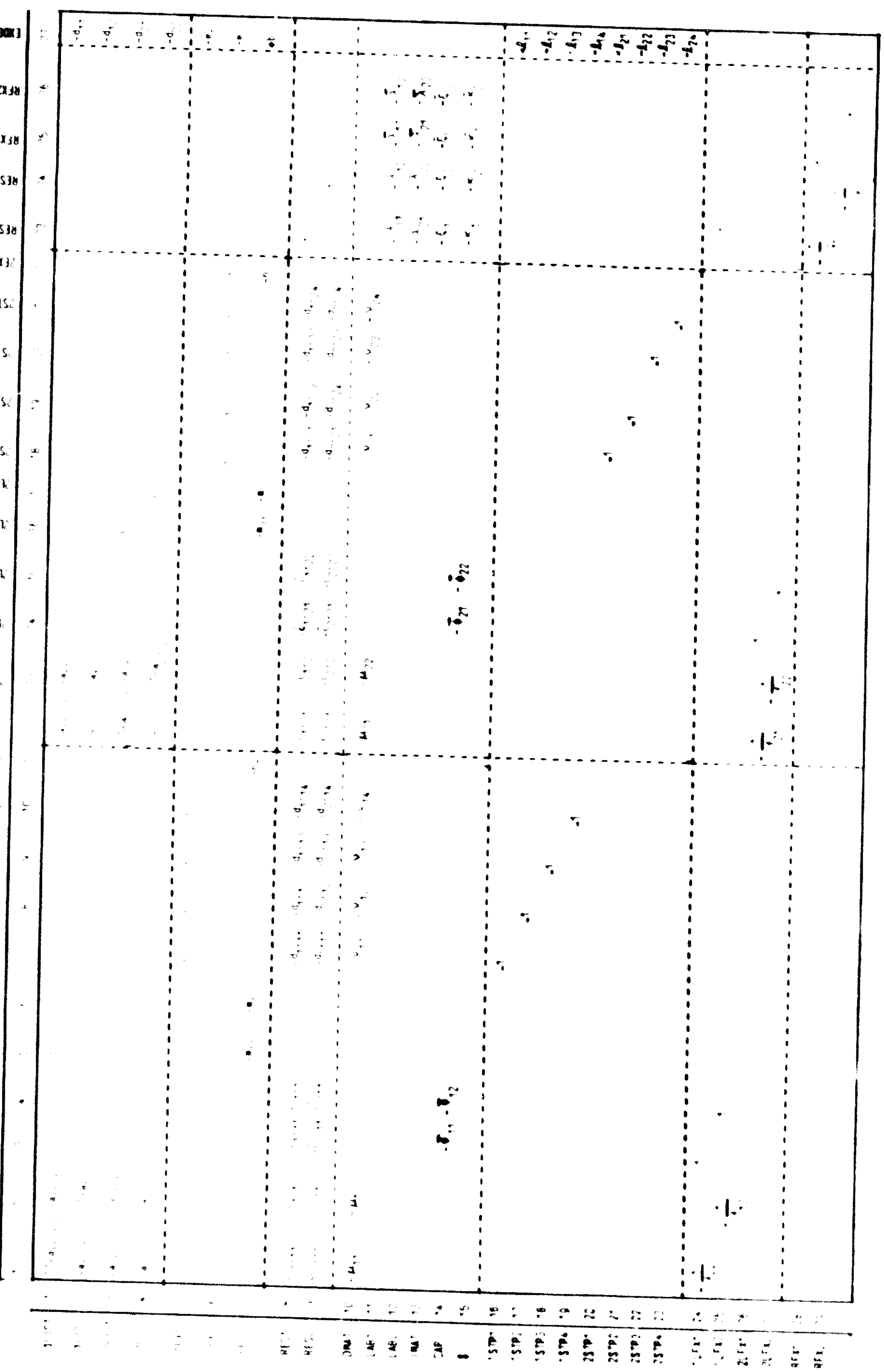
largest possible single production run. Figure 2 shows the relationship of fixed and variable costs on the assumption that multiple production runs can be undertaken. In each of these runs fixed costs are incurred once, and from there on variable costs are constant (a constant slope) up to the maximal production series. Incurring fixed costs twice will thus secure a maximal total output of (2).(f_{11}) and so on. In general, the tie-in parameter f or g is used to denote the upper limit on the scale of the variable-cost activity that corresponds to a single fixed-cost incurrence. If the scale of the variable-cost activity exceeds this parameter, the number of fixed-cost incurrences will be equal to the next larger integer.

4. A multi-branch model

36. Model 3 presents a generalization of the simplest model. It drops the unrealistic assumption that the sector can be programmed branch by branch and explicitly introduces the sharing of productive facilities (resource elements) between branches. In order to restrict Model to a manageable size, distinctions between listed products by typical product origin are now dropped, and only two listed products are shown for each of two branches. This modest amount of detail suffices to illustrate the principal novel points that emerge.

37. Model 3 is organized by branches: all production, fixed-cost, import, and extrapolated-product activities of a branch are brought together in a group. It will be noted that starting with the listed-product balances in the first four rows, intermediate-input requirements can be shown as inter-connecting the branches (rows 1-2 intersecting columns 12-13, and rows 3-4 intersecting columns 1-2). All other features of the entries in all rows of the first 22 columns and of the exogenous column remain essentially unchanged between Model 2 and Model 3, except for a slight generalization of the notation in order to allow the labelling of parameters by branch and, in the case of intermediate-input coefficients ($-a$), by branch both of origin and of destination. Apart from thus simultaneously showing more than one branch, the novelty of Model 3 is concentrated in columns 23-26. Columns 23-24 are resource-element-capacity maintenance activities labelled RES1 and RES2 that correspond to the identically labelled columns in Model 2, except that they are now tied in with the respective fixed-cost activities RFX1 and RFX2. These fixed costs have to be incurred whenever the capacity of a resource element is to be maintained at a level exceeding zero. The mathematical tie-in between the fixed-cost activity RES1 and the variable-cost activity RFX1 characterizing the first resource element, is

MODEL 3



precisely the same as the previously discussed tie-in between a production activity such as 1LIS1 and a fixed-cost activity such as 1LFX1. The tie-in is provided by the constraint of rows 28-29.

38. The fixed and variable costs associated with maintaining given resource-element capacities are intended as an approximation to the economies of scale that are known to occur when the total yearly capacity of a given resource element increases. With a given fixed cost and constant variable costs a larger capacity will imply lower resource inputs per unit capacity. The variable costs are broken up into specific resource inputs exactly as those in Model 2, while the fixed costs are given as lump-sum labour, material and capital requirements. There is an upper limit on capacity which corresponds to the empirical observation that given processing facilities are not built in indefinitely large sizes; if the size exceeds a certain limit, a duplication of facilities occurs. This can be represented mathematically by setting the fixed-cost tie-in parameter equal to the reciprocal of the capacity limit. In accordance with the earlier discussion on fixed cost constraints, this will push the scale of fixed-cost incurrence (e.g. the variable x_{25} corresponding to activity RFX1 in Model 3) up to at least x_{23}/g_1, the ratio of the scale of the variable-cost activity RES1 to the upper limit imposed on the capacity of resource element (1). If this ratio is between 0 and 1, the integrality requirement imposed on the fixed-cost activity scale x_{25} will push x_{25} all the way up to unity; if the ratio is greater than 1, the integrality requirement will push x_{25} up to the next larger integer. In this way the requirement for multiple facilities, together with multiple incurrences of fixed costs, is properly represented.

5. Economies of scale due to the seriality of production

39. This source of economies of scale is the only one included in Models 1 and 2 and is represented by fixed costs (or fixed resource inputs) tied to the scales of the respective production activities that embody variable costs. We shall now look more closely at the two kinds of fixed costs: (a) fixed costs associated with investment in tooling, jigs and fixtures; and (b) fixed costs associated with set-up operations for each production run.

40. The first kind of fixed costs creates no particular problems at the present stage of analysis. Later, it will be necessary to recognize these costs as leading to output requirements within the metalworking sector proper, and thus to a feedback between the required production scales of metalworking activities at a future date and the corresponding output of tools, jigs and fixtures intended for investment

purposes, during a given earlier time period t (see Chapter 10). At present it is important that fixed costs connected with a production activity also include set-up charges which have to be incurred for each separate production run. Thus the total amount of set-up charges depends on the connexion between yearly demand and the length of an individual production run.

41. In Model 1 all fixed costs including set-up charges, are expressed in monetary terms. In Models 2 and 3, investments in tooling, jigs and fixtures are given as lump-sum capital requirements, while set-up charges are approximated by giving the capacity requirements of resource elements corresponding to the actual yearly set-up time. Thus if a given resource element with a machine part of 50 units has a total of 300,000 effective yearly machine hours, set-up charges can be expressed as the number of machine hours required per run multiplied by the number of runs per year. This product is subtracted from the total number of effective machine hours available for production. Since Models 2 and 3 distribute a variety of yearly charges of resource elements (labour, investment, indirect material inputs) over total effective machine hours, this way of handling set-up charges is equivalent to the assumption that not only investment costs but also labour and indirect material costs per hour are the same regardless of whether set-up operations or production are being undertaken. This is probably a tolerable simplifying approximation. The main reason for separating direct from indirect material inputs is to avoid the more gross error that would be associated with assuming that the metal requirements were also proportional to total resource-element capacity utilized, without discrimination between the fixed and variable parts of the latter.

42. An important simplification introduced into the models is the fact that only one fixed investment in tooling, jigs and fixtures is provided for each productive process: there is only one variable-cost activity and one fixed-cost activity for the manufacture of each listed product by a given process.[10/] This avoids the problem of endogenously representing alternative degrees of complexity in the provision of tooling, jigs and fixtures. This problem is illustrated in Figure 4. With reference to Figure 4, x_1 (as in Models 1 and 2) represents the scale of the variable-cost activity in the production of the first listed product. There are now

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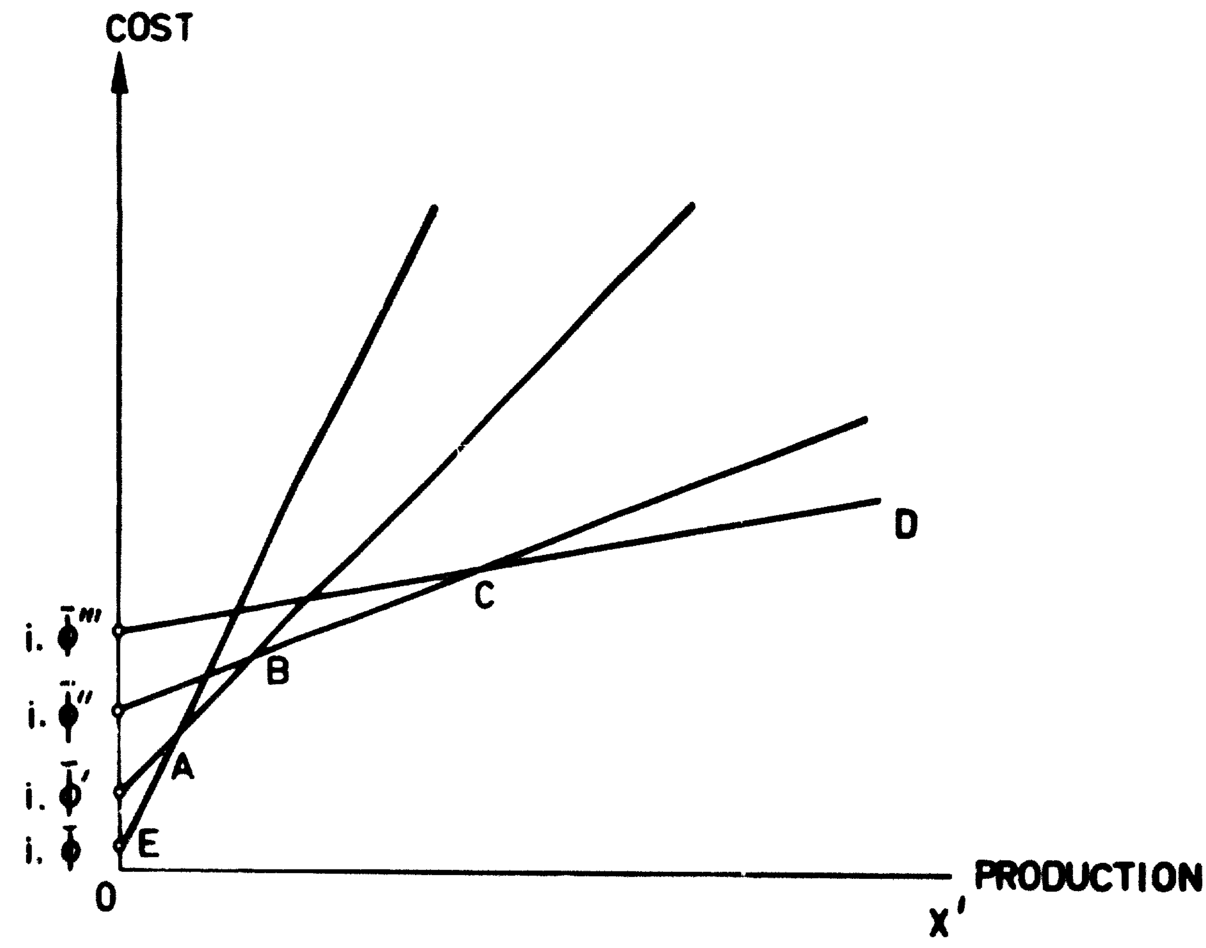
10/ This does not exclude the possibility of having two alternative techniques for producing a given product: e.g. forged or precision-cast crankshafts. The point raised in the text refers to each of these alternatives individually, and relates to the optimal extent of tooling.

Figure 4

The investment cost for tooling, jigs and fixtures, versus variable costs of production



four alternative degrees of tooling, represented by four separate fixed investments $\bar{F}$, $\bar{F}'$, $\bar{F}''$, $\bar{F}'''$ (in Model 2 there is only a single $\bar{F}$ parameter associated with each productive process). Annual fixed costs are obtained by applying appropriate capital charges $\underline{i}$ to these fixed investments. As the degree of tooling increases, the variable costs $\underline{k}$[11/] will decrease correspondingly[12/] as shown by the slope of the cost lines. Over varying ranges of x_1 different degrees of tooling become most efficient (lowest-cost). This is reflected by the broken line OEABCD which represents the production-cost frontier attainable by all techniques jointly. With respect to this production-cost frontier the models give only a single fixed-cost-to-variable-cost combination. Given sufficient empirical data, there is no difficulty in introducing alternatives into the models; in practice, however, it will generally be preferable to determine the optimal degree of tooling for a given x_1 by a side calculation. With exogenously given total demands the appropriate x_1 for this side calculation is the total yearly demand,[13/] as will be discussed in relation to set-up costs.

43. We now turn to fixed costs associated with set-up operations. If the fixed capacity requirements needed for setting up an individual run of a given listed product can be derived empirically, then the remaining piece of information needed for specifying yearly set-up costs is the number of production runs per year.

44. A simple engineering formula exists for calculating the optimal number of production runs per year so as to minimize the sum of set-up and inventory-carrying costs. Variable production costs (as earlier defined) and fixed investment costs

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11/ In Model 2 variable costs are given in terms of resource inputs. These can be converted to equivalent $\underline{k}$ values by applying appropriate prices to each resource input.

12/ Figure 4 shows only processes that are efficient (lowest-cost) over some range of x_1. It is possible for a process to be inferior to some other process at any x_1. Such a process would never be selected.

13/ Exogenous demands have to be increased by the amount of intermediate-input requirements to arrive at total demands (see Chapter 7).

for tooling, jigs and fixtures are excluded from the optimization formula, as these are not affected by the length of the individual production run:

$$r = \sqrt{\frac{d.k}{\bar{\bar{k}}} \cdot \frac{i}{2} \cdot \frac{p-d'}{p}} \quad ;$$

- r number of production runs per year at optimum;
- d demand, physical units per year;
- k variable production cost, dollar per physical unit;
- $\bar{\bar{k}}$ set-up cost per run in dollars;
- i inventory carrying charge, including interest, obsolescence, deterioration, handling, taxes, storage, insurance and pilferage;
- p production rate, physical units per day;
- d' demand, physical units per day.

45. The above expression gives the optimal number of runs per year.[14/] The expression $d.k/\bar{\bar{k}}$ is the ratio of yearly variable costs to the set-up charge of a

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14/ Total cost per year can be expressed as follows:

$$TC = FC + d.k + \bar{\bar{k}}.d/x_o + x_o.k.i.(p-d')/2p,$$

where in addition to the previous notation, TC is total cost, dollars/year; FC is fixed cost, dollars/year (not affected by the length of the production series); and x_o is the length of a production run. To optimize TC as x_o is varying,

$$0 = dTC/dx_o = 0 + 0 + (-d.\bar{\bar{k}}/x_o^2) + k.i.(p-d')/2p,$$

whence $$x_o = \sqrt{\frac{d.\bar{\bar{k}}}{k} \cdot \frac{2}{i} \cdot \frac{p}{p-d'}} \quad .$$

By $r = d/x_o$ the formula in the text follows immediately. In the expression for total costs, TC, the four terms correspond in turn to (a) fixed costs, such as yearly charges on investment in tooling, jigs and fixtures; (b) variable production costs; (c) set-up charges; and (d) inventory-carrying costs. The latter are obtained from average stock carried, which is one-half of the peak stock at the end of a production run, calculated as the product of the daily accumulation, p-d', and the length of a run, x_o/p days. Average stock is multiplied by variable production cost to convert it into value terms, and an inventory-carrying charge (per cent per year) is applied to the latter.

The expression for x_o, given before, can be rearranged after squaring to yield the equality at the optimum:

$$d\bar{\bar{k}}/x_o = x_o.k.i.(p-d')/2p;$$

thus yearly set-up charges are equal to yearly inventory-carrying costs at the optimum. For the derivation of optimal length of series, see for example Starr (1964).

single run, and the expression $(p-d')/p$ is the ratio of product accumulation to production. Total yearly set-up charges, expressed as $(r).(\bar{k})$, thus increase only as the square root of yearly demand and the set-up charges per unit output correspondingly fall with the square root of this demand. At the optimum, yearly set-up charges are exactly equal to yearly inventory-carrying costs; thus by doubling the set-up charges, the inventory-carrying costs can be exactly accounted for.

46. In Models 1 to 3 yearly set-up charges have been treated as fixed: it has been assumed that if a given listed product is not produced, no set-up charges would be required; while if there is production, the entire yearly set-up charges would be incurred, regardless of the actual amount produced. This is an approximation to the more complex engineering description.

47. As yearly demands of the listed products, excepting those of intermediate inputs, are in the first instance assumed to be exogenously given, the choice in regard to each listed product is generally narrowed to two alternatives: the product is either not produced at all, or it is produced at the maximum possible scale corresponding to total yearly demand.15/ Thus we can determine the optimal number of production runs per year by a side calculation based on total demand; this calculation will give yearly set-up costs. These costs, if doubled, can be taken to represent both set-up and inventory-carrying charges on a fixed, yearly basis. The maximal production in the fixed-cost constraints (e.g. f_{11} in Figure 2) must now be set to a value that is larger than yearly demand to ensure that yearly set-up charges are incurred only once.16/

48. The crucial simplifications employed in regard to the seriality of production in Models 1 to 3 are now readily apparent. The first simplification is the constancy of yearly demand; if this were made an endogenous variable of the system,

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15/ However, see Chapter 7 for the problems introduced by intermediate inputs.

16/ An alternative procedure is to provide two fixed-cost activities, one for investment-type fixed costs, and the other for set-up-type fixed costs. The tie-in parameter for the former can again be set to any value larger than yearly demand, while the tie-in parameter for the latter is the length of the optimal series, derived by a side calculation. Then set-up costs will be incurred in integer multiples, depending on the ratio of yearly demand to the length of the optimal series. The procedure in the text is both simpler and more exact, since the number of runs per year need not be an integer, while the incurrence of yearly fixed set-up charges is inherently an integer (0-1) variable.

alternate degrees of tooling as well as the square-root function connecting yearly set-up charges and yearly demand would have to be taken into account explicitly. The second simplification is the approximate anticipation of the productive structure for the determination of the yearly number of production runs by means of a side calculation. As the corresponding formula contains p, the daily production rate, a feedback exists between the structure of productive facilities (which determines the production rate) and the optimal seriality. While this feedback is not recognized within the structure of the models, in the course of programming it is possible, none the less, to make some allowance for the feedback by means of iterative revisions. Finally, the formula for the number of yearly production runs, r, reduces all costs to common monetary terms; in a programming model, however, many prices are themselves variables that cannot be used for side calculations prior to solving the problem as a whole. Moreover in an integer programming problem, the role of prices becomes subject to further qualifications, to be discussed below. Despite these observations, the side calculation is meant to be undertaken with prices that are assumed as given. Here again, iterative revisions may be employed. It should be noted that this pricing problem is not peculiar to the representation of seriality in the models; it will also be found in many other aspects of the operation of the models. All these problems will be discussed further in Chapter 10.

6. The representation of resource elements: the simplest case

49. Resource elements have been defined and discussed in great detail in an earlier report by the New School for Social Research (1967). In Models 1 to 3 resource elements enter in the simplest possible manner, namely with completely specified fixed and variable resource requirements. This begs the issues of connexions with semi-quantitative programming data and of the local adaptation of resource elements (the selection of an optimal machine park, the adoption of a proper degree of mechanization and automation in response to varying capital/labour prices, and the adaptation of the technology of production to the specific product assortment that is being produced). Nevertheless, it is asserted that the present conceptualization provides an adequate basis for more powerful generalizations which will be discussed subsequently. The form of Models 1 to 3 has been selected to provide an orderly sequence of presentation, as there are so many complexities operating simultaneously that they cannot be crammed into a single model that would still preserve some degree of overview of the problem.

50. In Model 1 resource elements remain implicit: the cost of production for each product is presented in fixed and variable parts, and all costs referrable to resource elements are already included in the variable parts of these dollar totals, thereby abstracting from all indivisibilities in resource-element investments. In Model 2 the costs associated with maintaining capacities of specific resource elements are broken up into physical input flows for labour and materials and into total capital requirements. When applying specific flow prices to the former and a capital charge to the latter, these are then converted into yearly money costs. All of these costs are expressed on the basis of a unit of resource-element capacity which is being maintained and are assumed to be fully proportional to total resource-element capacity: in other words, the postulate of complete divisibility (no lumpiness, no economies of scale) is still maintained for all resource elements. In addition, the machine park of a resource element (machine tools, hoists, furnaces etc.) and the required construction (buildings) are not itemized in the models: all individual capital goods and other capital assets are expressed in money values only. This will require generalization at a later stage to take into account the fact that

capital goods required for production at a future date lead to a demand for metalworking products at an earlier date. (A lathe is an output of the machine-tools branch of the metalworking industry.) Implicit in the pricing process is the assumption of exogenously given prices for performing the evaluation of investment requirements prior to the solution of the model; yet some of the required prices are included in the model itself and emerge only after the programming problem is solved. Thus the solution has to be anticipated in part while formulating the model. This problem is analogous to the pricing problem discussed in Chapter 5. It can be handled by making approximate estimates of the anticipated prices in formulating the model and revising these in an iterative fashion after the solution is obtained. If, alternately, it were desired to make the pricing process endogenous, all capital goods and construction would have to be itemized individually and balanced specifically within a multi-period model (see Chapter 10).

51. In Model 3 the assumption of perfect divisibility of resource-element capacity is superseded by the more realistic assumption that economies of scale exist in regard to such capacity. These economies of scale, within the confines of a static, single-period model, do not refer to the activity of constructing or expanding these resource elements, but only to the total yearly costs that are associated with maintaining given total resource-element capacities. There is reason to believe that total investments in the process facilities of the metalworking industries (and thus the yearly capital costs) are related to the size of these facilities (i.e. to the total capacity of a single facility) by means of a relationship of constant, less-than-unitary elasticity, of the form:

$$K_1/K_2 = (S_1/S_2)^e, \quad 0 < e < 1 \quad ;$$

where

K_1, K_2 total investment in process facilities (resource-element capacity) at two different sizes (capacities);

S_1, S_2 the corresponding resource-element capacities;

e elasticity, a constant exponent in the formula.

52. This relationship is shown in Figure 5 in natural and in logarithmic scale units; in the latter, the relationship reduces to a straight line. Evidence on the existence of such a relationship is largely indirect, and comes from the chemical-process industries where it is firmly established (US Bureau of Mines, 1949; Aries and Newton, 1955; Isard and Schooler, 1955; Chilton, 1960; UN - ECLA, 1963; Vietorisz, 1966). We also have quantitative evidence of a general trend toward

Figure 5

Assumed capacity-investment relationship for resource elements in (a) natural and (b) logarithmic scale units

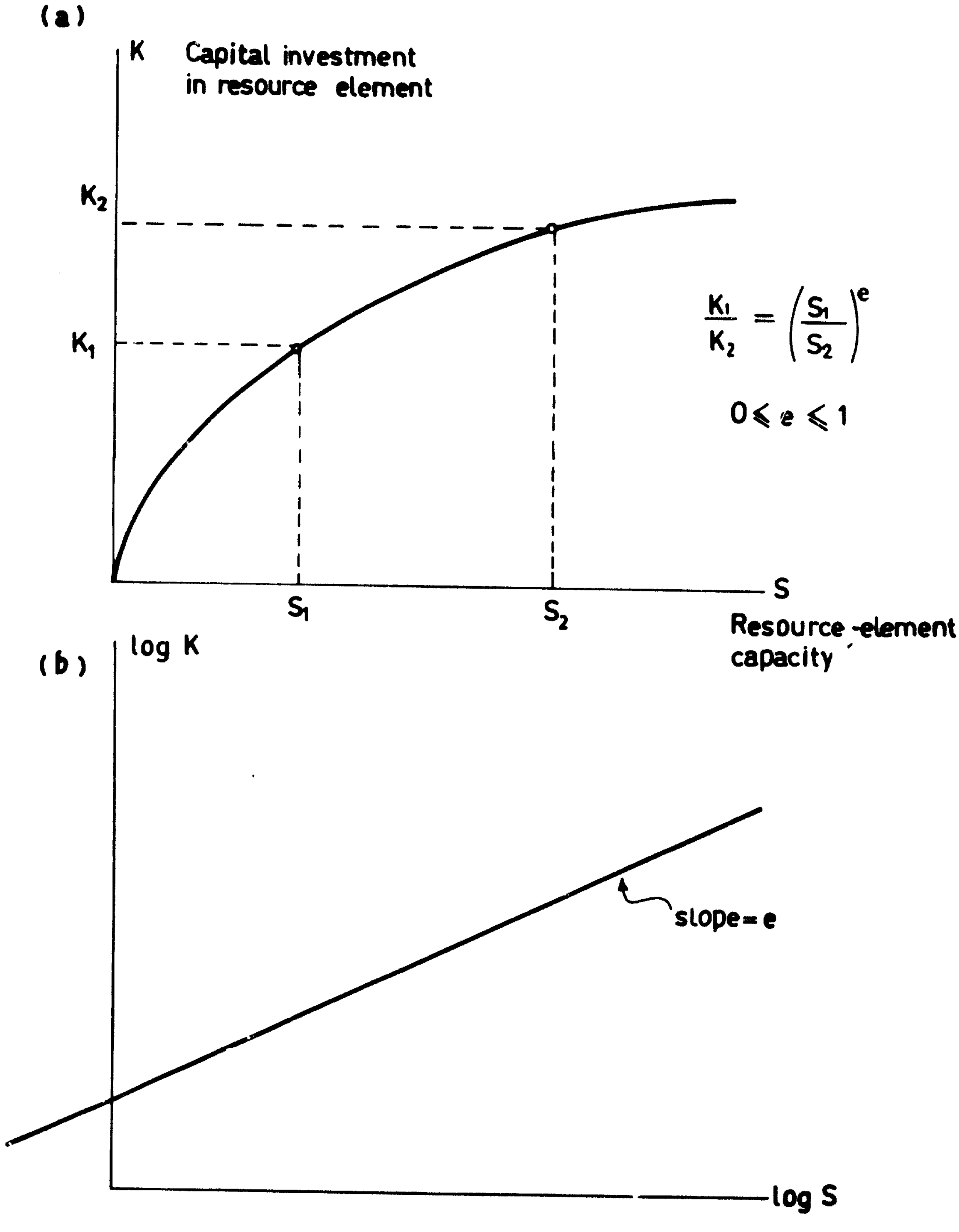
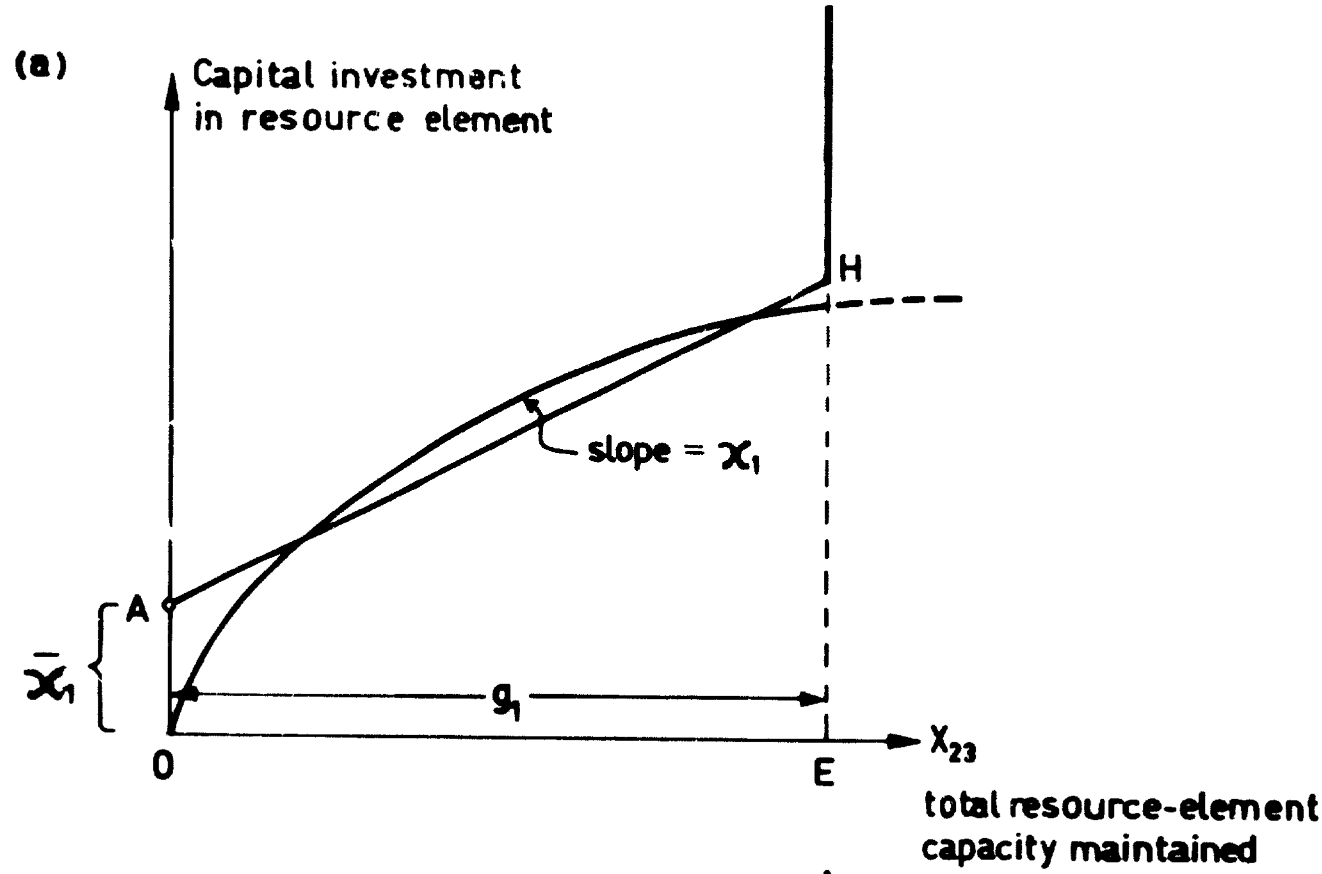
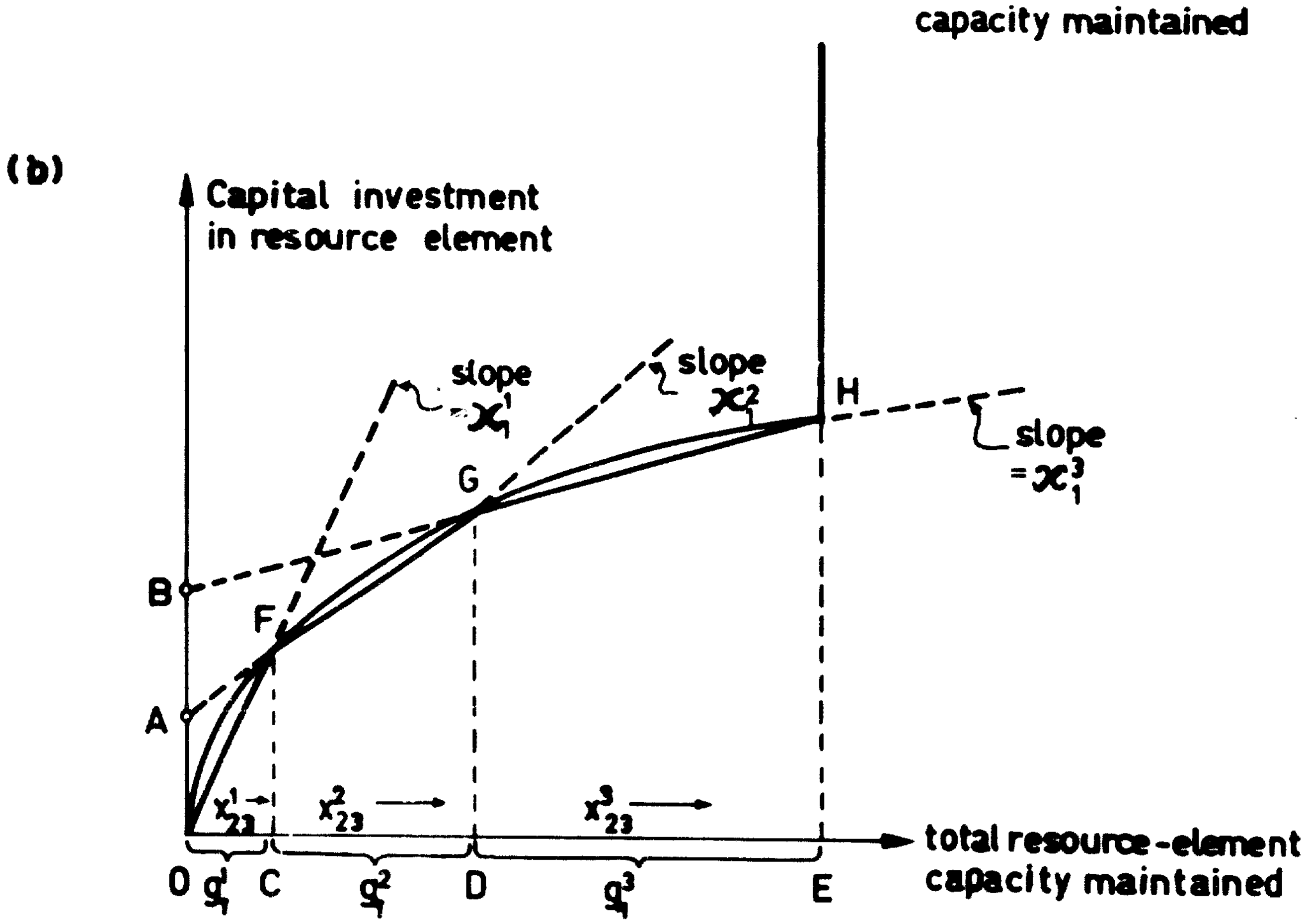


Figure 6

Two linear approximations to a smooth function representing constant elasticity of investment relative to size: (a) fixed cost and a single linear segment; (b) three linear segments





lower unit costs in larger metalworking processing facilities based on USSR data (Gallik, 1961) and a correlation of exactly the form given for a single kind of metalworking facility (jet engine production: Alpert, 19). Assuming that the form of the relationship is correctly specified, the programming problem is to include it in the model by means of some easy-to-handle approximation. Two possible approximations are shown in Figure 6. The first relies on a fixed cost plus a single linear segment; the second on three consecutive linear segments. In Model 3 the first, simpler approximation has been included. In this approximation total, investment (e.g. for the first resource element) equals $\bar{~}_1 \cdot x_{25} + {}_1 \cdot x_{23}$, where x_{25} is an integer variable. In both cases it is further postulated that the relationship representing economies of scale breaks off at some practical maximal size (capacity); if the total capacity requirements exceed this limit, there has to be a duplication of productive facilities. This problem has already been discussed in earlier Chapters and is handled by setting the g parameter (e.g. at the intersection of row 28 and column 23 in Model 3) to the maximal capacity of a single facility. If approximation by three linear segments is desired, the variable x_{23} representing resource-element-capacity requirement has to be split into three new variables, e.g. x^1_{23}, x^2_{23}, and x^3_{23}, each representing a marginal amount of capacity, with corresponding marginal investment requirements of ${}^1_1$, ${}^2_1$, and ${}^3_1$ which decrease in this order. Total investment equals ${}^1_1 \cdot x^1_{25} + {}^2_1 \cdot x^2_{25} + {}^3_1 \cdot x^3_{25}$. The capacity limit g_1 also has to be broken up into three corresponding parts g^1_1, g^2_1, and g^3_1 (see Figure 6), and two integer variables x^1_{25} and x^2_{25} are needed. The following relationships have to hold in order to ensure that the marginal amounts of investment are incurred in the proper sequence:

$$\frac{x^1_{23}}{g^1_1} \geq x^1_{25} \geq \frac{x^2_{23}}{g^2_1} \geq x^2_{25} \geq \frac{x^3_{23}}{g^3_1} \quad .$$

Thus four separate inequalities involving two integer variables are required to represent this approximation: after complementation by a surplus variable, each of the inequalities enters the model; the four inequalities jointly replace the single tie-in constraint between x_{23} and x_{25} in Model 3 (row 28).

53. The interpretation of the sequencing constraints is straightforward. Assume, e.g. that the total capacity requirement is greater than OE and leads to a duplication of facilities, and that the second facility has to operate at a capacity between OC and OD (see Figure 6). In the absence of sequencing constraints, x^3_{23}

(which has the lowest marginal cost g_1^3) would be used to provide the entire required capacity at this marginal cost (with no fixed costs of any kind), a procedure contrary to common sense. The sequencing constraints, however, intervene by forcing both x_{23}^1 and x_{23}^2 to be used in non-zero amounts as soon as x_{23}^3 is used; at this point the integer variables step in and force x_{23}^1/g_1^1 and x_{23}^2/g_1^2 up to unity, i.e. the first two segments are used to the full. If x_{23}^3/g_1^3 rose above unity, this would force both of the earlier segments to be incurred twice; to avoid this, x_{23}^3/g_1^3 is held exactly to unity, x_{23}^2/g_1^2 is set to a value between 1 and 2 and thus x_{23}^1/g_1^1 is forced up to 2. In sum, the first segment is incurred twice; the second segment, in the required amount between 1 and 2 times; and the third segment, once. This is in accord with natural sequencing.

7. The choice between domestic production and imports with fixed costs present

54. We shall begin the discussion of this problem with the simplest case presented by Model 1. Moreover, we shall initially relax the foreign-exchange constraint of row 9 and replace it by the inclusion of the foreign-exchange cost in the objective function at an exogenously given foreign-exchange rate. Thus the objective becomes the minimization of total money cost, including the cost explicitly represented in row 10 plus the domestic-currency equivalent of foreign-exchange inputs appearing in row 9.17/

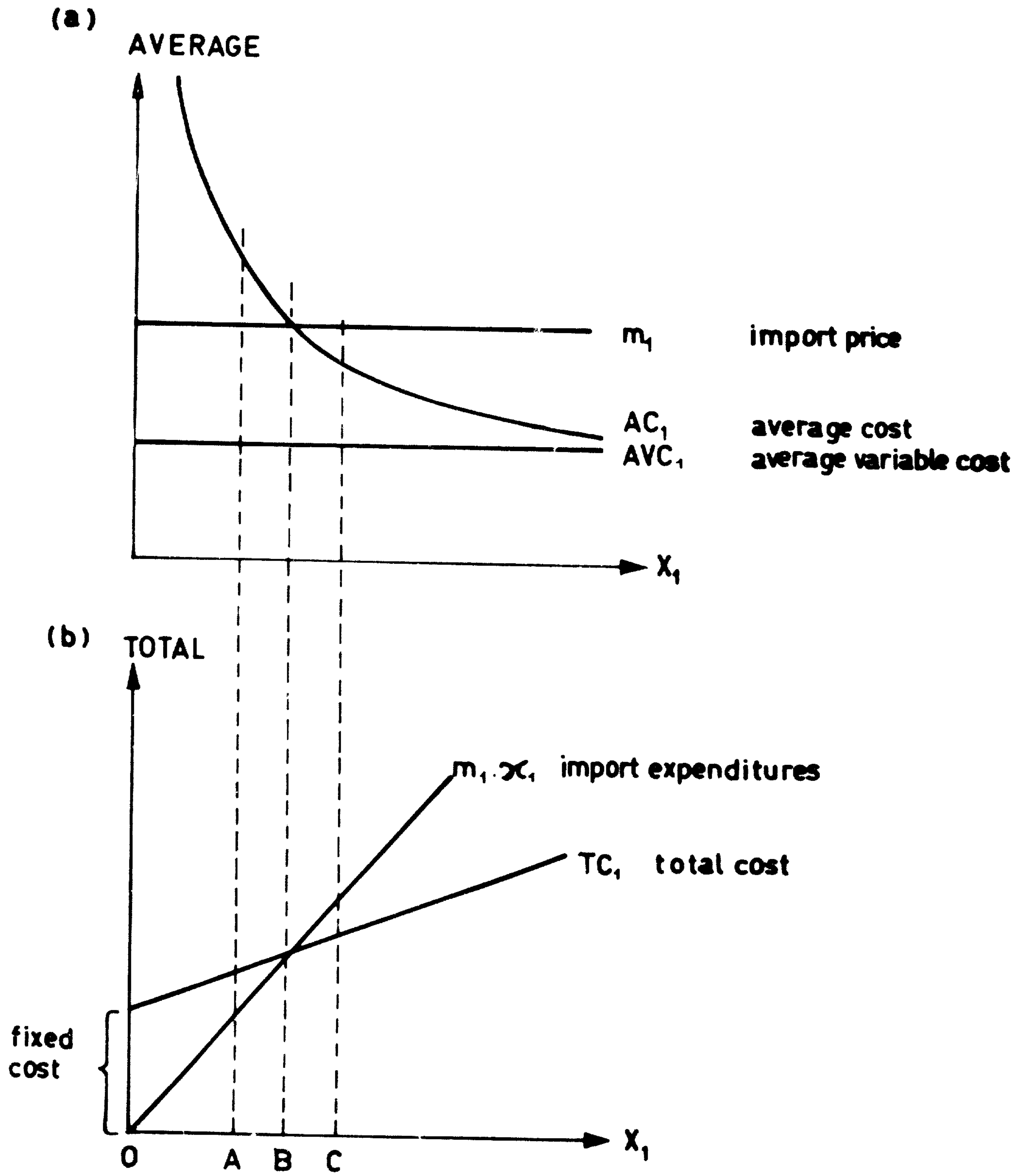
55. Given these assumptions, the alternatives of domestic production and imports can be individually considered for each product within the branch represented by Model 1. In particular, the full production cost, including fixed cost plus variable cost at the level of total demand, has to be compared with the import price m for each individual product.

56. With reference to Figure 7 the choice between domestic production and imports hinges on the level of total demand if domestic production costs and import prices are taken as given. If total demand is at the level OA, imports will be preferred, at OC, domestic production will be preferred and at OB the two alternatives are equivalent. If the listed product in question sells only to exogenous demand (i.e. it is not used as an input in any other production in the model), and if it has no inputs of other listed products, the conclusion for this product in isolation

17/ The exogenous foreign-exchange supply b does not affect the solution as it is a constant.

Figure 7

Domestic production cost and import cost of listed products:
(a) average; (b) total



is immediate. Often, however, the above restrictions are not true, especially if several branches are considered simultaneously, and they thereby increase the importance of intermediate transactions within the model. In such cases it is still possible to arrive at certain conclusions prior to recurring to formal integer programming solutions.

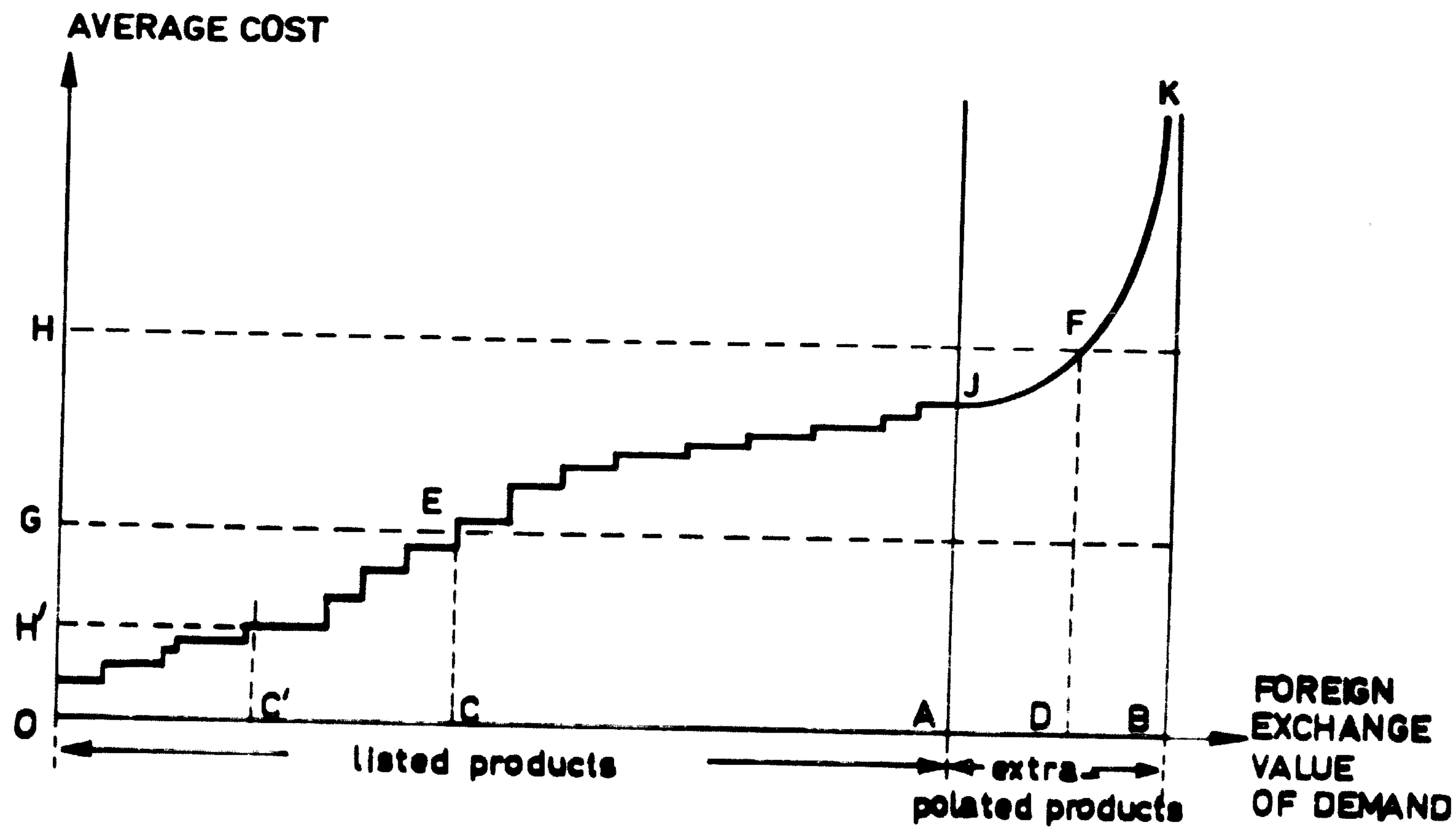
57. When the above simplifying restrictions hold for all products, the choice between domestic production and imports for the whole branch can be represented by Figure 8. In this Figure the listed products of the branch are lined up in the order of their domestic production costs per unit import value, showing production cost at the scale of exogenously given demand for each product. The costs represented are average costs at the latter scale: the hyperbolically falling trend of average costs (see Figure 7) for each product is not shown in Figure 8 where it is replaced by a straight line drawn at the level of average costs at the stated scale. The horizontal axis measures the cumulative foreign-exchange value of exogenous demand; the length of each step in the graph represents the foreign-exchange value of exogenous demand for the given listed product. Thus the distance OA represents the total foreign-exchange value of demands for all listed products within the branch measured at import prices, while the distance AB represents the total foreign-exchange value of demands for extrapolated products (the remaining products within the branch), also measured at import prices. Thus OB would be the total foreign-exchange expenditure if the demand within the branch were supplied entirely from imports, with no domestic production at all. Note that the extrapolated products generally represent the most specialized and lowest-seriality products in each branch, which in the developing countries are likely to be almost entirely imported; moreover, in statistical sources such products will almost never appear in individually itemized form, but rather as a residual (e.g. "other machine tools"). Thus the total foreign-exchange value at import prices (total import value) is a convenient way of representing these products in the aggregate.

58. Average costs in Figure 8 are measured in national currency, and they are standardized by reduction to a unit of import value. For example, if an electric motor costs 200 pesos to manufacture domestically, weighs 25 kilogrammes and can be imported for £100[18/] then the domestic manufacturing cost is 200/25 = 8 pesos per kilogramme of weight, or (calculating in the same way) 200/100 = 2 pesos per £ of

18/ The "peso" is used here as a national-currency unit, and the pound (£) as the unit of foreign exchange.

Figure 8

Profile of domestic production costs per unit import value for a given branch, including extrapolated products



import value. The latter measure is graphed in Figure 8. When the foreign-exchange rate (e.g. 5 pesos per £) is traced in at the level OG, the production-cost profile of the branch immediately discloses those products which are cheaper to produce domestically and those which are cheaper to import. If the foreign-exchange rate changes to a higher level, e.g. to OH, more products become attractive for domestic production: in fact, all listed products plus about one-half of the extrapolated products (measured at import value) should now be produced domestically. The crucial question of how to derive the part of the production-cost profile that represents the extrapolated products, without recurring to the (almost impossible) technique of listing and analyzing these individually, will be discussed in Chapter 8.

59. Figure 8 illustrates a related but somewhat more difficult problem. Supposing that the foreign-exchange rate is not given, but is made into an endogenous variable of the system, and that instead the branch is provided with a foreign-exchange allocation: how is the choice between domestic production and imports now to be undertaken? With reference to Figure 8, we assume that the foreign-exchange allocation is CB. The problem now becomes similar to the well-known mathematical "knapsack" problem[19/] and can be solved approximately by starting with full domestic production and successively selecting products for import in the order of decreasing domestic production costs until the foreign-exchange allocation is exhausted. In Figure 8 this occurs at C. In this example the foreign-exchange allocation CB happens to coincide with a step "riser" in the profile of domestic production costs, and thus the solution is exact; the corresponding foreign-exchange rate can be anywhere within the limits of the "riser" near the level OG. If, however, no such coincidence occurs, as with the allocation C'B that cuts a step over its horizontal stretch, the domestic production of the corresponding product must be undertaken at a scale that is less than the full exogenous demand, and the level of average costs will rise.

60. The approximation is a good one so long as the production rate of the last domestically produced product is close to the product's total demand. In fact, the rise of average cost times the actual output of this product provides an upper bound on the size of the error which might be committed. Figure 9 gives a numerical illustration of the kind of error that can occur. With a foreign-exchange allocation

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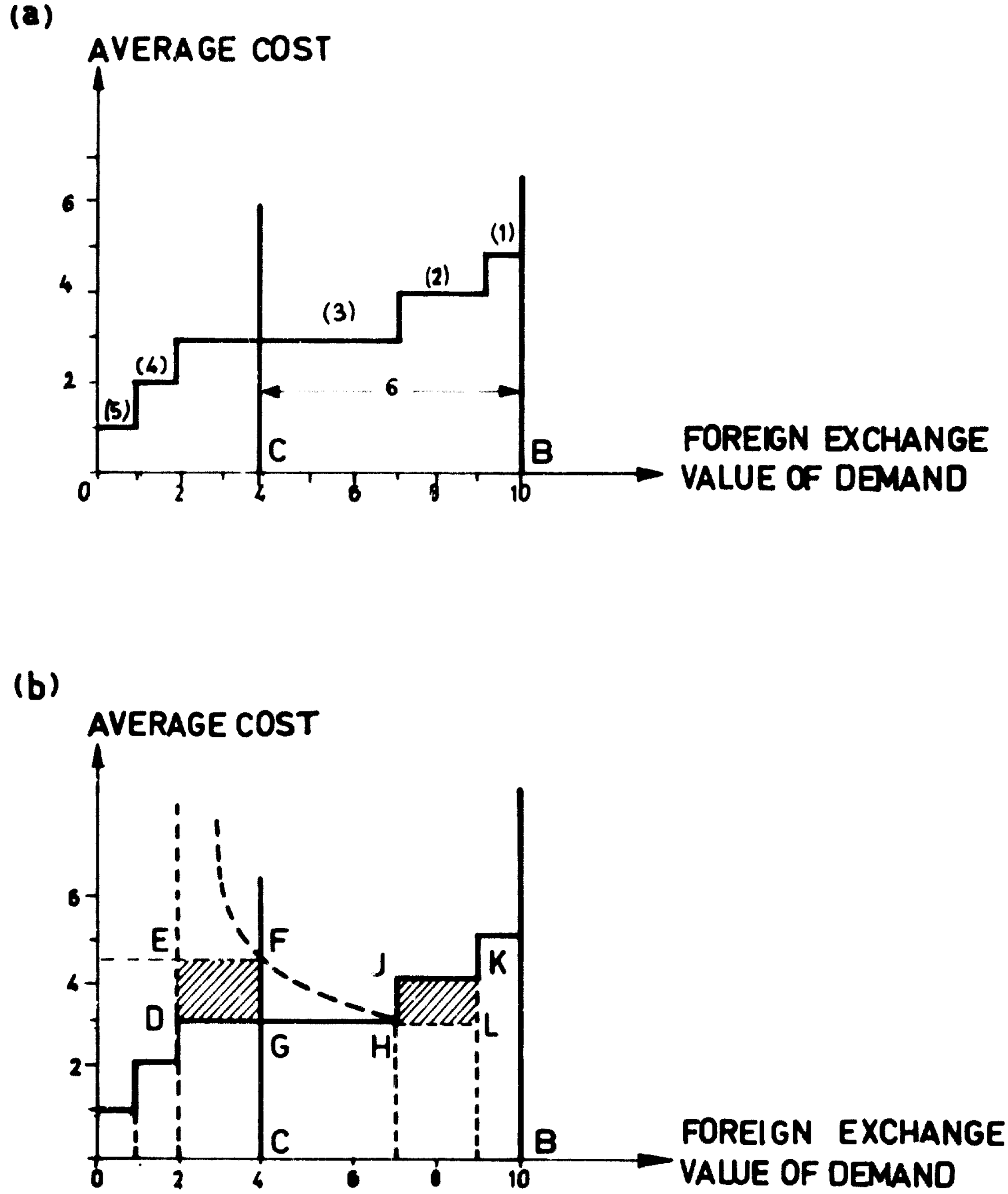
19/ See for example Dantzig (1963), 517 ff. The difference between the knapsack problem and the present problem is that in the knapsack problem production at a reduced scale (and elevated average costs) would not be possible; the choice for each product would be of the all-or-none kind.

(CB) of six units the approximate solution is to produce products (5), (4) and (3), the first two at full scale, the last at a reduced scale of two units. If the average cost of (3) did not rise, the total production cost for the three products (the area under the average-cost profile) would be nine units; with the indicated rise DEFG (Figure 9) the total cost is twelve units. Thus, in this case, nine units offer a lower bound on the current optimal solution, and the twelve units offer an upper bound. That the latter solution is not optimal can readily be seen by selecting products (5), (4) and (2) for domestic production, each at its full scale. Jointly these provide just enough domestic production to meet the foreign-exchange limit, and their total production cost is only eleven units. This is the optimal solution. In Figure 9 the shaded area DEFG represents the cost increase over the lower bound due to the rise of average costs at a reduced production scale of product (3): this cost increase of three units is a bound on the possible error. In the same figure, the area HJKL represents the cost increase over the lower bound that occurs when the production of product (2) at full scale is substituted for the production of product (3) at its original average cost that defines the lower bound. A comparison of HJKL with DEFG shows that the former is only two units; thus the corresponding solution is better than the solution obtained by the approximating procedure.

61. Figure 9 also shows that, no matter which of the two solutions is chosen, the role of the foreign-exchange rate as a guide to resource allocation is compromised. If the approximation is used, the exchange rate rises to 4.5 units; at this level it directs the inclusion of product (2) in domestic production. If the latter is included, the foreign-exchange rate drops to four units, and the production of product (3) is discontinued. However, at this exchange rate the production of (3) at its full scale (average cost = 3) appears attractive, and in a decentralized decision mechanism it will be undertaken, even though this leaves a large foreign-exchange slack and raises total production costs to eighteen units. Specific quantitative controls have to be introduced in either case: with the approximate solution, to keep the production scale of (3) restricted to two units and to keep (4) from being produced; with the optimal solution, to keep product (3) from being produced at all. None the less, the exchange rate still offers a valid guide for products (1), (2) and (5); if the problem were enlarged by extending the cost profile upward and downward by a number of additional steps, the exchange rate could be relied upon to control the majority of decisions with the exception of products (3) or (4) which would still require quantitative controls.

Figure 9

Approximate and optimal solution to foreign-exchange allocation problem: (a) profile of average costs for branch; (b) comparison of alternate solutions



62. What happens if intermediate inputs of listed products into each other are re-introduced? We shall first enquire into the production-versus-import choice under a given foreign-exchange rate. The problem centres on (a) how to price intermediate product inputs into a given listed product; and (b) how to determine the market expansion for a given product due to sales on intermediate account. To begin with, inputs of other listed products into a given listed product can be accounted for at import prices: this puts an upper limit on production costs, since in these programming models no product will be produced domestically at cost levels exceeding import prices. Second, the cost comparison can be made at the level of exogenous demand which cannot be lower than the level of total (including intermediate) demand. Under these conditions any product showing a cost advantage for domestic production will certainly be domestically produced.

63. Table 2 illustrates these principles. The model in Table 2 is organized in accordance with Model 1, except for the omission of the extrapolated products. The maximum production scale for each domestic production activity is assumed to be lower than 100, thus f_{kj} is set to 100 for all products. In making the approximating computations, however, f_{kj} is at all times set equal to the actual production scale; this production scale equals the known or estimated amount of demand. Thus average cost is $\bar{k}/d$, where d is initially the exogenous demand. In Table 2 the first tabulation of calculations sets <u>d</u> to exogenous demand for each product and values all intermediate inputs at import prices. Under these conditions, only production activity 2 attains a domestic production-cost estimate that is below import cost: 4.07 as compared with 5. Thus we can say that product (2) will be domestically produced.

64. Products failing to meet the above test can still meet the import-cost line (a) if their average variable costs are sufficiently reduced by accounting for intermediate inputs at lower domestic production costs rather than at import costs or (b) if their seriality is sufficiently increased by selling to other production lines. These adjustments can be carried on in combination in several rounds until no further improvement is possible by this technique. In Table 2 there are two rounds of adjustment. In the second tabulation intermediate inputs are re-priced to allow for a decrease of the cost of product (2) from 5 to 4.07 units which brings the production cost of product (1) below the import price of this product: 3.47 <u>versus</u> 3.50 units. The production cost of product (4) also falls, but not sufficiently to meet the import-price line. In the next round, the intermediate input requirements of both products (1) and (2) are added to exogenous demands, and intermediate inputs of these products are valued at their domestic production costs. Resulting from this

Table 2

A numerical illustration with intermediate inputs

	Domestic production: Variable cost				Fixed cost				Import				Exogenous demand	
1	1		.2						1				-17.2	Balance for listed products 1
2	-.2	1		-.1						1			-13.0	" " " " 2
3		-.2	1	-.1							1		- 5.8	" " " " 3
4	.2	-.2		1								1	-31.8	" " " " 4
5									-3.5	-4	-5	-2.5		" " foreign exchange
6	-1	-2	-2	-1.5	-20	-10	-15	-10						" " money costs
7	$-\frac{1}{100}$				1									Fixed-cost tie-in constraint 1
8		$-\frac{1}{100}$				1								" " " 2
9			$-\frac{1}{100}$				1							" " " 3
10				$-\frac{1}{100}$				1						" " " 4
	1	2	3	4	5	6	7	8	9	10	11	12	13	

1. Exogenous demand, import prices

	1	2	3	4		1	2	3	4
m	3.50	5.00	4.00	2.50					
k/d	1.16	.77	2.58	.31					
k	1.00	2.00	2.00	1.50					
1	-	-	.70	-	demand, estim.	17.2	13.0	5.8	31.8
2	1.00	-	-	.50	production, gross	-	-	-	-
3	-	.80	-	.40					
4	.50	.50	-	-					
	3.66	4.07	5.28	2.71					

2. Exogenous demand; adjust prices to production of listed products (2)

	1	2	3	4		1	2	3	4
m or p	3.50	4.07	4.00	2.50					
k/d	1.16	.77	2.58	.31					
k	1.00	2.00	2.00	1.50					
1	-	-	.70	-	demand, estim.	17.2	13.0	5.8	31.8
2	(.81)	-	-	(.41)	production, gross	-	-	-	-
3	-	.80	-	.40					
4	.50	.50	-	-					
	3.47	4.07	5.28	2.62					

3. Add to exogenous intermediate inputs of (1) and (2); adjust prices to production of (1) and (2)

	1	2	3	4		1	2	3	4
m or p	3.47	4.07	4.00	2.50					
k/d	1.16	.61	1.79	.26					
k	1.00	2.00	2.00	1.50					
1	-	-	.69	-	demand, estim.	17.2	16.4	8.4	37.8
2	.81	-	-	.41	production, gross	17.2	16.4	-	-
3	-	.80	-	.40					
4	.50	.50	-	-					
	3.47	3.91	4.48	2.57					

4. Simultaneous final solution: full production scales for all products

	1	2	3	4		1	2	3	4
p	3.24	3.72	3.72	2.49					
k/d	1.00	.48	1.07	.25					
k	1.00	2.00	2.00	1.50					
1	-	-	.65	-	demand, total	20	21	14	40
2	.74	-	-	.37	production, gross	20	21	14	40
3	-	.74	-	.37					
4	.50	.50	-	-					
	3.24	3.72	3.72	2.49					

Notes to Table 2

The model follows the format of Model 1, except for the omission of extrapolated products. There are four domestic-production activities, each complemented by a corresponding fixed-cost activity, and four import activities. Exogenous demands of the listed products are given in column 13. Balances are provided for each listed product, foreign exchange and money cost; tie-in constraints connect each production (variable-cost) activity with the corresponding fixed-cost activity. The usual f_{kj} coefficients in these tie-in constraints are set to 100 for every product. The objective is to minimize the sum of money-costs and foreign-exchange costs, where for the sake of simplicity, the foreign-exchange rate is pre-set to unity. Thus rows 5 and 6 can simply be merged into a single objective-function row.

In tabulation 1 the demand for each product is set to the exogenous demand, and its price is set to the import price. Average fixed costs ($\bar{k}/d$) for each product are calculated by dividing fixed cost by exogenous demand. "k" represents variable money costs, and rows 1 to 4 represent costs of intermediate listed-product inputs, each accounted for at its import price. The sum of production costs thus obtained is then compared to the import price: e.g. for product (2) the production cost is 4.07 and the import price 5.0; thus it is immediately selected for domestic production, while the other products are not.

In tabulation 2, the price of product (2) is dropped to 4.07, while demands are regarded as unchanged (for the moment). Product (1) now also shows an advantage on production.

In tabulation 3 allowance is made for the expansion of production due to intermediate demands and for price reductions. In general, the calculation of total production scales and of prices of domestically produced commodities requires a simultaneous solution for the latter, but the input-output structure here is simple enough to permit a step-by-step derivation. Products (3) and (4) are still cheaper to import.

In tabulation 4 the results of the optimal solution are given. To obtain this solution, it is necessary to assume that the simultaneous production of (3) and (4) in addition to (1) and (2) will make both (3) and (4) preferable to imports. The calculation confirms this.

the estimates of production costs for products (3) and (4) decrease, but since neither of these two products manages to meet the import-cost line, the adjustment process comes to an end.

65. Nevertheless, this is not necessarily the final solution, for several products in combination may still attain an advantage for domestic production. This will occur when the intermediate markets which they provide for one another will allow expanding production sufficiently to reduce the costs of the intermediate inputs below the import prices, and at the same time, due to the cost savings achieved by using lower-cost intermediate inputs, to reduce the prices of the input-utilizing products below the import prices (which is a condition for the creation of the postulated intermediate markets). Unless the interrelations between products are sparse, the number of possible combinations will be too large to explore without formal integer programming techniques. In any event, the burden of solution on the programming problem can be greatly reduced through a preliminary weeding out by means of the former techniques of obvious domestic production possibilities; the corresponding import activities can then be dropped and the fixed costs of domestic production made exogenous in the tie-in constraints: this cuts down the number of integer variables. In the problem of Table 2, for example, once it is decided that products (1) and (2) will be domestically produced, the only remaining combination to be explored is the simultaneous production of products (3) and (4), since the third round of calculations excludes both of these from being added on alone. The fourth tabulation in Table 2 shows the results of such a simultaneous solution for the domestic production of all four listed products. The resulting domestic production cost for each product is now below the import price.[20]

66. One feature of these computations merits further notice, as it calls attention to the nature of pricing in the presence of fixed costs. We have done the obvious thing by adding average fixed costs to variable production costs for each production activity. This, however, is not necessarily in formal accord with the specification of the model in linear programming format. In the latter format (see Model 1) fraction $1/f_{kj}$ of fixed costs is added on to variable costs;[21] this equals average fixed cost only if the solution value of the scale $\underline{x}$ of the corresponding variable-cost

20/ A similar numerical model of fixed-cost interaction in the iron and steel industry, based on Latin-American data, has been discussed in detail by Chenery (1959).

21/ If a product is de facto produced, the price-variable calculated for the fixed-cost tie-in row will equal the fixed cost itself; this price will then enter the value balance of the variable-cost activity.

activity happens to coincide with the pre-set value of the parameter f_{kj}. Such a coincidence can be achieved in the present illustration by hindsight if the f_{kj} parameters (all of which are 100 in Table 2) are re-set to 20, 21, 14 and 40 respectively, figures which are the production scales in the simultaneous solution. If this is not done and the model is solved by integer programming with the original f_{kj} parameters, the production scales will still be the same and all fixed costs will be incurred as required, but the price solution will be quite different and will not have the simple resource-allocating functions ascribed to prices in linear (and generally, in convex) models.[22/]

67. The question now arises: is it possible or useful to recur to the simple analysis of the production-cost for a branch (as shown in Figure 4) in the presence of intermediate inputs? In practice, the answer to this question hinges on the degree of interconnexion between the products. In the metalworking sector the structure of interconnexions is known to be very sparse: the majority of metal-working products are not required in the production of most of the other metal-working products, with a few specific exceptions. Thus semi-fabricates form a chain in which the linkage is highly specific and there are at most a few branchings only: e.g. a given clutch assembly may be used in more than one machine, but it will not be used in all the other hundreds of thousands of metalworking products. Some products are much more widely used, e.g. nuts, bolts, bearings and electric motors. For these we can take advantage of the fact of their wide distribution and relate their requirements to aggregate levels of branches within the sector, rather than linking them one by one to individual products. Thus the input requirements of many metalworking products can be costed out on the basis of reasonable preliminary guesses about the choice between production and imports which can be confirmed or corrected in a second round of calculations.

68. The method of constructing a production-cost profile for each branch is thus a highly useful pragmatic approximation that can relieve the formal programming models of a large fraction of the total burden of a detailed solution. There will be only

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22/ With the f_{kj} parameters re-set by hindsight, the linear programming model will achieve an optimal solution in which all fixed-cost incurrence activities appear with integer (unit) scales without any special mathematical devices to exclude fractional values. Otherwise fractional solutions have to be progressively weeded out by introducing new constraints (for example, Dantzig, 1963, Ch. 26). Each new constraint introduces a new price variable into the solution whose role is ambiguous from the point of view of resource allocation (Gomory and Baumol, 1960).

comparatively few cases in which large interactions between specific products will be evident enough to suggest the need for a simultaneous solution; these parts of the total problem can be relegated to solution by formal integer programming models. After an import-production choice is effected by the latter means, the production-cost profile of each branch can still be traced out for purposes of foreign-exchange allocation (if required) or for a branch-by-branch consideration of extrapolated products.

8. Extrapolated products

69. We can now return to Figure 8 taking into consideration that part of the diagram which refers to the extrapolated products of a branch. With the diagram as drawn, an appropriately high foreign-exchange rate (or low foreign-exchange allocation) will push import substitution within the branch beyond the individually listed products, and will cut into the extrapolated range AB. If (and this is the crucial point) we know the trend of domestic production costs per unit of import value for the extrapolated range, we shall immediately be able to identify the desirable extent of import substitution, AD; moreover, if there is a foreign-exchange allocation (rather than a fixed rate) we can also determine the now variable foreign-exchange rate.

70. First, to pass over a formal point rapidly, the continuous cost trend represented by the curve JFK (Figure 8) might be included in the models as such: this would transform these models into nonlinear programming models, for the solution of which (provided they are still convex, as in the present case) there are several convenient computational methods. For purposes of presentation (and often for computational purposes as well) it is just as satisfactory to approximate the curve JFK by a step function (not shown). The closeness of the fit can be adjusted to the requirements of precision imposed on the model. In Model 1, for example, this approximation involves just four steps. Formally, the cost profile for the branch is then transformed into a step function along its entire length; the steps, however, have a different meaning in the listed-product range than in the extrapolated range. In the latter they play the role of "virtual" products whose number, cost level and step length ("demand") can be adjusted to the requirements of an acceptable fit; in the former, however, they are specific individual products with given levels of demand.

71. How do we derive the extrapolated part of the cost profile, JFK? Within a branch we will be able to list individually those products that are predominant in production or imports. We assume that there is a sharp asymmetry in the frequency

distribution of these individual products, so that if a branch contains, say, 5,000 products, the first 100 or 2 per cent of the products within the branch might represent 60-70 per cent of the total demand. For convenience, in the plot of Figure 8 total demand is expressed at import values, in the form of total foreign-exchange requirements. If we list the first 200 individual products they might cover (hopefully) some 85-90 per cent of the demand within the branch.23/

72. The cost trend of the listed products is assumed to be rising for two fundamental reasons. First, as more and more rare and specialized products are considered for production, their seriality will be correspondingly lower, thereby raising their domestic production costs per unit of physical output, e.g. per ton. Yet this by itself would not be enough to give a rising cost trend (as plotted) per unit of import value if import prices per ton also rose correspondingly. It is, however, a reasonable supposition that import prices will not rise to the same extent, for in most developing countries the internal demand for each of these individual products will be considerably less than the scale at which typical production units within the world market are producing; thus while there is a seriality decrease both in the domestic market and in the world market, this decrease is apt to be sharper within the domestic market. Only the largest industrialized countries can be safely assumed to be exempt from this generalization. Second, as a product becomes more specialized and sophisticated it embodies a larger proportion of higher-grade technical production skills: these again are assumed to be proportionately higher priced in a developing country than in the advanced industrial areas serving the world market. For both of these reasons domestic production costs can be expected to be rising sharper than import prices and thus the cost profile of a branch plotted on the basis of unit import value will also be rising. The monotony of this rise is assured by lining up products in the proper sequence.

73. These considerations apply to listed and extrapolated products. If the most frequent and highest-value products are listed individually and lined up in the order of increasing production costs per unit of import value, there is good reason to expect that the remaining products which are not listed will cost more to produce and will continue approximately the trend observed to the left of point J on the cost profile. (This can of course be subjected to empirical testing in a number of

23/ This is confirmed in the case of one branch - electric motors - for which a pilot study has been undertaken (New School for Social Research, 1967).

concrete individual cases.)[24/] It is more difficult to anticipate how the trend will change as we cut further into the extrapolated range. Figure 8 illustrates a sharp up-turn of the trend as 100 per cent import substitution is approached. This is based on the common sense consideration that 100 per cent import substitution in say, one of the smaller African or Latin American developing countries would involve the production of items such as jet planes whose cost (if one could even speak of a reasonable cost estimate) would certainly be outlandish. In any given concrete instance it should be possible to arrive at a reasonable estimate of the order of magnitude of production costs for groups or classes of products within the extrapolated range by relying on the combined judgement of economists, planners, enterprise managers and other persons familiar with the local economy and in close contact with current operations of the metalworking sector. While this admittedly brings planning for the branch back into the realm of judgement and intuition (from which formal planning techniques were supposedly called upon to rescue it), the range of exclusive reliance on this art has been decisively narrowed none the less, and a much improved foundation has been laid from which judgement and intuition can take their departure. In any event, the break-off point in many branches will come before the extrapolated range is entered, and in those branches where this is not the case an effort can be made to expand progressively the list of individually listed items until the amount of extrapolation is reduced or eliminated.

74. In both Model 1 and Model 2, the activities associated with the step function of extrapolated products are not tied to any fixed-cost activities. This, of course, does not imply that extrapolated products have no fixed costs; on the contrary, high domestic production costs for individual items hinge in many cases precisely on high fixed costs in relation to the length of the potential production run. In the section JFK of the cost profile (Figure 8), however, each individual item contributes only a vanishingly short cost-step of its own, over which (as in the range OA of the same graph) production costs per unit import value are assumed to remain constant, with unit fixed costs at the level determined by total demand for the item. The decision is then whether to produce the item at the full scale of its available demand or not to produce it at all. Due to the very short step associated with each individual item (not with the approximating step function) we do not have to worry about less-than-full-scale production (as in the range OA). Thus fixed costs are

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24/ A similar trend for capital requirements within a branch has been postulated in a model by Chenery, 1955 (see also Chenery and Kretschmer, 1956), based on Italian data.

merged with variable costs over the entire length of the stretch JFK, and when this stretch is approximated by a rough step function, these approximating steps no longer require fixed-cost tie-ins.

75. In Model 3 the balancing of limited resource-element capacities makes it necessary to recur to some estimate of resource-element use by the extrapolated products. Strictly speaking, the same problem occurs also in Model 2 but there it can be by passed by assigning a direct money cost to resource-element-capacity usage, since by assumption such capacity can be provided in this model in any fractional amount required. This is not the case in Model 3, where the demand for certain capacities generated within the extrapolated range of a number of branches, if ignored, might seriously affect the resource-element-capacity balances. All the problems that have been mentioned in connexion with the extrapolation of money costs will be present to an even greater extent when the extrapolation of requirements for individual resource-element capacities and direct material inputs is attempted. All that can be said here is that it is probably best to by pass this problem in deriving a tentative solution, which will then call attention to those resource-element capacities for which an accurate estimate is essential. In a second round, maximum effort can be concentrated on improving the accuracy of the corresponding estimates, and a new solution will accordingly be derived incorporating an allowance for these capacities used by extrapolated products.

9. Organizational resource elements

76. Models 1 to 3 represent an oversimplification in one highly important respect: the omission of organizational resource elements. These consist of groups of engineers, technicians, administrators, market specialists etc., required to undertake the engineering, design, marketing, research and development, planning and administrative functions within individual enterprises and branches of the sector or within the sector as a whole. The exact location of some of these functions is somewhat ambiguous. In every country, even in those with the strongest committment to a market economy, there are important research functions supported by resources in the public domain that benefit many industrial enterprises. For example, in the United States of America the Bureau of Mines has long engaged in industrial research and development work, and of course there are many kinds of public support channelled to the universities that are prime sources of fundamental technological advances. Many of the above functions, on the other hand, are located in individual enterprises.

77. One of the most important attributes of these functions is that for their successful performance they require a group of skilled technicians forming a "critical mass" which has to reach a certain size and diversity before it can properly discharge its functions. For example, the production of agricultural machinery presupposes technical competence in the running of a variety of productive processes, adequate research and design skills, contact with markets and sales channels and so on. There is some flexibility in these requirements: for example, design skills can be replaced by reliance on the licensed production of foreign designs, and the group of skills as a whole can be scaled down if aspirations of meeting world market standards are lowered to simple import-substitution goals. The size of the group, however, is more or less independent of output up to a fairly large total volume and cannot be scaled down in proportion to reduced output needs. Hence the concept of "critical mass".

78. The simplest way of including these functions in the model is to treat them analogously to resource elements that serve specified groups of production activities. Thus a unique organizational resource element may be associated with each branch so that every activity of the branch draws on the capacity of this resource element. The critical-mass aspect can be readily represented by providing for large fixed-resource components tied to the capacity-maintenance activities for these resource elements. This still allows for an arbitrary marginal cost for maintaining larger capacity, and permits a cutoff at some maximum capacity in the same way as has been discussed in connexion with physical processing capacities.

79. With reference to Model 3, rows 8 and 9 can be re-interpreted as organizational resource elements associated with the two branches. If row 8 is to represent the organizational resource element associated with the first branch, then the entries in row 8 between columns 12 and 22 are dropped; analogously, if row 9 is to be associated with the second branch, entries in row 9 between columns 1 and 11 are dropped. Direct material inputs now become irrelevant, and all entries in row 10 are likewise dropped. (For illustrative purposes, we are now assuming that there are no scarce physical-processing resource-element capacities. In a practical model, of course, the organizational resource elements would be added on to the model rather than replacing existing physical-processing resource elements.) Columns 25 and 26 again represent the fixed resource inputs: among these, capital will now play a more subordinate role (associated with such items as typewriters or computers), and the principal entries will be new coefficients in more detailed labour-classification rows (at present there are only two classes of labour, rows 11 and 12). These

fixed labour resources represent the "critical-mass" aspect. In an extreme example, where a technical group of a given size with no expansion at all, can service any volume of production within the branch up to a stated limit g, all resource-requirement coefficients in columns 23 and 24 (which are the variable-cost activities for capacity maintenance) would be zero in the relevant resource rows (here, rows 11-15). In a less extreme illustration it may be assumed that there will be some expansion of the technical group with the volume of production in the branch, making these same coefficients somewhat larger than zero. In both cases g_j represents the capacity limit of the technical group serving branch j; beyond this limit the technical group has to be duplicated rather than being further expanded. All of these aspects are simple extensions of the behaviour of ordinary physical-processing resource elements.

80. It is not necessary to tie technical groups to individual branches. Some may be tied to groupings less comprehensive than a branch, others may interconnect several branches. The principles involved are not affected by these pragmatic variations.

10. Discontinuity and feedback in the models

81. Prior to a final generalization of the concept of resource elements and their interconnexion with the resource concept of the semi-quantitative programming stage, we have now arrived at the point where a crucial programming principle concerning economies of scale and indivisibilities can clearly be set forth. This will be referred to as the principle of minimum unavoidable discontinuity[25]: no variable in a model or in a programming procedure should be treated as a discontinuous (indivisible, integer) variable unless the estimated error committed by treating it as a rounded continuous variable exceeds the permissible error limit.

82. This principle is justified on three grounds. Most obvious, but not necessarily the most important, is the computing aspect. Integer variables impose an utterly disproportionate burden on computing facilities. It is not difficult to find relatively small problems (with about 50 integer variables) that will run for hundreds of hours on the largest available computers without arriving at a precise optimal solution. Since most of the data included in programming models as parameters are subject to considerable errors of their own, it is senseless to insist

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25/ Such a principle is implicit, e.g. in Vietorisz (1965).

on solutions in integers when solutions that are no worse in terms of over-all reliability can be arrived at with greatly reduced effort.

83. The second and more fundamental ground is connected to price mechanism and decentralization. Forced integer solutions (i.e. solutions in which certain variables are forced by special mathematical devices to assume integer values) generally play havoc with the simple resource-allocating functions of a price system. Even after such solutions are arrived at, it is not possible to define prices that will effectively decentralize all detailed decisions without, at the same time, relying on specific quantitative controls that will limit the options open to the decision units. (See for example, the discussion of Figure 9, para. 60-61.) The more variables can be treated as continuous, the fewer will be the instances in which quantitative controls have to play a key role.

84. The final ground, related to the specification of programming models, follows from the discussion of the role of prices in the models. It has become clear while discussing the detailed operation of the models that many aspects of reality can best be approximated by trial-and-error solutions. In other words, there are many feedbacks between the variables and the parameters of the models that could be made endogenous (i.e. modelled explicitly) only at the cost of intolerable complications introduced into the models which would make them next to impossible to compute and would moreover render them utterly opaque to the intuition. Even as it is, these models are complicated enough, requiring a real effort to follow their workings intuitively; yet a model, in the opinion of most practical planners, should never be relied upon unless its workings are transparent enough to be justified at least ex post on a common-sense basis. Now these trial-and-error approaches hinge on preliminary guesses concerning the solutions of the models, guesses which are built into the specifications of the models prior to starting the process of their solution. They almost always involve prices. Thus it is of the greatest importance to safeguard (in so far as possible) the role of prices within the model, even when this is accompanied by some sacrifice in terms of the error committed, rather than to disorganize utterly the simple resource-allocating functions of a price system by insisting on precise combinatorial solutions throughout.

85. In terms of the discussion of the foregoing sections this principle is translated into practice by treating as many as possible fixed-cost incurrence scales as continuous variables. In regard to the production of listed products, the effect of this procedure is to reduce average costs to level steps (such as are shown in

the cost profile of Figure 8) which distribute fixed costs over the largest estimated production scales; and in regard to resource elements, it allows capacity to be provided in exactly the required dosages. The latter is also the case in regard to skilled technical groups that can be treated as "organizational" resource elements. Thus quite early in the practical definition of the models it is convenient to work out trial solutions whose purpose is to segregate approximately the fixed costs that will be treated as continuously-divisible variables from those for which this would result in excessive error. As error estimates are possible only for the model as a whole, this has to be done largely on a common-sense, pragmatical basis by comparing solutions that are optimized in the presence of insufficient restrictions[26/] (and thus contain fractional solution values for inherently integer variables) with other solutions that observe all constraints including those of integrality, but that are not necessarily fully optimal.[27/] The difference between these bounds is an estimator of the over-all error that is being committed. No recognized method exists for estimating errors due to individual variables that are being treated as continuous even though they are inherently of the integer kind; thus the reduction of the over-all error to tolerable limits by a skilful selection of those variables that are de facto treated as integer variables involves a considerable exercise of judgement and skill. Needless to add, the derivation of the multiplicity of trial solutions involved in such a procedure, many of them involving integer variables, is greatly aided by a high-speed computer.[28/]

11. Resource-element definition and linkage to semi-quantitative work

86. So far, the machine park and other characteristics of each resource element in terms of which the models are formulated have been assumed to be exogenously given; yet it is clear that the selection of the proper resource elements according to the

26/ Technically, such solutions are termed as "dual-feasible". Linear programming solutions to integer programming models are always of this kind; so are solutions obtained by certain integer programming algorithms (e.g. the Gomory cutting-plane or all-integer methods: Gomory, 1958 and 1963) when these algorithms are interrupted before they reach the optimal solution. Since the latter converge rapidly to near the optimal solution and slow down more and more the closer they get to it, they are particularly well suited to refining the bound on the possible error that is being committed.

27/ Rounded fractional solutions are always of this kind. There are also other algorithms for identifying near-optimal solutions, for example, steepest-ascent methods modelled on convex programming, "branch-and-bound" methods based on a clever narrowing of potential combinations, and others (for example, Hadley, 1967).

28/ The reader familiar with systems analysis will readily perceive that this approach shows more than a little resemblance to the supposedly quite distinct method of programme choice based on computer simulation.

conditions of development of a given country is a key aspect of the planning of the metalworking sector and should therefore be made an integral part of the programming process.

87. The task of undertaking the detailed specification of resource elements ties the programming models to their earlier stage of semi-quantitative work that serves for preliminary orientation with regard to advantageous new lines of production and new productive facilities within the sector as shown in the report of the New School for Social Research (1967). Given the results of this preliminary work, the task of specifying resource elements can be stated as settling upon precise representations of the more general and comprehensive categories of processing facilities, termed resources, with which the semi-quantitative stage operates. In particular, it is necessary to specify the weight and seriality ranges of workpieces that a resource element can handle; the typical assortments of output that it can produce; the features of local adaptation, such as the degree of mechanization, which depend on the comparative prices of labour and capital; and others. Moreover, these features have to be made concrete by specifying the machine park and the material, labour and other flow input requirements of each resource element.

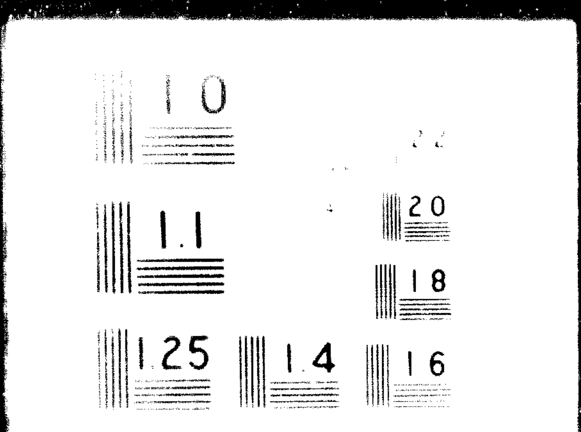
88. At the semi-quantitative stage, programming data are developed which associate individual listed products with inputs of intermediate commodities (subassemblies, components) and with the requirements of processing facilities stated in terms of the resources mentioned above. The available reserve capacities of the latter resources are then surveyed for a given country, and clusters of promising new lines of production are selected by matching the reserve capacities of resources against the processing requirements of various products. If the capacities of proposed new facilities are added to the existing reserve capacities, then the resulting clusters of products, whose production becomes advantageous with the investment in new facilities, can be used to judge the kind of capacity expansion that is likely to be of the greatest over-all benefit from the point of view of the sector or the economy. As some of the most immediately useful empirical information concerning the metalworking sector is now available at this semi-quantitative level, it is all the more essential to connect the modelling stage to the preceding semi-quantitative stage.

89. In general, each resource (casting, forging, heat treatment etc.) of the semi-quantitative stage will give rise to several resource elements at the stage of modelling. Every corresponding resource element will be adjusted to some range of each key processing parameter, usually weight of workpiece and seriality. Given these

OF 3

D O

0653



ranges, it is still necessary to specify the degree of mechanization and the typical output assortment of a resource element before it is possible to proceed with its engineering design and with the specification of its machine park. As many partially overlapping ranges of the key parameters are possible, with many degrees of mechanization and many kinds of product assortments, it is clear that there could be dozens, hundreds or perhaps even thousands of meaningful variants of resource elements associated with a given semi-quantitative resource. If it is desired to keep the number of resource elements within reason, say to a total of perhaps 100 to 150, then only a few major categories of resource elements can be associated with each resource, and the other variants have to be suppressed. The question is how to perform this selection most effectively, without prejudging the entire course of development of the sector.

90. For the sake of illustrating the problems involved, let us begin with a cluster of products that have emerged as promising candidates for further consideration from the semi-quantitative stage. Let us further suppose that the addition of medium-heavy forging capacity, for handling low to medium seriality of output, to the existing light forging capacity would open the door to the domestic manufacture of products within this cluster. At the semi-quantitative stage, however, these preliminary indications have not been translated into quantitative cost estimates, and the precise interrelations between various products and capacities have not yet been explored.

91. One of the first tasks in model building is the specification of the kinds of resource elements which will appear in the model. Thus the immediate question is: how do we proceed from the resources of the semi-quantitative stage to the required resource elements? The question is of decisive importance since the choice of resource elements may prejudge issues related to the degree of capital/labour intensity, local adaptation and the possibilities of keeping up with technological change. Even if the kind of resource can be approximately characterized by weight class (e.g. light, medium, heavy), seriality (low, medium, high, mass) and product assortment (by associating it with a particular cluster of products), the precise machine park included in its design will still depend on the most frequently encountered product weights, the most characteristic lengths of production runs and the nature of the particular products on which price pressure is strongest and for which top efficiency in production is most essential.

92. The problems of resource-element specification are illustrated in Table 3. The candidate products suggested by the semi-quantitative stage are represented here by just four items (A, 75 kilogrammes; B, 150; C, 300; and D, 350), an exceedingly small number as compared with the dozens or even hundreds that might be contained in typical clusters; they will, however, suffice to demonstrate several points. Among the various criteria of resource-element specification, the weight range of products handled by the "medium-heavy forge" has been singled out for attention in this example. It is assumed that, when restricting ourselves to this sole criterion, there is a choice of three variant-resource-element designs, each with a fully specified machine park and flow-input pattern, labelled RES1, ..., RES3. The corresponding assumed weight ranges of output are 50-250 kg, 75-350 kg, and 100-500 kg. The illustrative example contains all three variants in a simplified model patterned on Model 3: it postulates that the alternative variants of resource elements are included side by side in a model of the usual kind. It is emphasized at the outset that this is not intended as the suggested operational approach to the problem; if it were, the models would grow in practice to a size that would deprive them of all usefulness. The purpose of the illustrative example is precisely to point out some available shortcuts.

93. The model in Table 3 is greatly simplified by omitting all except variable capacity inputs for listed products; moreover, variable costs for the resource-element variants are set to zero and thus the latter are characterized by a single fixed cost that yields a specified capacity, in this case 60,000 effective machine hours per year for each of the three variant-resource elements.[29] As usual, these appear in the denominator of a fraction in the tie-in constraints (rows 9-11) connecting the fixed-cost and variable-cost activities for each variant-resource element. Fixed costs are given as a single dollar figure. Since the cluster of products contains candidates for domestic production, imports are omitted. The model is written out with two alternative exogenous demands in columns 17A and 17B. Only one of these demands may be used in deriving a particular programme; the scale of the other must be set to zero.

94. The scales of activities RFX1, ..., RFX3 (columns 14-16) are integer variables. The objective is the minimization of total dollar costs, consisting in this case entirely of fixed costs (row 5).

29/ In continuous operation there are 8,760 hours per year; allowing for maintenance, etc. 6,000 effective yearly hours per machine is a high estimate; our resource elements are set to contain 10 machines each.

Table 3

Resource element definition

		Listed products																	
		A (75 kg)		B (120 kg)			C (200 kg)			D (350 kg)		RES1	RES2	RES3	RFX1	RFX2	RFX3	EXOG.A	EXOG.B
1	A (75 kg)	1	1															[illegible]	1000[illegible]
2	B (120 kg)			1	1	1												[illegible]	[illegible]0000
3	C (200 kg)						1	1	1									[illegible]	30000
4	D (350 kg)									1	1							[illegible]	[illegible]
5	$, Million														-1	-1.[illegible]	-4		
6	RES1 (50-250 kg)	-15		-20			-25					1							
7	RES2 (75-350 kg)		-10		-15			-20		-[illegible]			1						
8	RES3 (100-500 kg)					-3			-5		-[illegible].5			1					
9	RFX1											$-\frac{1}{[illegible]}$			1				
10	RFX2												$-\frac{1}{[illegible]000}$			1			
11	RFX3													$-\frac{1}{[illegible]}$			1		
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17A	17B

Table 3 (continued)

Continuous solutions

Product	Plant	Exog. A				Exog. B			
		Hours	Scale	Cost	Cost	Hours	Scale	Cost	Cost
A	1	15,000	.250	.250		150,000	2.500	2.500	
	2	10,000	.167	.250	.250	100,000	1.667	2.500	2.500
B	1	400,000	6.667	6.667		4,000,000	66.667	66.667	
	2	300,000	5.000	7.500		3,000,000	50.000	75.000	
	3	60,000	1.000	4.000	4.000	600,000	10.000	40.000	40.000
C	1	75,000	1.250	1.250		750,000	12.500	12.500	
	2	60,000	1.000	1.500		600,000	10.000	15.000	
	3	15,000	.250	1.000	1.000	150,000	2.500	10.000	10.000
D	2	5,000	.083	.125		50,000	.833	1.250	
	3	1,500	.025	.100	.100	15,000	.250	1.000	1.000
					5.350				53.500

Rounded solutions

Plant	Continuous	Rounded	Cost	Continuous	Rounded	Cost
1	.250	1	1.000	–	–	–
2	–	–	–	1.667 a/	2	3.000
3	1.275	2	8.000	12.750	13	52.000
			9.000			55.000

a/ Alternative: # 1 at a scale of 2.50, rounded to 3, same cost.

95. It is immediately clear that this particular numerical example is dominated by the very large demand for product B[30] in both exogenous demand structures. Working with the exogenous demand structure A, the 20,000 units of demand for product B can be translated into individual variant-resource-element requirements which are calculated as 400,000, 300,000 and 60,000 hours respectively, or (in fractions of total capacity) as 6.667, 5.000 and 1.000. Thus it would take 6.667 replications of resource-element variant 1 to produce product B; or 5 replications of resource-element variant 2; or just one embodiment of resource-element variant 3. Comparing fixed costs, the total for resource-element variant 1 would be (6.667).(1.0) if fractional facilities could be built; in reality, however, the next larger integer number would have to be installed, i.e. 7. Due to the utter simplicity of interrelations within the model, similar calculations can be performed for each product resource-element-variant combination independently of all the others. These are summarized both for the fractional solutions that suppose the resource-element-variant scales to be continuously variable and for the rounded solutions. For exogenous demand structure A the continuous solution (which is sure to be an underestimate) is 5.35, while the rounded solution (which may be an over-estimate) is 9. Thus the maximum possible error is 3.65 units or 68 per cent of the lower bound. For exogenous demand structure B the continuous solution, 53.5, and the rounded solution, 55, together offer a far more favourable error bound: 1.65 units or some 3 per cent of the continuous cost estimate.

96. The optimal solution for exogenous demand A can be derived by hand, by the simple device of enumerating all plant combinations, starting with single plants (resource-element-variant capacities of unity taken individually) up to three plants (where the sums of resource-element-variant capacities are equal to 3). There are three single plants, six double combinations and ten triple combinations. Each of these can be rapidly checked to verify if the plant or plant combination suffices for servicing the stated exogenous demands of all four products. In this way the

30/ The demand for product B can be said to be "large" because the input requirements of product B are of the same order as those of the other products; thus the large number describing the extent of demand is not counteracted by an unusually small product size (or value).

combination (1,2,3) emerges as the best one with a total fixed cost of 6.5 units,31/ i.e. substantially below the cost of 9 estimated by rounding off the fractional solution.

97. What does this illustrative example demonstrate? Depending on the structure and size of demand, one particular resource-element variant among a number of closely related variants can dominate to such an extent that it practically eliminates the other variants from further consideration. With exogenous demand structure B, this is the situation with resource-element variant 3 of which 13 replications are required. The scale of this resource-element variant can be treated as a continuous variable, for any error introduced by so doing will evidently be small. Resource-element variants 1 or 2 can be retained for producing product A, but if these are dropped and product A is eliminated from further consideration, this is probably just as favourable a practical alternative. A final verdict depends on import prices, but at this stage of model specification it appears justified to concentrate on the investigation of the opportunities offered by resource-element variant 3.

98. The situation is more difficult with exogenous demand structure A. Resource-element variant 3 predominates here also, for if it were dropped, the production of product B alone would require a minimum (fractional) fixed cost of 6.667. There are, however, much more advantageous alternatives than sole reliance on this resource-element variant, since the latter would still fail to provide for the production of A, besides being a high-cost alternative. A combination of 3 and 2 appears favourable. It is low in cost (5.5 units) and drops the production of A and D, both of which are low in demand; this may be acceptable in practice.32/

99. The question is if, when confronted with the results of semi-quantitative programming work, it is necessary to recur to the use of this kind of model in making decisions about resource element specifications. It is suggested that this is not

31/ Any combination not containing resource element 3 can be immediately excluded, since the other two resource elements require huge capacities for producing product B. This eliminates 9 of the 13 combinations preceding combination (1,2,3) where these combinations are ordered in terms of ascending fixed costs. The remaining four are eliminated on almost as simple criteria. Since (1,2,3) with a cost of 6.5 proves feasible, subsequent combinations with higher costs need not be tested at all.

32/ The combination (3,2) is of course not a solution to the problem as stated in the model.

the case. In general, such models (of the type of Model 3) with full allowance for fixed costs for listed products and variable costs in representing the economies of scale of resource-element capacity maintenance activities, plus all the other complications discussed in earlier chapters, would become unworkable if exhaustive variations of all resource elements were included in them. First of all they would become entirely too large, and second, they would impose an impossible data-collection burden in calling for scores or hundred of variant-resource-element designs with fully specified machine parks and flow inputs. This road clearly leads to a dead end.

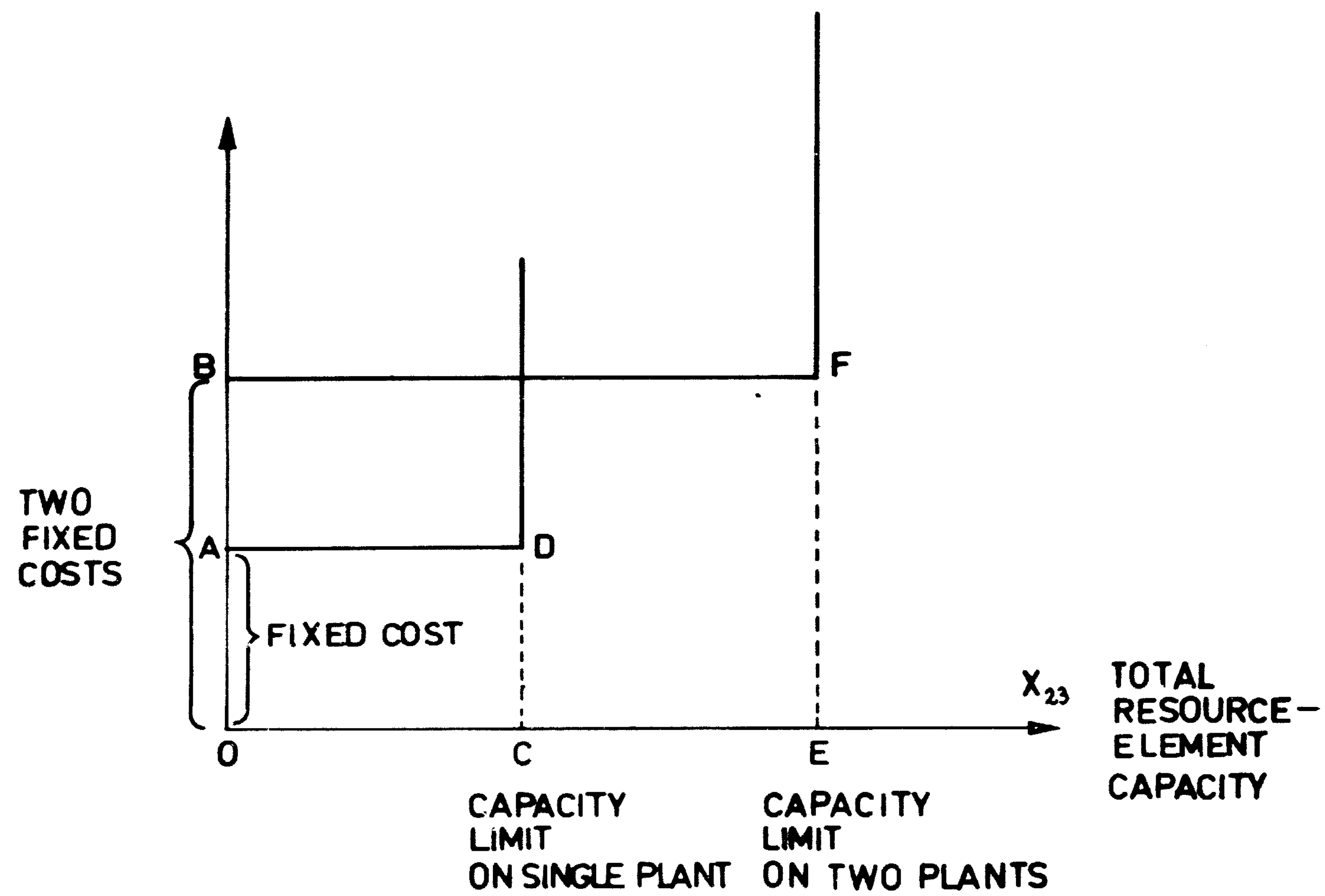
100. The model of Table 3 indicates, however, that an approximate approach can be defined in working towards reasonably specified resource elements. Some of the features introduced in Table 3 for numerical simplification can now be postulated deliberately for purposes of approximation. Thus resource-element variants can be characterized in an approximate way by their estimated annual fixed costs and capacities, without a closer description of their economies of scale and without engaging in detailed design and specification of machine parks, input flows and so on. On this basis individual products can be tested one by one to estimate which resource-element variant would allow them to be produced at lowest annual charge attributable to processing-capacity maintenance. In making this estimate fixed and variable capacity requirements of products (i.e. the coefficients $\underline{c}$ and $\underline{\bar{c}}$ in Model 3) may be merged into appropriate total capacity requirements at the level of demand given; direct material inputs and inputs of intermediate commodities can be ignored, as these typically have little influence on the choice of a resource-element variant; the variable inputs of resource-element-capacity-maintenance activities (of the type RES1) can likewise be ignored, since these reflect not a tradeoff of fixed against variable costs in production, but economies of scale in regard to the resource elements.33/ Thus the choice hinges on what it costs to maintain on an annual basis the fraction of the fixed capacity of the resource element that the given product actually requires. This fixed annual cost, it should be recalled, includes not only capital charges, but also labour and indirect material costs attributable to running the resource element at its capacity limit for a whole year.

101. Figure 10 illustrates the meaning of assuming zero variable costs in resource-element-capacity maintenance. This figure corresponds closely to Figure 3 except that the supply elasticity for resource-element capacity is assumed to be infinite

33/ If the data permit, the latter economies of scale can, however, be allowed for.

Figure 10

Resource element with zero variable costs for capacity maintenance



at the stated fixed level up to the capacity limit, at which point it becomes zero (the supply line shoots up vertically). The facility can be duplicated: this implies double fixed cost and double capacity limit. The variable on the horizontal axis is designated as x_{23} by reference to the numbering of activities in Model 3; correspondingly the scale of fixed-cost incurrence is x_{24}. This is the variable required to assume integer values in any feasible solution; fractional values appearing in a so-called continuous optimal solution always imply an underestimate of costs.

102. With the staged assumptions the approximation is closely analogous to the numerical illustration given in Table 3, and thus the individual testing of products against each separate resource-element variant derives its justification from the properties of this illustrative model. So long as demand is large enough to result in multiple facilities, the rounded continuous solutions are likely to be good approximations to the optimal integer solutions, and one-by-one testing is a reliable approximation. If this is not the case, one-by-one testing can lead to a larger error. Even in the more difficult case one-by-one testing can be used to weed out the less favourable variants. After this is accomplished, one can either try to obtain an improved estimate by setting up a small preliminary integer programming model such as shown in Table 3, or else one can include a very limited number of the most strategic resource-element variants as explicit alternatives in the specification of a major programming model of the type of Model 3.

103. If need be, perhaps as a result of a critical shortage of cost information on resource-element variants, the entire procedure described above can be further shortcut by simply matching product demands against ranges of resource-element characteristics, e.g. weight or seriality ranges. Then each resource element can be defined in such a way as to be centred on the most frequently occurring weight, seriality etc. ranges; jointly the resource elements have to cover the entire range of variation under consideration, even if in the course of programming some of these resource elements are eliminated as candidates for investment. This shortcut will not help in deciding on optimal degrees of mechanization in response to capital/labour price variations and on other issues of local adaptation whose study can be subsumed under the more elaborate preliminary estimates that have been discussed above; if the shortcut is resorted to, these issues have to be decided either on an intuitive basis or by means of special side studies.

104. In sum, the formulation of programming models such as Model 1 to 3 presupposes that the resource elements, in terms of which the models are formulated, are specified in advance. The task of undertaking this specification ties the programming models to the earlier stage of semi-quantitative work. Given the results of this preliminary work, the task of specifying resource elements can be stated as settling upon specific representations of the resources with which the semi-quantitative stage operates. Resources have to be tied down to detailed weight and seriality ranges, product assortments and other features of local adaptation such as the degree of mechanization. These features finally have to be made concrete by specifying the machine park and flow inputs of each resource element. In making the appropriate selection, it is necessary to recur to approximation procedures that either result in the direct specification of the resource elements or at least cut the number of alternatives to the bone. Two simple procedures have been suggested which offer a way of bridging the gap between the semi-quantitative and the model-building stages of programming.

12. Global-sectoral decomposition models

105. The relationship between sectoral and economy-wide programming can be studied by a variety of analytical devices including input/output tables, linear programming models and computer simulation. We shall choose a linear programming framework for the present discussion, since it is particularly well adapted to an intuitively clear presentation. We shall work with Dantzig-Wolfe type decomposition models (Dantzig and Wolfe, 1961) that bring out the key features of sectoral-global interrelations.

106. Table 4 presents the parameters and summarizes the key features of an illustrative global model with two different sectors. The organization of this model follows the principles laid down in Chapter 2 (see especially Table 1).
The distinguishing feature of the model is that the parameters of the two sectors follow a block-diagonal format (rows 3-6) except for the presence of two rows (rows 1-2) in which both sectors have entries. These interconnecting rows represent requirements of the primary factors, labour and capital; the remaining rows represent balances of goods that are specific to one of the two sectors. The activities of the model are production activities except for the last one which is the exogenous activity.

Table 4

An illustrative two-sector Dantzig-Wolfe type decomposition model (linear programming)

(a) Presentation of parameters

	SECTOR 1				SECTOR 2					
	Good A		Good B		Good C		Good D		Exogenous	
1	- 1.1	-1.25	- .3	-2.5	- 1.0	-2.5	- .6	- 3.0	350	Capital requirements
2	-12.5	-7.5	-6.0	-7.0	-15.0	-5.0	-4.0	-11.0	2,000	Labour requirements
3	1	1	- .5	- .2					-50	Good A balance
4		- .25	1	1					-50	Good B balance
5					1	1	- .8		-25	Good C balance
6					- .2	- .5	1	1	-25	Good D balance
	1	2	3	4	5	6	7	8	9	

(b) Listing of feasible basic solutions for each sector

		Sector	Activities and scales			Labour requirement	Capital requirement
Complex	A	1	1,3	75.0	50.0	-1,238	- 98
	B	1	2,3	85.7	71.4	-1,071	-129
	C	1	1,4	60.0	50.0	-1,100	-191
	D	1	2,4	63.2	65.8	- 934	-243
	E	2	5,7	53.6	35.7	- 946	- 89
	F	2	5,8	25.0	30.0	- 705	-115
	G	2	6,8	25.0	37.5	- 788	-138
	H	2	6,7	75.0	62.5	- 625	-225

107. Such a model is not fully general, since in practice the separation of sectoral coefficients into blocks is not quite as neat as this model would have it. The usual structure of input-output models, for example, leads us to expect an approximately triangular structure rather than the diagonal structure of this model. None the less, the simple properties of this model serve as a suitable take-off point for later modifications.

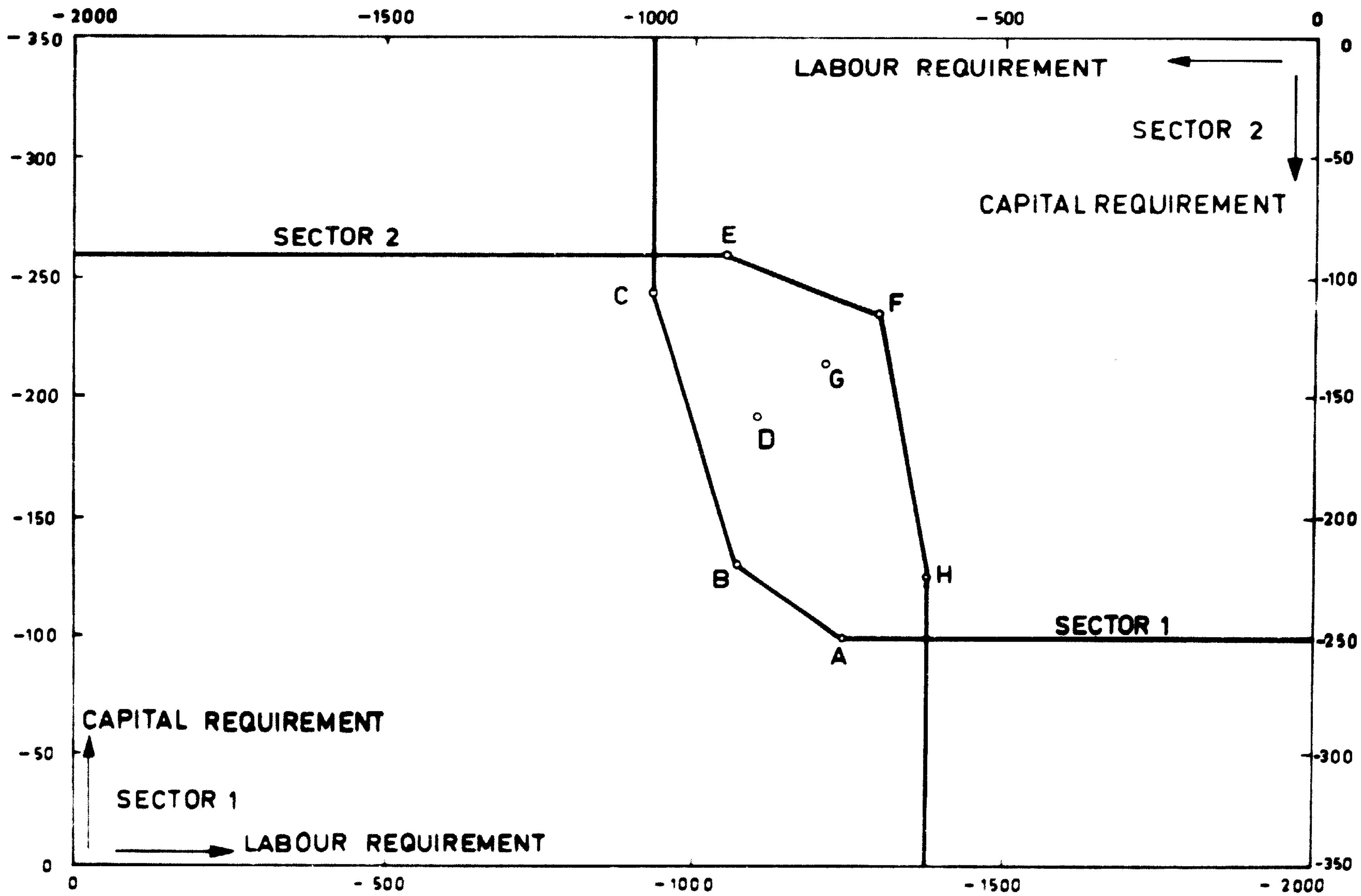
108. Table 4 lists eight different complexes which can be formed out of the production activities of the model. These complexes represent different ways of satisfying the final demands of each sector by using the least number of different production activities. The final demands are represented by negative entries in the exogenous column. For example, in Sector 1 the final demands for goods A and B are 50 units each. Good A can be produced by activity 1 or 2 and good B, by activity 3 or 4. Thus we must have at least two activities to satisfy the final demands of these two goods, and we can choose 1 and 3 (complex A), 1 and 4 (complex B), 2 and 3 (complex C) or 2 and 4 (complex 4). A similar choice can be made in Sector 2. Note that while we cannot satisfy sectoral final demands by a single activity, we could use, if we wanted, three or even all four activities of a sector for doing so. However, the restriction of each complex to two activities results in the simplest kind of productive structure. Technically, these complexes are referred to as feasible basic solutions to the sectoral sub-problems.

109. Figure 11 has been drawn to scale from the data of Table 4. Each complex is drawn as a point representing capital and labour input requirements, but these requirements are measured from the opposite corners of a box. Thus Sector 1's capital and labour needs are measured from the south-west corner and Sector 2's corresponding requirements from the north-east corner. The dimensions of the box represent combined economy-wide availabilities of the two factors. Such a diagram is known as an Edgeworth-Borley box diagram and is particularly well suited for studying the allocation of the two factors among the sectors of our illustrative, highly simplified economy. Clearly, joint allocations of either factor to the two sectors should remain within the limits of total factor availability.

110. The complexes are used to define sectoral isoquants in the diagram. These are obtained by interconnecting efficient complexes. G and C are not efficient, because they use more capital and labour than some other efficient point. The economic meaning of the connecting line is a production structure obtained by averaging complexes with different weights. Thus the midpoint between B and C is a 50-50 average

Figure 11

Interrelation of sectoral insoquants based on an illustrative decomposition model



of complex B and C, while the point 3/4 of the distance from F to H is an average consisting of 3/4 H and 1/4 F. The averaging pertains not only to factor inputs, but also to all other characteristics (e.g. activity scales) of the complexes. Beyond the averaged complexes, the isoquants are extended horizontally and vertically, e.g. to the right of A. These extensions can be interpreted as signifying factor-disposal activities, e.g. the extension to the right of A consists of a combination (not an average) of complex A with varying amounts of labour disposal, i.e. labour allocated to Sector 1 and left unutilized.

111. The sectoral isoquant is a generalization of the conventional production function to the operations of an entire sector. All points on the isoquant are in a sense equivalent, since they represent the job performance of the sector. In the present case this means meeting sectoral demands as specified by the exogenous activity; in other cases it might also mean remaining within specific sectoral supply or capacity limitations (which can also be specified in the exogenous activity). Thus the isoquant is a device for suppressing information with regard to sectoral detail such as precise activity levels, while conveying information on two items vital for analysing the global interrelations between sectors: first, the quantitative requirements (or supplies) of the basic resources interconnecting the sectors, in the present case of labour and capital; and second, the implied information that all underlying sectoral demand, supply or other types of specific balances and limitations are met.

112. Points that are not on the isoquant itself differ sharply in interpretation according to the side of the isoquant on which they appear. Points on the hollow side (i.e. above the isoquant of Sector 1 and below that of 2) are feasible from the technical/economic point of view but not efficient, since one can always find some point on the isoquant that uses less of both labour and capital. Conversely, the points on the other side of the isoquant are not attainable, since they signify a smaller use of labour and capital than some point on the isoquant that represents the best possible combination.

113. Global resource allocation can be represented in the diagram by choosing one allocation point for each sector and by checking whether the resulting allocation is (a) attainable and (b) efficient. For example, if we pick point B for Sector 1 and F for 2, the resulting allocation is efficient at the level of each sector and also globally attainable, since it uses less than the available amount of labour or capital. In order to specify the criterion of global efficiency, we have to select

an objective function. In Table 4, for example, the top row (capital) of the model can be selected as the objective-function row; the objective then is to maximize the surplus (minimize the use) of capital. In Figure 11 this objective can be served by maximizing the vertical separation between sectoral allocation points. B and F are clearly not the most efficient combination for this purpose; upon inspection the A-F combination is found to be more efficient. Note, however, that both the B-F and the A-F combinations leave some labour unutilized. Intuitively it is attractive to attempt full labour utilization for greatest efficiency in capital use. To this end one wishes to select two points that are precisely vertical, since the sum of labour allocations will then exhaust the available global supply. When one passes a ruler across the diagram while keeping it in the vertical position, the largest vertical separation is attained when the line passes through A and cuts the D-F segment of Sector 2. If minimizing the use of capital is the criterion, the optimal solution is the use of complex A for Sector 1 and a weighted average of D and F in Sector 2. We find a similar optimum for a different objective function, e.g. the minimization of labour; in the latter case we maximize the horizontal separation between allocation points and so on.

114. The geometric representation given above is intuitively transparent and is an excellent way of drawing attention to some of the key conceptual problems which arise in embedding sectoral programming decisions in an economy-wide resource allocation context. Yet it has the drastic limitation that the number of sectors and the number of connecting resources are restricted to two. For practical programming purposes we have to rely on a purely algebraic formulation of the model, a formulation that generalizes to any number of sectors and connecting resources. The Dantzig-Wolfe algorism is such a general, efficient computing device for obtaining rapid exact solutions. The need for some such computing device is all the more essential as generally it is impossible to proceed by the enumeration of sectoral complexes, whose number rises combinatorially with the number of distinct activities in the sectoral subproblems. Thus, instead of defining sectoral isoquants and relating them to each other, the general computational method takes a drastic shortcut that makes it unnecessary to enumerate specifically any but a few very highly selected sectoral complexes.

115. The basis of this shortcut is the definition of sectoral subprogrammes that are used to locate previously unknown sectoral complexes along the sectoral isoquants. Thus we can start with just a handful of complexes (e.g. one per sector;

but methods exist for locating even these starting complexes if they are not initially known) and additional complexes will accumulate in the course of the calculation. The sectoral isoquants can be thought of at all times as being approximated by connecting the already known complexes. Even a single complex will give an approximation of a sectoral isoquant: for example, when only complex B of Sector 1 is known, the estimate of the sectoral isoquant is an L-shape with the point of the L located at B. It is clear that approximations of the isoquant based on omitting some of the complexes will always be entirely on the attainable (technically feasible) side of the correct isoquant; thus they will be composed of feasible but generally inefficient points. The approximation improves as more complexes are identified; eventually, the method locates all complexes needed for defining the correct isoquants in the neighbourhood of the optimum. The shortcut consists in not identifying the overwhelming majority of the complexes in those regions of the correct isoquant that are not required for defining the optimum.

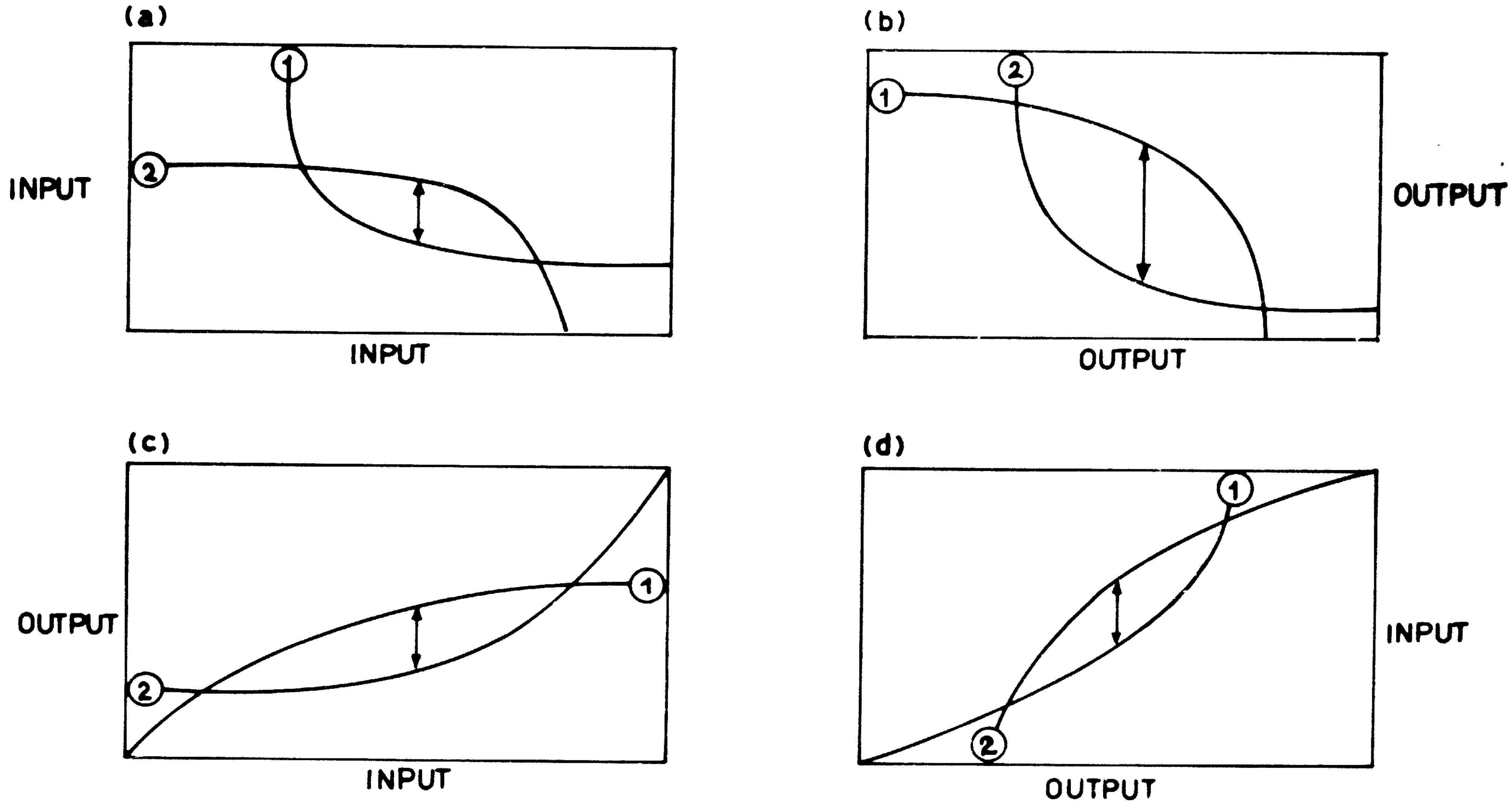
116. The sectoral subproblems used for locating new complexes consist of the intersection of the columns of the sector with the rows of the sector plus the interconnecting rows. Thus the subproblem for Section 2 consists of the intersection of columns 5-8 with rows 1-2 and 5-6 (Table 4). The intersection with rows 5-6 indicates the constraints specific to the sector, while the intersection with rows 1-2 is used for defining a sectoral objective function which consists of a weighted sum of these rows. Since the coefficients in these interconnecting rows represent primary factor requirements (labour and capital) we can interpret the weights as prices and regard the weighted sum as the total factor cost for the given sector. Thus the objective of the sectoral subprogrammes is the minimization of factor costs within each sector at given factor prices.[34/]

117. These factor prices, and the new complexes that are found in the course of sectoral suboptimization, form the connecting links of the entire computational procedure. At each step we determine a provisional optimum within the Edgeworth-Bowley box diagram, based on the estimated isoquants that connect the complexes already identified within each sector. The provisional optimum also identifies a corresponding set of factor prices. These prices are then used as weights for the objective functions of sectoral suboptimization problems. The latter identify new complexes which allow for a closer estimate of the sectoral isoquants, and the whole procedure

34/ In the two-dimensional diagram (Figure 11) the labour/capital factor price ratio is interpreted as the slope of a budget line connecting points of equal factor cost.

Figure 12

Sketch diagrams of two-sector decomposition models with two interconnecting resources. In each case, the surplus of the vertical dimension of the box is assumed to be maximized. The origin for Sector 1 is always the SW corner, for Sector 2 the NE corner. The connecting lines are in each case labelled with the number of the sector



is repeated. As long as the estimates of the isoquants keep improving, the provisional optimum also improves. The procedure ends when no further improvement is possible.

118. In the present illustrative case both of the interconnecting resources represented primary inputs. This need not always be the case. Figure 12 indicates the schematic structure of input-input, output-output and input-output combinations, where the segmented connecting lines are replaced by smooth curves. The first of these sketch diagrams corresponds to Figure 11. In this case the sectoral connecting lines correspond to the isoquants of neoclassical production functions. If both connecting resources are outputs, the sectoral connecting lines have the shape of neoclassical production possibility curves; in the case of one output and one input, the lines correspond to neoclassical factor input-product output diagrams, best known from applications to linear homogeneous production functions.

13. An illustrative model for the metalworking sector in an intersectoral decomposition system

119. Table 5 presents the parameters and other pertinent information relating to a decomposition model (discussed in para. 105-118) but specifically aimed at the requirements of the metalworking sector. This model is intentionally kept to a very small size to permit the gradual introduction of additional complexities.

120. The model shows only two products designated as goods 1 and 2, for the sector. Each of these may be imported or domestically produced. Required inputs are capital for domestic production and foreign exchange for imports. The import activity is reversible: in other words, it is assumed that at the prevailing world market price it is not only possible to import a commodity, but also to engage in negative imports, i.e. to export this commodity. This export opportunity, however, depends on a special quantitative limitation for each good, given in rows 5-6. The logic of the model permits the ready addition of further export activities at lower export prices: this procedure represents export demand in the form of a step function, with prices falling stepwise as exported quantities increase.

121. The complexes characterizing the model are given also in Table 5. There are only two significant alternatives for each good: it should be imported entirely or produced domestically and exported. The alternative of domestic production without any exports or imports is subsumed in the former alternatives. Thus there are four complexes that have to be considered individually. The capital (K) and foreign-exchange (FE) requirements of each complex are broken up by product.

Table 5

A small illustrative model for the metalworking sector in an intersectoral decomposition system

	Good 1: Product	Good 1: ± Imports	Good 2: Product	Good 2: ± Imports	Exo-genous	
1	1	1			-80	Product 1 balance
2			1	1	-30	Product 2 balance
3	-20		-15			Capital requirement
4		-10		-20		Foreign exchange balance
5		1			20	Export limit, good 1
6				1	6	Export limit, good 2
	1	2	3	4	5	

	Imports	Exports	FE(1)	FE(2)	Σ FE	K(1)	K(2)	ΣK
Complex A	1,2	-	-800	-600	-1,400	0	0	0
B	1	2	-800	+120	-680	0	-540	-540
C	2	1	+200	-600	-400	-2,000	0	-2,000
D	-	1,2	+200	+120	+320	-2,000	-540	-2,540

122. For the purposes of intersectoral analysis, capital and foreign exchange are regarded as the interconnecting resources; accordingly, the complexes have to be represented in an Edgeworth-Bowley type diagram with capital on the vertical and foreign exchange on the horizontal axis. The usual diagram, however, has to be slightly modified since both net surpluses (exports) and net deficits (imports) of foreign exchange in each sector have to be given consideration.

123. Figure 13 presents the corresponding diagram. Complexes A, B and D are found to be efficient. Note that the right-hand side of the diagram corresponds to the isoquant configuration such as that shown in Figure 11 or diagram (a) of Figure 12, while the left-hand side of the diagram corresponds to the input-output configurations given in diagrams (c) and (d) of Figure 12. The vertical line supplied as an extension from point D upward corresponds, as usual, to disposal or non-utilization of a factor, in this case of capital. Note that resource requirements are denoted by negative numbers; Figures 11 and 13 carry such negative numbers on the customary horizontal and vertical axes. In Figure 13, the opposite direction of the horizontal axis becomes meaningful and carries positive numbers to denote net foreign exchange outputs.

124. In Figure 14, a sketch diagram is given indicating the interaction between the metalworking sector (Sector 1) and the rest of the economy. It is assumed that the rest of the economy, like the metalworking sector, is capable of operating either on a net import or on a net export basis; thus the course of the sectoral connecting line for Sector 2 is similar to that for Sector 1. It is further assumed that a net positive balance of foreign exchange representing a net inflow of foreign investment, is available to the economy. This is the extent to which imports as a whole (foreign-exchange requirements) are permitted to exceed exports as a whole (endogenous foreign-exchange supplies). This net foreign investment defines the horizontal dimension of the Edgeworth-Bowley box; however, in the present case, the box is extended beyond the usual corner at which the net resource input of a sector becomes zero. The vertical dimension of the box is normal and represents the availability of capital In order to achieve the greatest possible capital economy, Sector 1 has to operate in the present case on a net export basis (see the position of the vertical arrow between the two sectoral lines) while Sector 2 operates on a net import basis. If net foreign investment becomes zero, the horizontal dimension of the Edgeworth-Bowley box collapses to a point, but the extensions to the net-export regions still operate; in the latter case it can thus be decided which sector will be a net exporter; the other sector will then have to have an equal net import. Finally, the diagram can also accommodate net foreign investment (capital outflow): in this case the origin for

Figure 13

Diagrammatic representation of a small illustrative model for the metalworking sector

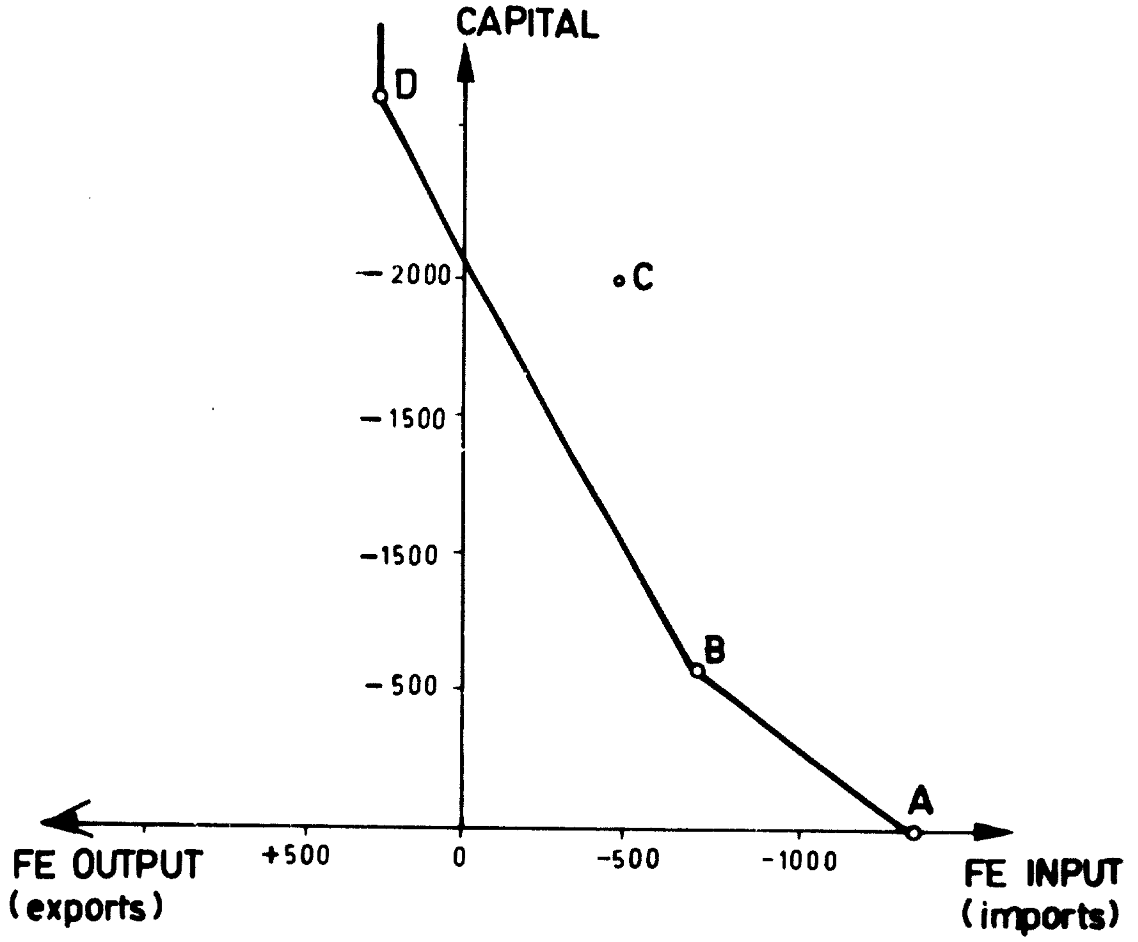
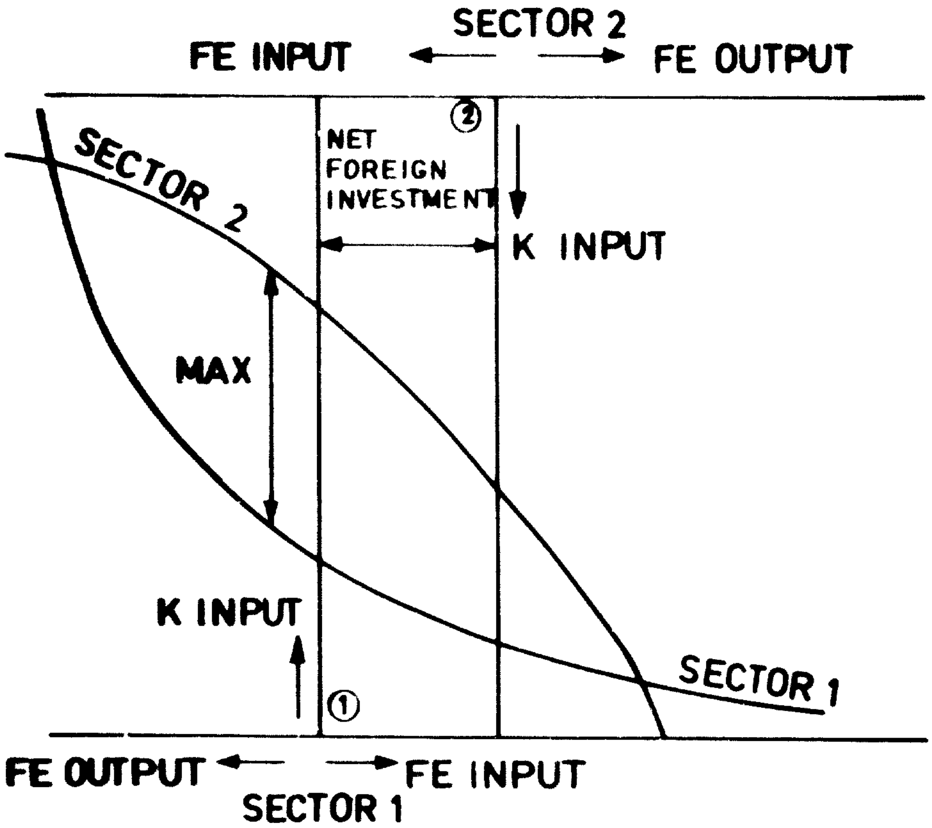


Figure 14

Sketch diagram of two-sector decomposition model where each sector may run either a net export or a net import surplus. Sector 1 corresponds to the numerical model diagrammed in Figure 13



Sector 2 (the NE corner of the box) moves to the left until it passes beyond the origin for Sector 1 (the SW corner of the box). The box thus has a negative horizontal dimension. This may appear unusual but is in fact very simple to interpret: either both sectors must have net export surpluses, or the net export surplus of one of the sectors has to be large enough to offset the net import surplus of the other and still leave sufficient foreign exchange to comply with the exogenous foreign-exchange requirement. In each of these cases the diagrammatic representation of a single sector follows the principles illustrated by the small numerical model, and the determination of the optimal position invariably rests on the identification of the vertical cut that gives the largest separation between the sectoral connecting lines.

14. Fixed costs in the model

125. We are now ready to introduce elements of indivisibilities and economies of scale into the model. These features are indispensable if the model is to be made realistic, since they play a key role in planning decisions affecting the sector. Some of their effects, at the level of planning within the sector, have already been discussed; it is, however, essential to give such a discussion more depth by placing it in a context that takes inter-sectoral relationships into explicit account.

126. The simplest way of introducing indivisibilities and economies of scale into the model is by adding fixed costs. This leaves the formulation of the model in linear programming notation essentially unchanged, except for the fact that some variables are restricted to integer values (Chapter 3). In the case of the small illustrative model for the metalworking sector, which we have just developed, we add the following parameters:

	Capital	Foreign exchange	Capacity limit
Fixed cost for production of good 1	200	200	160
Fixed cost for production of good 2	500	0	80

127. The model with these additions is shown in Table 6. The definition of fixed-cost activities and tie-in constraints follows exactly the principles laid out in previous chapters. The scales of activities 2 and 5 are integer variables.

128. The effect of the fixed costs is an increase of the capital and foreign-exchange requirements of the sector in a way that depends on the productive structure. Referring to Table 5, the listing of complexes indicates that complex A consists entirely of import activities, with no production within the sector; this

Table 6

A small illustrative model for the metalworking sector in an intersectoral decomposition system with fixed costs

	Good 1			Good 2				
	production		±	production		±	Exo-	
	variable	fixed	imports	variable	fixed	imports	genous	
1	1		1				-80	Product 1 balance
2				1		1	-30	Product 2 balance
3	-20	-200		-15	-500			Capital requirement
4		-200	-10			-20		Foreign exchange balance
5							20	Export limit, good 1
6						1	6	Export limit, good 2
7	$-\frac{1}{160}$	1						Fixed-cost tie-in, good 1
8				$-\frac{1}{80}$	1			Fixed-cost tie-in, good 2
	1	2	3	4	5	6	7	

complex will thus be unaffected by the introduction of fixed costs in production. Complex B, however, contains a productive activity for good 2 and accordingly has to incur the fixed cost for this activity, given as 500 units of capital. Thus the capital requirement of complex B rises from 540 to 1,040 units, while the foreign-exchange requirement remains the same.

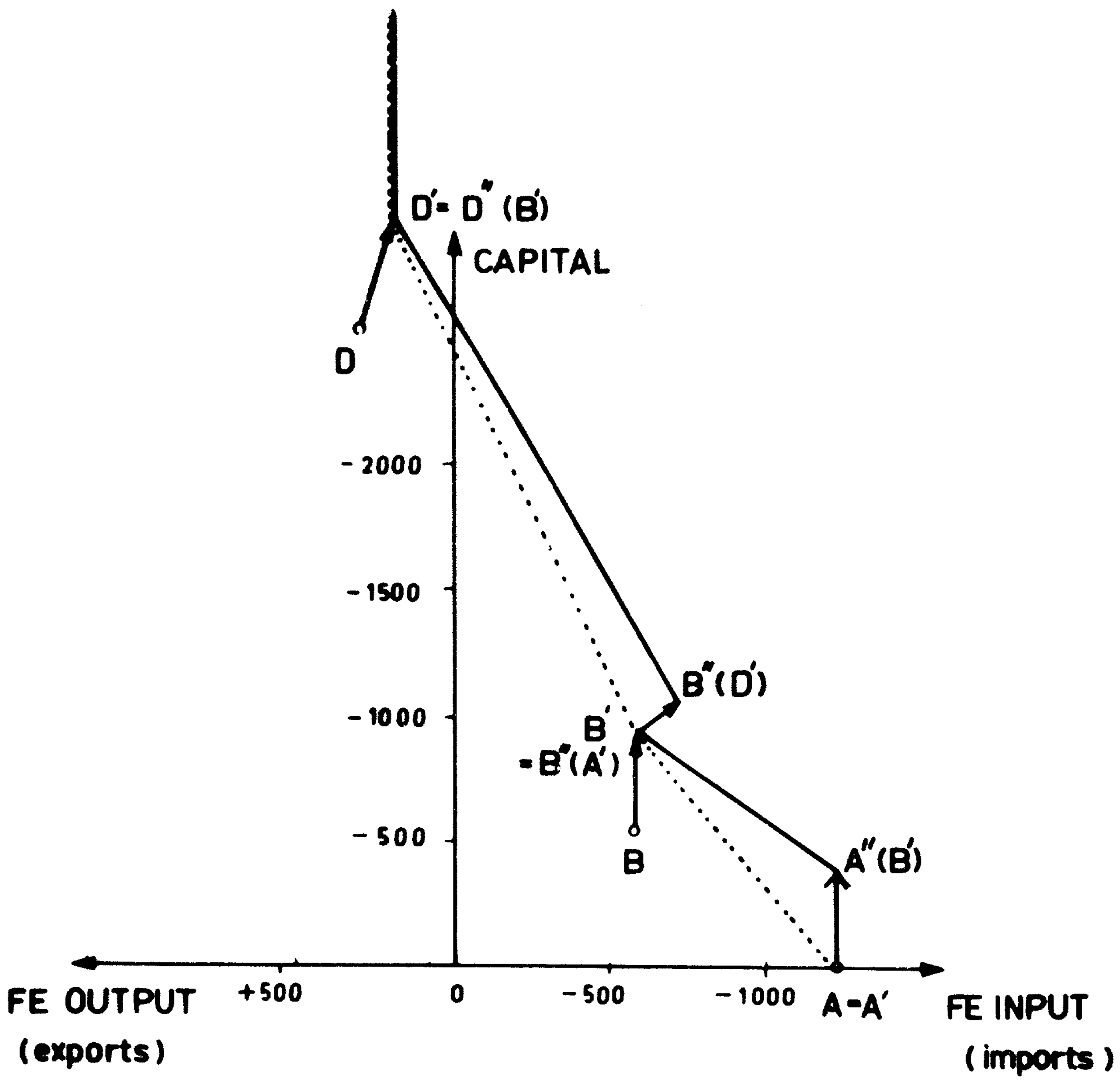
129. In Figure 15 the new complexes including the specified fixed costs are designated by A', B' and so on. (Ignore for the moment A", B" etc. occurring in this figure.) It can be seen that A coincides with A', while B' (designated by an arrow) is obtained from A by a vertical movement of 500 units, corresponding to the fixed capital requirement. Referring back to Table 5, complex D has both productive activities and is therefore assigned both fixed costs. These fixed costs add up to 700 units of capital and 200 units of foreign exchange. In Figure 15, accordingly, we move up vertically by 700 units (capital) and to the right by 200 units (foreign exchange) to obtain D' from D.

130. It may be thought that we can now connect the new complexes A', B', D' to get a new isoquant; this is not, however, the case, since it would imply averaging the fixed costs. For example, the midpoint between A=A' and B' would imply incurring half the total resource requirements of B'; this incurrence is all right as far as the variable costs are concerned, but the fixed costs have to be incurred once and for all as soon as there is any production of good 2. Yet the averaging procedure falsely assumes that the fixed costs, as much as the variable costs, can be cut in half in the latter case: in other words, this kind of averaging treats fixed-cost incurrences not as integer but as continuous (divisible) variables and will therefore be referred to as continuous averaging, and the resulting isoquant will be referred to as the continuous isoquant. In Figure 15 the latter is denoted by the dotted line connecting complexes A', B', D'.

131. For correct averaging we must include the respective fixed costs of any production activity in full, as soon as any production is undertaken. Thus in averaging complexes A' and B', we have to add on the fixed costs of production of good 2 (500 units of capital and no foreign exchange) for all averaged points, with the single exception of point A itself where the production of good 2 actually equals zero. This procedure is indicated in Figure 20 by erecting a fixed-cost arrow on A'. The end point of this arrow is A"(B') which will be referred to as the correct averaging point A" for B'. In Figure 15 the correct averaging line is seen to connect A"(B') with B' itself since there are no fixed costs at A' and therefore the correct averaging point B"(A') coincides with B'. Thus, starting with A' (which coincides with A

Figure 15

The introduction of fixed costs. The complexes A, B, D are carried by the fixed costs into points A', B', D'. These points cannot be directly averaged. The correct averaging points are shown as A", B", D"



where no fixed costs are incurred) the correct averaging line first rises to A"(B') (which corresponds to the smallest non-zero production level of good 2) and then proceeds to B'.

132. At B' the fixed costs for producing good 1 have to be added on before the averaging with D' can begin; accordingly an arrow representing 200 units of capital and 200 units of foreign exchange is erected on B', to get B"(D'). Note that this averaging point does not coincide with the averaging point B"(A'). At D' no further fixed costs need to be added since D' already allows for the fixed costs of both production activities. The correct averaging line thus runs from B"(D') to D'.

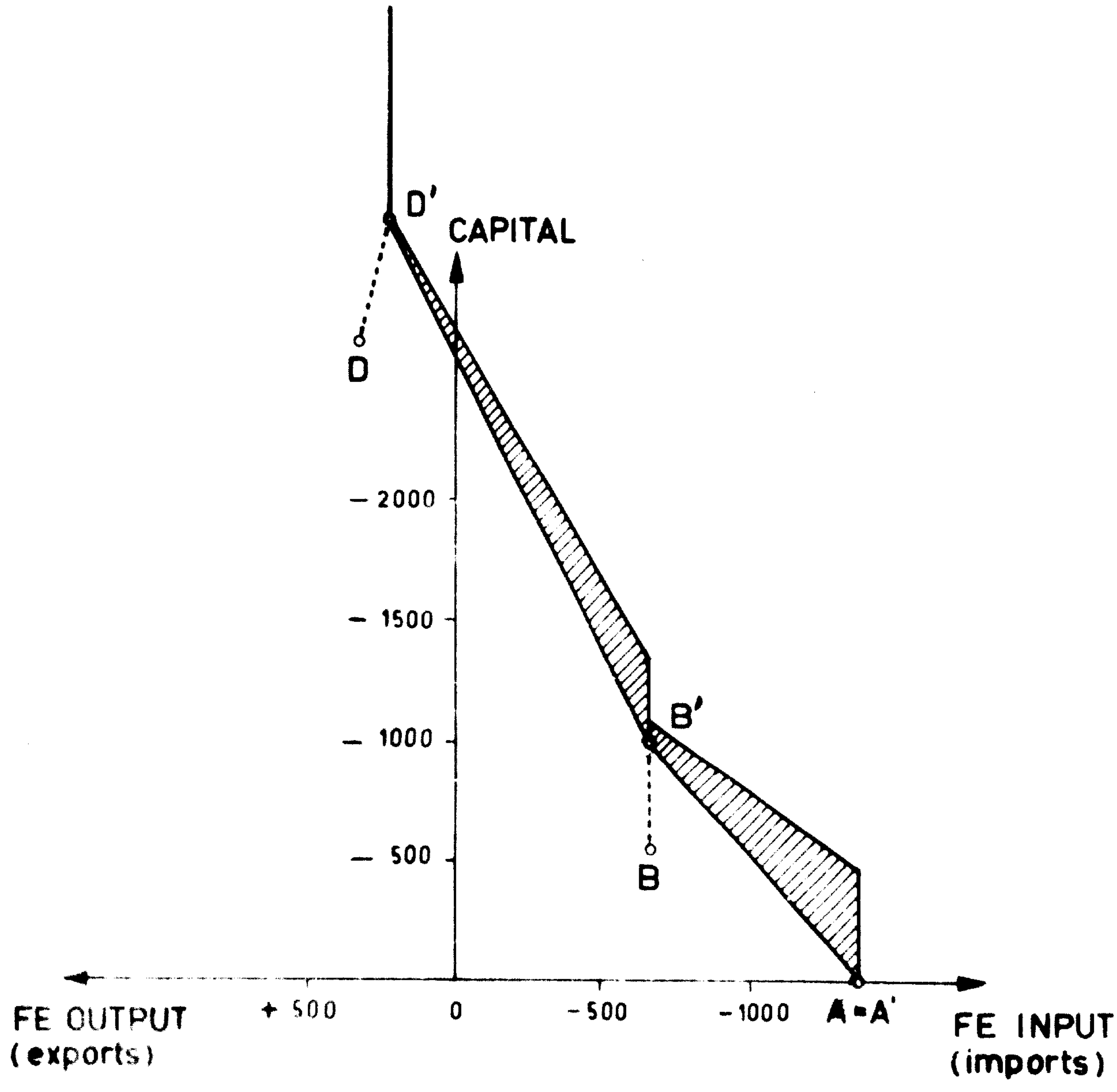
133. Figure 15 discloses that some points of line B"(D') to D' are inefficient. Thus point B"(D') itself and the points near to it are inefficient because they use more capital and foreign exchange than B' alone. They can therefore be superseded by B' combined with a disposal activity for capital. In other words, it is more efficient to use B' and to leave some capital unutilized than to use B"(D'), no matter what the direction of optimization happens to be. Figure 16 gives the final correct isoquant obtained after the elimination of inefficient points. The area between the correct isoquant and the continuous isoquant is shaded in the figure.35/

134. What information does the correct isoquant convey about the sector? First, it quantifies capital and foreign-exchange requirements. The precise meaning of this quantification has to be stated with some care, however, due to the jagged outline of the isoquant once fixed costs are introduced. Thus we define the correct isoquant in either one of two equivalent ways: (a) as the geometrical locus of least capital requirements corresponding to given foreign-exchange requirements or outputs; or (b) as the geometrical locus of least foreign-exchange requirements (where these are algebraically extended to include negative requirements, i.e. net outputs of foreign exchange) corresponding to given capital requirements. Second, the correct isoquant provides the implied information that all specific sectoral balances and constraints are correctly observed, including the fixed-cost tie-in constraints and the specifications that restrict certain variables to integer values. Evidently, the continuous isoquant violates the latter condition since it treats integer variables as continuously divisible.

35/ We have ignored complex C in deriving the correct isoquant. In general, one cannot state with certainty that an inefficient point of the no-fixed-cost problem will remain inefficient when the correct isoquant of the discrete problem is defined. It is necessary therefore to include such points in the correct averaging procedure until it can be shown that their correct averaging lines are everywhere inefficient.

Figure 16

The sectoral isoquant derived from the correct averaging line. Inefficient points are eliminated



135. Given the correct isoquant for the metalworking sector, we can define an intersectoral model on the same principles as in the purely linear case. With reference to Figure 14, the general structure of the problem remains exactly the same as before, with the simple modification that the isoquant of the metalworking sector (and possibly of the oth r sector as well) will now exhibit the kind of jagged indentations that occur in Figure 16. The optimum is still found geometrically by identifying the largest vertical separation between the two isoquants (for an objective function of capital-requirements minimization). The only essential difference is due to the fact that now there is a possibility for several local minima to occur, due to the indentations of the isoquant. Once this occurs, purely local optimality criteria are no longer adequate for the identification of the optimal solution; methods of finding such a solution are based on gradual improvements and consequently will break down. Note that the geometrical method of identifying the optimum by the criterion of the largest vertical separation between the isoquants relies on a complete scan of all possibilities summarized by the two isoquants!

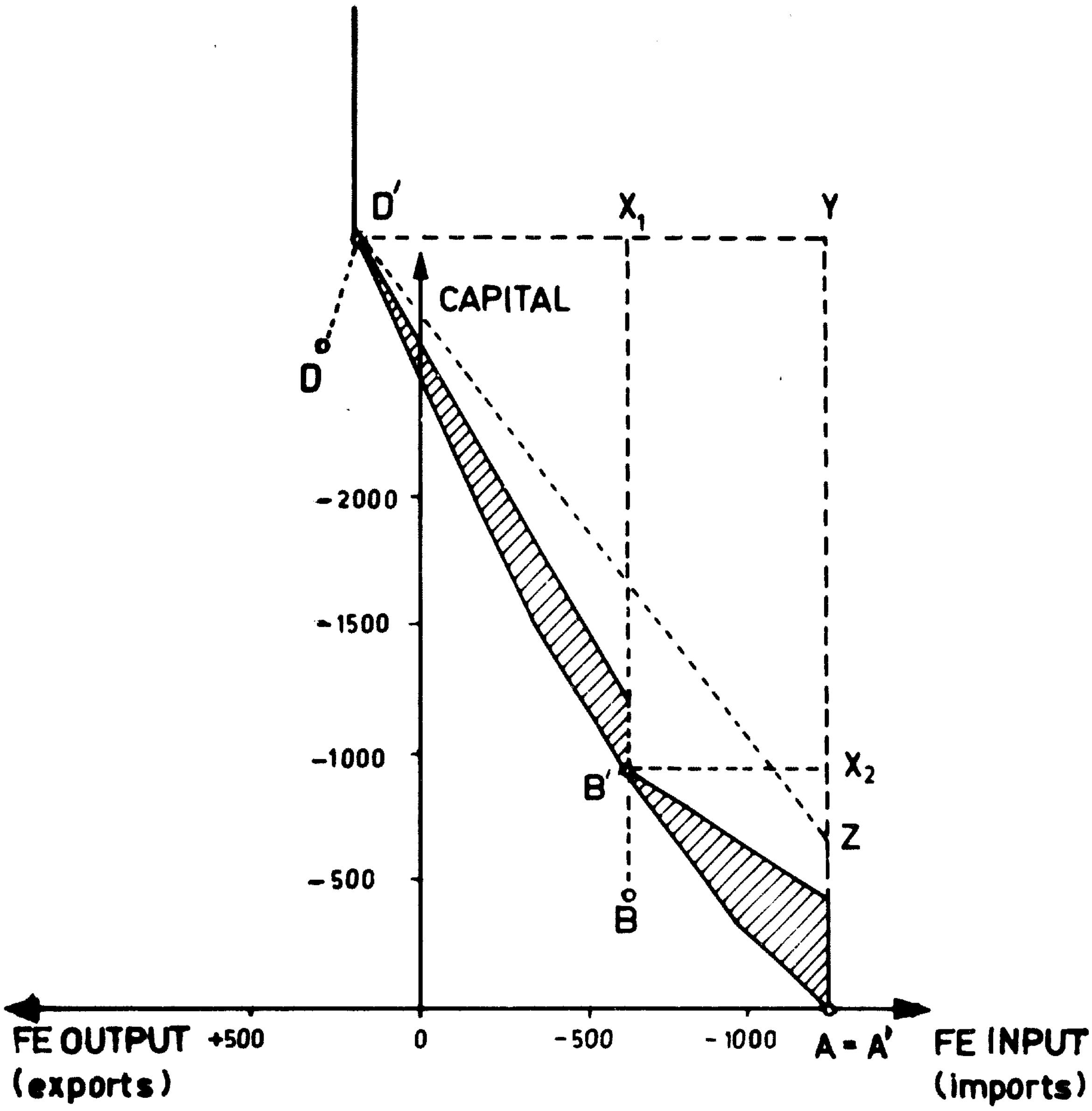
136. This key fact determines the different order of difficulties encountered in the linear and the discrete cases. In the purely linear case we can rely on powerful shortcut methods which avoid scanning all the combinatorial possibilities and which find their way directly to the optimum. In the discrete case the simple shortcut methods break down, and while a number of existing mathematical procedures show considerable improvements over the exhaustive enumeration of combinatorial possibilities, they involve much greater difficulties than the ones associated with the purely linear case. Above all, the simple relationships between resource allocation and pricing in the purely linear case (and also in nonlinear but convex models which do not involve indivisibilities and economies of scale) break down in the case of the discrete problem.

137. Fortunately, in practical planning tasks we do not require exact solutions. The parameters of the problem are themselves subject to considerable margins of error, and in any event a great many uncertainties about the future intrude upon exact formal solutions to programming models. Th practical planner is quite willing to accept approximations if he is provided with an adequate measure of control over the margins of error involved in such approximations.

138. With reference to Figure 16, we can define primal and dual approximations. Primal approximations always remain on the attainable (technically feasible) side

Figure 17

Primal approximations to the correct sectoral isoquant



of the correct isoquant, e.g. on the upper side in Figure 16. Thus they observe all constraints of the problem, but they may be inefficient. The search for the optimal solution based on such approximations will fall short of the true optimum; thus in the case of the minimization of capital requirements, the solution will be one that uses more capital than the attainable minimum. Many approximate solution methods to integer programming problems give such feasible but typically sub-optimal solutions. With reference to Figure 17, a solution method which found points A and D but missed point B might resort to a correct averaging procedure between the latter two points. The corresponding line would run higher than the correct isoquant, since it would connect D' with point Z and add the height of the step above B' to the step above A'. Taking this line as an approximation to the correct isoquant will necessarily result in sub-optimal solutions (see line D'ZA' in Figure 17). Another primal approximation might do away with averaging altogether and simply piece together solutions from unaveraged complexes. An isoquant corresponding to this strategy would connect A', B' and D' by large horizontal-vertical steps (see line $D'X_1B'X_2A'$ in Figure 17). If in addition some complexes remained unidentified, the steps would run at an even higher level (e.g. line D'YA' in Figure 17). To keep errors within bounds, it is clearly desirable to perform a correct averaging operation on complexes which have already been identified, and to keep from missing complexes which would allow significant improvements of the approximation.

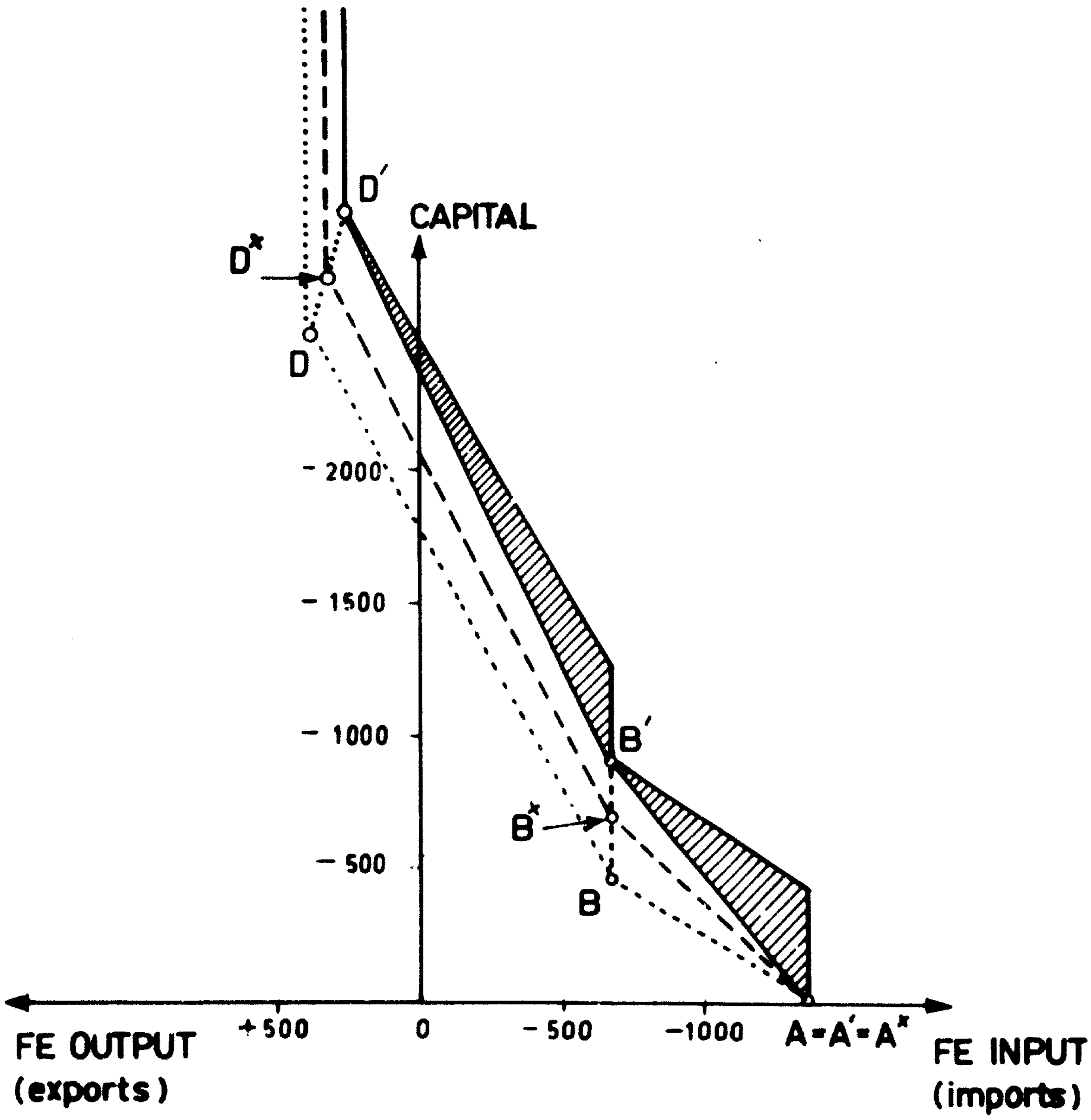
139. The essence of the dual approximations is that they close in on the correct isoquant from the infeasible (technically unattainable) side. The simplest approximation of this kind is the one that ignores fixed costs altogether and operates with the original isoquant ABD appearing in Figure 13 (see also Figure 18). A closer dual approximation takes fixed costs into account, but distributes them evenly over all units of available production capacity. A third and even closer approximation treats the fixed costs of the complexes as fully divisible for averaging purposes and leads to what has been labelled as the continuous isoquant.

140. It is important to clarify the distinction between the second and the third approximation. To this end, we calculated the following average fixed costs per unit of production capacity: good 1, 1.25 units of capital and 1.25 units of foreign exchange; good 2, 6.25 units of capital and no foreign exchange.[36] These average

36/ The calculations were made by dividing the fixed costs (200, 200) by the stated capacity of 160 for the production of good 1, and likewise dividing (500, 0) by 80 for good 2.

Figure 18

Dual approximations to the correct sectoral isoquant



fixed costs can be added to variable costs for each production process, and the resulting purely linear model can then be solved in the conventional way. Note, however, that this approximation generally understates the fixed costs associated not only with the averaged complexes but also with each complex in isolation. For example, complex B' (see Table 5) involves the production of 36 units of good 2 (30 for domestic demand, 6 for export); thus we calculate a fixed cost of (36).(6.25) = 225 units of capital instead of the actual 500 units. Likewise, for complex D' we calculate (100).(1.25)+(36).(6.25) = 350 units of capital and (100).(1.25)+(36).(0) = 125 units of foreign exchange instead of the correct figures of 700 and [illegible]00 respectively. These underestimated resource requirements for each complex will be denoted by the symbols B*, D*. Note that as A involves no fixed costs, A=A'=A*. Thus the isoquant estimated by the second dual approximation runs through A*, B*, D*, while the isoquant estimated by the third dual approximation is the continuous isoquant A'B'D'. It can be seen in Figure 18 that the third approximation is closer than the second, because the third is exact (has no error) at the points representing individual complexes and is in error only over the averaging stretches connecting the complexes. The second approximation is in error except at complexes such as A that have no fixed costs at all, while over the averaging stretches too, the error is larger than the one characterizing the third approximation.

141. In the general case the three dual approximations can be derived by mathematical programming techniques. The first is obtained when the sectoral subproblem is optimized while fixed-cost activities are omitted; the second, when fixed-cost activities are retained together with their respective tie-in constraints but the integrality conditions on the fixed-cost incurrence variables are suspended; and the third by integer programming and strict observance of the above mentioned integrality conditions.37/

142. The difference between the second and third dual approximations can be summarized as follows: while both work with average rather than marginal costs, as both take into account fixed costs which are ignored in the derivation of marginal costs,

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37/ In all three cases, the entire course of the approximating isoquant can be obtained only when the optimization is carried out repeatedly with gradually changing capital/foreign-exchange price ratios inserted in the sectoral objective functions, since each such optimization will lead to only one extreme point (complex) along the isoquant. This repetitive procedure can be formalized and shortcut in the case of linear programming and is technically referred to as parametric programming.

in the second approximation average costs are obtained by distributing fixed costs over the units of available capacity, while in the third approximation average costs are obtained by distributing fixed costs over the actual units of production characterizing each complex.

143. Jointly the primal and the dual approximations permit a satisfactory pragmatic approach to the programming of individual sectors within an over-all intersectoral model. The primal approximations will yield feasible solutions that are generally not optimal but close to the optimum; the dual approximations will permit placing an upper bound on the error that is being committed and th approximate pricing of the interconnecting resources.

15. A more comprehensive model

144. We shall now enlarge the model representing the metalworking sector in the global/sectoral decomposition system, in order to bring it closer in conception to the three models already discussed. In particular, we intend to represent some of the key features of Model 3 (see para. 36-38) that arise from the independent presence of economies of scale: first, at the level of the individual product (economies of long series), and second, at the level of resource elements (economies of large scale). We wish to study the interaction of these two kinds of indivisibilities in the derivation of the sectoral isoquant. In addition, we shall bring in variable exports which have not been included in Model 3. In Models 1 to 3 exports have been treated as exogenously determined, whereas in the small illustrative model discussed in Chapters 13 and 14 exports were already endogenous, because the scale of the import activity occurring in this model was treated as a free variable. Negative valued of this variable signified exports, and a specific limit was imposed on the extent of such exports. In the more comprehensive model exports will be treated as a falling step function of export price, with separate limits on the extent of exports at each of two price levels.

145. The model is specified in Table 7 in purely linear form without fixed costs. For each of the two goods two separate production activities are given, using one of two resource elements. There are also two activities representing the maintenance of resource-element capacity (columns 9-10). For each good there are five significant production-and-trade choices: (a) imports; (b) production by activity 1 and export up to limit of first step (row 7); (c) production by activity 1 and export up to limit of second step (row 8); (d) production by activity 2 and export up to limit

Table 7

A more comprehensive illustrative model for the metalworking sector in an intersectoral decomposition system. Resource element requirements are explicitly included

	Good 1 Production activity 1	Good 1 Production activity 2	Good 1 ± Imports	Good 1 Exports	Good 2 Production activity 1	Good 2 Production activity 2	Good 2 ± Imports	Good 2 Exports	Resource element 1	Resource element 2	Exogeneous	
1	1	1	1	-1							-80	Product 1 balance
2					1	1	1	-1			-30	Product 2 balance
3	-20				-15				1			Resource element 1 capacity balance
4		-15				-10				1		Resource element 2 capacity balance
5									-1	-1.25		Capital requirement
6			-10	+8			-20	+15				Foreign exchange balance
7			1								20	Export limit, good 1, step 1
8				-1							50	Export limit, good 1, step 2
9							1				6	Export limit, good 2, step 1
10								-1			20	Export limit, godd 2, step 2
	1	2	3	4	5	6	7	8	9	10	11	

Figure 19

Diagrammatic representation of a more comprehensive illustrative model for the metalworking sector. The numbers refer to individual complexes

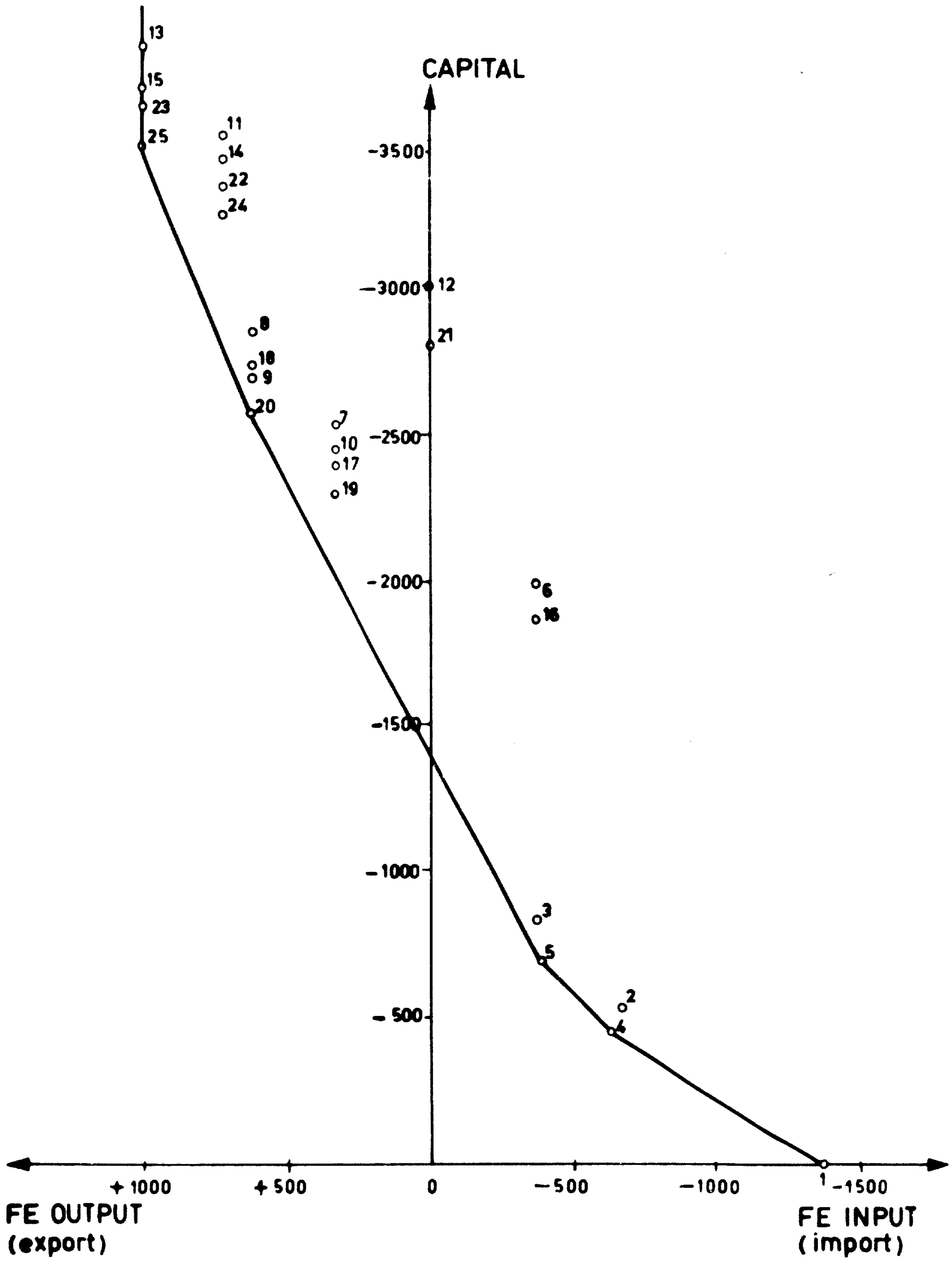


Table 8

Listing of complexes for the illustrative model given in Table 7

Complex	Activity for good 1	Activity for good 2	Resource requirements (-) or supplies (+): good 1 FE	good 1 R1	good 1 R2	good 2 FE	good 2 R1	good 2 R2	total ΣFE	total ΣR1	total ΣR2	total ΣK
1	I	I	-800			-600			-1400			
2	I	P1E1	-800			+120	-540		- 680	- 540		- 540
3	I	P1E2	-800			+420	-840		- 380	- 840		- 840
4	I	P2E1	-800			+120		-360	- 680		- 360	- 450
5	I	P2E2	-800			+420		-560	- 380		- 560	- 700
6	P1E1	I	+200	-2000		-600			- 400	-2000		-2000
7	P1E1	P1E1	+200	-2000		+120	-540		+ 320	-2540		-2540
8	P1E1	P1E2	+200	-2000		+420	-840		+ 620	-2840		-2840
9	P1E1	P2E1	+200	-2000		+120		-360	+ 320	-2000	- 360	-2450
10	P1E1	P2E2	+200	-2000		+420		-560	+ 620	-2000	- 560	-2700
11	P1E2	I	+600	-3000		-600			0	-3000		-3000
12	P1E2	P1E1	+600	-3000		+120	-540		+ 720	-3540		-3540
13	P1E2	P1E2	+600	-3000		+420	-840		+1020	-3840		-3840
14	P1E2	P2E1	+600	-3000		+120		-360	+ 720	-3000	- 360	-3450
15	P1E2	P2E2	+600	-3000		+420		-560	+1020	-3000	- 560	-3700
16	P2E1	I	+200		-1500	-600			- 400		-1500	-1875
17	P2E1	P1E1	+200		-1500	+120	-540		+ 320	- 540	-1500	-2415
18	P2E1	P1E2	+200		-1500	+420	-840		+ 620	- 840	-1500	-2715
19	P2E1	P2E1	+200		-1500	+120		-360	+ 320		-1860	-2325
20	P2E1	P2E2	+200		-1500	+420		-560	+ 620		-2060	-2575
21	P2E2	I	+600		-2250	-600			0		-2250	-2813
22	P2E2	P1E1	+600		-2250	+120	-540		+ 720	- 540	-2250	-3353
23	P2E2	P1E2	+600		-2250	+420	-840		+1020	- 840	-2250	-3653
24	P2E2	P2E1	+600		-2250	+130		-360	+ 720		- 610	-3263
25	P2E2	P2E2	+600		-2250	+420		-560	+1020		-2510	-3513

NOTES

FE: Foreign exchange
R1, R2: Resource element capacities
I: Imports
P1, P2: Production activity 1 or 2
E1, E2: Export step (limit) 1 or 2

Table 9

A more comprehensive model for the metalworking sector in an intersectoral decomposition system.

Fixed costs added

	Good 1 production activity 1 variable	Good 1 production activity 1 fixed	Good 1 production activity 2 variable	Good 1 production activity 2 fixed	Good 1 ± im-ports	Good 1 ex-ports	Good 2 production activity 1 variable	Good 2 production activity 1 fixed	Good 2 production activity 2 variable	Good 2 production activity 2 fixed	Good 2 ± im-ports	Good 2 ex-ports	Resource elements 1 variable	Resource elements 1 fixed	Resource elements 2 variable	Resource elements 2 fixed	Exo-genous	
	[illegible]		1		1	-1											-80	Product 1 balance
							1		1		1	-1					-[illegible]	Product 2 balance
	-[illegible]						-15						1					Resource element 1 balance
			-15						-1[illegible]						1			Resource element 2 balance
		-1[illegible]		-[illegible]				-5[illegible]		-100			-1	-250	-1.25	-100		Capital balance
					-10	+5					-20	+15						Foreign exchange balance
					1												[illegible]	Export limit, good 1, step 1
						-1											50	Export limit, good 1, step 2
											1						6	Export limit, good 2, step 1
												-1					20	Export limit, good 2, step 2
11	$-\frac{1}{1[illegible]}$	1																Fixed-cost tie-in constraints
12			$-\frac{1}{160}$	1														
13							$-\frac{1}{80}$	1										
14									$-\frac{1}{80}$	1								
15													$-\frac{1}{2000}$	1				
16															$-\frac{1}{500}$	1		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	

of first step; and (e) production by activity 2 and export up to limit of second step. The number of joint combinations for the two products is (5).(5) = 25; there are thus 25 complexes whose main characteristics are listed in Table 8. The capital/foreign-exchange isoquants implied by these 25 complexes are plotted in Table 8 which can be regarded as a generalized version of Figure 13. As the diagram indicates, there are only five efficient points with the given choice of parameters which determine the isoquant for the sector. Apart from pure imports for both goods (complex 1), the efficient choices always involve production by means of resource element 2 and comprise the following: imports of good 1 with production of good 2 up to either the first or the second export step (complexes 4 and 5); and production of good 2 up to the second export step with production of good 1 either up to the first or up to the second export step (complexes 20 and 25). All other complexes are inefficient. The isoquant has three sloping linear segments in the net import region and two in the net export region.

146. The model is extended by the introduction of fixed costs in Table 9. Fixed costs of 100 and 200 units respectively, are associated with the production activities for good 1 with a corresponding bound of 160 units of production for each of these, a bound which is not exceeded by any of the complexes. The corresponding parameters for good 2 are 50 and 100 units of fixed cost, with a production bound of 80 units. For the sake of simplicity, all fixed costs are given in terms of capital alone, even though it would be economically meaningful to define some fixed costs in terms of foreign exchange (necessarily imported components of productive capacity) or in terms of resource-element capacities. Similarly, fixed costs are also associated with resource elements: again these are given in terms of capital alone. The respective fixed costs are 250 and 100 units, with corresponding capacity limits of 2,000 and of 500 units. These limits are <u>not</u> bounds; for many complexes it becomes necessary to duplicate resource elements as total required capacity exceeds the limit associated with a single fixed-cost incurrence. Thus the integer variables for activities 14 and 16 will at times assume values of 2,3,4 and so on.

147. Table 10 lists the fixed costs of the complexes which are needed for drawing up the sectoral isoquant which is diagrammed in Figure 20. It will be noted that this figure is a more elaborately detailed variation on Figure 16.

148. The derivation of Figure 20 is laborious but straightforward. There are only two new features that emerge in comparison in Figure 16. First, it is no longer true that the continuous isoquant, as heretofore defined, will necessarily remain

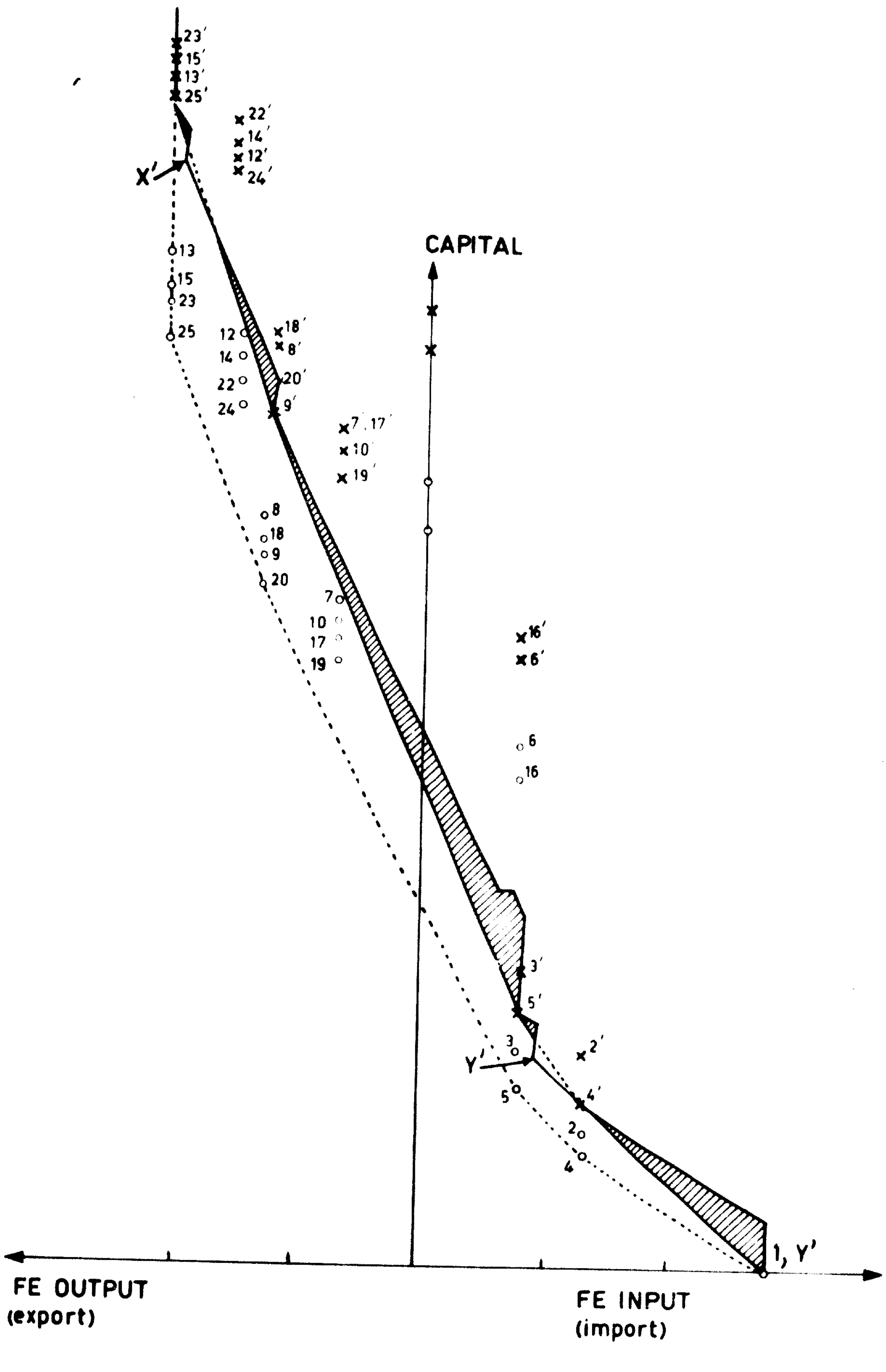
Table 10

Listing of fixed costs for complexes of a more comprehensive illustrative model for the metalworking sector

Complex	Act. 1 100	Act. 2 200	Act. 1 50	Act. 2 100	1 250/200	2 100/500	Total
1'							
2'			50		250		300
3'			50		250		300
4'				100		100	200
5'				100		200	300
6'	100				250		350
7'	100		50		500		650
8'	100		50		500		650
9'	100			100	250	100	550
10'	100			100	250	200	650
11'	100				500		600
12'	100		50		500		650
13'	100		50		500		650
14'	100			100	500	100	800
15'	100			100	500	200	900
16'		200				300	500
17'		200	50		250	300	800
18'		200	50		250	300	800
19'		200		100		400	700
20'		200		100		500	800
21'		200				500	700
22'		200	50		250	500	1000
23'		200	50		250	500	1000
24'		200		100		600	900
25'		200		100		600	900

Figure 20

Sectoral isoquants derived from the illustrative model presented in Table 9, showing effects of fixed costs



on the infeasible side of the correct isoquant. The former connects 1', 4', 5', 9', and 25', and as is readily seen it runs between 4' and 5', and again between 9' and 25', above the correct isoquant over a short distance. How this might happen can be easily followed in the case of 4' and 5'. These complexes have identical fixed costs except for resource element 2 which has to be duplicated in complex 5', while it appears only once in complex 4'. If it were not for this duplication in 5', the complexes 4' and 5' could be averaged correctly by a simple linear connexion. This linear connexion would run to a point 100 units below point 5' (if 5' had only 1 unit of resource element 2). As it is, the same line will be correct for 70 per cent of the distance from 4' to 5' because over this stretch the total required capacity of resource element 2 stays under 500 units, the limiting capacity for a single fixed cost! It is only at this point that an extra 100 units of fixed cost have to be incurred to permit the continuation of the correct averaging process. Thus there is a sudden jump of 100 units 70 per cent of the way from 4' to 5' after which the correct isoquant, maintaining the same slope as before the jump, runs into 5'. The direct route from 4' to 5', on the other hand, starts by anticipating the final effect of this jump and thus runs above the correct isoquant until the jump actually occurs. Such a situation can arise only when the fixed costs of a complex can be incurred stepwise.

149. As a result, the definition of the continuous isoquant has to be tightened for this case. As can be seen in Figure 20 the addition of new subcomplexes at X' and Y' allows a redefinition of the continuous isoquant. This redefinition satisfies the condition of having the continuous isoquant remain entirely on the infeasible side of the correct isoquant: the continuous isoquant will now connect the points 1'4'Y'5'9'X'25'. Such subcomplexes occur at integer multiples of the capacity limit associated with a single incurrence of a specific fixed cost of a given complex up to the number of actual incurrences minus one. When points along the continuous isoquant are identified by integer programming within the sectoral subproblems, this process will correctly identify subcomplexes such as X' and Y' that occur along the continuous isoquant.

150. Second, the complexes that define the continuous isoquant may not suffice to define the correct isoquant. The correct isoquant between 9' and 20' is not obtained by correctly averaging 9' and 25' but by using 20' instead of 9' in the correct averaging process because the latter averaging line runs at a lower level than the former. When using 9' there is an immediate jump of 200 units, whereas

when using 20' there is no such jump, as 20' and 25' have an almost identical fixed-cost structure (see Table 10). Inasmuch as 20' is only 1.5 units higher than 0', the latter is efficient when used alone but becomes inefficient as soon as averaging with 25' is undertaken.

16. Allocations and pricing in the presence of fixed costs

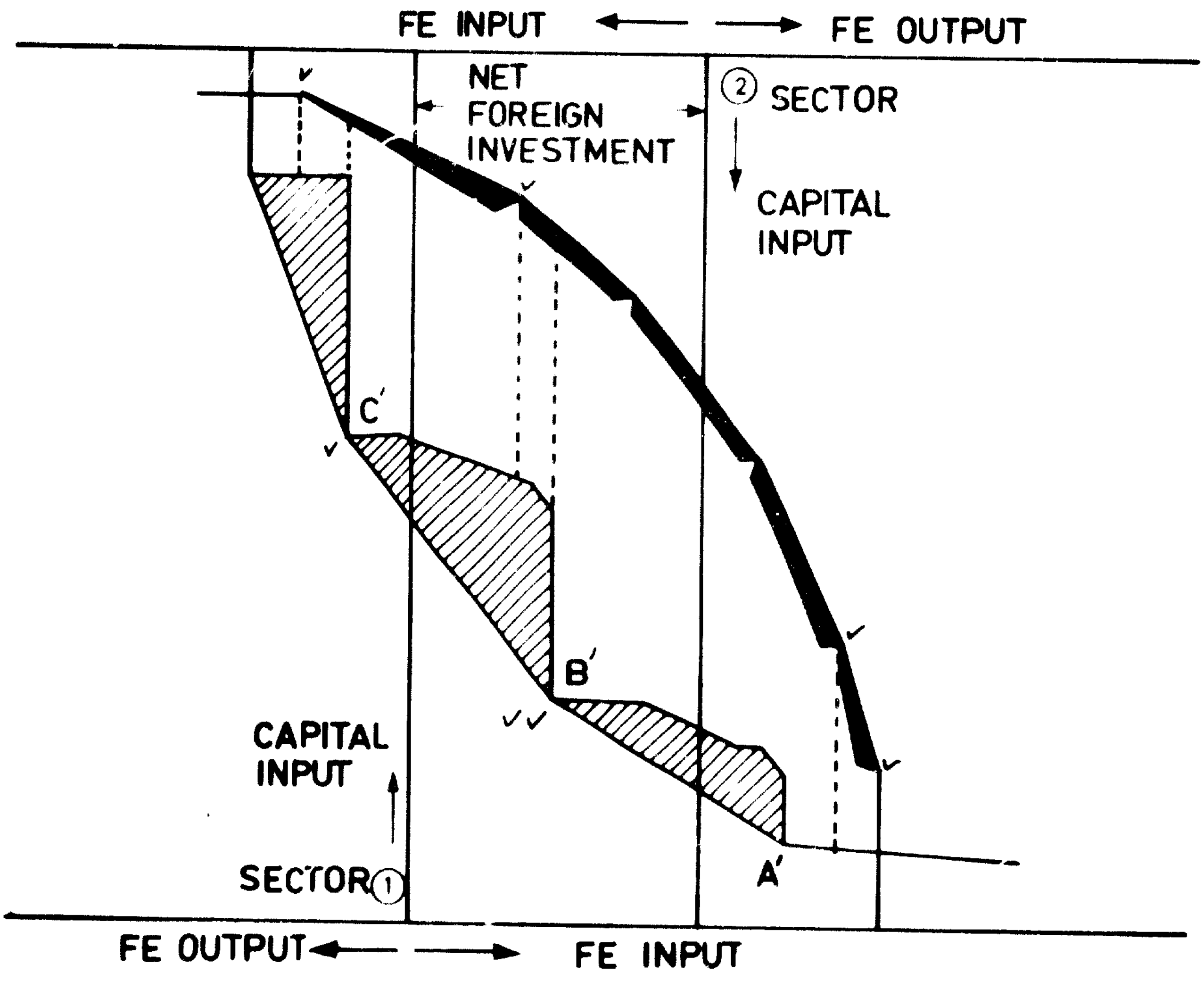
151. The precise definition of sectoral isoquants in the illustrative intersectoral model with two sectors and two resources (such as is shown for the continuous case in Figure 14) permits the identification of the optimal solution by scanning the diagram for the largest separation between the isoquants. If the least use of capital is the objective, the largest separation is required in the vertical direction. No matter how jagged the correct isoquant of each sector becomes as a result of correct averaging between complexes that have fixed costs, the search for the largest vertical separation is still a simple and quick operation (see Figure 21). Typically there will be multiple local optima - in Figure 21 there are six of these, identified by check marks - which have to be compared among themselves to find the over-all optimum, identified by a double check mark.

152. The separation between the correct isoquant and the continuous isoquant is indicated, as usual, by shading. Figure 21 shows the large indivisibilities in Sector 1 and the much smaller ones in Sector 2. If the continuous isoquant is used as a programming approximation to represent the correct isoquants of the sectors, the error committed in Sector 2 will be considerably smaller on the average than the error in Sector 1. At the same time, since correct local maxima tend to occur at complexes in one or the other of the sectors (because at these points there is no averaging and many fixed costs are saved) the approximation to the optimal solution based on the continuous isoquants will be burdened with less error originating in the sector with the larger indivisibilities than the average error in this sector. In Figure 21, for example, the maximum separation between the continuous isoquants occurs at B' which is also the point at which the correct over-all optimum is found. The difference between the value of the correct optimum and the approximation based on the continuous isoquant is determined only by the small error in Sector 2; there is in fact no error in Sector 1, even though the latter sector has a much larger average error over the course of the isoquant as a whole.

153. If there are more than two sectors and several of these have large indivisibilities which give rise to deeply indented isoquants, we no longer have any

Figure 21

Sketch diagram of two-sector decomposition model with fixed costs



guarantee that the continuous isoquants of the sectors will give a fair approximation to the optimum. It is essential therefore to complement this approximation by estimates of the optimum based on primal approximations (see Chapter 14), which will tend to underestimate rather than overestimate the optimum. The difference between the optimum derived from the primal and that derived from the dual approximation yields an upper bound on the error that might be committed.

154. The relationship between primal and dual approximations to the sectoral isoquants is of fundamental importance both to the construction of programming models and to practical planning decisions within the sector and the intersectoral system as a whole.

155. We now turn to the problem of small indivisibilities. To the extent that the difference between the correct isoquant and the dual approximation is within permissible error limits, we can replace the correct isoquant by the dual approximation. In practice this means that we do not have to build a sectoral model that allows for the last minute indivisibilities, but can be content with a model that treats the major indivisibilities in a discrete fashion while it uses dual-type approximation for the minor ones. Of the three dual approximations discussed in paragraphs 139-142 the first ignores fixed costs altogether, while the second and third distribute fixed costs either over full capacity or over actual production quantities in the complexes. The last two approximations, being variants of an average-cost type approach, are suitable as guides for the suppression of small indivisibilities. While both of these approximations are strictly of the dual type, i.e. they remain at all times on the infeasible side of the correct isoquant, when using them for eliminating small indivisibilities from further consideration in the model, it is best to modify them slightly and to distribute fixed costs over a production rate based on estimated capacity utilization. This will generally be less than 100 per cent. To begin with, even at points represented by individual complexes the capacities associated with fixed costs are typically less than fully utilized. With reference to Table 8, for example, complex 5 uses 560 units of capacity of resource element 2; the fixed cost associated with this resource element in the model of Table 9 is 100 units, with a capacity limit of 500 units. Thus to satisfy the capacity requirement of 560 units, fixed cost has to be incurred twice, and this leaves 440 units of unutilized capacity. When separate complexes are averaged, capacity utilization drops below 100 per cent even when the isolated complexes utilize the capacity fully. While it is not possible to predict accurately

what the capacity utilization of a particular resource element will be (as the estimate is required prior to the formulation of the model), one can gather some idea from the approximate relationship of total demand for capacity to capacity for a single fixed cost. The larger the number of units required, the higher the probable capacity utilization can be set. For n identical units, capacity utilization (assuming equal loading of all facilities) will be at least (1-1/n).

156. Given the estimated capacity utilization, fixed costs can be distributed over the corresponding units of production and added to variable costs. If the estimates turn out to be approximately correct, we arrive at an isoquant that runs on the average above the third dual approximation but below the correct isoquant. This approximation is neither a pure dual nor a pure primal approximation, but is within error limits of either. It has the merit of corresponding closely to the ordinary managerial practice of distributing fixed costs, for accounting purposes, over the anticipated production rate. If all fixed costs can be treated in this manner within prescribed error limits, the resulting model will be purely linear. In the solution of such a linear model all activities actually utilized will break even; thus the price solution of such a model will reflect full rather than marginal costs. Break-even at ex ante estimated capacity utilization levels is of course not the same as break-even at ex post realized levels. As the model is formulated exclusively in terms of the former, there is no feedback within the model between ex post realized and ex ante postulated capacity utilizations. In an actual decentralization mechanism based on average-cost pricing, the experience in regard to capacity utilization during a given period will modify the anticipated capacity utilization levels for the following period, and thus it becomes possible to define the behavioural prerequisites, at the level of the managerial decision of the firm, for an adaptive elimination of all ex post profits and losses.

157. We shall refer to the elimination of small indivisibilities from the model by means of an average-cost type approximation as the bridging-over of small indivisibilities. Thus in formulating a model for the metalworking sector, we shall begin by classifying production processes and resource elements into two groups: continuous (within error limits) and discrete. In the case of continuous production processes and/or resource elements, indivisibilities have to be bridged over by an average-cost type approximation.

158. Indivisibilities that are too large to be bridged over when formulating the model. These will enter the model in a discrete way, with their fixed costs specifically and separately accounted for in the manner indicated in the models discussed (see, for example, Table 9). The resulting model, containing continuous and discrete activities, may well be much too large to be solved directly by integer programming. In this case the primal and the dual approximations to the sectoral isoquants help to define approximate optimizing procedures that will yield both feasible but suboptimal solutions and upper bounds on the possible optimum. For example, the use of the third approximation for the sectoral isoquants involves the solution of integer programming models within the sectors that are much smaller than the intersectoral model as a whole. Each such solution contributes a point along the continuous isoquant. (The second approximation is even simpler: it involves only the solution of linear programming models, but it is less close.) These points can then be interrelated by a purely linear programming technique which is the exact counterpart of the Dantzig-Wolfe procedure (in which no fixed costs occur). The reason for this is that once we work with the continuous isoquant, we effectively linearize the intersectoral problem by bridging over all indivisibilities, no matter how large these are. The solution to this linearized intersectoral problem yields a set of prices for the interconnecting resources. This new set of prices is then used to define new objective functions for the sectoral subproblems (see Chapter 12). An integer programming solution for the latter will identify new complexes along the interconnecting isoquant and so on.

159. Simultaneously we can use primal approximations for the sectoral isoquants to obtain a feasible suboptimal solution to the intersectoral problem. In this task the difficulty is, in part, that of finding a sufficient number of sectoral complexes from which to construct approximate intersectoral programmes. In the simplest of these primal approximations such complexes are used one at a time for each sector without an attempt at averaging; in more sophisticated versions explicit account can be taken of the fixed costs occurring in the individual complexes and correct averaging can be applied. In any event, candidate complexes for these tasks can be supplied from two sources: the dual approximation and the large indivisibilities when these are contained in a sector. The relevant complexes constructed from the latter will be relatively few in number, and a partial enumeration strategy based on the knowledge of the structure of the sector can be expected to yield a good selection of candidate complexes. A comparison of solutions obtained by the primal and the dual approximations will define a bound on the possible error.

The approximations may have to be improved progressively until the optimal solution is obtained within an acceptable margin of error.

160. We will now discuss the problems of central allocations and decentralization by a price mechanism. The optimal solution to the intersectoral programming model identifies those discrete activities that have to be undertaken and separates them from those that will not be used. The solution to this model is the basis for taking central planning decisions with regard to fixed investments that are too large to be bridged over by an average-cost type decentralizing mechanism. The prices which can be associated with either the dual approximation or the primal integer-programming approximations are in general not suitable as guides to decentralized resource allocation decisions which will jointly arrive at the approximate optimum that has been identified, either because the prices bridge over excessively large indivisibilities, or because (in the case of the prices occurring in integer programming models) they cannot be uniquely associated with the resources whose decentralized allocation is desired. Thus it is inevitable that the planning decisions pertaining to discrete activities have to be undertaken centrally. Once it is decided, on the basis of the approximate optimal solution to the programming model, which fixed costs will be incurred and which not, the remainder of the problem becomes fully linear and can be decentralized by a price mechanism. To do this, reformulate that part of the model which remains after the central planning decision has been taken with regard to the discrete activities. Discrete activities whose fixed costs will not be incurred can be dropped out altogether, while those whose fixed costs will be incurred can be represented by their variable parts alone. The resource components of those fixed costs that will be incurred can then be subtracted from the respective exogenous resource availabilities (the sides of the Edgeworth-Bowley box). The model is now purely linear and can be solved by standard techniques. The price solution will yield correct decentralizing prices. These prices will prevail only on the assumption that the discrete part of the resource allocation problem has already been decided upon in one particular chosen manner.

161. In sum, the following planning strategy emerges from this discussion:

(a) Decide <u>ex ante</u> which indivisibilities can be bridged over within error limits by an average-cost type decentralizing mechanism. Estimate anticipated capacity utilization levels and distribute fixed costs over the corresponding number of units of the variable-cost activities. Thus the components of fixed cost are in effect added to the components of variable cost, and the resulting activities can be treated as continuously divisible.

(b) Build a model for each sector out of the continuously divisible and the discrete activities. Relate these models to each other via inter-connecting (intersectoral) resources (eg. foreign exchange and capital).

(c) Obtain an approximate optimal solution to the intersectoral model, using primal and dual approximations. Estimate the margin of error by the difference between the primal and the dual approximation. Refine these approximations until an optimal solution is obtained within an acceptable margin of error.

(d) Use this solution as a basis for making planning decisions with regard to the discrete activities. This amounts to deciding which fixed costs will be incurred and which not. The decision has to be put into effect by means of central resource allocations.

(e) If desirable, the rest of the resource allocation problem can be decentralized by means of an average-cost pricing approach that distributes fixed costs on the basis of estimated levels of capacity utilization. Profits or losses due to a divergence between anticipated and realized capacity utilization must be progressively eliminated by managerial decision rules that adjust the estimates for a given period on the basis of the experience of past periods.

17. Capacity allocation and pricing over time

162. The policy conclusions arrived at in paragraphs 151-161 have a corollary with regard to resource pricing. Since those fixed costs about which central decisions have to be taken do not enter the decentralizing price mechanism, they have no influence on the pricing of resources in the model. In other words, they are treated in effect as sunk costs for pricing purposes, while variable costs (including distributed costs in the case of small indivisibilities) alone determine the price structure. This does not mean that fixed costs have no effect on resource allocation. On the contrary, as the entire argument of the previous chapter has attempted to show, they have a crucial influence, but this influence cannot be exerted through the price mechanism (except for minor, bridged-over indivisibilities) and has to be given a chance to assert itself via a non-price type, essentially combinatorial, centrally controlled resource allocation mechanism. This mechanism not only complements, but underlies the decentralized pricing mechanism, since differing central allocations will give rise to different specific price structures.

163. This analysis also permits a simple resolution of the theoretical conflict between the relative merits of average-cost versus marginal-cost pricing. As far as small indivisibilities are concerned, average-cost pricing is found to be an attractive decentralizing device within error limits; the underlying rationale here is that the enormous savings of information handling, which accrue to decentralization, favour working with approximately optimal rather than exactly optimal outcomes.

larger indivisibilities must be handled by central decision-making based on models that summarize the main combinatorial alternatives open to the system as a whole. Once the key decisions about the major fixed costs have been made, further detail can be left to decentralized decision-making based on a price system that is built on pure marginal costs as far as the major fixed costs are concerned. Yet these marginal costs already incorporate average fixed costs derived from the smaller indivisibilities. The distinction between "small" and "large" indivisibilities depends entirely on the acceptable error limit with regard to the definition of the optimum.

164. All of the above conclusions are derived from static models. Dynamic features can be brought into the analysis by extending the models to cover several time periods. Model 4 has been constructed to illustrate some of these novel features while reducing the interrelations to their bare-bone essentials. The notation of Model 4 follows the notation for Models 1 to 3 given in the Annex, except for the omission of most of the subscripts and superscripts of the parameters. There are two production activities in each time period (columns 1-2, 9-10, 17-18); their fixed costs have been suppressed, as we wish to concentrate on resource-element capacities. Intermediate commodity inputs have also been suppressed. Resource-element-capacity requirements per unit of production <u>c</u> are shown in rows 5-6, 15-16, 25-26. The next two activities in each time period represent the incurrence of fixed costs associated with the building of new resource-element capacities; the following two, the corresponding variable building costs. Thus, before new capacity can be added, a fixed building cost must be incurred, and thereafter a variable building cost must be met for each unit of capacity built. This representation of economies of scale in regard to resource-element capacities has been discussed in paragraphs 51 to 53. The fixed and variable building costs are represented by the [illegible] and [illegible] parameters respectively, which refer to inputs of primary factors. It is assumed that the second resource element is continuous, i.e. it has no fixed building costs. For comparison, nevertheless, the fixed inputs have been denoted by parameters which are assumed to take on zero values. This is indicated by circling these parameters.

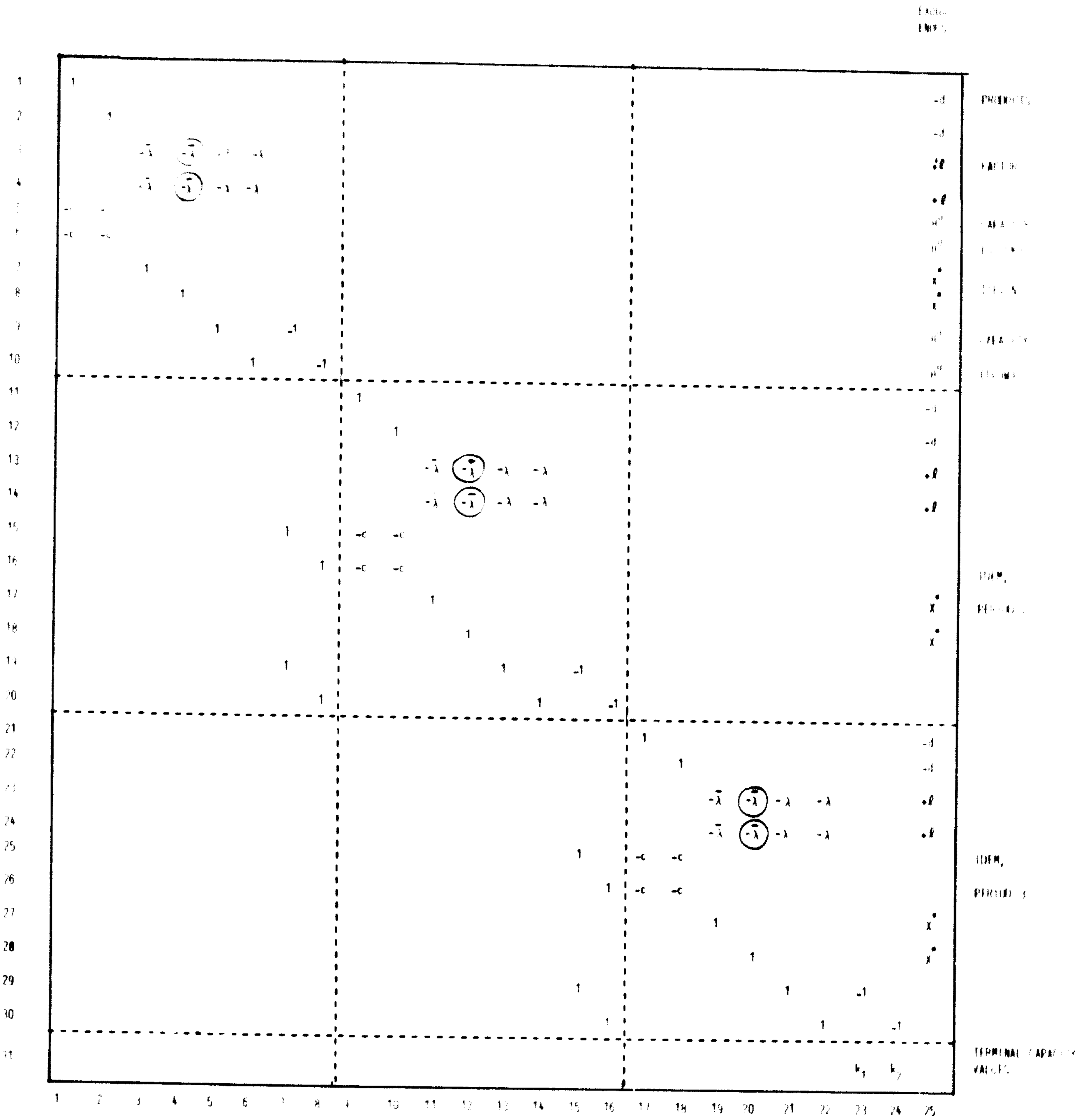
165. Finally, Model 4 includes a new kind of activity (columns 7-8, 15-16, 23-24), representing the holdover of existing capacity from period to period. Such activities are represented by a hypothetical purchase of capacity in period <u>t</u>; its renting out in period <u>t+1</u>, and its sale in the latter period. There is such a hold-over

MODEL 4

PERIOD 1 — PERIOD 2 — PERIOD 3

PRODUCTION | RESOURCE ELEMENT CAPACITY BUILDING | CAPACITY HOLDOVER

FIXED VARIABLE



activity for each resource-element capacity. Column 25 is, as usual, the exogenous column.

166. Among the rows of Model 4, the first two in each time period are product balances, with final demand in the exogenous column; the next two are primary-factor balances. Rows 5-6, 15-16 and 25-26 are capacity stock balances. These account for existing stocks, inherited from the previous time period, which are available for use in current production. The exogenous $\underline{H}^{o}$ entries in rows 5-6 represent stock inherited from the zero time period, while the stock availabilities of periods 2 and 3 depend on the scale of the hold-over activities in periods 1 and 2. The price variables associated with these rows (for an interpretation of price variables, see paragraph [illegible]) are capacity rentals. Rows 9-10, 19-20 and 29-30 are capacity flow balances. They account for the difference between inherited capacities and capacities passed on to the next time period. Since depreciation is suppressed, the above difference is the amount of capacity added during the period. The price variables associated with these balances are capacity flow prices, i.e. the buying and selling prices of a unit of capacity. Row 31 is the objective function and represents terminal (4th period) valuation of resource-element capacities, using the relative prices k_1 and k_2. This choice for the objective function is in accord with the usual formulation of multi-period stock (capacity) accumulation models. The k_1 and k_2 coefficients replace the capacity stock and flow balance entries which occur in each capacity hold-over activity in previous time periods.

167. Rows 7-8, 17-18 and 27-28 require special interpretation. Model 4 is taken to represent only the decentralized part of resource allocation, after the central decisions with regard to major fixed costs have been made. These rows, designated as tie-in rows after the usual tie-in constraints which they replace, set the scales of the fixed-cost activities to the predetermined integer values, represented by X* parameters in the exogenous column.38/ Where a fixed-cost activity is set to zero, the corresponding variable-cost activity is interpreted as also restricted to the zero scale.

168. Demand, represented by the exogenous entries in the product-balance rows, is assumed to increase from period to period. Instead of providing the extra capacity

38/ In these rows, in order to force exact equality between an X* parameter and the corresponding fixed-cost activity scale, no non-zero slacks are allowed. This can be handled by an elementary extension of the linear programming format shown in Chapter 2.

required for the additional demand of each period, it is generally advantageous to build ahead of demand, i.e. to add a larger amount of capacity during a given period than that required for satisfying the demand increase of that period. This occurs in capacity building whenever there are economies of scale which reduce the average cost of new capacity as the scale of the addition increases. Offsetting this advantage is the fact that expenses have to be incurred at an earlier time period than if some part of the additional capacity were built later, i.e. capital is tied up in currently unnecessary, idle capacity. For highly simplified cases it is possible to derive analytical solutions for the problem of optimal capacity addition (Manne, 1961); for more complex cases, such as the present one, recourse must be made to integer programming. The optimal solution to an integer programming formulation specifies the amount and kind of new capacity to be added in each period.

169. Given these results it is interesting to analyse the corresponding price implications, especially with regard to the rental price of capacity. Whenever there is slack capacity, the associated rental price will be zero. However, owing to the steady exogenous increase of demand, no capacity slack will persist indefinitely. Eventually the capacity limit will become binding and the rental price will rise above zero. In the intervening periods, however, the flow price of inherited and passed-on capacities of this particular kind will have remained constant, since in each two consecutive periods the stock (rental) price of capacity sets the difference between the corresponding flow prices. Thus if the capacity of resource element 1 has been redundant for n periods, beginning with period t, the stock-rental price for these periods will be zero, and the flow price of capacity in period t+n will be the same as in period t-1. In period t+n+1, when capacity becomes binding, the stock-rental price (see Model 4) equals

$$y_s(t+n+1) = y_f(t+n) - y_f(t+n+1) = y_f(t-1) - y_f(t+n+1),$$

where y_s and y_f refer to capacity stock (rental) and flow prices respectively, and the parentheses contain the index of the time period. Since the price solution of a multi-period programming model can be interpreted as representing discounted prices, in current values the flow price of slack capacity increases at compound interest. If we treat the flow price of the capacity of the second resource-element (which is continuous) as the numeraire in each time period for defining current prices, the rate of interest in the model will coincide with the rental of this continuous capacity.

170. If the technology of Model 4 is stable from period to period, the input requirements for producing additional capacity of resource element 1 will be the same in a later period as in an earlier one; thus the current flow price of the capacity of this resource element will drop to its initial value as soon as it is being added to. As a result, the drop in flow price has to be compensated by a high rental to give a rate of return on the holding of this capacity equal to the rate of return on the other capacity. The rental price obtained during the period (or periods) when the capacity is binding compensates for slack periods when rentals are zero. The result is a fluctuating price pattern for the capacity of resource element 1.

171. The flow price of the capacity of resource element 1 allows only for the variable part of building cost. As in a static model, fixed building costs are treated as sunk and do not enter the decentralized price formation mechanism. Resource allocation decisions pertaining to these fixed costs again have to be centrally taken. In more comprehensive models with a larger number of resource elements, there will probably be some resource elements whose fixed building costs are sufficiently minor to be bridged over by an average-cost approach, as discussed in connexion with the static models.

172. The amplitude of the price fluctuations on discontinuous resource-element capacities may be considerably reduced by secondary demands for these capacities. For instance, a large press may be indispensable for turning out refrigerator doors; the same press may however also be used for producing multiple units of smaller objects at a single stroke. If installed to enable the domestic production of high-grade refrigerators, this press may well have slack capacity for several years which can be taken up to manufacture smaller objects. These objects then constitute the secondary demand for the capacity of the large press which can be reduced as the primary demand increases, as the smaller objects can also be turned out on smaller presses. During a prolonged slack period the stock (rental) price of the capacity of the large press may well be zero: this condition serves to encourage any production activity that can generate economic value from the slack resource. As all demands, primary and secondary, increase over time, the slack period will eventually come to an end when the full range of production activities makes use of the press. When demand, however, increases, it will be necessary to cut out the lowest-grade uses, and to reserve the existing capacity for the most economic activities. This is achieved by allowing the stock (rental) price of capacity to rise to that point

where the lowest-grade uses are eliminated by their inability to meet the rental price. Further increases in demand will successively eliminate higher-grade secondary demands, until finally the scarcity of capacity will constrain even the primary demand which cannot be shifted to other capacities. At this point, if primary demand is inelastic, additional capacity has to be provided.

173. The hierarchy of primary and secondary demands defines a composite derived demand function that has a considerable price elasticity even when the individual demand functions are totally inelastic. Yet, if these demands have some elasticity of their own, the elasticity of composite demand for the capacity will further increase. Moreover, if some of the demand for a product that is a heavy capacity user can be covered from imports during periods of greatest capacity shortage, and conversely if the same product can be exported during periods of more ample capacity availability, a third influence is constituted, tending to make the composite demand for the discontinuous capacity more elastic. The greater the elasticity of this demand, the smaller will be the fluctuation of capacity rental prices for a fixed time-table of capacity additions; this analysis also suggests that the optimal size of capacity addition will increase with the improvement of capacity utilization.

174. The concept of interruptible secondary demand and of peak and off-peak load pricing is thoroughly familiar from studies on electric utilities, where the cycle is a daily one. In our case we are dealing with a cycle that exhibits a periodicity of several years between capacity additions, and this periodicity arises not from demand but from capacity fluctuations. None the less, the common element is the periodically fluctuating ratio of capacity to demand, and thus many of the familiar insights of the electric power load cycle can be applied to the long-range planning of discrete industrial capacity utilization and pricing. In particular, moderate long-term fluctuations in capacity and product prices, and structural fluctuations in the utilization of existing capacities and their complementation by exports and imports should be a normal part of long-range economic planning. The benefits that can be derived from such price and structural fluctuations have to be balanced against the disruptions caused by the continuous readjustments in production. These need not have, however, exclusively adverse effects. Cyclical readjustments facilitate the braking of rigidities and vested inefficiencies with which a stable production process often tends to be saddled. Such readjustments may also be of great help in the progressive introduction of technological innovations on which much of the genuine development of an economy so decisively rests.

18. Epilogue

175. The programming methodology developed in this report is based on an extensive investigation of the possibilities of technical/economic description for metal-working industries and on an analysis of the major planning problems raised by this vital but forbidding sector, presented in a voluminous earlier report entitled The Planning of Production and Exports in the Metalworking Industries (New School for Social Research, 1967), following a year-long study commissioned by UNIDO.

176. The best opportunity for testing these methods is offered by the two-level planning model, which has been developed for all sectors of the economy, including metalworking industries, by the Institute of Economic Science and the Centre of Computation Techniques of the Hungarian Academy of Sciences. A detailed description of the metalworking portions of this two-level model has recently been prepared under a UNIDO special service agreement (Deak, 1968); as far as known it is the only comprehensive two-level economy-wide programming model of its kind. As a clear example of convergent thinking on related problems, for years the author of this report has independently been relying on two-level planning models as a frame-work for thinking about sectoral planning problems. In the course of a UNIDO-sponsored visit to Hungary he had the opportunity to discuss his approach with the economists who were responsible for the construction of the Hungarian planning model.39/ The methodology suggested in this report is largely consistent with the conceptual framework of the Hungarian planning model and can in many ways be thought of as constituting a more detailed third level for the metalworking sector that could be constructed under the existing two levels. It should be noted that the construction of such a third level for the mining sector has already been underway for some time, as the model has been found to offer background information of increasing relevance for practical planning decisions. While many details of the author's suggested methodological approach will undoubtedly be rejected or modified if and when a UNIDO-sponsored country study of the sector in Hungary finally comes to fruition, it is hoped that it will at least provide a take-off point for a practical test. It would be particularly interesting if some of the suggestions for the handling of fixed costs in an essentially linear-programming type decomposition model could be made the subject of experiments with the aid of the existing two-level planning model.

39/ In particular Dr. János Kornai, Director of the project.

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3 OF 3

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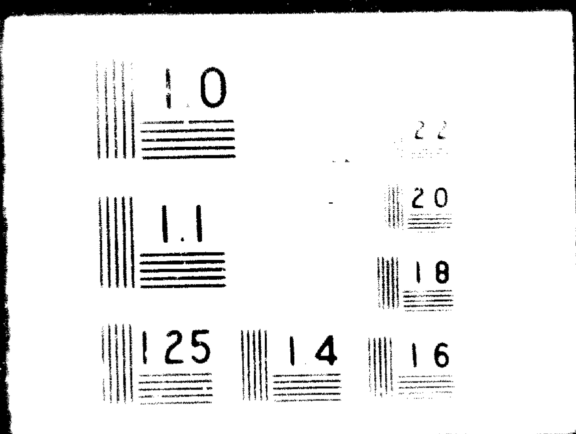

177. One question that must be left in a somewhat unsatisfactory state pertains to a practical price system. While it is being suggested to bridge small indivisibilities, major fixed costs are still left outside the price structure. For administrative and incentive reasons, though, it may be indispensable to distribute many of the major fixed costs that occur in the metalworking sector, even if they are made subject to central decisions. It has been found inadvisable to provide enterprises with free resources. How, then, is the process of decentralization affected if major fixed costs, after central decision-making, are distributed over the units of capacity or of output? Since fixed-cost incurrence is centrally decided, it will be unaffected by the change; but the burdening of low-grade secondary activities with average fixed costs will discourage enterprises from using slack capacities, and will thus be anti-economical. To what extent is the acceptance of such adverse effects justified for the sake of avoiding wasteful capacity utilization that is gradually hardened into a vested inefficiency? The answer to this question leads well beyond the usual technology-centred formulation of the problem.

178. There are of course many other questions in an equally unsatisfactory state. Problems of technical innovation, labour training, production scheduling, productivity and many others have not even been touched upon. The centre of attention was occupied by the problem of technical/economic description of meaningful production alternatives for the sector under a given technology, in a predominantly static framework, and with no institutional constraints from the side of labour skills and so on. The framework of linear and integer programming was utilized for organizing the available alternatives in one particularly simple and obvious fashion, without an implied commitment to this framework as necessarily the last word in the organization of this kind of information. All in all, despite a constant effort to simplify the problems, the very nature of the sector is such that it piles complication on complication in a seemingly endless way. Quite possibly it will be necessary to complement the essentially synoptic approach taken here -- the amassing of data for a grand decision -- with an adaptive-control type approach having a totally different orientation in that it treats major parts of the system as "black boxes" whose internal workings are fundamentally inaccessible to description and analysis.

170. None the less, when all this has been said, it is clear that so far only a modest effort has been directed by economists toward coping with the universe of problems posed by sectoral planning. If the present report excites enough criticism and dissent to stimulate some additional effort in this field, it will have achieved its purpose.

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ANNEX

Notation for Models 1 to 3

Rows

1LIS1,...,1LIS7	Listed-product balances. The first numeral is the serial number of the branch of the sector, the last is the serial number of the product within the branch.
1EXT, 2EXT	Extrapolated products, treated as a single item for each branch. The first numeral is the serial number of the branch. These rows refer to supply-demand balances of the extrapolated products.
FE	Foreign-exchange balance.
RES1, RES2	Resource-element-capacity balances. The last numeral is the serial number of the resource element.
DMAT	Direct material input. Refers to material input into production that is accounted for directly in connexion with a product, rather than indirectly via the material input requirements of resource elements.
LAB1, LAB2	Labour input into resource elements. Last numeral is the serial number of the class of labour.
IMAT	Indirect material input, via resource elements. Includes tools, lubricants, form sand, etc. Here only one item is carried, in physical units, but several items may be added or total cost may be carried as a single money sum.
cap	Capital requirement. This is the total capital stock tied down, measured in money terms. The price applicable to this resource is the capital carrying charge, consisting of the rate of interest plus any other charges.
$	Annual money cost, accounted directly (a flow).
1STP1,...,1STP4	Step function limits for extrapolated products. First numeral refers to branch, last to the serial number of the step.
1LFX1,...,1LFX7	Fixed-cost constraints for set-up charges in production of listed products. First numeral refers to branch, last to serial number of product.
RL1, RL2	Resource-element capacity limits. The last numeral is the serial number of the resource element.

RFX1, RFX2	**Fixed-cost constraints for resource-element capacities. The last numeral is the serial number of the resource element.**

Columns

1LIS1,...,1LIS7	Production of listed products. First numeral refers to branch, last to serial number of product.
1LFX1,...,1LFX7	Fixed-cost incurrence activity for production of listed products. Represents the incurrence of set-up charges for a given production series. First numeral refers to branch, last to serial number of product.
1LM1,...,1LM7	Import activities for listed products. First numeral refers to branch, last to serial number of product.
1EST1,...,1EST4	Production step in producing extrapolated products of a branch. Total production scale is sum of successive steps. First numeral refers to branch, last to serial number of step.
[illegible], 2EXTM	Import of extrapolated products. First numeral refers to branch.
[illegible], [illegible]	Resource-element capacity maintenance. These activities indicate the inputs needed for maintaining (<u>not</u> building) given resource-element capacities. In static one-period models no building activities occur. The last numeral is the serial number of the resource element.
[illegible]1, [illegible]	Fixed-cost incurrence for resource-element capacities. The scales of these activities measure the fraction of fixed cost actually incurred. The last numeral is the serial number of the resource element.
EXOG	Exogenous activity specifying fixed supplies and demands of different resources.

Parameters

a^i_j (Models 1-[illegible])	Input of listed product <u>j</u> into another listed product (serial number not specified). The superscript <u>i</u> refers to the typical product from which the coefficients of the given listed product have been derived. As an input this coefficient is provided with a negative sign. Conceivably a situation might occur where two or more listed products are produced by the same process: in this case by-products would be designated by positive <u>a</u> coefficients.

$a_{ij,k\ell}$ (Model 3) — Input of listed product no. j of branch i into another listed product, no. k of branch ℓ. Typical products from which the given listed product is derived are not distinguished in this notation. Other comments given above for a^i_j also apply here.

A — Matrix of a^i_j coefficients, of order 7x7.

I — Identity matrix, of same order (7x7) as A. The identity matrix has (+1) entries along the main (NW-SE) diagonal.

k^i — Portion of variable production cost of a listed product expressed in money terms, per unit of output. Superscript: see a^i_j.

$\bar{k}^i$ — Yearly fixed cost associated with production of given listed product. Consists of yearly capital charges of tooling, jigs and fixtures, and time (capacity) cost of setting up the required number of yearly production runs. Superscript: see a^i_j.

c^i_j (Models 1-2) — Variable capacity requirement of j-th resource element in the production of a given listed product. "Variable" means that portion of total capacity requirement that varies directly with scale of production, as distinguished from fixed requirement. Superscript: see a^i_j.

$c_{i,jk}$ (Model 3) — Analogous to former parameter. Subscripts: i, serial number of resource element; j, branch of listed product; k, serial number of listed product.

$\bar{c}^i_j$ (Models 1-2) — Fixed capacity requirement of j-th resource element in the production of a given listed product. Consists of share of time fund of given resource element devoted to setting up the required number of yearly production runs. Superscript: see a^i_j.

$\bar{c}_{i,jk}$ (Model 3) — Analogous to former parameter. Subscripts: see $c_{i,jk}$.

μ^i (Models 1-2) — Direct material input into production of given listed product. That portion of all material inputs that is accounted for separately for each listed product, as distinguished from indirect material inputs accounted for via resource-element-capacity utilizations. Superscript: see a^i_j.

μ_{jk} (Model 3) — Analogous to former parameter. Subscripts: j, branch of listed product; k, serial number of listed product.

ϕ_i (Models 1-2)	Capital investment in tooling, jigs and fixtures required for production of a given listed product. Superscript: see $\underline{a}^i_j$.
ϕ_{jk} (Model 3)	Analogous to former parameter. Subscripts: see μ_{jk}.
m_j (Models 1-2)	Foreign-exchange requirement in importing a given listed product, per unit amount; i.e. the foreign-exchange import price. The subscript refers to the serial number of the listed product. As an input this coefficient is provided with a negative sign. At times the corresponding activity might be permitted to run in reverse, signifying an export; then $\underline{m}_j$ becomes the export price.
m_{jk} (Model 3)	Analogous to former parameter. Subscripts: see μ_{jk}.
n (Models 1-2)	Foreign-exchange requirement for importing a unit amount of the extrapolated products, of the branch: i.e. the import price of the extrapolated products, treated as a single aggregate commodity: as an input this coefficient is provided with a negative sign. At times the corresponding activity might be permitted to run in reverse, signifying an export: then $\underline{n}$ becomes the export price.
n_j (Model 3)	Analogous to former parameter. Subscript: $\underline{j}$, branch of extrapolated product.
γ_j	Production cost per unit of extrapolated products treated in aggregate terms. The subscript $\underline{j}$ refers to the serial number of the step in the step-function used to represent the rising trend of these money costs.
d_j	Yearly demand for the $\underline{j}$-th listed product.
b	Exogenous foreign-exchange allocation to or availability for the model. If negative, it signifies a net requirement; in the latter case imports have to be treated as free variables, permitted to take on negative values in order to allow foreign-exchange generation by export.
f_j (Models 1-2)	Limit for individual step $\underline{j}$ in step function for extrapolated products. See γ_j.
f_{ij} (Model 3)	Analogous to former parameter. Subscripts: $\underline{i}$, branch of extrapolated product; $\underline{j}$, serial number of individual step in step function.
λ_{ij}	Variable part of labour of classification $\underline{i}$ utilized per year in maintaining a unit capacity of resource element $\underline{j}$.

$\bar{\lambda}_{ij}$	Fixed part of labour of classification $\underline{i}$ utilized per year in maintaining any capacity in excess of zero of resource element $\underline{j}$.
ξ_j	Variable part of indirect material input utilized per year in maintaining a unit capacity of resource element $\underline{j}$.
$\bar{\xi}_j$	Fixed part of indirect material input utilized per year in maintaining any capacity in excess of zero of resource element $\underline{j}$.
κ_j (Kappa)	Variable part of capital stock tied up in maintaining a unit capacity of resource element $\underline{j}$.
$\bar{\kappa}_j$ (Kappa)	Fixed part of capital stock tied up in maintaining any capacity in excess of zero of resource element $\underline{j}$.
f_{kj}	Upper bound on production scale of listed product $\underline{j}$ in branch $\underline{k}$.
ε_j	Limit on capacity of a single resource element $\underline{j}$.

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