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SOME QUESTIONS CONCERNING THE PLANNING OF BUSES WITH CHASSIS[1/]

by

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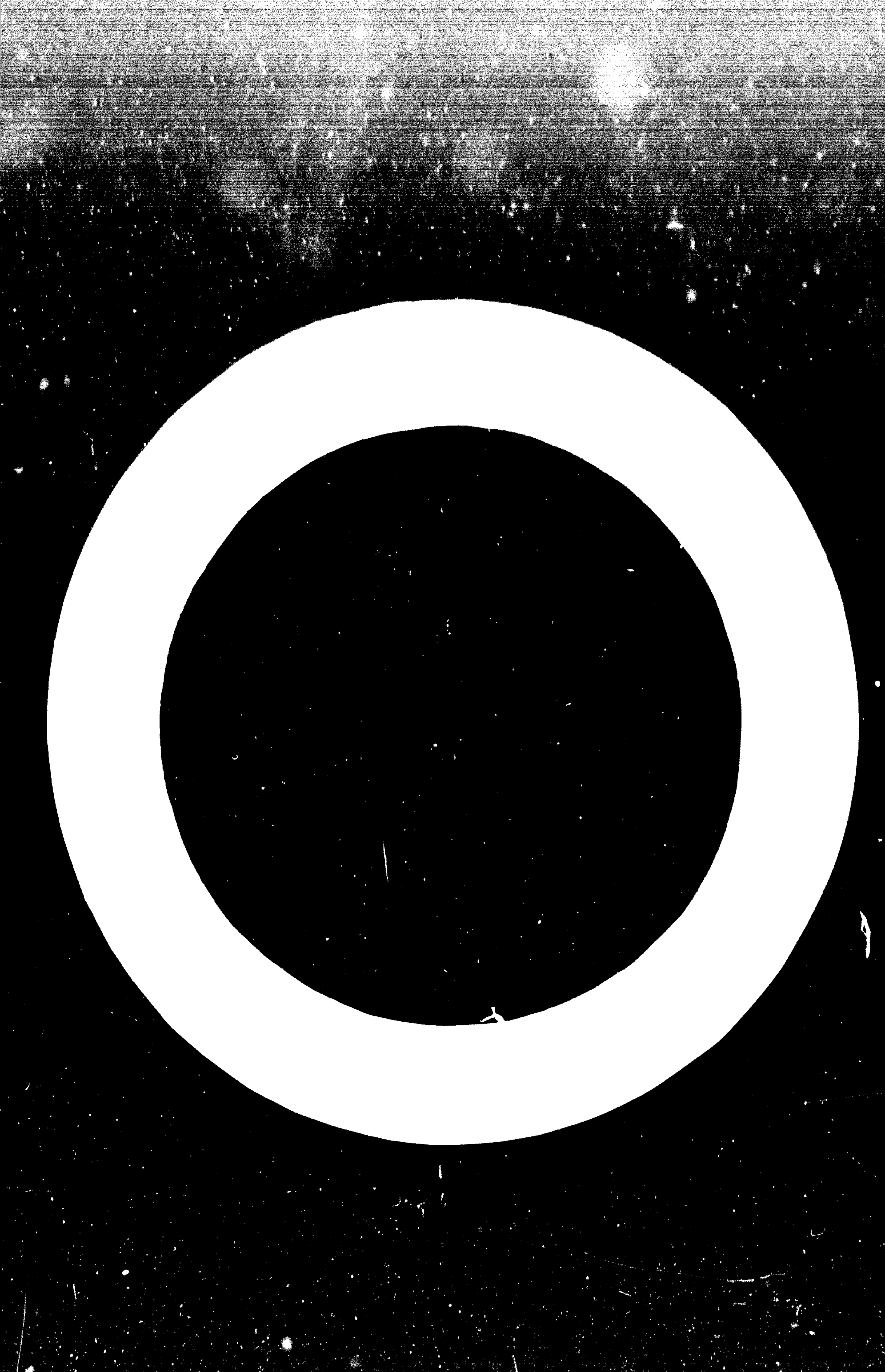

1. Introduction

Both passenger and goods traffic has been rapidly increasing for some time all over the world. The manufacture of motorcars is also developing constantly.

Commensurate with the tasks involved in traffic, the ratio of participation of some branches of communication has also changed. In general, road and air transport have gradually come into the foreground to the detriment of rail and ship transport. Naturally, delivery tasks also increase for rail traffic at an absolute value. This proves the progressivity of road transport even more. The development of road transport can be clearly proved by the fact that between 1955 and 1965 the number of motor cars has almost doubled. A simple multiplication of the motor vehicle pool does not solve the problem of the quantitative increase of traffic tasks. Gradual modification as regards quality of the motor vehicle pool is necessary.

During the qualitative transformation of the motor vehicle pool, the constant increase in dimensions, loadability and capacity of motor vehicles can be established as a tendency. Even the loadability of the roads cannot permanently halt this growth. A typical example of this is that the average length of buses was between 8 and 10 m after the Second World War. In the 1960's, most countries standardized the max. length of buses as 11 m. In the [illegible]

[illegible]

[illegible] a few years ago, most [illegible] 12 m long.

The gradual increase of the permissible axle load enabled the increase of vehicle sizes. For many years the permissible axle load was 10 t in many European countries. This has been raised in many countries to 12 t from [illegible] on. In fact, a 14 t axle load has been permitted in the Danubian countries.

Commensurate with the increased dimensions and capacities of utility vehicles, the manufacturers have aimed at lengthening their period of operation without major repairs. This developmental tendency is also manifested in that the main aggregates of the vehicle are over-dimensioned since the durability of aggregates which work below their load-bilities can be considerably raised. In parallel with the problems which involve durability, the analysis of the behaviour of vehicles in accidents and their safety has become more important.

The increase of durability and the extension of safety seemingly prevent the use of one of the most effective methods of transport capacity, that of the reduction of the dead weight /tare/. Reduction of the weight of vehicles not only results in an important and appreciable economic saving in their production, but a continuous additional profit results during the operation of the vehicle. With an identical investment of energy, more useful load can be carried on a lighter vehicle, i.e. with an identical load, the necessary energy for transport decreases. A lighter vehicle accelerates more rapidly at an identical engine output. On impact, the lighter vehicle of smaller mass represents a

..., ... the ..., etc.

The ... , the recognition of the importance of safety ..., ... more and more important concerning stability dimensioning. The specialisation of vehicles and the ... variants rapidly increase the number of stability calculations.

The problem is even more complex in the manufacture of vehicles in developing countries. In many cases it is not vehicles of individual design that are being manufactured, but the constructions of developed countries are adopted which may have been planned for quite different road conditions. Manufacture is frequently divided. Mechanical fittings and chassis are purchased, and probably only the body work is built independently.

However, vehicles with chassis involve many problems even in developed countries. Well proved chassis designed for trucks can be incorporated in series with closed support structures or bus bodies, In order to clarify this matter, the combined behaviour of the chassis and the body has to be investigated as a result of external load. Without the analysis of this, in my opinion, developing countries cannot start satisfactorily with body building on foreign chassis.

2. Stability characteristics of body structures

In order to obtain the most favourable weight relations, the body and the chassis have to be produced with uniform stability and spread by the distribution of the material in the outer threads.

Of the two conditions, the outer, functional dimensions of the support or the use of the structural elements of constant wall thickness prevent the achievement of a shape of uniform strength. In conventional metal structures, the shape of uniform strength can at best be approached by steps. With the extension of the use of plastics, most probably [illegible] possibility [illegible] elements of constantly changing wall thicknesses can be used.

Apart from the outer dimensions, the loss of the stability of the structure also hinders its spreading. Extremely thin structural elements, evade greater compressive and shearing load locally or they bend outwards entirely. Obviously, stiffeners, local thickening or the use of sandwich structures can extend or increase the limits of stability.

However, a spreading structure has another limit, apart from the problem of stability; without stiffening, it is only suitable for the uptake of distributed load. It cracks locally as a result of concentrated forces or it breaks locally on the effect of a tensile load.

With the spread of the structure and commensurate thinning, our support scarcely resembles the support model used in theoretical stress analysis. The cross-sectional dimensions of the support are almost identical in magnitude to the longitudinal dimensions. The material has to be tested carefully; apart from stresses, the magnitude of deformations have to be investigated. In fact, in certain cases the analysis of the deformation supplies more information about the support than the rated tensile value which frequently only approaches the real tension.

The theory of de Saint-Venant is not valid, or only to a very limited extent for light structures; according to which not too far from the force effect, the concrete force system can be replaced with the resultant. Given local loads from the concentrated forces affect a considerable part of light structures. Tasks which involve the introduction of forces appear because of the invalidity of the theory of de Saint-Venant.

In general, [illegible] theoretical [illegible] are not valid/constant for tension and compression, and linear for bending and twisting/.

The theoretical stress distribution for more complex load is not true even for prismatic supports and for supports without trimmings./ e.g. hindered twisting or obstructed bending/.

The openings and trimmings on the structure completely change the picture of force distributions in their vicinity, and considerable tension peaks arise.

In many cases, the theory of superposition is not valid within the limits of elasticity. The picture of the tension column can be completely re-arranged with the increase of the load of the structure. As an example, we mention the classic Wagner support /Fig.1/. It is known that on load a clean shearing appears in the plate of the Wagner support, in its horizontal belts tension or stress occur, and the vertical stiffeners are free of load. With an increase of the load, the load of some of the structural elements does not change proportionally, but the load becomes completely re-arranged. The plate receives a tension load. The belt rods bend apart from the tension or stress, and the vertical stiffeners initially free of load become pressed, and may buckle.

The calculation characteristic listed, the large number of stiffeners in the structure and the openings equally complicate the stress calculations.

Light structures are generally uncertain supports: their inner stresses can only be determined with complex equations of many unknowns. After having determined the inner

[illegible]

value of stability of the structure or a part of it, whether [illegible] the [illegible] stage [illegible] the entire calculation may have to be repeated.

The calculations are greatly complicated as the primary function of light structures, unlike engineering structures, is usually not static, but intended for traffic and delivery. Therefore, the construction of the structure is frequently not governed by statical, but by communication, aesthetic or even fashion trends.

The loads, compared with the static loads of buildings, are extraordinarity varied. Certain load types require different structural constructions. Possible synchronism of different loads requires separate investigations. The principle of superposition can be used generally only to a very limited extent for these investigations.

Briefly, light structures, compared with general machine structures, need very circumspect, through and complex tensile analysis.

Involuntarily the questions may arise whether these calculations have to be carried out, whether they are worth-while and even possible, owing to the complexities.

All three questions have to be answered in the affirmative.

The set aim, for truly light structures, can only be achieved with the improvement of the tensile analytic model, and, unfortunately, with the obligatory complexity of the calculations. It may be suggested that measurements should be taken instead of the complex calculations. Fundamentally this is incorrect. Tensiometric measurements are

[illegible]

[illegible] systems [illegible] ments can only be carried out according to the calculations. [illegible] methods have to be checked constantly with measurements.

3. Problems of the selection of a static model

Operational conditions and the inner forces resulting in the structure are manifold and interrelated with one another in a complex manner, e.g. the use of the screw of the carriage body does not depend only on the condition of the road, but on the rate of wheels, the twisting rigidity of the bus, its springs, gear system and mass distribution. These complex relations cannot be left out of consideration on dimensioning.

Extensive simplifications are necessary in order to handle these connections. A simplification of this kind is that we trace back the loads on the carriage body to two basic load groups. These are the bending stress and the twisting stress.

A simplification of the loading relations can be considered as a model which obviously has to be improved during operational measurements. However, we do not deal with the models of loading in this study, merely with conventional "bending" and "twisting" and the investigation of pretensions which originate from manufacturing inaccuracies for the entire tensile calculations and measurements which govern reliability.

Even with these simplified load suppositions, the complex body structure cannot be considered in its real geometrical dimensions in the calculations. Therefore, the engineering structure has to be simplified to a static and finally a calculation model which are approachable for our [illegible] fundamental question for the selection of a model is, how much its behaviour should approach that of the real structure. [illegible], it seems natural to select the static model as a true replica of the real structure as far as possible, since calculating machines are suitable to solve very complicated equations with several hundred unknown factors. If we consider the rough estimation of the external stresses which affect the structure, the proportion of the material, the force re-arrangements due to technological solutions, then the selection of a less accurate but more easily approachable model for calculations seem expedient, since no calculation can be more accurate than the initial information used during the calculations.

Where should we draw the limit between the two views? The prevalent limit depends on many factors; obviously, with a knowledge of more accurate external stresses and a proved calculation method, a more accurate model should be selected and vice versa. Thus the selection of the model constantly develops, changes from the simpler to the more complicated, and follows the general development of engineering knowledge. However, the question may arise whether there is no rule of absolute validity for the selection of the model. Today, it can be modestly stated that the model qualitatively well reflects the behaviour of the real structure.

If the static model of the vehicle has been selected according to the above basic principles, it can be established that in no type can we speak of a unified static model suitable for the investigation of all load kinds. The models used for various load types differ considerably from one

[illegible] at [illegible] the [illegible]
of the [illegible] for their own load type. The same model, probably under slightly different conditions [illegible] does not resemble the behaviour of the real structure. The static model in certain cases follows in its geometrical arrangement the given body structure. In other instances the abstraction of the model results in such a simplified "structure" that it does not even resemble the geometrical arrangement of the real structure. The variability of the models can be readily studied, for example, on a bus with chassis.

If in the side view drawing of the bus the components are marked, the force factors which indicate the weights of the passengers, it seems obvious that the entire, complex spatial support system /Fig. 2/ can be modelled with a simple double-support straight beam /possibly with a beam of changing rigidity/. Whilst we search for the reaction forces, wheel loads and stress loads on the carriage body, our model is satisfactory. However, from the bending stresses, thus established the utility of certain structural elements can no longer be determined, e.g. directly with the help of the inertias for the cross-sections of certain bodies. Our tensiometric measurements of the carriage body clearly showed that the cross-sections of the body did not even remain flat during bending, that the tension derived from the bending does not change linearly along the height of the support structure, in fact it rapidly changes signs several times /Fig. 2.c/.

A more thorough investigation of the measurement results led to the recognition that the bus, instead of a simple beam, has to be considered as a more complex support system, a so-called hinged connection support lattice /Fig. 3./ which consists of separate longitudinal and cross supports. These support elements more or less well follow the theoretical stress analysis connections, the tensile distribution is adequate with the exception of the curved passages according to the Navier equation, i.e. if some of the support elements are grids, then the measured rod forces satisfactorily agree with the calculated values.

Mostly, however, the total of the bending stress measured on the longitudinal supports and side walls in 5-10 % less than the total bending compression measured on the same cross-section. This quantitative deviation - always in the same direction - shows that whilst the model gives an accurate magnitudinal picture of the play of force, it neglects some effect - support element. This neglect means not having taken into consideration the window area and the roof structure; nor have we considered that the two longitudinal supports and the centre line of the two walls are not in a plane, but the centre line of the side walls is considerably higher, by approx. 400-500 mm, than the centre lines of the longitudinal and cross supports. As a result of the complete deformation of the support grid, a further secondary load occurs on the support structure which the primary calculation /neglecting the load change as a result of the deformation/ cannot take into consideration. Of the two omissions, the influence of the window and roof structure is more important. The primary calculation can take into consideration /Fig. 4/ the importance of the window and roof structure, but the number of the statically non-determined amounts considerably increase, and the equation system for their determination become considerably more complicated, and to solve it is extremely difficult owing to the deviating magnitude of the unknown factors. Therefore it is expedient to trace back the investigation of the bending load to the original and simpler support grid model, and to consider the effect of the window area and the roof structure by a separate correction. The side walls seem to be more rigid than deduced from the geometrical data because of the presence of the window and roof structure. In consideration of the usual measurement relations, we carried out small-sample tests to assess the effect of the increase of rigidity.

In the case of lattice structures /which is most frequent in the side walls of buses/, the roof structure results in an approx. 10-15 % increase of rigidity /Fig. 5/. Thus, the support lattice calculations should be carried out in the equation with a 10-15 % increase for the side walls, instead of the rated rigidity. As a result of the calculation, obviously the side wall of the bus receives a greater partial load than deduced from the original rigidity. With the load obtained in this way, irrespective of the support lattice, the importance of the window area and the roof structure should be investigated. The separate investigation of the window area can be carried out in several ways, depending on whether we consider the upper end of the window column suitably compressed and rigid on the roof according to reality or whether we visualize it as a hinged connection. The two types of models exclusively influence the load of the window columns, the entire frame support is insensitive to this model variation. The two, tests, independent from each other, lead to simple and readily solved equation systems; true, that because of the independent solutions the compatibility conditions do not materialize with due rigour. However, the theoretically rigourous solution also neglects the secondary phenomena, and the unsatisfactory materialisation of compatibility is secondary in practice, thus identical with the calculation errors which take into consideration the contribution to deformation.

We have not mentioned the contributory effects of the doors located on one side of the bus which thoroughly distort the hitherto satisfactory picture of the model. Luckily, however, the participation of the doors can be followed well mathematically, and, similar to the window area, with separate calculations, but due theoretical rigour, their influence on the force play of the structure can be determined.

The door opening located on one side upsets the symmetrical relation /Fig. 6/ of the structure. The structure remains symmetrical along most of its length, with the exception of the door. The calculations of these slightly asymmetrical /quasisymmetrical/ supports can be traced back to the investigation of a symmetrical and a separate interference support.

The hinged connection, simple support lattice model designed for bending, evades the twisting load because of the hinged connection, i.e. the model is unstable.

The real bus body, however, is able to take up the twisting load. In fact, according to measurements /fig. 7/, its twisting rigidity can reach a considerably high value, $J_tG = 4-8.10^9$ kgf/cm².

We have ascertained from the measurements that the applications of linear mechanics are not valid for the twisting load of buses, i.e. the deformations arising are not proportional with the load.; a more detailed analysis of this has not been prop[illegible] out.

According to the deformation measurements, naturally it is desirable to consider the body as a thin-walled tubular structure /[illegible]/ which [illegible], the internal forces that can be calculated from theoretical equations. However, the thin-walled tubular structure can only be taken as a rough approach, for several reasons:

1. The twisting load does not affect the ends of the tubular structure, but at several intermediate cross-sections, depending on the arrangement and suspension of the wheel. In case of general cross-section data, an impeded twist results instead of a clean twist, and so in the cross-section corners of the bus which can be considered more or less as a paralellogram, additional normal forces arise.

2. The tubular structure is lacking in the windows and doors, consequently, in these passages instead of the shearing processes which can be calculated theoretically, it is necessary to take up the twisting loads originating from the inner forces through the bending of the window and the door frames. This internal force distribution is mostly statically indeterminate.

3. The cross walls, including the end walls, of the bus suffer deformation to a considerable extent as a result of the twisting load, hence the shape holding /cross-section holding/ of the structure is insatisfactory. During complete deformation measurements, in addition to the slewing of some of the cross-sections, the def

from [illegible] observed.

All these deviations make the justification of the use of [illegible]. In spite of this, the literature, almost without exception, advocates the use of the Bredt equation.

The tubular structure model, apart from its own internal errors and problems, does not completely conform to real bus structures because the longitudinal and cross-supports and possible luggage racks are completely neglected, whereas their own twisting rigidity is very considerable. /$I_t G$ = 1-3.10^9 kgf/cm²/.

Consideration of the twisting rigidity of the bottom frame and the luggage racks make the static calculations very difficult. Therefore, the calculation should be carried out in two stages, similar to that of the window area. It can be assumed in the calculation that the tubular model of the body and the bottom structure distribute the complete twist load in proportion to their twisting rigidities. This supposition usually does not satisfy the conditions of compatibility for the whole of the structure, but the calculated play of force conforms adequately with the measurements.

For the analysis of the self-load of the bottom structure, the support lattice supposed to be hinged at bending loads has to be replaced by a rigidly connected support lattice.

The compressive rigid connection reflects the play of force more accurately in bending, too, but the difference is small between the two models on bending, whereas the difference is fundamental on twisting, i.e. it transforms to stability from a labile structure. The static calculation in itself is [illegible] for the compression-rigid connection support grid. However, for the calculation of rigidity conditions of the bottom structure which occur in buses, readily approachable methods are available with which the distribution of internal forces and the magnitude of impeded twisting most important for the tensile condition, can be easily determined.

It should be mentioned that several details of the play of force are unclarified in the investigation of the twisting load.

The neglect of this topic has, of course, other objective reasons, apart from its complexity. The twisting load in the operation of the bus is less important than the bending load. It is only important for buses which run on bad roads. The secondary importance of the twisting load is shown by the fact that the twisting load which belongs to the 3-point support only occurs on extraordinarily bad roads, through the simultaneous diagonal lift of the two axles, taking into consideration the springs and the body rigidity of the bus. A max. 25-30 % of the max. twisting load can occur on the roads under consideration.

The determination of the inner forces derived from the inaccurate matching of the chassis and the carriage body can lead to very interesting static models.

For a known magnitude of manufacturing inaccuracy or for a conscious assumption, the use of a hinged-connection support-grid model /Fig. 9/ is expedient for bending. By knowing the accurate dimension deviations of certain connection points, separate compatibility equation systems can be written up for every one. Thus, the internal forces can be determined with the superposition of a large but finite number of equation systems. Some of the equation systems differ only as to their load factors; Thus, they require only a single matrix inversion.

This direct form of the task needs no further analysis. A much more difficult question, so far unsolvable in practice with the existing models is the task of indirect variation. Let us compose this indirect task:

What manufacturing accuracy and what tolerance field has to be prescribed for the designing of chassis and carriage bodies so that no greater internal force arises in the body on incorporation than that regulated by us.

The difficulty is that we are looking for a universal prescription for manufacturing accuracy which determines an identical tolerance field for every fixing point because the technological properties do not permit a difference between certain points; moreover, the results should be valid not only for one type of bus, but in general for all buses with large chassis. As a final difficulty we mention that the components during manufacture are not produced for the centre, lower or upper limits of the tolerance field, but they may take up any value within the limits of the tolerance range, and so the entire indirect task can only be approached with the methods of probability calculations.

These viewpoints necessitated the development of simpler and more regular models than those used hitherto. The model can be simplified in two stages. First, we consider the support lattice used so far as regular, i.e. we assume that the longitudinal supports and rigidity of the side walls are constant along their lengths, we assume a balanced cross-support distribution, and consider the rigidity of the cross supports to be equal. This sort of simplification of the model makes the use of mathematical, i.e. differential equations, possible to determine the internal forces. The differential equation for the real structure is a parametric, fourth-order, homogeneous linear equation; more accurately, a simultaneous equation system whose results can be compounded by solving the separate equations.

For the established values of the rigidity relation of the cross-supports and longitudinal support elements, this differential equation can be reduced to a second order one.

As regards the pure differential equations, they formally agree with the flexible support in the fourth-order case, or with the fixed support continuous equation of the second order. Naturally, the rigidity of this artificial support agrees with the reciprocal sum of the rigidity of the longitudinal supports /Fig. 10./.

If we consider that the solution of the differential equation for more than 5 supports, scarcely differs in practice from that for an infinite number of supports - a considerable simplification - the abstraction of the model is even more conspicuous. However, this abstraction is necessary, if we wish to consider the static distribution of certain manufacturing tolerances.

4. Problems of static calculations

In vehicular structures, we usually determine the play of force by force methods as the uncertainty degree of the structures is smaller than the shift of the multiple junction. However, in the force method, the setting up and solution of the compatibility equation system involves a great deal of time, and this greatly depends on the experience of the person who makes the calculation compared with the mobile system.

In mechanized calculation, the reduction of time needed for the calculation programme is a new problem or possibly a modification of the calculation which makes the use of an existing programme possible.

These three problems /setting up the equation system, its solution and the use of an existing programme/ make equally necessary the investigation of the fundamental question of the force method and the selection of the body support.

The body support is usually taken in practice by a suitable number of assumed crosscuts of the original support, so that the new support obtained should be statically defined. Compared with this custom, in our opinion, it is a necessary and sufficient condition for the force method that, for the formation of the correct body support with the assumed cross cuts at a given external load and during the activity of the superficial connection forces, the body support should not be labile. The definition permits the labile statically defined and statically undefined use of the body support, only it may not behave labilely at the given load.

A detailed discussion of the unsuitability of the body support which is labile at a given external or superfluous connection force is unnecessary, since as a result of the load a support of this kind deforms to an extent which results either in the collapse of the structure or in fundamentally a different shape from the original geometrical design.

We prove the correctness of a labile body support which remains stable at a given external load and superfluous connection forces, with the example /Fig. 11/ of an arbitrarily selected labile body support. In the first two examples, the labile body support did not represent a special

[illegible]

[illegible]

The deliberately chosen labile body support can simplify the static calculations in many instances.

In our definition we have unambiguously determined the conditions for the correct choice of the body support, but the selection of a favourable body support for calculation techniques needs further instructions. Thus, e.g. it is important that the play of force of the body support owing to the external load should "resemble" the real play of force as the compatibility equation system for such a body support is, generally, less sensitive to the numerical accuracy of the calculation. In vehicular structures this "similarity" can be understood more accurately. The body support of the vehicular structure possesses the "fundamental properties" of the structure. The fundamental properties of vehicles, according to our investigations are:

1. Single or several axled structural symmetry
2. The periodic /rhythmic/ repetition of the structure, and
3. The combination of the independent support systems of the structure.

The use of these three basic properties listed above, demands the generalization of the definition quoted of the body support as, e.g. the structural symmetry and the structural rhythm often results only in lability, or it can be ensured with the use of a statically undetermined body structure /Fig. 12/.

We have made the work of the designer considerably easier with the definition of the body support and the introduction of the demands for "similarity". For complicated structures, [illegible] od which other [illegible] are not of equal value as regards calculation techniques.

We are looking for a method with which the most favourable body support can be sel[illegible], more advantageous than the original, at modest demands.

Let us mark the superfluous connection forces with $\underline{\underline{m}}$ of a statically n-times indefinite structure suitably selected for a body support. /In the following, for the sake of brevity we shall exclusively consider the resultant deformations due to bending stresses, but our deductions are valid if the shearing deformations due to forces, normal forces, twisting compressions, etc. cannot be neglected./ Obviously,

$$\underline{\underline{m}}^{*} = \left[M_1^1 /s/,\ M_2^1 /s/, \ldots M_i^1 /s/, \ldots M_n^1 /s/ \right]$$

if we mark with s the approved coordinate of the support structure.

Let M_1^1 $M_2^1 \ldots\ldots M_i^1 \ldots\ldots M_r^1$ mean the order 1...2 ...n in the hitherto visualized crosscut the internal force derived from the acting unit connection load pair as a function of s. We mark the use derived from the external load of the identical body support as scalar M_o /s/.

The unknown connection forces

$$\underline{x}^{*} = \left[X_1,\ X_2, \ldots X_i, \ldots X_n \right]$$

are given with vectors. If in the s coordinate point of the support the rigidity is JE, the compatibility equation system can be written as follows:

$$\int_{/s/} \underline{\underline{m}}\ \underline{m}^{*} \frac{ds}{JE}\ \underline{x} + \int_{/s/} \underline{\underline{m}}\ M_o \frac{ds}{JE} = \underline{0}$$

or briefly

$$\underline{R}\,\underline{x} + \underline{a} = \underline{0} \qquad /1/$$

Let $\underline{T}$ be an invertible $n \times n$ type quadratic matrix and let us replace value $\underline{x}$ with $\underline{T}^{-1}\underline{y}$. Obviously, we can write

$$\underline{T}\,\underline{R}\,\underline{T}^{-1}\underline{y} + \underline{T}\,\underline{a} = \underline{0} \qquad /1'/$$

or in its expanded form, with some re-arrangement:

$$\int_{/s/} (\underline{T}\,\underline{g})(\underline{T}\,\underline{g})^{*} \frac{ds}{JE}\,\underline{y} + \int_{/s/} (\underline{T}\,\underline{g})\, M_o \frac{ds}{JE} = \underline{0}$$

By marking the expression $/\underline{T}\,\underline{g}/$ as $\underline{h}$ in the following:

$$\int_{/s/} \underline{h}\,\underline{h}^{*} \frac{ds}{JE}\,\underline{y} + \int_{/s/} \underline{h}\,M_o \frac{ds}{JE} = 0 \quad \text{or briefly}$$

$$\underline{S}\,\underline{y} + \underline{b} = \underline{0} \qquad /2/$$

Expression /2/ completely agrees in shape with expression /1/, and can be understood as an equation system belonging to another body support. The use of the entire M of the support structure, as clearly shown, can be, obviously, expressed with the use of either equation /1/ or /2/:

$$M_{/s/} = M_o/s/ + \underline{x}^{*}\underline{g} = M_o/s/ + \underline{y}^{*}\underline{h} \qquad /3/$$

The set out relationships make the transformation possible from a selected body support to a more favourable one which can be considered a linear transformation of the original body support. The above linear transformation hitherto has been exclusively understood as the transformation of the unknown $\underline{x}$.

The advantage of the new understanding is that instead of the complicated mathematical consideration of $\underline{T}$ transformation matrix we can select the $\underline{h}$ stress by way of a simple comparison.

It can be shown that it is possible to produce an $\underline{n}$-times statically indetermined task from a selected body support, but according to the definitions of a body support with linear transformation, provided we select the $\underline{\underline{T}}$ transformation matrix from the entire mass of the n x n type quadratic invertable matrices.

We have mentioned earlier that the use of statically indeterminate body support is expedient in many cases. The correct use of the indeterminate body support can be proved mathematically. The proof reveals that the use of the statically indeterminate body support can be considered a characteristic instance of body support transformation. Here, we determine the $\underline{\underline{T}}$ transformation matrix from the original $\underline{\underline{D}}$ coefficient of the compatibility equation.

If

$$\underline{\underline{D}} = \begin{bmatrix} \underline{\underline{D}}_{11} & \underline{\underline{D}}_{12} \\ \underline{\underline{D}}_{21} & \underline{\underline{D}}_{22} \end{bmatrix} \quad \text{and } \underline{\underline{D}}_{11} \text{ and } \underline{\underline{D}}_{22} \text{ are quadratic}$$

then

$$\underline{\underline{T}} = \begin{bmatrix} \underline{\underline{E}} & \underline{\underline{0}} \\ -\underline{\underline{D}}_{21}\underline{\underline{D}}_{11}^{-1} & \underline{\underline{E}} \end{bmatrix} \quad \text{where } \underline{\underline{E}} \text{ is the unit matrix} \qquad /4/$$

As can be easily seen the $\underline{\underline{T}}\,\underline{\underline{D}}\,\underline{\underline{T}}^{x}$ matrix are diagonal hypermatrices whose inversion is carried out in blocks.

The assumption of the linear transformation of the statically indefinite body support can be used for the investigation of the statically indefinite support system connection. A classical case of this is the incorporation of the trestle chassis and the body work of the bus.

Let $\underline{\underline{D}}_{11}$ and $\underline{\underline{D}}_{22}$ be separate statically indeterminate supports /e.g. chassis and carriage body/ as the coefficient of the compatibility equation, and let us connect the two indetermined supports with another statically indefinite linkage [illegible] the compatibility equation [illegible] entire structure [illegible] matrix form:

$$\begin{bmatrix} \underline{\underline{D}}_{11} & \underline{\underline{0}} & \underline{\underline{D}}_{13} \\ \underline{\underline{0}} & \underline{\underline{D}}_{22} & \underline{\underline{D}}_{23} \\ \underline{\underline{D}}_{31} & \underline{\underline{D}}_{32} & \underline{\underline{D}}_{33} \end{bmatrix} \begin{bmatrix} \underline{\underline{X}}_1 \\ \underline{\underline{X}}_2 \\ \underline{\underline{X}}_3 \end{bmatrix} + \begin{bmatrix} \underline{\underline{d}}_1 \\ \underline{\underline{d}}_2 \\ \underline{\underline{d}}_3 \end{bmatrix} = \begin{bmatrix} \\ \underline{\underline{0}} \\ \\ \end{bmatrix} \qquad /5/$$

The matrix which transforms equation /5/ into a diagonal hypermatrix on the basis of connection /4/ assumes the following simple form:

$$\underline{\underline{T}} = \begin{bmatrix} \underline{\underline{E}} & \underline{\underline{0}} & \underline{\underline{0}} \\ \underline{\underline{0}} & \underline{\underline{E}} & \underline{\underline{0}} \\ -\underline{\underline{D}}_{31}\underline{\underline{D}}_{11}^{-1} & -\underline{\underline{D}}_{32}\underline{\underline{D}}_{22}^{-1} & \underline{\underline{E}} \end{bmatrix}$$

On carrying out the transformation with matrix $\underline{\underline{T}}$, the coefficient third block of the diagonal hypermatrix obtained conforms precisely to the statically indefinite body support.

This method can be extended to the determination of internal forces which originate from the connection of more than two, but independent basic system connections.

[illegible] the connecting members is one, and the numbers of the basic systems are 1, 2, 3 ... /Fig. 13/. The parallel combination of the connection members does not involve any new theoretical work, and the two elements connected in parallel can always be replaced by a single connection member of a higher degree of indetermination.

This result is used for the recalculation of statically indeterminate supports, if, during measurement, it is revealed that the measurements are unsuitable owing to [illegible] or over dimensioning. By considering the entire support as a basic system, the supplementary support elements connected in parallel with the unsuitably dimensioned elements /which ensure the adequate cross section/ can be taken for connection elements. By varying the rigidity of the connection members, we can find the most suitable rigidity stability data, without having to recalculate the basic system in its entirety.

This method is suitable for the calculations of structural irregularities, trimmings , errors of symmetry and rhythm.

Up to now the elements of the matrices have been mathematical functions. When using calculating machines, it is advisable to dispense with these markings, and write so that the elements of the matrices are numbers.

Let us replace the column vector with $\underline{\underline{m}}$ /s/

$$\underline{\underline{M}} = \begin{bmatrix} M_1^{1a} & M_1^{1b} & M_1^{2a} & M_1^{2b} \ldots & M_1^{ma} & M_1^{mb} \\ M_2^{1a} & M_2^{1b} & M_2^{2a} & M_2^{2b} \ldots & M_2^{ma} & M_2^{mb} \\ \vdots & & & & & \\ M_n^{1a} & M_n^{1b} & M_n^{2a} & M_n^{2b} \ldots & M_n^{ma} & M_n^{mb} \end{bmatrix}$$

rectangular matrix where M_1^{ka} is the utility originating from the inner pair of forces effecting the i assumed cross-cut the network of the holding line /s/ at the $\underline{k}$ beginning of the passage and the M_i^{kb} ending of the same passage. M_i changes linearly /Fig. 14a/ between the two passage limits.

Instead of the M_o /s/ scalar function which originates from the external load, let us consider the moment values of M_o /s/ at the beginning and end of every holding passage as a vector, i.e. let us introduce the marking:

$$\underline{\underline{m}}_o^* \quad M_o^{1a} \quad M_o^{1b} \quad M_o^{2a} \quad M_o^{2b} \ldots\ldots M_o^{ma} \quad M_o^{mb}$$

Let us compile the flexible properties of the support passages in a diagonal hypermatrix /spring matrix/:

$$\underline{\underline{R}} = \left\langle \underline{\underline{R}}_1 \quad \underline{\underline{R}}_2 \quad \underline{\underline{R}}_3 \ldots\ldots \underline{\underline{R}}_k \ldots\ldots R_m \right\rangle$$

where matrix $\underline{\underline{R}}_1$ is the spring constant of k holding passage:

$$\underline{\underline{R}}_k = \frac{r_k}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$r_k = \frac{l_k}{J_k E_k}$$

l_k, J_k and E_k are the lengths of k holding passage, the inertia and flexible modulus of its cross-section, respectively.

With the help of these

$$\underline{\underline{D}} = \underline{\underline{M}}\,\underline{\underline{R}}\,\underline{\underline{M}}^* \quad \text{and} \quad \underline{d} = \underline{\underline{M}}\,\underline{\underline{R}}\,\underline{m}$$

assuming that only the bending stresses cause deformation on the holder and that the bending stresses can be disconnected along the holding line network to passages which change linearly between two passage limits each, and that the rigidity of the support is constant on the given passage.

Of course, this method can be extended to cases when M compression is not linear along the passage, but changes parabolically. In second order parabolas, however it is not sufficient to give the values of the inner tube on the passage limits, but the M^c value taken at the middle of the passage has to be given, see /Fig. 14b/.

$$\underline{\underline{m}}_i^* = \ldots\ldots M_i^{ka} \quad M_i^{kc} \quad M_i^{kb} \ldots\ldots$$

and at the same time the $\underline{\underline{R}}_k$ element of the spring matrix changes:

$$R_k = \frac{r_k}{30} \begin{bmatrix} 4 & 2 & -1 \\ 2 & 16 & 2 \\ -1 & 2 & 4 \end{bmatrix}$$

5. Summary.

In the design of bus body and underframe construction the statical dimensioning of its elements is very important. The basic question of the force method solution for statically indeterminate structures is the selection of a correct basic system. For this purpose, the usual definition of the basic system must be reviewed, and a definition should be introduced with a wider interpretation than the former one. In some cases, the use of statically indeterminate and instable basic systems, respectively, should also be permitted. The known linear transformation of the compatibility matrix equation may be regarded, in general, as the transformation of the basic systems /the change of the basic systems/. The generalization of statically indeterminate basic systems permits the reduction of the statical calculation of complex jointed-beam systems /i.e. bus body and underframe/ to the repeated calculation of their elements. In the case of a limited number of external loads, this may be regarded as an iteration consisting of a finite number of steps; for a generalized external load, there is no iteration required because the bending moments resulting from the units of redundancies are given. The process may also be utilized for redimensioning statically indeterminate structures.

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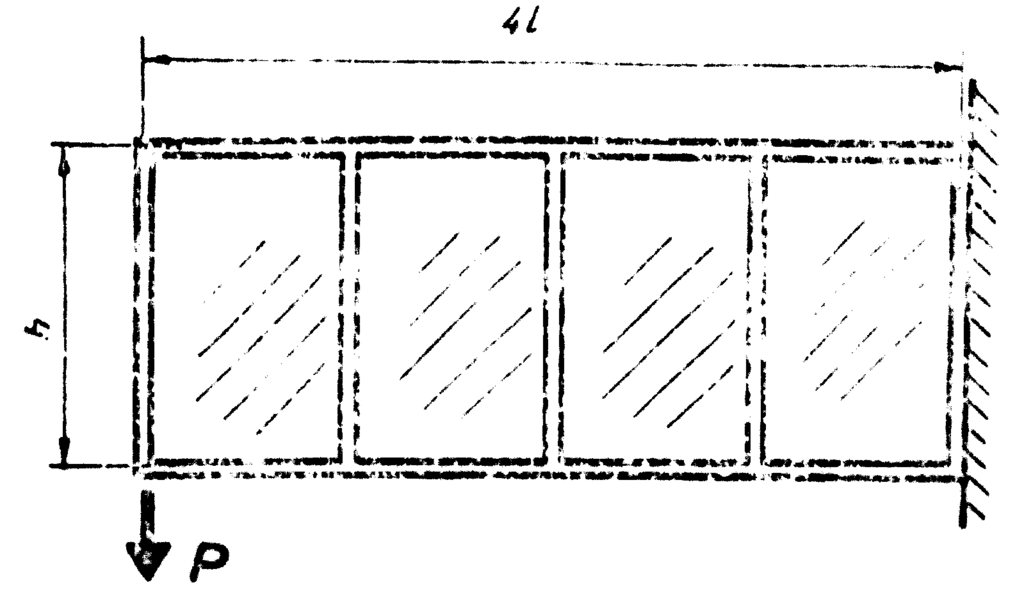

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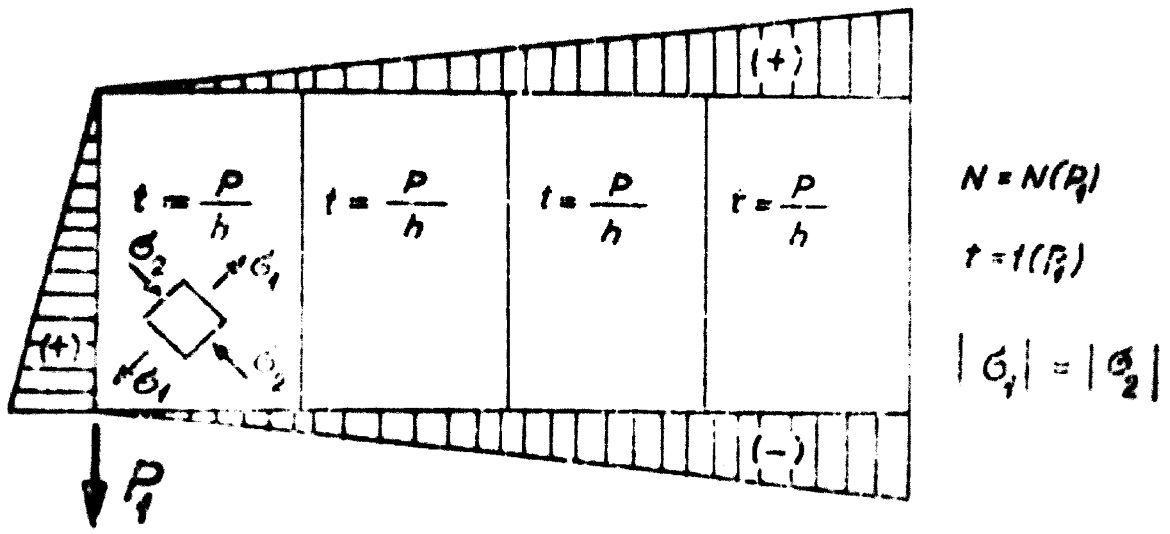

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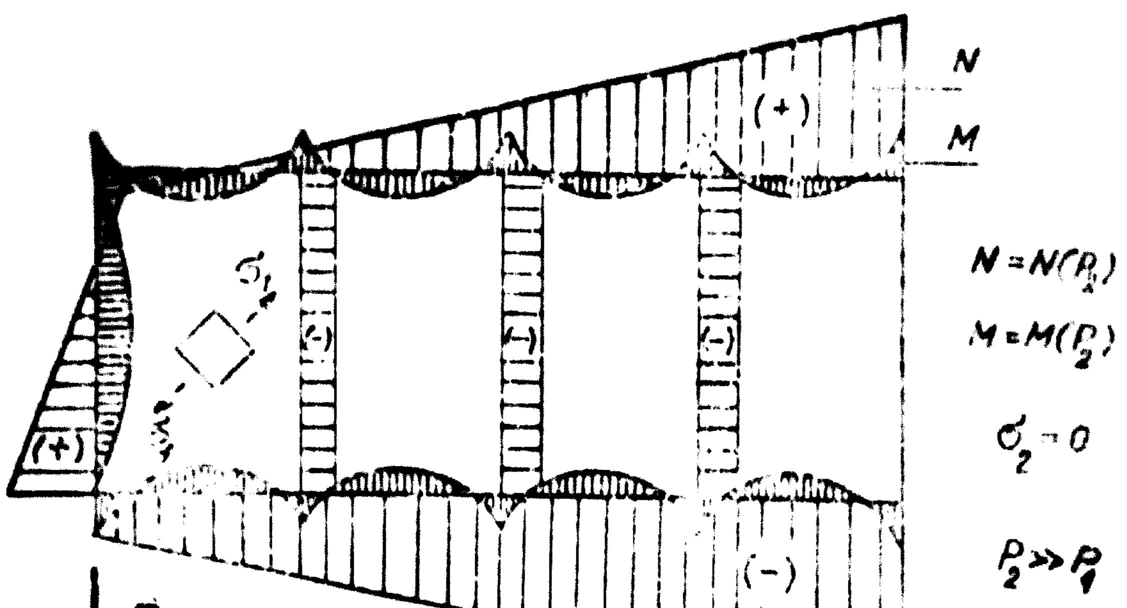
$t = \frac{P}{h}$
$t = \frac{P}{h}$
$t = \frac{P}{h}$
$t = \frac{P}{h}$
σ_2
σ_1
σ_1
σ_2
(+)
(−)
P_1
$N = N(P_1)$
$t = t(P_1)$
$|\sigma_1| = |\sigma_2|$

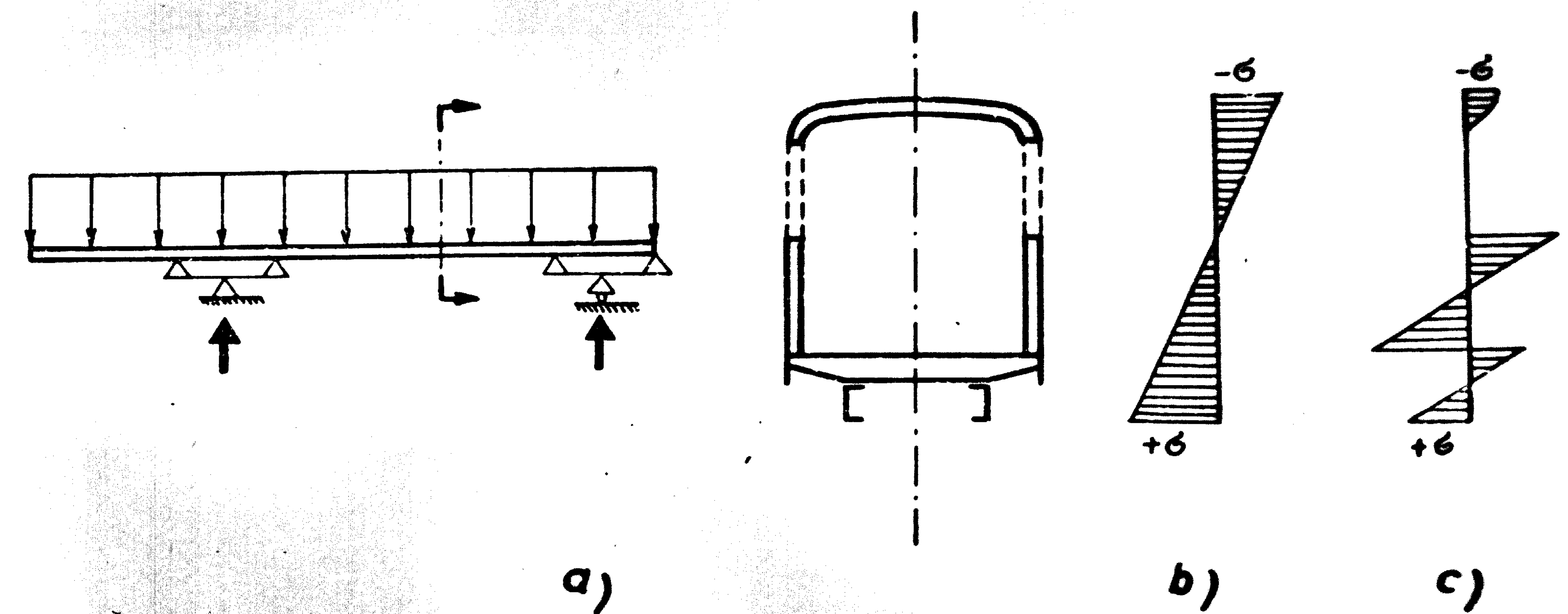
a)



N
M
(+)
σ_1
(-)
(-)
(-)
(+)
(−)
P_2
$N = N(P_2)$
$M = M(P_2)$
$\sigma_2 = 0$
$P_2 \gg P_1$

b)

1.



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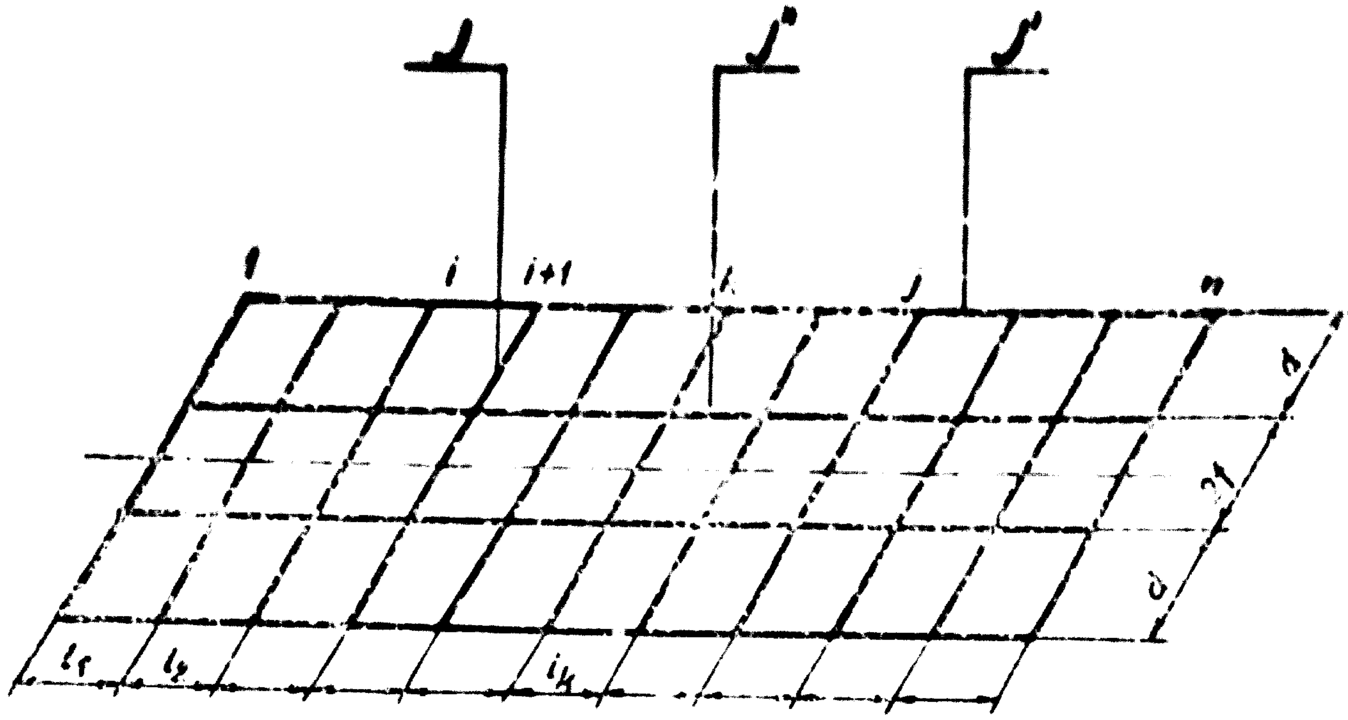

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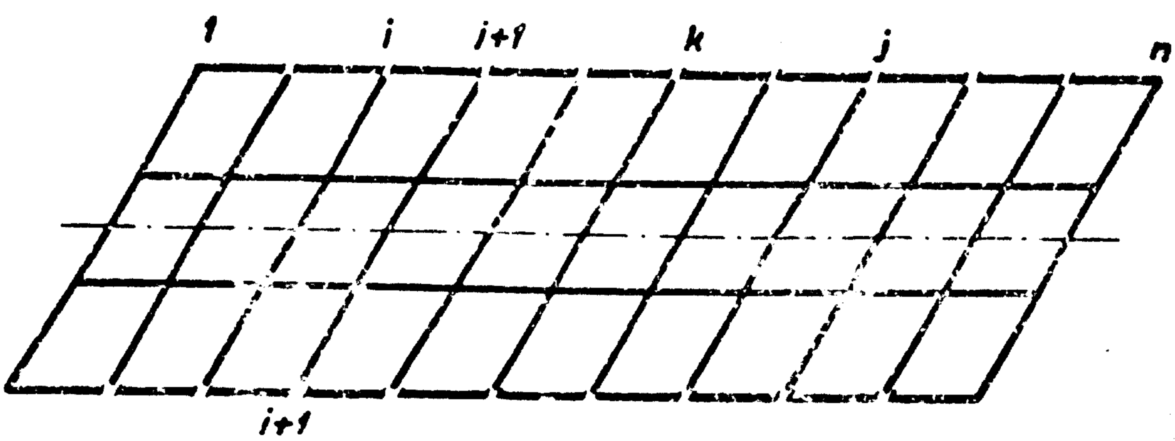

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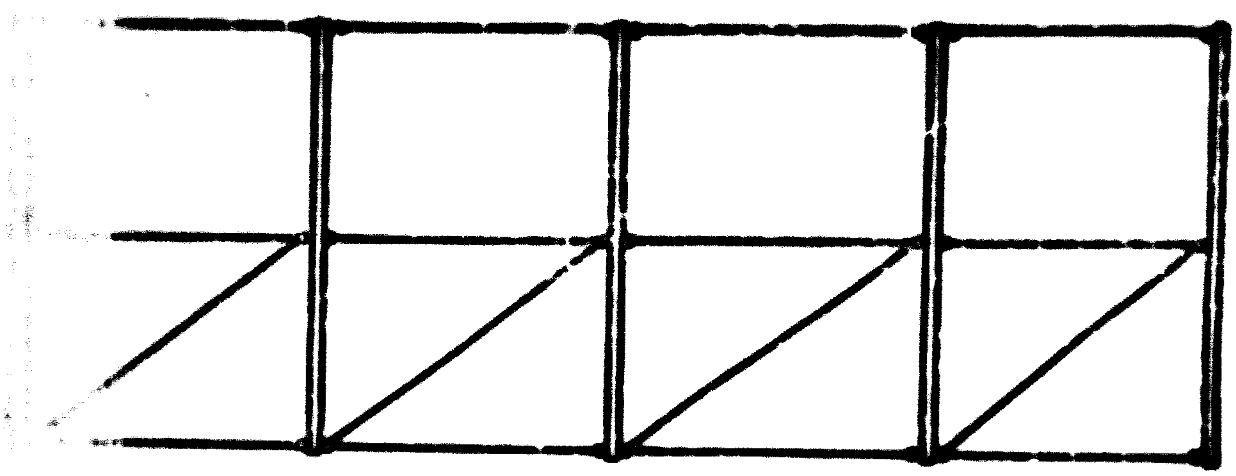

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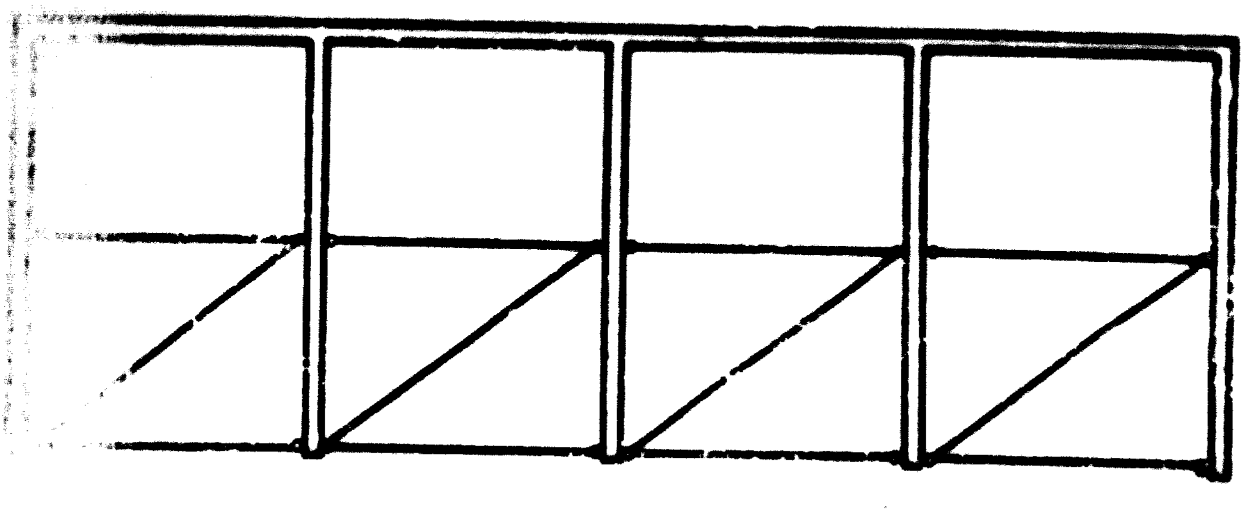

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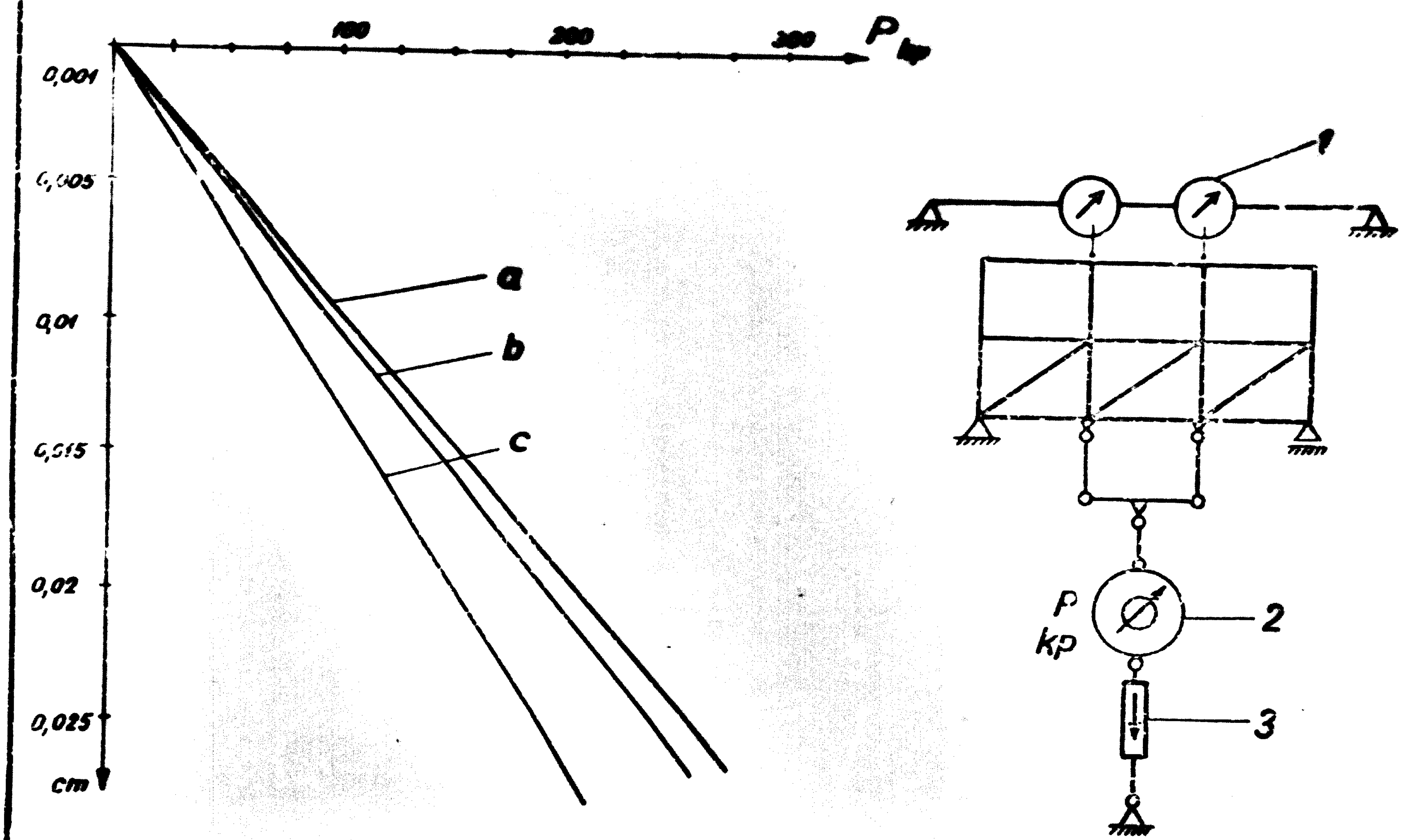

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300
P kp
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cm
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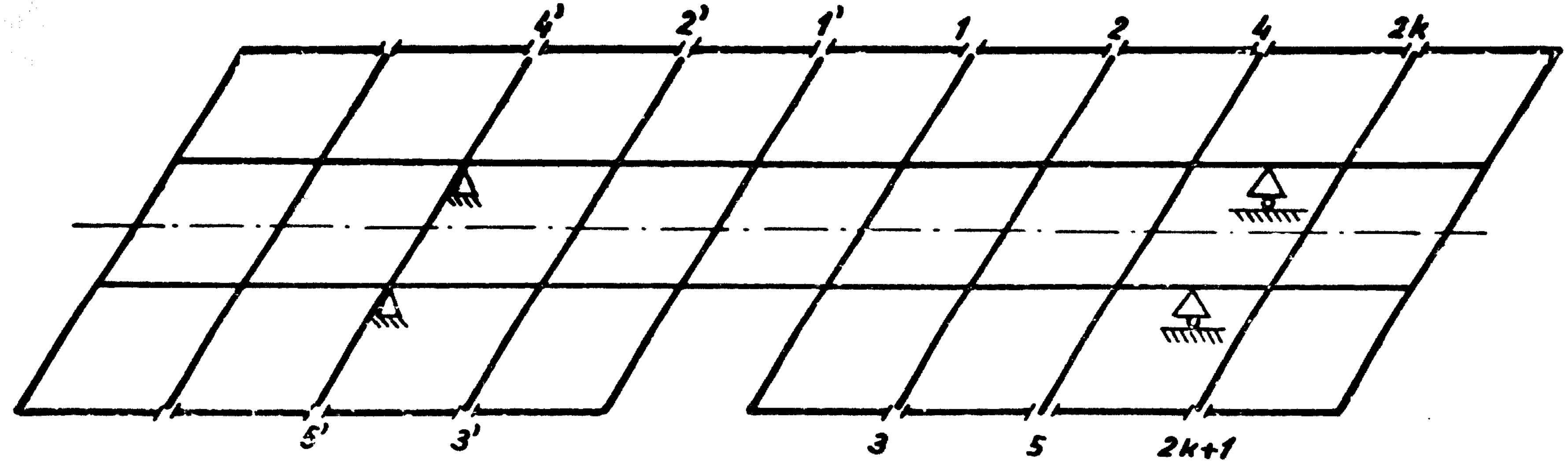
3
P
kp



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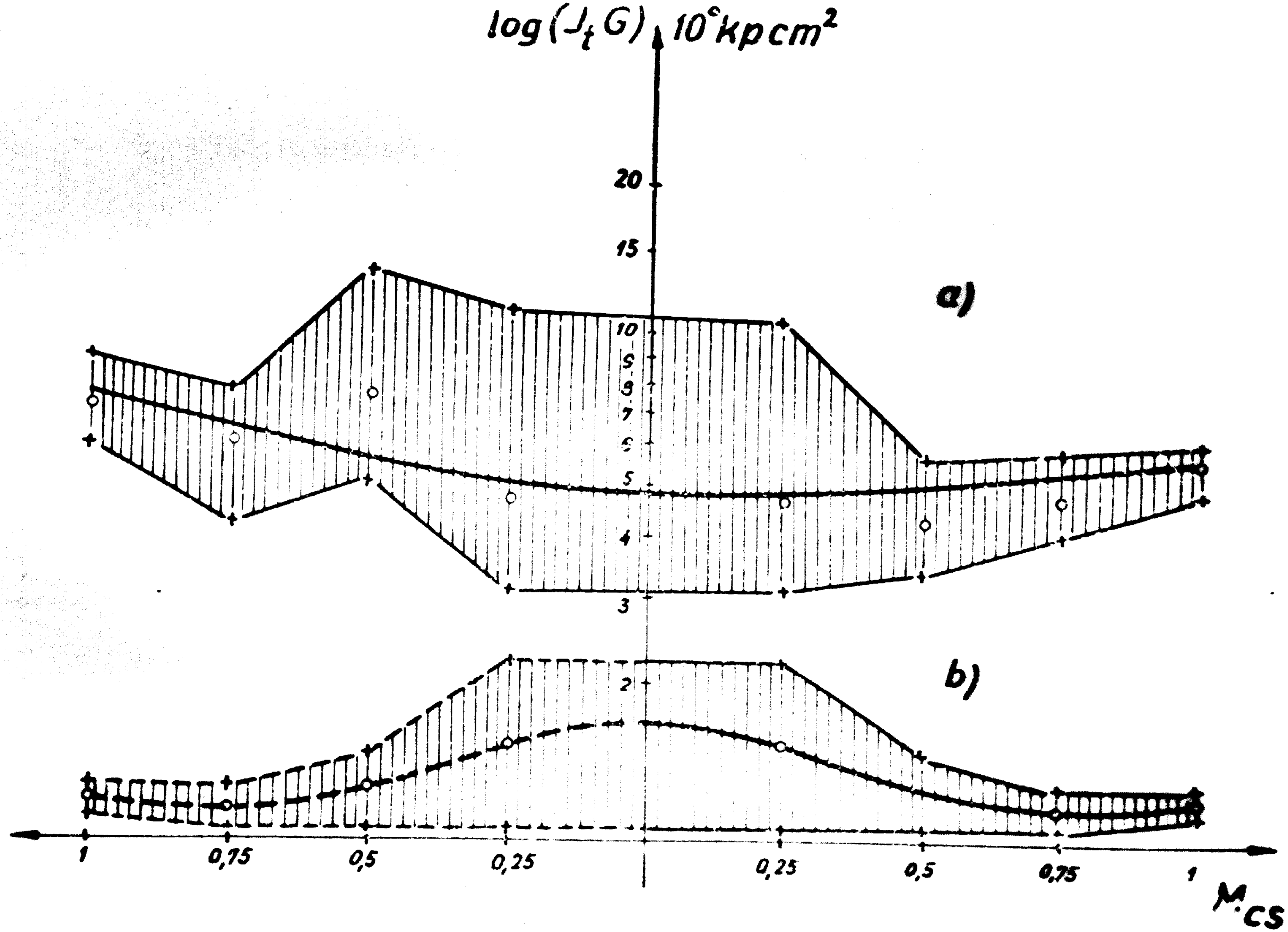
log(J_t G) 10^6 kp cm^2
20
15
10
9
8
7
6
5
4
3
2
a)
b)
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0,5
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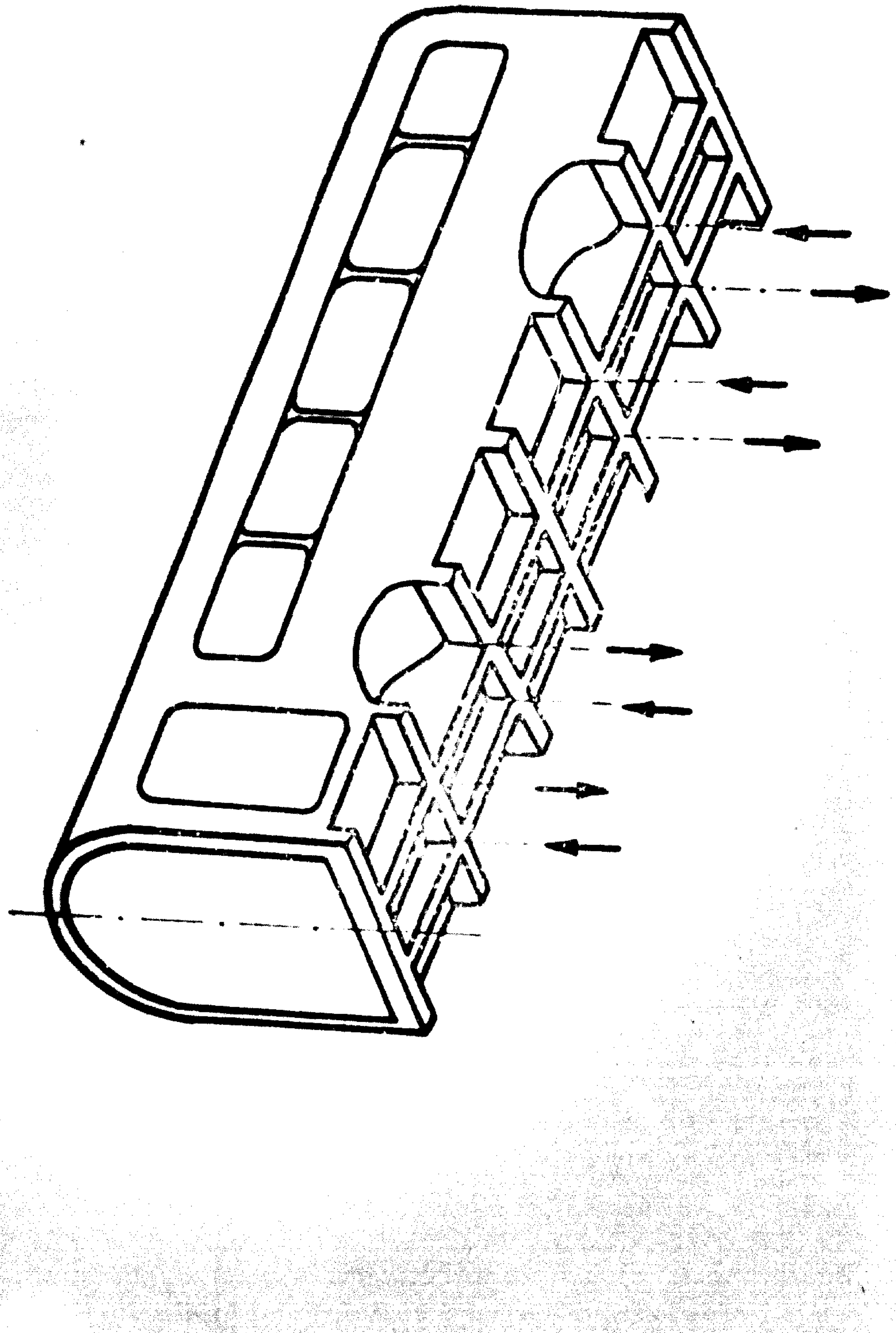
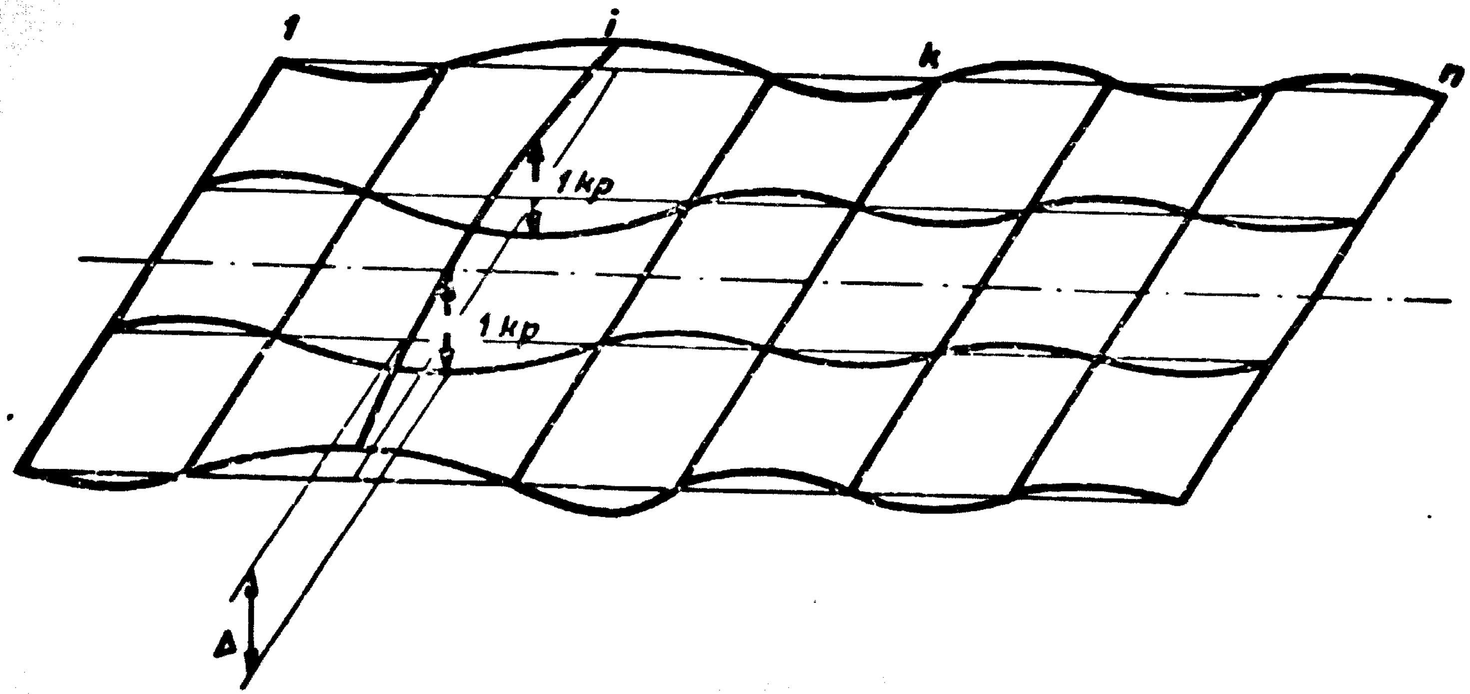
1
μ_cs





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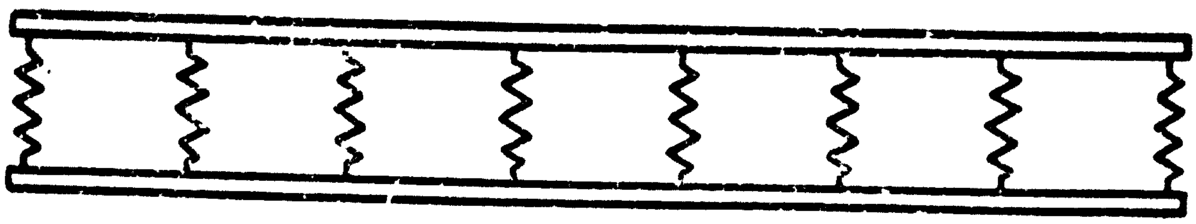

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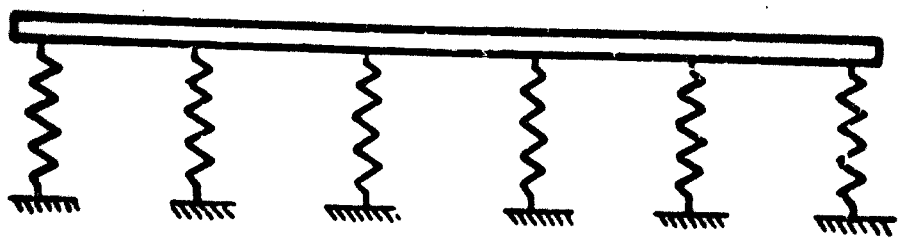

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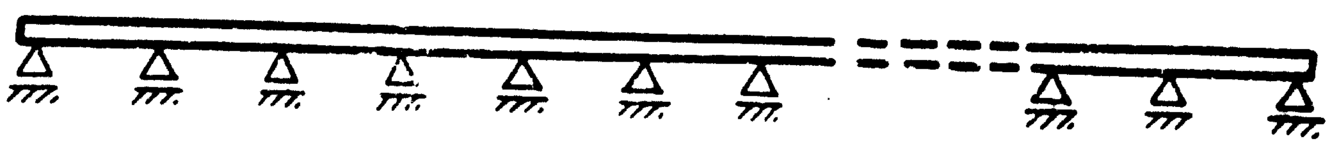

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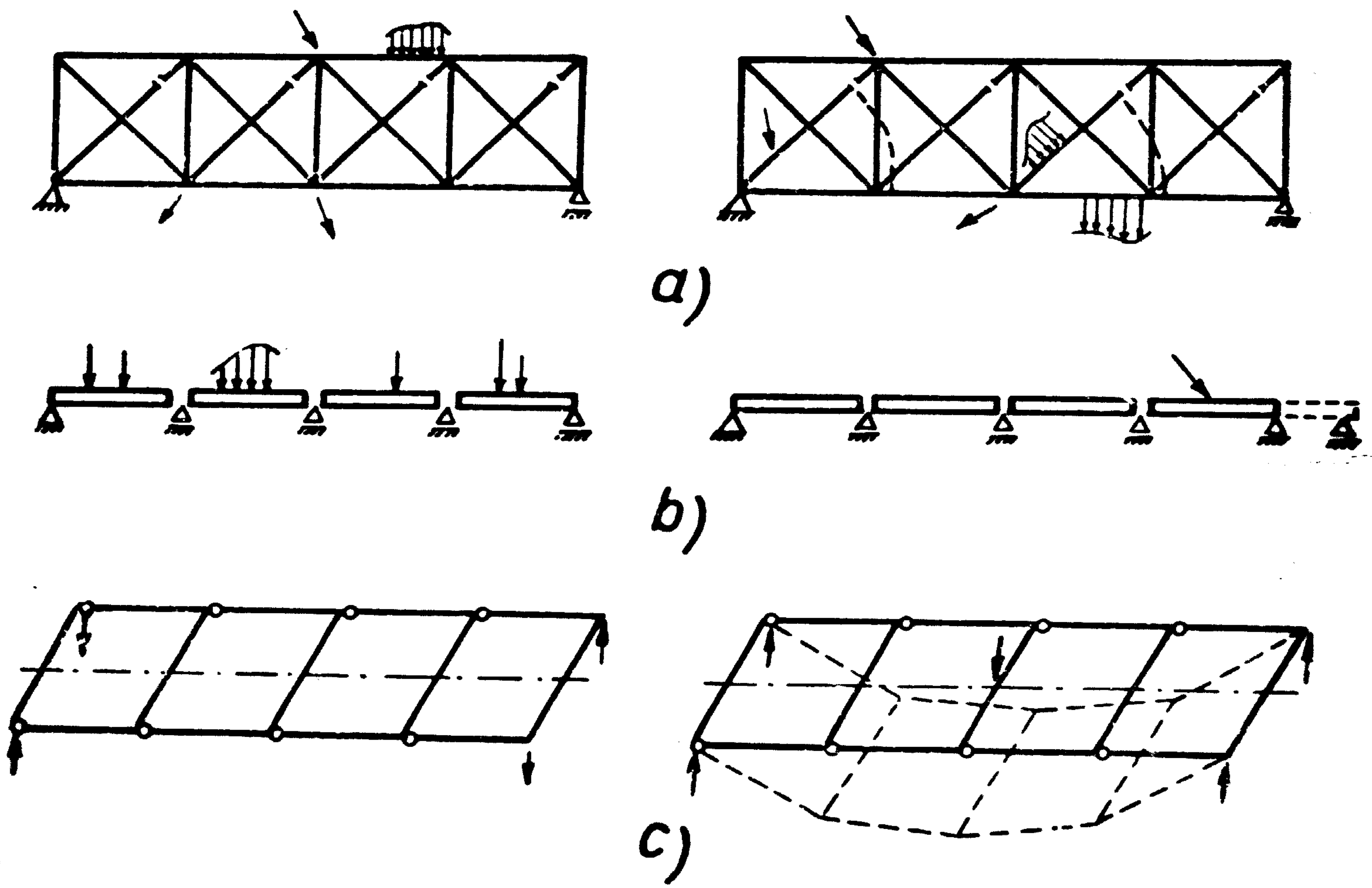
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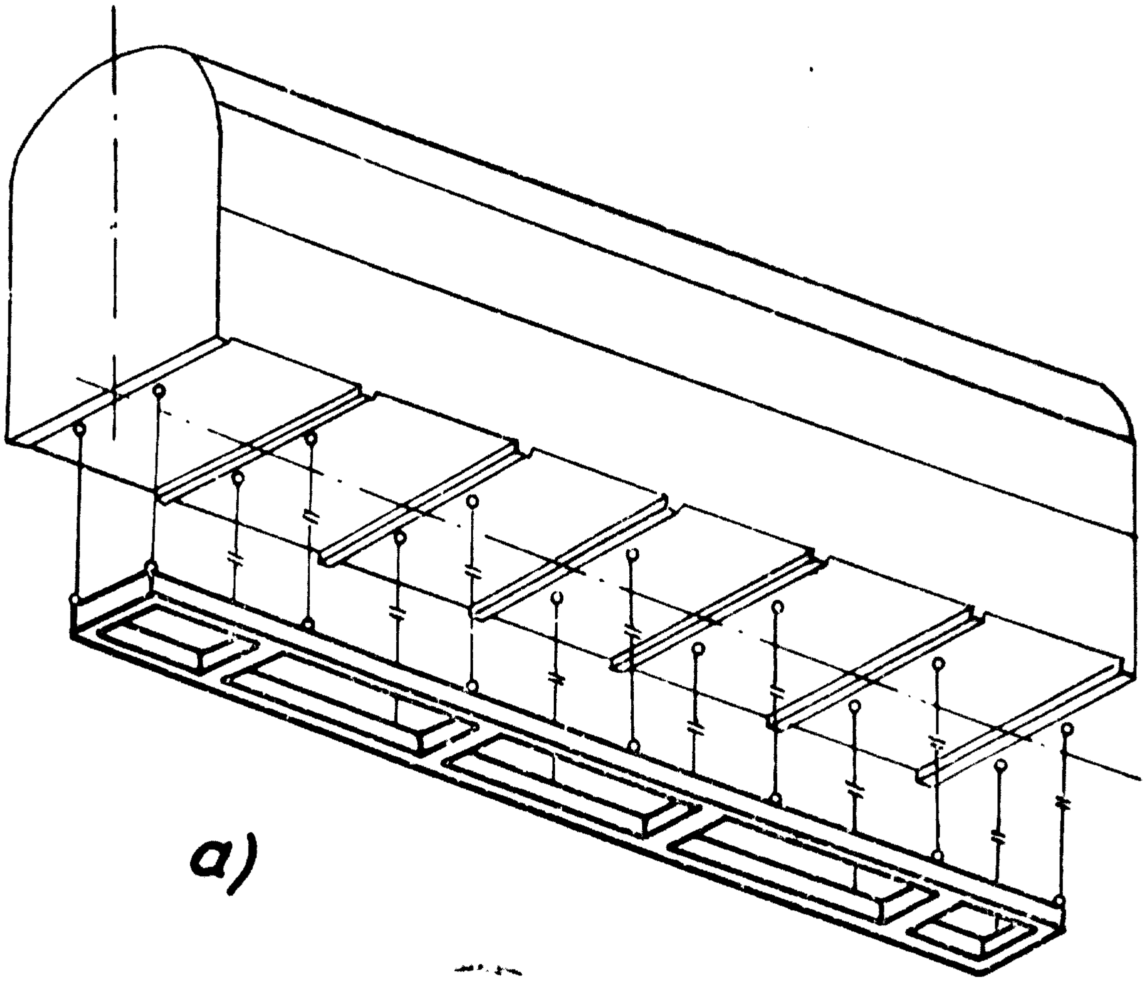

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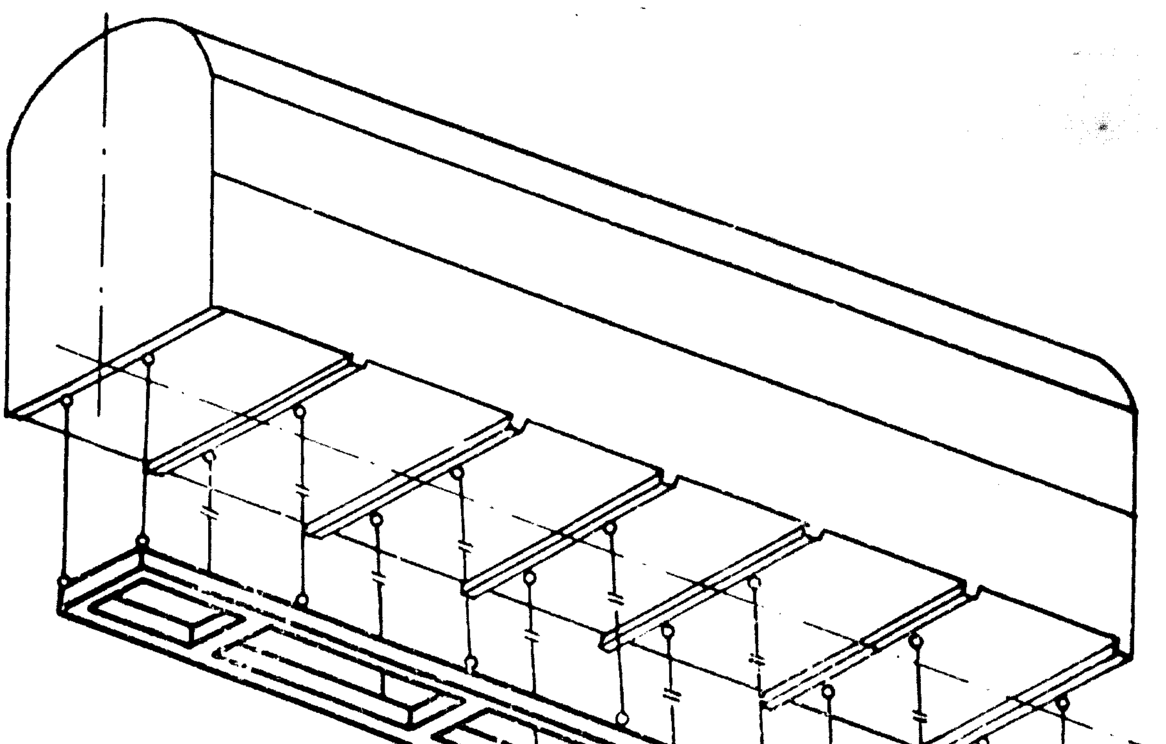

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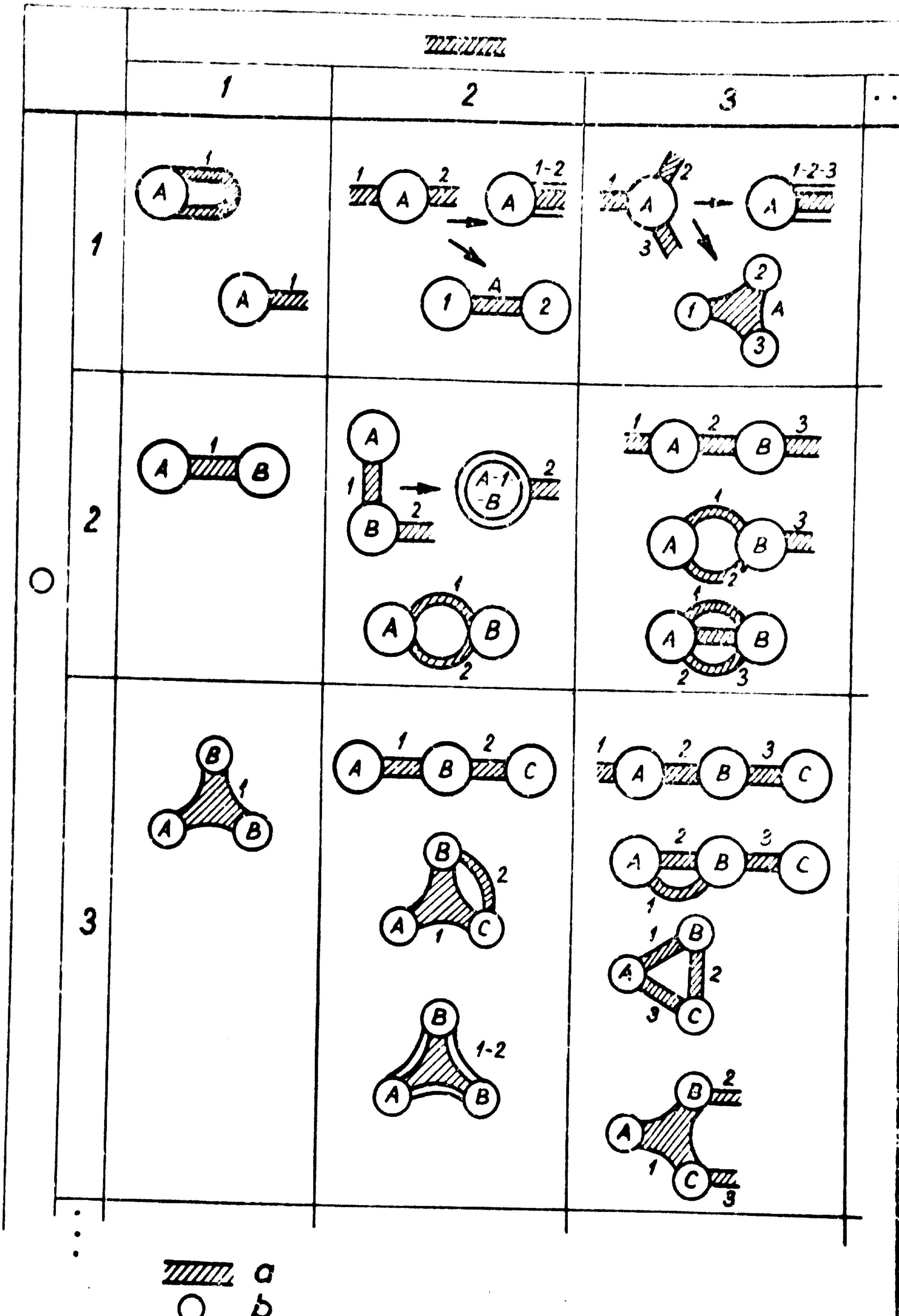
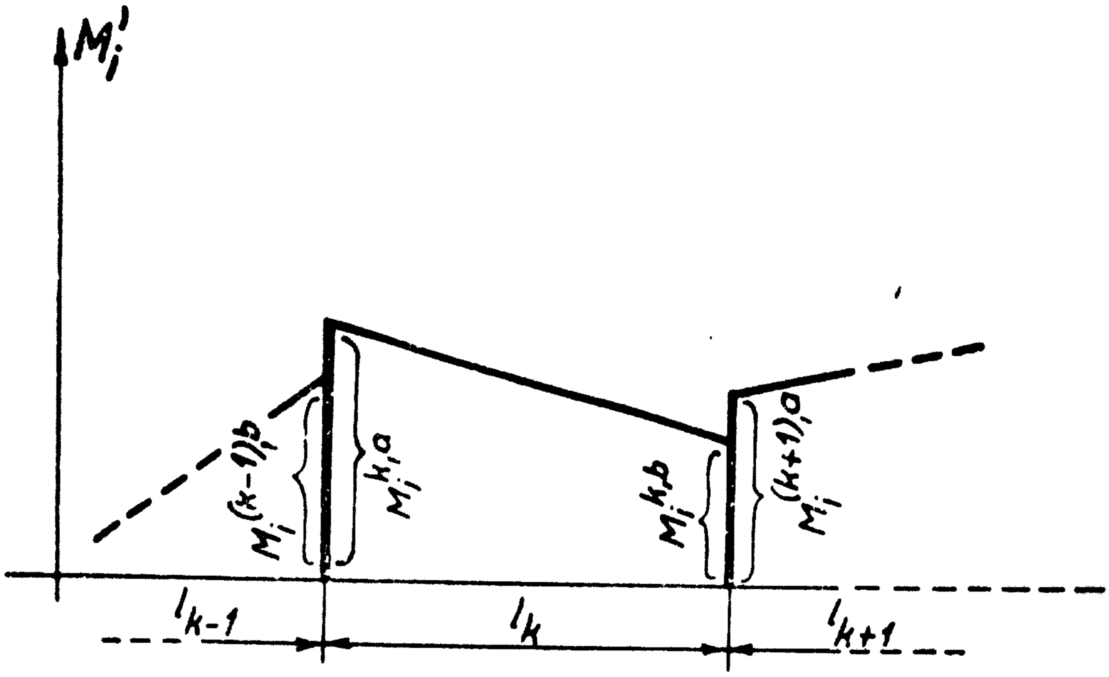
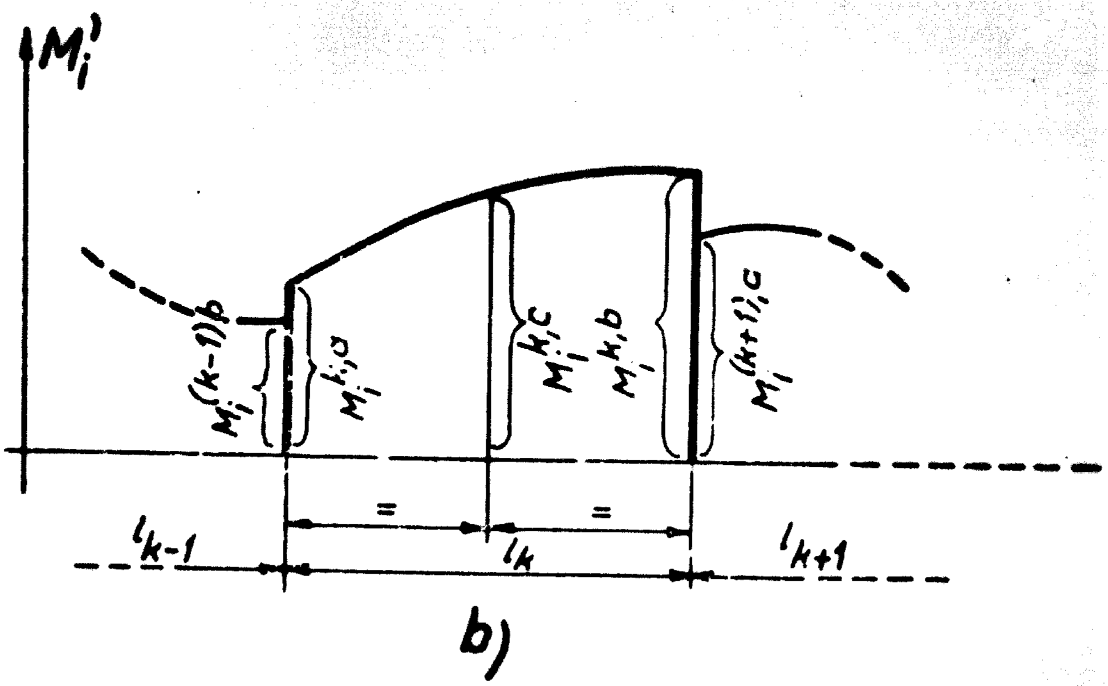

a
b



M'i
Mi(k-1),b
Mik,a
Mik,b
Mi(k+1),a
lk-1
lk
lk+1

a)



M'i
Mi(k-1),b
Mik,a
Mik,c
Mik,b
Mi(k+1),a
=
=
lk-1
lk
lk+1

b)

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